ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

DEPARTMENT OF STATISTICS

POSTGRADUATE PROGRAM

THE AGE PATTERN OF MORTALITY IN BALKAN COUNTRIES: COMPARISONS OVER TIME AND SPACE

By

Ioannis G. Andritsos

A THESIS
Submitted to the Department of Statistics
of the Athens University of Economics and Business
in partial fulfilment of the requirements for
the degree of Master of Science in Statistics

Athens, Greece
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ISBN: 960-8287-01-4

ISBN: 960-8287-03-0

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ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ΤΜΗΜΑ ΣΤΑΤΙΣΤΙΚΗΣ

ΔΙΑΧΡΟΝΙΚΗ ΜΕΛΕΤΗ ΚΑΙ ΣΥΓΚΡΙΣΗ ΤΩΝ ΚΑΤΑ ΗΛΙΚΙΑ ΕΠΙΠΕΔΩΝ ΘΝΗΣΙΜΟΤΗΤΑΣ ΤΩΝ ΒΑΛΚΑΝΙΚΩΝ ΧΩΡΩΝ.

Ιωάννης Γ. Ανδρίτσος

ΔΙΑΤΡΙΒΗ

Που υποβλήθηκε στο Τμήμα Στατιστικής
του Οικονομικού Πανεπιστημίου Αθηνών
ως μέρος των απαιτήσεων για την απόκτηση
Μεταπτυχιακού Διπλώματος Ειδίκευσης στη Στατιστική

Αθήνα
Μάιος 2001

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September 2002

DEDICATION

To my Family

ACKNOWLEDGEMENTS

I am immensely indebted to my supervisor Dr. Anastasia Kostaki for her valuable help and advice during the preparation of this thesis. She awoke my interest in Demography and she initiated my studies in that field.

I wish also to thank my Family for their support and encouragement.

VITA

I was born in Nikaia Peraias in May 1964. In 1982 I entered the Military Academy of Evelpidon in Athens and I received my Degree in June 1986. In October 1999 I began the Master's Program in Statistics in the Department of Statistics, Athens University of Economics and Business.

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ABSTRACT

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May 2001

The purpose of this work is to study and compare the development of the age-specific mortality patterns, of the Balkan countries. In that, we use empirical data of Hellas, Bulgaria, Romania for the period 1975-1995 and Yugoslavia for the period 1975-1990. In our analysis, we initially use empirical data given in five-year age groups. Departing from the empirical frequencies of dying given in five-year age groups, we use an expanding technique proposed by Kostaki (1991), in order to estimate the age-specific probabilities of dying. We thus, provide estimations of the graduated age-specific mortality pattern of each country, for the years 1975, 1980, 1985, 1990, 1995 and therefore form complete life tables for each population considered. The reason that we used grouped data, instead of the empirical ones is related to the quality of the empirical evidence. An evaluation of the analytical data sets has shown that, these are effected of the problem of age heaping. Grouping them in five-year age groups, the effect of this problem is eliminated. Then using an expanding technique, one estimates the analytical probabilities of dying.

The expanding technique used is a parametric one, in the sense that in its frame a parametric model that represent mortality as a function of age, is utilized. In the present work the classical eight-parameter formula of Heligman and Pollard (1980) is used. Many applications of this model have shown that, it is efficient for representing the mortality pattern of the total life span. Moreover, each one of the eight parameters incorporated in the model has particular demographic interpretation. Thus, studying the levels as well as the evolution of the parameter values, we provide accurate comparisons over space and time. The results of our calculations led us to interesting findings for the evolution of the mortality patterns considered.

ABSTRACT

[illegible]

THE AGE PATTERN OF MORTALITY IN BALKAN COUNTRIES

[illegible]

May 2001

[illegible]

[illegible]

ΠΕΡΙΛΗΨΗ

Ιωάννης Ανδρίτσος.

ΔΙΑΧΡΟΝΙΚΗ ΜΕΛΕΤΗ ΚΑΙ ΣΥΓΚΡΙΣΗ ΤΩΝ ΚΑΤΑ ΗΛΙΚΙΑ ΕΠΙΠΕΔΩΝ ΘΝΗΣΙΜΟΤΗΤΑΣ, ΤΩΝ ΒΑΛΚΑΝΙΚΩΝ ΧΩΡΩΝ.

Μάιος 2001

Σκοπός της εργασίας αυτής είναι η διαχρονική μελέτη των επιπέδων της κατά φύλο και ηλικία θνησιμότητας του ελληνικού πληθυσμού για την περίοδο 1975-1995 και η σύγκρισή τους με τα αντίστοιχα επίπεδα των Βαλκανικών χωρών Βουλγαρίας, Ρουμανίας, Γιουγκοσλαβίας, για την ίδια χρονική περίοδο. Για τις ανάγκες αυτής της διερεύνησης, χρησιμοποιήθηκαν οι παρατηρούμενες συχνότητες θνησιμότητας κατά πενταετείς ομάδες ηλικιών. Στα δεδομένα αυτά, εφαρμόστηκε η τεχνική εξάπλωσης συνεπτυγμένων πινάκων επιβίωσης της Κωστάκη (1991). Με αυτό τον τρόπο επιτυγχάνεται, εκτίμηση των κατά ηλικία πιθανοτήτων θανάτου, από τις αντίστοιχες εμπειρικές συχνότητες, του συνεπτυγμένου πίνακα επιβίωσης, για κάθε χώρα και για τα έτη 1975, 1980, 1985, 1990, 1995. Για τις ανάγκες της διερεύνησης μας, δεν χρησιμοποιήθηκαν οι κατά ηλικία εμπειρικές συχνότητες θνησιμότητας, αλλά οι αντίστοιχες κατά πενταετείς ομάδες ηλικιών συχνότητες. Η επιλογή αυτή έγινε με σκοπό την ελαχιστοποίηση προβλημάτων που συνδέονται με συστηματικές πηγές σφαλμάτων που επιφορτίζουν τα εμπειρικά κατά ηλικία δημογραφικά δεδομένα. Συστηματικά σφάλματα όπως ανακρίβειες των κατά ηλικία δεδομένων, πρόβλημα το οποίο συνδέεται με τις λανθασμένες δηλώσεις ηλικιών, γνωστό στη διεθνή δημογραφική βιβλιογραφία ως φαινόμενο 'age heaping', επιφορτίζουν κυρίως τα κατά ηλικία δεδομένα. Έτσι κάνοντας χρήση της τεχνικής αποομαδοποίησης των δεδομένων, επιτυγχάνεται εκτίμηση των κατά ηλικία πιθανοτήτων θανάτου.

Για τις ανάγκες αυτής της διερεύνησης, χρησιμοποιείται μία μέθοδος εξάπλωσης συνεπτυγμένων πινάκων επιβίωσης, στα πλαίσια της οποίας εφαρμόζεται το παραμετρικό μοντέλο θνησιμότητας των Heligman και Pollard (1980). Οι διάφορες εφαρμογές της μεθόδου, έδειξαν ότι παρέχει μια πολύ ικανοποιητική περιγραφή της κατά ηλικία θνησιμότητας. Η εφαρμογή αυτής της μεθόδου, παρέχει ως αποτέλεσμα εκτιμήσεις των κατά ηλικία πιθανοτήτων θανάτου των πληθυσμών. Ακόμα με τη βοήθεια των εκτιμήσεων των παραμέτρων του μοντέλου, παρέχεται η σύγκριση των επιπέδων θνησιμότητας.

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CHAPTER 1

INTRODUCTION

"1 death is tragedy, but 1 million deaths is statistic."

Mortality has shown a remarkable decline throughout the last century in all human populations. This decline has in general been more intense for infants and children while relatively lower for the older ages. Furthermore, this decline has at most been more dramatic for females than for males.

The above makes clear that for a complete examination of the mortality development of a population one should study the mortality pattern by age and sex while the calculation of some overall indices is insufficient.

The age-specific mortality pattern has a typical shape, common in all human populations, which is fairly complex. At the beginning of life (age zero), the level of mortality is very high (comparable to the level of mortality at the later adult ages), then the mortality pattern exhibit a rapid exponential decline obtaining a minimum level at the beginning of the puberty. Then, it shows an exponential increase with increasing rate until a highest attainable age where it reaches its maximum. At the early adult ages the mortality pattern exhibits a hump related to the advanced risks of accident mortality at these ages as well as to maternal mortality for females.

Except of its typical age-specific shape, the mortality pattern of all human populations shows a typical progression over time. The level of infant and childhood mortality decreases by time while the rate of mortality decline at these ages shows a falling progression by time, as infant mortality shows a stronger decline than mortality at the other childhood ages. The accident hump of the mortality curve at the early adult ages is more pronounced in males than females while in most populations, it becomes more intense for both sexes in recent years. Additionally, the level of adult and senescent mortality is also decreased by time while the rate of increase of the mortality pattern at these ages becomes higher in recent years as more and more people survive until senescence but there has not been any progression in the maximum life length. Besides, the intensity the spread and the location of the accident hump are also variable through time, exhibiting a trend to be more pronounced and located in younger ages in recent years.

This thesis provides an examination and a comparison of the development of the age and sex specific mortality patterns in Balkan countries: Greece, Bulgaria, Yugoslavia, and Romania, during the period 1975-1995. In doing that we use the empirical abridged death frequencies of each population and expand them using a technique proposed by Kostaki (1991). This technique is a parametric one in the sense that within its frame a parametric model, which represents mortality as a function of age, is utilized. In our calculations the classical Heligman-Pollard eight parameter formula is used. Many authors e.g. Heligman and Pollard (1980), Hartmann (1987), Forfar and Smith (1987), Kostaki (1992), Gongdon (1993), Karlis and Kostaki (2000) have used this formula in order to efficiently graduate mortality data sets.

Using the expanding technique we provide estimations of the age-specific probabilities of dying of each population. At the sequel, using the sets of parameter estimates, we provide comparisons of the mortality patterns through time and space.

In Chapter 2 we refer to useful concepts and formulae of demography and the theory of life tables.

Chapter 3 focuses on the parametric models that are extensively reviewed in Section 3.1. Section 3.2 refers to statistical methods for estimating the parameters of these models and Section 3.3 presents the main uses of them, i.e. graduation, comparison and forecasting. In Section 3.4 are cited some additionally uses of these models.

Chapter 4 provides a description, of the expanding methods, for estimating the age specific mortality pattern.

A description of the empirical data sets used is also provided, in Chapter 5.

An analysis over the years 1975 to 1995, of the age-specific mortality pattern of the Balkan countries is presented in Chapter 6, while a comparison between the age-specific mortality pattern of the Balkan countries (Hellas, Bulgaria, Romania, Yugoslavia) over the period 1975 to 1995 is presented in Chapter 7.

Chapter 8 provides some concluding remarks.

CHAPTER 2

THEORY OF LIFE TABLES

2.1 Introduction

In this chapter we refer to useful concepts and formulae of demography and the theory of life tables.

In human biology, the whole number of inhabitants occupying an area (such as a country or the world) and continually being modified by increases (births and immigrations) and losses (deaths and emigrations).

The study of human populations is called demography a discipline with intellectual origins stretching back to the 18th century, when it was first recognized that human mortality could be examined as a phenomenon with statistical regularities. Demography casts a multidisciplinary net, drawing insights from economics, sociology, statistics, medicine, biology, anthropology, and history. Its chronological sweep is lengthy: limited demographic evidence for many centuries into the past, and reliable data for several hundred years are available for many regions. The present understanding of demography makes it possible to make comparisons between different populations, and project (with caution) population changes several decades into the future.

A basic tool of the mortality analysis is the life table. A life table provides a description of mortality, survivorship and life expectancy for a specified population. A life table is a statistical model, designed essentially to measure mortality. In practice it is employed by a variety of specialists in a variety of ways. Life tables are used by actuaries, vital statisticians and medical researchers to determine life insurance premiums, pension values, gains in life expectancy of a people and decreased probabilities of dying from improved medicines and surgical techniques. In its simplest form, the entire table is generated from age-specific mortality rates (m_x), resulting to a set of useful functions, which in general determine mortality, survivorship and life expectation. Life tables are in a sense, one form of combining mortality rates of a population at different ages into a single statistical model.

2.2 Categories of Life Tables

Life Tables are distinguished in two general categories.

A. *Cohort Life Tables.*

B. *Period Life Tables.*

In general the existence of a number of l_0 live births is assumed. This is of a *"closed"* nature (a closed population).

A demographer will observe, how this cohort of l_0 persons, diminutes as it grows older. This will conclude to the construction of a *Cohort Life Table*.

The other case, which is more realistic from the point of the ability to construct it, introduces the case of a *Period Life Table*. Since the first case may take 100 to 110 years or more to be constructed, we do an approximation by case B.

This is a more practical solution, since it can give as a table for every year. Thereafter, when we refer to a life table, we will mean a period one.

Because of problems arizing in empirical data, life tables are distinguished in two categories:

A. Complete Life Tables

They present the age - specific mortality experience by single age x.

B. Abridged Life Tables

They present mortality for groups of ages. The usual representation is for age 0 separately and for the groups 1-4, 5-9, 10-14, ..., 85+.

2.3 The Construction of an Abridged Life Table

We start from l_0=100.000, or 10.000, which is the *hypothetical* cohort. It is referred, as the *root* (radix) of the table.

The basic life table function is l_x, $0 \leq x \leq w$ (age where the cohort extincts), which describes the survivors of death exposure at the exact age x. Usually from the death registrations, we get a central death rate m_x, for any value x in [x , x +n). From that we can approximate q_x, the probability of dying, which is the only information we need to know in order to construct our life table.

The *Abridged life table* **formulae** are introduced here. It is simple to derive from the following, the expressions corresponding to a full life table. It only requires to equate n with *one (=1).*

${}_nq_x$: this is the probability of someone of age x to die before reaching age $x+n$, i.e. to die in the age interval $[x, x+n)$

${}_np_x$:this is the probability of someone of age x to survive through the interval $[x, x+n)$.

With,

$$ {}_np_x = 1 - {}_nq_x $$

l_x : this is the number of people surviving at exact age x.

With,

$$ S(x) = \frac{l_x}{l_0} $$

denoting the survival probability for the age interval $(0,.\ x]$.

${}_nd_x$: this is the number of deaths for the age interval $[x, x+n)$.

With,

$$ {}_nd_x = l_x - l_{x+n} $$

or,

$$ {}_nd_x = l_x\,{}_nq_x $$

${}_nL_x$: this the total number of years of life, that the l_x-people of the population experience in the age interval $[x, x+n)$.

Each person that survives through the age interval $[x, x+n)$, contributes n-years of life, and each one that dies (since a uniform distribution of deaths is assumed) contributes approximately $n/2$ of years of life.

Then,

$$ {}_nL_x = nL_{x+1} + \frac{1}{2}d_x $$

and since,

$$ {}_nd_x = l_x - l_{x+1} $$

then,

$$ {}_nL_x = \frac{1}{2}(l_x + l_{x+1}) $$

In the continuous case, we will have,

$$ {}_nL_x = \int_x^{x+n} l(t)dt = \int_0^n l(x+t)dt \cong n\left(\frac{l_x + l_{x+1}}{l_x}\right) $$

T_x: this is the total number of years, that the l_x-people of the population are about to live in the interval $[x, w)$, where w is an age difficult to reach (w-1, the greater age).

With,

$$ T_x = \sum_{i \geq x} L_i $$

also,

$$ T_{x+n} = T_x - {}_nL_x $$

and,

$$ {}_nL_x = T_x - T_{x+n} $$

And in the continuous case,

$$ T_x = \int_0^{w-x} l(x+t)dt $$

$\overset{0}{e}_x$: is called expectation of life or life expectancy at age x. It is the expected remaining life of persons of age x.

With,

$$ \overset{0}{e}_x = \frac{T_x}{l_x} $$

Then the expected life of a person of age x is equal to, $x + \overset{0}{e}_x$

And in the continuous case,

$$ \overset{0}{e}_x = \frac{1}{l_x}\int_0^{w-x} l(x+t)dt $$

2.4 Approximations of Life Table functions.

We present here some useful relations between mortality measures ${}_nq_x$ and, ${}_nm_x$. It is usual in practise to be unable to obtain ${}_nq_x$ directly by the empirical data, but one can approximate it by ${}_nm_x$.

Assume that ${}_na_x$ is the expected number of years that a random person of age x will be surviving in the age interval ($0 \prec {}_na_x \prec x$). Then,

$${}_nf_x = \frac{{}_na_x}{n}, 0 \leq {}_nf_x \leq 1$$

is the expected percent of years lived by such person in the interval $[x, x+n)$.

So,

1. n, the number of years that each person which survives through the age interval $[x, x+n)$ contributes to ${}_nL_x$. The number of l_{x+n} people will have total contribution of a number of nl_{x+n} years of life.

2. ${}_na_x$, the number of years that each person which dies in the age interval $[x, x+n)$, contributes to ${}_nL_x$. Then their total contribution will be a number of $({}_na_x).({}_nd_x)$ years of life.

and,

$$\begin{aligned} {}_nL_x &= nl_{x+n} + {}_na_{x\,n}d_{x\,n}L_x = \\ &= n(l_x - {}_nd_x) + n\,{}_nf_{x\,n}d_x \Rightarrow \\ &\Rightarrow {}_nL_x = n[l_x - {}_nd_x(1 - {}_nf_x)] \end{aligned}$$

From the previous, the mortality ratio ${}_nm_x$, will take the following form,

$${}_nm_x = \frac{{}_nd_x}{{}_nL_x} \Rightarrow {}_nm_x = \frac{{}_nd_x}{n[l_x - {}_nd_x(1 - {}_nf_x)]}$$

But, since ${}_nd_x = l_{x\,n}q_x$,

$${}_nm_x = \frac{l_{x\,n}q_x}{nl_x[1 - {}_nq_x(1 - {}_nf_x)]} \quad {}_nm_x = \frac{{}_nd_x}{n[1 - {}_nq_x(1 - {}_nf_x)]}$$

Assuming now uniform distribution of deaths in each age interval we will have, ${}_na_x = \frac{n}{2}$ and, ${}_nf_x = \frac{1}{2}$ so,

$${}_nm_x = \frac{{}_nq_x}{n[1 - {}_nq_x\frac{1}{2}]}$$

or,

$$ {}_{n}q_{x} = \frac{{}_{n}m_{x}}{n\left[1 - {}_{n}m_{x}\,\frac{n}{2}\right]} $$

The age - specific ${}_{n}m_{x}$ and ${}_{n}q_{x}$ values can be derived from the above by setting $n=1$. Such approximations are very useful in the application of certain expanding methods (see Pollard, 1989, Reed (see Valaoras, 1984, King, 1914)).

CHAPTER 3

LAWS OF MORTALITY

3.1 A Presentation

In this chapter we consider the history of attempts to summarize mortality age schedules and move on to consider parameter estimation (though more details regarding estimation will appear in subsequent chapters). We then consider the scope for particular applications such as forecasting and expansion of abridged tables.

Parametric models, which present mortality as a function of age x, are widely used both in Demography and Actuarial Science.

Attempts to model mortality curves can be traced back to about (1725 when A. De Moivre presented the mortality intensity μ_x as a function of age

$$\mu_x = 1/(w - x)$$

where w represents the highest attainable age. This model gives almost good results at the older ages but is inadequate for the younger, a remark made firstly by De Moivre himself.

The most popular perhaps model or mortality law was the one developed by *Gompertz* (1825)

$$\mu_x = B\, c^x \qquad (3.1.1)$$

who argued that the intensity of mortality (in his terms the average exhaustion of man's power to avoid death) gained equal proportions in equal intervals of age. The parameter c usually has a value around 1.09. The survivorship function under this law is

$$l_x = k\, a^{c^x}$$

with $\alpha = exp\ (-B/ln\ c) < 1$ and K chosen so as to conform with the desired radix of life table. This model has been extensively used by demographers and actuaries for graduating are – specific mortality rates and preparing life tables.

Hartmann (1987) remarked that because of the little numerical difference between the estimated mortality intensity μ_x and the estimated mortality rate q_x, expression (3.1.1) can be used to model either of the functions. Pollard (1991) pointed out that Gompertz model can be used for many accurate and quick approximate calculations in life table probabilities and functions, even though the data concerned are either limited or are not quite of the Gompertz shape; a familiar fact more to actuaries than to demographers.

Makeham (1867) made a development of Gompertz's law by the addition of a constant

$$\mu_x = A + Bc^x$$

a law reflected the division of causes of death into those due to chance and those due to deterioration. This model proved to be inappropriate for modeling the whole life span because although appears to have good fit to middle ages it overestimates mortality of older ages (Perks, 1932) and also fails to describe the younger.

Oppermann in 1870, who was the founding father of actuarial science in Denmark, proposed a mortality law appropriate only for infant and child mortality

$$\mu_x = \frac{\alpha}{\sqrt{x}} + \beta + \gamma\sqrt{x}$$

The Danish statistician and actuary *Thiele* (1872) introduced an innovative idea that deaths fall into three distinct categories; those affecting childhood, middle life and old-adult life. He proposed the composition

$$\mu_x = \mu_{1\ x} + \mu_{2\ x} + \mu_{3\ x} \qquad (3.1.2)$$

with,

$$\mu_{1x} = \alpha_1 \exp\{-\beta_1 x\}$$
$$\mu_{2\ x} = \alpha_2 \exp\left\{-\frac{1}{2}\beta_2^2 (x-\gamma)^2\right\}$$
$$\mu_{3\ x} = \alpha_3 \exp\{\beta_3 x\}$$

where $\alpha_1, \beta_1, a_2, \beta_2. \gamma, a_3, \beta_3$ are all positive parameters. The Gompertz law, model childhood and adult age, while the middle age is modeled by the normal – apart from a scale factor – probability density function. The functions $\mu_1, \mu_2. \mu_3$ have main contribution to μ_x for the periods of childhood, middle and adult ages respectively whereas the other additive terms provide insignificant contribution. All three jointly give a model of mortality for all ages.

About 10 years later the German statistician and actuary *Wittstein* (1883) suggested the formula

$$q_x = \frac{1}{m} \alpha^{-(mx)^n} + \alpha^{-(w-x)^n},$$

where w is the highest attainable age and m, n the parameters to be estimated. Wittstein's law divided mortality into the two components of childhood and adult age on the basis of a generalization of De Moivre's law. Wittstein illustrated his law to male Danish data of ages 5 to 85, which had been collected by Oppermann.

Hartmann (1987) in order to compare the laws of Thiele and Wittstein fitted them to Swedish female experience of the period 1961 – 1970 by means of the unweighted method of least – squares. The former provided a closer fit to the data although it occasionally fit the middle life component to the wrong part of the mortality curve (misplace the accident hump as it is named to demographic and actuarial literature. Wittstein's law was used by Statistics Sweden for smoothing the death rates at advanced ages (Hartmann, 1987).

Makeham in 1889 (Kostaki, 1992a) presented another expression of his earlier model with the addition of another term

$$\mu_x = \mathrm{A} + \mathrm{H}x + \mathrm{B}c^x$$

Perks (1932) proposed the formulae

$$\mu_x = \frac{\mathrm{A} + \mathrm{B}c^x}{1 + Dc^x}$$

and

$$\mu_x = \frac{\mathrm{A} + \mathrm{B}c^x}{\mathrm{K}c^{-x} + 1 + Dc^x}$$

which are the divisions of Makeham's first expression by one and two exponential respectively.

Beard (1951) derived the formula

$$q_x = \frac{A + Bc^x}{Ec^{-2x} + 1 + Dc^x}$$

which is clearly related to the Perks family of curves. He used for the estimation of parameters the trial and error method. This formula did not attempt to reproduce mortality decreasing with increasing age, a fact observed at the youngest ages 30.

Barnett (CMI Committee, 1974), a British actuary, proposed the expression

$$\frac{q_x}{1-q_x} = A - Hx + Bc^x,$$

when it came to the problem of choosing an appropriate formula for graduating the assured lives mortality experience for 1967 – 1970. Although it produced a satisfactory graduation it was criticized for not readily allowing a comparison of its parameters with other tables at different times or other countries.

The first model which gave unusually close fits to empirical mortality at all ages was the one suggested by *Heligman and Pollard* (1980):

$$\frac{q_x}{p_x} = A^{(x+B)^C} + D\exp\left\{-E\left(\log\frac{x}{F}\right)^2\right\} + GH^x, \qquad (3.1.3)$$

where q_x is the probability of dying within a year, $p_x = 1 - q_x$ the complement of this and *A,B,.....H* are the parameters. In the case where x=0 the right hand side is interpreted as $A^{B^C} + G$.

This model is an enhanced version of Thiele's law it is based on the same idea of dividing mortality into three components. The first component concerns the infant and early childhood ages, the second refers to the middle age mortality (the accident hump) and the third, which is actually the Gompertz model, refers to the exponential rise of mortality at the adult ages.

All the parameters have demographic interpretations. A measures the level of mortality and it is almost equal to q_1 (the probability that one will die before reaching age 1). B is an age displacement to account for infant mortality. C measures the rate of mortality, decline in childhood cause to adaptation of children to environment. D represents severity of the accident term E and F indicates its spread and location respectively. G represents the base level of senescent mortality and finally H reflects the rate of increase of that mortality.

The three components of mortality and their contribution to total mortality are illustrated graphically in Figure 1.

In the same study (Heligman and Pollard, 1980), some variations of the basic mortality curve (3.1.3) were also proposed and fit to post-war Australian national mortality data

$$q_x = \mathrm{A}^{(x+\mathrm{B})^C} + D\exp\left\{-\mathrm{E}\left(\log\frac{x}{F}\right)^2\right\} + \frac{GH^x}{1+GH^x} \quad (3.1.4)$$

$$q_x = \mathrm{A}^{(x+\mathrm{B})^C} + D\exp\left\{-\mathrm{E}\left(\log\frac{x}{F}\right)^2\right\} + \frac{GH^x}{1+KGH^x} \quad (3.1.5)$$

$$q_x = \mathrm{A}^{(x+B)^C} + D\exp\left\{-\mathrm{E}\left(\log\frac{x}{F}\right)^2\right\} + \frac{GH^{x^K}}{1+GH^{x^K}} \quad (3.1.6)$$

The reason was to produce alternatives which could deal satisfactory with the probability that appeared to Australian females 1946 – 1948 data set of a false existence of an accident hump at the older ages while at the same time the true accident hump near age 20 was almost non-evident. The first form (3.1.4) is almost indistinguishable from (3.1.3) curve except that q_x may theoretically assume values greater than unity, although probably never seen in practice. The other two variations improved the fitting because the introduction of allowed for curvature at the older ages. Formula (3.1.6) provided a better fit than (3.1.5) to Australian males where the curvature is concave downward; the opposite was true to females where the curvature is concave upward.

Forfar and Smith (1987) fitted the Heligman – Pollard law to English Life Tables ELT1 to ELT13 for both males progression in the eight parameters. The thirteen ELT tables were based on deaths during the period between 1841 and 1972. The researchers resulted that all data sets gave a reasonably good fit and that there was a reasonable progression of parametric values from one life table to the next apart from females ELT12 (period 1960-1962). This life table presented unusually a very modest hump around age 20 and the beginning of a more pronounced one starting around age 70.

Hartmann, too (1983, 1987) examined the fit of the Heligman – Pollard law to Sweden life tables for males and females between 1900 and 1970 and concluded that it is a realistic model of human mortality through the entire life interval. The comparisons he made between the mortality curve (3.1.3) and Thiele's law have shown that the Heligman Pollard's law is mush less likely to produce a false accident hump than Thiele's . Therefore, it seems that the ''lognormal'' nature of the middle term of (3.1.3) fits experience better than the corresponding ''normal density'' term used by Thiele.

A quite recently mortality law, suggested by *Mode and Busby* (1982), determines the survival function *l (x)* according to

$$l(x)=\begin{Bmatrix} l_0(x), & 0\le x\le \delta_0 \\ l_0(\delta_0)l_1(x-\delta_0), & \delta_0\le x\le \delta_1 \\ l_0(\delta_0)l_1(\delta_1-\delta_2)l_2(x-\delta_1), & x\ge \delta_1 \end{Bmatrix}$$

where

$$l_0(x)=\exp(\alpha_0(\exp(-\beta_0 x)-1)),$$
$$l_1(x)=\exp\left(\frac{\beta_1\gamma_1^3}{3}-\alpha_1 x+\frac{\beta_1}{3}(x-\gamma_1)^3\right),$$
$$l_2(x)=\exp(-a_2 x-\beta_2(\exp(\gamma_2 x)-1))$$

Here, $l_0(x)$ is the probability that an individual is alive at age $x(\le\delta_0), l_1(x-\delta_0)$ is the conditional probability that an individual who survived to age δ_0 is alive at age $x(\delta_0\le x\le\delta_1)$and finally $l_2(x-\delta_1)$ is the conditional probability that an individual who survived to age δ_1 is alive at age $x(\ge\delta_1)$. A disadvantage of this model is the arbitrariness derived from the necessary choice of ages δ_0 and δ_1.

Even though several applications have shown that the Heligman – Pollard model gives the most satisfactory representation of the age pattern of mortality for the whole life interval, it has already been noted that it produces systematic errors, too. This formula raises a systematic deviation from the empirical values at the early adult ages where the second term dominates. Thus, the estimated accident hump is located at a higher age than the empirical one. This source of systematic errors is eliminated by a nine-parameter version of the model, an improvement developed by *Kostaki* (1992b) with the addition of one more parameter to formula (3.1.3).

$$\frac{q_x}{p_x} = \begin{cases} A^{(x+B)^c} + D\,\mathbf{exp}\left\{-E_1\left(\mathbf{log}\dfrac{x}{F}\right)^2\right\} + GH^x, \text{for } x \le F \\ A^{(x+B)^c} + D\,\mathbf{exp}\left\{-E_2\left(\mathbf{log}\dfrac{x}{F}\right)^2\right\} + GH^x, \text{for } x > F \end{cases} \quad (3.1.7)$$

In this version only the middle term has changed by the substitution of parameter E. The spread of the accident hump indicated by E is now reflected by the two parameters E_1 *and* E_2 for the left and right of its top respectively.

3.2 Estimating the parameters

The parameters that appear in laws of mortality can be estimated by the following statistical methods (Benjamin and Pollard, 1980):

- The method of maximum likelihood.
- The method of least – squares.
- The method of minimum chi-square.
- Bayesian MCMC method.

3.2.1. Maximum likelihood method

The principle of maximum likelihood is to use as estimate of a parameter the value that maximizes the likelihood of the observed event. As has been already mentioned the number D_x of deaths at age x is binomially distributed with parameters R_x (the exposed to risk at age x) and q_x (the mortality rate at age x) : $B(R_x, q_x)$. Deaths at various ages in the experience are assumed to be mutually independent.

The likelihood is

$$L = \prod_x \binom{R_x}{D_x} q_x^{D_x} (1-q_x)^{R_x - D_x} \quad (3.2.1)$$

where R_x, D_x are known and q_x is a function of the unknown parameters of a particular formula. The maximum likelihood estimates of the unknown parameters are the values that maximize (3.2.1) or also the logarithm of (3.2.1):

$$\log L = \sum_x \left[\log\binom{R_x}{D_x} + D_x \log q_x + (R_x - D_x)\log(1-q_x) \right] \quad (3.2.2)$$

By equating to zero the first partial derivatives of

$$\Lambda = \sum_x \left[D_x \log q_x + (R_x - D_x)\log(1-q_x) \right] \quad (3.2.3)$$

(the first term of equation (3.2.2) is known and fixed so it is omitted from (3.2.3)) with respect to the unknown parameters we get the simultaneous equations for the evaluation of the maximum likelihood estimates of these parameters.

3.2.2. Least – squares method

Let $\overset{o}{q}_x$ be the observed mortality rate at age x and $q_x = F(x)$ the value we attempt to fit at that age. The parameters of the formula F(x) should be chosen so that the fitted curve passes as close as possible to the observed $\left\{ \overset{o}{q}_x \right\}$. The least-square curve minimizes the sum of squares of the distances between the fitted values and the corresponding observed values

$$\sum_x \left[F(x) - \overset{o}{q}_x \right]^2 \quad (3.2.4)$$

The above unweighted least squares approach is appropriate for the case where the observed rates are reliable which means that all have the same variance. However, this is rarely true. Therefore, in the case where the variance of the observation at age x is proportional to w_x^{-1} the appropriate approach is the weighted least-square curve obtained by minimizing the expression

$$\sum_x w_x \left[\overset{0}{q}_x - F(x) \right]^2 \qquad (3.2.5)$$

Thus, weighting is according to the stability of the observed case. A large weight w_x, which at a particular point x corresponds to a small variance (more precision), will make the fitted curve to pass very close to that point whereas a small weight will make the curve pass not as closely; attributes which are desired.

The variance of q_x *is* $\tau_x^2 = q_x(1-q_x)/R_x$ or approximately q_x / R_x, since $(1-q_x) \cong 1$.

The mortality rate q_x is unknown so we approximate it either by the mortality rate q_x^s of a suitable standard table or by $\overset{0}{q}_x$.

The approach which is followed for the fitting of a non-linear model q(x)=F(x) is usually an iterative one. Either of the weights $\left\{ R_x / \overset{o}{q}_x \right\}$ or $\left\{ R_x / q_x^s \right\}$ can be used for the calculations of initial mortality rates $\left\{ q_x^{(1)} \right\}$ and the final graduated mortality rates will be computed by using as weights the $\left\{ R_x / q_x^{(1)} \right\}$. Although further iteration will unlikely produce substantial changes in the graduated rates, if it is a attempted and converged then the result would coincide with the minimum chi-square solution.

The simultaneous equations for the evaluation of the (weighted) least-squares estimates of the parameters are obtained by equating to zero the first partial derivatives of expression (3.2.5) (3.2.4).

3.2.3. Minimum chi square method

The number D_x of deaths at age x is binomial distributed with parameters R_x and q_x

Provided that the expected number of deaths $R_x q_x$ is not too small (smaller than five) the distribution of D_x is approximately normal with mean $R_x q_x$ and variance $R_x q_x(1-q_x)$ and

$$\chi^2 = \sum_x \frac{(D_x - R_x q_x)^2}{R_x q_x (1-q_x)}$$

has the chi-square distribution. The value we attempt to fit is $q_x = F(x)$ and the parameters of the formula $F(x)$ are chosen to be the ones that minimize the chi-square value for the experience.

Both parameter estimates and standard errors are conditional on the choice and use of the loss function. For example, the unweighted least-squares option will give different estimates for the first and second term parameters of the Heligman-Pollard model if compared with those obtained from the binomial weighting of the least-squares curve (weights inversely proportional to binomial sampling variances, $w_x = R_x / (q_x(1-q_x))$;

also the standard errors of the former be highly inflated (Congdon, 1993).

Chi-square minimization gives estimates which are close to those from a binomial weighting but it overstates standard errors, too. Forfar et al. (1988) presented a comprehensive comparison between chi-square and maximum likelihood methods in the broader context of estimation system.

Several researchers, who have applied the weighted least-squares method to fit Make-ham's form of $-\ln p_x$ during the period from 1872 to 1966, have used for weight the consequently approximation of the variance

$$w_x = 1/\sigma_x^2 \cong R_x / p_x q_x$$

(Seal, 1979). The AF French Committee in 1895, though, used the incorrect weight R_x and its example has been followed. Chandler (1872) used the correct weight for q_x, $R_x/(q_x(1-q_x))$, but an approximate Makeham formula

$$q_x = A + Bc^{x+\frac{1}{2}}.$$

Pocock et al. (1981) suggested an intermediate technique between weights equal for all ages (unweighted graduation) and weights inversely proportional to sampling variance. It is well Known that areas studied, usually vary considerably in size of population and number of deaths involve. Those with large population provide more reliable mortality rates and hence should receive greater weights in any analysis. The researchers of this study argued that the weighted least-squares method tends to over-weight the larger towns, since the residual mean square from the resultant regression will usually be larger than would be expected from sampling variation. In the intermediate case the weights are inverse to $\tau_x^2 + \sigma^2$, with τ_x^2, such as above and σ^2 such that the weighted residual sum of squares equals its expectation N-p (N is the number of age groups and p the number of parameters in the graduating function).

An alternative error structure mentioned in the actuarial literature is the Poisson model for death rates μ_x calculated by using central exposures (Forfar at al., 1988 Renshaw, 1991). Experiments with mortality have shown that parameter estimates and standard errors are very similar for binomial and Poisson assumptions, although the weighted error sum of squares under the Poisson model is quite lower under the binomial one.

Heligman and Pollard (1980) estimated the parameters of their model (3.1.3) by minimizing the following sum of squares

$$S^2 = \sum_x \left(\frac{\hat{q}_x}{q_x} - 1\right)^2 \tag{3.2.6}$$

where $\hat{q}_x$ is the estimated form the formula mortality rate and q_x is the empirical mortality rate. This version of the classical least-squares minimization criterion could be regarded as the weighted sum of squares (3.2.4), where $w_x = 1/(q_x)^2$.

Forfar and Smith (1987) used the same function (3.2.6) to examine the goodness of fit of the Heligman-Pollard model over the English Life Tables 1 to 13 for both males and females and to estimate the parameters without making any comment.

Hartmann (1983) in his attempt to fit the same model to Swedish mortality data tried firstly the minimization of the unweighted sum of squares.

$$\sum_x \left(\frac{\hat{q}_x}{\hat{p}_x} - \frac{q_x}{p_x}\right)^2 \tag{3.2.7}$$

The computations he made resulted frequently in unacceptable negative estimates of parameter B. Therefore, he used the recommended by Heligman and Pollard sum of squares (3.2.6) for making his conclusions about the fit of the model. With this procedure acceptable estimates were always obtained. In a later work Hartmann (1987) indicated that:

If the exact number of the exposed to risk R_x at each age x is known, then the squared standardized residuals could be computed and checked in order to see if the recorded rates are statistically commensurate with the ones obtained from the model.

Kostaki (1992 a) experimented to fit the Heligman-Pollard model by means of a nonlinear least-squares procedure using the algorithm E04FDF of the NAG library. She tried unsuccessfully to estimate the parameters of the model using as criteria the minimization of the unweighted sum of squares (3.2.7) and – since $\hat{q}_x$, q_x are very small values and $\hat{p}_x$, p_x are near unity – the approximately equal to (3.2.7) unweighted sum.

$$\sum_x (\hat{q}_x - q_x)^2 \tag{3.2.8}$$

She argued that the problem with this last sum (3.2.8) might have arisen because of the nature of it as well as the shape of Heligman-Pollard formula. The third term of (3.1.3) is linear in logarithms, which means that the rate of increase of the q_x values for the old ages is assumed by the formula to be constant, while the rate of increase of the empirical q_x values in many cases appears to change with time. This problem has concerned Economov et al. (1989) too, in their study of white male and female mortality data from 48 states of the United States. In the case where this minor systematic difference at the senescent ages between the real shape of mortality and the one derived from the formula appears, probably high residuals at these ages will appear too. The form (3.2.6) gives more magnitude to younger ages, where the weights are heavier as they correspond to smaller q_x values. On the contrary, form

(3.2.8) gives equal regard to residuals from the whole range of ages along with the problem of fit of the specific formula the minimization process becomes more cumbersome than with form (3.2.6).

The E04FDF NAG algorithm proved also inefficient to provide parameter estimates for the Heligman-Pollard model when the statistically consistent use of form (3.2.5) with weights the usual reciprocals of the binomial sampling variances of q_x or the sum

$$\sum_x \frac{1}{q_x}(\hat{q_x} - q_x)$$

were tried. Only when the function to be minimized was the one proposed by Heligman and Pollard (3.2.6), the algorithm gave satisfactory results (Kostaki 1992a).

Kostaki (1992a) in order to overcome the inadequacy of the least-squares NAG-algorithm developed an alternative stepwise procedure for estimating the parameters. Compared with the conventional iterative routines this method proved advantageous because it is less cumbersome (initial parameter values are not required) and more flexible (different criteria can be minimized). However, it provides estimates of a somewhat lower quality, which seems quite satisfactory for purposes such as the description of the mortality pattern.

Finally, in order to present and evaluate the nine-parameter version of Heligman – Pollard formula (3.1.7), Kostaki (1992b) used the least-squares minimization of (3.2.6) with the NAG-algorithm.

3.2.4. Bayesian MCMC method

Congdon (1993) argues that the use of parametric modeling of mortality data is necessary in many practical demographical problems. Dellaportas (1995), focus on a form of model introduced by Heligman and Pollard (1980) and adopt a Bayesian analysis, using Markov chain Monte Carlo simulation, to produce the required posterior summaries. This opens the way to richer more flexible inference summaries and avoids the numerical problems encountered with classical methods.

For a greater detail someone may consult Dellaportas, et. al. (1997).

3.3 The uses of mortality laws

3.3.1 Graduation - Description

Most mortality empirical schedules are based on limited samples coming from much larger populations. Being the samples insufficiently small, the age-specific mortality rates calculated from them vary unevenly from age to age affected by random statistical fluctuations and thus, form unstable estimates of the true mortality pattern underlying the data. Congdon (1993) pointed out that the raggedness of mortality rates generally increase as the size of the denominator population at risk diminishes, the smaller the time interval over which the event (death) is observed and more infrequent this event is. Brass (1979) also mentioned that the mortality data for about 80% of the population of the world is extremely scanty.

Graduation is the removal of the awkward irregularities and inconsistencies appeared to the data. Actuaries have been too interested in graduation methods because insurance tables seem more reasonable when the premium rises steadily rather than irregularly with age. Demographers, facing the same problem, have focused on graduation techniques, too. The use of a parametric model ensures that the mortality data will be ideally smoothed to provide more reliable and precise estimates. A variety of parametric models, already listed in section 3.1, have been proposed from the actuarial and demographic literature for the graduation of the observed mortality rates. The majority of these mortality laws are applicable only to the adult ages above about the age of 30. They do not take into account the high but rapidly falling mortality at the early childhood ages, nor the accident hump in early adult ages. Laws which are applicable to the entire age range is the one proposed by Thiele in 1872 (3.1.2), the more recent one proposed by Heligman and Pollard in 1980 (3.1.3), and a nine-parameter version of this last formula suggested by Kostaki in 1992 (3.1.7).

Several applications of the Heligman – Pollard model on mortality experiences in Australia (Heligman and Pollard, 1980), in USA (Mode and Busby, 1982), in England (Forfar and Smith, 1987) and in Sweden (Hartmann, 1987, Kostaki, 1992 a) have shown that:

It provides a very satisfactory representation of the age pattern of mortality and is probably the best law among those, which had been suggested in time before its appearance. However, this model produces systematic deviation from the experience due to the misplacement of the estimated accident hump. The nine-parameter version of the Heligman – Pollard model (Kostaki, 1992b) provides a closer fit to empirical mortality data eliminating this source of systematic error.

3.3.2 Comparison

Comparison of demographic patterns is one that is facilitated by the use of a parametric model, which includes into a limited number of parameters the essential features of mortality for all ages.

Keyfitz (1982) referred to the beneficiary contribution of mortality laws to comparison since they summarize the whole procedure to differences between a few parameters instead of a large number of age-specific rates.

Congdon (1993) too, numbered this property of the process among the advantages of representing a mortality pattern with a mathematical formula. However, he indicated the problem of overparameterization in time series comparisons of parameters.

3.3.3 Forecasting - Projection

Forecasting is based on the idea investigation of mortality patterns in the past might help to forecast the future development accurately.

Keyfitz (1982) pointed out that forecasting supposes the existence of typical directions of movement through time. The usual mortality component of population forecasts is provided by extrapolation of past trends. Mathematical formulae facilitate the forecasting of mortality into the future. One is focused only on the progression and projection of the limited number of parameter estimates. Given that the parameter estimates for subsequent periods develop according to a simple pattern, then this pattern can be extrapolated in order to give forecasts. However, when parameter instability over time is observed, the ability for parametric forecasting is reduced.

Pollard, in his paper published in 1987, reviewed a variety of methods which have been suggested by actuaries and demographers in order to project age-specific mortality rates : projection by extrapolation of mortality rates (or transformation of mortality rates) at selected ages, projection by reference to a law of mortality, projection by reference to model life tables, projection by reference to another ''more advanced'' population, projection by reference to an ''optimal'' life table attainable under ideal conditions, projection by cause of death and finally combinations of these methods. He remarked that the choice of method must depend on the type, the extend and the quality of the data available and also on the purpose for which the projection is required. About the concerned method of projection by a mathematical law, he mentioned that:

Some disadvantages may be the difficulty in selecting a suitable law, which would be applicable to the whole life span. A possible lack of a clearly discernible pattern for the estimates of parameters over time, an inability to be applied when only data at a single epoch is available and the fact that independent extrapolation of individual parameters may lead to unreasonable projected mortality rates.

Congdon (1993) referred to the helpful use of parametric models for forecasting because of their attribute to facilitate the assessments of mortality trends over time, as these are undertaken in terms of the assumptions about parameters instead of examining the projected population totals. Congdon as well as Rogers (1986) stated that a parsimonious model should be preferred for forecasting purposes, even if there is a slight loss in goodness of fit, because of the problem of overparameterization (see Section 4.2), which has implications in forecasts to future years. An alternative simpler method for forecasting and also for comparisons proposed by Congdon is the relational approach. At its simplest it involves a logit regression with two parameters (formula (6) in Subsection 2.2.3). The aim is to relate a mortality schedule for a particular year or area to a chosen standard schedule. Then the trends in the parameters can be assessed and extrapolated in order to obtain forecasts of the mortality.

Especially about the use of the Heligman – Pollard model for forecasting, Hartmann (1987) pointed that it is an ideal and useful model for making population projections because of the well behaved nature of the parameter estimates and the unusually good fits it had given.

At the same period Forfar and Smith (1987) used this model to make projections. After fitting the Heligman – Pollard model (3.1.3) to English Life Tables for both males and females for the period 1838 – 1972, they checked the progression of its eight parameters over time. They estimated the parameters of this period and then produced estimates for the future life tables of the years 1981 and 1991. While their paper was prepared the mortality tables of the year 1981 were published, so they included a comparison of the parameters projection with those calculated from the tables. Using as loss function the one proposed by Heligman and Pollard (3.2.6), they concluded that the projection estimates were very good for females but much poorer for males. At the end they produced, as well, estimates for:

- The parameter values of the years 1991 and 2001 using formula (3.1.3) for females and
- The alternative formula (3.1.4) for males, adding the remark that these projections are based on historical data with no allowance for future, unknown trends as for example the effect of aids.

3.4 Diverse uses

3.4.1 Expanding an abridged life table

A mortality pattern can be utterly described by a complete life table in which the data are presented for every year of age. However, there are cases where the available empirical mortality data are incomplete or unreliable, so that their quality does not permit the computation of a complete life table. In these cases an abridged life table is constructed, which contains data tabulated usually in five-year intervals, except for the first five years that are usually presented in the two intervals [0,1) and [1,5). Consequently, often arises the problem of construction of a full life table from the corresponding abridged one. Parametric models can be used as the chief component in the procedure of expanding an abridged life table.

A demonstration and evaluation of such a technique is presented by Kostaki (1991) which used the Heligman – Pollard formula to estimate directly the age-specific probabilities of dying q_x , while later (Kostaki, 1992a) used the nine-parameter variant of this law for the same application. Kostaki (1991) argues that the main advantage of this procedure, in comparison with the conventional interpolation formulae applied to tabulate $l(x)$ - values (survival probabilities), might be the use of a formula, which efficiently describes the age pattern of mortality.

3.4.2 Degrouping mortality data

Parametric models are entailed, as well, in a technique for estimating, from grouped empirical death data, the age – specific numbers of deaths d_x (Kostaki and Lanke,

1999a). This is primarily of great interest in some countries of Southern Europe and in the Third World where the available data are provided in a grouped form. Such form in the data is result of age misstatements in age recording (heaping), rounding to the nearest integer divisible with five. Thus, these over – understatements which affect the data sets will efface each other into the five-year age group.
The technique can be utilized in order to eliminate the age heaping in the empirical death data as well as the age heaping in the empirical population data by the occasional use of the method of extinct generations on the previous results. In the frame of this technique the Gompertz law was used.

3.4.3 Accurate approximations to life table probabilities and functions

One of the parametric models of mortality, the Gompertz law, has appeared extremely useful, because it allows for many quick and accurate approximations, more familiar among actuaries than demographers, in life contingency calculations (Pollard, 1991). Therefore, formulae for:

- The probability of surviving from age x to age x+t;
- The probability of first death;
- Joint lifetime of two persons;
- Median time to death;
- Percentiles;
- Modal age of death;
- Complete expectation of life;
- Standard deviation of the time to death;
- Life annuities;
- The proportion of a stationary population above a given age and finally
- The proportion of a stable population with growth rate r aged x, are provided by invoking the Gompertz law even when the data are not strictly of the Gompertz shape.

CHAPTER 4

METHODS FOR EXPANDING AN ABRIDGED LIFE TABLE

4.1. Methods – A Brief Overview

In this chapter we provide a description of the expanding methods for estimating the age specific mortality pattern.

The problem of estimating a complete life table, when the data are provided in age intervals has been extensively discussed in demographic, biostatistical, as well as in actuarial literature. An abridged life table presents the mortality pattern by age groups. The main reasons for providing data in an abridged form are related to the phenomenon of "age heaping", caused by certain age misstatements in data registration and also the unstable mortality probability estimates provided by insufficiently small samples. The most typical case of age-misstatements is that of the preference of ages ending in multiples of five or zeros. Such misstatements cause the appearance of age heaps.

Several methods have been suggested in the literature for reproducing the age-specific mortality pattern. A suggested solution is the application of some graduation formula to the observed data. Since data may contain great "systematic" fluctuations except of "random" ones the above solution is not preferred.

In the following lines, we consider methods that have appeared so far in the literature as tools of expanding an abridged life table to a complete one. Valaoras (1984) presents in detail an old method which was initially presented by Reed (any reference on other publication of the method is not given by Valaoras, 1984). J. Pollard (1989) presents a relatively new one, which requires five-year age grouping. Kostaki (1991) presented a method to which a parametric model of mortality is utilised. Dellaportas, et. al., (1997) introduce a Bayesian version of the application of the famous Heligman & Pollard formula. Kostaki (1998) presents a new method, which is non-parametric, in the sense that it does not require the use of a parametric model. It is a method, which relates the target abridged life table with an existed complete one. The rest of the methods that bibliography suggests introduce the application of some interpolation formula to the survivors function l_x values of the abridged life table. Such a conventional technique is the application of the famous six-point Lagrangean interpolation (see, Johnson & Johnson, 1980). Beers (1944) presents a set of some other six- point interpolation formulae. King (1914) presented a method of separate interpolation on death and exposed to the risk population values of the abridged life table. This is of an oscillatory piecewise interpolation technique as suggested by the constraints applied to points where the fitted polynomials join. Spline interpolation (see e.g. Hsieh, 1991, McNeil, et. al., 1977, or Wegman and Wright, 1983) is a case of an oscillatory interpolation technique which has lately received great attention. Great literature is referring lately to splines, as tools for expanding an abridged life table.

Here we consider in more detail the known eight-parameter (HP8) as an expanding tool and we also consider and describe the case of a reduced form of that model, when this is required. Bibliography suggests that parametric models and splines are the more preferred methods in the case of expanding an abridged life table. Except their basic difference, which is that the one is parametric and the other non-parametric, the two methods exhibit other interesting dissimilarities and similarities. A contrast between spline interpolation and the parametric model application in general is made, and it is described within our application.

4.2. The Parametric Method – Parametric Models of Mortality as Expanding Tools.

4.2.1. Parametric Models of Mortality.

Mathematical descriptions of schedules of mortality rates, called laws of mortality, offer usually an efficient means of condensing the amount of information to be specified as a set of assumptions. The last is imposed by a set of parameters and functions. The information that is required for a parametric model adopted to represent a mortality schedule is included in a "Life Table".

The problem posed here is that of reproducing the age-specific pattern from incomplete or grouped data. Kostaki (1991) solves this problem by proposing the use of an adequate parametric model. The rationale of the proposed method is simple. If there is a model that adequately represents mortality of a population, then this model can also estimate in an adequate way the complete mortality experience, when an abridged one is the only available.

The search for a mortality law has occupied the attention of statisticians and demographers for over a century. The attempt of representing mortality via a parametric model starts by Gompertz (see, Pollard (1991)). This is the well known Gompertz law of mortality initially proposed for modeling mortality at the elderly, but also adopted for the earlier adult ages. The earliest attempt to represent mortality at all ages is that of Thiele (1872), who combined three functions each one representing a different part of the mortality schedule. In the same sense, Heligman & Pollard (1980) set out analogously a function of mortality, as represented by the odds of mortality q_x / p_x at age x as an eight-parameter formula of age. This followed to be the famous eight-parameter HP model for representing the age pattern of mortality.

There is no particular choice of a parametric model. Its adequacy of representing mortality is only required. Kostaki (1991) presents how such model choice can form an efficient solution for estimating age - specific mortality schedules. She introduces the method by adopting the application of the classical 8-parameter model by Heligman & Pollard (1980) hereafter HP, but another adequate model choice is also permissible.

