ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ
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ΠΡΟΓΡΑΜΜΑ ΜΕΤΑΠΤΥΧΙΑΚΩΝ ΣΠΟΥΔΩΝ

A comparative analysis of multivariate Value at Risk models: Evidence from Developed and Emerging markets

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051289

**Εργασία υποβληθείσα στο
Τμήμα Λογιστικής και Χρηματοοικονομικής
του Οικονομικού Πανεπιστημίου Αθηνών ως
μέρος των απαιτήσεων για την απόκτηση
Μεταπτυχιακού Διπλώματος Ειδίκευσης**

Αθήνα

Οκτώβριος 2006

Εγκρίνουμε την εργασία του

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ΟΝΟΜΑ ΕΠΙΒΛΕΠΟΝΤΟΣ ΚΑΘΗΓΗΤΗ **ΥΠΟΓΡΑΦΗ**

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ΟΝΟΜΑ ΣΥΝΕΞΕΤΑΣΤΗ ΚΑΘΗΓΗΤΗ **ΥΠΟΓΡΑΦΗ**

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2/11/06

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Abstract

Value at Risk (VaR) approach has received a great attention from both regulatory and academic fronts. Numerous papers have studied various aspects of VaR methodology. However, different methodologies of computing VaR generate widely varying results, suggesting that the choice of VaR model is very important. I use daily equity index returns from twenty two developed markets and from twenty six emerging markets over the past twelve years in order to examine the exposure to market risk of nine hypothetical global portfolios and to evaluate the relative predictive performance of alternative multivariate Value at Risk models. Performance evaluation is based on a three-step testing procedure for correct conditional coverage of the interval forecasts and a regulatory loss function that compares two alternative models. I present empirical evidence that the filtered historical simulation and the extreme value theory models are more accurate especially at higher quantiles than other well-known modeling approaches.

Acknowledgements

Special thanks to Professor Manolis Kavussanos for giving me the opportunity to treat this important topic and to Mr. Dimitris Dimitrakopoulos for his insightful, critical and constructive comments. I, nevertheless, retain responsible for any remaining errors.

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1. Introduction

Modeling market risk, as precisely as possible, is perhaps the most challenging goal of an international financial organization, investing in developed and emerging capital markets. Risk is defined as the degree of uncertainty of future net returns. This uncertainty takes many forms, which is why most participants in the financial markets are subject to a variety of risks. Market risk arises from movements in the level or volatility of market prices. The most widely used approach for quantifying market risk is Value at Risk (VaR) methodology. VaR tools allow financial organizations to quantify market risk in a systematic fashion. A large body of literature exists describing and examining various VaR models, either in each developed market or in each emerging market separately. However, in practice, institutional investors and financial institutions manage large portfolios, which consist of a combination of different markets. In this paper, I use daily equity index returns from twenty two developed markets and from twenty six emerging markets over the past twelve years in order to examine the exposure to market risk of nine hypothetical global portfolios and to evaluate the relative predictive performance of alternative multivariate Value at Risk models. I present empirical evidence that multivariate GARCH models, which perform well in developed markets in 95% confidence level, are inefficient when applied in emerging markets and that the filtered historical simulation and the extreme value theory models are more accurate especially at higher quantiles than other well-known modeling approaches.

Over the past decade the growth of trading activity in global capital markets have resulted in a re-analysis of the market risks faced by financial institutions and how it is measured. Developed markets, which are less risky, with high degree of information efficiency and higher market liquidity, have always been an attractive choice for global investors. On the other hand, emerging markets in Europe, Latin America, Asia, the Mideast and Africa provide a new menu of opportunities for investors. A stock market is classified as "emerging" if it is located in a low or middle income economy as defined by the World Bank or its investable market capitalization is low relative to its most recent gross domestic product. The growth of emerging markets has received much attention in the past few years. Investors have been attracted to the potentials for high returns along with diversification benefits of such markets. Even managers and trustees of pension funds have begun to commit a portion of their pension assets to emerging markets equity securities.

Recently, developed and emerging markets have experienced several extreme market events. Examples include the U.S. stock market crash of October 1987, when prices at the New York Stock Exchange fell by one third over five trading days. The Japanese stock price bubble finally deflated at the end of 1989, sending the Nikkei index from 39,000 to 17,000 three years later. A total of $2.7 trillion in capital was lost, leading to an unprecedented financial crisis in Japan. The Latin American crisis of 1994 involving Argentina, Brazil and Mexico created substantial financial losses. The Asian turmoil of 1997 wiped off about three fourth of the dollar capitalization of equities in Indonesia, Korea, Malaysia and Thailand. The Russian default in August 1998 sparked a global financial crisis that culminated in the near failure of a big hedge fund. Before the 1990s, financial crises were seen as events only affecting the country in which they had originally occurred. After the 1990s financial crises started to spread rapidly beyond the countries where they had originated to others with different economic structures and institutions. Consequently, effective use of Value at Risk methodology has emerged as a response to the increased volatility in global financial markets. However, different methodologies of computing VaR have generated widely varying results because of the different characteristics of developed and emerging markets, suggesting that the choice of VaR model is very significant.

2. Literature review

Emerging markets are commonly associated with extraordinary high average returns compared to developed markets, high volatility, a degree of return predictability, and low correlation with developed markets, offering diversification benefits to global investors. Literature review indicates the different characteristics of emerging equity markets. Errunza (1977), Errunza and Rosenberg (1982) and Errunza (1983) are among the earlier studies on emerging markets, pointing out the potential benefits of investing in emerging markets. Bailey and Stulz (1990) and Bailey et al. (1990) have shown that the potential benefits through diversification from the Pacific Basin stock markets are substantial. An effective diversification through investing in emerging markets may also result in reducing risk significantly (see Divecha, Drach and Stefek 1992, Wilcox 1992, Speidell and Sappenfield 1992, Mullin 1993, Errunza 1994). In a study of twenty new equity markets in emerging economies, Harvey (1995) found that the inclusion of emerging securities significantly reduces portfolio risk and increase expected returns. He also explained the high volatility of emerging markets due to the lack of diversification in the country index, high risk exposures to volatile economic factors and time-variation in the risk exposures or incomplete integration into world capital market. Moreover, he concluded that the amount of predictability found in the emerging markets is greater than that found in developed markets.

It has been documented that when markets are individually volatile, the correlation between the returns of the various different markets increases. Karolyi and Stulz (1996) use data from American Depository Receipts of Japanese stocks traded on the New York Stock Exchange and a matched sample of US stocks, and find that comovements are high when contemporaneous absolute returns of the national markets indices are high. Bekaert and Harvey (1997) investigate the emerging market time-varying volatility and explored the forces that determine the difference of volatility in various emerging markets. They argue that those forces are asset concentration, market capitalization, size of the trade sector, cross-sectional volatility of individual securities, turnover and foreign exchange variability. Bekaert, Erb, Campbell and Viskanta (1998) detailed the distributional characteristics of emerging markets and explore how these characteristics change over time. They argued that emerging markets are highly non-normal. Masters (1998) investigated the emerging market indexes and found them inherently inefficient and concluded that building a portfolio around a particular index may be less desirable in emerging markets than in other asset classes. Erb, Harvey and Viskanta (1998) pointed out that correlation varies depending on both the state of economy and the state of the equity markets in each country. Aggarwal, Inclan, and Leal, (1999) examined the events which cause major shifts in emerging markets' volatility. They found that, unlike developed markets, large changes in volatility seem to be related to country-specific events. Ang and Bekaert (1999) go further to suggest that equity markets correlations increase more in volatile bull than volatile bear markets. Recently, several authors have investigated the volatility of Central and Eastern European stock markets. Kasch-Haroutounian and Price (2001), Poshakwale and Murinde (2001) found that significant autocorrelation, high volatility persistence, significant asymmetry, lack of relationship between stock market volatility and expected return and non-normality of the return distribution are basic characteristics of stock market volatility in transition countries.

Taking into account all the above characteristics of emerging markets, it could be argued that a number of Value at Risk models that perform well in developed markets may be inefficient, when applied in emerging markets. There is a huge number of studies concentrated on different VaR method. However, most of these studies have been tested on developed markets such as those in the US and Europe without taking into account the idiosyncratic nature and the volatility peculiarities of emerging markets. For instance, Guermat and Harris (2002) use representative equity portfolios for the US, UK and Japan. They show that exponentially weighted maximum likelihood based VaR forecasts are generally more accurate than those generated by both the EWMA and GARCH models, particularly at high VaR confidence levels. Cotter (2004) argues that extreme value theory models, applied in five equity indexes from Ireland, UK, France, Germany and Spain, dominates alternative approaches in tail estimation as it avoids model risk.

The most recent studies in emerging markets are concentrated to Extreme Value Theory models for computing VaR. Jondeau and Rockinger (1999) studied the tail behavior of stock returns in five mature markets, nine Asian, six Eastern European, and seven Latin American emerging markets using extreme value theory. De Melo Mendes

(2000) found that in Latin American markets the combinations of robust estimation of GARCH models and an extreme value theory model results in more precise conditional risk estimates than those obtained using classic estimation procedures. Ho, Burridge, Cadle, and Theobald, (2000) used extreme value theory to model the tails of the return distributions of six Asian financial markets during the recent volatile market conditions. They found that the value at risk measures generated by an extreme value framework are different to those generated by variance covariance and historical methods, particularly for markets characterized by high degrees of leptokurtosis such as Malaysia and Indonesia. Seymour and Polakow (2003) argue that both extreme value theory and volatility updating (via GARCH-type modeling) on a representative portfolio of South African stocks, provide significantly better results than established methods such as the historical simulation. Da Silvaa and de Melo Mendes (2003) use the extreme value theory to analyze ten Asian stock markets, identifying which type of extreme value asymptotic distribution better fits historical extreme market events. Their results suggest that the extreme value method of estimating VaR is a more conservative approach to determine capital requirements than traditional methods. Gencay and Selcuk (2004) investigate the relative performance of VaR models using daily stock returns of nine different emerging markets. Their results indicate that extreme value theory based VaR estimates are more accurate at higher quantiles than well-known modeling approaches, such as variance covariance method and historical simulation. Bao, Lee and Saltoglu (2006) investigate the predictive performance of various classes of VaR models for the stock markets of five Asian economies that suffered from the 1997-1998 financial crises. They argue that extreme value theory models behave reasonably well in the crisis period and that filtering often appears to be useful for some models, particularly for the extreme value theory models.

3. Defining Value at Risk

Value-at-Risk is a measure of the worst expected loss of a portfolio of financial instruments over a target horizon under normal market conditions with a given probability (Jorion 2002). In other words, VaR describes the quantile of the projected distribution of gains and losses over the target horizon. If $(1-a)$ is the selected confidence level, VaR corresponds to a lower tail of the distribution and represents $a\%$ chance that the actual loss in the portfolio's value is greater than the VaR estimate.

[3.1] $\Pr ob(\Delta P(\Delta t, \Delta s) < VaR) = a$

where $\Delta P(\Delta t, \Delta s)$ is the change in the market value of the portfolio, expressed as a function of the forecast horizon Δt and the change in the underlying asset price Δs. In this paper, Value at Risk of equity portfolios is computed over a one day horizon for both 95% and 99% confidence levels.

The concept and use of value at risk is recent. Value at risk was first used by major financial firms in the late 1980's to measure the risks of their trading portfolios. The term found its way through the Group of Thirty report published in July 1993. Apparently, this was the first widely publicized appearance of the term value at risk. Since that time period, the use of value at risk has exploded and it is increasingly being used by smaller financial institutions, non-financial corporations and institutional investors. In the last few years Value at Risk has become a very popular methodology for quantifying market risk. It has been adopted by many different types of financial organizations. An important reason for this is J.P. Morgan's decision in 1994 to make their RiskMetrics database freely available to all market participants providing a tremendous impetus to the growth in the use of Value at Risk. Another reason is the climate created by derivatives disasters such as Procter and Gamble, Metallgesellschaft, Orange County, and Barings. A third reason is the fact that Regulators have also become interested in Value at Risk. In April 1995, the Basle Committee on Banking Supervision proposed allowing banks to calculate their capital requirements for market risk with their own Value at Risk models, using certain parameters provided by the committee. In January 1996, the Basel Committee on

Banking Supervision of the Bank of International Settlement issued a revised consultative proposal on an "Internal Model-Based Approach to Market Risk Capital Requirements" that represents a big step forward in recognizing the new quantitative risk estimation techniques used by the banking industry. These proposals recognize that current practice among many financial institutions has superseded the original guidelines in terms of sophistication, and that financial institutions should be given the flexibility to use more advanced methodologies.

At the time Value-at-Risk burst on the scene, it was devised initially as a method to report financial market risk. However, VaR is not only useful for reporting purposes but also as a risk control tool. VaR limits can be used to control the risk of traders, as a supplement to traditional limits on notional amounts. Such limits can also be used at the level of the overall financial institution. Another application of VaR can be found in investment management. Institutional investors can monitor their portfolio risk and compare the market risk arising from different financial instruments. Moreover, VaR can be used to evaluate the performance of risk takers on a return/risk basis and to estimate capital levels required to support risk taking.

More than one VaR model is currently being used and most practitioners have selected an approach based on their specific needs, the types of positions they hold, their willingness to trade off accuracy for speed (or vice versa) and other considerations. There are four widely used methods for computing VaR, the variance covariance approach, historical simulation, Monte Carlo simulation and the extreme value theory. The variance covariance approaches are based on the assumption that market returns have a joint normal distribution. The variance covariance matrix is forecasted using several volatility models and VaR of a portfolio is estimated using the properties of normal distribution. Historical simulation, which is usually applied under a full valuation model, makes no explicit assumptions about the distribution of asset returns. This approach involves using historical changes in market prices to construct a distribution of potential future portfolio profits and losses. Monte Carlo simulation methods attempt to generate paths of market returns using a defined stochastic process. The extreme value theory models only the tails of the distribution rather than the entire one. Therefore, it focuses on the parts of the distribution that are essential for the VaR.

4. Value at Risk models

Methods to Value at Risk basically can be classified into two groups. The first group includes the parametric approach, which attempts to fit a parametric distribution such as normal to the data. VaR is then measured directly from the standard deviation. The second group includes the non-parametric approach, which is based on the empirical distribution or a simulated stochastic distribution and its sample quantile. Parametric methods include the variance covariance approach. Non-parametric methods include the historical simulation models and the Monte Carlo simulation models. The extreme value theory models are assumed to be semi-parametric.

The parametric methods that are used in this paper are the fixed weight approach, the exponentially weighted moving average, the constant correlation GARCH, the dynamic conditional correlation GARCH, the orthogonal GARCH and the exponential GARCH. All models assume that returns are normally distributed and their main difference is how the variance covariance matrix is estimated. The non-parametric methods that are used in this paper are the historical simulation, the weighted historical simulation, the filtered historical simulation, the Monde Carlo simulation using normally distributed random variables, the mixed of normal distributions Monde Carlo and the extreme value theory. The following section provides a detailed description of each of these models.

All the above models calculate Value at Risk using daily continuously compound returns. Let r_t denote the continuously compound rate of return and define P_t to be the closing price of each asset at time t, then

$$[4.1] \quad r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Financial returns often exhibit particularly fat tails and excess kurtosis (see Appendix I). This means that extreme price movements occur more frequently than implied by a normal distribution. Duffie and Pan (1997) identify jumps and stochastic volatility as possible causes of kurtosis. The peak of the return distribution is higher and narrower than that predicted by the normal distribution. Note that this characteristic, often referred to as the "thin waist", along with fat tails is a characteristic of a leptokurtotic distribution. Moreover, it is often found that financial daily returns are autocorrelated but the magnitude of the autocorrelation is too small to be economically significant. For longer return horizons (beyond a year), however, there is evidence of significant negative autocorrelation (Fama and French, 1988). Furthermore, financial squared returns often have significant autocorrelations.

4.1. Fixed weight approach

The equally weighted moving average assumes that market returns are normally distributed, the return variance covariance matrix Σ_t is constant over the period of estimation and that all observations carry the same weight in the volatility estimate. The principal reason for preferring to work with volatility is the strong evidence that the volatility of financial returns is predictable. Therefore, if volatility is predictable, it makes sense to make forecasts of it to predict future values of the return distribution. The elements of variance covariance matrix Σ_t can be estimated by:

$$[4.2] \quad \sigma_i^2 = \frac{1}{N-1}\sum_{t=1}^{N}(r_{it} - \bar{r}_i)^2, \qquad \bar{r}_i = \frac{1}{N}\sum_{t=1}^{N} r_{it}$$

$$[4.3] \quad \sigma_{ij} = \frac{1}{N-1}\sum_{t=1}^{N}(r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j)$$

where σ_{it}^2 is the variance of the ith risk factor and σ_{ijt} is the covariance between the ith and the jth risk factor. N is the length of the window of observations. For this approach a moving window of 200 observations is used for better calibration of the model. The next step is to compute the volatility of the portfolio of the risk factors, which is less than the sum of volatilities as Markowitz (1952) had shown and is given by:

[4.4] $\sigma_p^2 = w\Sigma_t w'$

where w is a one by number of assets vector of weights and w' represents the transpose of vector w. Ultimately, the portfolio variance can be translated into a VaR measure for one monetary unit. If all individual security returns are assumed normally distributed, the portfolio return, a linear combination of normal random variables, is also normally distributed, so the VaR estimate is obtained using percentile points on the normal distribution. For 95% confidence level VaR can be estimated by:

[4.5] $VaR_p^{95\%} = 1.65\sqrt{\sigma_p^2}$

and for 99% confidence level

[4.6] $VaR_p^{99\%} = 2.33\sqrt{\sigma_p^2}$

The strong side of the equally weighted moving average is that it is flexible, simple and widely used. It also enables the addition of specific scenarios and enables the analysis of the sensitivity of the results with respect to the parameters. However, it relies heavily on the important assumption that all of the major market parameters are normally distributed. The issue is whether the normal approximation is realistic. Moreover Σ_t is assumed to be time invariant. Empirical work (Engel and Gizycki (1999)) indicates that the Σ_t does vary through time.