The Expanding Procedure

We have our model for the mortality experience $\frac{q_x}{p_x} = F(X;\Theta)$, where $F(X;\Theta)$ the right hands side of the equation with Θ being the vector of the parameters of the model, $\Theta = (A, B, ..., H)$.

From the model we get for the one-year odds of dying and as concluded for the one-year probabilities of dying,

$$q_x = \frac{F(x;\Theta)}{1+F(x;\Theta)} = G(x;\Theta)$$

In addition, the relation implies the following model for the death probabilities in the abridged life table

$$ {}_nq_x = 1-\prod_{i=0}^{n-1}(1-q_{x+i}) \Rightarrow {}_nq_x = 1-\prod_{i=0}^{n-1}(1-G(x+i,\Theta)) = {}_nG(x;\Theta) $$

where ${}_nq_x$: is the probability of someone of age x to die before reaching age $x+n$, i.e. to die in the age interval $[x, x+n)$. We consider ${}_nG(x;\Theta)$ as an explicit but complicated function of Θ, x, n.

Then given the abridged (grouped) mortality experience one starts the process of expanding the abridged table by minimizing,

$$ \sum_x \left(\frac{{}_nG(x;\Theta)}{{}_nq_x}-1\right)^2 $$

This is the loss function of the estimation algorithm where the summation is over all relevant values of x.

Get the estimates of the parameters as they come from this minimisation procedure and insert them to the mortality formula. The estimated model will produce now an expanded abridged life table.

Two cases of a parametric model application are introduced so far by the bibliography.

4.2.2 The Eight - parameter H&P Formula as an Expanding Tool (HP8)

The HP8 is a non-linear model composed by eight-parameters. It relates the odds of dying with age x. The model's rationale is simple. It is distinguished in three parts representing, three classes of death cause. So, three causes can be seen, namely those affecting childhood, early and middle adult life, and the old age.

The formula utilises a model that presents the odds of mortality $\frac{q_x}{p_x}$ as a parametric function of age x according to the formula,

$$ \frac{q_x}{p_x} = A^{(X+B)^C} + D\exp(-E(\ln(x/F))^2) + GH^x \tag{4.1} $$

q_x is the modelled probability of an individual of age x to die before reaching age $x+1$. The modelled quantity, as referred previously, are the odds of dying than not dying before age $x+1$, when the individual of study is of age x.

Heligman and Pollard (1980), Hartmann (1987), Forfar and Smith (1987), Kostaki (1992) fit the formula (4.1) to empirical data sets of several countries and different time periods. In all these investigations, the parameters of the model are estimated using an iterative routine of the Nag library that is based upon a modification of the Gauss-Newton algorithm. A detailed description of this procedure is provided in Gill and Murray (1978). In all these investigations the parameters of the model are estimated by minimising the sum of squares of the relative deviation between the estimated and the observed probabilities of dying,

$$\sum_x \left(\frac{\hat{q}_x}{q_x} - 1\right)^2 ,$$

a loss function proposed by Heligman and Pollard (1980).

The expanding technique

Using the notation β for the parameters of the model (4.1) and the notation $F(x;\theta)$ for the right hand side of (4.1), we get

$$q_x = \frac{F(x;\theta)}{1+F(x;\theta)} = \tag{4.2}$$

$$= G(x;\theta)$$

say, and hence the relation

$$_n q_x = 1 - \prod_{i=0}^{n-1} (1 - q_{x+i})$$

implies the model

$$ {}_n q_x = 1 - \prod_{i=0}^{n-1} (1 - G(x+i;\theta) $$

$$ = {}_n G(x;\theta) $$

say, the later being an explicit but complicated function of θ, x, and n.

Thus given the values of ${}_n q_x$ of the abridged life table, one provides estimates of θ minimizing

$$ \sum_x \left(\frac{{}_n G(x;\theta)}{{}_n q_x} - 1 \right)^2 $$

where the summation is over all relevant values of x. Then inserting this θ into (4.2) estimates, $\tilde{q}_x$, of the one-year q_x-values are produced.

4.2.3 The Nine - parameter H&P Formula as an Expanding tool (HP9)

In general, HP8 is an adequate solution to the expanding of an abridged life table. Anyway, it fails to reproduce correctly the accident hump, since it estimates its beginning at a later age. That is more obvious in the cases of data where a severe accident hump exists.

In such cases, a nine - parameter version of the H&P formula which is introduced by Kostaki (1991) will perform better, since it improves the estimates provided by HP8 at that part of the curve. The HP9 model, makes the model more flexible in the accident hump by modifying the middle model term. The suggested formula is:

$$ \frac{q_x}{p_x} = \begin{cases} A^{(X+B)^C} + D\exp(-E_1(\ln(x/F))^2) + GH^x, & x \le F \\ A^{(X+B)^C} + D\exp(-E_2(\ln(x/F))^2) + GH^x, & x > F \end{cases} $$

The new parameters are the E_1, E_2 terms related to the spread of the accident hump at the left and right respectively.

Model is estimated by nonlinear least squares and the fit is again connected to the problem of overparameterisation. Kostaki (1992) presents the HP9 as a tool for expanding an abridged life table and as a solution to incomplete data problems. It is simple to define the method since it is an application of the originally proposed parametric also by Kostaki (1991), where now as the model $F(x;\Theta)$ we use HP9 (vector Θ is now of length nine).

4.2.4. Bayesian HP8 formula

Dellaportas, et. al. (1997) adopt a Bayesian inference approach to the eight- parameter H&P (HP8) fit which according to the authors, it has several advantages compared to the classical solution. Firstly, it resolves the problem of overparameterisation, very usual to a least squares fitting of a model, by the use of an informative prior distribution. Secondly, the non-normality of the likelihood surface in the parameterization usually adopted means that the least square estimates is inadequate. Thirdly, it is applied as an expanding tool by routinely applying a simulation - based Bayesian computation methodology. Here we carry out a small reference from a theoretical only viewpoint, since this thesis extends only to solutions of classical statistics.

The Expanding Procedure

From a Bayesian perspective, the abridged life table problem can be seen as an incomplete data problem, or as adopted by the authors as a constrained parameter problem. A general approach using an MCMC strategy is used, see Gelfand et al. (1992). As for applying graduation procedures to statistical data (e.g., demographic data) using a bayesian estimation strategy, see Carlin (1992).

In general, we assume that the eight - parameter H&P (HP8) model describes the true underlying the data age - specific pattern (the one year $q_x's$)). We also consider as Θ = (A,B,C,D,E,F,G,H) the parameter vector, and by d_x the single year of age death counts which is binomially distributed with par's E_x and q_x. The only known non-random quantities are the grouped population counts (the exposed to the risk of death for each group) E_x, and the population with age 0 and the only observed data the grouped death counts, ${}_nd_x$.

Quantities as Θ, $d_x's$,(or $q_x's$) are considered as unknowns.

Let ${}_nE_x$, ${}_nd_x$, E_x, d_x, be the vectors of the previous. E_x, d_x, do not contain the values for age $x = 0$. So, the full model for the data and the unknowns given the known quantities is of the form,

$$p({}_nd_x, d_x, d_0, E_x, \Theta / {}_nE_x, E_0) = p({}_nd_x / d_x, E_x, \Theta, {}_nE_x, E_0, d_0) p(d_x / E_x, \Theta, {}_nE_x, E_0, d_0)$$
$$p(d_0 / E_x, \Theta, {}_nE_x, E_0) p(E_x, \Theta / {}_nE_x, E_0)$$

We note that ${}_nd_x$, depends only on d_x , since

$${}_nd_x = \sum_{i=x}^{x+n} d_i$$

so for the first conditional density we get a product of indicator functions, ranging over $x=1,5,10,\ldots,w$ (w is the age, where a generation extinct).

$$p({}_nd_x / d_x, E_x, \Theta, {}_nE_x, E_0, d_0) = \prod p({}_nd_x / d_x, \ldots\ldots, d_{x+k})$$

When E_x, Θ are known the next conditional density is derived where d_x depends only on the previous, with each components being independent binomial distributions.

$$p(d_x / E_x, \Theta, {}_nE_x, E_0, d_0) = p(d_x / E_x, \Theta) = \prod_{x=1}^{n} p(d_x / E_x, q_x),$$

where q_x the HP8 one year estimated probabilities.

Similarly,

$$p(d_0 / E_x, \Theta, {}_nE_x, E_0) = p(d_0 / E_0, q_0)$$

q_0 the first year HP8 estimate.

And finally, we consider α-priori that Θ is independent of anything else, and that E_x given ${}_nE_x$ is independent of Θ.

$$p(E_x, \Theta / {}_nE_x, E_0) = p(\Theta) p(E_x / {}_nE_x, E_0)$$

$p(\Theta)$ is the prior distribution for θ, the HP8 model parameters.

$p(E_x / {}_nE_x, E_0)$ is the prior for the exposed to the risk of death population counts.

4.3 The Adjusted Parametric Model Application

Theoretically, the expanded one-year probabilities of dying constructed by our technique, or any expanding technique, it is expected to have corresponding n-year probabilities which have values close to the original abridged life table values. We may change the one - year probabilities by values that satisfy the desired property. Kostaki (1991) proposed the following, simple solution.

$$\hat{q}'_{x+i} = 1-(1-\hat{q}_x)^K$$

where $K = \dfrac{\ln(1-{}_{n}q_{x})}{\sum_{i=0}^{n-1}\ln(1-\hat{q}_{x+i})}$ and $\hat{q}'_{x+i}$ the new values of the one year probabilities.

It is easy to explain the rationale behind this particular choice of adjustment. It amounts to assuming that the force of mortality $\mu'(x)$ is in each n-year interval $[x, x+n)$ a constant multiple, say $K\mu(.)$, of one say $\mu(.)$, of those infinitely many forces of mortality which produce the complete life table obtained by the expansion process.

4.4. The Pollard's Model

The usual representation of abridged data is the one with groups of age x=0,1-4,5-9,...,75+,or 85+. Pollard (1989) proposes a method for deriving a full life table from mortality data given for each single year until the age of 5 and then for five - year age groups. The usual form of grouping is close to the one the method requires. To describe the method we also assume that deaths during a given calendar year are reported according to age x of last birthday at time of death. Because of non-uniform exposure of deaths over the 5-year age group, ${}_5\hat{m}_x$ as produced by the mean population over an age interval will provide a biased estimate of the mortality rate. To overcome this problem two assumptions are adopted:

Exponential variation of the force of mortality μ_x (at age **x**) within the age interval and that the population remains stable within this age interval.

Pollard's method, adopts the use of the classical Gompertz's law of mortality (Benjamin and Pollard (1980)), for degrouping mortality in the whole life span.

Gompertz's law assumes the above described exponential variation of μ_x (at x) within an age interval, i.e.

$$\mu_x = Bc^x$$

This was usually applied only to increasing mortality at adult ages, now it is revised as a tool for describing the decreasing mortality of younger age groups too. In the last case we assume that c<1.

4.4.1 The Procedure

Having values of deaths and central populations for the ages 1, 2, 3 and 4, allow the immediate estimation of μ(1.5), μ(2.5), μ(3.5), μ(4.5). Values of μ(x) at the pivotal ages 7.5, 12.5, ..., 82.5 are provided by the ratios, $\mathring{e}_5, {}_5\mathring{m}_{10}, \ldots, {}_5\mathring{m}_{80}$, , without adjustment at the younger ages, and after adjustment according to formula,

$$\mu(x+2.5) = \frac{{}_5\mathring{m}_x}{\left[1 - \frac{25}{12}\left({}_5\mathring{m}_x + r\right)\ln c + \frac{25}{24}(\ln c)^2\right]}$$

for ages as above, say, 30. In order to have values of m(x) at intervening ages we may apply *interpolation* (described later). If no smoothing or graduation is required then, linear interpolation on $\ln\mu_\chi$ is recommended. Also for ages x<7.5 we may apply an *extrapolation*.

Then **interpolation** and **extrapolation** appear to be the main tools of this method. Since μ_x implies of a theoretical and not an empirical valued measure, any value given refers to this function's approximation.

To describe the construction of the adjustment formula, we outline the following.

In the μ_x, we start from the assumption of exponential variation.

- Exponential variation of μ_x

The survival probability from age x to x+t can take the form,

$${}_tp_x = \exp\left\{-\int_0^t \mu_{\chi+\upsilon} du\right\}$$

and after some calculations we deduce to,

$$ {}_t p_x = \exp\left[\frac{t(\mu_x - \mu_{x+t})}{\ln(\mu_{x+t}/\mu_x)}\right] $$

Which is an exact formula when μ_x varies exponentially.

Concentrating now on the application on the adult ages, we get by $c^t = e^{\ln c}$, and ln c approximated by 0.09, that,

$$ {}_t p_x = \exp\left\{-\mu_x\left[t + \frac{1}{2}t^2 \ln c\right]\right\} $$

The last assumes *quinquennial age groups*.

Now under the stability assumption over the age range (x,x+5) with intrinsic growth rate *r*, the number of persons in the age interval may be written,

$$ P(x, x+5) = K \int_{-2.5}^{2.5} e_x^{-rt}\, p_{x+2.5}\, dt $$

and under μ_x exponentially varying we may write,

$$ P(x, x+5) = 5K\left[1 - \frac{25}{24}\mu_{x+2.5}\ln c + \frac{25}{12}(\mu_{x+2.5} + r)^2\right] $$

and,

$$ D(x, x+5) = K \int_{-2.5}^{2.5} e_t^{-rt}\, p_{x+2.5}\,\mu_{\chi+2.5+t}\, dt = $$

$$ 5K\mu_{x+2.5}\left[1 - \frac{25}{24}\mu_{x+2.5}\ln c + \frac{25}{12}(\mu_{x+2.5} + r - \ln c)^2\right] $$

Writing ${}_5\overset{o}{m}_x$ for D(x,x+5)/P(x,x+5), dividing the two above formulae and replacing $\mu_{x+2.5}$ in the correction term by , we conclude to the adjustment formula.

4.5 The Reed's Model

A simple tool for expanding was also introduced by *Reed* (see Valaoras, 1984). This method's main tool is the five - year mortality rate.

$$ {}_5m_x = \frac{{}_5d_x}{{}_5M_x} $$

where ${}_5d_x$ represents the number of deaths in the five year age interval [x,x+5) and ${}_5M_x$ the mean population at the same age interval.

Valaoras (1984) who presented an analytical description of this method adopted this method in the construction of some Greek Life Tables and it's performance on Greek data.

4.5.1 The Procedure

We have the five - year mortality rates, ${}_5\mu_x$. Then the one – year probabilities of dying $q_7, q_{12}, q_{17}, \ldots\ldots$ are calculated, using the following approximate formula,

$$ q_{x+2} = \frac{2\,{}_5m_x}{2+{}_5m_x} $$

In order to estimate the full series of the one-year probabilities the following formula is then adopted,

$$ \frac{q_x}{K^x} = a + bx + cx^2 + dx^3, x \geq 5 $$

Using the already estimated one year probabilities ($q_7, q_{12}, q_{17}, \ldots\ldots$,) the complete q_x series is estimated by least squares.

In order to estimate the complete probability series the following procedure is used.

The formula is fitted twice. First, in order to produce the complete probability series for the age range $5 \leq x \leq 20$ it is fitted to $q_7, q_{12}, q_{17}, q_{22}$ values with K=0.989943. Then for $x \leq 25$, the formula is fitted to using the $q_{22}, q_{27}, q_{32}, \ldots$ values with K=1.0251234.

Finally, for the ages 21 to 24, a linear combination of the two fitted equations is used in order to estimate the one-year probabilities:

$$q_{21} = 0.8q'_{21} + 0.2q''_{21}$$

$$q_{22} = 0.6q'_{22} + 0.4q''_{22}$$

$$q_{23} = 0.4q'_{23} + 0.6q''_{23}$$

$$q_{24} = 0.2q'_{24} + 0.8q''_{24}$$

Where the first and second terms of the above equations are the fitted probability values for K=0.989943 and K=1.0251234 respectively. Reed proposed the two of K values.

4.5.2 Properties and Problems

This technique is not applicable to the early childhood ages (x<5). Adequate as well as it does not produce successful results for the early adult ages (see Kostaki, 1992). However, it is effective for the adult ages.

4.6 The Non Parametric Method

Kostaki (1998) describes a non-parametric method in the sense it does not require the adaptation of a parametric model, adequate for describing the mortality pattern, like in Kostaki (1991) and (1992).
It is considered as a **relational technique** since it *relates* an abridged life table with another complete one used as a reference table. The *reference* table is not necessary to be a standard one. The performances of the method will always differ as the reference table changes in period or in the population it describes. Anyhow the differences, as application demonstrates, will be of slight extent.

4.6.1 The Procedure

We have the ${}_nq_x$ probabilities, elements of our abridged life table and the $q_x^{(R)}$ of the reference (e.g. a standard table) life table, which is of complete form.

Under the assumption that the force of mortality μ_x, underlying the abridged life table is, in each n-year age interval [x,x+n), a constant multiple of the one underlying the reference (complete) life table in the same interval $\mu_x^{(R)}$, i.e.,

$$\mu(x) = {}_nK_x \mu^{(R)}(x)$$

where,

$$K = \frac{\ln(1 - {}_nq_x)}{\sum_{i=0}^{n-1} \ln\left(1 - q_{x+i}^{(R)}\right)}$$

Then the one year probabilities of dying q_{x+i}, i=0,1,...,n-1 are in each n-year interval equal to

$$1 - (1 - q_{x+i}^{(R)})^{nKx}$$

Therefore from the ${}_nq_x$ and the ${}_1q_x^{(R)}$ we calculate ${}_nK_x$ and then estimate the one - year probabilities, q_x underlying the target abridged life table.

4.7 Polynomial or Piecewise Polynomial Interpolation

4.7.1 Interpolation Techniques

Since mortality data grouped in single years (or even narrower) intervals usually are not available we study techniques of expanding the interval data (abridged data) to single year values, based on the application of *mathematical interpolation*.

A usual method is to fit to the interval data a single polynomial. We interpolate values of a function *f(x)*, where *x=age*, which we have tabulated and produce function's values at intervening ages.

Interpolation

The notion of interpolation is to estimate a non-tabulated value of a function from tabulated values. This method is useful when the corresponding function, here usually a survivors function l_x, is not mathematically specified (usual case in life tables, e.g. we have certain values on certain ages only).

4.7.2 Lagrangean interpolation of six terms (points).

The power of an interpolation formula is denoted by the number of terms that is consisted of six term formulae are most famous in the literature, that means formulae that comprise six successive data points or else six successive tabulated values of a function.

In general Lagrangean interpolation formula assumes that the fitted polynomial, the function of interest say *u(x)* which is of degree *k*, is a linear combination of *k+1* tabulated values of this function. When only two tabulated values are interpolated by another then *k+1=2* and *k=1*, the fitted polynomial is of first degree and the interpolation is called *linear*. With the same rationale we have cubic interpolation when the values to be interpolated are four and the polynomial of third degree. So,

$$u(x) = \sum_{i=1}^{k} \frac{\prod_{j \neq i} (x - x_j)}{\prod_{j \neq i} (x_i - x_j)} . u(x_i)$$

Then if we have $x_1 = 1, x_2 = 3$, the formula takes the above expression with coefficients, -0.5 and 0.5 respectively for u_{x_1} and u_{x_3} when the interpolated value is u_{x_2}. These are the coefficients for the linear interpolation of u_{x_1} and u_{x_3} and the following is the above derived linear interpolation formula :

$$u(x) = \frac{(x - x_2)}{(x_1 - x_2)} u(x_1) + \frac{(x - x_1)}{(x_2 - x_1)} u(x_2)$$

It is interesting to observe that coefficients add up to one. That happens for every value that *k+1* and then *k* take. It is justified by the fact that if all tabulated values have to be equal to a constant then the non tabulated ones have to be equal to the same constant also.

A conventional interpolating technique that is usually applied, is the six point Lagrangean interpolation method. It is very applicable, fast, and very simple to handle, since it does not require great computational skills by the researcher.

The six term Lagrangean interpolation formula applied on the existing values of the survivors function l_x expresses each non tabulated value as a linear combination of six particular polynomials in x, each of degree five. As Johnson & Johnson (1990) describe,

$$l(x)=\sum_{i=1}^{6}\frac{\prod_{j\neq i}\left(x-x_j\right)}{\prod_{j\neq i}\left(x_i-x_j\right)}.l(x_i)$$

where $x_1, x_2, \ldots, x_6$ are the tabular ages nearest to x.

When the x_i's are equally spaced this formula can be expressed in simpler forms.

Elandt and Norman Johnson (1990), and Abramowitz, and Stegun (1972), tabulated the above equation's coefficients, which we also provide in the of this study. Lagrangean interpolation is a very famous case of interpolation, adopted by several authors, like *e.g., Namboodiri, Suchindran (1987)*, when conversation comes to expanding an abridged life table.

4.7.3 Other Six-Term Formulae for (Actuarial) interpolation.

The literature on interpolation techniques contains too many formulae from which few are used to real data applications. The most applied, is the one previously described, Lagrangean formula. Formulae that comprise six terms of the interpolated function as said before, are more famous.

Henry Beers (1944) gathers some of these and compares their performance. It is reasonable that all comprise cases of piecewise polynomial interpolation. The formulae briefly stated are:

1. The elementary fifth difference formula.

2. The curve-of-sines osculatory formula.

3. Sprague's fifth-difference osculatory formula.

4. Shovelton's tangential formula.

5. The minimized-fifth-difference formula.

6. Henderson's famous near-osculator\nolinebreak y formula.

7. Jenkin's formula.

As *f(s)* we present the function to be interpolated, where our usual choice is the survivors function l_{x+s} or l(x+s).

In general **six - point interpolation formulae** *(most of them here)* have *three common characteristics*.

They are *symmetric* in the sense that they give the same results whether they are applied forward or backward, i.e. *f(s)* or *l(x+s)*, is the same function of *f(-2)*, *f(-1)*, *f(0)*, *f(1)*, *f(2)* and *f(3)* as *f(-s)* is of *f(2)*, *f(1)*, *f(0)*, *f(-1)*, *f(-2)* and *f(-3)* with *s* positive and less but not equal to *1*, and with *f(s)* denoting the value u_{x+s} (or, l(x+s)) of the function we interpolate. The rest *f(.)*values take similar expressions. They are *correct up to four differences*, Δ^4f, i.e. when fifth differences are zero, they give the same results as the classical elementary fourth difference formula applied to the same given values. They are *correct to fifth differences on the average*, Δ^5f, i.e. the sum of the interpolated values in each age interval is equal to the sum computed from the same given values by the classical elementary fifth difference formula (*presented later*). This characteristic is usually a sufficient guaranty of the reliability of the results of interpolation performed by one of these formulae. The reader interested to mathematical details on those characteristics may follow the already referred work by, Beers (1944).

Formulae Mathematical Expressions

1. **The elementary fifth differences formula** is considered as the best formula when fifth differences are constant.

Formula:

$$l_{x+s} = f(s) = \frac{(s+2)(s+1)s(s-1)(s-2)}{120}$$

1. **The curve of sines osculatory formula** was used in the course of the construction from population statistics of some of the early English life tables. It is called "osculatory" because first and second derivatives of consecutive curves are equal at points of junction. The term osculatory will be considered in detail. later, when the category of "osculatory " interpolation techniques will be described.

Formula:

$$l_{x+s} = f(s) = \frac{(s+1)s(s-1)(s-2)(1-\cos \pi s)}{48}$$

1.**Sprague's fifth difference osculatory formula** has the same first and second derivatives at each point of junction as the fourth degree curve through that point and the two given values on each side of it.

Formula:

$$l_{x+s} = f(s) = s^3 \frac{(1-2)(7-5s)}{24}$$

1. **Shovelton's tangential formula** is characterized like that, as tangential, because it makes the first derivatives (but not the second) of consecutive curves equal at points of function.

Formula:

$$l_{x+s} = f(s) = s^2 \frac{(1-s)(5-s)}{48}$$

1. **The minimized fifth difference formula** was derived for the special purpose of minimizing the sum of squares of the fifth differences of the interpolated values. It has no algebraic expression for l_{x+s}, or $f(s)$.

2. **Henderson's** (famous, as given by the author) **near-osculatory formula** (presented in TASA, Transactions of the American Society of Actuaries, Vol. IX, 219-20) is expressed as

Formula:

$$l_{x+s} = f(s) = l_x + s\Delta l_x + \frac{s(s-1)}{2}\left(\Delta^2 l_{x-1} - \frac{1}{6}\Delta^4 l_{x-2}\right)$$

$$+\frac{(s-1)s(s+1)}{6}\left(\Delta^3 u_{x-1} - \frac{1}{6}\Delta^5 l_{x-2}\right)$$

1. **Jenkin's formula** (presented in RAIA, the Records of the American Institute of Actuaries, Vol. XV, 89) is expressed as

$$l_{x+s} = f(s) = l_x + s\Delta l_x + \frac{s(s-1)}{2}\Delta^2 l_{x-1}$$

$$+\frac{(s+1)s(s-1)}{6}\Delta^3 l_{x-1}$$

$$\frac{s^3(s-1)}{12}\Delta^4 l_{x-1}$$

$$-\frac{s\ (s-1)^3}{12}\Delta^4 l_{x-2}$$

Measures of comparison and formulae evaluation.

We compare the smoothness of formulae 1 to 5 by calculating their fifth differences, $\Delta^5 f$. In general differences of a certain degree may be adopted as a comparison tool of two, or more interpolation formulae (*for formulae 1 to 7*). As Beers (1944) suggested, for routine actuarial interpolation it is highly desirable, or even necessary to have a formula that can be applied with some assurance without preliminary analysis of the particular characteristics of the differences of the given values of the function to be interpolated. Now, if fifth differences are negligible in size, there is no problem since formulae with that characteristic will provide almost the same results. Problems arise when the fifth differences of the given values are too large to neglect but too irregular to be plausible. Another criterion of the smoothness of an interpolation formula is required. Beers (1944), suggested the smallness of the sum of squares of the fifth differences of the interpolated values.

$$\sum\left(\Delta^5 l_x\right)^2,$$

where l_x the interpolated function.

Some of the formulae presented by Beers (1944) are also osculatory. That means, that they osculate at points of junction x. This is the case we deal within the next sections.

4.8 Osculatory Interpolation

The usual case of interpolation is the one called, *ordinary piecewise interpolation*. As it is described we fit polynomials to pieces of the data which we connect, one by one, at points called joins, and produce a continuous function. The problem is that derivatives at the joints are discontinuous. Osculatory interpolation techniques overcome this problem by ensuring that important derivatives (usually the first two) will be continuous to the whole range of values. In that case polynomials join smoothly or "kiss", hence the name "osculatory". Osculatory interpolation techniques introduce here, a method by *King* and the one of the application of *Spline Models*.

4.8.1 King's Method

This is the method of osculatory interpolation that was proposed by George King (1914), mainly in order to reduce the effects of age misstatements in mortality data. It was applied so to graduate mildly the data and produce some of the published English Life Tables. The method itself is less applicable and largely for historical interest. It requires the assumption of a small effect of age misstatement since it has a small graduating power. The original method is applied separately to the exposed to risk population ${}_nE_x$ and death counts ${}_nd_x$ although it can be applied directly to the mortality probabilities ${}_nq_x$, but only with the assumption of mild amount of error to suffice. If ${}_nE_x$, ${}_nd_x$ do not obtain a typical pattern, then they do not require just a mild graduation. In such case King's technique will apply better to the ${}_nq_x$ - values of the abridged life table. The probability of dying usually demonstrates a typical behavior.

The Method.

Originally the method alternates in the next five steps:

1. The Exposed to risk are grouped into quinquennial age groups.

2. A pivotal exposed to risk value is calculated for the central age of each group, using *King's pivotal value formula*.

3. Graduated Exposed to risk values at the remaining ages are found by osculatory interpolation, using *King's osculatory formula*.

4. Graduated deaths are obtained by applying steps 1 to 3 to the observed deaths.

5. Graduated mortality rates are found by division.

Next the method's two basic tools are presented via mathematical expressions.

A. King's pivotal value formula.

We consider a third degree polynomial u_x and define,

$$w_{-1} = [n]u_{-n};\ w_0 = [n]u_0;\ w_1 = [n]u_n$$

u_x is the value of our function for age x (e.g., ${}_nE_x, {}_nd_x or\ {}_nq_x$).

[n] is a summation operator and it means the *n-term* simple moving average applied on the series of values of u_x.

In general the method uses the notion of simple moving averages applied to a series of function values (here, ${}_nE_x$, ${}_nd_x$ or ${}_nq_x$)

The usual case is **n=5**, which implies the use of quinquennial age groups. So,

$$w_{-1} = u_{-7} + u_{-6} + u_{-5} + u_{-4} + u_{-3}$$
$$w_0 = u_{-2} + u_{-1} + u_0 + u_1 + u_2$$
$$w_1 = u_3 + u_4 + u_5 + u_6 + u_7$$

A formula for u_0 in terms of $w_{-1}, w_0 w_1$ is then considered,

$$w_0 = [n]u_0 = nu_0 + \frac{n(n^2-1)}{24}\Delta^2 u_{-1} \tag{4.3}$$

$$w_{-1} + w_0 + w_1 = [3n]u_0 = 3nu_0 + \frac{3n(9n^2-1)}{24}\Delta^2 u_{-1} \tag{4.4}$$

Where (4.3) is an alternative expression for a simple moving average using the Δ^2 – operator (difference operator). The last denotes the **i-th** differences} of a series of function values.

By subtracting three times equation (4.3) from equation (4.4) we come up with the following,

$$\Delta^2 w_{-1} = n^3 \Delta^2 u_{-1}$$

Solve the above with respect to $\Delta^2 u_{-1}$ and substitute the result to equation (4.5). We finally solve with respect to u_0 and conclude to King's pivotal value formula,

$$u = \frac{1}{n}\left\{ w_0 - \frac{(n^2-1)}{24n^2}\Delta^2 w_{-1} \right\} \tag{4.5}$$

B. King's osculatory interpolation formula.

In order to describe the formulae construction, assume as Benjamin & Pollard (1980) do in their work, that we have four successive values of a function *f(s)* or u_x (the function we want to interpolate, e.g., E_x, d_x, q_x. The four points may be denoted as **A,B,C,D** with that their physical order.

The idea is to fit a quadratic through points **A,B,C,**

$$u_x = (1+\Delta)^{x+1} u_{-1} = u_{-1} + (x+1)\Delta^2 u_{-1}$$

with slope at position **B** equal to,

$$\left[\frac{d}{d_x} u_x\right]_{x=0} = \Delta u_{-1} + \frac{1}{2}\Delta^2 u_{-1}$$

Plus a quadratic through **B,C,D,**

$$u_x = (1+\Delta)^x u_0 = u_0 + x\Delta u_0 + \frac{1}{2}x(x+1)\Delta^2 u_0$$

with slope at point *C* equal to,

$$\left[\frac{d}{d_x} u_x\right]_{x=1} = \Delta u_0 + \frac{1}{2}\Delta^2 u_0 = \Delta u_{-1} + \frac{3}{2}\Delta^2 u_{-1} + \frac{1}{2}\Delta^3 u_{-1}$$

And a cubic through **B,C**:

$$ax^3 + bx^2 + cx + d$$

with gradient,

$$3ax^2 + 2bx + c$$

The pivotal values or the joins of the three polynomials are points B,C. As it is reasonable, it is unable to estimate a function of third degree using two values. We do that by imposing constraints on the slope values of the fitted polynomials.

By equating ordinates and gradients at the pivotal points *B* and *C*, we deduce that:

$$d = u_0 \qquad \text{(ordinate at B)}$$

$$c = \Delta u_{-1} + \frac{1}{2}\Delta^2 u_{-1} \qquad \text{(gradient at B)}$$

$$u_1 = a + b + c + d \qquad \text{(ordinate at C)}$$

$$3a + 2b + c = \Delta u_{-1} + \frac{3}{2}\Delta^2 u_{-1} + \frac{1}{2}\Delta^3 u_{-1} \qquad \text{(gradient at C)}$$

and by solving with respect to **a,b,c,d** we deduce the interpolating formula of King,

$$u_x = u_0 + x\Delta u_{-1} + \frac{x + x^2}{2}\Delta^2 u_{-1} + \frac{x^2 - x^3}{2}\Delta^3 u_{-1}$$

Benjamin & Pollard(1980) describe all the above theoretical considerations via a numerical example.

4.9 Spline Functions

Piecewise polynomials that osculate at specific values known as *knots*, produce a polynomial function called *spline*. They present interesting properties when applied as a smoothing tool but also as an interpolation tool.

CHAPTER 5

SOURCE - DATA.

5.1 Source

In this chapter we provide a description of the empirical data sets used.

We use empirical data sets of the male and female populations of Balkan countries (Hellas-Bulgaria and Romania) during the periods 1975-1995 and Yugoslavia during the period 1975-1990. The original data sets are taken from the University of Thessaly and derive from sources and publish like:

- National Statistical Services of each country,
- Statistical Service of European Union,
- Annual publish international organizations (ONU, Annuaire Demographique and Population prospects 1950-2050),
- European Council.

5.2 Data Sets

In this thesis in order to avoid the influence of age heaping we use empirical data given in five-year age groups. The empirical death frequencies for each age group have been calculated as the ratio of deaths and the exposed-to-risk populations. Especially for the age zero, the death frequency has been calculated as the ratio of deaths at age zero and births. For our study, we have empirical data given in five-year age groups ${}_5q_x$, and the analytical one-year q_x. Departing from the abridged set of values, for each one of the populations, we produce estimates of the one-year probabilities, using the formula:

$$ {}_5q_x = 1 - \prod_{i=0}^{4} (1 - q_{x+i}) . $$

Then we compare the resulting q_x with the empirical ones. As a result, they are different each other, so we decide to use the grouped data. There are two fundamental reasons for providing the data in-groups of ages, rather than by age. The first one is related with the random variations incorporated in the observed data set, while the second one is associated to the systematic sources of errors which affect the data sets mainly those of developing countries. Most mortality empirical schedules, especially those used in actuarial practice and in biostatistical applications but even those used in population analysis in general, are based on limited samples coming from much larger populations. Being the samples insufficiently small, the age specific mortality rates calculated from them are heavily affected by significant random fluctuations forming thus unstable estimates of the true mortality pattern underlying the data. This problem is common in actuarial practice, when the actuary needs to provide analytical and accurate mortality estimations having information based only on a limited sample comprising the policy-holders of a life Office. Demographers face also the same problem. As Brass (1979) mentioned for about 80% of the population of the world the mortality data is extremely scanty, therefore researchers face the problem to provide mortality estimates from very limited and unstable empirical evidence. Considering the data in wider age intervals, often in five-year ones, the size of these random fluctuations is minimized and the estimates of the mortality probabilities are less unstable. Therefore more reliable under the realistic assumption that the random errors affecting the sizes of both the number of deaths and the exposed to risk population for a number of subsequent ages in some extension out each other into each age group. The systematic source of errors effected mortality rates is mainly related to the phenomenon of heaping, i.e. the appearance of local misstatements of age recording in both death and population data. The most typical such misstatements observed in both census and death registration forms is a preference for ages ending in multiples of five and to some extend at the even ages. The common approach to overcome this source of systematic errors is to group the data in five year age groups having a similar motivation as before, i.e. that these over and understatements which affect the age specific data sets will efface each other into the five year age group.

CHAPTER 6

DIACHRONIC ANALYSIS (1975-1995) OF THE AGE SPECIFIC MORTALITY PATTERN BETWEEN THE BALKAN COUNTRIES

6.1 Results

In our analysis, we initially use empirical data given in five-year age groups. Departing from the empirical frequencies of dying given in five-year age groups, we use an expanding technique proposed by Kostaki (1991), in order to estimate the age-specific probabilities of dying. We thus, provide estimations of the graduated age-specific mortality pattern of each country, for the years 1975, 1980, 1985, 1990, 1995 and therefore form complete life tables for each population considered. The reason that we used grouped data instead of the empirical analytical ones is related to the quality of the empirical evidence. An evaluation of the analytical data sets has shown that these are affected of the problem of age heaping. Grouping them in five-year age groups the effect of this problem is eliminated. Then using an expanding technique, one estimates the analytical probabilities of dying. The expanding technique used is a parametric one in the sense that in its frame a parametric model that represents mortality as a function of age, is utilized. In the present work the classical eight-parameter formula of Heligman and Pollard (1980) is used. Many applications of this model have shown that it is efficient for representing the mortality pattern of the total life span. Moreover, each one of the eight parameters incorporated in the model has particular demographic interpretation. Thus, studying the levels as well as the evolution of the parameter values we provide accurate comparisons over space and time. The results of our calculations led us to interesting findings for the evolution of the mortality patterns considered.

We applied the expanding procedure of Kostaki (1991), to the empirical probabilities of dying ${}_5q_x$, $80 \geq x$ for the Balkan countries Hellas – Bulgaria – Romania and Yugoslavia, over the period 1975-1995. The parameters of the formula have been estimated by means of a non-linear least-squares procedure. The H&P algorithm E04FDF, part of the NAG library, was used in order to calculate the unconstrained minimum of the sum of square (3.2.6). Table 1 in the Appendix-A displays the values of (3.2.6) on the exit of the iterative procedure, for males and females when the HP8 model is fitted.

Since the Heligman & Pollard model is a non-linear model of age x a non-linear least squares procedure was adopted in order to estimate the parameters. This is accomplished using an iterative estimation algorithm. E04FDF routine of the NAG library is an easy-to-use algorithm for finding the unconstrained minimum of the supplied sum of squares which is defined here by formula (3.2.6). The applied Heligman & Pollard models require constraints for the parameters. Since this algorithm searches for the unconstrained solution, it is expected to occur in some cases one or more negative parameter values. All parameters are constrained to take non negative values. Neither of our datasets demonstrated such problems. Additionally to the use of the subroutine LSFUN1 in order to calculate the value of the loss function at the exit of the iterative procedure.

In order to provide adequate starting values for the parameters the UNABR procedure of the MORTPAK package is used. In some few cases MORTPAK does not suggest adequate initial values. In such cases the algorithm fails to converge and stops.

In order to obtain estimates for the standard errors of the parameters of HP8, we used E04YCF routine also supplied by the NAG library. The E04YCF routine provides estimates of the elements of the variance-covariance matrix of the estimated parameters. The estimates are derived from the Jacobian of the loss function's value at the given solution. In all cases that we got estimates for the parameters, we didn't have problem to obtain an estimate for their standard errors. A floating point occurred by the algorithm in some cases but we got adequate results for the parameters and for their standard errors. Several applications of the HP8, like Rogers (1986), or Forfar and Smith (1987), suggest that the model is overparameterized. An overparameterized model means that one or more parameters are superfluous and these parameters are proved statistically insignificant. Congdon (1993), considered the statistical stability of the parameters and thoroughly discussed the problem of over parameterization of mortality models while he provided estimates of parameters and their standard errors fitting a nine parameter version of the Heligman and Pollard formula to an empirical data set of Greater London 1980-82. For his calculations he used the SPSS-X package. Additionally Karlis and Kostaki (2001) provide an alternative way for estimating the standard errors of the parameters of mortality models by utilization of a bootstrap approach. This leads to more flexible inference and overcomes problems related to the calculations involved in asymptotic standard errors, described above.

Moreover, confidence bands for the entire curve can be constructed via bootstrap reflecting the uncertainty of the model.

Tables A2-A7 in the Appendix-A show the estimated parameters of HP8 model and their asymptotic standard errors.

The parameter correlations are calculated using the of SPSS statistical package. Table A.8 in Appendix A and Figure B.2 in Appendix B display the noticeable correlation between the pairs of parameters A-B, A-C, B-C and G-H, for the entire data set of life tables. The highly negative correlation between parameters G and H indicate the almost perfect inverse linear relation of them; a feature which could be helpful in forecasting as only one of G and H would need to be extrapolated whereas the other could be estimated from their linear relation. Both G and H consist in the third term of the model, which reflects the rise in mortality in the later adult ages. While the base level of mortality (represented by G) decreases with time, the rate of increase of that mortality (reflected by H) naturally increases.

Let us now use the results of our calculations, in order to describe the evolution of the age specific mortality pattern, of each country.

6.2 Hellas

6.2.1 Population-Health

Like the rest of the European countries at the post-war years, Hellenic population has exhibited a change in its demographic profile, experienced falling fertility and longer life expectancy. In 1997 (the latest empirical data given by the National Statistical Service of Greece) there were 2071 more births than deaths. Total population is expected to continue to grow mainly as a result of immigration. However, as in the other countries of the European Union, the population is ageing.

There is a system of national health and medical care funded through compulsory public health insurance, but many people are covered by additional private insurance for health care or pay for the various medical and hospital private services.

The National Health Service provides free primary care in hospitals and rural health centers. Supplementary care is paid for by the insurance funds. There are a few private hospitals and clinics but legislation passed in 1983 forbids the opening of any new ones. With the exception of a few highly professional private establishments, the standard of care is usually higher in the public sector. However, remuneration in the public sector is low and 'tips' for doctors to secure faster or better treatments are commonplace. There is only a limited number of general practitioners in Hellas. As a result, most care is provided through the outpatient departments of hospitals and clinics or by private specialists. The rural health centers have modern diagnostic equipment but frequently lack technicians to operate it. Most are staffed with generalists. A reform bill introduced in 1997 aims to tackle many of these problems by instituting a countrywide network of primary-care providers, better management of resources and the creation of a public health service to promote preventive medicine.

6.2.2 Comments on Figures

Figures B.3 and B.4 show the age specific mortality probabilities for males and females during 1975 to 1995 over the whole age range. Mortality in Hellas has declined over the last 2 decades. The decline in mortality between the 1975 and 1990 has occurred at all ages and for both sexes, but the extend and timing of improvements has varied. Whilst mortality has declined during the period, the improvements has not occurred uniformly at all ages. During the examined period the decline in mortality probabilities have been more intense for infants and children while relatively lower at the older ages.

Figures B.11 and B.12 show the probabilities of dying, for ages 0 to 15. We can see that the most striking reductions in mortality occurred in infancy (age 0) and childhood (1-9 years of age), where death probabilities declined markedly for both sexes. As expected, for the mortality curve, we have the higher value during 1975. In other words the probability of dying during the first year of life in 1975 is higher than 20 years later.

Figure B.19 and B.20 show the probabilities of dying, for ages 15 to 40. The mortality at the young adult ages as reflected in the accident hump become more pronounced in recent years for both sexes. This is a finding which require further research but may possibly be attributed to the accelerated traffic accident mortality while for females this fact is mainly related to their increased participation in the labor force.

Figures B.27 and B.28 show the probabilities of dying, for ages 40 to 80 in detail. It is observed that, the level of adult and senescent mortality is also decreased by time while the rate of increase of the mortality pattern at these ages becomes higher in recent years as more and more people survive until senescence.

6.2.3 Progression of parameters

Figures B.75-B.82 show the progression of the parameters A to H overtime for both males and females, in Hellas. Using the sets of parameter estimates, we provide comparisons of the mortality patterns through time.

Infant & childhood mortality

It includes three parameters: Parameter A, which reflects the level of infant mortality has fallen by over 50% from 1975 to 1995 indicating that for both males and females infant mortality has declined considerably. Throughout the period males have experienced higher child mortality than females. In most recent years, the parameter A for males and females are converging.

Parameter B is indicative of the mortality level at age zero and C related to the rate of mortality decreases in childhood ages. Parameters B and C, for females, while at the beginning (1975 to 1980), they decrease, then they remain constant from 1980 to 1990, while during the last period (1990-1995), they increase. This might be explained by the fact that, Hellas has reached to very low levels of mortality, during the recent years, naturally some random fluctuations are expected. The estimates of B and C for males remain fairly stable, with a low variation (increase from 1975-1980 and decrease from 1990-1995).

Therefore, the infant and childhood mortality in Hellas has declined over the last two decades.

Young adult ('accident') mortality

The second term in HP8 model represents the 'accident hump'. When we compare the 'accident hump' for males and females, we see that males experienced considerably higher 'accident mortality', with D taking values about four times greater for males than for females. We may say that, the progression of D parameter for both sexes remains about stable.

The increase in E represents the increasing severity of the hump in the curve. The result from the standard error for the middle term parameter E, has shown that (Table A.6), in the case of Hellas female 1975-1980-1995 population, it is too large, to give statistical significance for this parameter.

The location of the hump however for males has remained more or less constant near age 20 (parameter F), while for females overtime we will notice a decrease in the parameter. The location of the accident hump, may be at a younger age for females than for males, during 1995.

The accident hump for the female population of Hellas has obviously been more intense in recent years, a finding related to the greater participation of the females in the labor force.

The accident hump becomes more intense for Hellenic males too in recent years. This phenomenon might be related to the accelerated traffic accidents in Hellas. This finding has also been noticed in other countries e.g. Australia, (Tickle, L.,1996) and Great Britain (Pollard, J.H., 1996).

Senescent mortality

The third term in HP8 represents the ageing of the body (senescent mortality), and its parameters describe the age pattern of mortality at the older ages.

Parameter G reflects the level of later adult mortality and H related to the rate of mortality increases at the later adult ages.

For both males and females the estimates of G behave similarly, i.e. they change smoothly over the examined period. The level of senescent mortality for both sexes declined as the years past.

The values of G for males are higher than for females, throughout the period, indicating higher male mortality throughout the senescent age span.

It is noticeable that H representing the near geometric progression of mortality with age, which has increase for both sexes.

So it seems that diachronic, the level of adult and senescent mortality decreased by time, while the rate of increase of the mortality pattern at these ages becomes higher in recent years, as more and more people survive until senescence.

6.2.4 Expectation of life

Life expectancy at birth is the number of years a new-born infant can be expected to live if prevailing patterns of mortality at the time of its birth remain the same throughout its life. Life expectancy reflects social factors such as health care, disease control, immunization, overall living conditions, and nutrition.

Tables A.9 – A.14 and Fig. B.147 – B.149, show the expectation of life at birth, age 25 and age 65 for males and females based on 1975 to 1995 Hellas mortality rates. For males the life expectancy at birth at 1975 was 72.09 years, increased to 75.1 years at 1995. For females the change has been even more dramatic, the expectation of life having increased from 76.43 years to 83.53 years in 1995.

The progression in life expectancy at age 25 was similar with that at birth. For males the life expectancy at birth at 1975 was 50.04 years, increased to 51.52 years at 1995. For females the change has been even more dramatic, the expectation of life having increased from 53.72 years to 60.38 years in 1995.

The expectation of life at age 65 has also improved for both sexes during the period examined. For males the life expectancy at birth at 1975 was 14.81 years, increased to 16.23 years at 1995. For females the change has been even more dramatic, the expectation of life having increased from 16.74 years to 22.41 years in 1995.

Therefore in Hellas the expectation of life, for both sexes has improved over the last 2 decades, especially during the infant period, with bigger rate for females than males.

6.3 Bulgaria

6.3.1 Population-Health

A national health-care system provides free medical care to all citizens, but facilities are often not well equipped. Private treatment is now also available.

A fallen birth rate and a rising death rate, have combined in recent years to deepen the natural decline in population that has prevailed since the beginning of the 1990's. The causes lie mainly in the various effects of economic hardship:

A high abortion rate and a low marriage rate alike reflect pessimism about future economic conditions, while high infant mortality arises mainly from deficient health-care. Number of health-care personnel declined markedly between 1990 and 1992, under the influence of emigration and shortage of funds, although some recovery took place in 1993 in most categories, and doctors were still considerably more numerous than in 1985. The number of hospital beds, however, increased by 3,2% between 1990 and 1993. High infant mortality rates and falling life expectancy in recent years in part reflect falling standards of health-care, although the fact that infant mortality has fallen from its peak in 1991 presumably reflects recovery from the immediate post-communism crisis.

Until the 1920s, peasants relied on traditional medicine and went to a doctor or hospital only as a last resort. Traditional healers believed that many illnesses were caused by evil spirits and could therefore be treated with magic, with chants against the spirits, with prayers, or by using medicinal herbs. The knowledge of healing herbs was highly valued in village society. For healing one could also drink, wash, or bathe in water from mineral springs, some of which were considered holy. Even in post communist Bulgaria, some resorted to herbal medicine or to persons with reputed extrasensory healing powers. Because of the scepticism of conventional doctors, little research was done on the validity of traditional herbal medicine, but in 1991 doctors began to consider rating skilled herbalists as qualified specialists.

Beginning in 1944, Bulgaria made significant progress in increasing life expectancy and decreasing infant mortality rates. In 1986 Bulgaria's life expectancy was 68.1 years for men and 74.4 years for women. In 1939 the mortality rate for children under one year had been 138.9 per 1,000; by 1986 it was 18.2 per 1,000, and in 1990 it was 14 per 1,000, the lowest rate in Eastern Europe. The proportion of long-lived people in Bulgaria was quite large; a 1988 study cited a figure of 52 centenarians per 1 million inhabitants, most of whom lived in the Smolyan, Kurdzhali, and Blagoevgrad regions.

The steady demographic ageing of the Bulgarian population was a concern, however. In the 1980s, the number of children in the population decreased by over 100,000. The prenatal mortality rate for 1989 was 11 per 1,000, twice those in West European countries. In 1989 the mortality rate for children of ages one to fourteen was twice as great as in Western Europe. The mortality rate for village children was more than twice the rate for city children. However, in 1990 some Bulgarian cities had mortality rates as low as 8.9 per 1,000, which compared favourably with the rates in Western Europe.

Poor conditions in maternity wards and shortages of baby needs worried new and prospective mothers. Hospital staff shortages meant that doctors and nurses were overworked and babies received scant attention. Expensive neonatal equipment was not available in every hospital, and transferral to better-equipped facilities was rare. In 1990 the standard minimum weight to ensure survival at birth was 1,000 grams, compared with the World Health Organization standard of 500 grams.

The number of medical doctors, nurses, and dentists in Bulgaria increased during the 1980s. Bulgaria had 27,750 doctors in 1988, almost 6,000 more than in 1980. This meant one doctor for every 323 Bulgarians. Some 257 hospitals were operating in 1990, with 105 beds per 1,000 people.

Like other aspects of society, health services underwent significant reform after 1989. In 1990 health officials declared that the socialist system of polyclinics in sectors serving 3,000 to 4,000 people did not satisfy the public's need for more complex diagnostic services. They claimed the system was too centralized and bureaucratic, provided too few incentives for health personnel, and lacked sufficient modern equipment and supplies. Thereafter, new emphasis was placed on allowing free choice of a family doctor and providing more general practitioners to treat families on an ongoing basis. Beginning in 1990, Bulgaria began accepting donations of money and medicine from Western countries. During the reform period, even common medicines such as aspirin were sometimes in short supply. Prices for medicines skyrocketed. Shortages of antibiotics, analgesics, dressings, sutures, and disinfectants were chronic.

In November 1989, the Council of Ministers decreed that doctors could be self-employed during their time off from their assigned clinics. Doctors could work for pay either in health facilities or in patients' homes, but with significant restrictions. When acting privately, they could not certify a patient's health or disability, issue prescriptions for free medicine, perform outpatient surgery or abortions, conduct intensive diagnostic tests, use anaesthetics, or serve patients with infectious or venereal diseases. In 1990 the National Assembly extended the right of private practice to all qualified medical specialists, and private health establishments and pharmacies were legalized. Church-sponsored facilities were included in this provision. The 1990 law did not provide for a health insurance system, however, and establishment of such a system was not a high legislative priority for the early 1990s.

In 1991 the government created a National Health Council to be financed by 2.5 billion leva from the state budget plus funds from donors and payments for medical services. The goal of the new council was to create a more autonomous health system. Also in 1991, the Ministry of Health set up a Supreme Medical Council and a Pharmaceutics Council to advise on proposed private health centers, pharmacies, and laboratories and to regulate the supply and distribution of medicine.

In 1988 the top three causes of death in Bulgaria were cardiovascular illnesses, cancer, and respiratory illnesses. An expert estimated that "socially significant diseases" caused 88 percent of all deaths. That resulted from an unhealthy lifestyle and was thus preventable. Strokes, the most prevalent cause of death, killed a higher percentage of the population in Bulgaria than anywhere else did in the world. In 1985 nearly 58,000 Bulgarians suffered strokes, and nearly 24,000 of them died. The mortality rate for strokes was especially high in northern Bulgaria, where it sometimes exceeded 300 fatalities per 100,000 persons. In villages the rate was three times as high as in the cities. Doctors cited unhealthy eating habits, smoking, alcohol abuse, and stress as lifestyle causes of the high stroke rate.

In 1990 about 35 percent of Bulgarian women and 25 percent of men were overweight. Sugar provided an average of 22 percent of the calories in Bulgarian diets, twice as much as the standard for balanced nutrition. Another 35 percent of average calories came from animal fat, also twice as much as the recommended amount. That percentage was likely much higher in the villages, where many animal products were made at home. Modernization of the food supply generally led to increased consumption of carbohydrates and fats. In contrast, the traditional Bulgarian diet emphasized dairy products, beans, vegetables, and fruits. Large quantities of bread were always a key element of the Bulgarian diet. Average salt consumption was also very high. In 1990 the average Bulgarian consumed 14.5 kilograms of bread, 4.4 kilograms of meat, 12.6 kilograms of milk and milk products, 15 eggs, and 15 kilograms of fruits and vegetables per month.