4.2. Exponentially weighted moving average

In order to capture the dynamic features of volatility an exponential moving average of historical observations is used, where the latest observations carry the highest weight in the volatility estimate. Although the exponentially weighted moving average estimation ranks a level above the fixed weight approach in terms of sophistication, it is not complex to implement. This approach was proposed by J.P. Morgan and Reuters (1996) in their RiskMetrics. According to a survey of Deloitte & Touche (2002) the most widely used approach is this parametric, popularized by the JP Morgan. The exponentially weighted moving average has two important advantages over the fixed weight approach. First, volatility reacts faster to shocks in the market as recent data carry more weight than data in the distant past. Second, following a shock (a large return) the volatility declines exponentially as the weight of the shock observation falls. For a window of N observation, the exponentially weighted volatility is computed by the formulas:

[4.7] $$\sigma_i^2 = (1-\lambda)\sum_{t=1}^{N} \lambda^{t-1}(r_{it} - \bar{r}_i)^2$$

[4.8] $$\sigma_{ij} = (1-\lambda)\sum_{t=1}^{N} \lambda^{t-1}(r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j)$$

The exponentially weighted moving average can be written in recursive form and can be used to make a one day volatility forecasts. In order to derive the recursive form, it is assumed that infinite amount of data are available and that the sample mean is zero.

[4.9] $$\sigma_{it+1}^2 = \lambda\sigma_{it}^2 + (1-\lambda)r_{it}^2$$

[4.10] $$\sigma_{ij,t+1} = \lambda\sigma_{ijt} + (1-\lambda)r_{it}r_{jt}$$

The parameter λ is often referred to as the decay factor. This parameter, which is between zero and one, determines the relative weights that are applied to the observations and the effective amount of data used in estimating volatility. The optimal decay factor minimizes the root mean squared error of volatility forecasts. In this paper, $\lambda = 0{,}94$ is used and a windows of 200 observations.

It is obvious that volatilities and covariances vary with time. Implicitly, what we are assuming in modeling the variances and covariances as exponentially weighted moving averages is that the variance process is nonstationary. RiskMetrics model is a private case of an integrated generalized autoregressive conditional heteroscedastic process (Nelson 1990, Lumsdaine 1995) without a drift and with constant parameters. Using the above equations we compute the elements of variance covariance matrix Σ_{t+1}. Assuming that market returns are normally distributed, the one day forecast of Σ_{t+1} is used to obtain the VaR estimate, for different confidence levels using equations [4.4] and [4.5].

4.3. Constant Correlation GARCH

A common feature of financial time series returns is volatility clustering (volatility may be higher for certain time periods and low for other periods). A second characteristic is that volatility evolves over time in a continuous manner and it is often stationary (volatility varies within some fixed range). A variety of econometric models are available in the literature for modeling volatility. These models are referred to as conditional heteroscedastic models and can be used to estimate the variance covariance matrix. The constant correlation GARCH of Bollerslev (1990) assumes that the risk factors correlation is time invariant. This model estimates each diagonal element of the variance covariance matrix Σ_t using a univariate GARCH model (Bollerslev 1986). The autocorrelation function of the squared individual return series indicates a statistically significant lag-1 and lag-2 autocorrelation in most emerging markets. However, the magnitude of the autocorrelation in developed markets is less significant. In a daily *AR(r)-GARCH (p,q)* model the conditional volatility today depends on the previous days' conditional volatility and on the previous days' squared forecast error. For choosing the appropriate order of the model, the required coverage probability is considered. The *AR(2)-GARCH (1,1)* model, which most successfully captures the required coverage probability a in the $(1-a)\%$ confidence level, is given by:

$$[4.11] \quad r_{it} = \phi_0 + \phi_1 \cdot r_{i,t-1} + \phi_2 \cdot r_{i,t-2} + e_{it} \qquad e_t = \sigma_t \varepsilon_t$$

[4.12] $\sigma_{it}^2 = \omega_0 + \alpha \cdot e_{i,t-1}^2 + \beta \cdot \sigma_{i,t-1}^2$

Let N to be the number of risk factors. Define R to be the N by N correlation matrix, which is time invariant, and $D_t = diag\left\{\sqrt{\sigma_{it}^2}\right\}$ (diagonal matrix of GARCH volatilities), the variance covariance matrix Σ_t is given by:

[4.13] $\Sigma_t = D_t \cdot R \cdot D_t$

There are three parameters to be estimated for each variance using maximum likelihood techniques. By assuming that correlation between risk factors is constant, this model reduces the total number of parameters to be estimated to $(3N + N(N-1)/2)$. However, as the number of risk factors increases, the model became too computationally time consuming for practical applications. For all multivariate GARCH models a moving window of 700 observations is used in order to compute the elements of the variance covariance matrix Σ_t. Once the matrix Σ_t has been estimated, the standard variance covariance approach described above is used to estimate the VaR.

4.4. Dynamic Conditional Correlation GARCH

The assumption of constant conditional correlation is arguably too restrictive over long time periods. Engle and Sheppard (2001) and Engle (2002) generalized Bollerslev's constant correlation model to obtain a dynamic conditional correlation GARCH. This model differs only in allowing the correlation matrix R_t to be time varying. DCC GARCH, which parameterizes the conditional correlations directly, is naturally estimated in two steps. The first is a series of univariate GARCH estimates for each asset and the second is to use transformed residuals resulting from the first step to estimate a conditional correlation estimator. This method has clear computational advantages over other multivariate GARCH models in that the number of parameters to be estimated in the correlation process is independent of the number of series to be correlated. Thus potentially very large correlation matrices can be computed. The conditional variance covariance matrix is given by:

[4.14] $\Sigma_t = D_t \cdot R_t \cdot D_t$

where R_t is the correlation matrix, which is time varying and $D_t = diag\left\{\sqrt{\sigma_{it}^2}\right\}$. The elements in D_t follow the univariate GARCH(1,1) processes as described in equation [4.12]. Engle's particular $DCC(m,n)$ structure, where m is the lag length of the innovation term in the DCC estimator and n is the lag length of the lagged correlation matrices, can be written as:

[4.15] $R_t = Q_t^{*-1} Q_t Q_t^{*-1}$

[4.16] $Q_t = (1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} \beta_n)\overline{Q} + \sum_{m=1}^{M} a_m (\varepsilon_{t-m} \varepsilon_{t-m}^{'}) + \sum_{n=1}^{N} \beta_n Q_{t-n}$

where ε_τ is a vector containing the residuals standardized by their conditional standard deviation, $\overline{Q}$ is the unconditional variance covariance matrix of the standardized residuals resulting from the first stage estimation and Q^* is a diagonal matrix containing the square root of the diagonal elements of Q_t. For choosing the appropriate $DCC(m,n)$ structure, the required coverage probability is considered. The $DCC(1,1)$ structure, which most successfully captures the required coverage probability, is used in all cases. The dynamic conditional correlation GARCH framework can be estimated using the maximum likelihood method. Value at Risk is computed using a one day forecast of matrix Σ_t, assuming that the return series is conditionally normally distributed.

4.5. Orthogonal GARCH

The multivariate GARCH models described so far are too computationally time consuming for large portfolios. A solution proposed by Engle, Ng and Rothschild (1990), which exploits factor analysis to enable a small number of factors to describe a high proportion of the structure of the variance covariance matrix. Later, Alexander and Chibumba (1998) propose an 'orthogonal' GARCH model that utilized principal component analysis in order to orthogonalize the original returns. These orthogonal returns are known as the 'principle components'. Since these are, by definition, orthogonal to each other, the number of parameters need to be estimated is reduced substantially because we no longer need to measure the covariances.

Define T to be the length of the window of observations and N the number of risk factors. Then the T by N matrix R contains the full set of historical returns. Let W be a N by N matrix of eigenvectors of $R'R$. The orthogonal principle components are then the columns $[P_1 ... P_N]$ of:

[4.17] $P = [P_1 ... P_N] = RW$

Solving for matrix R and using the property of matrix W that its inverse is equal to its transpose, it is possible to write the change in risk factor i as a linear combination of the principle components where the weights are given by the elements of the ith eigenvector:

[4.18] $R = PW'$

$$\Rightarrow r_i = w_{i1} p_1 + w_{i2} p_2 + ... + w_{iN} p_N$$

where r_i is the return of the ith risk factor, p_j is the jth zero mean principal component and w_{ij} are the weights. The variance covariance matrix Σ_t is given by:

[4.19] $\Sigma_t = W \cdot V \cdot W'$

where and $V_t = diag\{\text{var}(P)\}$ (diagonal matrix of variances of principal components). It should be noted that in order to obtain the variance covariance matrix Σ_t, only the eigenvectors of $R'R$ and the diagonal elements of V need to be estimated and each of the principal component variances is modeled independently, in a univariate setting, using a GARCH(1,1) framework.

4.6. Exponential GARCH

An alternative approach, which has great computational advantages, is to use an asymmetric univariate GARCH. Instead of estimating the variance covariance matrix of all the portfolio components, it is possible to estimate only the variance of the portfolio returns. Brooks and Persand (2003) compare the forecasting performance of univariate and multivariate forecasting models for financial asset return volatility. They argue that the gain from using a multivariate GARCH model for forecasting volatility is minimal. This result is true both under standard statistical and risk management evaluation measures. Given the complexity, estimation difficulties, and computer-intensive nature of multivariate GARCH modeling, they conjecture that unless the conditional covariances are required, the estimation of multivariate GARCH models is not worth while. In the context of portfolio volatility, more accurate results can be obtained by aggregating the portfolio constituents into a single series, and forecasting that, rather than modeling the individual component volatilities and the correlations between the returns.

Define T to be the length of the window of past returns and N the number of risk factors. Then the N by T matrix R contains the full set of historical returns. Let w to be a one by N vector of portfolio weights. Then the sample of historical returns of the portfolio is given by:

[4.20] $R_p = w \cdot R$

The Nelson's (1991) exponential GARCH (1,1) model is a specification for modeling asymmetric dependence of volatility on past returns, and it is given by:

$$[4.21] \quad \ln(\sigma_t^2) = a_0 + \gamma\left(\frac{e_{t-1}}{\sigma_{t-1}}\right) + \lambda\left[\left(\frac{|e_{t-1}|}{\sigma_{t-1}}\right) - \left(\frac{2}{\pi}\right)^{0.5}\right] + \beta \ln(\sigma_{t-1}^2)$$

Variance is asymmetric if $\lambda \neq 0$. Impact of the most recent residual is exponential rather than quadratic. There is evidence that this specification is too strong for large shocks.

4.7. Historical Simulation

Historical simulation is perhaps the most simple to implement non parametric method. It does not require any statistical assumption about the distribution of portfolio returns. This technique uses past price movements to calculate a hypothetical future distribution of profit and loss on the current portfolio. This provides a series of changes in portfolio value that would have been realized had the current portfolio been held over the period in question. The Value at Risk is then set equal to a percentile of the empirical distribution of historical returns given a required level of confidence. At least one year of recent daily returns must be used in historical simulation. A longer period is more appropriate when available. In this paper, a moving window of 600 daily returns is used for each risk factor for better calibration of the model.

Historical simulation assumes that past and present moments of the density function of returns of each risk factor are constant and equal. Given the data set of historical portfolio returns, which is computed using equation [4.20], a distribution of potential future portfolio profits (positive returns) and losses (negative returns) is constructed and the VaR number is derived for a specific confidence interval $(1-a)\%$ as the corresponding quantile of this distribution.

$$[4.22] \quad VaR_{t+1}^{a} = Quantile\left\{\{r_t\}_{t=1}^{T}, 100a\right\}$$

Historical simulation is powerful because of its simplicity and its relative lack of distributional assumptions. This method accommodates non-normal distributions and therefore it accounts for fat tails and excess skewness. This simple approach does not come without a cost, as the choice of the sample length T affects the estimates. If T is too large, the most recent observations, that probably are describing better the future distribution, carry the same weight with the earliest returns which are not equally important as the new ones. On the other hand, if T is too small, then either too few or insufficient extreme events will be observed. In both cases, the sample size is a hinter factor, as the VaR is either underestimated or overestimated. This remark was confirmed by Van den Goorbergh and Vlaar (1999) as they argued that the VaR estimates for Dutch equity index were extremely sensitive to the sample length. Historical simulation's ability to predict future losses may be however undermined if the distribution of any risk factors is not independent and identically distributed. Stationarity implies that the probability of occurrence of a specified loss is the same for each day. Independence implies that the size of price movement in one period will not influence the movement of any successive prices.

4.8. Weighted Historical Simulation

Weighted historical simulation is a hybrid approach proposed by Boudoukh, Richardson and Whitelaw (1998). It exploits the non parametric nature of historical simulation while imposing an exponential weighting scheme on the historical returns. While the historical simulation attributes equal weights to each observation in building the conditional empirical distribution, the hybrid approach attributes exponentially declining weights to historical returns. Most recent observations are assigned a bigger weight.

The approach starts with calculating the historical returns of the portfolio using equation [4.20]. The next step is to assign a weight k_t to each return according to how far in the past the observation is. All the weights sum to unity.

$$[4.23] \quad k_{t-n+1} = \frac{\lambda^{n-1}(1-\lambda)}{(1-\lambda^n)}, \qquad n = 1 \ldots T$$

The returns are then arranged in ascending order. To obtain Value at Risk of the portfolio for a given confidence level $(1-a)\%$, we start from the lowest return and we keep accumulating the weights until $a\%$ is reached. Linear interpolation is used between adjacent points in order to achieve exactly $a\%$ of the distribution. In this paper, a moving window of 600 observations is used and the value of λ equals to 0,99.

The two main shortcomings of weighted historical simulation are that it depends on a particular historical data set and that it can not accommodate changes in market structure. Pritsker (2000) reviews the assumptions and limitations of historical and weighted historical simulation. He points out that both methods associate risk with only the lower tail of the distribution. In an example, he showed that after the crash of 1987 the estimated VaR of a short equity portfolio, as computed by historical simulation or weighted historical simulation, did not increase. The reason is that the portfolio recorded a huge profit during the day of the crash. Pritsker goes further by formulating some interesting properties of the historical and weighted historical simulation.

4.9. Filtered Historical Simulation

Filtered historical simulation, which was presented by Hull and White (1998) and by Barone-Adesi, Giannopoulos and Vosper (1999), takes into account the changes in past and current volatilities of historical returns and make the least number of assumptions about the statistical properties of future price changes. It consists of drawing random standardised returns from the portfolio's historical sample and after rescaling these standardised historical returns with the current volatility, they are used as innovations for generating scenarios for future portfolio returns. This method not only generates scenarios that conform to the past history of the current portfolio's profits and losses, but also overcomes an additional limitation of the variance-covariance model. It allows both past and future volatility to vary over time. Filtered historical simulation retains the nonparametric nature of historical simulation by bootstrapping (sampling with replacement) from the standardized residuals. One of the appealing features of the filtered historical simulation is its ability to generate relatively large deviations (losses and gains) not found in the original portfolio return series.

The bootstrapped filtered historical simulation method requires the observations to be approximately independent and identically distributed. However, most financial return series exhibit some degree of autocorrelation and, more importantly, heteroskedasticity. To produce a series of independent and identically distributed observations, a first order autoregressive moving average model *ARMA(1,1)* is fitted to the conditional mean of the portfolio returns and a *GARCH(1,1)* model (Bollerslev, 1986) is fitted to the conditional variance. For choosing the appropriate order of the *ARMA(r,m)-GARCH (p,q)* model, the required coverage probability, which most successfully approximates the true coverage probability, is considered. The first order autoregressive moving average model compensates for autocorrelation, while the *GARCH(1,1)* model compensates for heteroskedasticity. The portfolio return series is computed using equation [4.20]. The residuals and conditional volatilities are given by:

$$[4.24] \quad r_t = \phi_0 + \phi_1 r_{t-1} + \theta_1 e_{t-1} + e_t \qquad e_t \sim N(0, \sigma_\tau^2)$$

$$[4.25] \quad \sigma_t^2 = \omega_0 + \alpha_1 e_{t-1}^2 + \alpha_2 \sigma_{t-1}^2$$

Having filtered the model residuals from the portfolio return series, each residual is standardized by the corresponding conditional standard deviation.

$$[4.26] \quad z_t = \frac{e_t}{\sqrt{\sigma_t^2}}$$

These standardized residuals represent the underlying zero-mean, unit-variance, independent and identically distributed series. The independent and identically distributed property is important for bootstrapping and allows the sampling procedure to safely avoid the pitfalls of sampling from a population in which successive observations are serially dependent. The filtered historical simulation bootstraps standardized residuals to generate paths of future asset returns and, therefore, makes no parametric assumptions about the probability distribution of those returns. To do this, each bootstrapped standardized residual is scaled by the deterministic volatility forecast one day ahead:

[4.27] $\varepsilon_{t+1} = z_t \cdot \sqrt{\sigma_{t+1}^2}$

Forecasts of volatility for one day ahead σ_{t+1}^2 are simulated by the recursive substitution of scaled residuals into the variance equation (GARCH). Using the bootstrapped standardized residuals as the independent and identically distributed input noise process, future portfolio returns are generated using forecast from the autoregressive moving average model:

[4.28] $r_{t+1} = \phi_0 + \phi_1 r_t + \theta_1 \varepsilon_t + \varepsilon_{t+1}$

Value at Risk is calculated as a quantile of the future portfolio return distribution for a given confidence level.

4.10. Monde Carlo Simulation

Monte Carlo simulation is an alternative to traditional historical simulation approaches. The main difference is that Monte Carlo, using a defined stochastic process and a statistical distribution, attempts to generate a large number of correlated returns for each risk factor that were not, in fact, observed over the historical period, but they are just as probable to occur in the future. Individual returns are then used to construct a distribution of hypothetical portfolio profits and losses. Finally, the value at risk is determined as the corresponding quantile of this distribution.

The first and most crucial, step in the simulation process consists of choosing an appropriate theoretical distribution that will conform to the empirical distribution of each asset. This implementation of the Monte Carlo simulation employs the assumption that returns are normally distributed. In a multivariate case we should also take into account that returns are correlated. To account for this correlation, Cholesky decomposition is used. Let R to be the correlation matrix computed from historical returns. Define C to be a unique lower triangular matrix, produced by Cholesky decomposition, such that:

[4.26] $C \cdot C' = R$

The population of correlated random variable used in the simulation process is given by:

[4.27] $Z = C \cdot U$

where U is a vector of uncorrelated random variables that are normally distributed with zero mean and unit variance.

The next step is to choose a particular stochastic model for the behavior of equity prices. A commonly used model is the geometric Brownian motion. Small movements in asset prices can by described by:

[4.25] $dS = \mu \cdot S \cdot dt + \sigma \cdot S \cdot dz$

where S is the asset price, and the parameters μ and σ represent the expected drift and volatility, which are estimated from historical data using exponential smoothing method. Finally 20.000 possible returns for each individual risk factor, over an interval of length dt, are given by:

[4.28] $\frac{dS}{S} = \mu \cdot dt + \sigma \cdot dz = \mu \cdot dt + \sigma \cdot \varepsilon \cdot \sqrt{dt}$

where ε represents a random drawing from a standardized normal distribution. Hypothetical portfolio returns are computed using equation [4.20] and from the distribution of portfolio returns, the appropriate percentile is determined to provide the VaR estimate for a given confidence level.