In the 1980s, Bulgaria ranked tenth in the world in per capita tobacco consumption. Tobacco consumption was growing, especially among young people. Each Bulgarian consumed 7.34 liters of alcohol per month, not including huge amounts of home-made alcoholic beverages. Between 1962 and 1982, recorded alcohol consumption increased 1.6 times.

In 1990 an estimated 35 percent of the population risked serious health problems because of environmental pollution. In the most polluted areas, the sickness rate increased by as much as twenty times in the 1980s. By 1990, pollution was rated the fastest-growing cause of "socially significant diseases," particularly for respiratory and digestive disorders. Doctors in the smelting center of Srednogorie found that the incidence of cancer, high blood pressure, and dental disorders had increased significantly in the 1980s.

Pollution had an especially adverse effect on the immune systems of children. In the first few years of the Giurgiu plant's operation, the number of deformed children born across the Danube in Ruse increased 144 percent. From 1985 to 1990, this number increased from 27.5 to 39.7 per 1,000. Miscarriages, stillbirths, and premature, low-weight births doubled during that period. The infant mortality rate in Srednogorie was three times the national average in 1990. Excessive lead in the soil and water at Kurdzhali had caused a great increase in skin and infectious diseases in children there. In 1990 environmental authorities named the village of Dolno Ezerovo, near Burgas, the "sickest village in Bulgaria" because over 60 percent of its children suffered from severe respiratory illnesses and allergies.

In 1987 Bulgarian health authorities instituted limited mandatory testing for human immuno deficiency virus (HIV), which causes acquired immune deficiency syndrome (AIDS). All prospective marriage partners, all pregnant women, and all transportation workers arriving from outside Bulgaria were required to be tested. Hemophiliacs, Bulgarian navy sailors who had traveled abroad after 1982, and students and workers visiting vacation resorts also fell under this rule. As of October 1989, some 2.5 million people in Bulgaria, including about 66,000 foreigners, had been tested for HIV, and 81 Bulgarians were diagnosed as HIV positive. According to government figures, six of that number had contracted AIDS. Foreigners diagnosed as HIV positive were ordered to leave the country. Bulgaria estimated it would spend over US$4 million to treat AIDS and HIV-positive patients in 1991.

6.3.2 Comments on Figures

Figures B.5 and B.6 show mortality rates by age for males and females during 1975 to 1995 over the whole age range. Figure B.13 to B.14 show the rates for ages 0 to 15, Figure B.21 to B.22 show the rates for ages 15 to 40, and Figures B.29 to B.30 shows the rates for ages 40 to 80 in detail. Mortality in Bulgaria has slightly declined over the last two decades for females while this has been in higher levels for males especially at the later adult ages. At the Especially for males the age pattern of mortality at the later adult ages exhibit a higher level by time with declined rate of increase. A curious finding requiring further research is that the accident hump for females is almost disappeared though the Bulgarian females experienced high labor force participation rates throughout the examined period. The diminishing of the accident humps in Bulgaria might be related to the intense emigration from that country.

6.3.3 Progression of parameters

The three components of mortality and their contribution to total mortality are illustrated graphically in Figure B.83-B.90. All the parameters have demographic interpretations.

Infant & childhood mortality

Parameter A has remain constant for males, while for females it has fallen, indicating that only for females infant mortality has declined considerably.

The parameters B and C for males increase from 1975 to 1995. For females the parameters remain fairly stable, with a low variation (decrease from 1975 to 1990 and increase from 1990 to 1995). It is evidence that male mortality has not declined, while female mortality has declined over the period 1975 to 1995.

Young adult ('accident') mortality.

When we compare the 'accident hump' for males and females, it seems that, even though they have the same starting point for D at 1975, then males experienced considerably higher 'accident mortality', with D taking values about three times greater for males than for females. We may say that, the progression of parameter D for both sexes remains about constant, which means that after 1980 we have almost decreasing parallel lines for both sexes.

The increase in E represents the increasing severity of the hump in the curve, its spread. We may say that E remains constant, with higher spread for males than for females. The result from the standard error for the middle term parameter E, has shown that (Table A.6), in the case of Bulgarian males - females populations during 1975, is too large, to give statistical significance for this parameter.

The location of the hump however for both males and females, has remained more or less constant close to 20 years (parameter F). The result from the standard error for the middle term parameter F, has shown that, in the case of Bulgaria females 1975 and 1990 population, it is too large, to give statistical significance for this parameter.

The fact of large standard errors in the middle term parameters of the female populations, is mainly related to the nature of the age specific age pattern of mortality, that exhibits a much lighter accident hump compared with that of the male ones, which on the other hand is very predominant. Moreover, in several cases in the female populations this accident hump becomes nearly non-existent. Therefore, in such cases the addition of one more parameter in the middle term of the model is superfluous, since the contribution of this term to the model fitting is nearly negligible.

It is evidence that the accident hump is almost disappeared from the mortality curves of Bulgarian females at the years 1975, 1990 and 1995.

This is a curious finding requiring further research. The diminishing of the accident humps in Bulgaria might be related to the intense emigration from that country.

Senescent mortality

The values of G for males are higher than for females, throughout the period, indicating higher male mortality throughout the senescent age span. Although the values for females have remained relatively constant, for males they have increased.

It is noticeable that for both sexes, the estimates of H behave similarly: H decreases with time at the same rate, with starting point bigger for females than males.

Therefore, the level of later adult and senescent mortality increases by time for Bulgarian males, while they remain fairly stable for the corresponding female population. The causes possibly lie in the significant lowering of the economic and social status in Bulgaria.

6.3.4 Expectation of life

A measure of mortality that is not distorted by changes in the age structure of the population is the expectation of life. Tables A.9 – A.14 and Fig. B.150 – B.152, show the expectation of life at birth, age 25 and age 65 for males and females. For males the life expectancy at birth at 1975 was 68.61 years, but by 1995 this had decreased to 67.55 years. For females the change has been in the opposite direction, the expectation of life having increased from 73.41 years in 1975 to 75.45 years in 1995.

The progression in life expectancy at age 25 was similar with that at birth. The improvement for females, being 2.5 years throughout the examined period, while for males it has been reduced by roughly two years.

Bulgarian females experienced an improvement in their expectation of life over the last two decades, which is almost entirely due to improvements in the year after birth and for ages 25 and over, with the greatest improvement occurring in the senescent ages.

6.4 Romania

6.4.1 Population-Health

Romania's population has fallen every year since 1990 as a result of a combination of declining birth rates, increasing mortality rates and emigration.

There are many health problems in Romania. As a result of the practice of giving newborn babies blood transfusions if they appeared anemic, a large number of children contracted acquired immune deficiency syndrome (AIDS) from contaminated needles and blood. Many women are also infected. Hepatitis B is also widespread. Illnesses associated with heavy pollution are common. Health facilities are often poorly equipped and understaffed. Large groups of orphaned children do not receive adequate attention or care, and conditions in psychiatric hospitals are often poor. The government has been addressing these problems with foreign assistance.

The age-specific death rates for all age groups over 35 increased between 1990 and 1997, resulting in an increase in the overall death rate from 10.6 per 1000 inhabitants in 1990 to 12.2 in 1997. Although this improved to 11.8 per 1000 inhabitants in 1999, Romania's death rate is one of the higher in Eastern Europe, after Bulgaria, Hungary and Estonia. Male life expectancy has stabilized in the 1990's at just over 73 years. Romania continues to experience high infant and maternal mortality rates, even by the standards of many middle-income developing countries, although some improvement gas been recorded since 1994.

Romania's healthcare system has deteriorated in recent years, as hospitals have lost funding and expert staff as a result of public-spending cuts, and is considerably below the standards of western Europe. Most households are unable to afford private alternatives. This is reflected in the country's infant mortality rate, which remains one of Europe's highest, despite declining from 26.9 deaths per 1000 live births in 1989 to 22.1 per 1000 in 1997. Among east European countries outside the Commonwealth of Independent States (CIS), only Albania and FYROM have equivalent or higher infant mortality rates. The maternal mortality rate, at 60 per 1000 births, is six times that of Poland or Hungary and three times that of Russia, according to UN. Health expenditure accounts for only 3% of total government expenditure (compared with an EU average in 1997 of 7.8%), the lowest proportion in Eastern Europe. In Romania the number of medical staff has been falling: in 1996 there was 46893 physicians (including dentists) compared with 48530 in 1990, giving a ratio of 490 inhabitants per physician. Equipment is inadequate and physical conditions in much of the hospital system are deteriorating.

6.4.2 Comments on Figures

Figures B.7 and B.8 show the probabilities of dying by age for males and females during 1975 to 1995 over the whole age range. Only childhood mortality in Romania has somewhat declined over the last two decades. The accident hump has almost disappeared through time for both sexes. The shape of the mortality curve at ages 15 to 25 for females is more flat than for males, as it was expected, because of the negative accession of the women to the work force. It is evident that male mortality has not declined, at the adult ages.

6.4.3 Progression of parameters

Figures B.91-B.98 show the progression of the parameters A to H overtime for males and females, in Romania.

Infant & childhood mortality

In passing, one will notice that parameter estimates of A, B and C, for both sexes vary much more, as one might expect. Parameter A, has fallen by over 50% from 1975 to 1995 for females, indicating that female child mortality, has declined considerably. The parameter estimates in this model, B and C for both sexes, while at the beginning (1975 to 1985), it seems that they decrease, then in 1990 increases, while during the recent years, they decrease.

The interpretation of these results is not clear. It seems that the level of infant mortality decreases by time. We have noticed an unusually high value of infant-childhood mortality, in 1990. We can explain that by the social-economic problems Romania has faced during this period.

Young adult ('accident') mortality.

The middle term reflects accident mortality and it includes three parameters. Parameter D related to the severity of the accident mortality for males and females remains about stable. The result from the standard error for the middle term parameter D, has shown that for Romanian female population in 1995, is too large, to give statistical significance for this parameter.

The parameter E related to the spread of the accident hump for females remains stable.

The result from the standard error for the middle term parameters E, has shown that for females during 1995 and for males over the period 1980 to 1995, is too large, to give statistical significance for this parameter.

The location of the hump however, has remained more or less constant close to the age of 20 years, (parameter F). The result from the standard error for the middle term parameters F, has shown that for Romania of males and females during 1995 is too large, to give statistical significance for this parameter. It is evident that the accident hump of the two Romania populations becomes disappeared at recent years.

The diminishing of the accident humps in Romania might be related to the intense emigration from that country.

Senescent mortality

The parameter estimates G in this model for males are higher than for females, throughout the period, indicating higher male mortality, throughout the senescent age span. The values for females have remained relatively constant, while for males they have increased.

The estimates of H for males, have fallen over the period 1975-1995, while for females, they have remain almost stable.

An unusual though plausible finding is that the levels of later adult and senescent mortality increase by time for Romanian males, while they remain fairly stable for the corresponding female populations. The causes possibly lie in the significant lowering of the economic and social status Romania.

6.4.4 Expectation of life

Tables A.9 – A.14 show the expectation of life at birth, age 25 and age 65 for males and females. In order to assist interpreting the results, it is helpful to construct a graph of the expectation of life at each combination. This graph is shown in Figures B.153-B.155. For males the life expectancy at birth at 1975 in Romania was 67.68 years, but by 1995 this had decreased to 65.58 years. For females the expectation of life has been increased from 72.83 years in 1975, to 73.56 years in 1995.

The life expectancy at age 25 for females, has remain constant, when for males it has reduced over this period. It is evident that male mortality has not declined, when females mortality has remain constant, over the period 1975 to 1995.

6.5 Yugoslavia

6.5.1 Population-Health

Despite significant decreases in mortality in recent decades, the level and structure of mortality of the Yugoslav population remains much less favorable than in developed countries. According to data for 1990-91, life expectancy was 69 for males and 74 for females (in contrast to 53.5 and 56 respectively in the 1950's); 50% of the post war increase is accounted for by the 1950/51-1960/61 period. The infant mortality rate has been decreasing throughout the post war period from 117.2 per 1000 in 1950 to 14.3 per 1000 in 1996. Nevertheless, this compares poorly with the infant death rate in most European countries. There was an increase in the infant mortality rate in 1992-1995, when it rose from about 21 per 1000 in 1991 to 23.7 per 1000 in 1995. This is explained by factors associated with the introduction of international sanctions and the economic deterioration of the country. Besides Yugoslavia, in Europe such high rates prevailed only in Albania and Romania.

Inter war Yugoslavia was noted for the endemic presence of malaria, typhus, typhoid, syphilis, dysentery, and trachoma. By the 1980s, these scourges had been reduced to individual cases. Still, Serbia and Montenegro suffer from significant health problems. Before the civil unrest of the 1990s, infant mortality was more than 63 per 1,000 live births in Kosovo and averaged 35 per 1,000 throughout Serbia and 22 per 1,000 in Montenegro. Only the Vojvodina, with an infant death rate of 12 per 1,000, approached standards of central and Western Europe.

Despite marked improvements in medical services, Yugoslavia's population suffers from crowded housing conditions, poor nutrition, and lack of sanitary services.

The communist regime introduced a health insurance program in 1945. Currently, pregnant women, infants, and children up to age 15 receive complete health care, as do students up to age 26. All citizens also are entitled to treatment for infectious diseases and mental illness. Still, about one-fifth of the population remains outside the health care system.

The regime has placed great emphasis on training doctors. Before World War II, one doctor served every 12,000 inhabitants in Yugoslavia. In 1990 there was one doctor for every 400 residents of Serbia and one for every 530 residents of Montenegro. Kosovo was the most poorly served region, with one physician for every 900 residents in 1990.

An extensive national health system covers the rural and urban populations. Private practice was legalized in 1990, and a growing number of doctors, dentists, and nurses now choose to operate in the private sector. However, most people cannot afford private medical or dental treatment, and public hospitals are often short of supplies and equipment. In general, the country has experienced a lowering of health standards. With food shortages, the average calorie intake of children has dropped from 3,200 to 2,100 per day.

6.5.2 Comments on Figures

Figures B.9 and B.10 show the probabilities of dying by age for males and females during 1975 to 1990 over the whole age range. During this period the mortality pattern exhibit a reduction for the whole life span. The accident hump of females that was intense at 1980 and 1985, becomes disappeared at 1990.

6.5.3 Progression of parameters

Figures B.99-B.106 show the progression of the parameters A to H overtime for both males and females, in Yugoslavia.

Infant & childhood mortality

At a glance, one will notice that the parameter A has fallen by over 50% from 1975 to 1990 indicating that for both males and females infant mortality has declined considerably.

The parameter B, which is indicative of the rate of infant mortality while at the beginning (1975 to 1980), it seems that we have a fallen, then we have an increase from 1980 to 1985, and during the recent years, it has declined.

The estimates of C for males and females remain fairly stable, with a low variation: decrease from 1975-1980, then increase from 1980-1985 and during the recent years decrease.

It is an evident that in Yugoslavia the infant and childhood mortality, over the period 1975 to 1990, has declined.

Young adult ('accident') mortality.

When we compare the 'accident hump' for males and females, we see that males experienced considerably higher 'accident mortality', with D taking values about two times greater for males than for females. We may say that, parameter D for both sexes reduces with almost the same rate.

The result from the standard errors of all estimates of the parameter E, has shown that in Yugoslavia it is too large (larger than the half the corresponding estimated values of the parameters), to give statistical significance for this parameter.
The diminishing of the accident humps in Yugoslavia might be related to the intense emigration from these countries.

Senescent mortality

For both males and females the estimates of G behave similarly, i.e. they change smoothly as the level of high life mortality changes.
The estimates of G for males are higher than for females, throughout the period, indicating higher male mortality throughout the senescent age span.
Parameter H for both sexes males and females it has fallen at the beginning from 1975-1980, while at the period 1980-1990 it remains constant.

As expected in Yugoslavia, the senescent mortality remains stable during the last 2 decades, with bigger level for males,.

<u>6.5.4 Expectation of life</u>

Tables A.9 – A.14 and Fig. B.156 – B.158, show the expectation of life at birth, age 25 and age 65 for males and females. For males the life expectancy at birth on 1975 – 1990 data for Yugoslavia was 66.98 years, but by 1990 this has increased to 69.4 years. For females the change has been even more dramatic, the expectation of life having increased from 71.77 years to 75.65 years in 1990.

The progression in life expectancy at age 25 was similar with that at birth. The improvement for females, being two years, when for males it is 0.5 years. It is evident that in Yugoslavia mortality has declined especially for, over the period 1975 to 1990.

<u>6.6 Conclusions on Comparisons over time</u>

Like the rest of the European countries at the postwar years, Hellenic population has exhibited a change in its demographic profile, experienced falling fertility and longer life expectancy. In 1997 (the latest empirical data given by the National Statistical Service of Greece) there were 2071 more births than deaths. Total population is expected to continue to grow mainly as a result of immigration. However, as in the other countries of the European Union, the population is aging.

A fallen birth rate and a rising death rate, have combined in recent years to deepen the natural decline in population that has prevailed since the beginning of the 1990's. The causes lie mainly in the various effects of economic hardship:
A high abortion rate and a low marriage rate alike reflect pessimism about future economic conditions, while high infant mortality arises mainly from deficient health-care. Number of health-care personnel declined markedly between 1990 and 1992, under the influence of emigration and shortage of funds, although some recovery took place in 1993 in most categories, and doctors were still considerably more numerous than in 1985. The number of hospital beds, however, increased by 3,2% between 1990 and 1993. High infant mortality rates and falling life expectancy in recent years in part reflect falling living standards and standards of health-care, although the fact that infant mortality has fallen from its peak in 1991 presumably reflects recovery from the immediate post-communism crisis.

Romania's population has fallen every year since 1990 as a result of a combination of declining birth rates, increasing mortality rates and emigration.

There are many health problems in Romania. As a result of the practice of giving newborn babies blood transfusions if they appeared anemic, a large number of children contracted acquired immune deficiency syndrome (AIDS) from contaminated needles and blood. Many women are also infected. Hepatitis B is also widespread. Illnesses associated with heavy pollution are common. Health facilities are often poorly equipped and understaffed. Large groups of orphaned children do not receive adequate attention or care, and conditions in psychiatric hospitals are often poor. The government has been addressing these problems with foreign assistance. Romania continues to experience high infant and maternal mortality rates, even by the standards of many middle-income developing countries.

Romania's healthcare system has deteriorated in recent years, as hospitals have lost funding and expert staff as a result of public-spending cuts, and is considerably below the standards of western Europe. Most households are unable to afford private alternatives. This is reflected in the country's infant mortality rate, which remains one of Europe's highest, despite declining from 26.9 deaths per 1000 live births in 1989 to 22.1 per 1000 in 1997. Among east European countries outside the Commonwealth of Independent States (CIS), only Albania and FYROM have equivalent or higher infant mortality rates. The maternal mortality rate, at 60 per 1000 births, is six times that of Poland or Hungary and three times that of Russia, according to UN. Health expenditure accounts for only 3% of total government expenditure (compared with an EU average in 1997 of 7.8%), the lowest proportion in Eastern Europe. In Romania the number of medical staff has been falling: in 1996 there was 46893 physicians (including dentists) compared with 48530 in 1990, giving a ratio of 490 inhabitants per physician. Equipment is inadequate and physical conditions in much of the hospital system are deteriorating.

Despite significant decreases in mortality in recent decades, the level and structure of mortality of the Yugoslav population remains much less favorable than in developed countries. The infant mortality rate has been decreasing throughout the post war period from 117.2 per 1000 in 1950 to 14.3 per 1000 in 1996. Nevertheless, this compares poorly with the infant death rate in most European countries. There was an increase in the infant mortality rate in 1992-1995, when it rose from about21 per 1000 in 1991 to 23.7 per 1000 in 1995. This is explained by factors associated with the introduction of international sanctions and the economic deterioration of the country. Besides Yugoslavia, in Europe such high rates prevailed only in Albania and Romania. Inter war Yugoslavia was noted for the endemic presence of malaria, typhus, typhoid, syphilis, dysentery, and trachoma. By the 1980s, these scourges had been reduced to individual cases. Still, Serbia and Montenegro suffer from significant health problems. Before the civil unrest of the 1990s, infant mortality was more than 63 per 1,000 live births in Kosovo and averaged 35 per 1,000 throughout Serbia and 22 per 1,000 in Montenegro. Only the Vojvodina, with an infant death rate of 12 per 1,000, approached standards of central and Western Europe.

Mortality in Hellas has declined over the last two decades. The decline in mortality between the 1975 and 1990 has occurred at all ages and for both sexes, but the extend and timing of improvements has varied. Whilst mortality has declined during the period, the improvements has not occurred uniformly at all ages. During the examined period the decline in mortality probabilities have been more intense for infants and children while relatively lower at the older ages. We can see that the most striking reductions in mortality occurred in infancy (age 0) and childhood (1-9 years of age), where death probabilities declined markedly for both sexes.

The accident hump for the female population of Hellas has obviously been more intense in recent years, a finding related to the greater participation of the females in the labor force.

The accident hump becomes more intense for Hellenic males too in recent years. This phenomenon might be related to the accelerated traffic accidents in Hellas. This finding has also been noticed in other countries e.g. Australia, (Tickle, L.,1996) and Great Britain (Pollard, J.H., 1996).

The expectation of life at birth, age 25 and age 65 in Hellas, has improved for both sexes during the period examined.

Mortality in Bulgaria has slightly declined over the last two decades for females while this has been in higher levels for males especially at the later adult ages. At the Especially for males the age pattern of mortality at the later adult ages exhibit a higher level by time with declined rate of increase. A curious finding requiring further research is that the accident hump for females is almost disappeared though the Bulgarian females experienced high labor force participation rates throughout the examined period. The accident hump is almost disappeared from the mortality curves of Bulgarian females at the years 1975, 1980, 1985 and 1990 being slight at 1995.

Only childhood mortality in Romania has somewhat declined over the last two decades. The accident hump has almost disappeared through time for both sexes. The shape of the mortality curve at ages 15 to 25 for females is more flat than for males, as it was expected, because of the negative accession of the women to the work force. It is also observed that the mortality pattern for males strongly increases by time at the adult ages. The accident hump of the two Romanian populations becomes disappeared at recent years.

An unusual though plausible finding is that the levels of later adult and senescent mortality increase by time for Bulgarian and Romanian males, while they remain fairly stable for the corresponding female populations. The causes possibly lie in the significant lowering of the economic and social status in the two countries especially in Romania.

During this period the mortality pattern exhibit a reduction for the whole life span, in Yugoslavia.

The accident hump of Yugoslavian females that was intense at 1980 and 1985 becomes disappeared at 1990.

The diminishing of the accident humps in Bulgarian, Romanian and Yugoslavian populations might be related to the intense emigration from these countries.

The estimates of G for males are higher than for females, throughout the period, indicating higher male mortality throughout the senescent age span.

CHAPTER 7

COMPARISON OF THE AGE SPECIFIC MORTALITY PATTERN BETWEEN THE BALKAN COUNTRIES

7.1 Comparison of the age specific mortality pattern between the Balkan countries in 1975.

7.1.1 Comments on Figures

Figures B.35 and B.36 show the probabilities of dying by age for males and females during 1975 over the whole age range. In general, the probability for someone to die, in Hellas is smaller than the other countries. We have the lowest values for Hellas, then for Bulgaria, Romania and Yugoslavia.

Figure B.45 and B.46 show the probabilities of dying for ages 0 to 15. For the mortality curve, we have the lowest value for Hellas, then for Bulgaria, Yugoslavia and Romania. In other words, the probability for someone to die during the first year of life is higher in Romania than in Yugoslavia, Bulgaria or in Hellas.

Figure B.55 and B.56 show the probabilities of dying for ages 15 to 40. The mortality patterns for young adults are generally lower in Hellas. As expected, the shape of the mortality curve at ages 15 to 25 for females during 1975 is more flat than for males.

As can be seen from Figures 55 and 56 of Appendix B, the accident hump that corresponds to the early adult ages, does not appear. Thus the middle term of the model (parameters D, E and F) is unnecessary and superfluous in this case and to that extent the model is overparameterized.

Figures B.65 and B.66 show the probabilities of dying for ages 40 to 80 in detail. As expected, the mortality patterns for females are generally lower than males. In general, the probability for someone to die in Hellas is smaller than the other countries. We have the lowest value for Hellas, then for Bulgaria, Romania and Yugoslavia.

7.1.2 Progression of parameters in 1975

Figures B.107-B.114 show the progression of the parameters A to H during 1975, for all countries.

Infant & childhood mortality

The parameters A, B and C describe the pattern of mortality during the infant and childhood ages. It is clear from the plot of the parameters A, B and C, that in 1975, Romania has a score which is substantially higher than that in the other countries, indicating that for both males and females infant mortality has declined considerably from country to country.

Therefore, the country with the lowest value of infant mortality during 1975 is Hellas then Bulgaria, Yugoslavia and Romania.

Young adult ('accident') mortality.

The parameter estimates D, E and F in HP8 model represent the 'accident hump'. When we compare the 'accident hump' for males and females, we see that males experienced considerably higher 'accident mortality', with D taking higher values for males than for females. The progression of parameter D between the countries, for females, vary much more than for males.

The result from the standard error for the middle term parameter E, has shown that it does not, give statistical significance for this parameter for Yugoslavia females.

The location of the hump however for males has remained more or less constant near age 20 (parameter F), for all countries, except Yugoslavia, Bulgaria and Hellas females which are not statistical significant.

Senescent mortality

The third term in HP8 represents the ageing of the body (senescent mortality), and its parameters describe the age pattern of mortality at the older ages.

The values of G for males are higher than for females, indicating higher male mortality than for females, throughout the senescent age span.

The values for females have remained relatively stable, while for males we have the lowest value for Hellas then for Bulgaria – Romania and Yugoslavia, indicating that the level of senescent mortality differs from country to country.

It is noticeable that H representing the near geometric progression of mortality with age, has remained relatively stable for males, while for females we have the higher value for Bulgaria - Hellas then for Yugoslavia and Romania.

7.1.3 Expectation of life

Tables A.9 – A.14 and Fig. B.159 – B.161, show the expectation of life at birth, age 25 and age 65 for males and females based on 1975 Hellas – Bulgaria – Romania – Yugoslavia mortality rates. For males the life expectancy at birth in 1975 data in Hellas was 72.09 years, while in Bulgaria it was 68.61, in Romania 67.68 and Yugoslavia 66.98 years. The expectation of life respectively for females in Hellas was 76.43 years, in Bulgaria 73.41 years, in Romania 72.83 years, in Yugoslavia 71.77 years.

The progression in life expectancy at age 25 was similar with that at birth. For males the life expectancy at age 25 on 1975 data for Hellas was 50.04 years, while in Bulgaria it was 46.6, in Romania 46.95 and Yugoslavia 46.22 years. For females, the expectations of life in Hellas was 53.72 years, in Bulgaria 50.66 years, in Romania 51.46 years and Yugoslavia 50.6 years.

For males the life expectancy at age 65 on 1975 data for Hellas was 14.81 years, while in Bulgaria it was 12.76, in Romania 13.44 and Yugoslavia 12.75 years. For females, the expectation of life in Hellas was 16.74 years, in Bulgaria 14.35 years, in Romania 15.92 years and in Yugoslavia 14.7 years.

7.2 Comparison of the age specific mortality pattern between the Balkan countries in 1980

7.2.1 Comments on Figures

In order to assist in interpreting the results, it is helpful to construct a graph of the mortality patterns by age for males and females during 1980, over the whole age range. These graphs are shown in Figures B.37 and B.38. In general, the probability for someone to die, in Hellas is smaller than the other countries.

Figure B.47 and B.48 show the probabilities of dying for ages 0 to 15. From inspection of these graphs, we see that, we have the lowest value for Hellas, then for

Bulgaria, Yugoslavia and last Romania. The interpretation of these results is that, the probability of dying during the first year of life is higher in Romania than Yugoslavia-Bulgaria and Hellas. We can see that the most striking reductions in mortality occurred in infancy (age 0) and childhood (1-9 years of age), where mortality patterns declined markedly and remarkably uniformly for both sexes.

Figures B.57 and B.58 show the probabilities of dying for ages 15 to 40. The mortality patterns for young adults are generally lower in Hellas. As expected, the shape of the mortality curve at ages 15 to 25 for females during 1980 is more flat than for males.

Figures B.67 and B.68 show the probabilities of dying for ages 40 to 80 in detail. As expected, the mortality patterns for females are generally lower than males. In general, the probability for someone to die in Hellas is smaller than the other countries. We have the lowest value in Hellas, then for Bulgaria, Yugoslavia and Romania.

7.2.2 Progression of parameters in 1980

Figures B.115-B.122 show the progression of the parameters A to H during 1980, for all countries.

Infant & childhood mortality

At a glance, one will notice that we have the lowest values of the parameter A, for Romania then for Yugoslavia - Bulgaria and Hellas, indicating that for both males and females infant mortality has declined considerably from country to country. As can be seen, the parameter estimates for both males and females are converging.

The values of parameter C for both sexes are lower for Romania then Yugoslavia - Bulgaria and Hellas, indicating that for both males and females childhood mortality vary from country to country.

We might thus conclude that, the country with the lower infant mortality during 1980 is Hellas, then Bulgaria-Yugoslavia and Romania.

Young adult ('accident') mortality.

From inspection of the 'accident hump' for males and females, we see that males experienced considerably higher 'accident mortality', with D taking higher values for males than for females.

The result from the standard error for the middle term parameter E, has shown that it does not, give statistical significance for this parameter for Hellas, Yugoslavia females and Romania males.

The location of the hump however for males has remained more or less constant near age 20-25 (parameter F), for all countries, taking values higher for females than for males.

Senescent mortality

The estimates of G for males are higher than for females, throughout the period, indicating higher male mortality throughout the senescent age span. We have the lowest values for Hellas then for Bulgaria - Yugoslavia and Romania, indicating that the level of older people mortality differs from country to country.

For the parameter H, we have the highest values for Hellas then for Bulgaria - Yugoslavia and Romania.

Therefore the probability for someone to die in Hellas is smaller than the other countries. We have the lowest value for Hellas then for Bulgaria, Romania and Yugoslavia.

7.2.3 Expectation of life

Tables A.9 – A.14 and Fig. B.162 – B.164, show the expectation of life at birth, age 25 and age 65 for males and females based on 1980 Hellas – Bulgaria – Romania – Yugoslavia mortality rates. For males the life expectancy at birth at 1980 data in Hellas was 73.13 years, while in Bulgaria it was 68.44, in Romania 67.04 and last Yugoslavia 67.81 years. Respectively the expectations of life for females in Hellas was 77.72 years, in Bulgaria 74.1 years, in Romania 72.87 years, in Yugoslavia 73.49 years.

The progression in life expectancy at age 25 was similar with that at birth. For Hellas males the life expectancy was 50.52 years, while in Bulgaria it was 46.2, in Romania 45.77 and Yugoslavia 46.25 years. For females, the expectations of life in Hellas was 54.53 years, in Bulgaria 51.17 years, in Romania 50.87 years and Yugoslavia 51.58 years.

For males the life expectancy at age 65 for Hellas was 15.14 years, while in Bulgaria it was 12.68, in Romania 13.29 and Yugoslavia 12.93 years. For females, the expectation of life in Hellas, was 17.36 years, in Bulgaria 14.96 years, in Romania 15.58 years and in Yugoslavia 15.66 years.

Therefore, a female in 1980 would expect to live slight more than a male and for someone in Hellas would expect to live slight more than anyone who lives in Bulgaria or Romania, or Yugoslavia.

7.3 Comparison of the age specific mortality pattern between the Balkan countries in 1985

7.3.1 Comments on Figures

Figures B.39 and B.40 show the probabilities of dying by age for males and females during 1985 over the whole age range. In general, the probability for someone to die, in Hellas is smaller than the other countries.

Figure B.49 and B.50 show the mortality patterns for ages 0 to 15. For the mortality curve, we have the lowest value for Hellas, then for Bulgaria, Yugoslavia and Romania. This suggests that the probability for someone to die during the first year of life in Romania is higher than in Yugoslavia, Bulgaria or Hellas. We can see that the most striking reductions in mortality occurred in infancy (age 0) and childhood (1-9 years of age), where mortality patterns declined markedly and remarkably uniformly for both sexes.

Figures B.59 and B.60 show the probabilities of dying for ages 15 to 40. The shape of the mortality curve at ages 15 to 25 for Bulgaria – Romania females in 1985, is more flat than for males.

Figures B.69 and B.70 show the mortality patterns for ages 40 to 80 in details. As expected, the mortality patterns for females are generally lower than males. In general, the probability for someone to die, in Hellas is smaller than the other countries. We have the lowest value for Hellas, then for Bulgarian, Yugoslavia and Romania.

7.3.2 Progression of parameters in 1985

Figures B.123-B.130 show the progression of the parameters A to H during 1985, for all countries.

Infant & childhood mortality

From inspection of these Figures, one will notice that the estimates of A, B and C change at the same way for both sexes. We have the highest values of parameter A, in Romania then Yugoslavia, Bulgaria and Hellas, for both sexes, indicating that for both males and females infant mortality has declined considerably from country to country. As can be seen, the parameter estimates for both males and females are converging. The parameters B-C in Romania are higher than in Bulgaria, Yugoslavia and Hellas. Therefore the country with the lowest level of infant mortality during 1985, is Hellas-Bulgaria, then Yugoslavia and Romania.

Young adult ('accident') mortality.

The values of parameter D for males are lower for Hellas then Bulgaria, Yugoslavia and Romania. For females we have the highest values for Romania then for Bulgaria - Yugoslavia and Hellas.
The result from the standard error for the middle term parameter E, has shown that it does not, give statistical significance for this parameter in Yugoslavia females and Romania males.
The location of the hump however for males has remained more or less constant near age 20-25 (parameter F), for all countries, taking almost higher values for females than for males.

Senescent mortality

The estimates of parameter G for males are higher than for females, indicating higher male mortality throughout the senescent age span. We have the lowest values for Hellas then for Bulgaria - Yugoslavia and Romania, indicating that the level of senescent mortality differs from country to country.

The values of parameter H, for both males and females are higher for Hellas then Bulgaria - Yugoslavia and Romania, indicating that the rate of increase of the older people mortality differs from country to country.

Therefore, the country with the lowest level of senescent mortality during 1985, is Hellas then Bulgaria - Yugoslavia and Romania.

7.3.3 Expectation of life

Tables A.9 – A.14, show the expectation of life at birth, age 25 and age 65 for males and females. To assist in interpreting the results, it is helpful to construct a graph of the expectation of life, at each combination. These graphs are shown in Figures B.165 – B.167. The life expectancy at birth on 1985 data for Hellas males, was 73.42 years, while in Bulgaria it was 68.14, in Romania 66.89 and in Yugoslavia 68.06 years. Respectively, the expectation of life for Hellas females, was 78.88 years, in Bulgaria 74.44 years, in Romania 73.28 years and in Yugoslavia 74.12 years.

The results indicate that, the life expectancy at age 25 on 1985 data for Hellas males, was 50.46 years, while in Bulgaria it was 45.4, in Romania 45.3 and in Yugoslavia 46.21 years. The expectation of life for Hellas females, was 54.25 years, in Bulgaria 51.5 years, in Romania 51.08 years and in Yugoslavia 52.64 years.

The life expectancy at age 65 on 1985 data for Hellas males, was 15.15 years, while in Bulgaria it was 12.76, in Romania 13.44 and in Yugoslavia 12.75 years. The expectation of life for Hellas females, was 17.97 years, in Bulgaria 15.05 years, in Romania 15.57 years and in Yugoslavia 15.87 years.

Therefore, a female in 1985 would expect to live slight more than a male. For someone in Hellas would expect to live slight more than anyone does whom lives in Bulgaria or Romania, or Yugoslavia.

7.4 Comparison of the age specific mortality pattern between the Balkan countries in 1990

7.4.1 Comments on Figures

Figures B.41 and B.42 show the age specific mortality patterns for males and females in 1990, over the whole age range. As expected Hellas experience much lower levels of mortality than the other Balkan countries while the gap between the level of the mortality patterns of Hellas and the other three countries has been larger than in 1975. Now Romania experiences the highest mortality levels than the other countries. Figures B.51 and B.52 show the probabilities of dying for ages 0 to 15. For the mortality curve, we have the lowest value for Hellas, then for Bulgaria-Yugoslavia and Romania. In other words the probability for someone to die during the first year of life is higher in Romania than Yugoslavia, Bulgaria or Hellas. We can see that the most striking reductions in mortality occurred in infancy (age 0) and childhood (1-9 years of age), where the probabilities of dying declined markedly and remarkably uniformly for both sexes.

Figures B.61 and B.62 show the probabilities of dying for ages 15 to 40. The shape of the mortality curve at ages 15 to 25 for females during 1990 is more flat than for males.

Figures B.71 and B.72 show the rates for ages 40 to 80 in detail. As expected the mortality rates for females are generally lower than males. In general, the probability for someone to die, in Hellas is smaller than the other countries. We have the lowest value for Hellas, then for Bulgaria, Yugoslavia and Romania.

7.4.2 Progression of parameters in 1990

Figures B.131-B.138 show the progression of the parameters A to H in 1990, for both males and females.

Infant & childhood mortality

At a glance, one will notice that the values of parameter A for both sexes are higher for Romania then Yugoslavia - Bulgaria and Hellas, indicating that for both males and females childhood mortality has declined considerably from country to country.
The values of Parameter B are lower for Hellas then Bulgaria - Yugoslavia and Romania.
The values of parameter C, which measures the rate of childhood mortality, are higher for Hellas then Bulgaria-Yugoslavia and Romania.
Therefore the country with the lowest level of infant mortality during 1990 is Hellas then Bulgaria, Yugoslavia and Romania.

Young adult ('accident') mortality.

The result from the standard error for the middle term parameter E, has shown that it does not, give statistical significance for this parameter in Yugoslavia males-females, and Romania males. Parameter E takes higher values for males than for females.
The location of the hump however for males has remained more or less constant near age 20 (parameter F), for males, while for females takes almost higher values. The result from the standard error for the middle term parameter F, has shown that it does not, give statistical significance for this parameter for Bulgaria females

Senescent mortality

The parameter estimates of G for males are higher than for females, throughout the period, indicating higher male mortality throughout the senescent age span. The values for both sexes are lower for Hellas then for Bulgaria - Yugoslavia and Romania, indicating that the level of older people mortality differs from country to country.
The values of parameter H for both males and females are lower for Hellas then for Bulgaria - Yugoslavia and Romania.

Therefore, the country with the lowest level of senescent mortality during 1990 is Hellas then Bulgaria - Yugoslavia and Romania.

7.4.3 Expectation of life

Tables A.9 – A.14 and Fig. B.168 – B.170, show the expectation of life at birth, age 25 and age 65 for males and females based on 1990 Hellas – Bulgaria – Romania – Yugoslavia mortality rates. The life expectancy at birth on 1990 data for Hellas males, was 74.8 years, in Yugoslavia 69.4 years, in Bulgaria 68.01, in Romania 67.18, while respectively the expectation of life for Hellas females, was 79.75 years, in Yugoslavia 75.65 years, in Bulgaria 74.86 years, in Romania 73.58 years.

The progression in life expectancy at age 25 was similar with that at birth. The life expectancy for Hellas males, was 51.35 years, while in Yugoslavia 46.72 years, in Bulgaria it was 45.33, and in Romania 45.48. The expectation of life for Hellas females, was 55.89 years, in Yugoslavia 52.64 years in Bulgaria 51.5 years, in Romania 51.08 years.

The life expectancy at age 65 on 1990 data for Hellas males, was 15.91 years, while in Bulgaria it was 12.78, in Romania 13.67 and last in Yugoslavia 13.17 years. The expectation of life for Hellas females, was 18.32 years, in Yugoslavia 16.41 years, in Bulgaria 15.3 years, in Romania 15.42 years.

7.5 Comparison of the age specific mortality pattern between the Balkan countries in 1995

7.5.1 Comments on Figures

Figures B.43 and B.44 show the mortality patterns by age for males and females during 1995 over the whole age range. In general, the probability for someone to die, in Hellas is smaller than the other countries.

Figure B.53 and B.54 show the mortality patterns for ages 0 to 15. For the mortality curve, we have the lowest value in Hellas, then Bulgaria, and Romania. In other words the probability of dying during the first year of life is higher in Romania than Bulgaria or Hellas. We can see that the most striking reductions in mortality occurred in infancy (age 0) and childhood (1-9 years of age), where the mortality patterns, declined markedly and remarkably uniformly for both sexes.

Figure B.63 and B.64 show the probabilities of dying for ages 15 to 40. The mortality patterns for young adult are generally lower in Hellas. The shape of the mortality curve at ages 15 to 25 for females during 1975 is more flat than for males.
Figures B.73 and B.74 show the mortality patterns for ages 40 to 80 in detail. As expected, the mortality patterns for females are generally lower than males. In general, the probability for someone to die, in Hellas is smaller than the other countries. We have the lowest values for Hellas, then for Bulgaria and Romania.

7.5.2 Progression of parameters in 1995

Figures B.139-B.146 show the progression of the parameters A to H during 1995, for Hellas-Bulgaria-Romania.

Infant & childhood mortality

At a glance, one will notice that, the values of parameter A are higher for Romania then for Bulgaria and Hellas, for both sexes, indicating that for both males and females child mortality has declined considerably from country to country.
The values of parameter B are lower for Hellas then for Romania and Bulgaria.
The country with the lowest level of infant mortality during 1995 is Hellas then Bulgaria and Romania.

Young adult ('accident') mortality.

We can say that, parameter D for both males-females remains stable. The result from the standard error for the middle term parameter D, has shown that it does not, give statistical significance for this parameter in Romania females.
The result from the standard error for the middle term parameter E, has shown that it does not, give statistical significance for this parameter for all countries.
The location of the hump however for males has remained more or less constant near age 20 (parameter F), for males-females. The result from the standard error for the middle term parameter F, has shown that it does not, give statistical significance for this parameter for Romania males-females

The accident hump of the Bulgarian, Romanian and Yugoslavian populations becomes disappeared at recent years.

The diminishing of the accident humps might be related to the intense emigration from these countries.

Senescent mortality

The estimates of G for males are higher than for females, throughout the period, indicating higher male mortality throughout the senescent age span. The estimates for females have remained relatively constant, while for males are lower in Hellas then to Bulgaria and Romania, indicating that the level of male senescent mortality differs from country to country.

The level of female senescent mortality is the same for all the countries.

7.5.3 Expectation of life

Tables A.9 – A.14 and Fig. B.171 – B.173, show the expectation of life at birth, age 25 and age 65 for males and females based on 1995 Hellas – Bulgaria – Romania mortality rates. For males the life expectancy at birth on 1995 data for Hellas was 75.1 years, while in Bulgaria it was 67.55, in Romania 65.58 years, while the corresponding expectation of life for females in Hellas was 83.53 years, in Bulgaria 75.45 years, and in Romania 73.56 years.

The progression in life expectancy at age 25 was similar with that at birth. For males the life expectancy at age 25 on 1995 data for Hellas was 51.52 years, while in Bulgaria it was 44.71, in Romania 43.52 years. For females, the expectation of life in Hellas was 60.38 years, in Bulgaria 52.1 years, and in Romania 50.93 years.

For males the life expectancy at age 65 on 1995 data for Hellas was 16.23 years, while in Bulgaria it was 12.51, in Romania 12.44 years. For females, the expectation of life in Hellas was 22.41 years, in Bulgaria 16.16 years, and in Romania 15.71 years.

Therefore, a female in 1995, would expect to live slight more than a male and for someone in Hellas would expect to live slight more than anyone who lives in Bulgaria or Romania.

7.6 Conclusions on Comparisons over space

As expected Hellas experience much lower levels of mortality than the other Balkan countries while the gap between the level of the mortality patterns of Hellas and the other three countries has been larger than in 1975. The general mortality levels are lower for Hellas following by Bulgaria, Yugoslavia and Romania, which has experienced the highest mortality levels in comparisons to the other three countries.

Expectation of life

Life expectancy at birth is the number of years a new-born infant can be expected to live if prevailing patterns of mortality at the time of its birth remain the same throughout its life. Life expectancy reflects social factors such as health care, disease control, immunization, overall living conditions, and nutrition.

A female would expect to live slight more than a male and for someone in Hellas, would expect to live slight more than anyone who lives in Bulgaria, Romania or Yugoslavia.

CHAPTER 8

CONCLUSIONS

- Mortality in Hellas has declined over the last 2 decades. The decline in mortality between the 1975 and 1995 has occurred at all ages and for both sexes, but the extend and timing of improvements has varied.
- The general mortality levels are lower for Hellas following by Bulgaria, Yugoslavia and Romania, which has experienced the highest mortality levels in comparisons to the other three countries.
- An unusual though plausible finding is that the levels of later adult and senescent mortality increase by time for Bulgarian and Romanian males, while they remain fairly stable for the corresponding female populations. The causes possibly lie in the significant lowering of the economic and social status in the two countries especially in Romania.
- The accident hump for the female population of Hellas has obviously been more intense in recent years, a finding related to the greater participation of the females in the labor force.
- The accident hump becomes more intense for Hellenic males too in recent years. This phenomenon might be related to the accelerated traffic accidents in Hellas. This finding has also been noticed in other countries e.g. Australia, (Tickle, L.,1996) and Great Britain (Pollard, J.H., 1996).
- Yugoslavia experienced highest mortality than the other three countries at 1975 while at 1990 Romania exhibited the highest mortality levels.
- The accident hump is almost disappeared from the mortality curves of Bulgarian females at the years 1975, 1980, 1985 and 1990 being slight at 1995.
- The accident hump of the two Romanian populations becomes disappeared at recent years.
- The accident hump of Yugoslavian females that was intense at 1980 and 1985 also becomes disappeared at 1990.
- The diminishing of the accident humps in Bulgarian, Romanian and Yugoslavian populations might be related to the intense emigration from these countries.

APPENDIX A

Tables

COUNTRY	YEAR	MALES	FEMALES
HELLAS	1975	0,01274	0,03705
	1980	0,01331	0,09045
	1985	0,02088	0,08452
	1990	0,02634	0,04179
	1995	0,02263	4,0511
BULGARIA	1975	0,01879	0,02118
	1980	0,01033	0,05476
	1985	0,01062	0,06466
	1990	0,01491	0,09892
	1995	0,05189	0,10569
ROMANIA	1975	0,01223	0,02946
	1980	0,02831	0,05000
	1985	0,01790	0,02903
	1990	0,02385	0,02391
	1995	0,24075	0,13665
YUGOSLAVIA	1975	0,01672	0,04667
	1980	0,01897	0,06012
	1985	0,02049	0,07186
	1990	0,06604	0,06378

Table A.1: Sums of squares of the relative deviations between the empirical and the fitted ${}_5q_x$-values

COUNTRY	YEAR	$Ax10^4$	$SEx10^8$	$Bx10^3$	$SEx10^5$	$Cx10^3$	$SEx10^5$
HELLAS	1975	10,13	78,72	0,11	203,59	69,92	12,77
	1980	9,21	0,32	1,20	0,05	86,44	4,03
	1985	7,43	0,30	2,07	0,18	87,64	5,62
	1990	5,08	0,20	3,04	0,52	84,60	8,13
	1995	2,60	0,10	0,10	0,01	60,97	17,37
BULGARIA	1975	17,85	1,49	3,86	0,44	97,61	5,71
	1980	18,64	0,93	13,20	1,46	117,71	3,43
	1985	15,42	0,73	17,04	2,73	110,91	3,87
	1990	17,78	1,81	52,55	22,00	147,05	9,95
	1995	17,05	8,63	77,80	195,43	168,80	73,13
ROMANIA	1975	32,96	4,14	13,97	2,25	133,14	6,13
	1980	29,39	7,36	21,15	8,84	139,03	13,55
	1985	33,35	7,06	48,99	22,12	156,55	11,70
	1990	46,83	34,40	165,78	206,13	214,35	34,32
	1995	24,13	52,51	48,27	371,53	158,62	248,49
YUGOSLAVIA	1975	32,07	3,82	22,03	3,14	159,09	5,46
	1980	18,27	1,47	7,16	0,77	126,62	5,94
	1985	18,40	1,65	11,63	1,78	133,68	6,81
	1990	10,85	2,09	8,24	4,61	118,26	26,71

Table A.2: Estimated values for parameters A, B, C and their standard errors for Hellas-Bulgaria-Romania-Yugoslavia males respectively, using the HP8 formula

COUNTRY	YEAR	$Dx10^4$	$SEx10^{11}$	E	S.E.	F	S.E
HELLAS	1975	6,5	3,84	6,89	0,16	21,88	0,15
	1980	7,44	128,32	7,59	0,80	21,25	0,08
	1985	9,62	230,18	8,52	0,94	22,01	0,09
	1990	10,11	356,40	10,11	1,64	22,06	0,08
	1995	8,64	340,43	7,68	1,58	22,74	0,24
BULGARIA	1975	7,05	280,16	6,33	2,07	22,63	0,38
	1980	6,77	165,48	8,72	2,19	21,47	0,13
	1985	5,74	302,25	16,54	18,13	20,76	0,12
	1990	6,4	350,27	12,26	10,19	20,27	0,18
	1995	5,94	3834,45	31,09	741,56	20,69	0,76
ROMANIA	1975	6,33	284,84	4,53	1,33	22,14	0,72
	1980	4,74	1182,52	17,72	134,90	20,88	0,78
	1985	3,77	799,66	19,46	143,04	23,36	1,15
	1990	3,51	9682,01	32,83	6466,87	19,77	0,83
	1995	1,93	45773981,94	260,29	3000875027,21	21,63	56499,21
YUGOSLAVIA	1975	7,12	279,74	6,73	2,16	23,45	0,42
	1980	6,07	374,87	13,51	13,69	21,58	0,22
	1985	4,79	338,94	13,38	20,05	21,64	0,34
	1990	5,2	984,50	13,63	49,74	21,80	0,88

Table A.3: Estimated values for parameters D, E, F and their standard errors for Hellas – Bulgaria - Romania-Yugoslavia males respectively, using the HP8 formula

COUNTRY	YEAR	$Gx10^6$	$SEx10^{13}$	$Hx10^2$	$SEx10^6$
HELLAS	1975	48,94	2,40	109,73	2,15
	1980	38,63	44,02	110,38	1,87
	1985	32,48	57,29	110,63	2,16
	1990	30,26	97,96	110,98	2,59
	1995	29,79	517,57	111,2	11,52
BULGARIA	1975	53,1	231,24	110,29	2,66
	1980	69,75	156,96	109,91	1,10
	1985	102,28	256,31	109,37	0,88
	1990	126,27	540,26	109,01	1,19
	1995	146,99	2085,88	108,85	3,78
ROMANIA	1975	74,09	672,18	109,64	4,27
	1980	139,79	2215,16	108,75	4,50
	1985	163,8	2031,68	108,54	2,92
	1990	204,79	2683,09	108,12	2,65
	1995	209,23	18596,77	108,34	14,74
YUGOSLAVIA	1975	69,11	362,73	109,90	2,43
	1980	84,96	343,04	109,56	1,65
	1985	84,58	349,89	109,58	1,71
	1990	78,21	961,76	109,62	5,43

Table A.4: Estimated values for parameters G, H and their standard errors for Hellas – Bulgaria - Romania-Yugoslavia males respectively, using the HP8 formula

COUNTRY	YEAR	$Ax10^4$	$SEx10^8$	$Bx10^3$	$SEx10^5$	$Cx10^3$	$SEx10^5$
HELLAS	1975	12,13	2,25	8,96	8,81	118,76	51,03
	1980	6,85	0,88	0,30	0,02	69,91	11,18
	1985	4,64	0,50	0,25	0,03	65,38	21,45
	1990	3,57	0,22	0,22	0,02	62,95	13,57
	1995	3,11	0,34	12,18	20,29	111,84	62,29
BULGARIA	1975	21,00	4,35	39,93	46,31	139,51	33,73
	1980	19,37	6,59	42,82	59,82	136,11	30,86
	1985	15,82	4,56	34,27	45,00	120,50	24,00
	1990	12,90	5,35	16,77	36,85	101,45	46,98
	1995	14,15	6,07	34,02	69,20	119,25	35,51
ROMANIA	1975	54,91	38,76	137,21	130,32	204,33	19,74
	1980	29,46	12,46	45,91	38,24	152,63	17,85
	1985	27,60	6,22	40,06	20,23	137,08	9,37
	1990	34,54	13,10	107,11	103,51	163,83	16,37
	1995	17,28	17,80	12,05	299,79	106,82	1060,79
YUGOSLAVIA	1975	34,19	14,49	36,42	25,74	169,76	25,43
	1980	17,45	3,52	7,20	1,80	122,32	11,42
	1985	19,55	5,44	21,47	11,12	140,21	14,67
	1990	10,01	1,52	5,93	2,22	106,51	16,44

Table A.5: Estimated values for parameters A, B, C and their standard errors for Hellas-Bulgaria-Romania-Yugoslavia females respectively, using the HP8 formula

COUNTRY	YEAR	Dx10^4	SEx10^{11}	E	S.E.	F	S.E
HELLAS	1975	3,37	394,78	0,93	0,47	31,39	72,02
	1980	2,49	328,62	26,17	146,45	21,47	0,45
	1985	2,14	111,57	4,55	4,24	21,70	1,82
	1990	2,18	126,51	7,77	14,98	23,84	1,85
	1995	1,46	138,06	17,30	91,55	16,33	0,59
BULGARIA	1975	7,26	39238,80	0,46	0,30	78,95	12261,70
	1980	3,69	370,68	1,61	0,90	27,35	24,33
	1985	3,2	193,22	2,94	1,87	23,39	4,80
	1990	3,42	3491,89	1,65	3,92	32,61	260,18
	1995	2,8	448,53	4,28	9,88	24,48	10,17
ROMANIA	1975	5,1	564,54	2,41	0,86	26,80	8,59
	1980	4,09	535,21	5,08	7,37	24,97	4,09
	1985	4,29	1382,10	3,27	3,76	30,46	22,72
	1990	5,33	12407,64	1,27	1,28	41,44	538,37
	1995	179,02	509301789990,32	0,61	375,16	401,07	973852094,15
YUGOSLAVIA	1975	6	11615,25	0,95	0,73	46,94	965,95
	1980	2,19	195,57	11,63	51,58	20,99	0,96
	1985	2,97	923,97	24,16	548,60	19,40	0,34
	1990	1,29	94,40	6,37	23,11	20,83	3,44

Table A.6: Estimated values for parameters D, E, F and their standard errors for Hellas – Bulgaria – Romania-Yugoslavia females respectively, using the HP8 formula

COUNTRY	YEAR	$Gx10^6$	$SEx10^{13}$	$Hx10^2$	$SEx10^6$
HELLAS	1975	17,35	26,37	110,91	16,73
	1980	12,27	25,32	111,6	8,24
	1985	8,3	24,45	112,27	11,27
	1990	8,09	292,59	112,89	48,04
	1995	6,98	385,20	111,8	14,72
BULGARIA	1975	6,23	30,84	113,10	18,92
	1980	11,64	93,72	112,04	18,97
	1985	14,49	88,34	111,70	12,73
	1990	14,75	599,74	111,53	74,28
	1995	22,02	427,14	110,86	27,96
ROMANIA	1975	23,71	454,93	110,82	24,71
	1980	33,87	779,73	110,34	22,95
	1985	26,31	774,44	110,72	33,03
	1990	23,36	1381,67	110,76	64,79
	1995	26,74	884542,72	110,31	20563,08
YUGOSLAVIA	1975	11,25	189,84	112,14	35,86
	1980	23,09	81,05	110,89	5,30
	1985	24,9	82,55	110,73	4,92
	1990	20,65	83,44	110,89	6,54

Table A.7: Estimated values for parameters G, H and their standard errors for Hellas – Bulgaria - Romania-Yugoslavia females respectively, using the HP8 formula

	A	B	C	D	E	F	G	H
A	1,000	,773	,849	,028	,080	,422	,365	-,169
B	,773	1,000	,828	-,088	,308	,285	,405	-,225
C	,849	,828	1,000	-,107	,369	,287	,483	-,276
D	,028	-,088	-,107	1,000	-,066	,130	,028	-,042
E	,080	,308	,369	-,066	1,000	-,530	,841	-,797
F	,422	,285	,287	,130	-,530	1,000	-,299	,457
G	,365	,405	,483	,028	,841	-,299	1,000	-,828
H	-,169	-,225	-,276	-,042	-,797	,457	-,828	1,000

Table A.8: Correlation Matrices of A-B-C-D-E-F-G-H parameters

YEAR	COUNTRY			
	HELLAS	BULGARIA	ROMANIA	YUGOSLAVIA
1975	72.09	68.61	67.68	66.98
1980	73.13	68.44	67.04	67.81
1985	73.42	68.14	66.89	68.06
1990	74.80	68.01	67.18	69.40
1995	75.10	67.55	65.58	

Table A.9: Expectation of life at birth in years for (Hellas – Bulgaria – Romania - Yugoslavia) males, over 1975 to 1995.

YEAR	COUNTRY			
	HELLAS	BULGARIA	ROMANIA	YUGOSLAVIA
1975	76.43	73.41	72.83	71.77
1980	77.72	74.10	72.87	73.49
1985	78.88	74.44	73.28	74.12
1990	79.75	74.86	73.58	75.65
1995	83.53	75.45	73.56	

Table A.10: Expectation of life at birth in years for (Hellas – Bulgaria – Romania - Yugoslavia) females, over 1975 to 1995.

YEAR	COUNTRY			
	HELLAS	BULGARIA	ROMANIA	YUGOSLAVIA
1975	50.04	46.60	46.95	46.22
1980	50.52	46.20	45.77	46.25
1985	50.46	45.40	45.30	46.21
1990	51.35	45.33	45.48	46.72
1995	51.52	44.71	43.52	

Table A.11: Expectation of life at age 25 in years for (Hellas – Bulgaria – Romania - Yugoslavia) males, over 1975 to 1995.

YEAR	COUNTRY			
	HELLAS	BULGARIA	ROMANIA	YUGOSLAVIA
1975	53.72	50.66	51.46	50.60
1980	54.53	51.17	50.87	51.58
1985	54.25	51.22	50.99	51.83
1990	55.89	51.50	51.08	52.64
1995	60.38	52.10	50.93	

Table A.12: Expectation of life at age 25 in years for (Hellas – Bulgaria – Romania - Yugoslavia) females, over 1975 to 1995.

YEAR	COUNTRY			
	HELLAS	BULGARIA	ROMANIA	YUGOSLAVIA
1975	14.81	12.76	13.44	12.75
1980	15.14	12.68	13.29	12.93
1985	15.15	12.53	13.20	12.88
1990	15.91	12.78	13.67	13.17
1995	16.23	12.51	12.44	

Table A.13: Expectation of life at age 65 in years for (Hellas – Bulgaria – Romania - Yugoslavia) males, over 1975 to 1995.

YEAR	COUNTRY			
	HELLAS	BULGARIA	ROMANIA	YUGOSLAVIA
1975	16.74	14.35	15.92	14.70
1980	17.36	14.96	15.58	15.66
1985	17.97	15.05	15.57	15.87
1990	18.32	15.30	15.42	16.41
1995	22.41	16.16	15.71	

Table A.14: Expectation of life at age 65 in years for (Hellas – Bulgaria – Romania - Yugoslavia) females, over 1975 to 1995.