The main drawback of Monte Carlo methods is their computational time requirements. Moreover, the generation of the scenarios is based on random numbers drawn from a theoretical distribution, often normal, which not only does not conform to the empirical distribution of most asset returns, but also limits the losses to around three or four standard deviations when a very large number of simulation runs is carried out.

Finally, another potential weakness is model risk. Monde Carlo simulation relies on a specific stochastic model and therefore, it is subjected to the risk that the model is wrong.

4.11. Mixed of Normal Distributions Monde Carlo

Mixture of normal distributions is used to model situations where the data can be viewed as arising from two or more distinct populations. Zangari (1996) has proposed a practical application of mixture of normal distributions to incorporate fat tails in the VaR calculation. The mixture of normal distributions can accommodate the observed skewness and kurtosis of the financial time series and hence can describe them better than the standard normal distribution.

Under a mixture model, the probability p that a given day is quiet and the probability $1-p$ that a given day is hectic are specified. Conditionally, returns are normally distributed, but with low volatility on quiet days and with high volatility on hectic days. The resulting unconditional distribution, the mixture normal distribution, exhibits heavy tails due to the random nature of the volatility. In order to fit a mixture of normal distribution, we need to estimate five parameters: two means, two standard deviations, and the probability of having a quiet day. Mathematically, the standardized return distribution is generated according to the following probability density function:

$$[4.29] \quad PDF = p \cdot N_{\alpha}(\mu_{\alpha},\sigma_{\alpha}^{2}) + (1-p) \cdot N_{\beta}(\mu_{\beta},\sigma_{\beta}^{2})$$

The parameters of the mixture of normal distributions can be estimated by maximum likelihood techniques, under the assumption that both distributions have zero mean. The likelihood function is given by:

$$[4.30] \quad l(p,\sigma_{\alpha},\sigma_{\beta}) = \frac{1}{\sqrt{2\pi}} \sum_{t=1}^{T} \left[\frac{p}{\sigma_{\alpha}} \exp(-\frac{r_t^2}{2\sigma_{\alpha}^2}) + \frac{(1-p)}{\sigma_{\beta}} \exp(-\frac{r_t^2}{2\sigma_{\beta}^2}) \right]$$

where r_t is the historical returns of each risk factor. Unfortunately, as pointed out by Hamilton (1991), a global maximum does not exist for this function. Consequently, attempting to use this approach in order to estimate the parameter may lead to instability, local solutions, and non convergence problems. Venkataraman (1997) used the quasi-Bayesian maximum likelihood estimation approach (first suggested by Hamilton) in order to estimate the parameters of the mixture of normal distributions, which is computationally simpler than the techniques suggested by Zangari. The method is to maximize the following variant to the likelihood function:

$$[4.30] \quad l(p,\sigma_\alpha,\sigma_\beta)-\frac{\alpha_\alpha}{2}\log(\sigma_\alpha^2)-\frac{\alpha_\beta}{2}\log(\sigma_\beta^2)-\frac{b_\alpha}{\sigma_\alpha^2}-\frac{b_\beta}{\sigma_\beta^2}-\frac{c_\alpha m_\alpha^2}{2\sigma_\alpha^2}$$

where $l(p,\sigma_\alpha,\sigma_\beta)$ is the likelihood function defined in equation [4.29] and $\{\alpha_\alpha,\ \alpha_\beta,\ b_\alpha,\ b_\beta,\ c_\alpha, m_\alpha\}$ are (nonnegative) constants that reflect one's prior beliefs about the parameters that are being estimated. The calibration in the multivariate case is more difficult, as we must estimate two variance covariance matrices corresponding to the quiet and hectic days and the probability p. For simplicity, the parameters of the mixture of distributions for each risk factor are computed separately as in univariate case. Random numbers are drawn from each mixture of distributions with probability p from the first distribution and with probability $1-p$ from the second distribution and using Cholesky decomposition we create correlated random numbers. Once the random numbers have been obtained, the standard Monte Carlo approach described above is used to estimate VaR.

4.12. Extreme Value Theory

Extreme Value Theory, which has recently received much attention in the risk management literature, provides a formal framework to study the tail behavior of the fat-tailed distributions. It has, therefore, the potential to perform better than other approaches in terms of predicting unexpected extreme changes. Instead of forcing a single distribution for the entire sample, it is possible to investigate only the tails of the sample distribution, because only the tails are important for estimating Value at Risk

The oldest group of extreme value theory models is the Block Maxima models, in which one divides the data into consecutive blocks and focuses on the series of maxima in these blocks (see Longin 1996, 2000). As an alternative, a more modern group of models is the Peaks over Threshold models, in which one looks at those events in the data that exceed a high threshold and model these separately from the rest of the observations (see Danielsson and de Vries (1997), Embrechts et al (1999), McNeil and Frey (2000), Bali (2003)). The Peaks over Threshold models are generally considered to be the most useful for practical applications, due to their more efficient use of the (often limited) data on extreme values. This paper will concentrate on the Peaks over Threshold (POT) method, which is based on the Generalized Pareto Distribution.

Modeling the tails of each risk factor's distribution with a Generalized Pareto Distribution, which can describe the behavior of the extreme observations, requires the observations to be approximately independent and identically distributed. If the returns are not independent and identically distributed, the estimated parameters of the Generalized Pareto Distribution will be biased. Following McNeil and Frey (2000), the return series of each risk factor is filtered via a GARCH(1,1) process:

[4.31] $r_t = \mu + e_t \qquad e_t \sim N(0, \sigma_\tau^2)$

[4.32] $\sigma_t^2 = \omega_0 + \alpha_1 e_{t-1}^2 + \alpha_2 \sigma_{t-1}^2$

Having filtered the model residuals from each return series, the residuals are standardized by the corresponding conditional standard deviation:

[4.33] $z_t = \dfrac{e_t}{\sqrt{\sigma_t^2}}$

The next step in the overall modeling process is to fix a high threshold u, assuming that excess residuals over this threshold have a Generalized Pareto Distribution (GPD). The GPD is a two parameter distribution with distribution function:

$$[4.30] \quad G_{\xi,\beta}(y) = \begin{cases} 1-(1+\frac{\xi \cdot y}{\beta})^{-1/\xi} & if \quad \xi \neq 0 \\ 1-\exp(-\frac{y}{\beta}) & if \quad \xi = 0 \end{cases}$$

where $\beta > 0$ is a scale factor and $y \geq 0$ when $\xi \geq 0$ and $0 \leq y \leq -\beta / \xi$ when $\xi < 0$. This distribution is generalized in the sense that it subsumes certain other distributions under a common parametric form. ξ is the important shape parameter of the distribution. The case $\xi > 0$ corresponds to the heavy-tailed distributions whose tails decay like power functions such as the Pareto. The case $\xi = 0$ corresponds to distributions like the normal, exponential, gamma and lognormal, whose tails decay exponentially and the case $\xi < 0$ corresponds to short-tailed distributions with a finite right endpoint like the uniform and beta distributions. The distribution of excesses losses over a high threshold u is defined to be:

$$[4.28] \quad F_u(y) = P\{X - u \leq y \mid X > u\}$$

for $0 \leq y < x_0 - u$ where x_0 is the finite right endpoint of F. The excess distribution represents the probability that a loss exceeds the threshold u by at most an amount y, given the information that it exceeds the threshold. It is very useful to observe that it can be written in terms of the underlying F as

$$[4.29] \quad F_u(y) = \frac{F(y+u) - F(u)}{1 - F(u)}$$

Balkema and de Haan (1974) and Pickands (1975) showed for a large class of distributions F that it is possible to find a positive measurable function $\beta(u)$ such that

[4.30] $$\lim_{u \to x_0} \sup_{0 \le y < x_0 - u} | F_u(y) - G_{\xi,\beta(u)}(y) | = 0$$

That is, for a large class of underlying distributions F, as the threshold u is progressively raised, the excess distribution F_u converges to a generalized Pareto. Since $F_u(y)$ converges to the generalized Pareto distribution for sufficiently large u and since $x = u + y$ for $x > u$, using equation [4.29] we have

[4.31] $$F(x) = (1 - F(u))G_{\xi,\beta}(x - u) + F(u)$$

After determining a high threshold u, the last term on the right hand side can be estimated by $(n - N_u)/n$, where N_u is the number of exceedances and n is the sample size. As a result, we have the following tail estimator

[4.32] $$\hat{F}(x) = 1 - \frac{N_u}{n}(1 + \hat{\xi}\frac{x - u}{\hat{\beta}})^{-1/\hat{\xi}}$$

where $\hat{\xi}$ is the estimated shape parameter and $\hat{\beta}$ is the estimated scale parameter of the Generalized Pareto Distribution using the maximum likelihood estimation. Notice that the tail estimator is valid only for $x > u$. For a given confidence level $a > F(u)$, the VaR estimate is calculated by inverting the tail estimator in Equation [4.32] to obtain

[4.33] $$VaR_a = u + \frac{\hat{\beta}}{\hat{\xi}}((\frac{n}{N_u}(1 - a))^{-\hat{\xi}} - 1)$$

Following Longin (2000), after having estimated Value at Risk using Peaks over Threshold method for each individual risk factor, the portfolio VaR is given by:

[4.34] $VaR_p = \sqrt{v \cdot R \cdot v'}$

where R is the correlation matrix and v is a one by number of risk factors vector with individual VaR estimates.