APPENDIX B

Figures

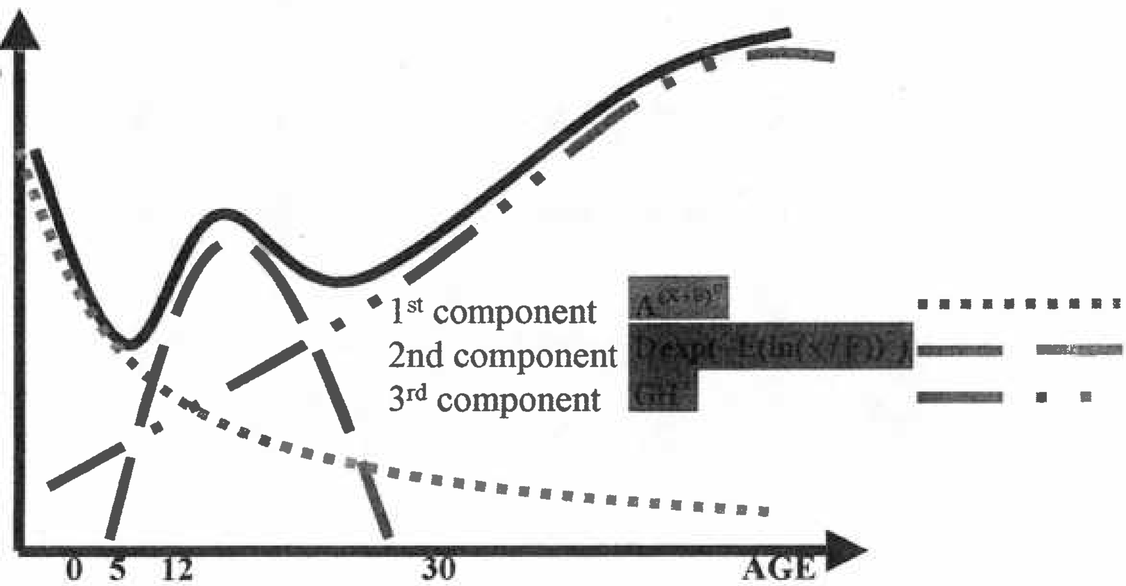

Fig. B. 1: The graduated q_x curve and its three components

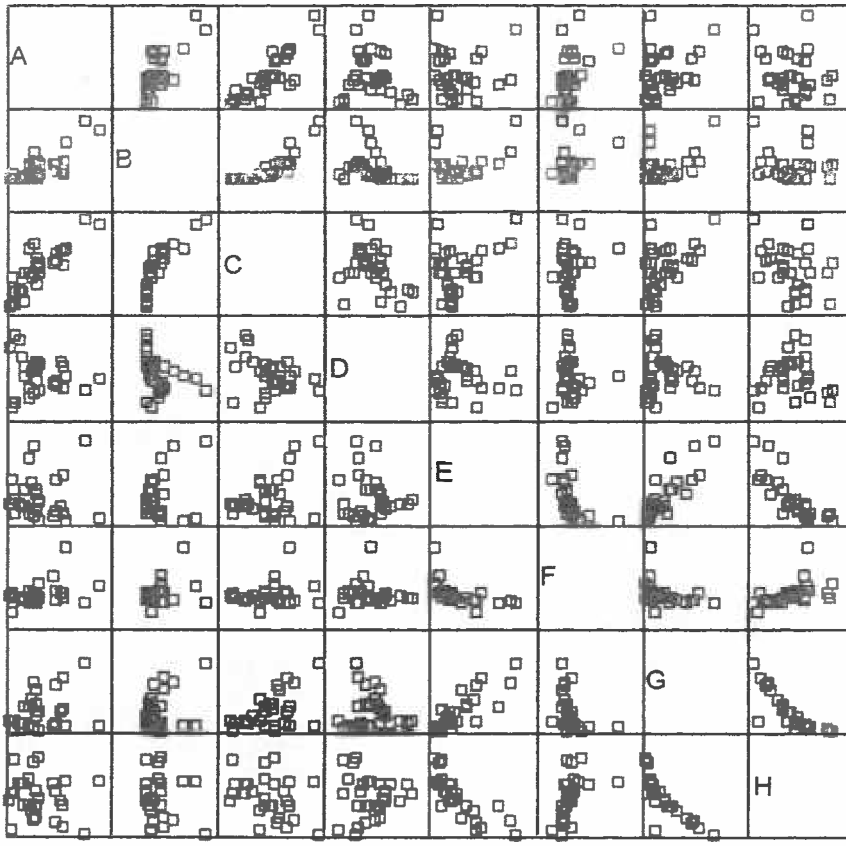

Fig. B. 2 : Correlation matrices of parameters A-B-C-D-E-F-G-H

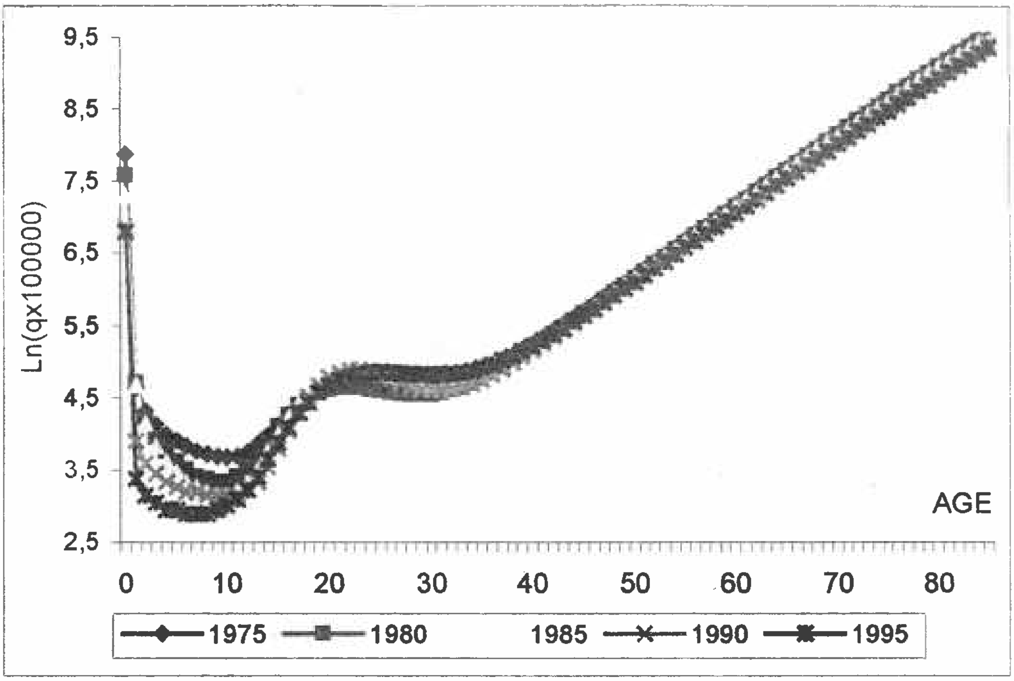

Fig. B.3: Graduated values q_x for Hellas males, over 1975-1995

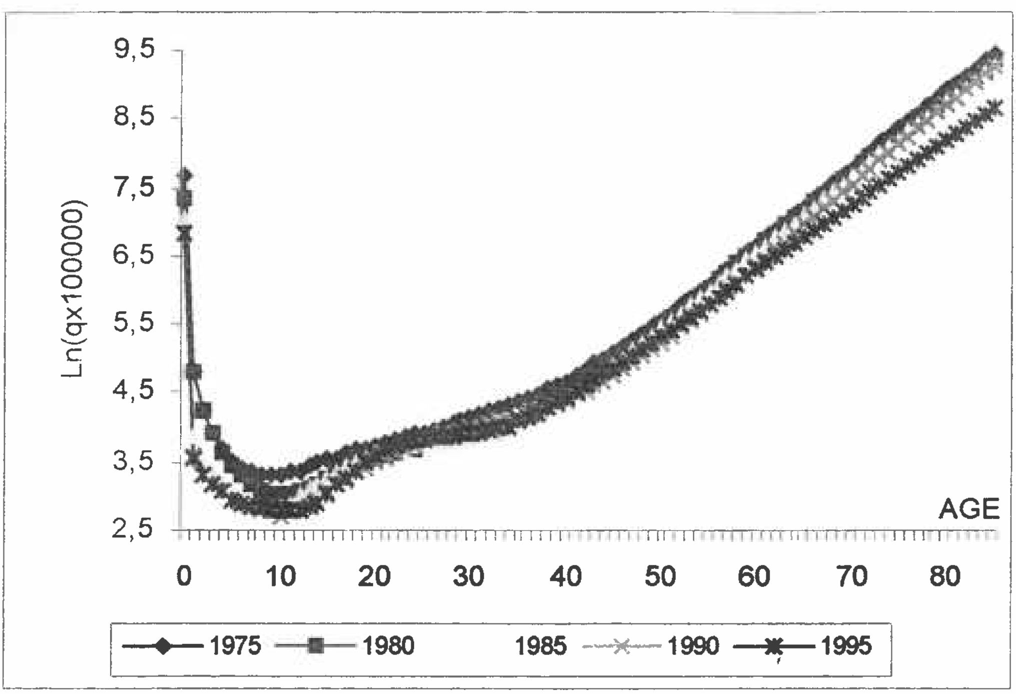

Fig. B.4: Graduated values q_x for Hellas females, over 1975-1995

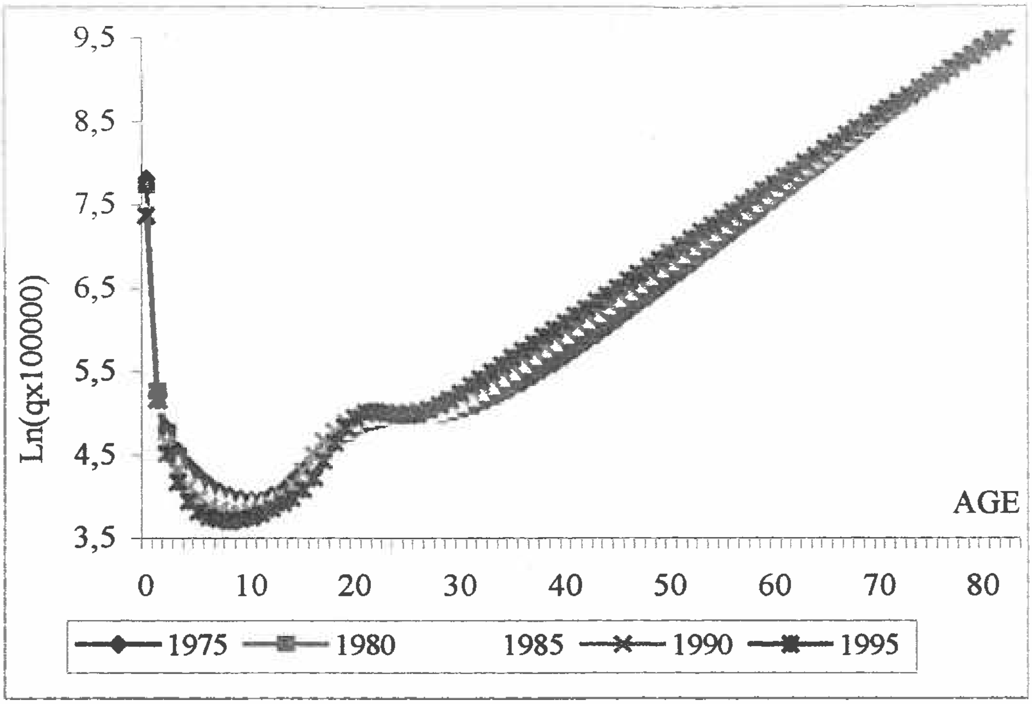

Fig. B.5: Graduated values q_x for Bulgaria males, over 1975-1995

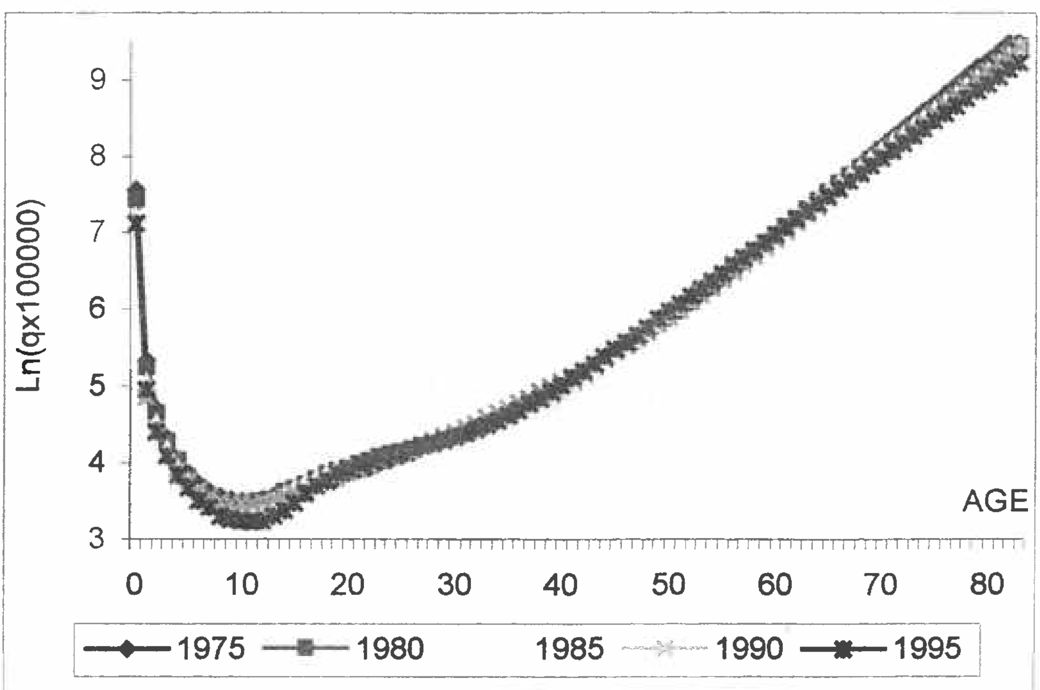

Fig. B.6: Graduated values q_x for Bulgaria females, over 1975-1995

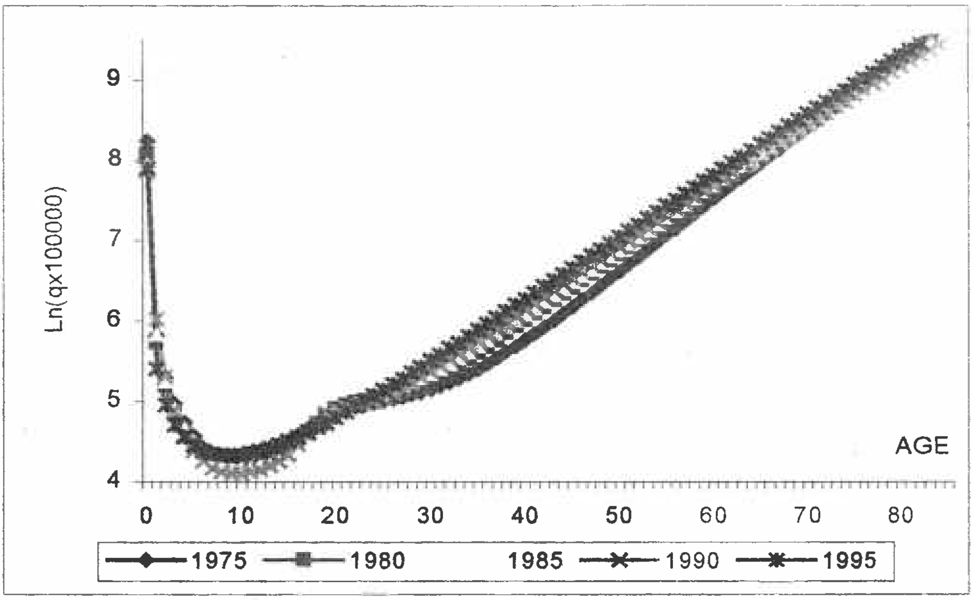

Fig. B.7: Graduated values q_x for Romania males, over 1975-1995

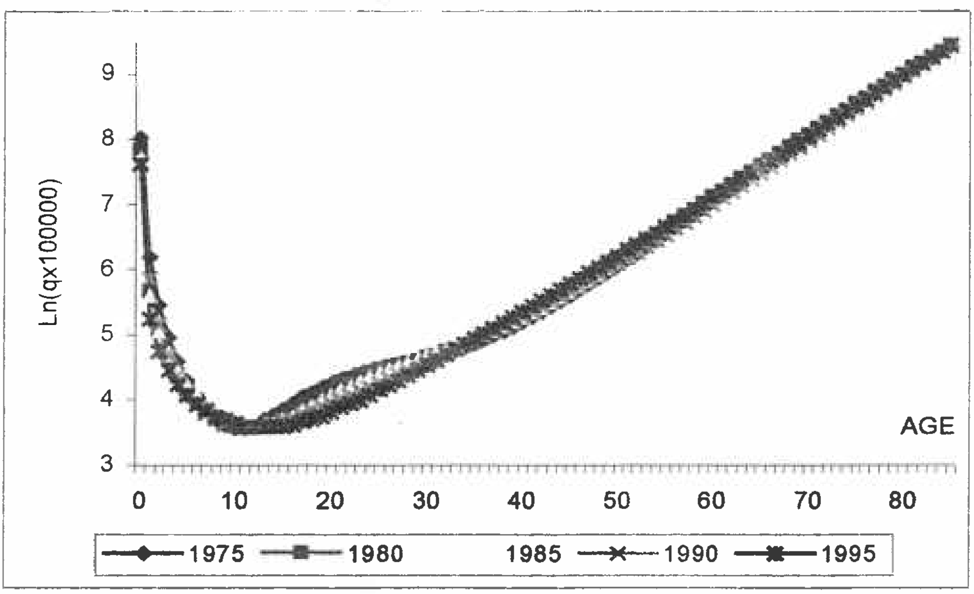

Fig. B.8: Graduated values q_x for Romania females, over 1975-1995

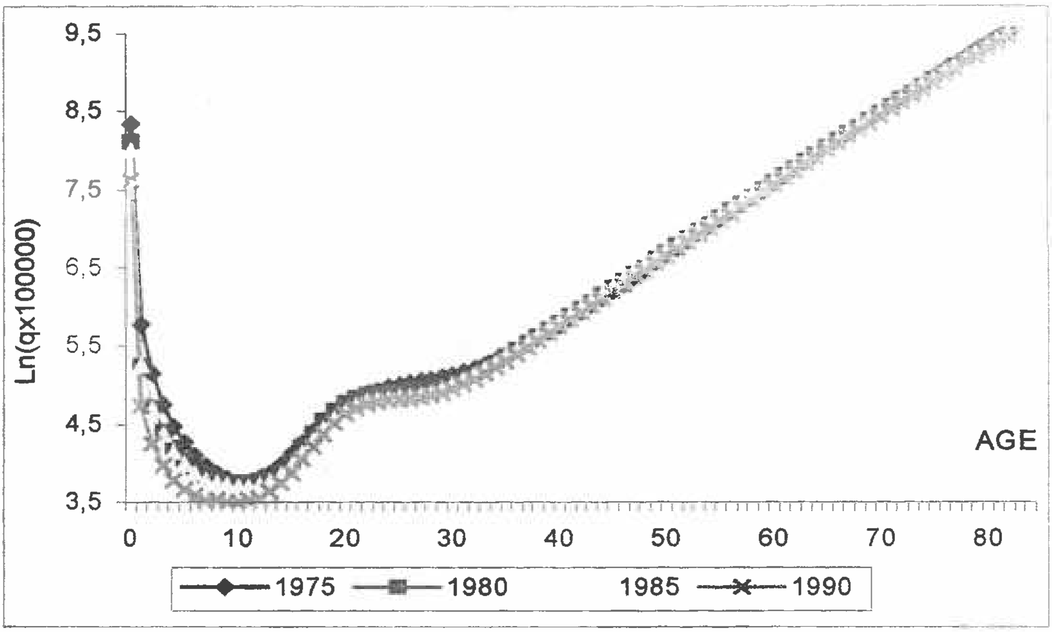

Fig. B.9: Graduated values q_x for Yugoslavia males, over 1975-1990

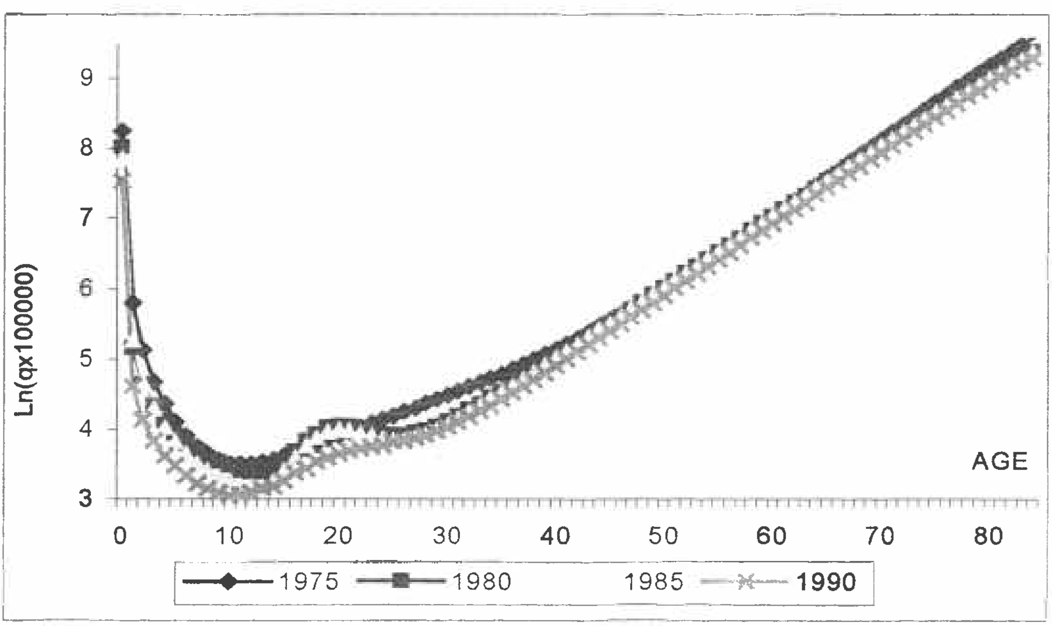

Fig. B.10: Graduated values q_x for Yugoslavia females, over 1975-1990

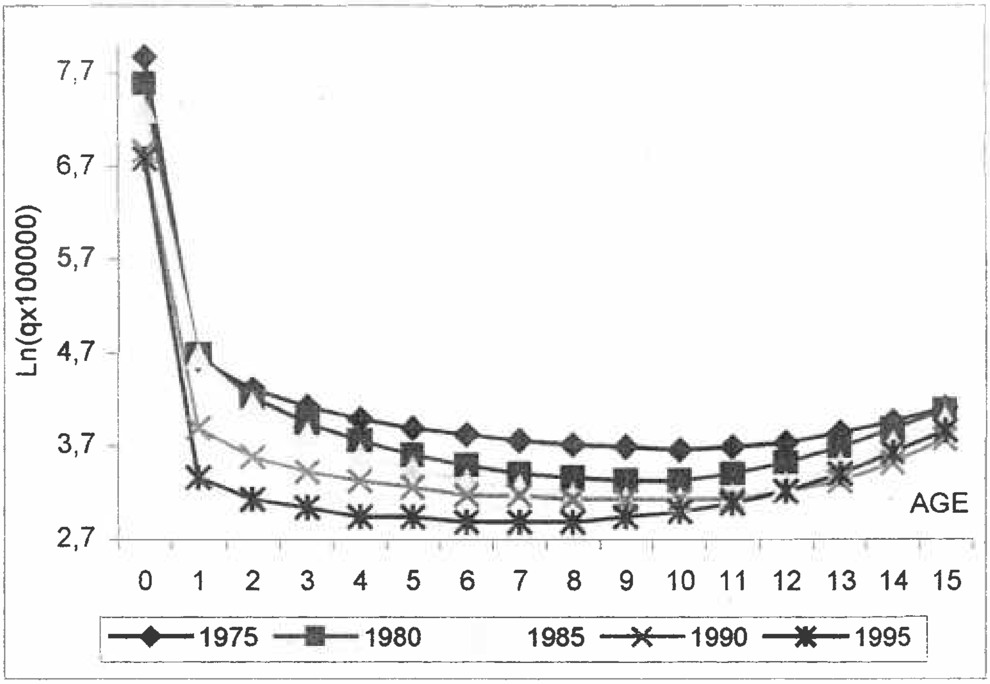

Fig. B.11: Graduated values q_x for Hellas males aged 0-15, over 1975-1995

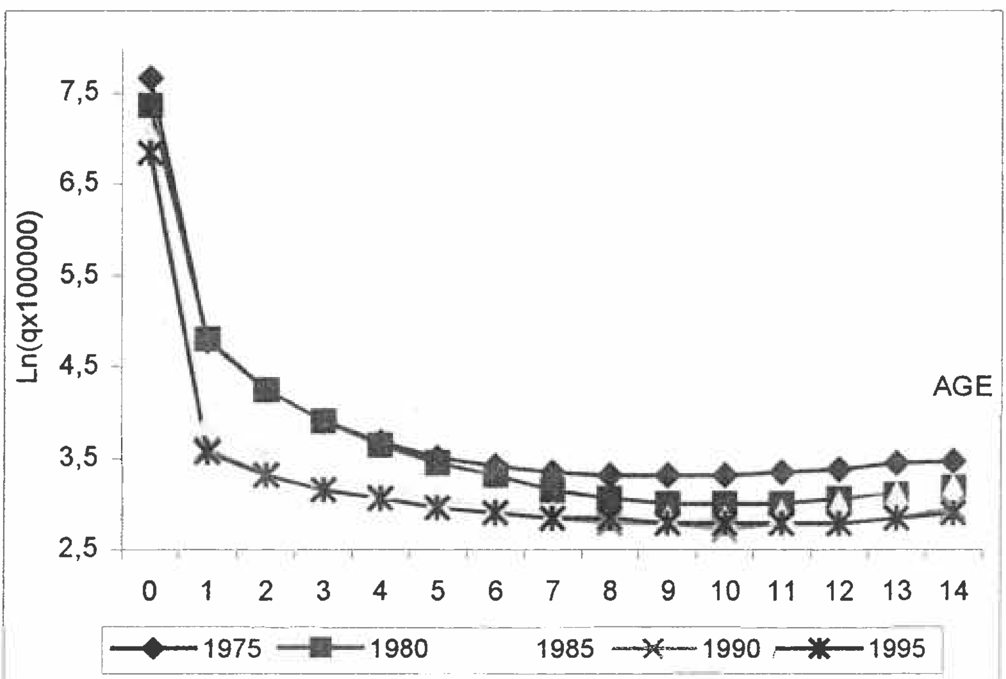

Fig. B.12: Graduated values q_x for Hellas females aged 0-15, over 1975-1995

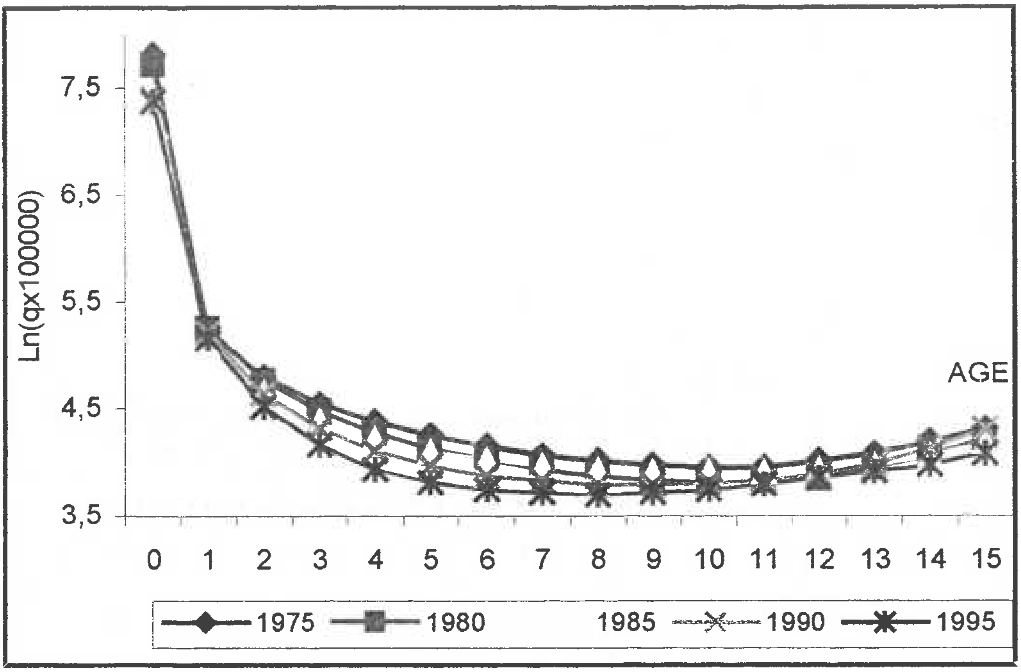

Fig. B.13: Graduated values q_x for Bulgaria males aged 0-15, over 1975-1995

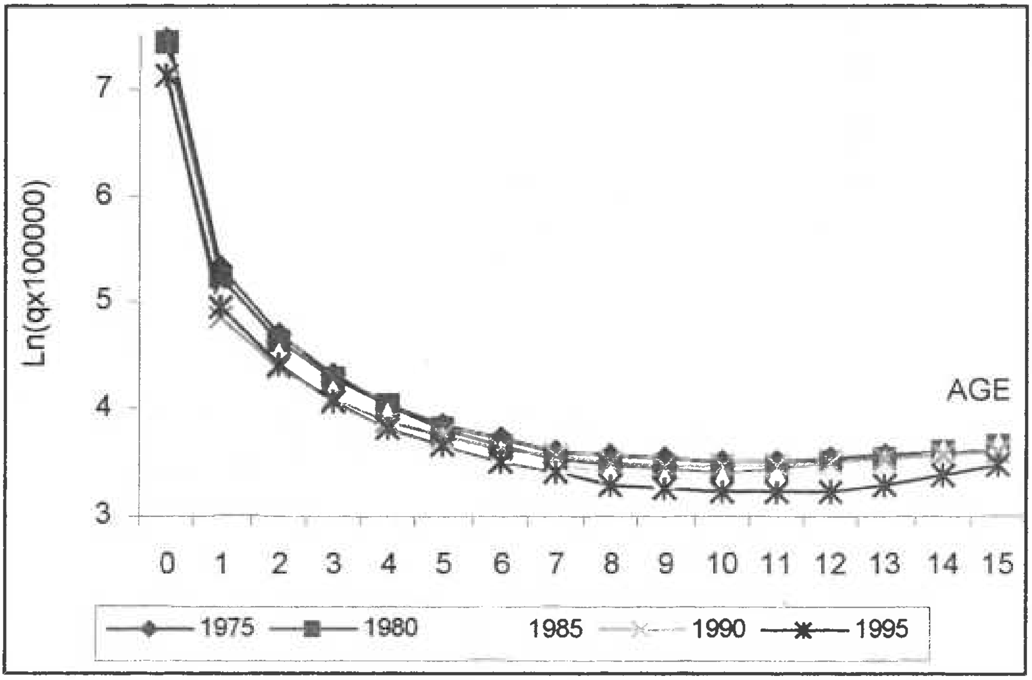

Fig. B.14: Graduated values q_x for Bulgaria females aged 0-15, over 1975-1995

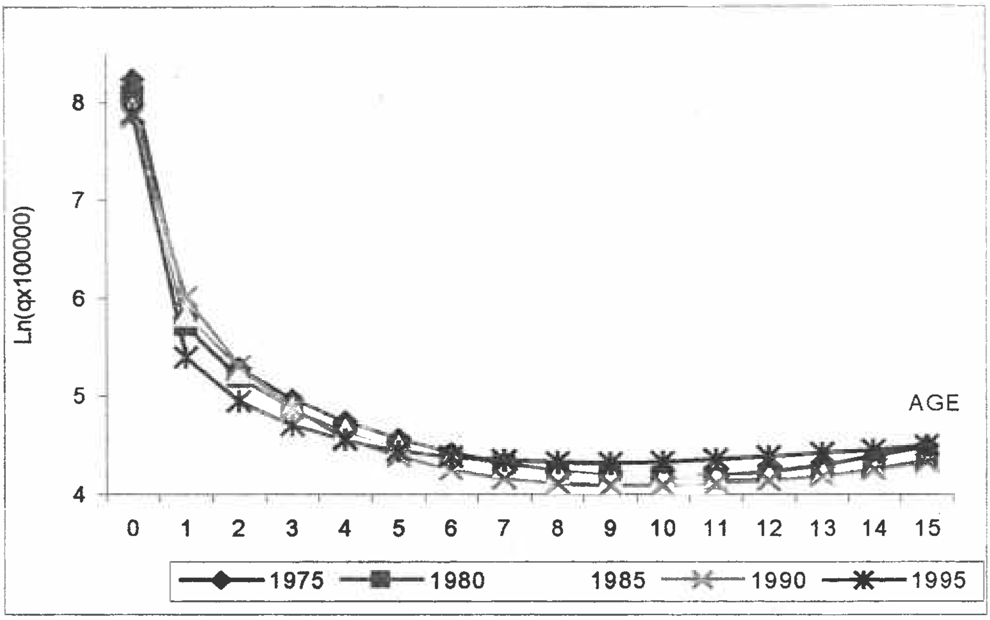

Fig. B.15: Graduated values q_x for Romania males aged 0-15, over 1975-1995

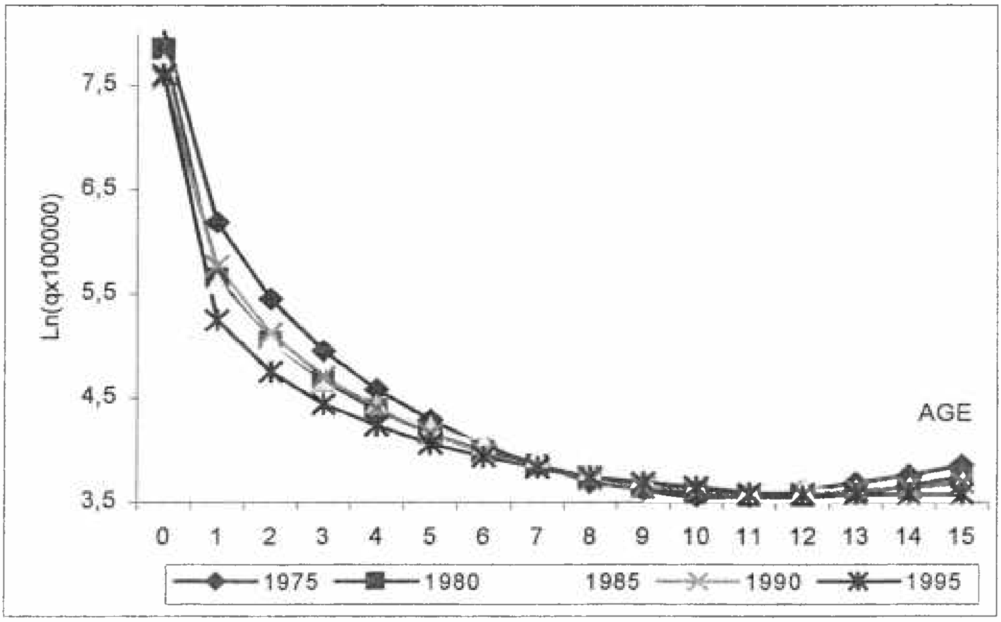

Fig. B.16: Graduated values q_x for Romania females aged 0-15, over 1975-1995

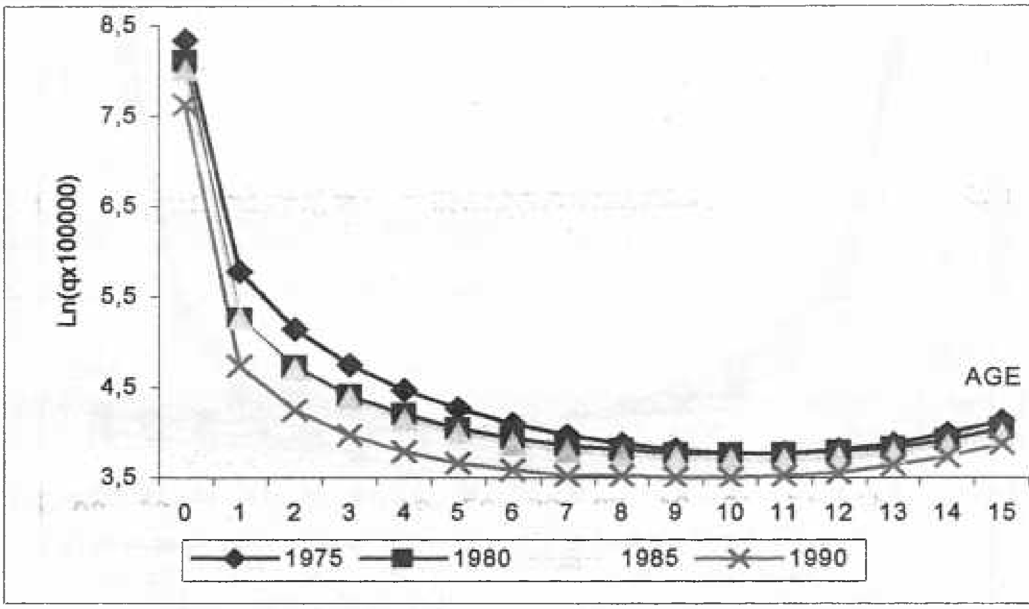

Fig. B.17: Graduated values q_x for Yugoslavia males aged 0-15, over 1975-1990

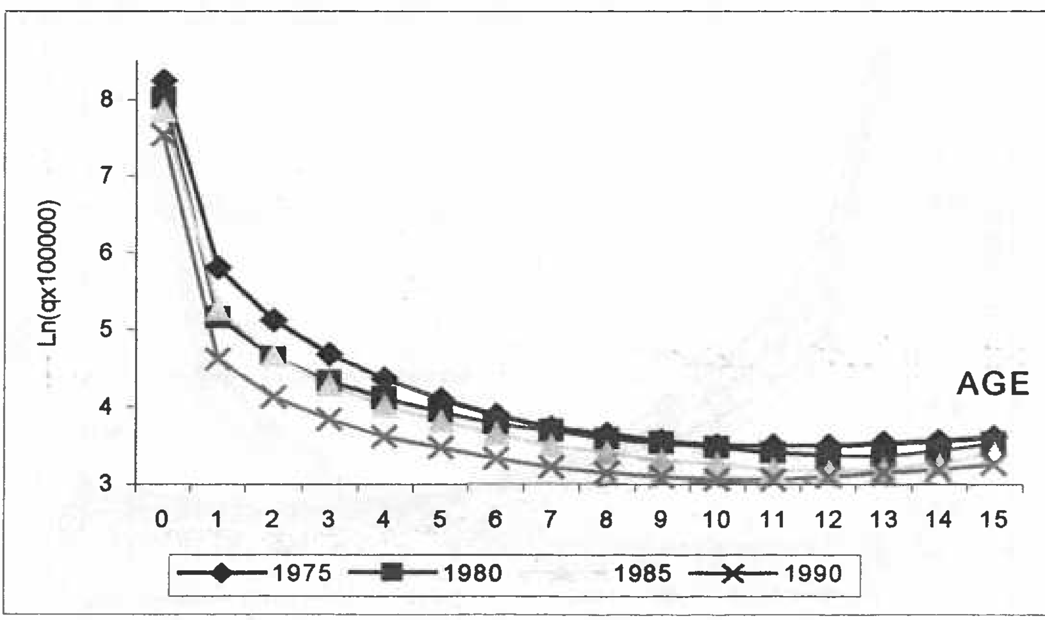

Fig. B.18: Graduated values q_x for Yugoslavia females aged 0-15, over 1975-1990

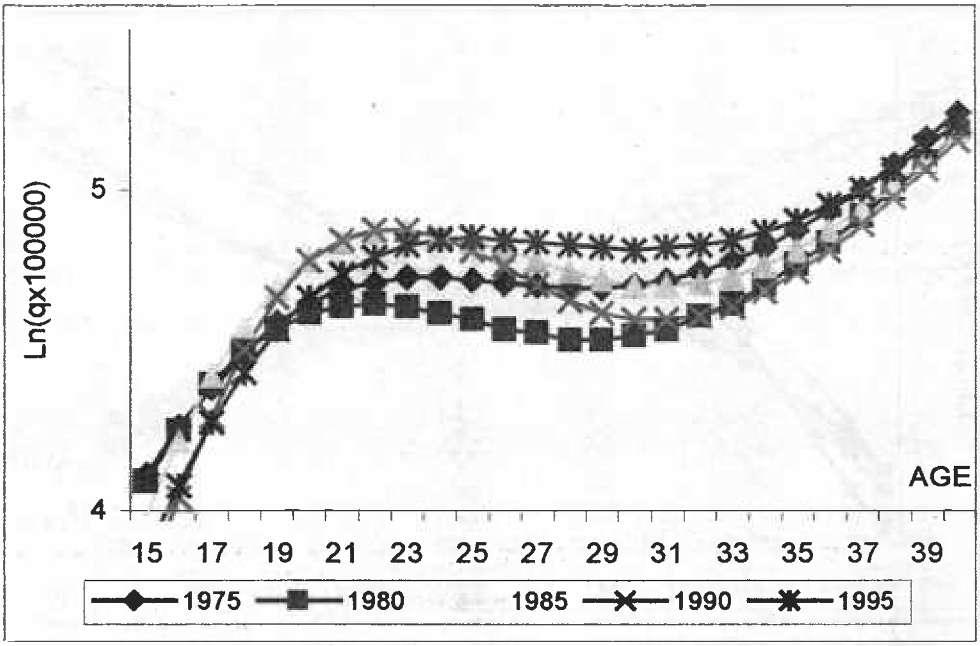

Fig. B.19: Graduated values q_x for Hellas males aged 15-40, over 1975-1995

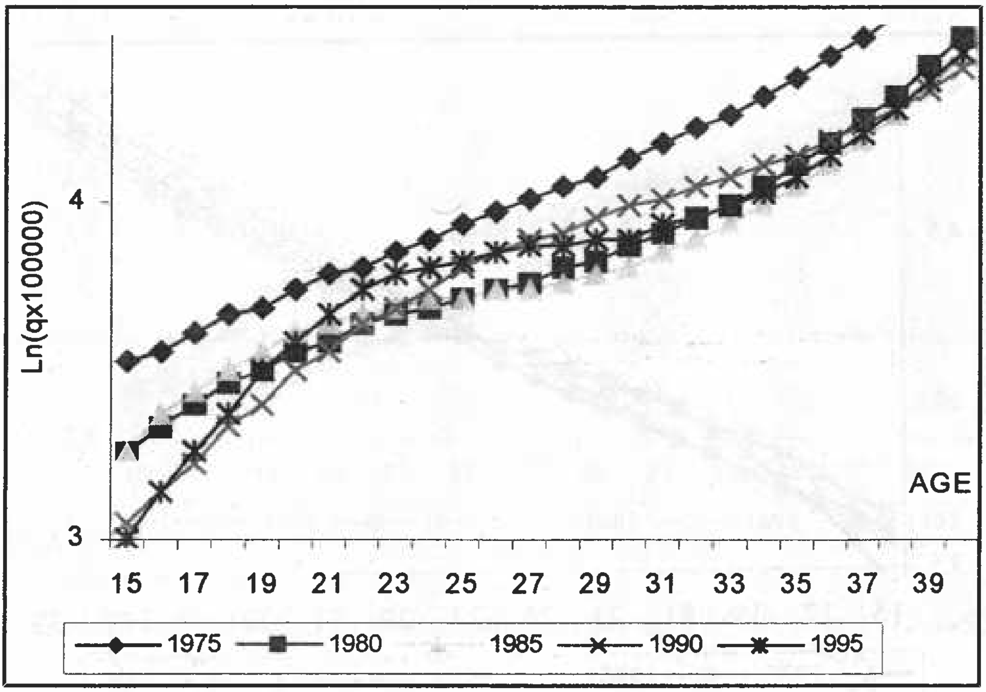

Fig. B.20: Graduated values q_x for Hellas females aged 15-40, over 1975-1995

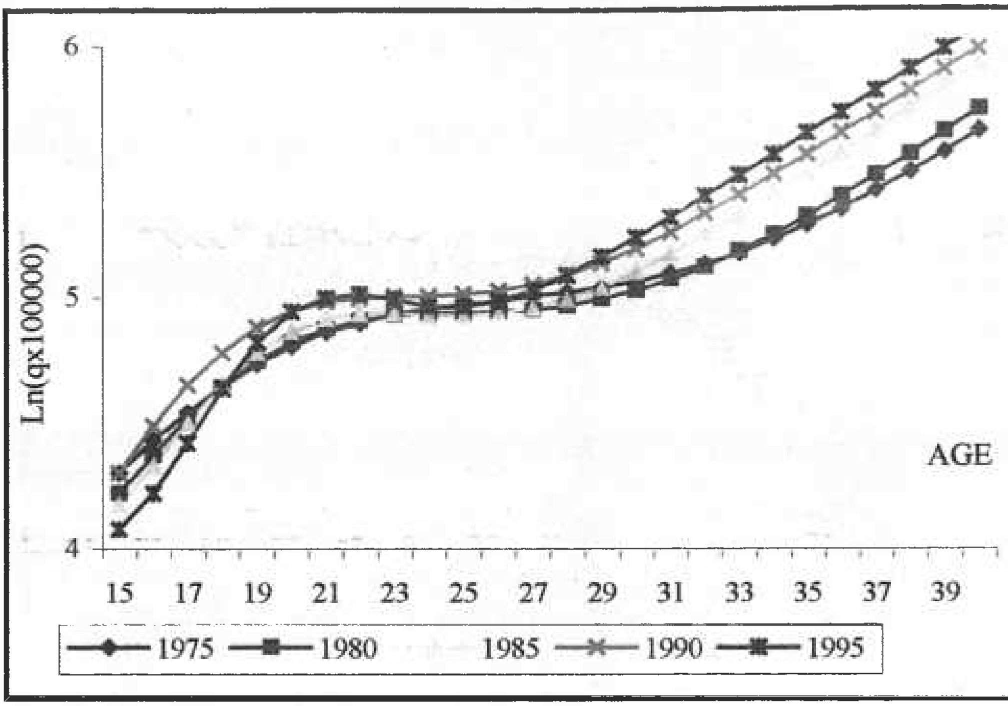

Fig. B.21: Graduated values q_x for Bulgaria males aged 15-40, over 1975-1995

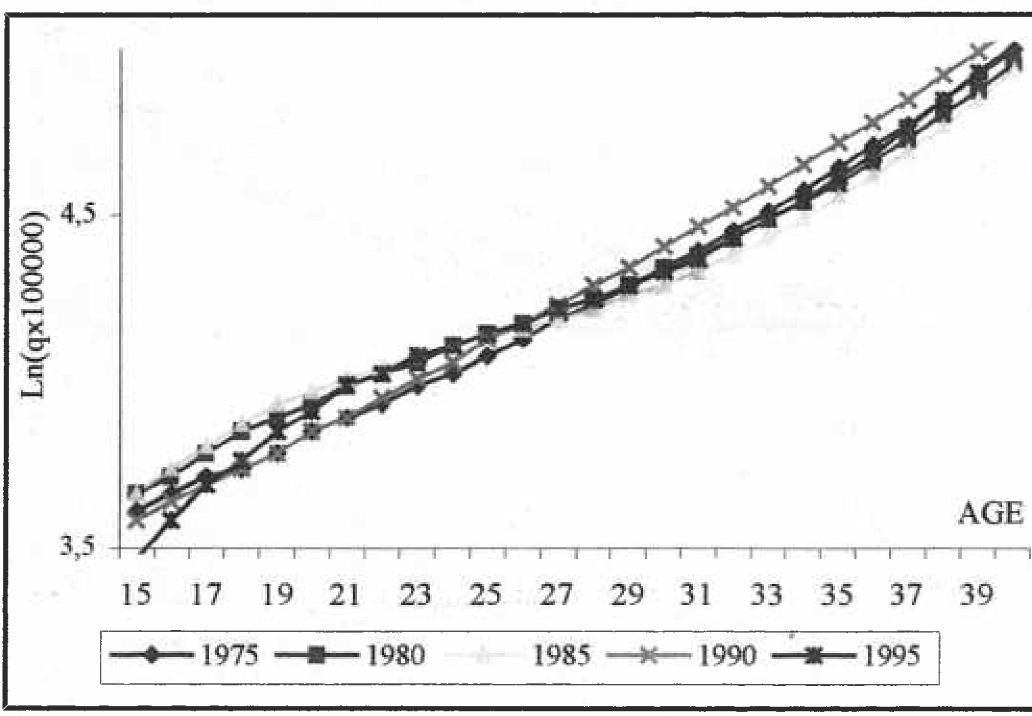

Fig. B.22: Graduated values q_x for Bulgaria females aged 15-40, over 1975-1995

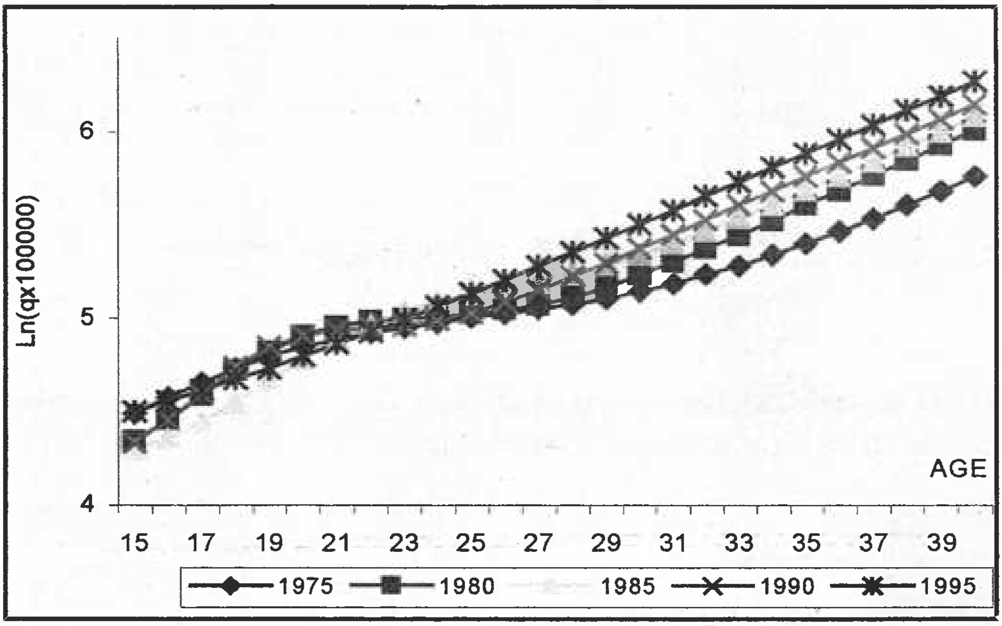

Fig. B.23: Graduated values q_x for Romania males aged 15-40, over 1975-1995

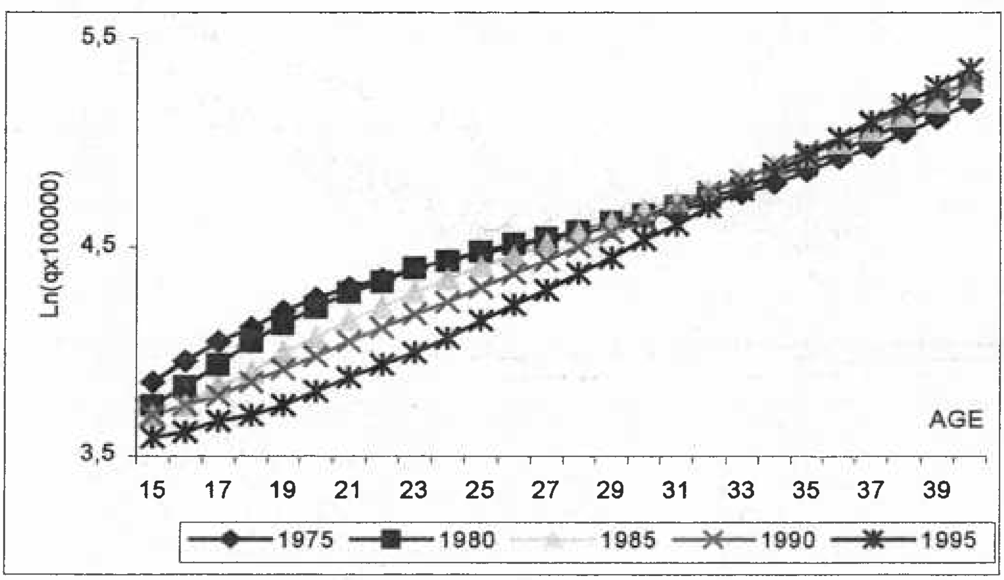

Fig. B.24: Graduated values q_x for Romania females aged 15-40, over 1975-1995

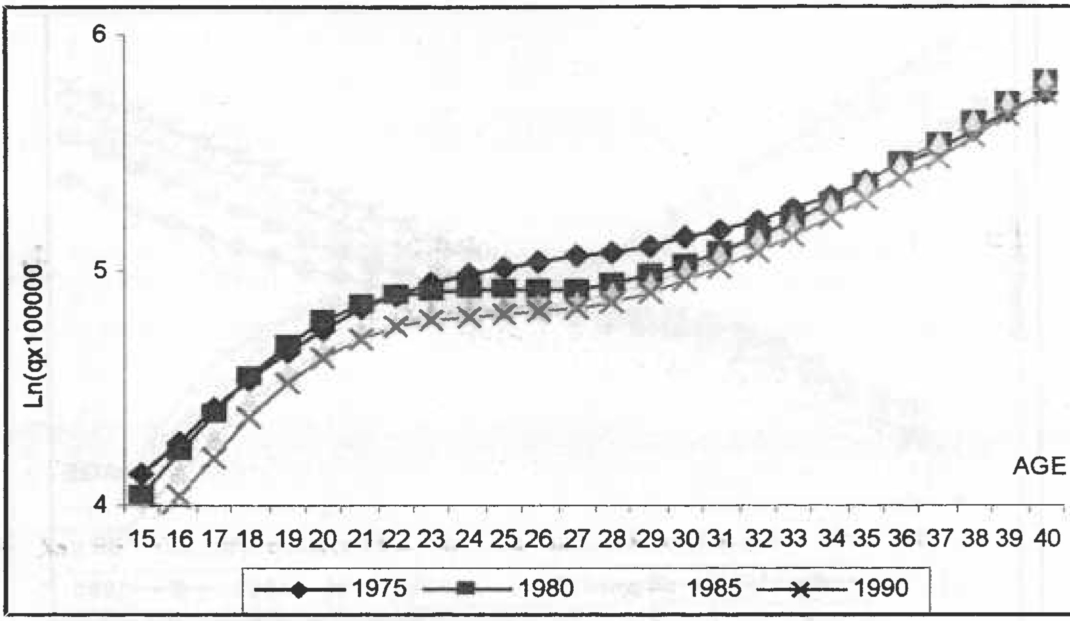

Fig. B.25: Graduated values q_x for Yugoslavia males aged 15-40, over 1975-1990

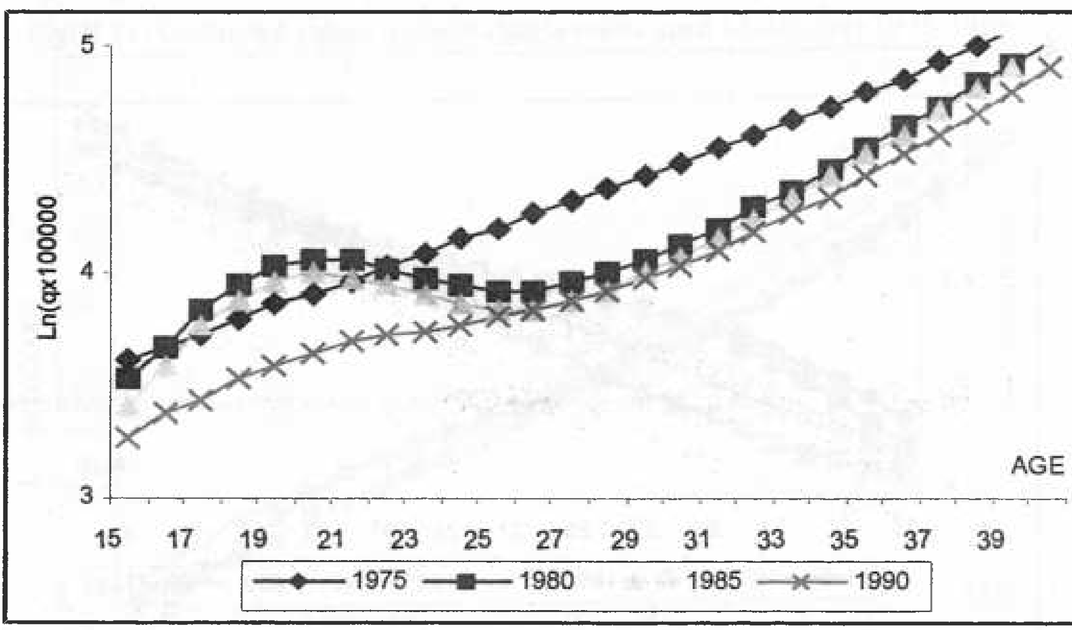

Fig. B.26: Graduated values q_x for Yugoslavia females aged 15-40, over 1975-1990

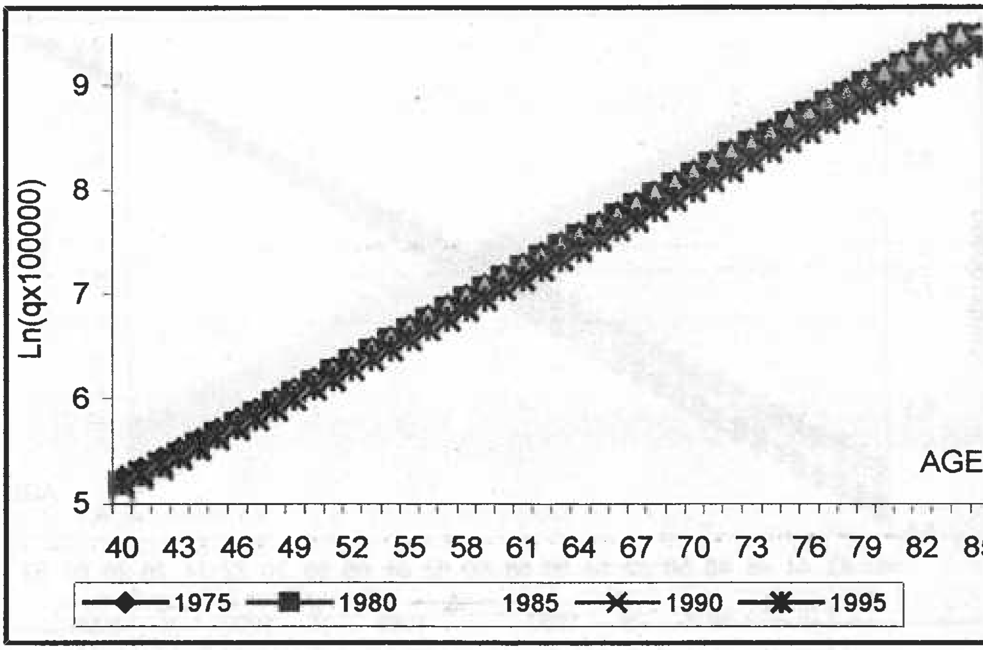

Fig. B.27: Graduated values q_x for Hellas males aged 40-80+, over 1975-1995

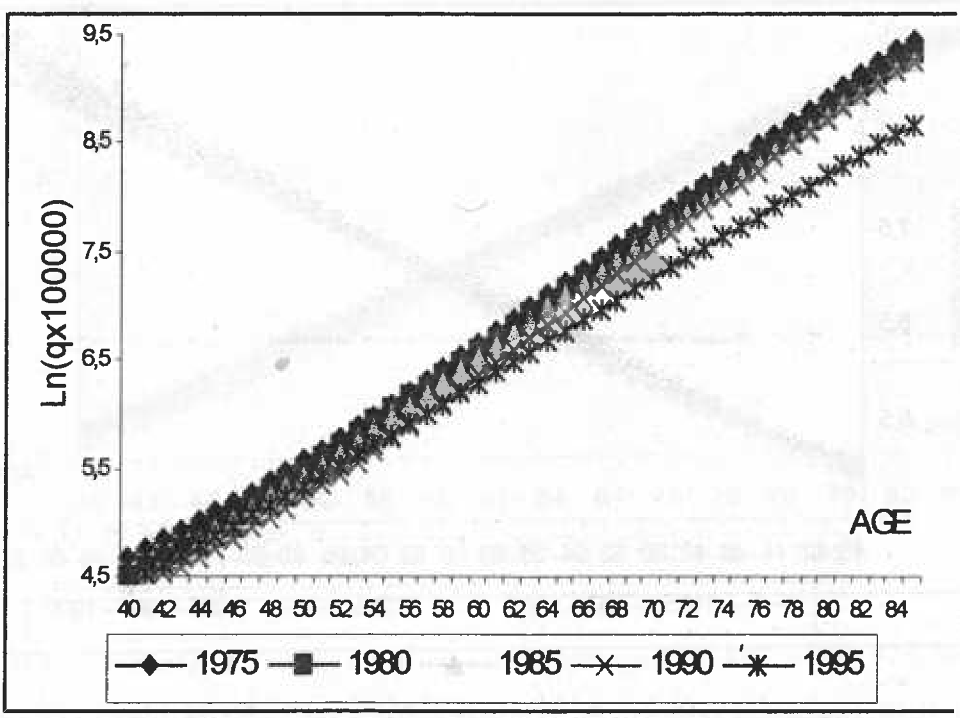

Fig. B.28: Graduated values q_x for Hellas females aged 40-80+, over 1975-1995

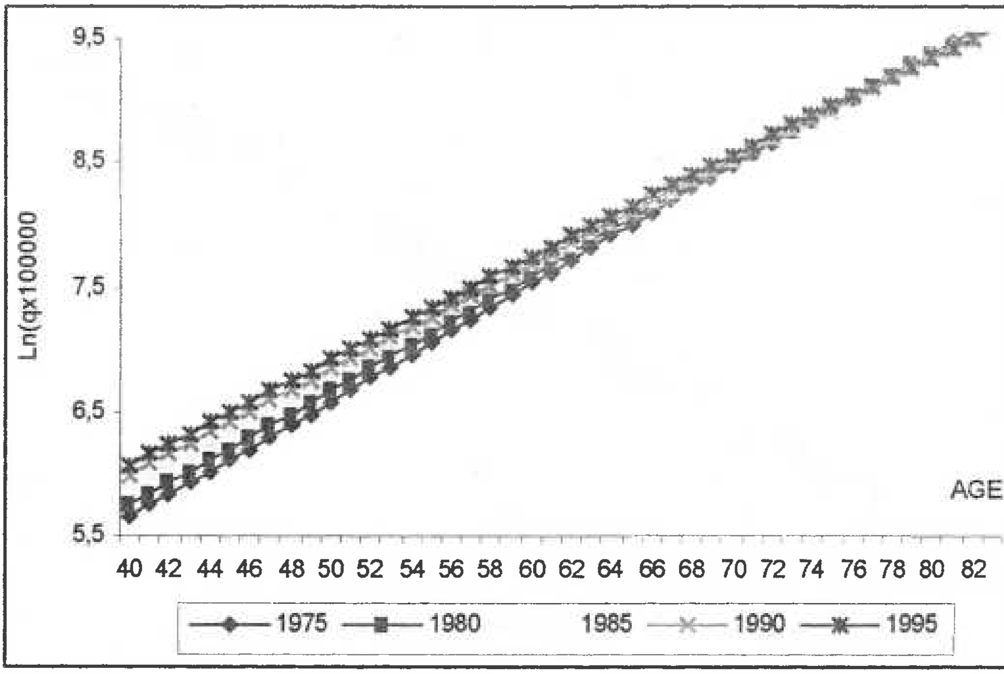

Fig. B.29: Graduated values q_x for Bulgaria males aged 40-80+, over 1975-1995

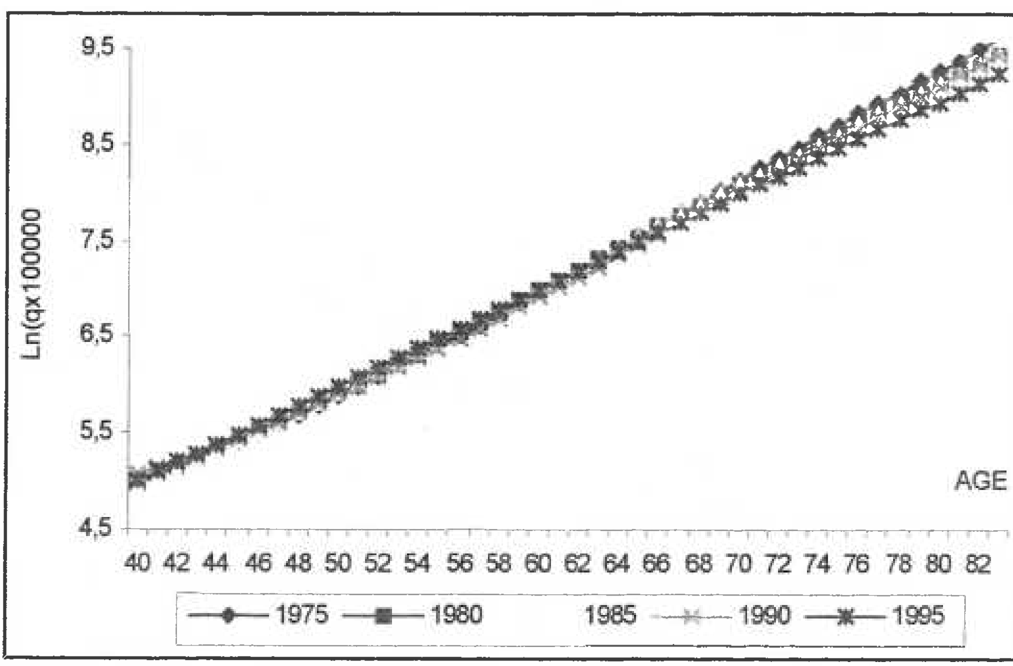

Fig. B.30: Graduated values q_x for Bulgaria females aged 40-80+, over 1975-1995

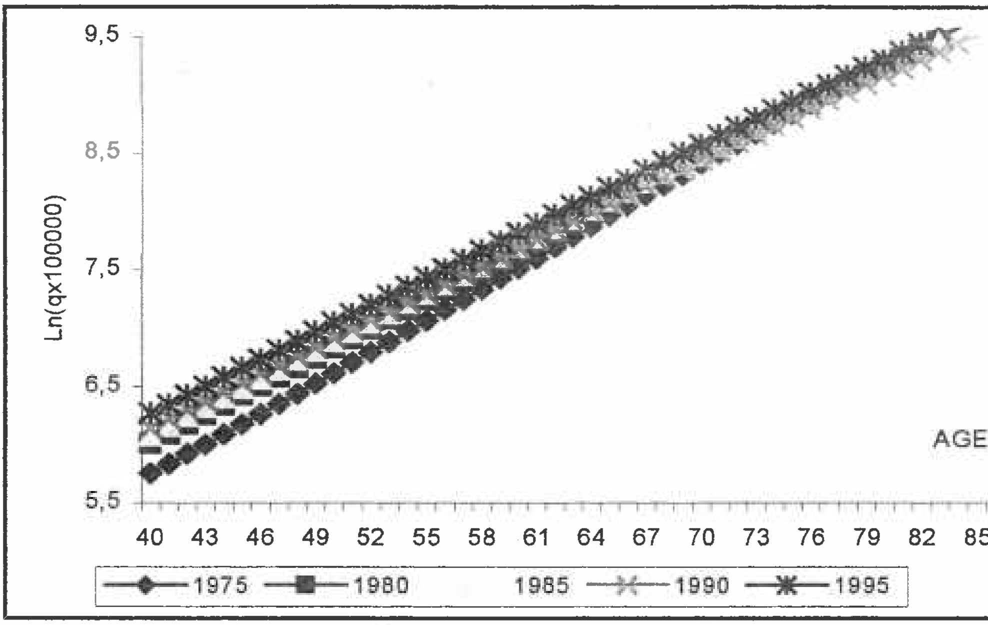

Fig. B.31: Graduated values q_x for Romania males aged 40-80+, over 1975-1995

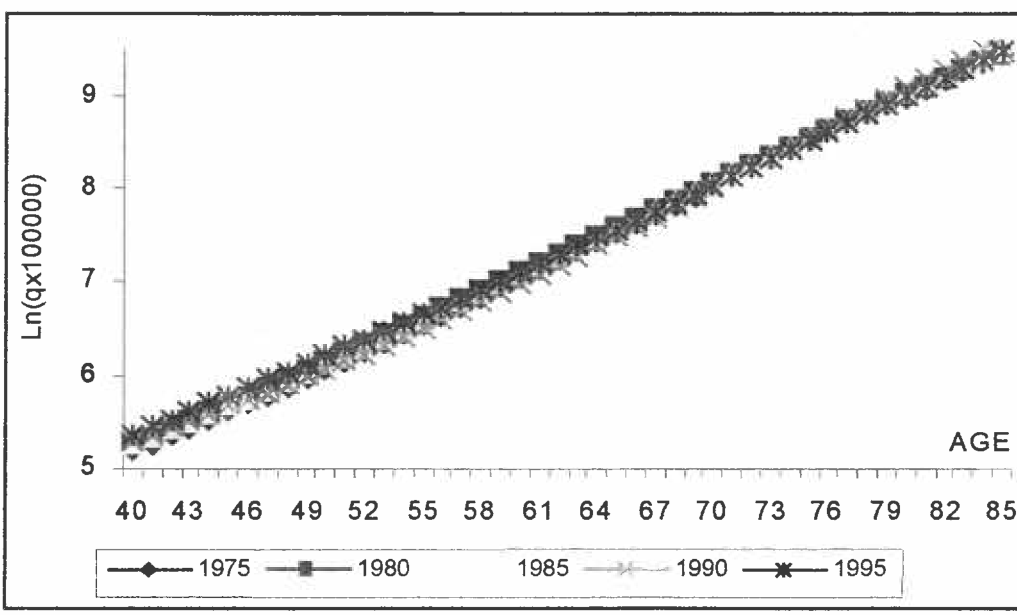

Fig. B.32: Graduated values q_x for Romania females aged 40-80+, over 1975-1995

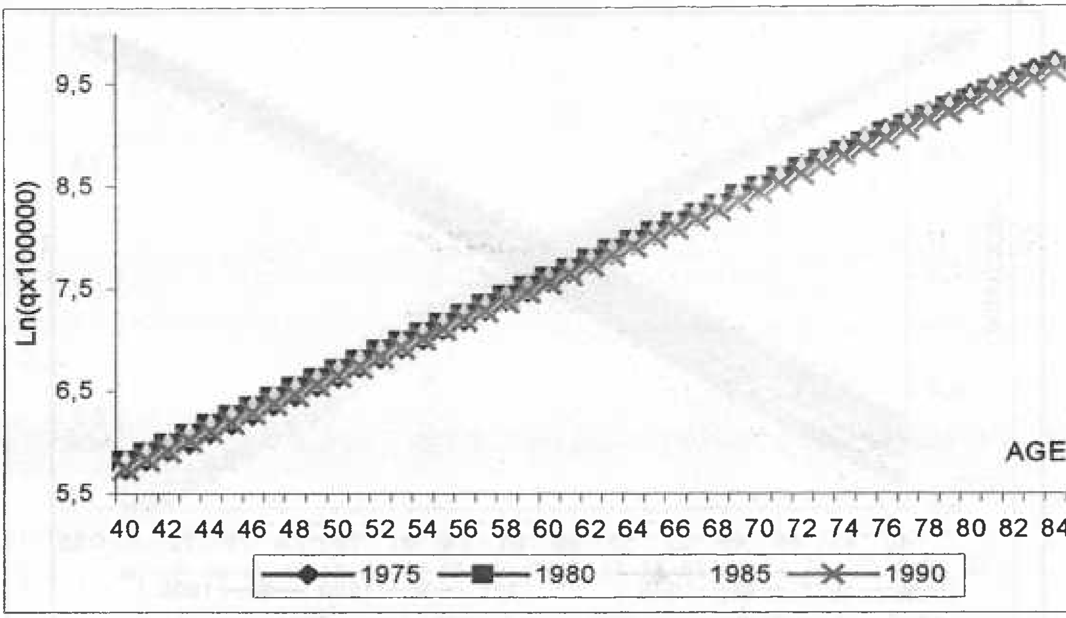

Fig. B.33: Graduated values q_x for Yugoslavia males aged 40-80+, over 1975-1990

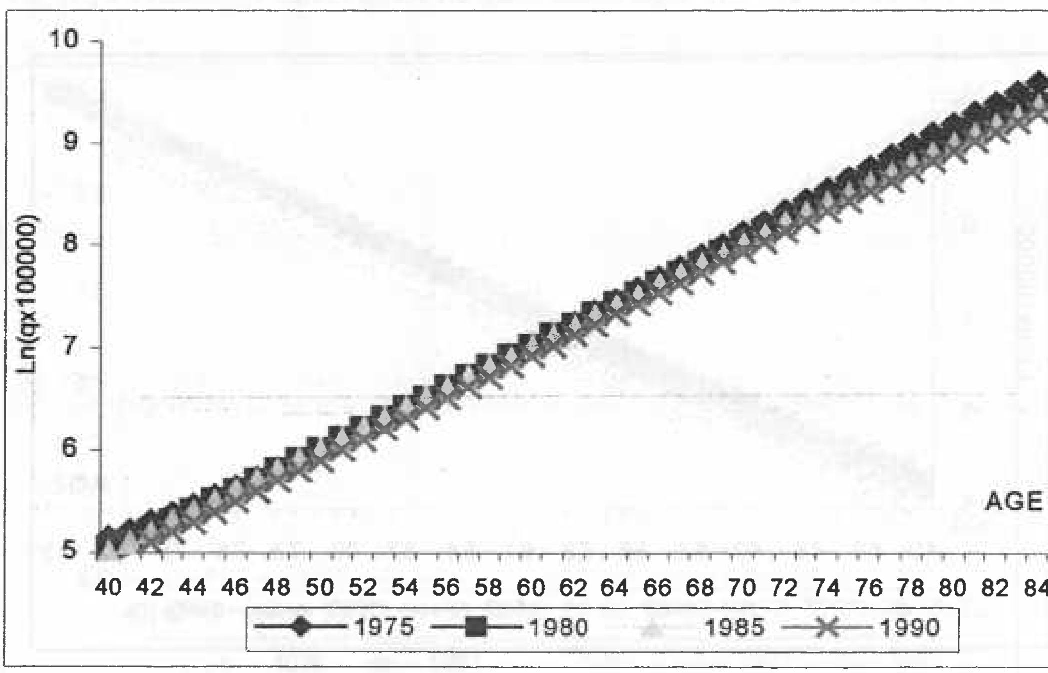

Fig. B.34: Graduated values q_x for Yugoslavia females aged 40-80+, over 1975-1990

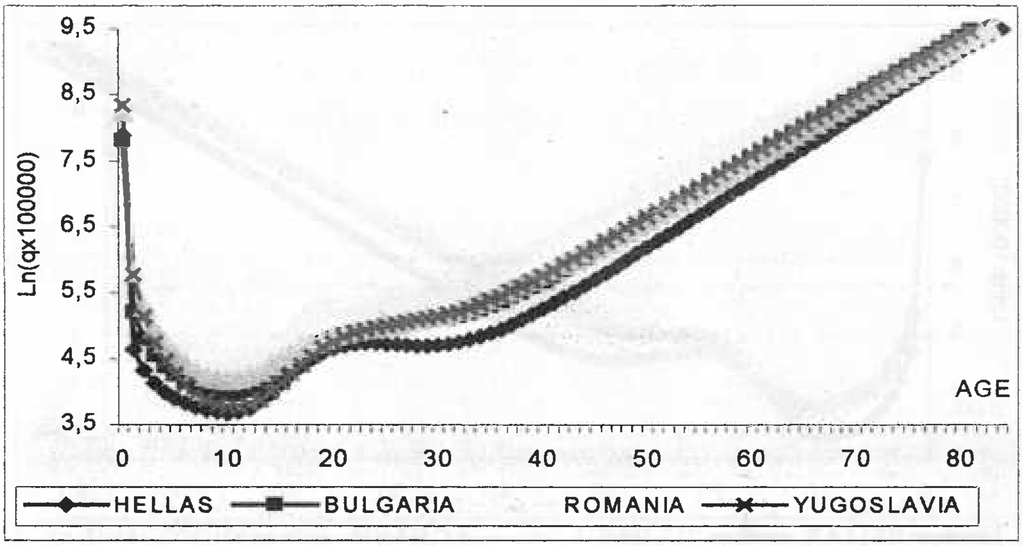

Fig. B.35: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males, in 1975

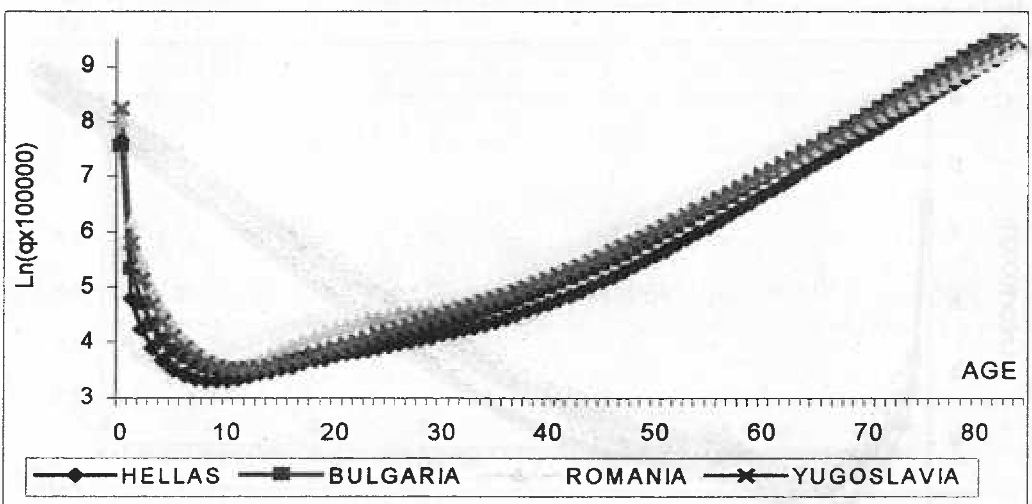

Fig. B.36: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females, in 1975

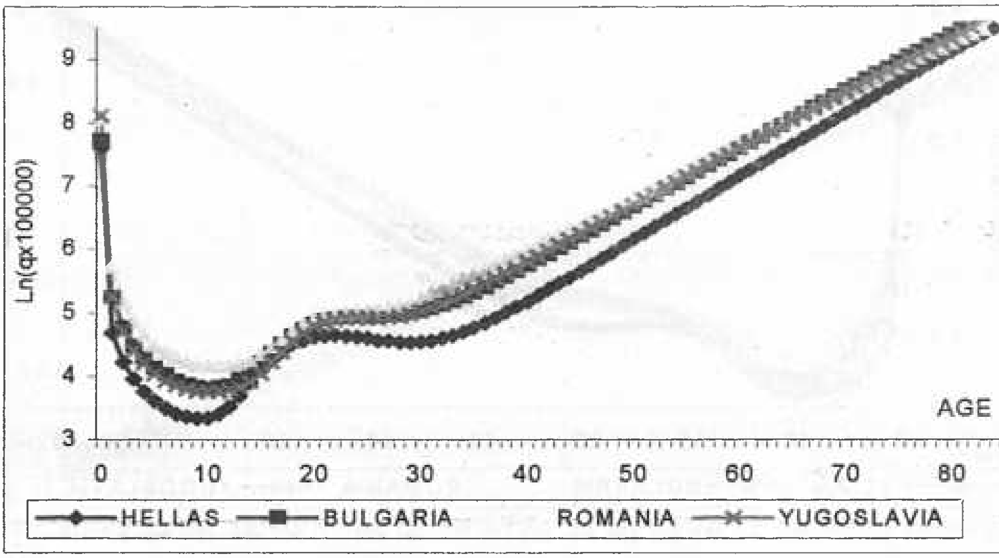

Fig. B.37: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males, in 1980

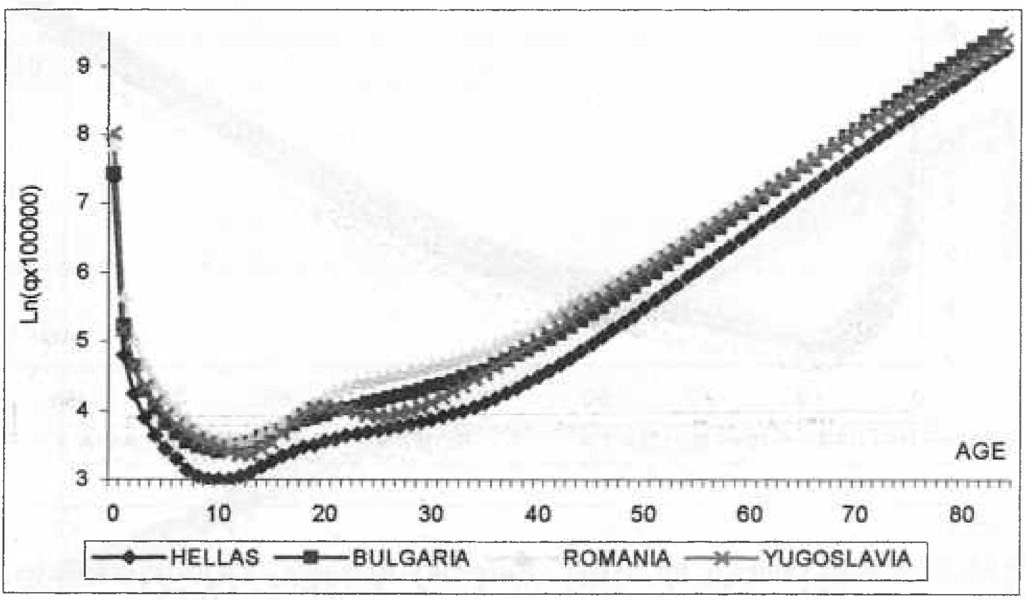

Fig. B.38: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females, in 1980

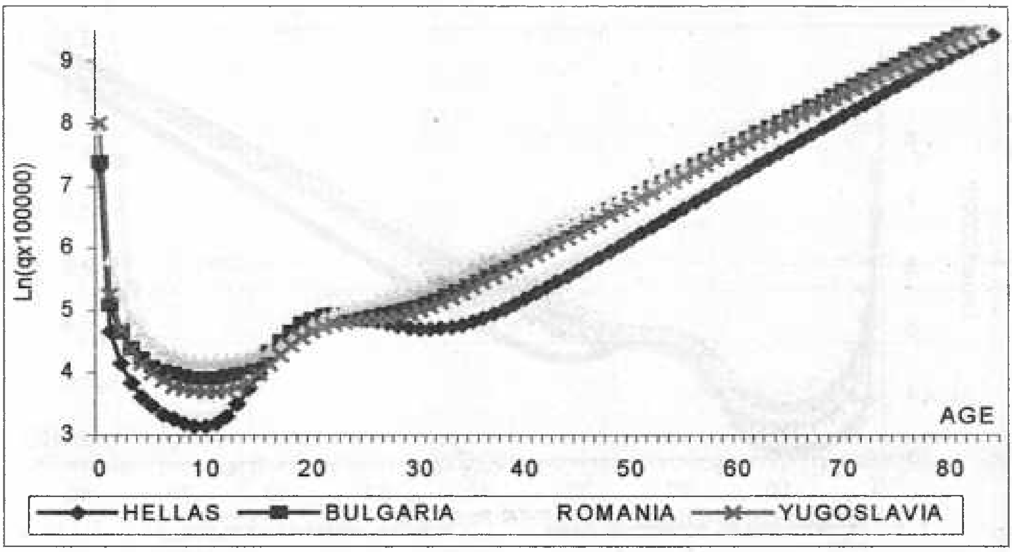

Fig. B.39: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males, in 1985

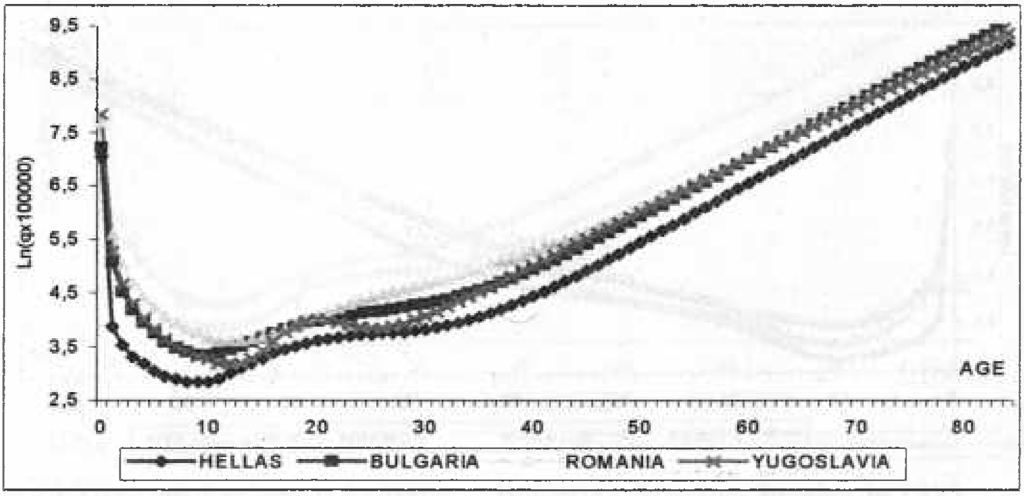

Fig. B.40: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females, in 1985

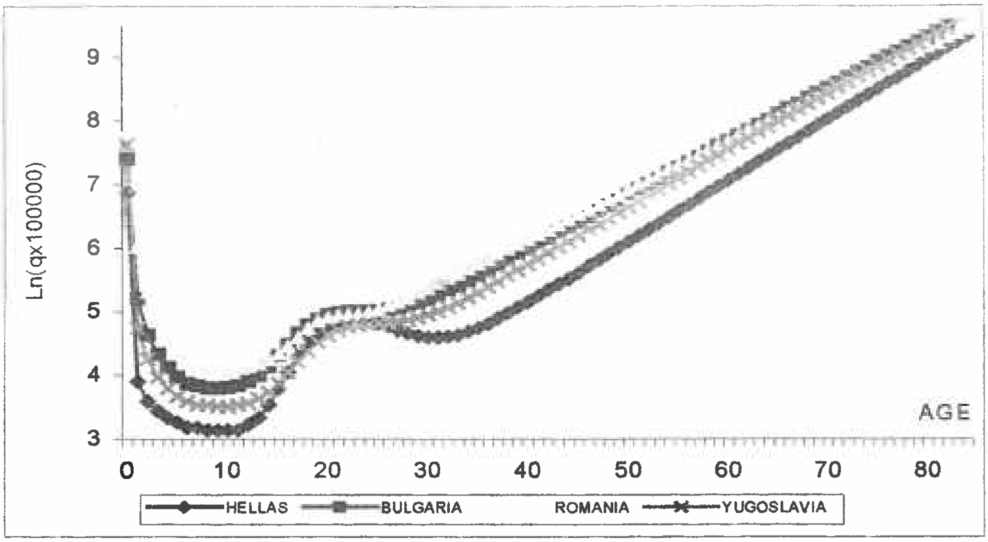

Fig. B.41: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males, in 1990

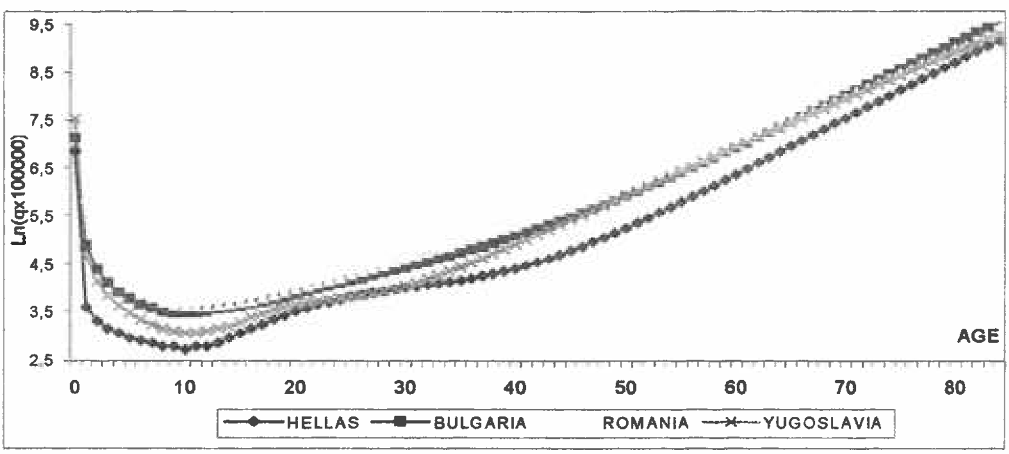

Fig. B.42: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females, in 1990

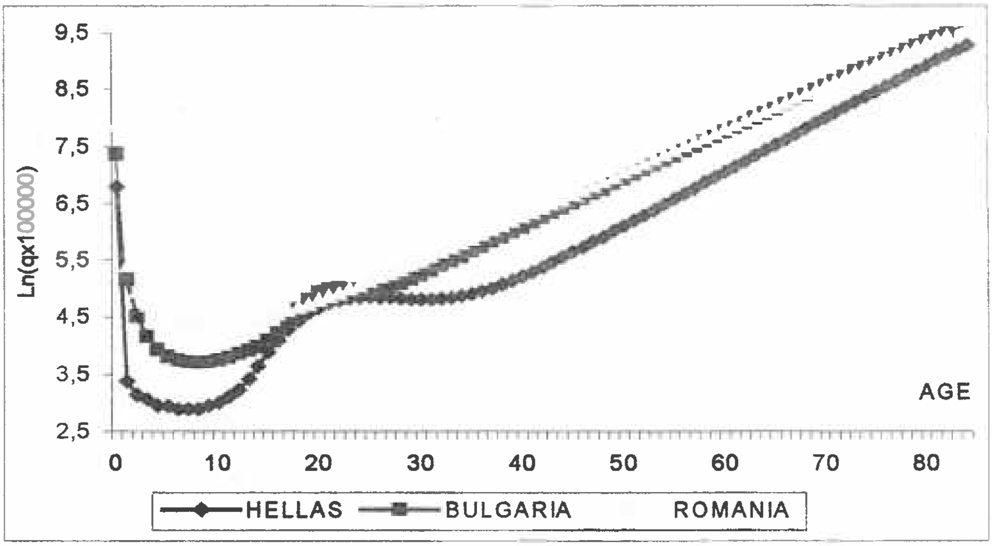

Fig. B.43: Graduated values q_x for Hellas – Bulgaria – Romania males, in 1995

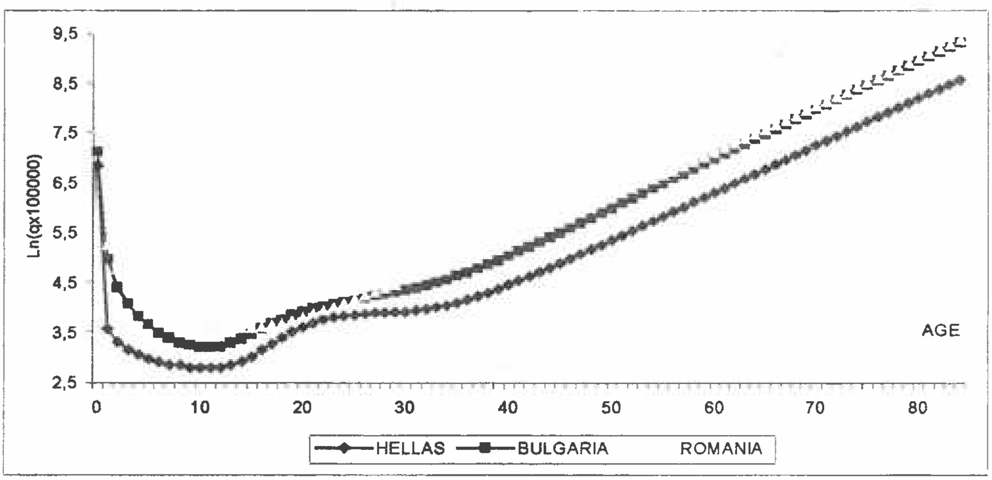

Fig. B.44: Graduated values q_x for Hellas – Bulgaria – Romania females, in 1995

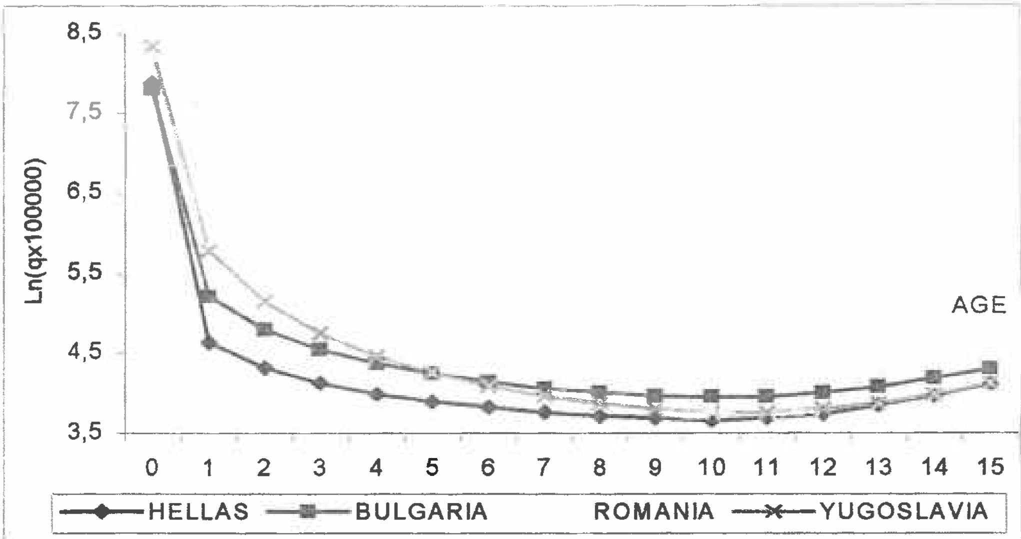

Fig. B.45: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 0-15, in 1975

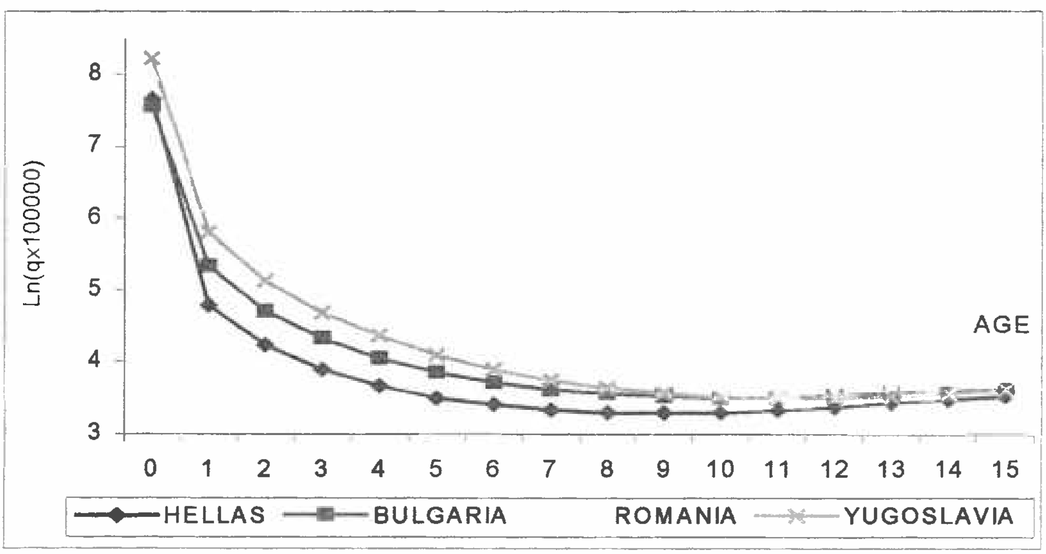

Fig. B.46: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 0-15, in 1975

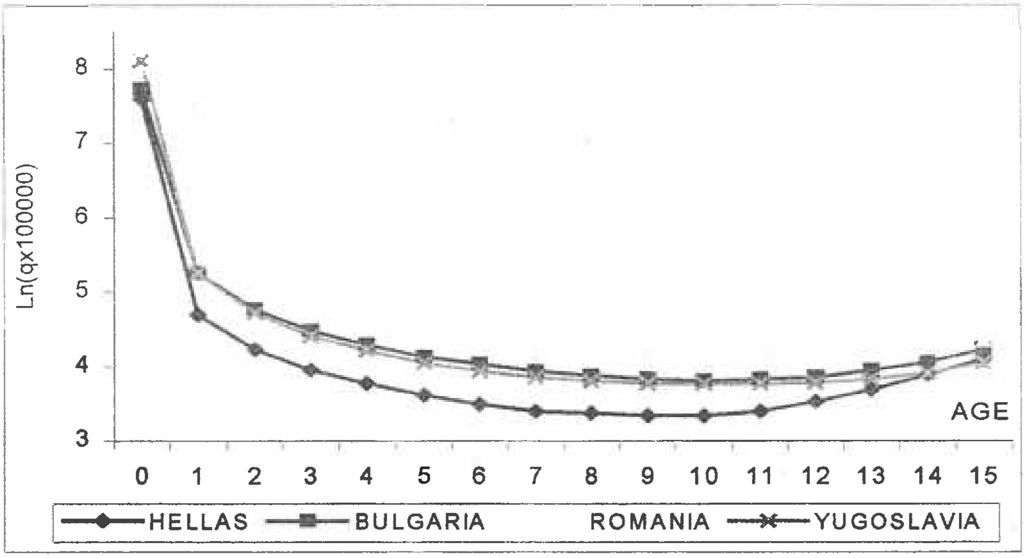

Fig. B.47: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 0-15, in 1980

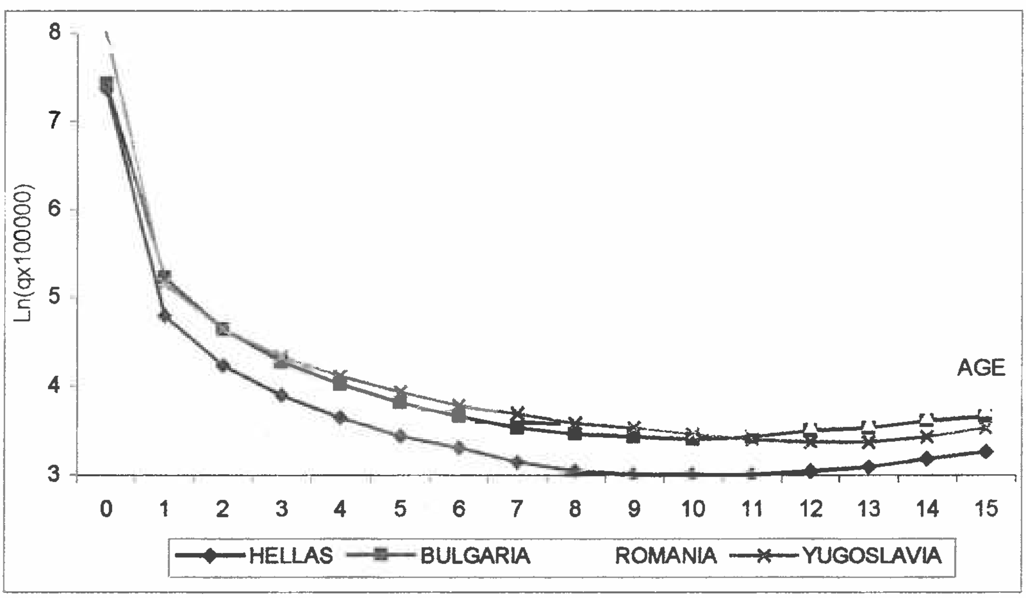

Fig. B.48: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 0-15, in 1980

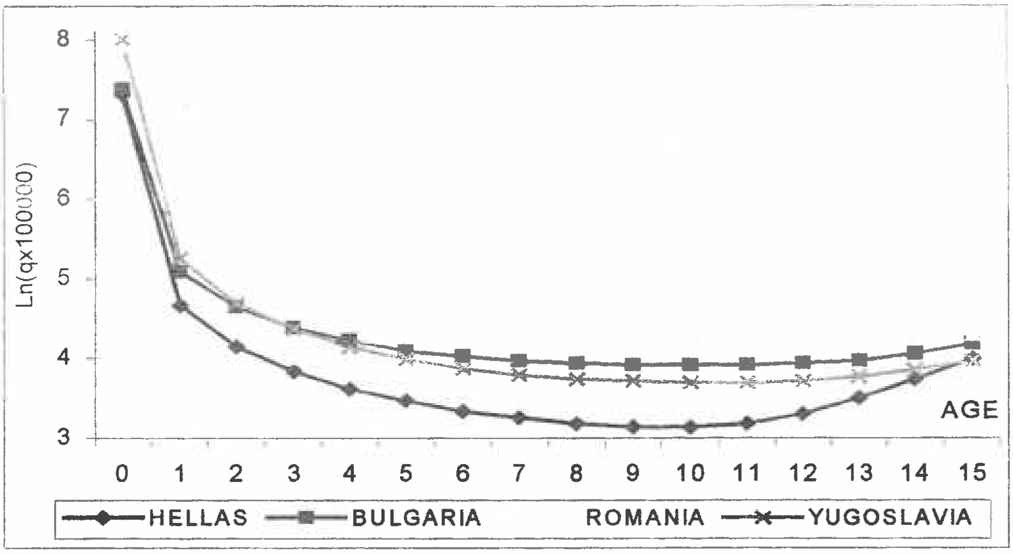

Fig. B.49: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 0-15, in 1985

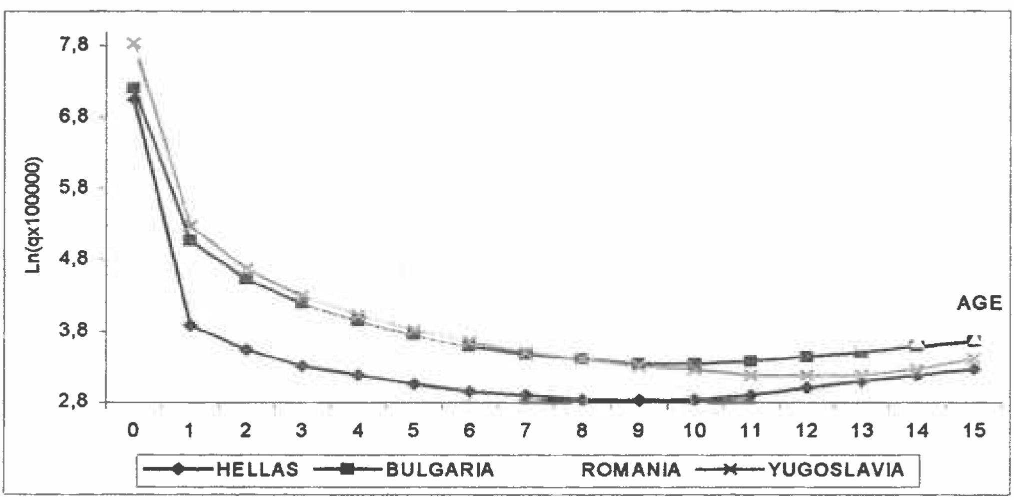

Fig. B.50: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 0-15, in 1985

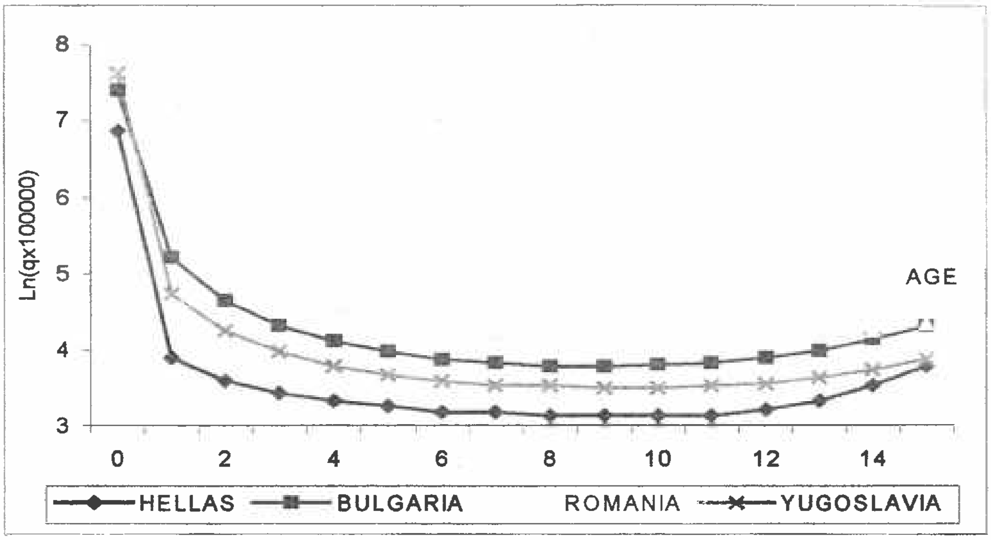

Fig. B.51: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 0-15, in 1990

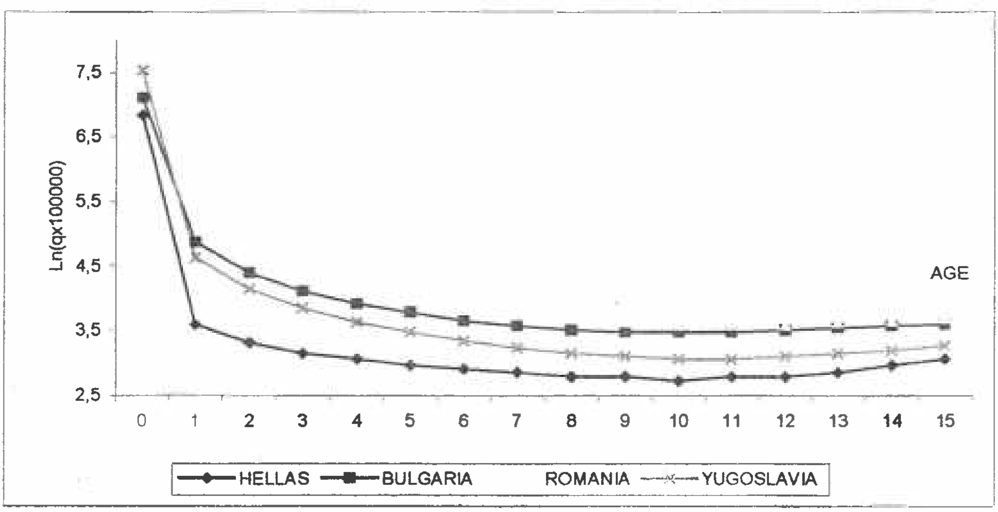

Fig. B.52: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females **aged 0-15, in 1990**

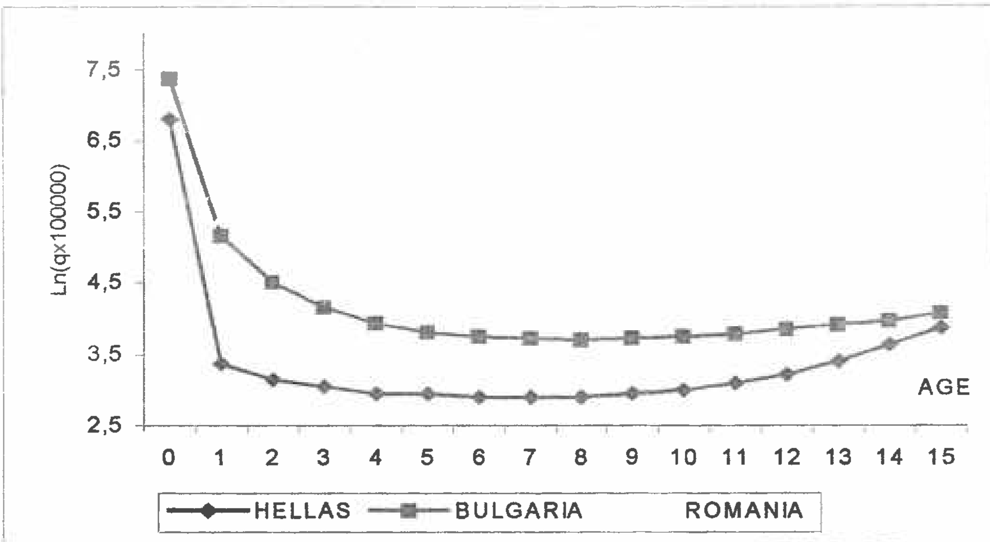

Fig. B.53: Graduated values q_x for Hellas – Bulgaria – Romania males aged 0-15, in 1995

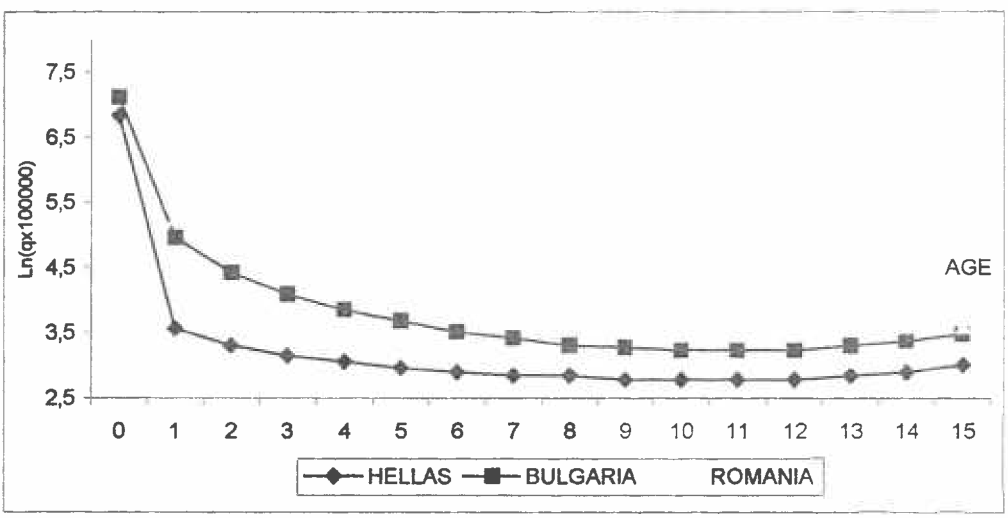

Fig. B.54: Graduated values q_x for Hellas – Bulgaria – Romania females aged 0-15, in 1995

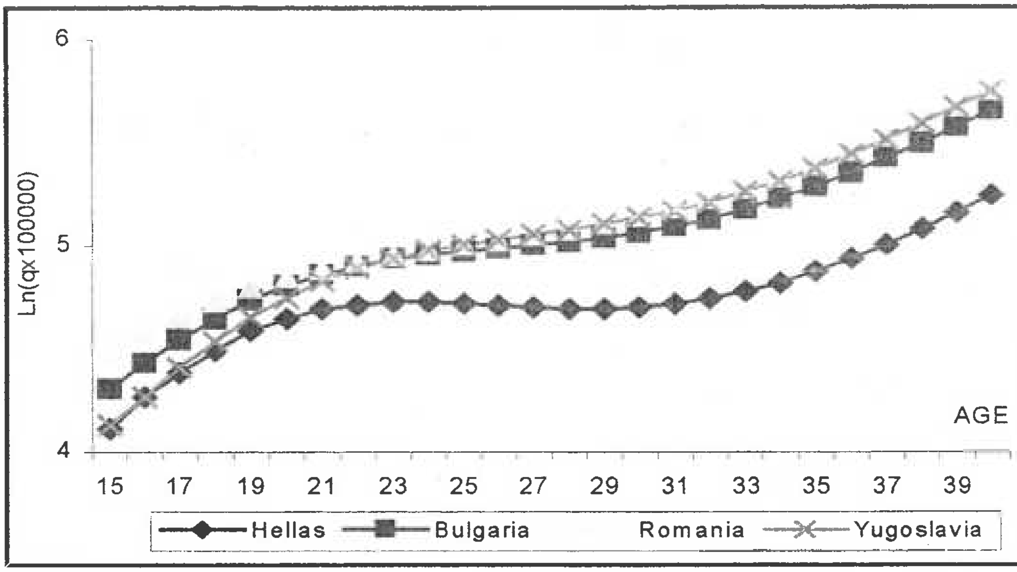

Fig. B.55: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 15-40, in 1975

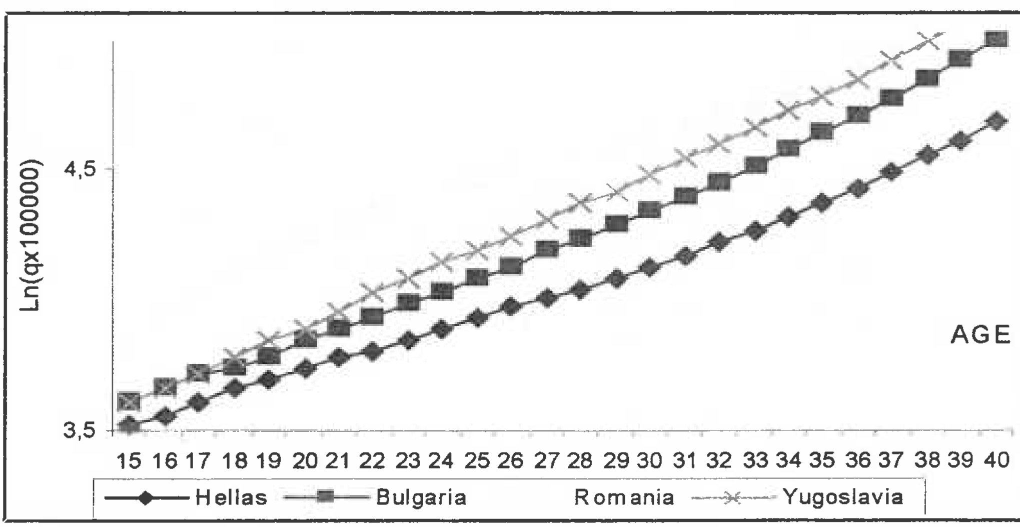

Fig. B.56: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 15-40, in 1975

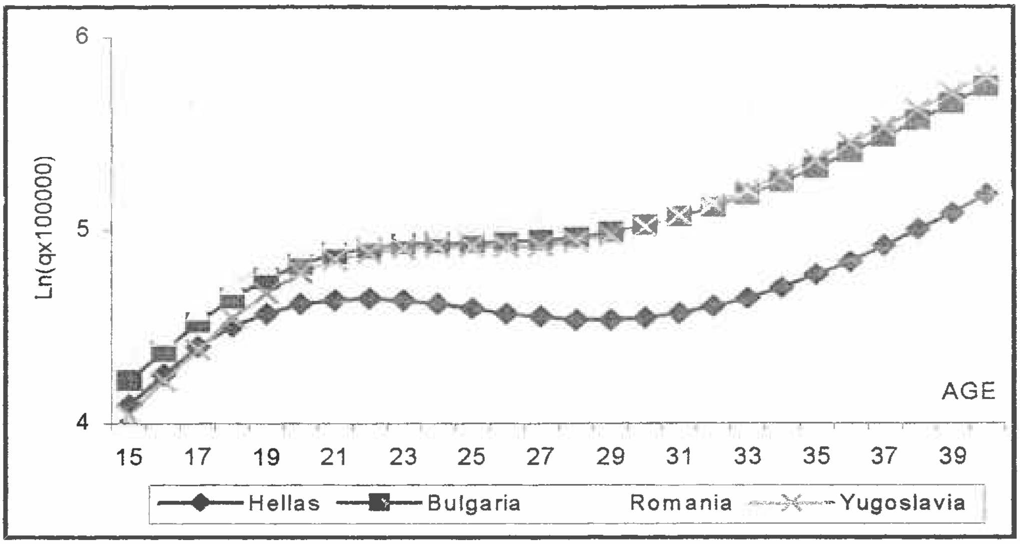

Fig. B.57: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 15-40, in 1980

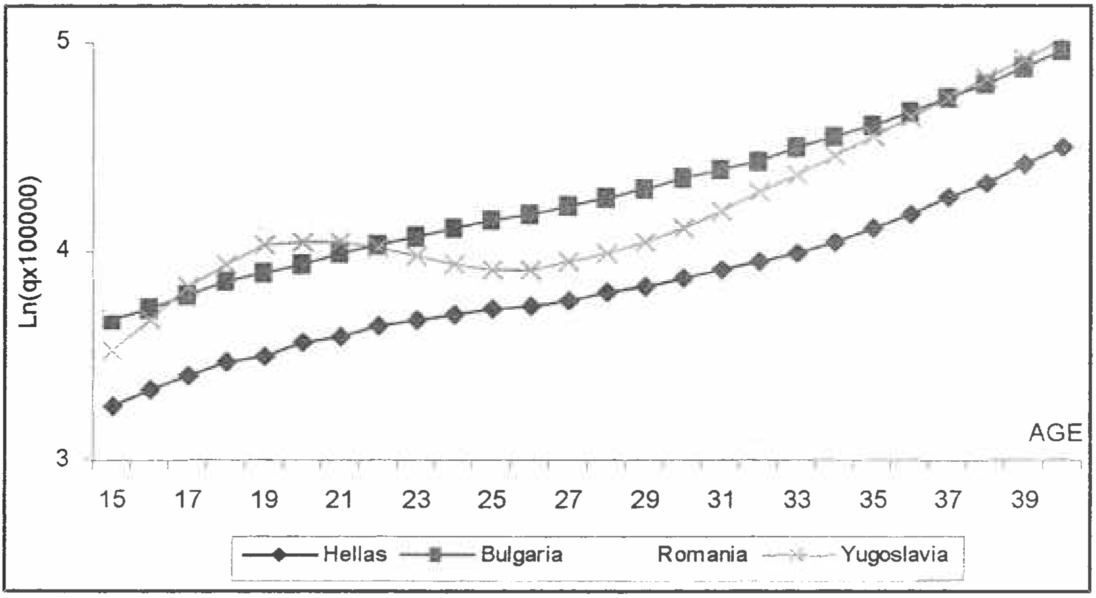

Fig. B.58: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 15-40, in 1980

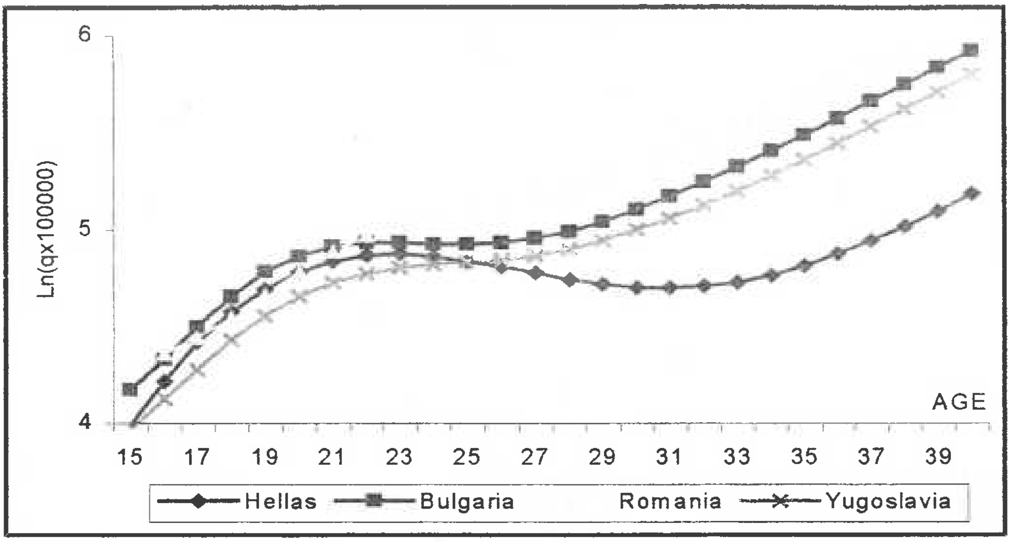

Fig. B.59: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 15-40, in 1985

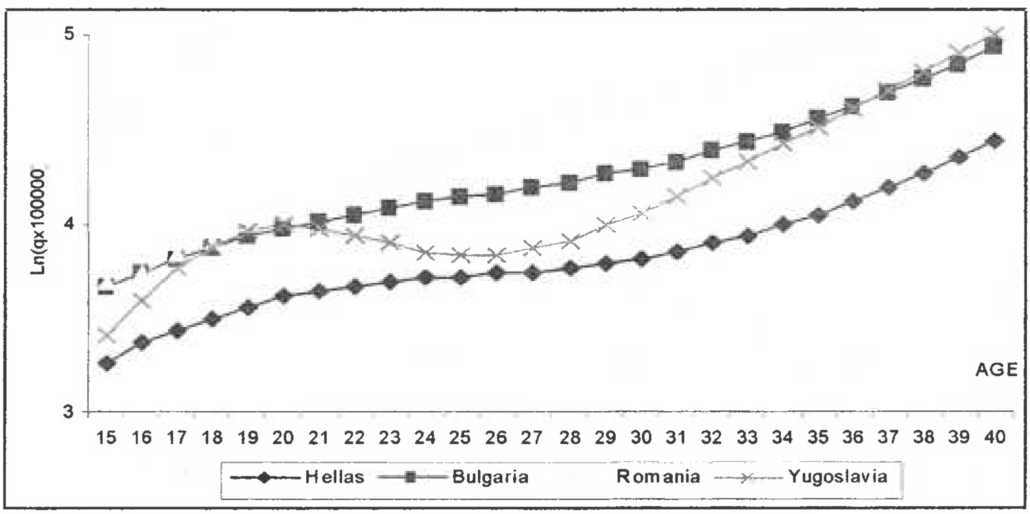

Fig. B.60: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 15-40, in 1985

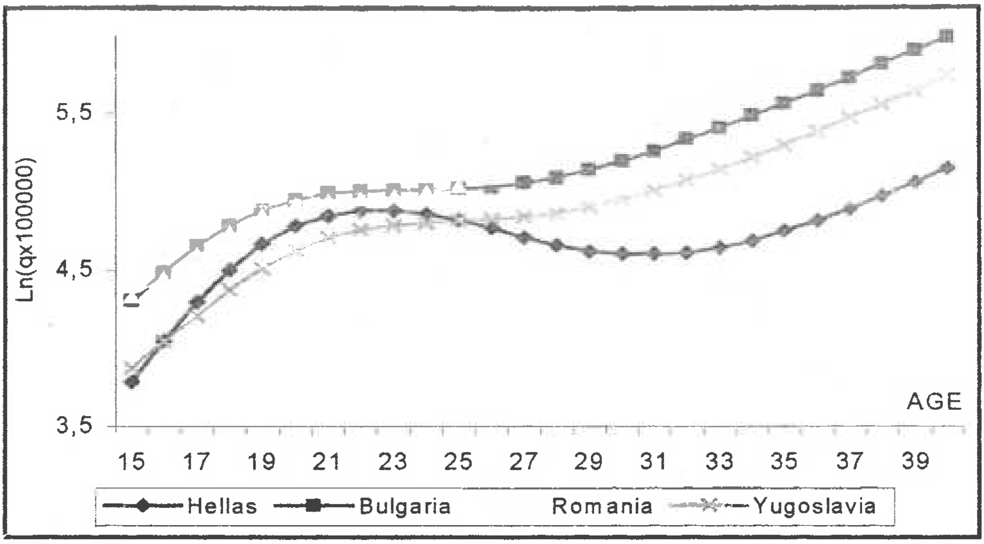

Fig. B.61: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 15-40, in 1990

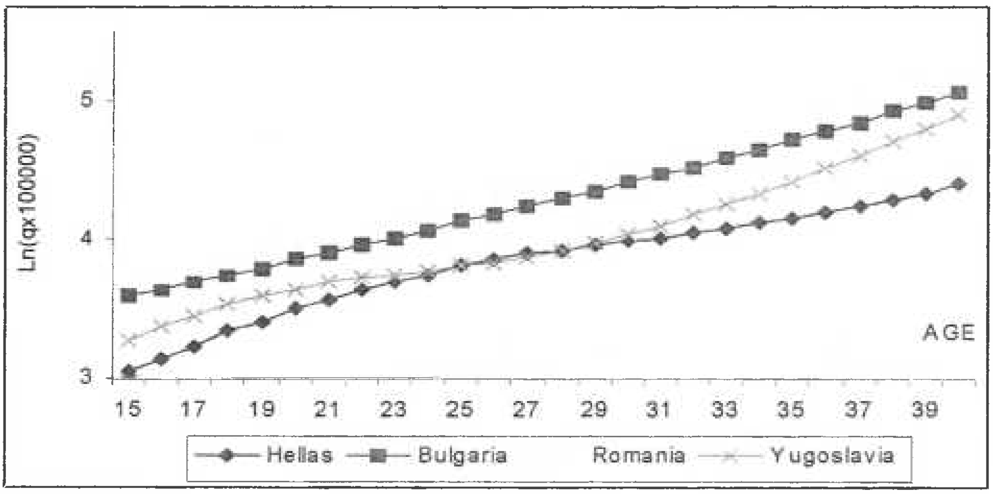

Fig. B.62: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 15-40, in 1990

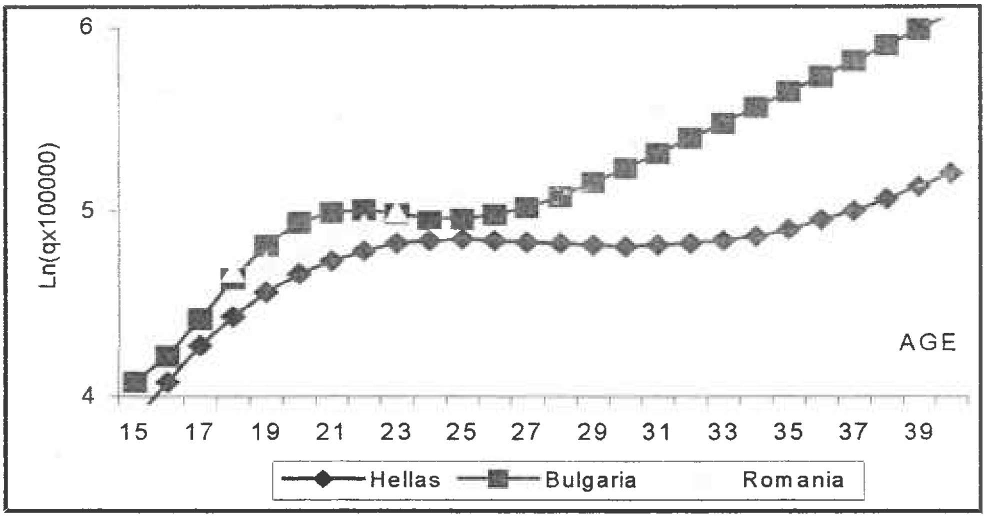

Fig. B.63: Graduated values q_x for Hellas – Bulgaria – Romania males aged 15-40, in 1995

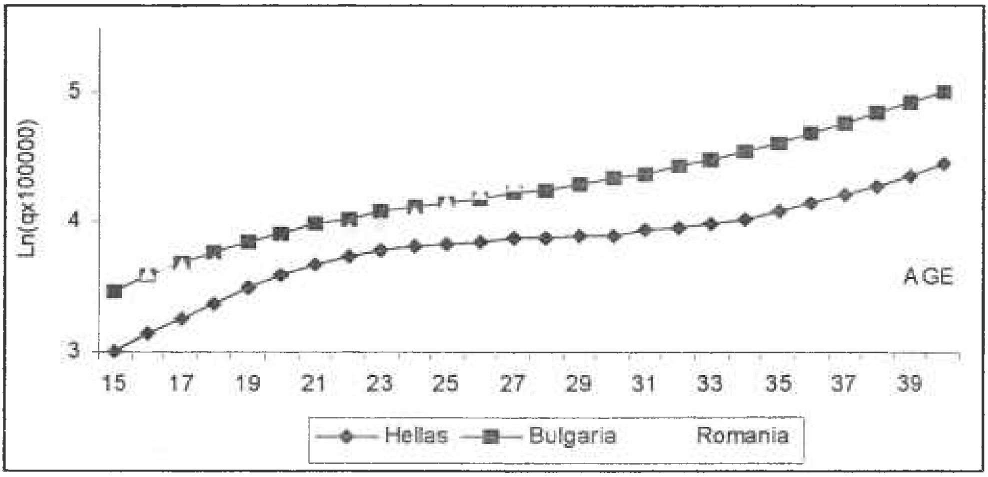

Fig. B.64: Graduated values q_x for Hellas – Bulgaria – Romania females aged 15-40, in 1995

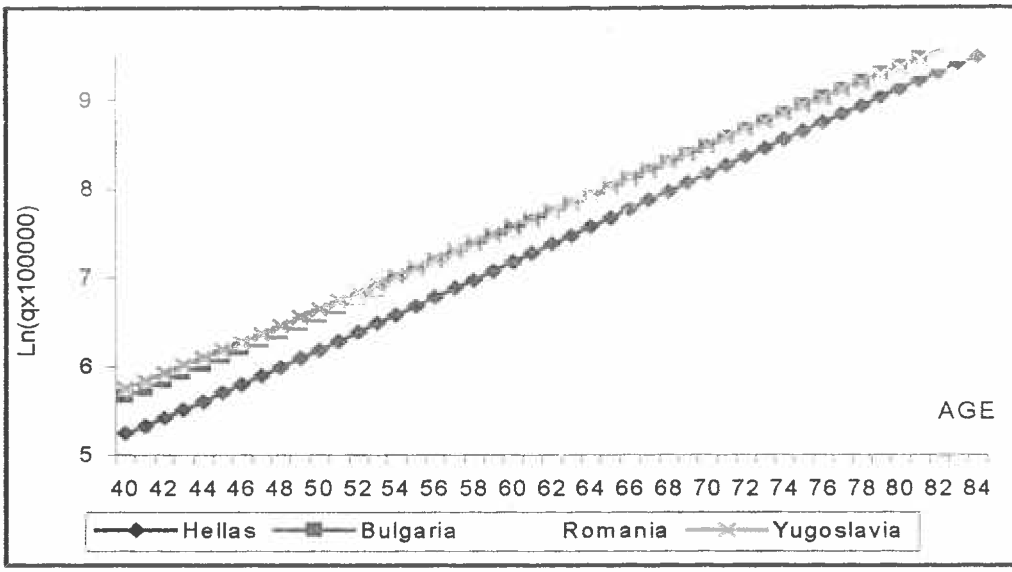

Fig. B.65: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 40-80+, in 1975

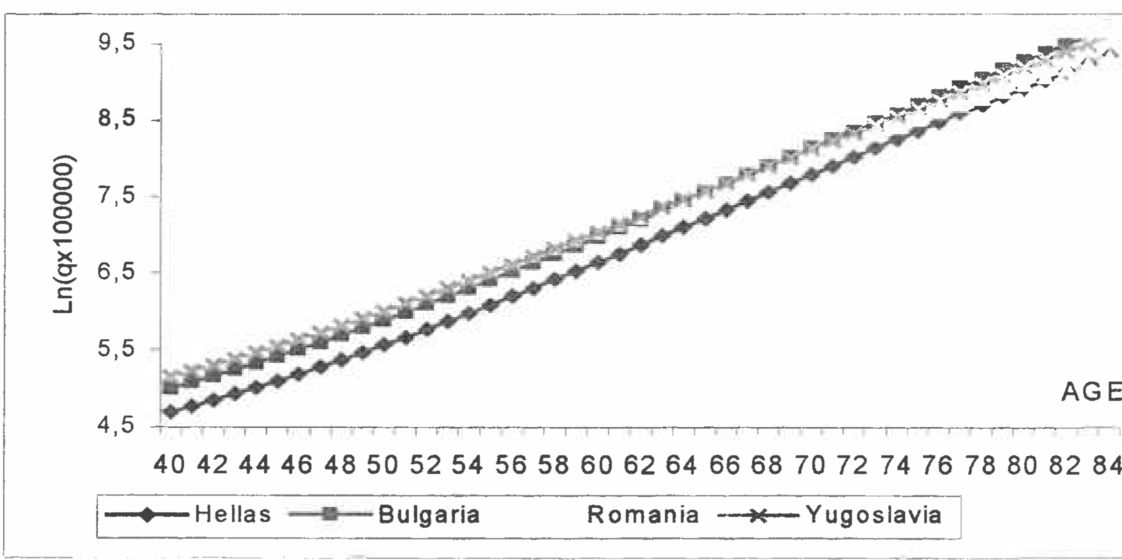

Fig. B.66: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 40-80+, in 1975

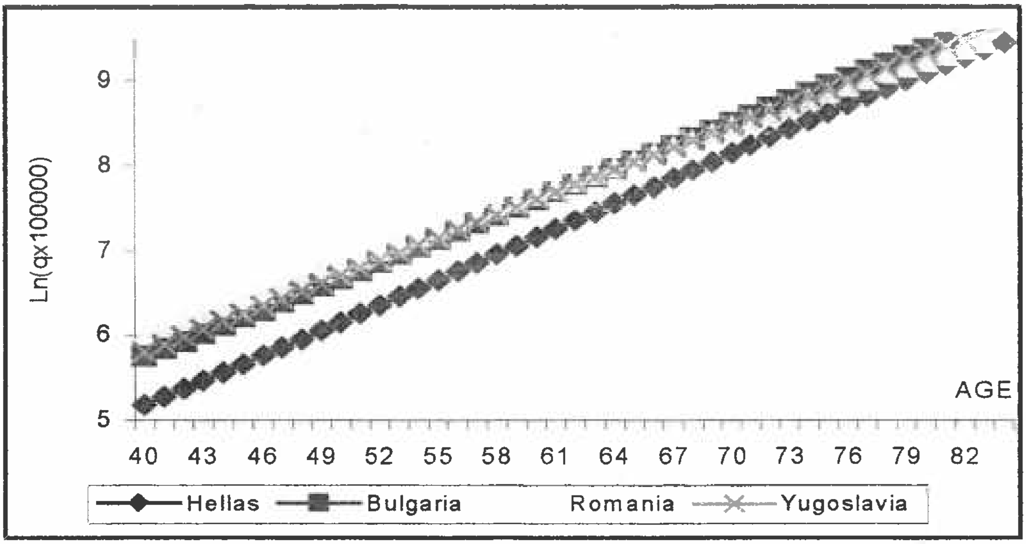

Fig. B.67: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 40-80+, in 1980

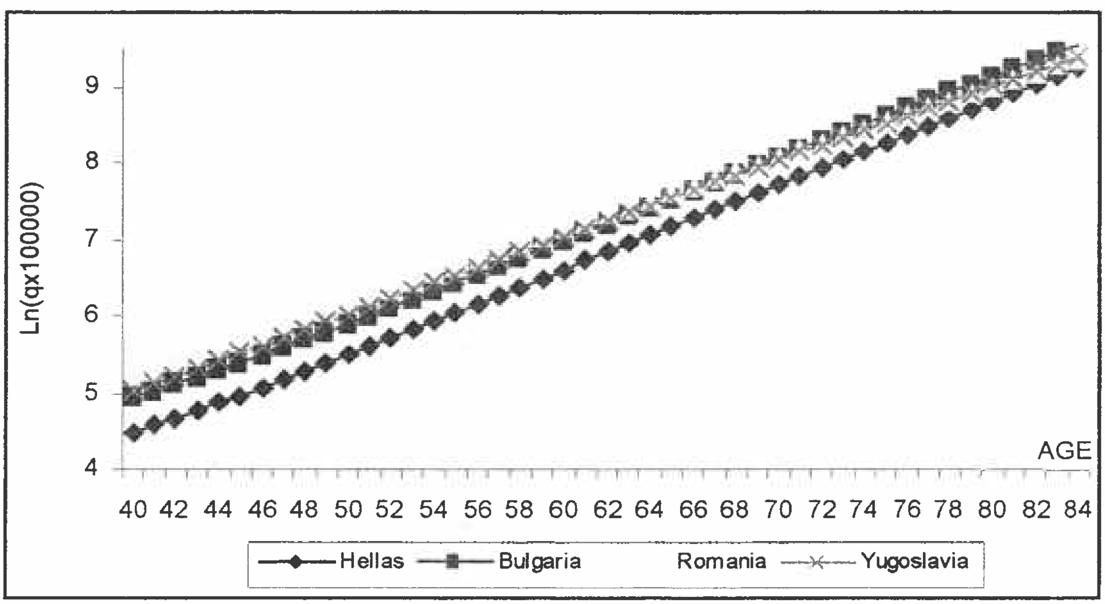

Fig. B.68: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 40-80+, in 1980

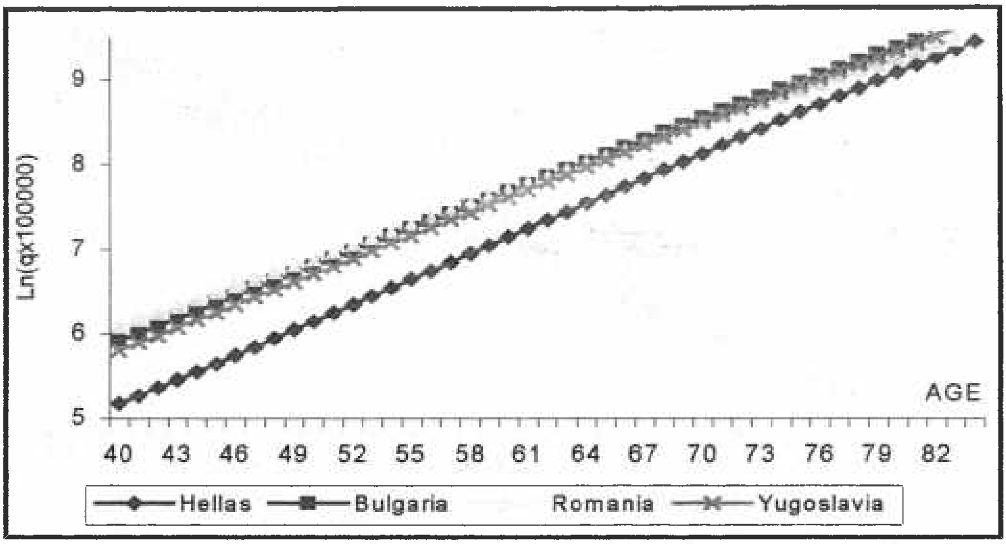

Fig. B.69: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 40-80+, in 1985

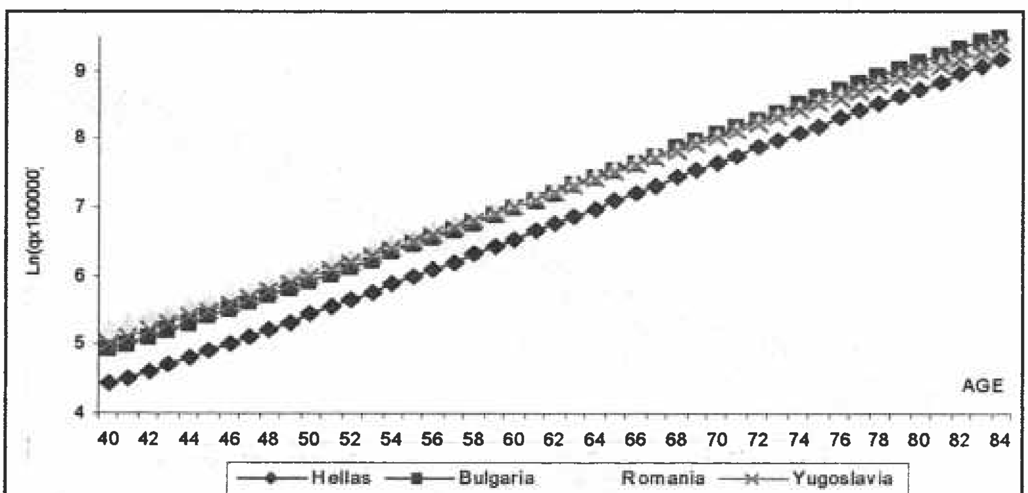

Fig. B.70: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 40-80+, in 1985

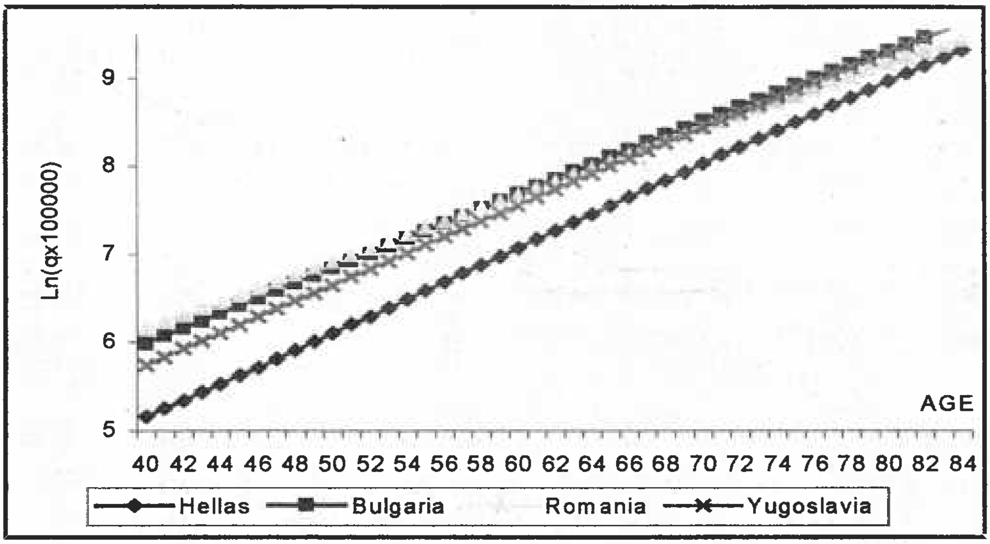

Fig. B.71: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia males aged 40-80+, in 1990

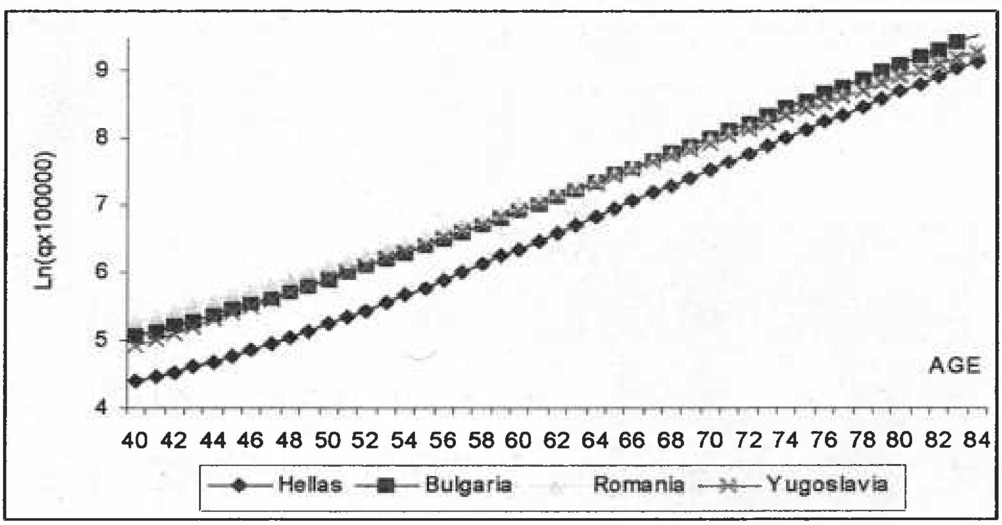

Fig. B.72: Graduated values q_x for Hellas – Bulgaria – Romania - Yugoslavia females aged 40-80+, in 1990

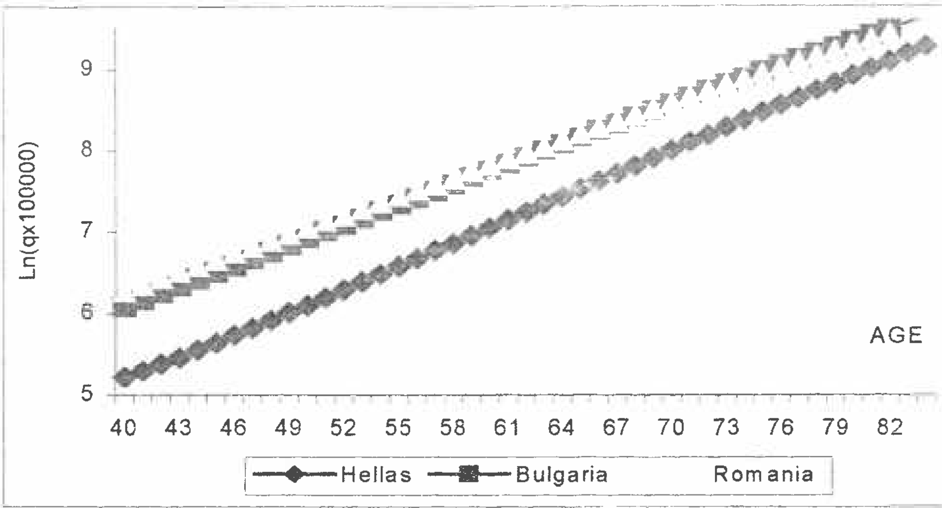

Fig. B.73: Graduated values q_x for Hellas – Bulgaria – Romania males aged 40-80+, in 1995

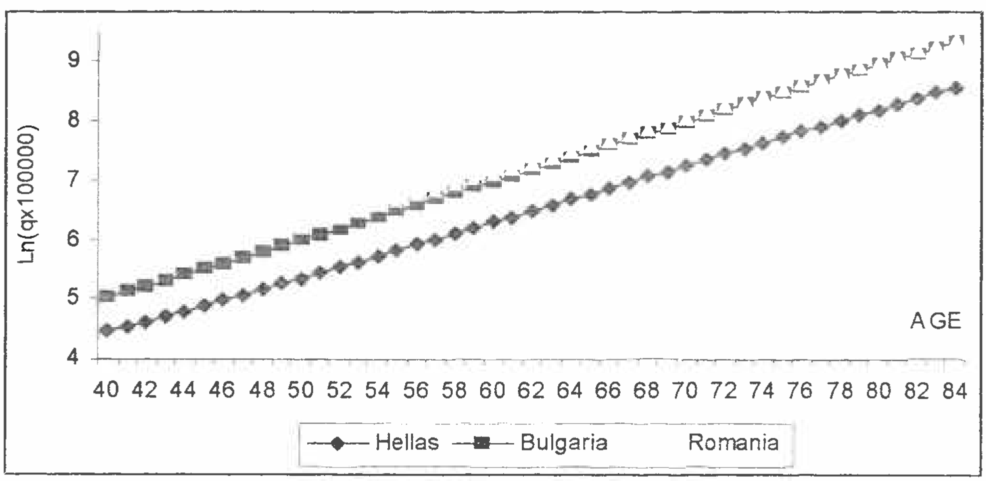

Fig. B.74: Graduated values q_x for Hellas – Bulgaria – Romania females aged 40-80+, in 1995