A key to the abbreviated model names that appear in the following sections is provided in Table 1. The number in brackets indicates the length of the moving window of observations for each model.

Table 1: The Value at Risk Models

Abbreviation	**Description**
FWA(200)	Fixed weight approach
EWMA(200)	Exponentially weighted moving average
CC-GARCH(700)	Constant correlation GARCH
DCC-GARCH(700)	Dynamic conditional correlation GARCH
OGARCH(700)	Orthogonal GARCH
EGARCH(700)	Exponential GARCH
HS(600)	Historical simulation
WHS(600)	Weighted historical simulation
FHS(600)	Filtered historical simulation
MC(200)	Monde Carlo simulation
MNMC(200)	Mixed of Normal Distributions Monde Carlo
EVT(700)	Extreme Value Theory

This table provides the abbreviated model names. The number in brackets indicates the length of the moving window of observations, which is used to estimate Value at Risk.

5. Model Evaluation

Backtesting is a formal statistical framework that consists of verifying whether a Value at Risk model is adequate. It is very important to evaluate the relative predictive performance of alternative VaR models prior to adoption, because different VaR implementations are known to yield fairly different VaR forecasts for the same portfolio, sometimes leading to significant errors in risk measurement. It will be expensive in terms of both time and money for a financial organization to change, once any one model has been adopted. The choice of an inadequate VaR model is called 'model risk', and is recognized as an important issue in risk management.

The first stage of the evaluation process involves testing all VaR models for statistical accuracy. Christoffersen (1998) has designed a three step procedure for the evaluation of interval forecasts: a test for "unconditional coverage", a test for "independence" and a test for "conditional coverage". All three tests are performed using the likelihood ratio framework. In the second stage of the evaluation process, a regulatory loss function, which expresses the goals of a financial regulator, is used in order to compare the VaR models that are not rejected in the first stage and to select the best model, which minimizes the total loss in a statistically meaningful way.

5.1. Unconditional Coverage Test

A Value at Risk measure, estimated at a significant level $1-p$, achieves the correct unconditional coverage if the actual portfolio's losses exceed the VaR measures p percent of the time in very large samples. If the loss of the portfolio on a particular day is larger than the VaR number predicted in advance for that day, we have a violation or failure. Kupiec (1995) developed a framework for unconditional coverage that tests a null hypothesis that the probability of failure $\hat{\pi}$ is equal to p against an alternative that the probability differs from p assuming the failure process is independently distributed. The likelihood ratio statistic for unconditional coverage is given by:

$$[5.1] \quad LR_{uc} = -2\ln\left(\frac{(1-p)^{T_0} p^{T_1}}{(1-\hat{\pi})^{T_0}\hat{\pi}^{T_1}}\right) \sim X_1^2$$

where: T_0 is the number of times in the sample when the VaR forecast is not exceeded,

T_1 is the number of violations in the sample,

T is the number of observation in the sample,

$$\hat{\pi} = \frac{T_1}{T}$$

The likelihood ratio statistic is asymptotically distributed chi-square with one degree of freedom. Thus we would reject the null hypothesis ($H_0 : \hat{\pi} = p$) at 95% confidence level if $LR_{uc} > 3.8416$. It should be noted that the choice of confidence region for the test is not related to the confidence level p selected for VaR. This quantitative level refers to the decision rule to accept or reject the model.

Alternatively, we can calculate the P-value associated with our test statistic for 95% confidence level. The P-value is defined as the probability of getting a sample which conforms even less to the null hypothesis than the sample we actually got, given that the null hypothesis is true. In this case the P-value is calculated as:

$$[5.2] \quad \text{P-value} = 1 - F_{X_1^2}(LR_{uc})$$

where $F_{X_1^2}(LR_{uc})$ denotes the cumulative density function of a X^2 variable with one degree of freedom. If the P-value is below 5%, we reject the null hypothesis.

5.2. Independence Test

The Value at Risk forecasts should be small in periods exhibiting low volatility and larger in more volatile periods. Occasions when the loss actually exceeds the VaR forecast, known as a failure or violation, should therefore be spread across the sample and not appear in clusters. A VaR model, which does not capture the volatility dynamics of the underlying portfolio return distribution, will exhibit a clustering of violations but may still exhibit correct unconditional coverage. If the VaR forecasts are used in order to estimate capital levels required to support risk taking, the clustering of violations is connected to solvency risk. A financial organization will be unable to cover a series of consecutive losses that exceed VaR estimates. An independence test has been developed by Christofferson (1998), who extends the LR_{uc} statistic to specify that the violations must be serially independent. In the test for independence, the hypothesis of an independently distributed failure process is tested against the alternative hypothesis of a first-order Markov failure process. The likelihood ratio statistic for this test is given by:

$$[5.3] \quad LR_{ind} = -2\ln\left(\frac{(1-\hat{\pi}_2)^{(T_{00}+T_{10})}\,\hat{\pi}_2^{\,(T_{01}+T_{11})}}{(1-\hat{\pi}_{01})^{T_{00}}\,\hat{\pi}_{01}^{\,T_{01}}\,(1-\hat{\pi}_{11})^{T_{10}}\,\hat{\pi}_{11}^{\,T_{11}}}\right) \sim X_1^2$$

where: T_{00} is the number of times in the sample when a non violation is followed by an other non violation, T_{01} is the number of times in the sample when a non violation is followed by a violation, T_{10} is the number of times in the sample when a violation is followed by a non violation, T_{11} is the number of times in the sample when a violation is followed by an other violation. The probabilities $\hat{\pi}_{01}$, $\hat{\pi}_{11}$ and $\hat{\pi}_2$ are given by:

$$\hat{\pi}_{01} = \frac{T_{01}}{T_{00}+T_{01}}$$

$$\hat{\pi}_{11} = \frac{T_{11}}{T_{10}+T_{11}}$$

$$\hat{\pi}_{2} = \frac{T_{01}+T_{11}}{T_{00}+T_{01}+T_{10}+T_{11}}$$

In large samples, the distribution of the LR_{ind} test statistic is also chi-square with one degree of freedom. Thus we would reject the null hypothesis ($H_0 : \hat{\pi}_{01} = \hat{\pi}_{11}$) at 95% confidence level if $LR_{ind} > 3.8416$. The P-value is also calculated for this test.

5.3. Conditional Coverage Test

Ultimately, we care about simultaneously testing if the VaR violations are independent and the average number of violations is correct. We can test jointly for independence and correct coverage using the conditional coverage test (Christoffersen 1998). This test is done by testing the null hypothesis of an independent failure process with failure probability p against the alternative hypothesis of a first-order Markov failure process with a different transition probability matrix. The likelihood ratio statistic for conditional coverage test is given by:

$$[5.4] \quad LR_{cc} = -2\ln\left(\frac{(1-p)^{T_0} p^{T_1}}{(1-\pi_{01})^{T_{00}} \pi_{01}^{T_{01}} (1-\pi_{11})^{T_{10}} \pi_{11}^{T_{11}}}\right) \sim X_2^2$$

Note that the LR_{cc} test takes the likelihood from the null hypothesis in the LR_{uc} test and combines it with the likelihood from the alternative hypothesis in the LR_{ind} test. We therefore have that:

$$[5.5] \quad LR_{cc} = LR_{uc} + LR_{ind} \sim X_2^2$$

So that the joint test of conditional coverage can be calculated by simply summing the two individual tests for unconditional coverage and independence. The LR_{cc} test statistic is distributed chi-square with two degrees of freedom. Thus we would reject the null hypothesis ($H_0 : \hat{\pi}_{01} = \hat{\pi}_{11} = p$) at 95% confidence level if $LR_{uc} > 5.9915$. The P-value is also calculated for this test.

It is probably unrealistic to expect that a VaR measure will provide exactly correct conditional coverage. However, we would at least hope that the VaR estimate would increase when risk appears to increase. Christoffersen's basic framework is limited in that it only deals with first-order dependence. Christoffersen and Diebold (2000) suggest a regression-based approach that can be used to test for the existence of various forms of dependence structures.

5.4. Regulatory loss function

The accuracy of the Value at Risk estimates can also be evaluated using the general loss function approach of Lopez (1998, 1999). VaR models are assessed by comparing the values of the loss function. A VaR model which minimizes the total loss is preferred to other models. Lopez proposed three loss functions which might reflect the utility function of a regulator: the binomial loss function, the magnitude loss function and the zone loss function. The latter two penalize failures more severely as compared with the binomial loss function. In this paper, a regulatory loss function, which pays attention to the magnitude of the losses that exceed the VaR estimate, is used to reflect the regulator's utility function. The regulatory loss function is defined as:

$$[5.6]\quad l_t = \begin{cases} (\Delta P_t - VaR_t)^2 & \text{if } \Delta P_t < VaR_t \\ 0 & \text{if } \Delta P_t \geq VaR_t \end{cases}$$

where ΔP_t is the change in the market value of the portfolio at time t. The quadratic term in the above loss function ensures that large failures are penalized more than the small failures. Diebold and Mariano (1995) propose and evaluate explicit tests of the null hypothesis of no difference in the accuracy of two competing forecasts. Sarma, Thomas and Shah (2003) use such a non-parametric sign test to evaluate the superiority of a VaR model from another in terms of the loss function values. The superiority of model i over model j with respect to the regulatory loss function can be tested by performing a one-sided sign test. The null hypothesis is $(H_0 : \theta = 0)$ against the one-side alternative hypothesis ($H_0 : \theta < 0$). θ is the median of the distribution of z_t, defined as $z_t = l_{it} - l_{jt}$, where l_{it} and l_{jt} are the values of the regulatory loss function generated by model i and model j respectively, for the day t. Here, z_t is known as the loss differential between model i and model j at time t. Negative values of z_t indicate a superiority of model i over j.