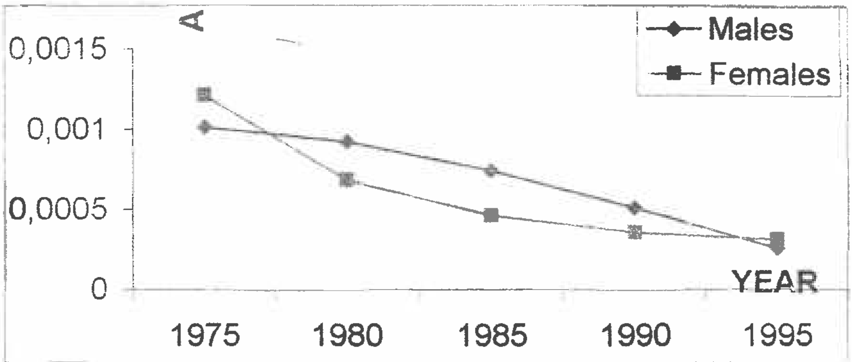

Fig. B.75 : Progression of parameter estimates A, for Hellas

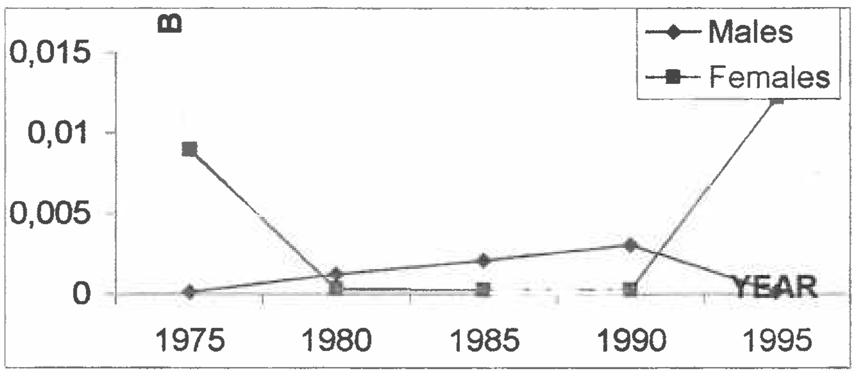

Fig. B.76 : Progression of parameter estimates B, for Hellas

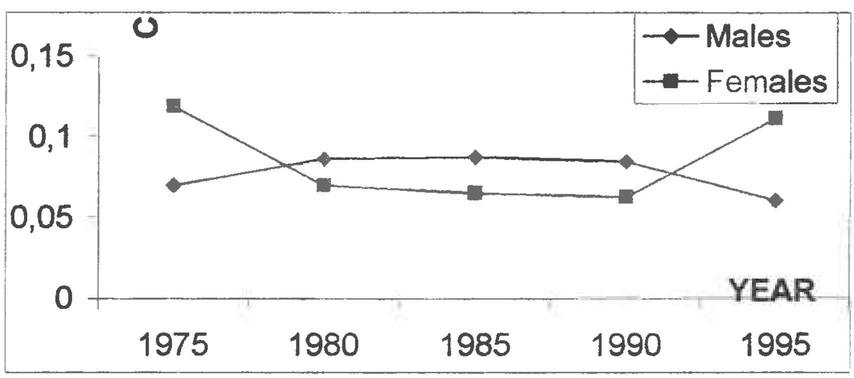

Fig. B.77 : Progression of parameter estimates C, for Hellas

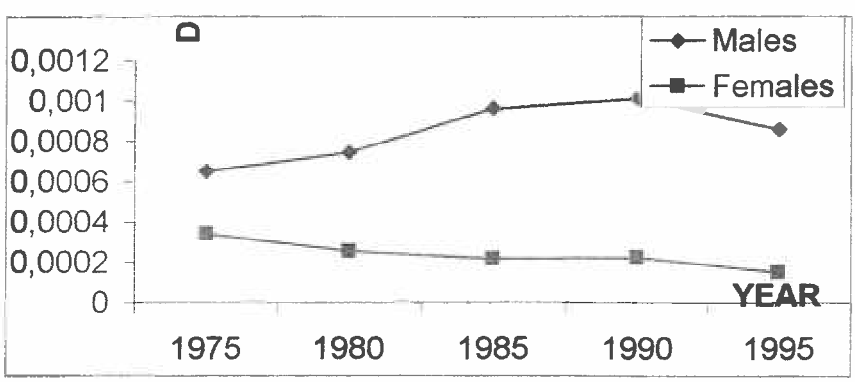

Fig. B.78 : Progression of parameter estimates D, for Hellas

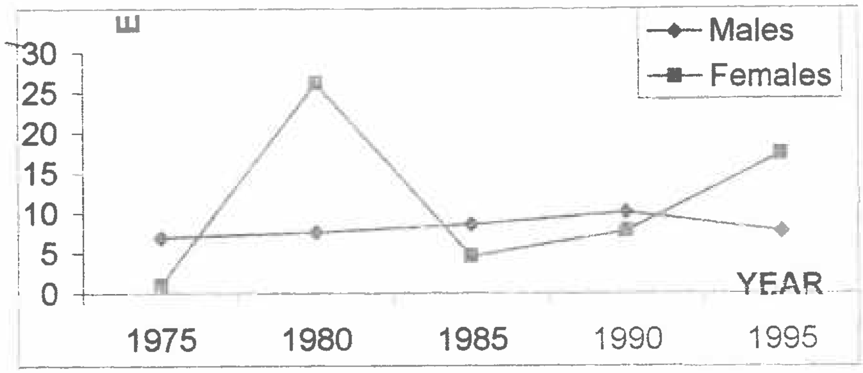

Fig. B.79 : Progression of parameter estimates E, for Hellas

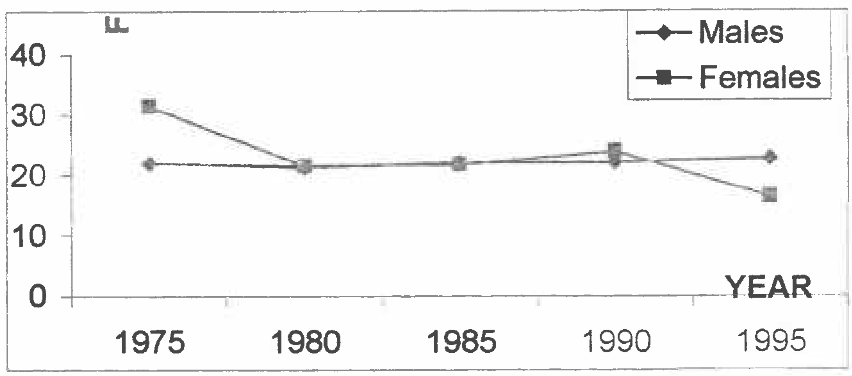

Fig. B.80 : Progression of parameter estimates F, for Hellas

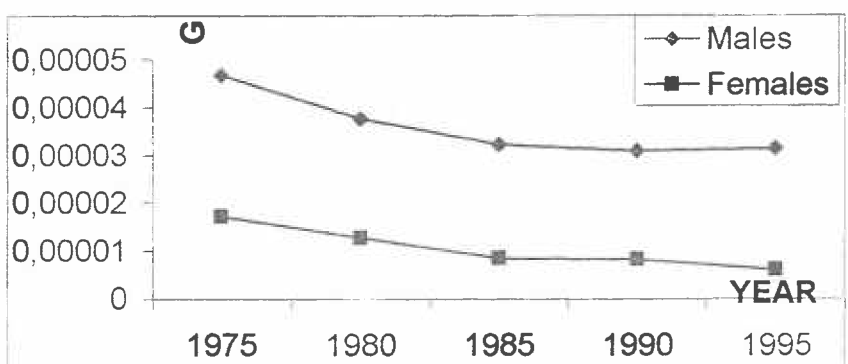

Fig. B.81 : Progression of parameter estimates G, for Hellas

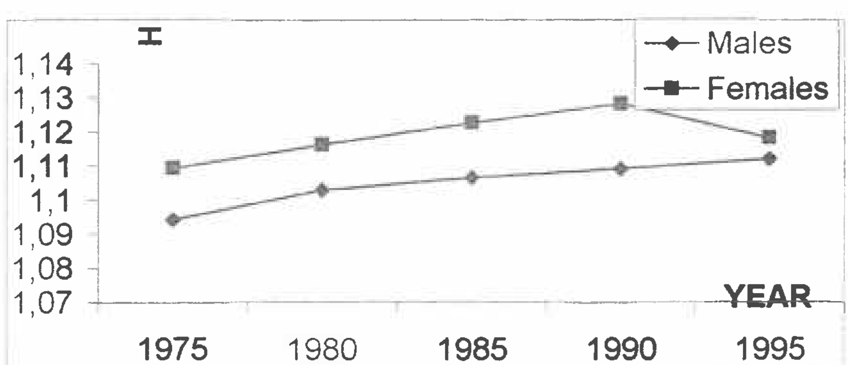

Fig. B.82 : Progression of parameter estimates H, for Hellas

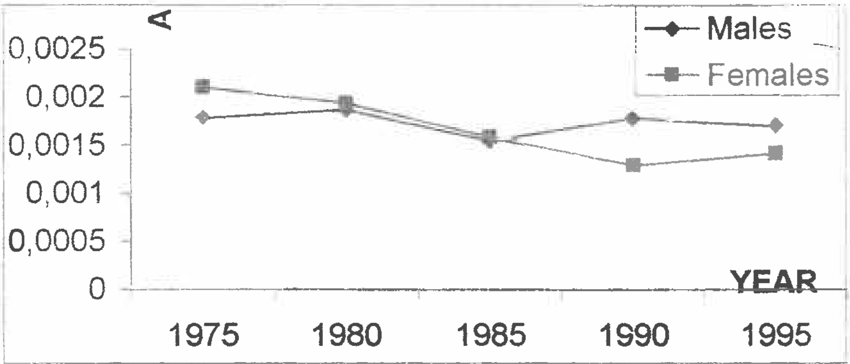

Fig. B.83 : Progression of parameter estimates A, for Bulgaria

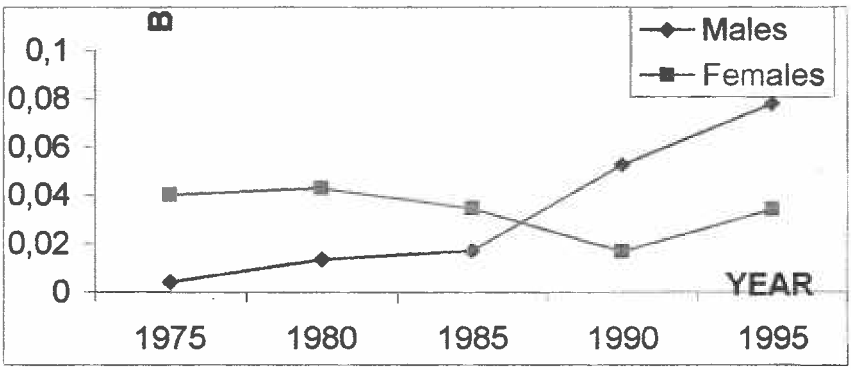

Fig. B.84 : Progression of parameter estimates B, for Bulgaria

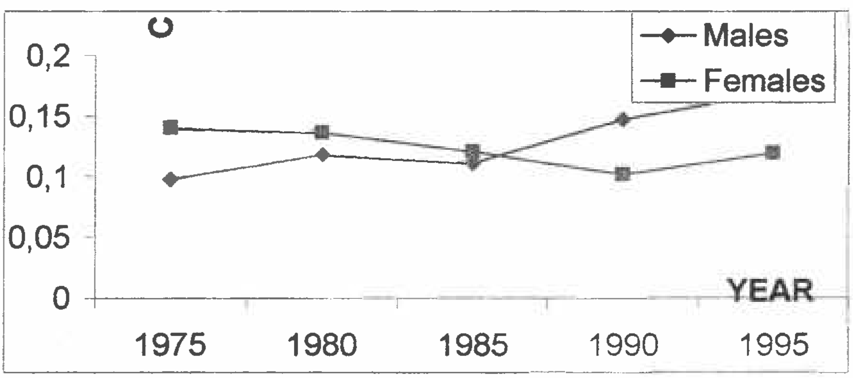

Fig. B.85 : Progression of parameter estimates C, for Bulgaria

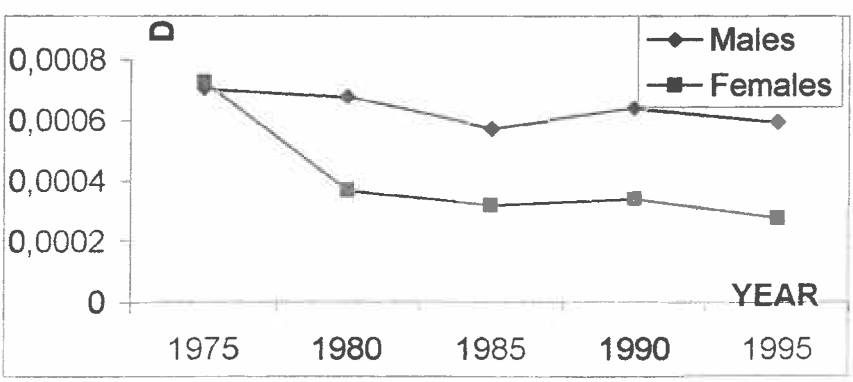

Fig. B.86 : Progression of parameter estimates D, for Bulgaria

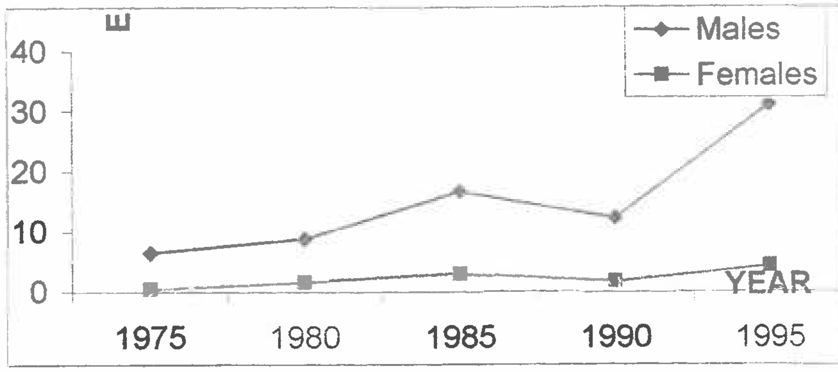

Fig. B.87 : Progression of parameter estimates E, for Bulgaria

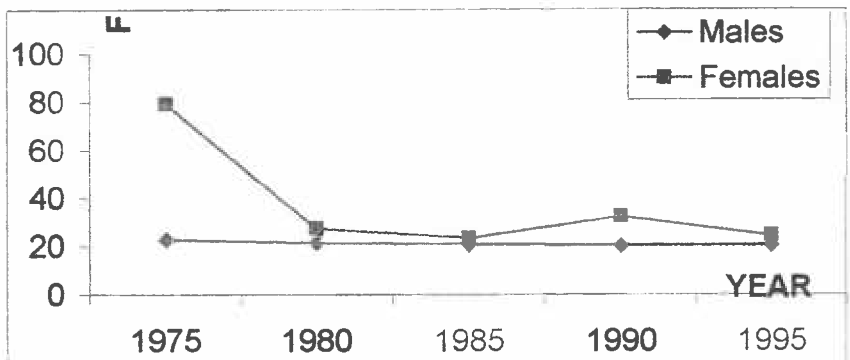

Fig. B.88 : Progression of parameter estimates F, for Bulgaria

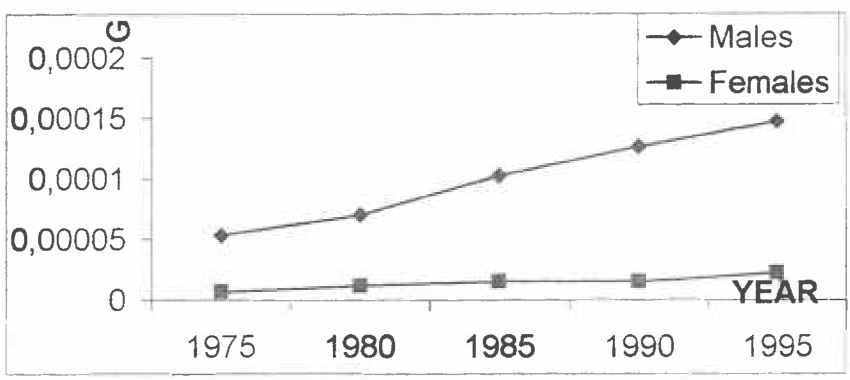

Fig. B.89 : Progression of parameter estimates G, for Bulgaria

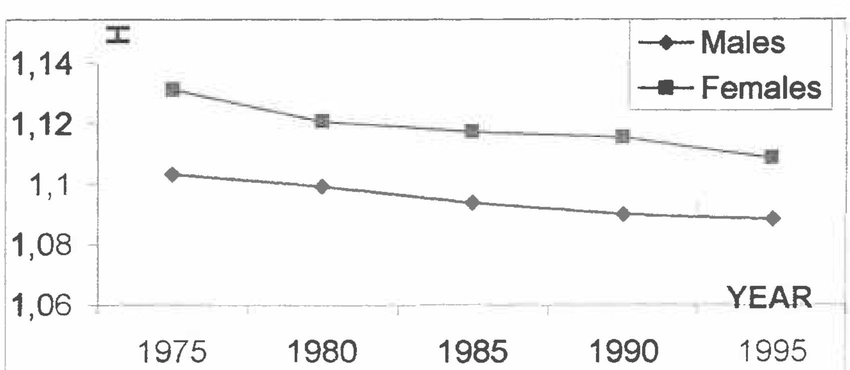

Fig. B.90 : Progression of parameter estimates H, for Bulgaria

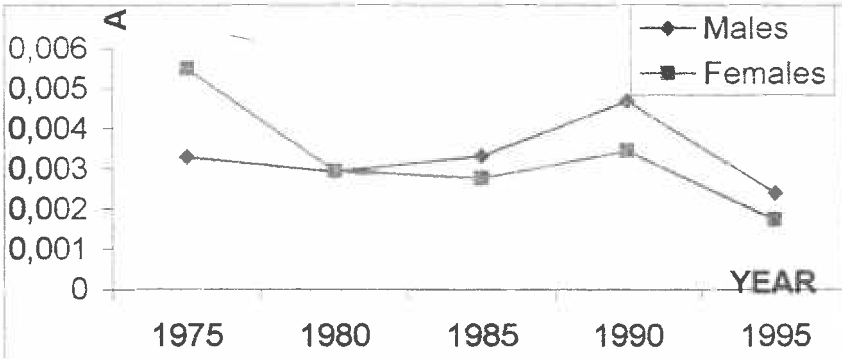

Fig. B.91 : Progression of parameter estimates A, for Romania

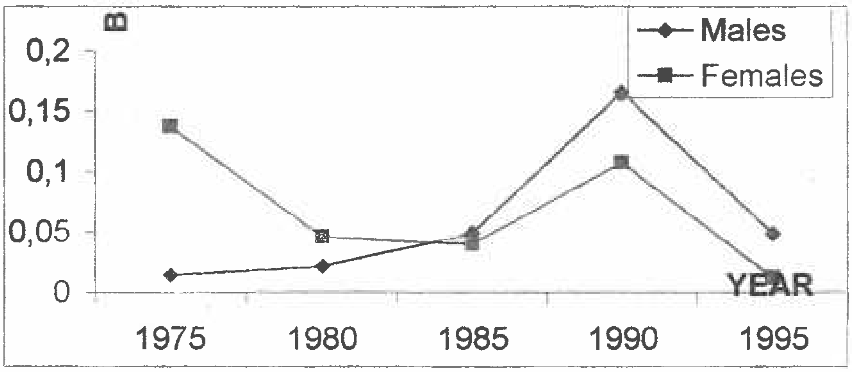

Fig. B.92 : Progression of parameter estimates B, for Romania

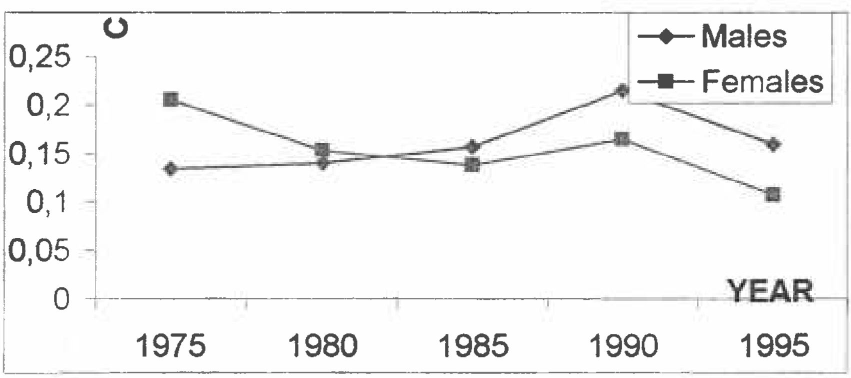

Fig. B.93 : Progression of parameter estimates C, for Romania

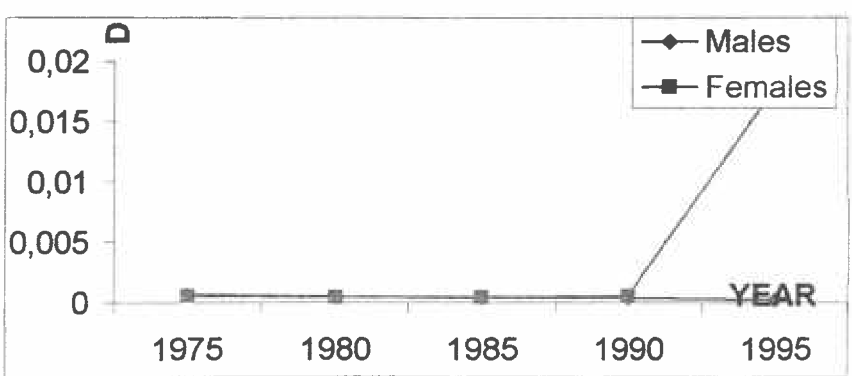

Fig. B.94 : Progression of parameter estimates D, for Romania

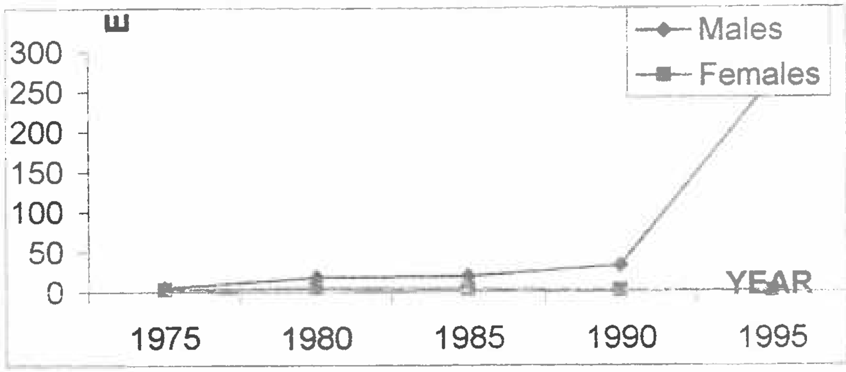

Fig. B.95 : Progression of parameter estimates E, for Romania

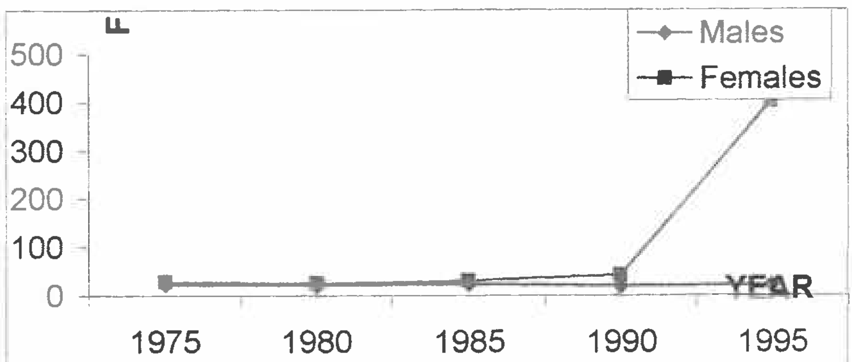

Fig. B.96 : Progression of parameter estimates F, for Romania

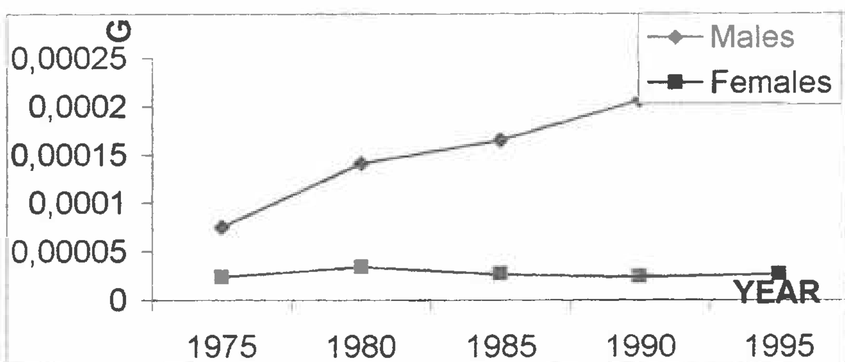

Fig. B.97 : Progression of parameter estimates G, for Romania

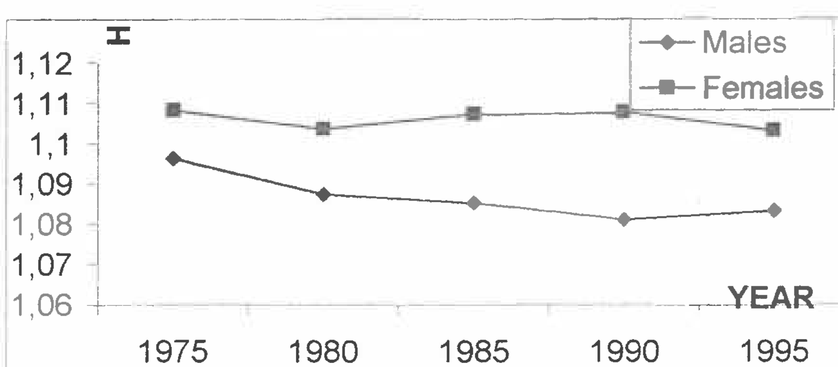

Fig. B.98 : Progression of parameter estimates H, for Romania

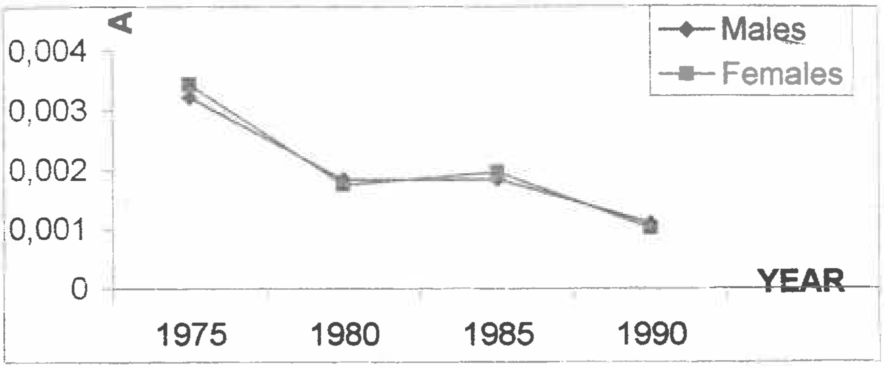

Fig. B.99 : Progression of parameter estimates A, for Yugoslavia

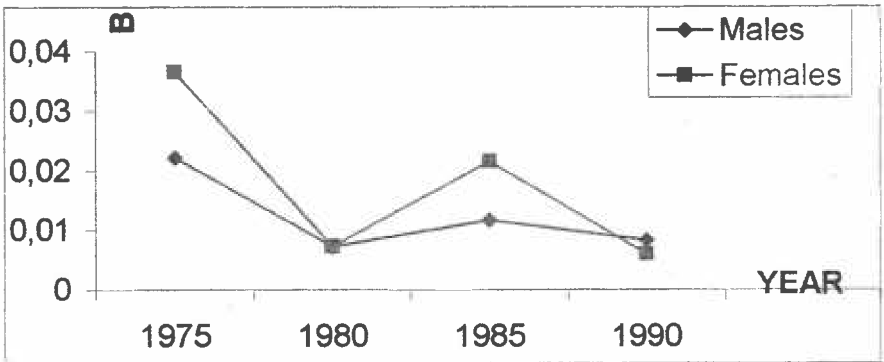

Fig. B.100 : Progression of parameter estimates B, for Yugoslavia

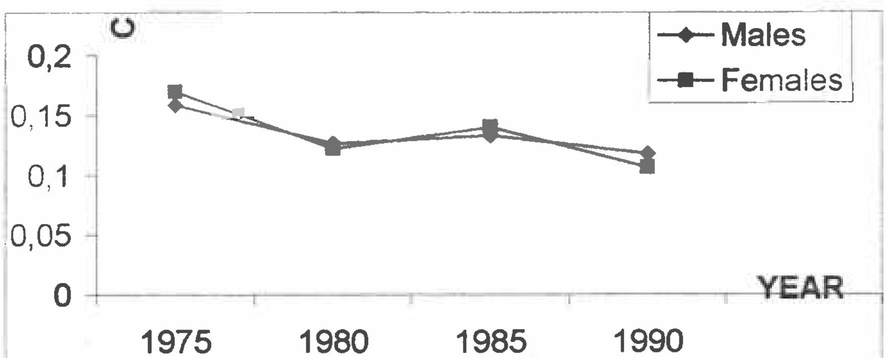

Fig. B.101 : Progression of parameter estimates C, for Yugoslavia

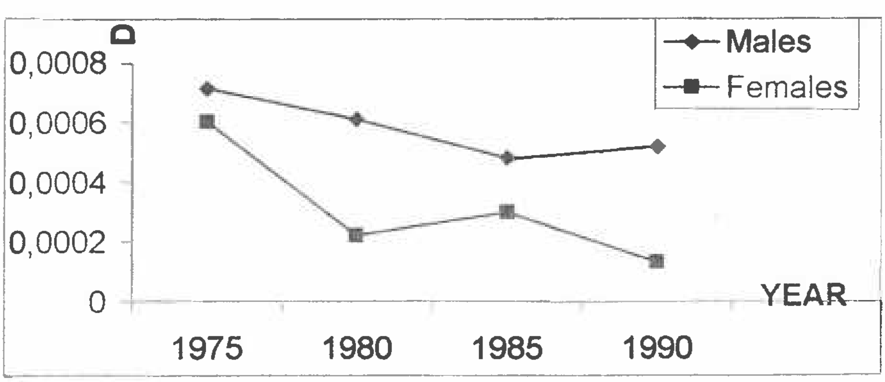

Fig. B.102 : Progression of parameter estimates D, for Yugoslavia

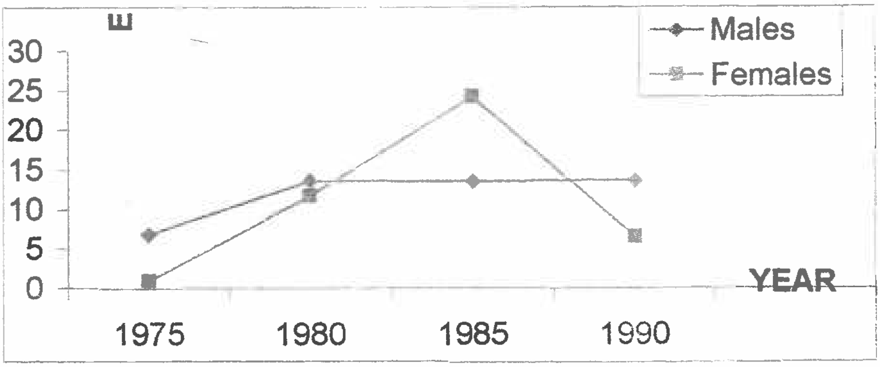

Fig. B.103 : Progression of parameter estimates E, for Yugoslavia

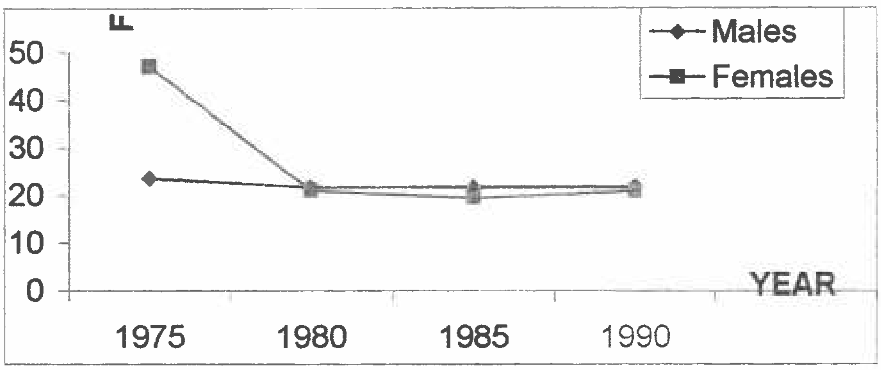

Fig. B.104 : Progression of parameter estimates F, for Yugoslavia

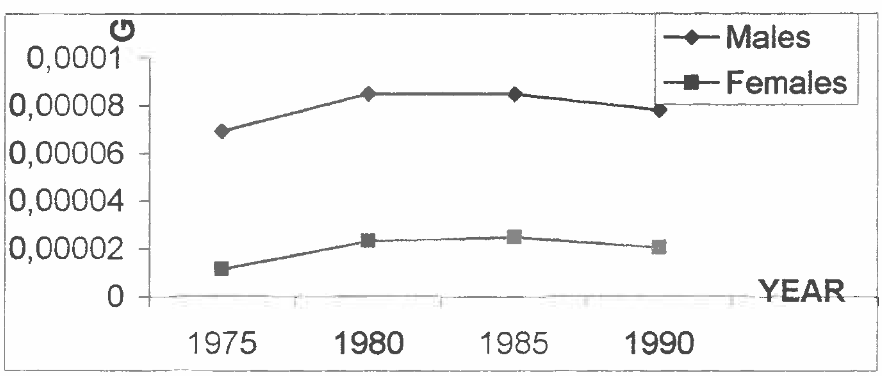

Fig. B.105 : Progression of parameter estimates G, for Yugoslavia

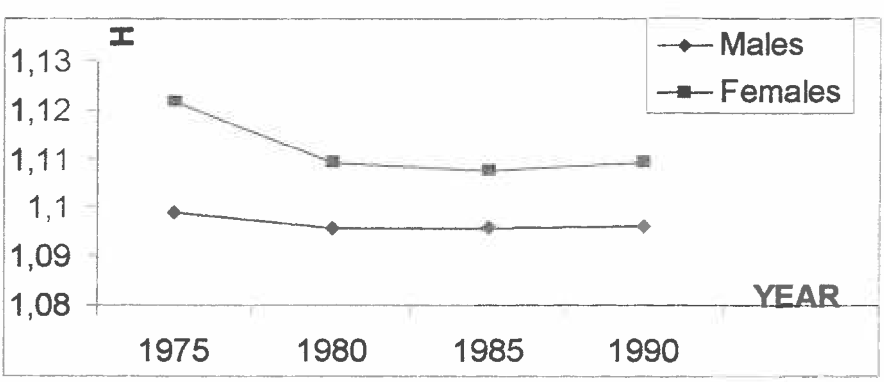

Fig. B.106 : Progression of parameter estimates H, for Yugoslavia

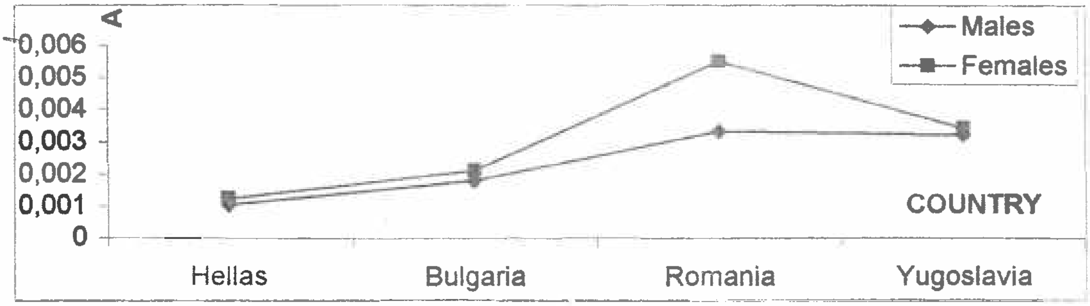

Fig. B.107 : Progression of parameter estimates A, in 1975, for Hellas - Bulgaria – Romania - Yugoslavia

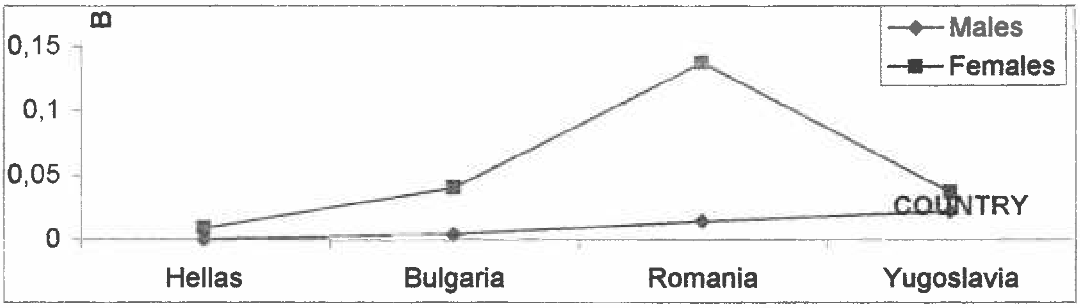

Fig. B.108 : Progression of parameter estimates B, in 1975, for Hellas - Bulgaria – Romania - Yugoslavia

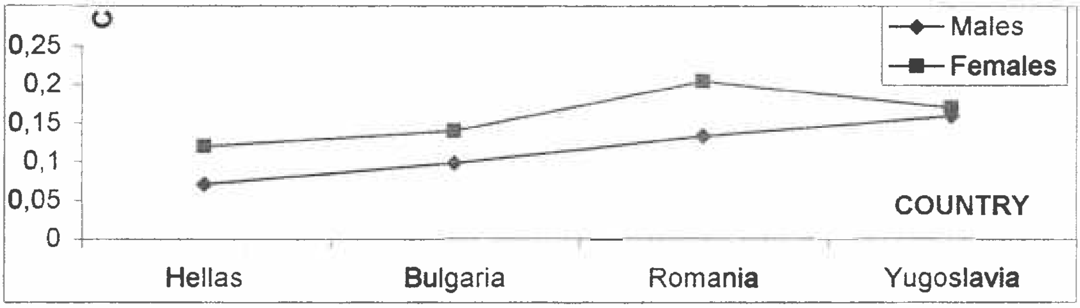

Fig. B.109 : Progression of parameter estimates C, in 1975, for Hellas - Bulgaria – Romania - Yugoslavia

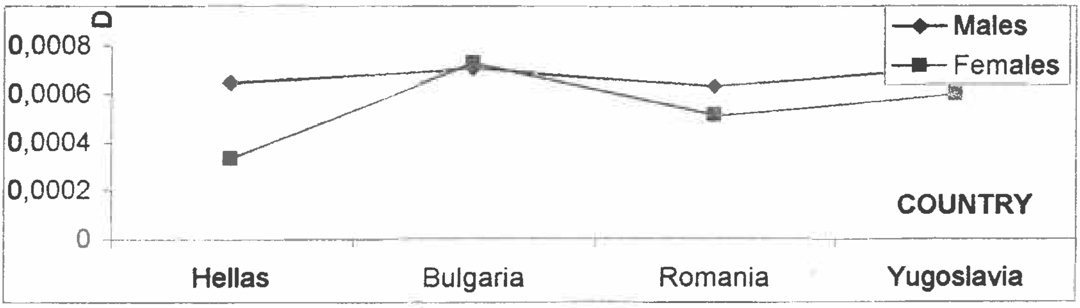

Fig. B.110 : Progression of parameter estimates D, in 1975, for Hellas - Bulgaria – Romania - Yugoslavia

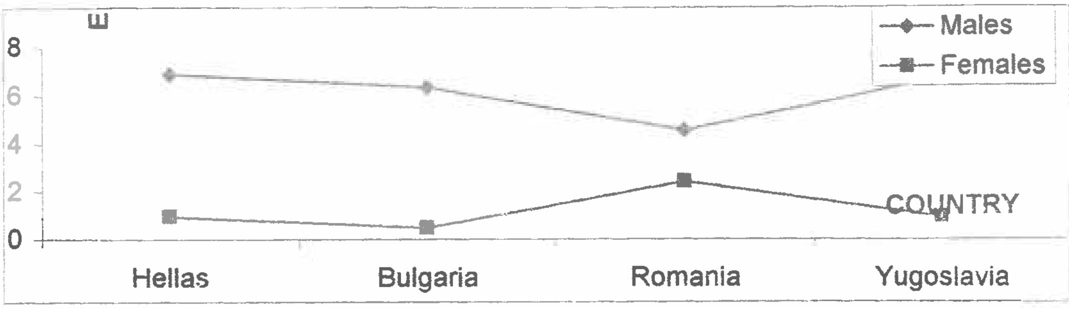

Fig. B.111 : Progression of parameter estimates E, in 1975, for Hellas - Bulgaria – Romania - Yugoslavia

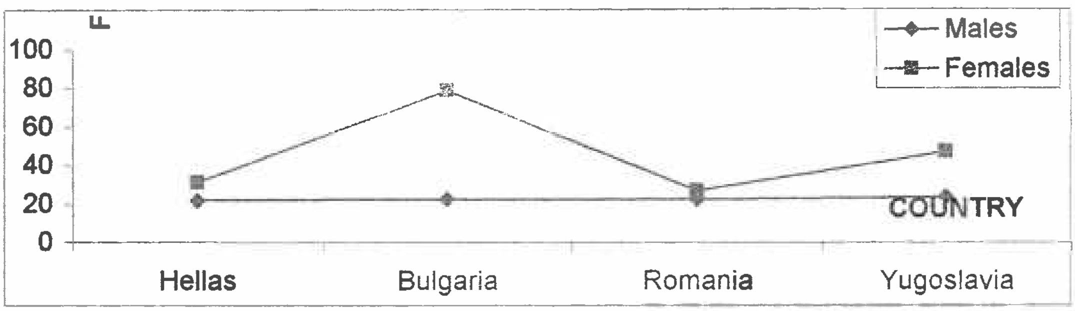

Fig. B.112 : Progression of parameter estimates F, in 1975, for Hellas - Bulgaria – Romania - Yugoslavia

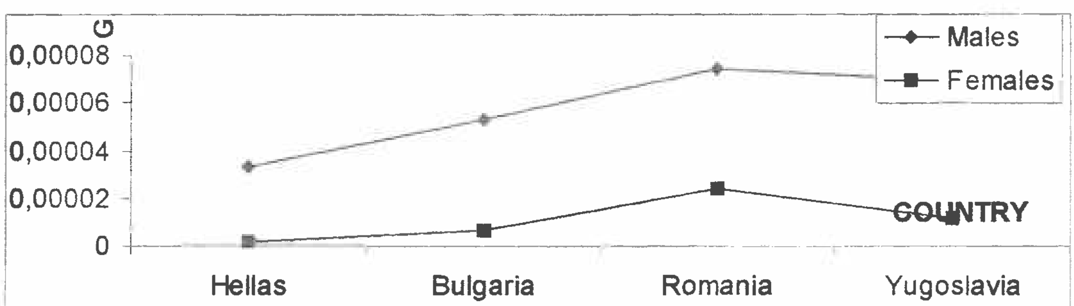

Fig. B.113 : Progression of parameter estimates G, in 1975, for Hellas - Bulgaria – Romania - Yugoslavia

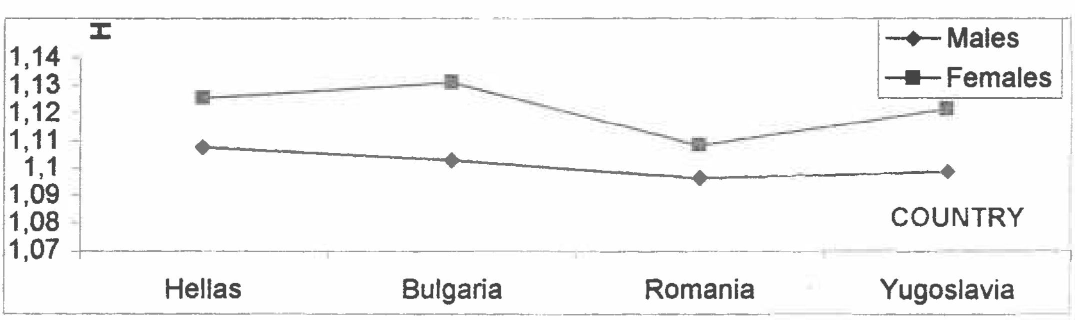

Fig. B.114 : Progression of parameter estimates H, in 1975, for Hellas - Bulgaria – Romania - Yugoslavia

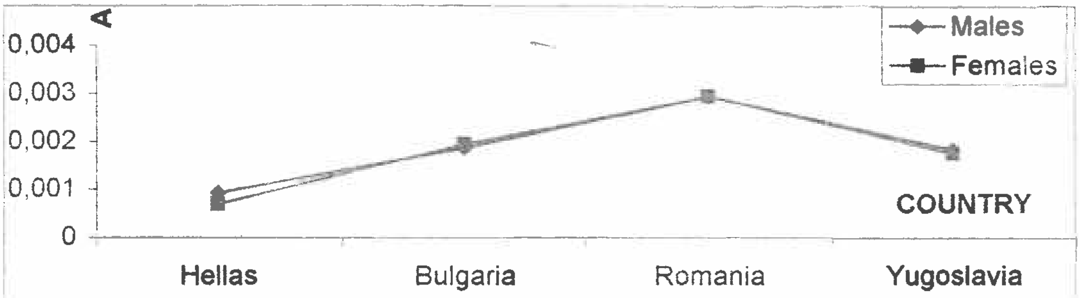

Fig. B.115 : Progression of parameter estimates A, in 1980, for Hellas - Bulgaria – Romania - Yugoslavia

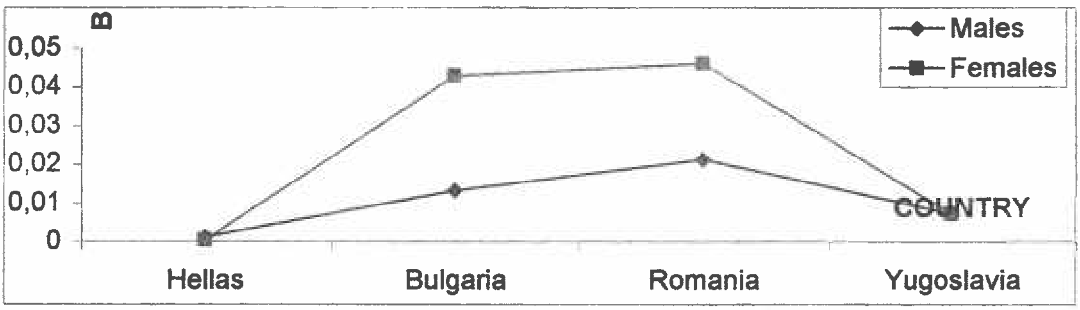

Fig. B.116 : Progression of parameter estimates B, in 1980, for Hellas - Bulgaria – Romania - Yugoslavia

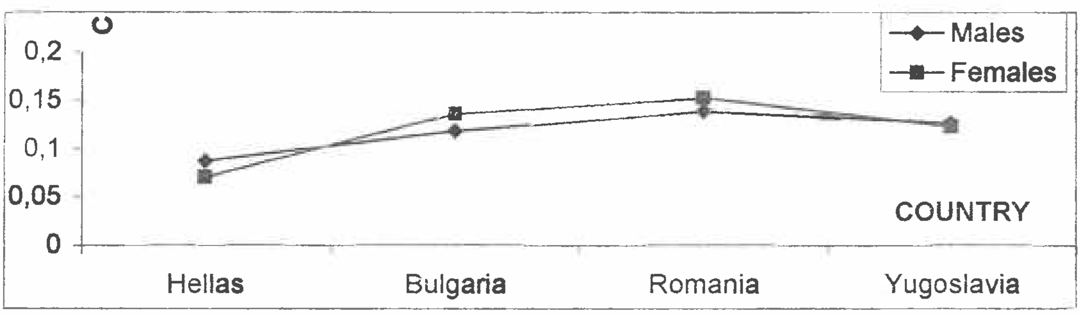

Fig. B.117 : Progression of parameter estimates C, in 1980, for Hellas - Bulgaria – Romania - Yugoslavia

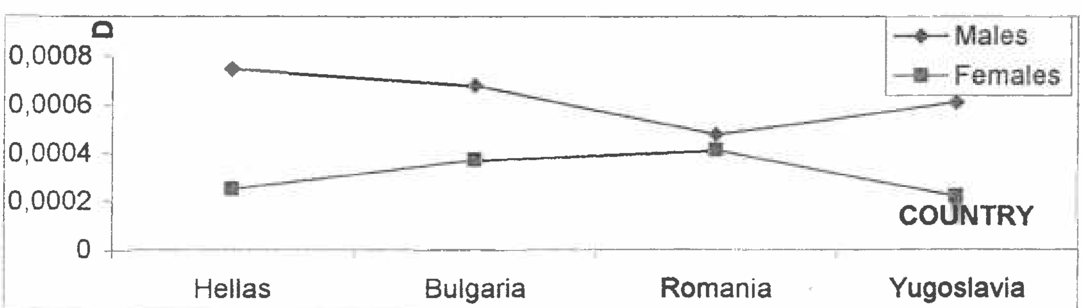

Fig. B.118 : Progression of parameter estimates D, in 1980, for Hellas - Bulgaria – Romania - Yugoslavia

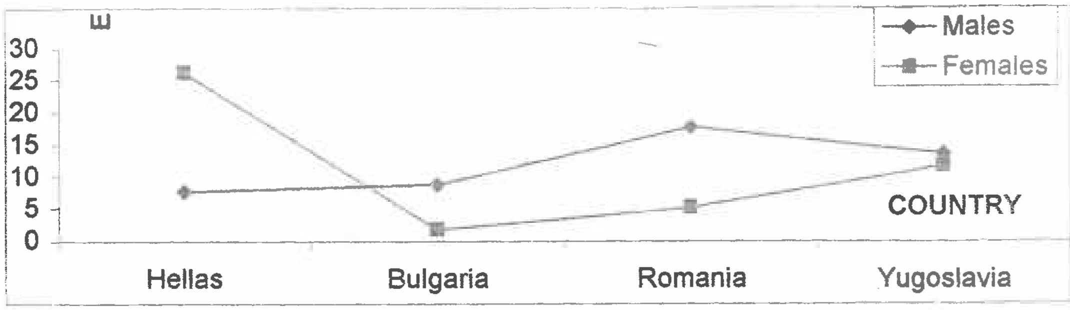

Fig. B.119 : Progression of parameter estimates E, in 1980, for Hellas - Bulgaria – Romania - Yugoslavia

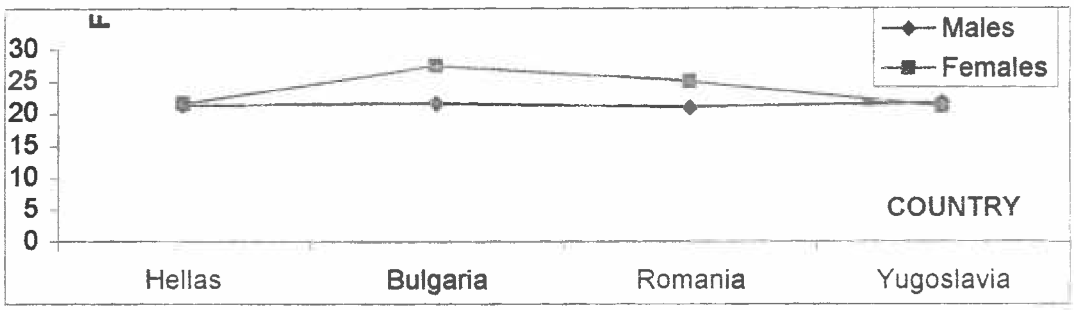

Fig. B.120 : Progression of parameter estimates F, in 1980, for Hellas - Bulgaria – Romania - Yugoslavia

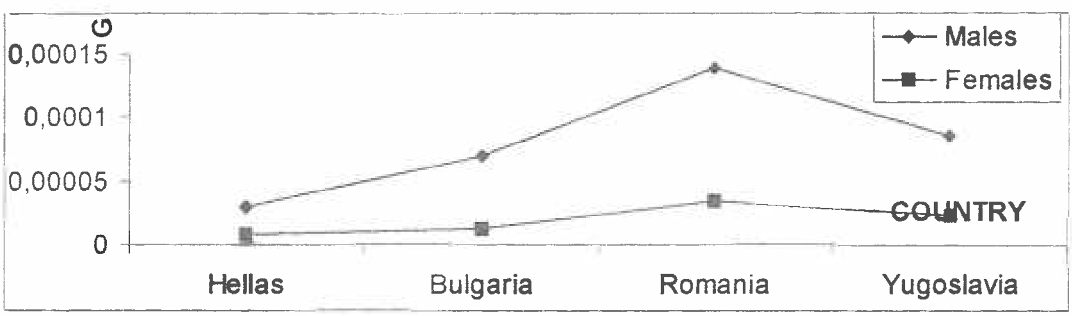

Fig. B.121 : Progression of parameter estimates G, in 1980, for Hellas - Bulgaria – Romania - Yugoslavia

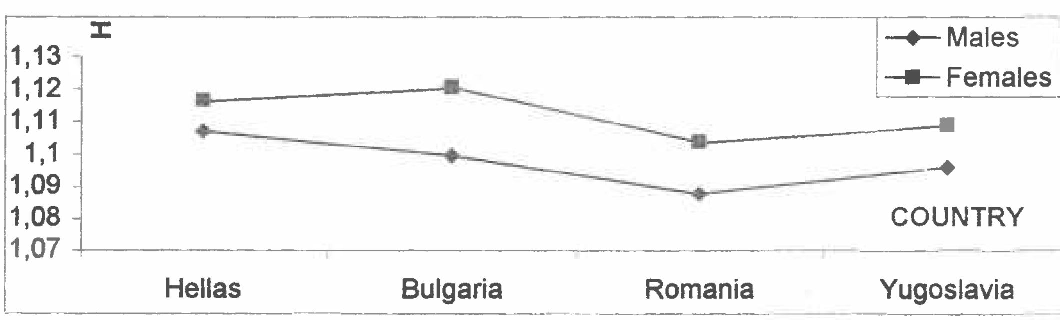

Fig. B.122 : Progression of parameter estimates H, in 1980, for Hellas - Bulgaria – Romania - Yugoslavia

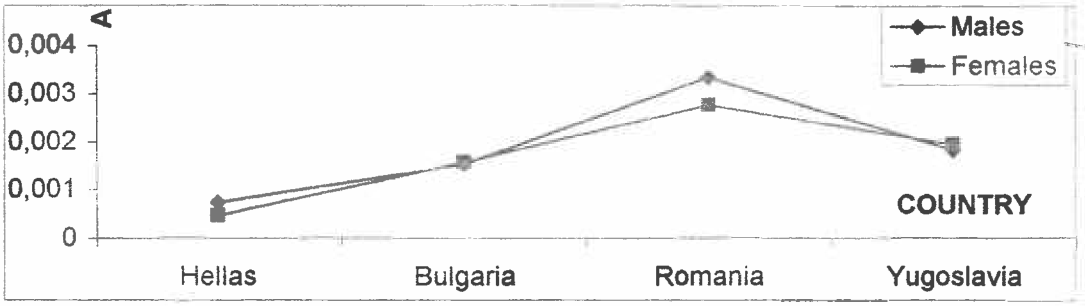

Fig. B.123 : Progression of parameter estimates A, in 1985, for Hellas - Bulgaria – Romania - Yugoslavia

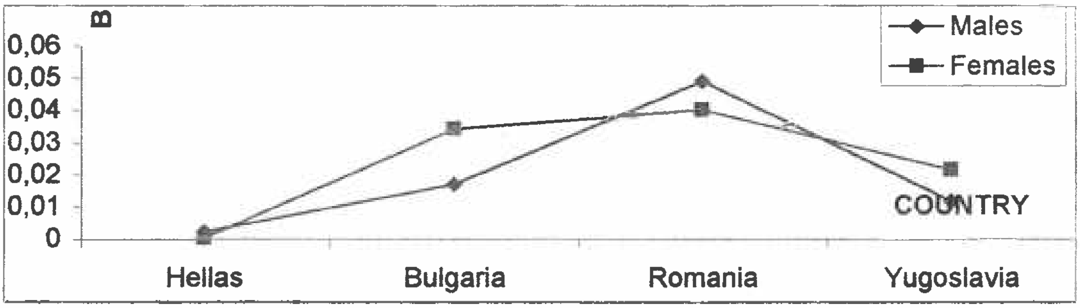

Fig. B.124 : Progression of parameter estimates B, in 1985, for Hellas - Bulgaria – Romania - Yugoslavia

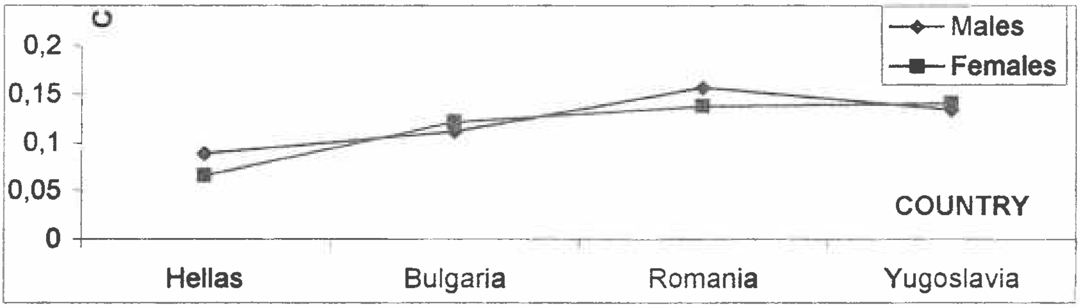

Fig. B.125 : Progression of parameter estimates C, in 1985, for Hellas - Bulgaria – Romania - Yugoslavia

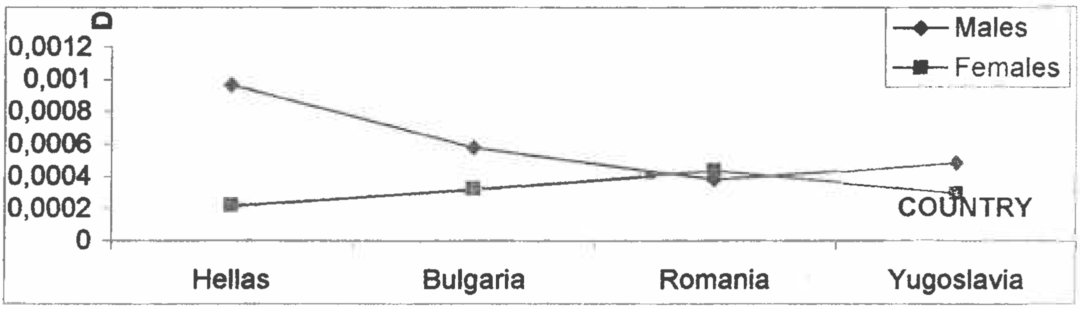

Fig. B.126 : Progression of parameter estimates D, in 1985, for Hellas - Bulgaria – Romania - Yugoslavia

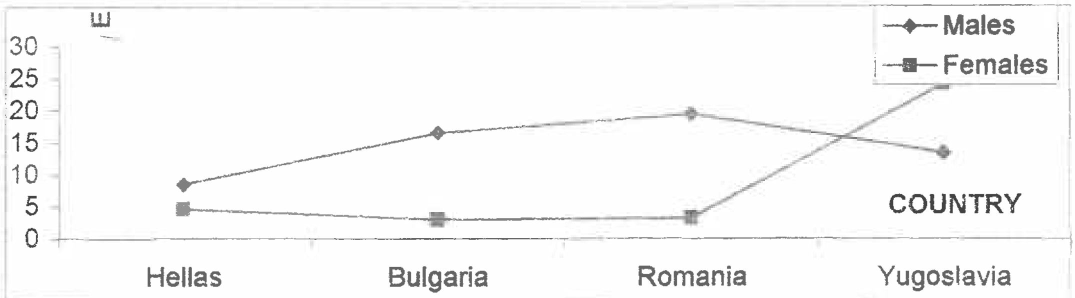

Fig. B.127 : Progression of parameter estimates E, in 1985, for Hellas - Bulgaria – Romania - Yugoslavia

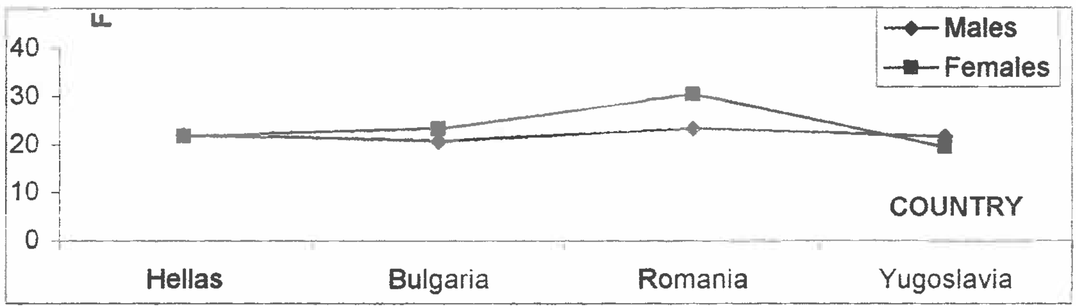

Fig. B.128 : Progression of parameter estimates F, in 1985, for Hellas - Bulgaria – Romania - Yugoslavia

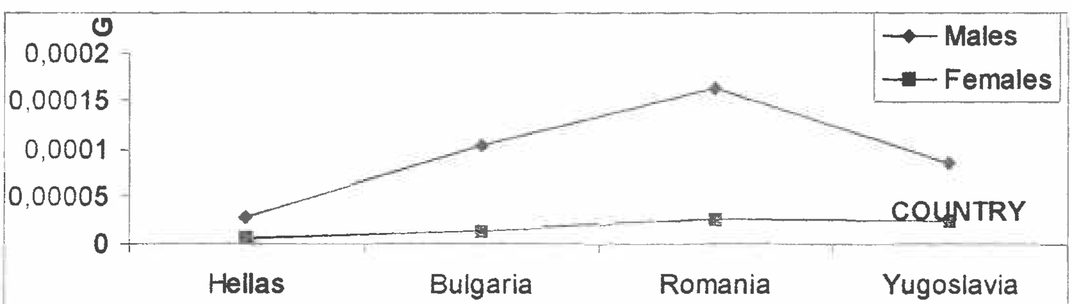

Fig. B.129 : Progression of parameter estimates G, in 1985, for Hellas - Bulgaria – Romania - Yugoslavia

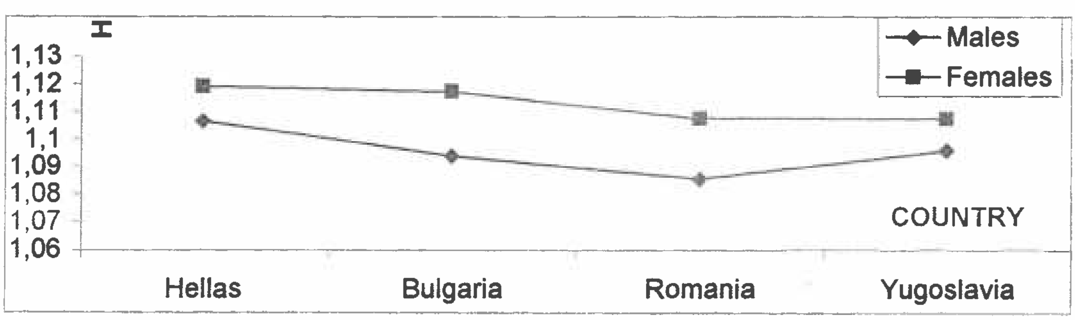

Fig. B.130 : Progression of parameter estimates H, in 1985, for Hellas - Bulgaria – Romania - Yugoslavia

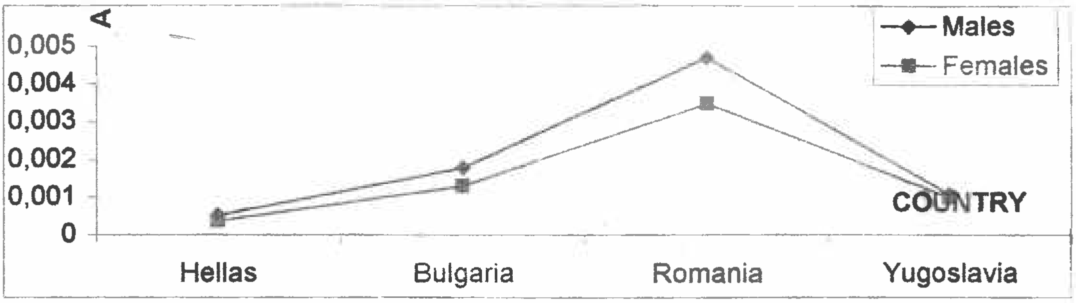

Fig. B.131 : Progression of parameter estimates A, in 1990, for Hellas - Bulgaria – Romania - Yugoslavia

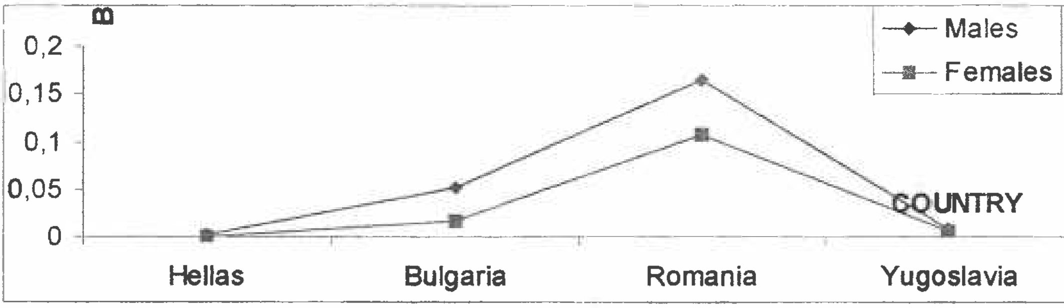

Fig. B.132 : Progression of parameter estimates B, in 1990, for Hellas - Bulgaria – Romania - Yugoslavia

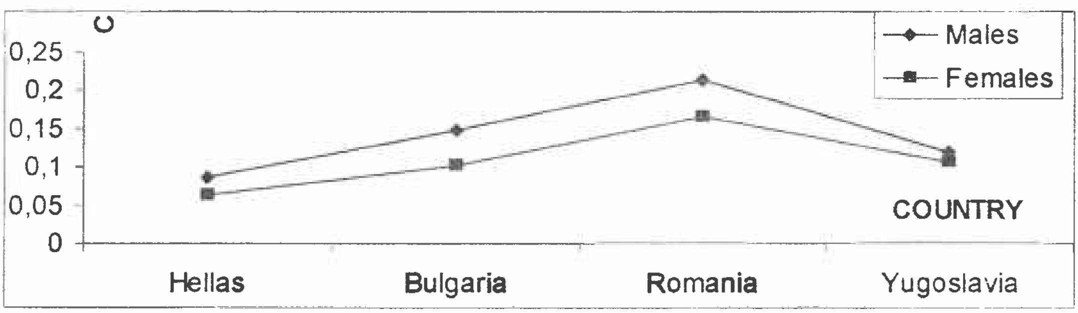

Fig. B.133 : Progression of parameter estimates C, in 1990, for Hellas - Bulgaria – Romania - Yugoslavia

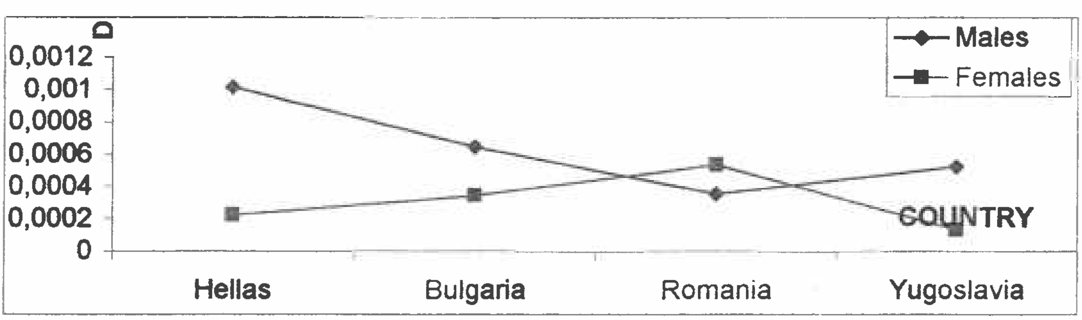

Fig. B.134 : Progression of parameter estimates D, in 1990, for Hellas - Bulgaria – Romania - Yugoslavia

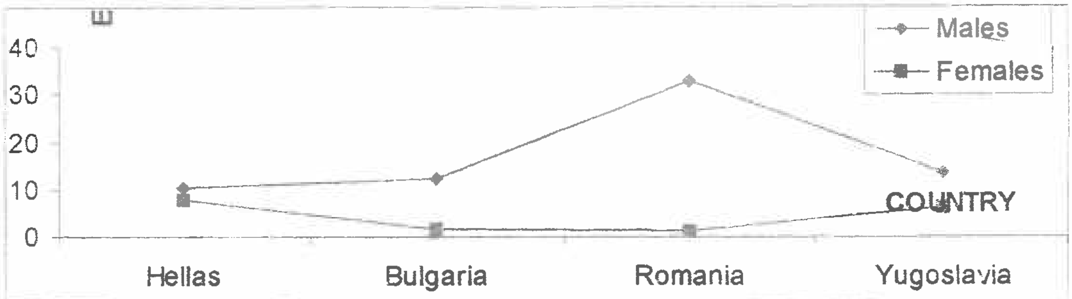

Fig. B.135 : Progression of parameter estimates E, in 1990, for Hellas - Bulgaria – Romania - Yugoslavia

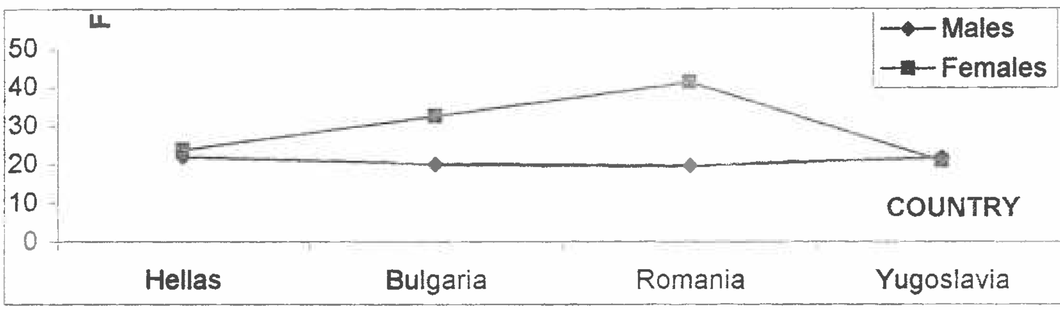

Fig. B.136 : Progression of parameter estimates F, in 1990, for Hellas - Bulgaria – Romania - Yugoslavia

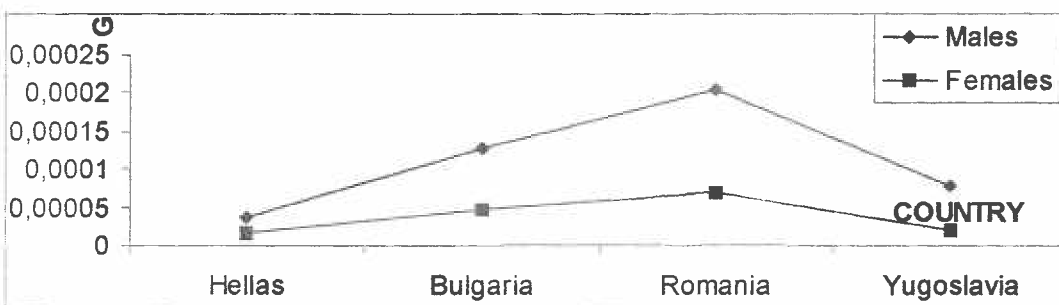

Fig. B.137 : Progression of parameter estimates G, in 1990, for Hellas - Bulgaria – Romania - Yugoslavia

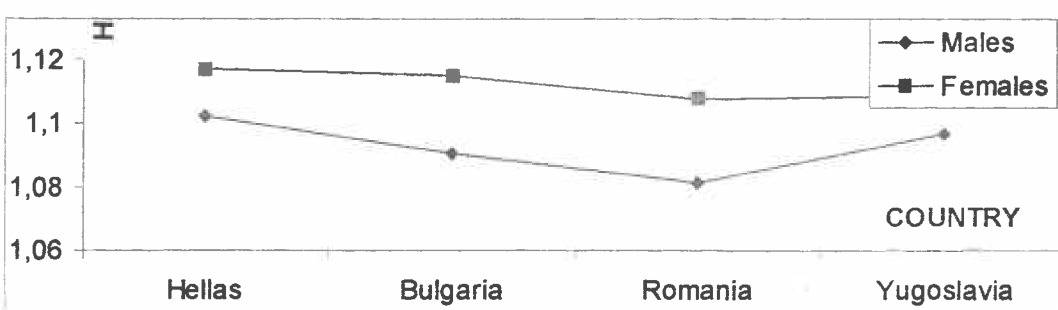

Fig. B.138 : Progression of parameter estimates H, in 1990, for Hellas - Bulgaria – Romania - Yugoslavia

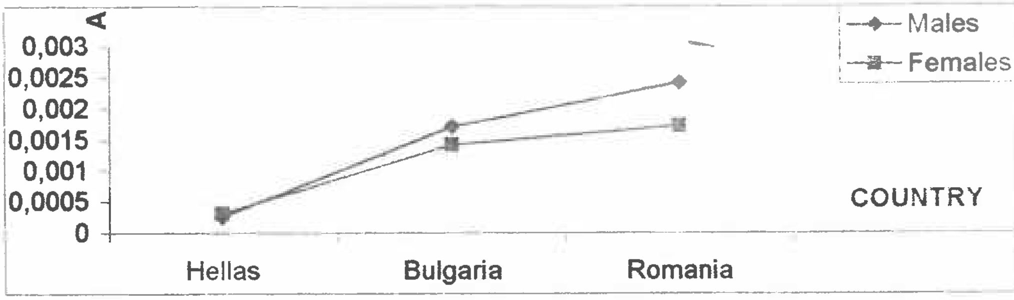

Fig. B.139 : Progression of parameter estimates A, in 1995, for Hellas - Bulgaria – Romania

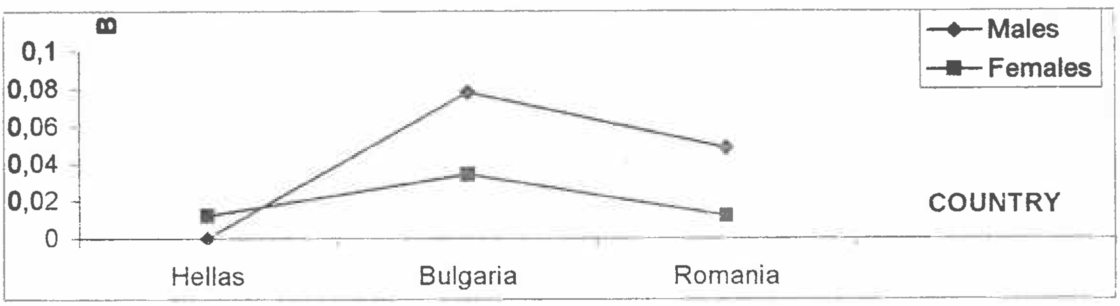

Fig. B.140 : Progression of parameter estimates B, in 1995, for Hellas - Bulgaria – Romania

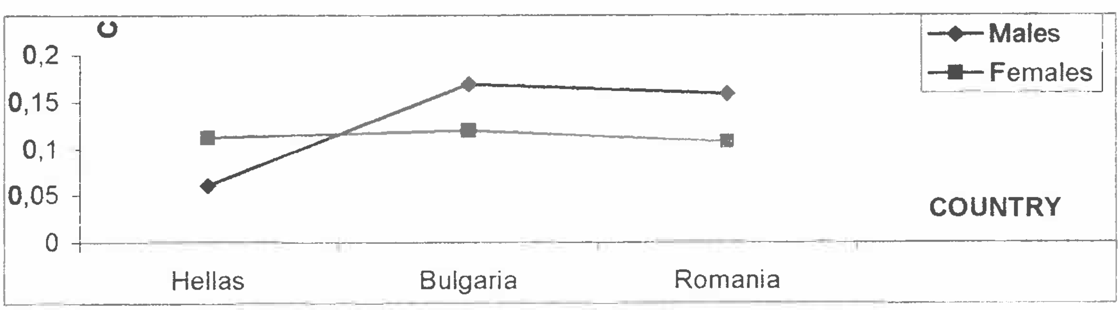

Fig. B.141 : Progression of parameter estimates C, in 1995, for Hellas - Bulgaria – Romania

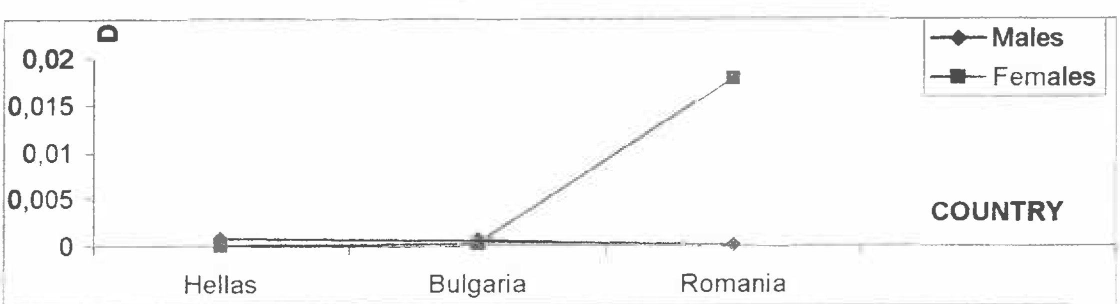

Fig. B.142 : Progression of parameter estimates D, in 1995, for Hellas - Bulgaria – Romania

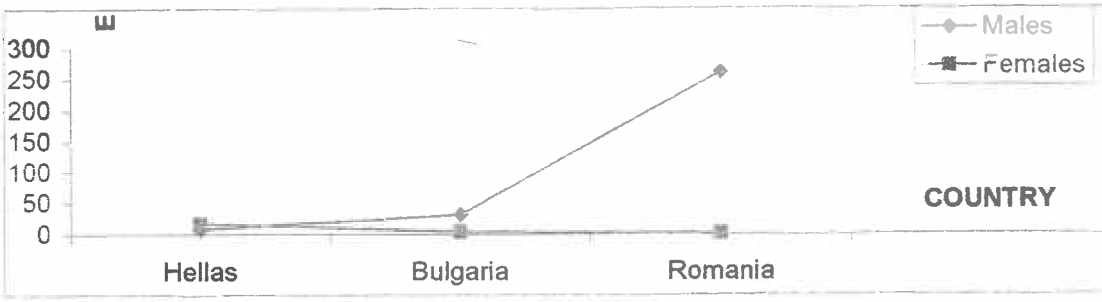

Fig. B.143 : Progression of parameter estimates E, in 1995, for Hellas - Bulgaria – Romania

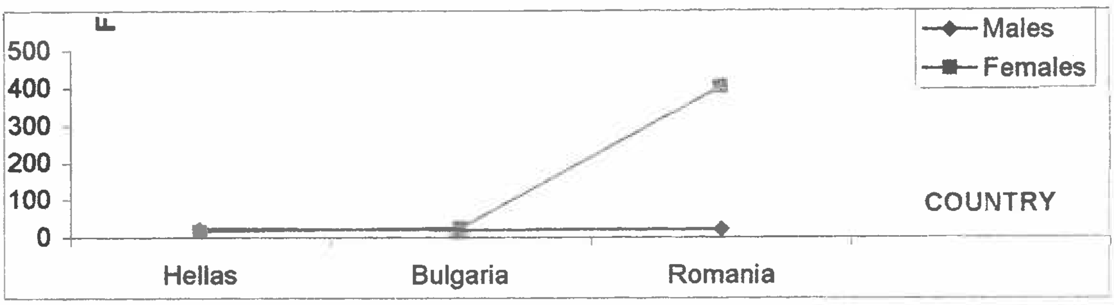

Fig. B.144 : Progression of parameter estimates F, in 1995, for Hellas - Bulgaria – Romania

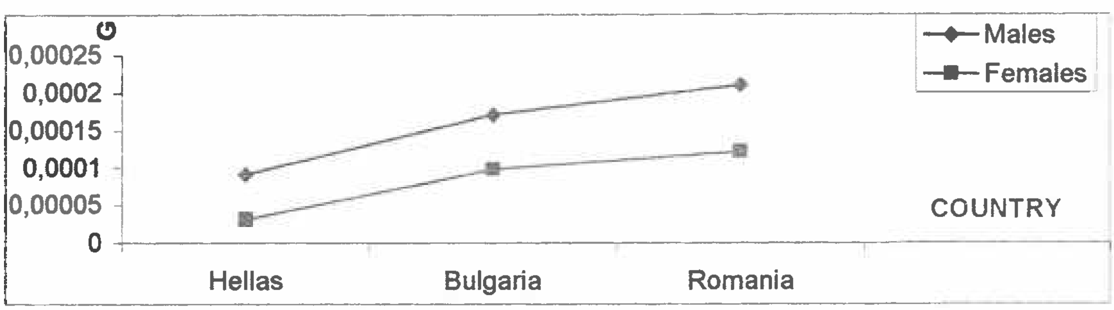

Fig. B.145 : Progression of parameter estimates G, in 1995, for Hellas - Bulgaria – Romania

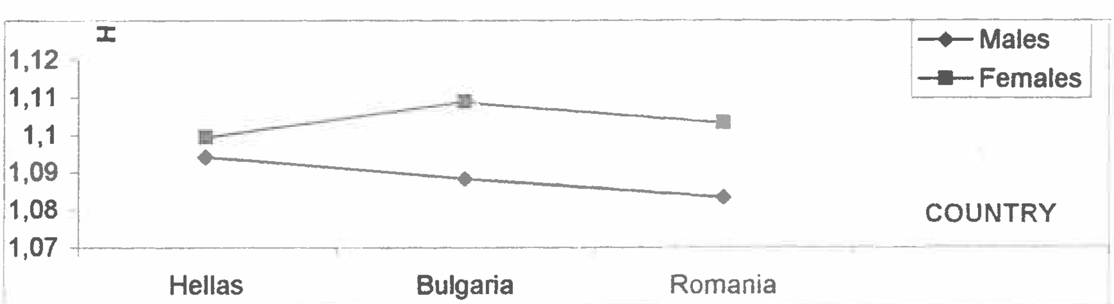

Fig. B.146 : Progression of parameter estimates H, in 1995, for Hellas - Bulgaria – Romania

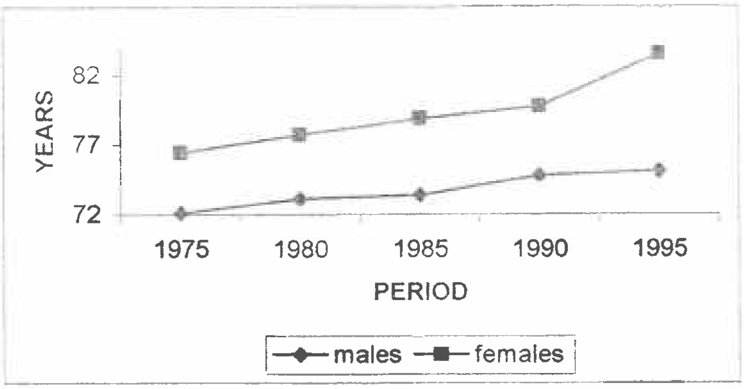

Figure B.147: Expectation of life at birth in years for Hellas males - females, over 1975 to 1995.

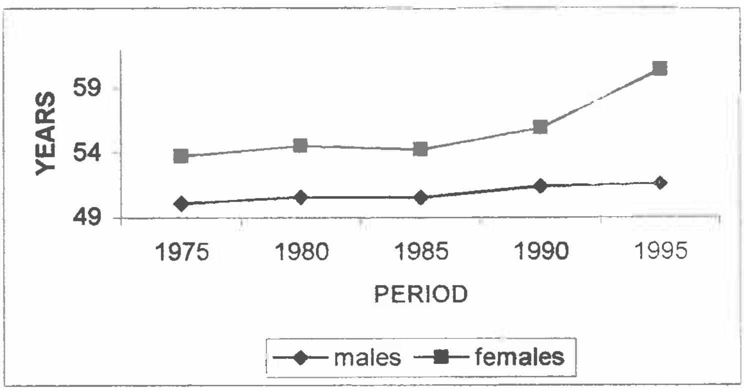

Figure B.148: Expectation of life at age 25 in years for Hellas males - females, over 1975 to 1995.

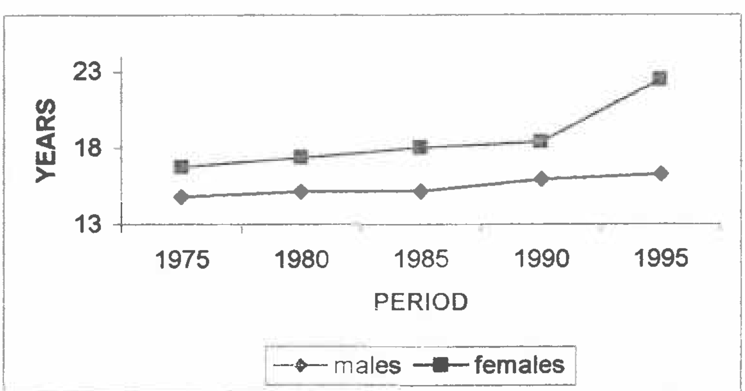

Figure B.149: Expectation of life at age 65 in years for Hellas males - females, over 1975 to 1995.

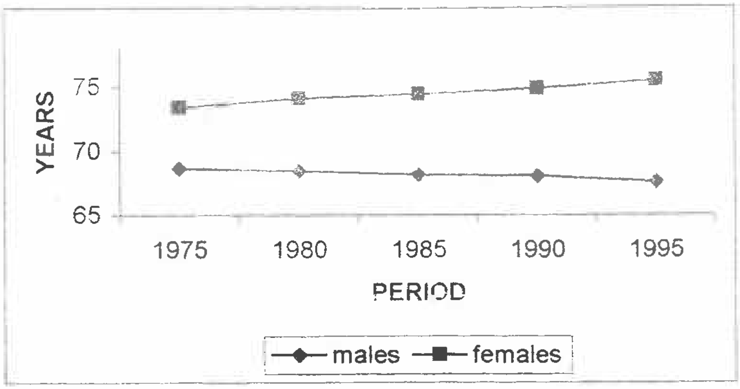

Figure B.150: Expectation of life at birth in years for Bulgaria males - females, over 1975 to 1995.

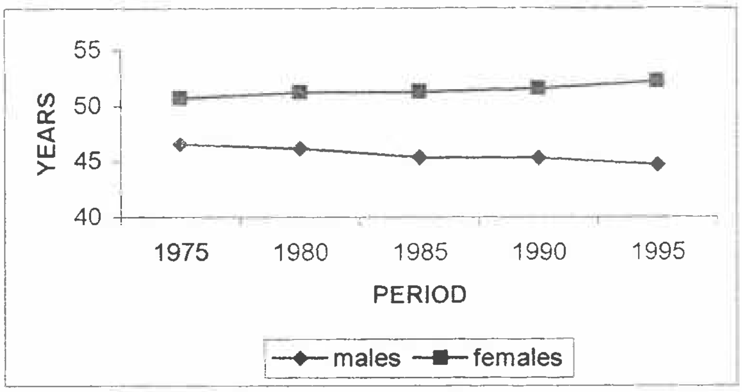

Figure B.151: Expectation of life at age 25 in years for Bulgaria males - females, over 1975 to 1995.

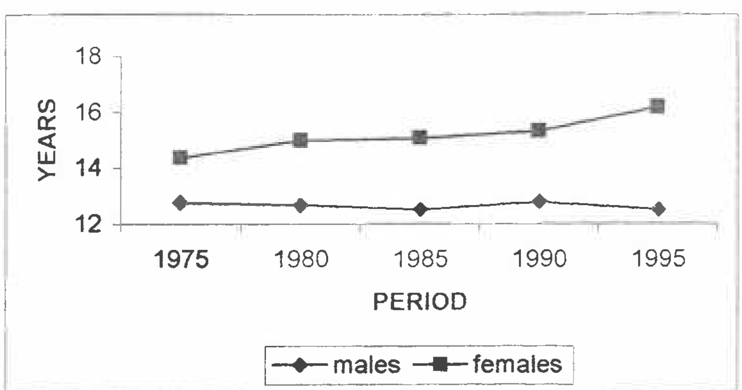

Figure B.152: Expectation of life at age 65 in years for Bulgaria males - females, over 1975 to 1995.

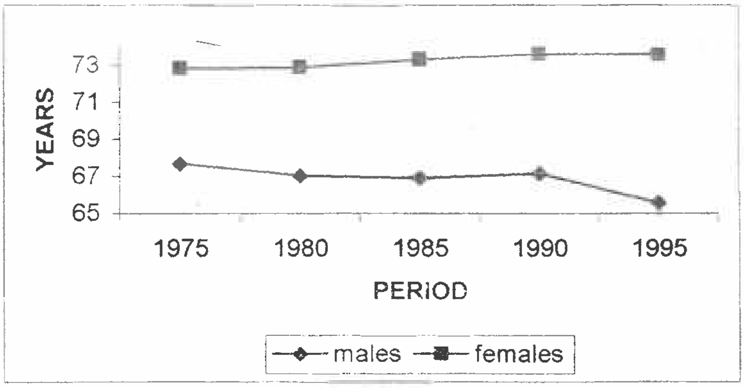

Figure B.153: Expectation of life at birth in years for Romania males - females, over 1975 to 1995.

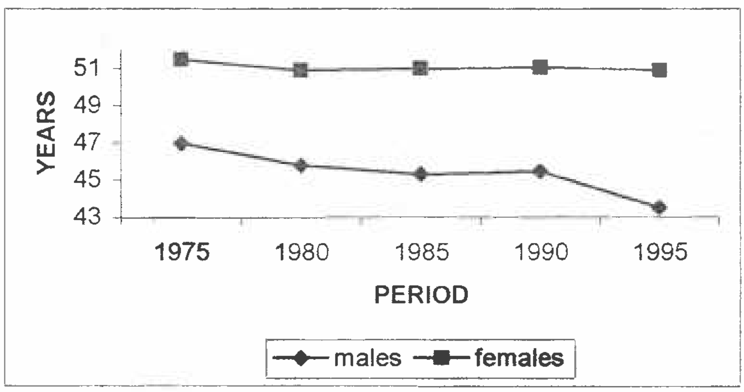

Figure B.154: Expectation of life at age 25 in years for Romania males - females, over 1975 to 1995.

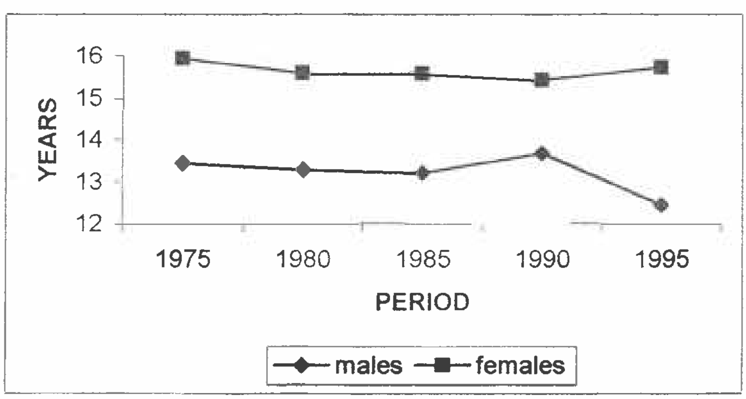

Figure B.155: Expectation of life at age 65 in years for Romania males - females, over 1975 to 1995.

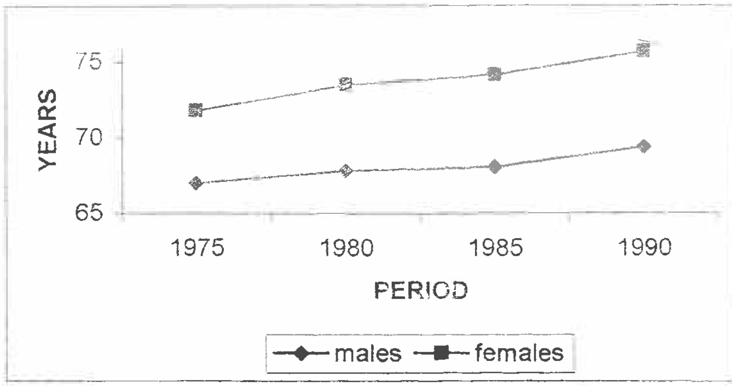

Figure B.156: Expectation of life at birth in years for Yugoslavia males - females, over 1975 to 1990.

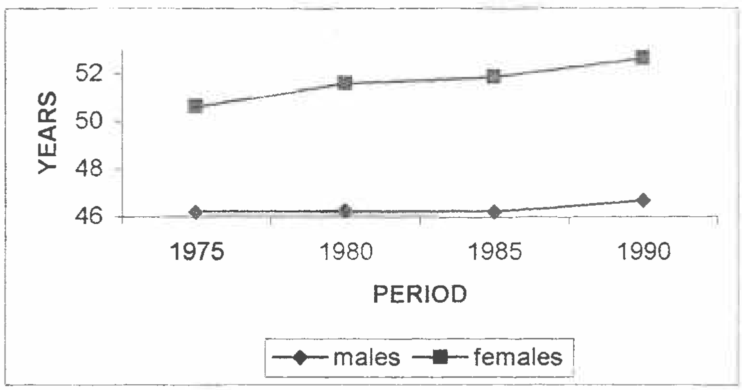

Figure B.157: Expectation of life at age 25 in years for Yugoslavia males - females, over 1975 to 1990.

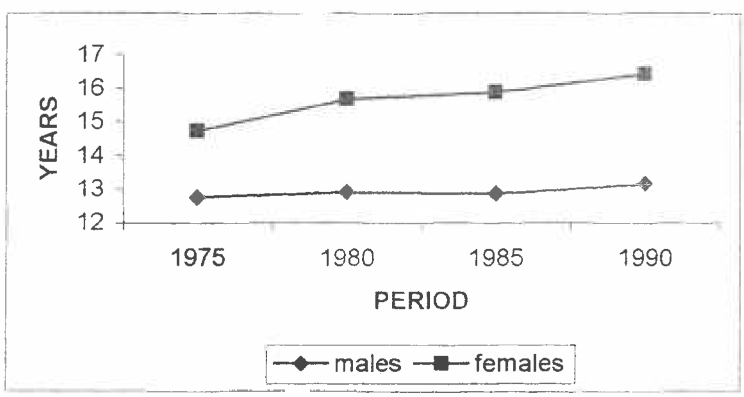

Figure B.158: Expectation of life at age 65 in years for Yugoslavia males - females, over 1975 to 1990.

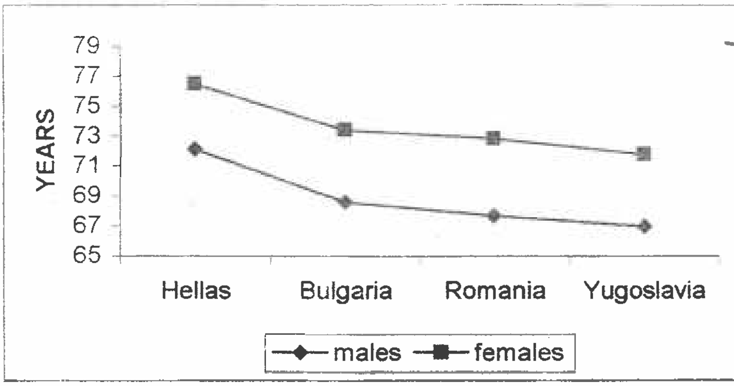

Figure B.159: Expectation of life at birth in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1975.

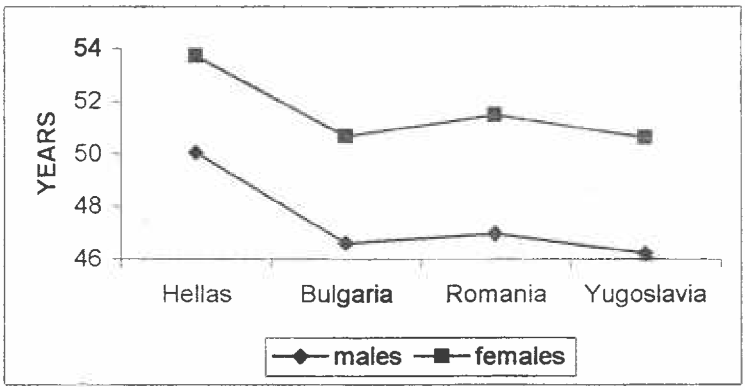

Figure B.160: Expectation of life at age 25 in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1975.

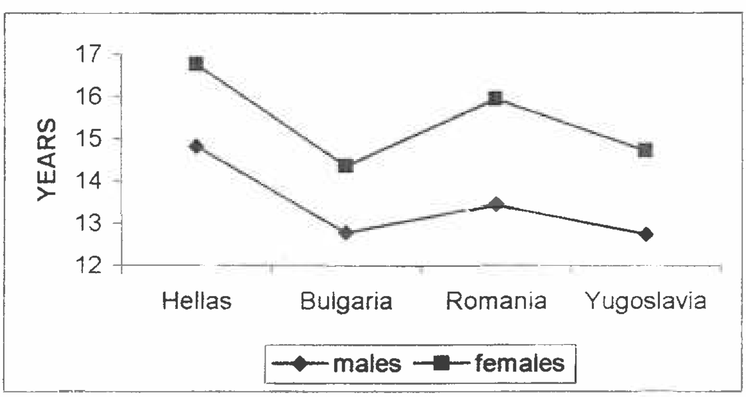

Figure B.161: Expectation of life at age 65 in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1975.

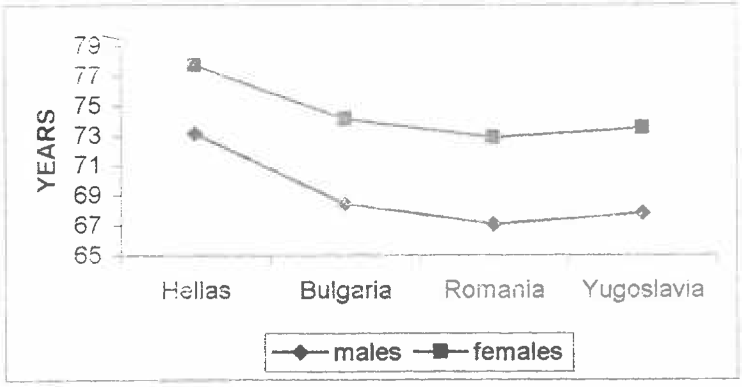

Figure B.162: Expectation of life at birth in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1980.

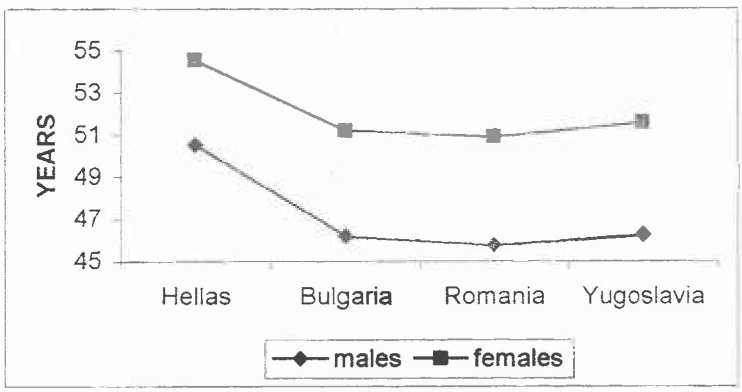

Figure B.163: Expectation of life at age 25 in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1980.

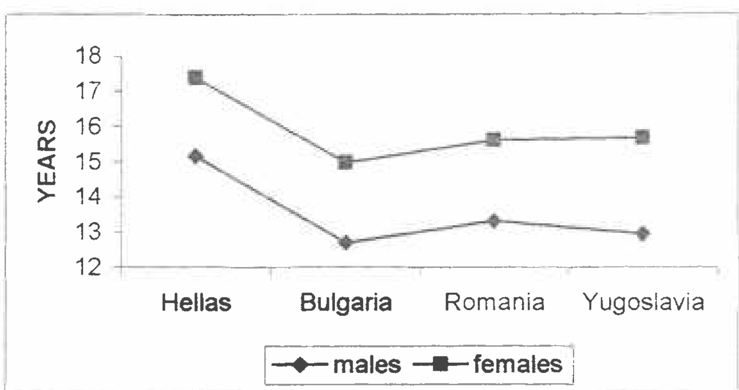

Figure B.164: Expectation of life at age 65 in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1980.

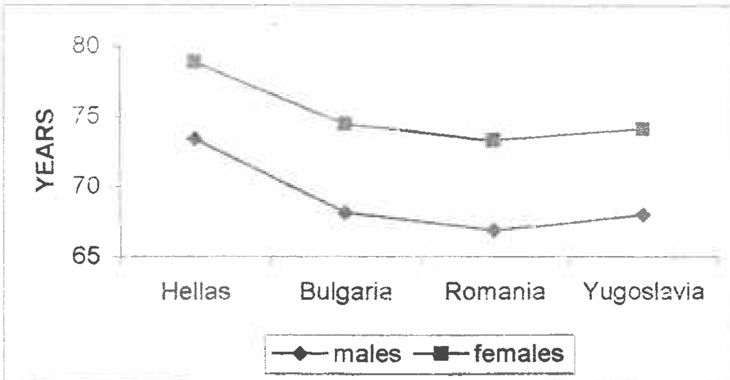

Figure B.165: Expectation of life at birth in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1985.

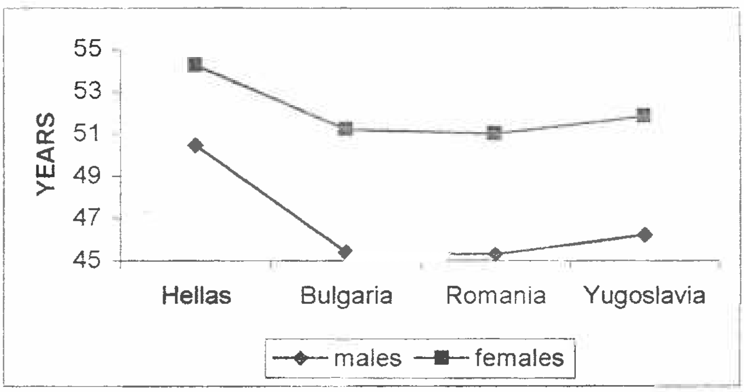

Figure B.166: Expectation of life at age 25 in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1985.

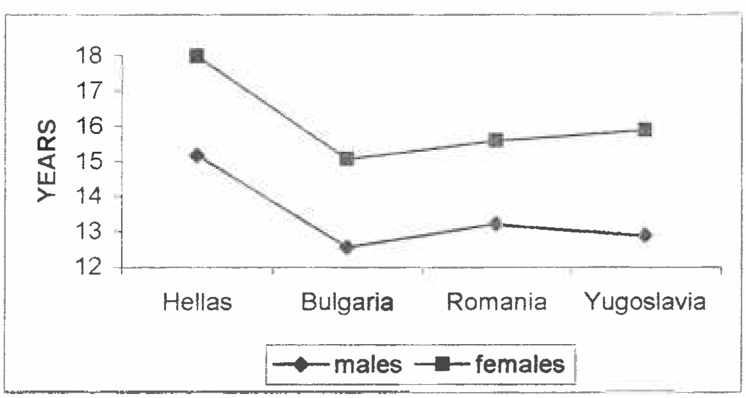

Figure B.167: Expectation of life at age 65 in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1985.

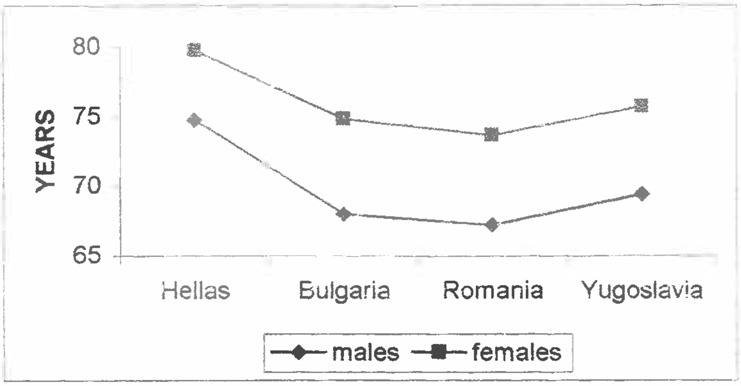

Figure B.168: Expectation of life at birth in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1990.

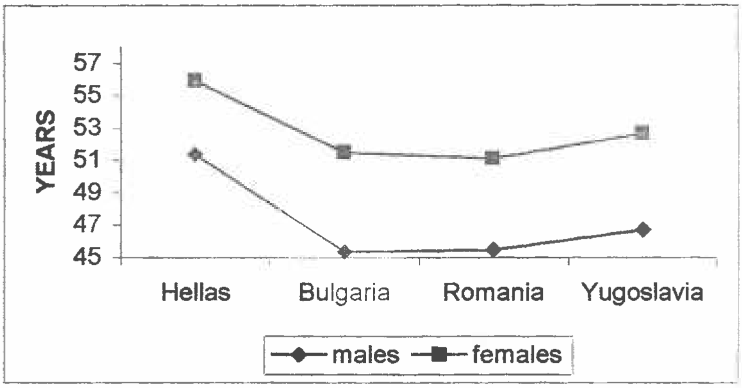

Figure B.169: Expectation of life at age 25 in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1990.

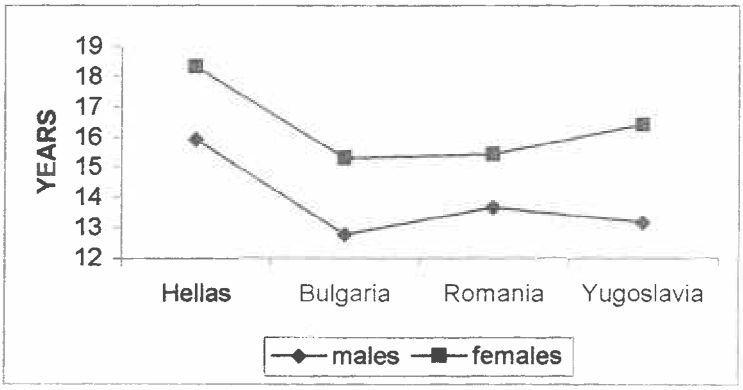

Figure B.170: Expectation of life at age 65 in years for (Hellas – Bulgaria – Romania – Yugoslavia) males - females, in 1990.

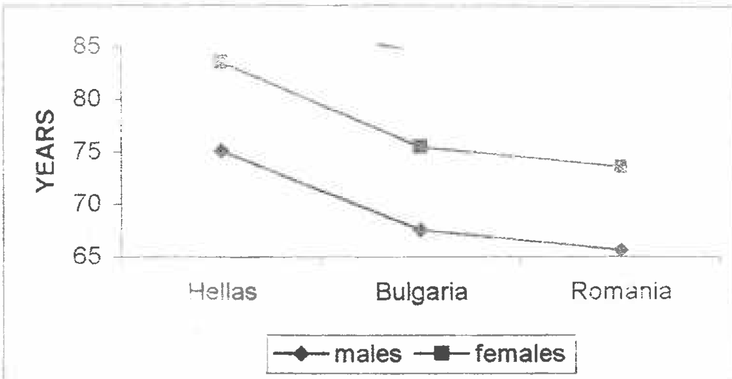

Figure B.171: Expectation of life at birth in years for (Hellas – Bulgaria – Romania) males - females, in 1995.

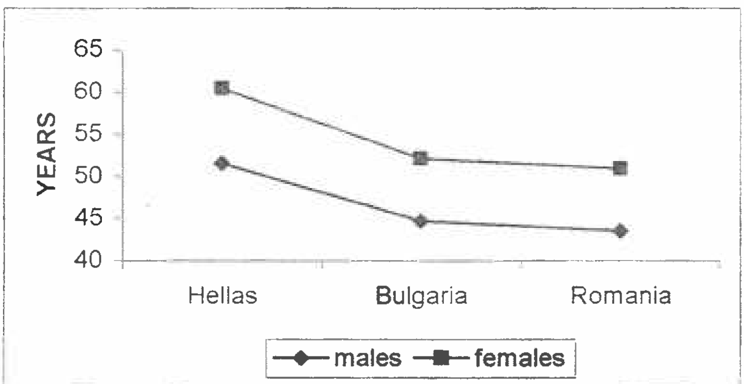

Figure B.172: Expectation of life at age 25 in years for (Hellas – Bulgaria – Romania) males - females, in 1995.

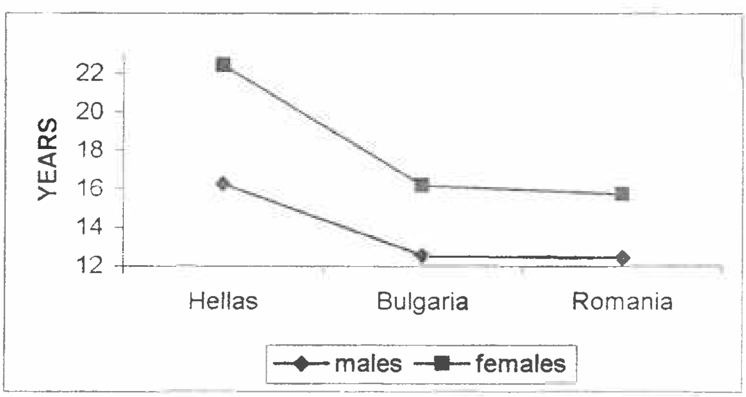

Figure B.173: Expectation of life at age 65 in years for (Hellas – Bulgaria – Romania) males - females, in 1995.

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