Define T to be the total number of z_t's and S_{ij} to be the number of non-negative z_t's. If z_t is independent and identically distributed then the exact distribution of S_{ij} is binomial with parameters $(T,0.5)$ under the null hypothesis. For large samples, the standardized version of the sign statistic S_{ij} is asymptotically standard normal:

$$[5.7] \quad S_{ij}^{a} = \frac{S_{ij} - 0.5T}{\sqrt{0.25T}} \sim N(0,1) \text{ asymptotically}$$

The null hypothesis is rejected at the 5% level of significance if $S_{ij}^{a} < -1.66$. Rejection of the null hypothesis would imply that model i is significantly better than model j in terms of the particular loss function under consideration. Otherwise model i is not significantly better than model j.

6. Portfolio composition

Global institutional investors and financial institutions have become popular investment vehicles that combine individual securities, with specific characteristics, into portfolios with a specific investment objective. These portfolios are created to reflect a particular appetite for risk. In this paper, I examine the exposure to market risk of global hypothetical equity portfolios, with securities from developed or emerging markets.

The data consist of daily equity index returns for twenty two developed markets and for twenty six emerging markets between 2/1/1995 and 31/8/2006 (source: DataStream). Descriptive statistics for each market are presented in Appendix I (Table 1 and Table 2). All indices are derived from Morgan Stanley Capital International (MSCI). The MSCI Standard Equity Index Series captures 85% of the total market capitalization while it accurately reflects the economic diversity of the market. The MSCI Equity Indices measure the performance of a set of equity securities over time. They are calculated using the Laspeyres' concept of a weighted arithmetic average together with the concept of chain-linking. The MSCI Equity Indices are calculated in "local currency". While the local currency series of country indices cannot be replicated in the real world, it represents the theoretical performance of an index without any impact from foreign exchange fluctuations, a continuously hedged portfolio.

The MSCI Indices are constructed with a view to being fully investable from the perspective of international institutional investors. This includes representing in each index securities of reasonable size and liquidity at weights that can easily and cost effectively be reflected in global institutional portfolios. Moreover, MSCI indices reflected investable opportunities for global investors by taking into account local market restrictions on share ownership by foreigners. These restrictions may have assumed several forms. Specific classes of shares excluded from foreign investment. Specific securities or classes of shares for an individual company may have had limits for foreign investors. The combination of regulations governing qualifications for investment, repatriation of capital and income, and low foreign ownership limits may have created a difficult investment environment for the foreign investor. Finally, specific industries, or classes of shares within a specific industry, may have been restricted to foreign investors.

Using MSCI Standard Equity Indices, four hypothetical global portfolios with developed markets and five with emerging markets are created in order to reflect different risk preferences. Descriptive statistics for each portfolio are presented in Appendix I (Table 3). The main reason that global investors include in their portfolios markets with high volatility is that those markets on average offer high expected returns. The hypothetical global portfolios that invest in emerging markets are created according to the following approach. Emerging markets are sorted form highest volatility market to lowest volatility market. Assuming that portfolios' weights are fixed throughout the risk horizon (no rebalancing) and that any transaction cost is ignored, the first five to six emerging markets with highest volatility are chosen and a combination of them constitutes the first portfolio with emerging markets. The next five to six markets constitute the second portfolio and so on. The correlation between two different emerging markets is also taken into account. All hypothetical portfolios are efficient portfolios according to the Modern Portfolio Theory. It should be noted that these hypothetical portfolios are a close approximation of actual portfolios managed by global institutional investors. Despite the fact that most global portfolios consist of a large number of different markets, seventy to eighty percent of the total market value of these portfolios is composed of only five to six markets. The same approach is also used for developed markets. A more detailed description of each portfolio's elements is provided in Table 2 and Table 3.

Table 2: Developed markets portfolios

Abbreviation	Description
Portfolio (I) DM	Australia, Austria, Switzerland, United Kingdom, USA
Portfolio (II) DM	Belgium, Canada, Denmark, Ireland, Japan, Portugal
Portfolio (III) DM	Hong Kong, Netherlands, New Zealand, Norway, Singapore
Portfolio (IV) DM	Finland, France, Germany, Italy, Spain, Sweden

This table provides the constituents of the hypothetical developed markets portfolios. All portfolios are efficient and represent different risk preferences. Portfolio (I) is less risky because it has the lowest volatility, while Portfolio (IV) has the highest volatility. These portfolios are a close approximation of actual portfolios managed by global institutional investors.

Table 3: Emerging markets portfolios

Abbreviation	Description
Portfolio (I) EM	Chile, Colombia, India, Israel, South Africa, Taiwan
Portfolio (II) EM	Czech republic, Egypt, Hungary, Morocco, Sri Lanka
Portfolio (III) EM	Brazil, Mexico, Peru, Philippines, Poland
Portfolio (IV) EM	China, Indonesia, Malaysia, Pakistan, Thailand
Portfolio (V) EM	Argentina, Korea, Russia, Turkey, Venezuela

This table provides the constituents of the hypothetical emerging markets portfolios. All portfolios are efficient and represent different risk preferences. Portfolio (I) is less risky because it has the lowest volatility, while Portfolio (V) has the highest volatility. These portfolios are a close approximation of actual portfolios managed by global institutional investors.

7. Empirical application in developed markets

In the following sections, the backtesting methodology described above is applied in order to evaluate the performance of alternative VaR models for the hypothetical global portfolios with developed markets. The backtesting period consist of roughly 2400 daily observations and it is the same for all VaR models. In the first stage of the evaluation process the VaR estimates are tested for correct conditional coverage and in the second stage, the best model for each of the four different portfolios is selected according to the regulatory loss function for both 95% and 99% confidence intervals. A detailed presentation of the results from Christoffersen (1998) tests for correct conditional coverage corresponding to various VaR models in developed markets portfolios and the average values of the regulatory loss function, can be found in Appendix II (Table 1 to 4).

7.1. Model selection for 95% confidence interval

An important finding is that five out of twelve VaR models are always rejected in developed markets portfolios at 95% confidence level for lacking the property of independence, despite the fact that they exhibit correct unconditional coverage. These models are the fixed weight approach, the orthogonal GARCH, the historical and the weighted historical simulation and the mixed of normal distributions Monde Carlo. The clustering of violations created by these models indicates that they are unable to capture the dynamic time-varying volatility in daily equity returns or if risk dependent on higher order moments, these models may be efficient in forecasting conditional variance but they are unable to capture the true risk. The risk forecasting performance of exponentially weighted moving average turns out to be much worse than that of GARCH models, which often exhibit correct conditional coverage, indicating that such conditional heteroscedastic models are appropriate for modeling volatility in developed markets. This finding is in line with West and Cho (1995), who found that GARCH models outperform the exponentially weighted standard deviation estimates only in short time horizons. Another important finding is that the difference between the forecasting performance of the univariate exponential GARCH model, which is estimated by aggregating the portfolio constituents into a single series and the multivariate GARCH models, is minimal. These results are in line with Brooks and Persand (2003), who highlight that accurate results can be obtained using univariate GARCH models in the portfolio return series. The filtered historical simulation and the extreme value theory models have shown the most satisfactory performance in terms of accuracy and efficiency. An explanation may be that these models allow both past and future volatility to vary over time and that they are able to generate relatively large deviations (losses and gains) not found in the original portfolio return series. According to the regulatory loss function, the best model, which minimizes the loss, is the extreme value theory in three out of four developed markets portfolio. The exponential GARCH and the filtered historical simulation approaches are selected only in the third portfolio. Neither model is superior to the other according to the sign test statistic.

7.2. Model selection for 99% confidence interval

Interestingly, results obtained for 99% confidence interval suggest that the predictive power of all the parametric approaches used in this paper and the predictive power of Monde Carlo simulation are not satisfactory, because these models are always rejected for lacking the property of correct unconditional coverage, despite the fact that they satisfy the backtesting property of independence. An explanation may be that in contrast with those models' assumption that portfolio returns are normally distributed, the empirical distributions of all the portfolios' returns exhibit particularly fat tails and excess kurtosis (see Appendix I, Table 3). This means that extreme price movements occur more frequently than implied by a normal distribution. Note that the normality assumption seems to work well in 95% confidence level. However, at higher quantiles, the empirical distribution of portfolio returns exhibit particular fat tails that differ significantly from the tails of the normal distribution. Furthermore, the historical simulation does not produce superior risk forecasts than that in 95% confidence level as it is not rejected only in the fourth developed markets portfolio. One reason is the strong assumption of historical simulation that the future distribution of portfolio returns depends on the past distribution. However, developed markets portfolios often exhibit extreme returns not having been observed in the past. Moreover, if the window omits important events, the tails will not be well represented. The models, which have shown the most satisfactory performance, are the weighted and the filtered historical simulation, the mixed of normal distributions Monde Carlo and the extreme value theory. These models do not depend on a particular statistical distribution. Consequently, they produce more accurate risk forecast of the future potential loss. In the first and the second portfolios the VaR models that minimize the loss are the weighted and the filtered historical simulation. According to the regulatory loss function neither of these two models is superior to the other. In the case of the third portfolio the best models are the filtered historical simulation and the extreme value theory. Finally, in the fourth portfolio the best models are the weighted historical simulation and the mixed of normal distributions Monde Carlo. Again both models are assumed to be equal according to the sign test statistic.

8. Empirical application in emerging markets

The empirical evidence in emerging markets portfolios highlights the difficulties in risk modeling in markets with non-normality of the return distribution, high volatility persistence, significant asymmetry and lack of relationship between stock market volatility and expected return. The backtesting period match with the period used in the developed markets portfolios and consists of roughly 2400 daily observations between 16/9/1997 and 31/8/2006. A detailed presentation of the results from Christoffersen (1998) tests for correct conditional coverage corresponding to various VaR models in emerging markets portfolios and the average values of the regulatory loss function, can be found in Appendix II (Table 5 to 9).

8.1. Model selection for 95% confidence interval

One of the most important findings is that parametric VaR models such as the constant correlation GARCH, the dynamic conditional correlation GARCH and the exponential GARCH, which perform well in 95% confidence interval in developed markets portfolios, are now rejected for lacking the property of independence, despite the fact that they exhibit correct unconditional coverage. The results differ significantly among portfolios with different excess kurtosis and great asymmetry. Emerging markets exhibit more often critical events which cause major shifts in volatility. As a result, GARCH based models are rejected because they exhibit clusters of violations, when such a critical event takes place. It should be noted that in the case of the third portfolio neither model produces satisfactory conditional coverage probabilities because this portfolio has very strong characteristics of a leptokurtotic distribution with significantly fat tails. In this case, the limitations of Value at Risk methodology are observed and the need to supplement Value at Risk methods with stress testing process. Normal distribution based Monde Carlo simulation is found to satisfy both the unconditional and independence property in three portfolios, though unconditional coverage test rejects the mixed of normal distributions Monde Carlo, because it overestimates market risk offering a coverage probability much lower than five percent. Moreover, the filtered historical simulation and the extreme value theory models work relatively well in emerging markets too. They are rejected only in one out of five portfolios. These models possess the ability to predict unexpected extreme changes in the market value of the portfolios, which often occur in emerging markets. The model selected with the regulatory loss function is the filtered historical simulation for the first and fourth emerging markets portfolio and the extreme value theory for the second and fifth portfolio. In the case of the third portfolio, the results are striking, because all models are rejected. None of them is found to satisfy the independence property.

8.2. Model selection for 99% confidence interval

The weighted historical simulation model, which has been proved to be successful in 99% confidence level in the developed markets, is now rejected in four out of five emerging markets portfolios for not offering the property of independence while it is found to possess the property of unconditional coverage. An explanation may be that the distributional characteristics of emerging markets change more often over time. As a result, the weighted historical simulation is unable to accommodate for future extreme returns that are not found in the estimation sample. Furthermore, the variance covariance models, the historical simulation and the Monde Carlo simulation underestimate market risk as they do not produce satisfactory coverage probabilities. The reason is that market risk arising from emerging markets contains significant time variation. These models usually miss situations with temporarily elevated volatility or they are very slow to incorporate 'structural breaks'. The mixed of normal distributions Monde Carlo works relatively well in 99% confidence interval. In most cases, it accommodates the observed skewness and kurtosis of the portfolio return series and produces correct conditional coverage. The filtered historical simulation and the extreme value theory modeling approaches provides VaR estimates that have greater accuracy and constitute an efficient risk measure especially at higher quantiles for all the emerging markets portfolios. Both models minimize the total loss in the first and third portfolio according to the regulatory loss function. Neither of these two models is superior to the other. Finally, for the rest of the portfolios, extreme value theory stands alone as the best model.

9. Conclusion

In this paper, a comprehensive predictive assessment of twelve different VaR models is studied for nine global equity portfolios, which reflect a particular appetite for risk. The data consist of daily equity index returns for twenty two developed markets and for twenty six emerging markets between 2/1/1995 and 31/8/2006. Performance evaluation is based on a three-step testing procedure for correct conditional coverage of the interval forecasts. Among the VaR models that exhibit correct conditional coverage, the best model, which minimizes the total loss for each portfolio, is selected using a regulatory loss function, which is assumed to reflect the regulator's utility function.

As a general conclusion drawn from this analysis is that the filtered historical simulation and the extreme value theory models produce superior risk forecasts especially at higher quantiles than that of other well-known modeling approaches for both developed and emerging markets portfolios. Normal distribution based models perform well in 95% confidence interval. However, at higher quantiles, these models are rejected for lacking the property of correct unconditional coverage, despite the fact that they exhibit independence. The constant correlation GARCH, the dynamic conditional correlation GARCH and the exponential GARCH models, which produce accurate risk forecasts in developed markets at 95% confidence level, are inefficient when applied in emerging markets. Furthermore, the weighted historical simulation, which generate reasonable well VaR forecast in developed markets at 99% confidence level, fails to possess the independence property in emerging markets portfolios. Finally, it should be noted that widely used models such as the fixed weight approach, the exponentially weighted moving average and the historical simulation, should be used by institutional investors and financial institutions with caution because they exhibit very poor predictive power and most often clustering of violations.

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APPENDIX I: Descriptive statistics

Table 1: Descriptive statistics for Developed Markets

	Mean	Median	Standard Deviation	Skewness	Kurtosis	Min	Max
USA	0,030%	0,014%	0,993%	-0,1139	4,3343	-6,967%	5,610%
United kingdom	0,023%	0,019%	1,000%	-0,1534	3,5459	-6,011%	5,810%
France	0,032%	0,015%	1,258%	-0,1326	2,8956	-7,227%	6,571%
Germany	0,027%	0,052%	1,364%	-0,2421	3,7585	-8,667%	7,450%
Switzerland	0,043%	0,053%	1,081%	-0,2425	4,7561	-7,390%	7,120%
Italy	0,035%	0,015%	1,325%	-0,1070	2,3825	-7,416%	7,045%
Spain	0,044%	0,031%	1,291%	-0,1502	3,1449	-7,582%	6,534%
Netherlands	0,032%	0,045%	1,248%	-0,2175	4,8809	-7,696%	7,839%
Japan	0,001%	0,000%	1,236%	0,1336	2,9731	-6,512%	7,409%
Canada	0,035%	0,028%	0,975%	-0,5541	7,2495	-9,261%	5,315%
Australia	0,028%	0,014%	0,826%	-0,2943	3,1775	-6,755%	5,222%
Sweden	0,051%	0,018%	1,515%	0,1141	3,9490	-8,690%	11,536%
Hong Kong	0,028%	0,000%	1,598%	0,0166	9,2320	-13,792%	15,980%
Finland	0,068%	0,054%	2,226%	-0,4395	7,3987	-20,931%	16,863%
Belgium	0,031%	0,033%	1,055%	0,1753	7,2124	-5,609%	9,999%
Norway	0,037%	0,001%	1,271%	-0,1089	5,2727	-7,301%	10,648%
Ireland	0,032%	0,020%	1,124%	-0,4563	6,4366	-9,015%	8,033%
Denmark	0,036%	0,022%	1,069%	-0,3344	2,8090	-6,422%	4,917%
Austria	0,028%	0,010%	0,975%	-0,5442	4,5196	-7,457%	5,905%
Portugal	0,030%	0,000%	0,960%	-0,3789	5,9724	-7,823%	6,087%
Singapore	0,016%	0,000%	1,219%	0,1305	7,4669	-8,999%	10,974%
New Zealand	0,013%	0,000%	1,101%	-0,6581	17,6457	-16,338%	11,143%
Average values	0,032%	0,020%	1,214%	-0,2072	5,5006	-8,812%	8,364%

This table provides descriptive statistics of the daily log returns of twenty two developed markets, used in the construction of the portfolios, between 2/1/1995 and 31/8/2006. The number of observation used in calculations for each market is roughly 3100.

Table 2: Descriptive statistics for Emerging Markets

	Mean	Median	Standard Deviation	Skewness	Kurtosis	Min	Max
Korea	0,032%	0,001%	2,074%	0,1231	3,7655	-13,097%	11,445%
Taiwan	0,011%	0,000%	1,659%	0,0930	2,4705	-10,309%	9,172%
Brazil	0,278%	0,116%	2,337%	0,4749	7,1393	-13,627%	24,656%
South Africa	0,048%	0,006%	1,211%	-0,5380	6,7084	-12,208%	6,750%
Russia	0,084%	0,088%	3,237%	-0,3648	8,8455	-28,097%	24,220%
Mexico	0,070%	0,020%	1,522%	0,0477	4,8761	-12,689%	12,137%
China	-0,028%	0,000%	1,927%	0,0769	5,1698	-14,457%	12,725%
India	0,044%	0,000%	1,547%	-0,3095	3,9362	-12,050%	8,099%
Israel	0,028%	0,033%	1,458%	-0,2303	4,3174	-8,868%	8,428%
Malaysia	0,016%	0,000%	1,597%	0,8400	42,3921	-24,159%	23,263%
Turkey	0,174%	0,069%	2,833%	-0,0443	4,2040	-19,715%	17,816%
Thailand	0,004%	0,000%	1,934%	1,0650	10,7825	-10,374%	21,430%
Indonesia	0,042%	0,002%	1,926%	-0,1046	12,3636	-19,145%	16,829%
Poland	0,084%	0,000%	2,136%	-0,0852	4,7378	-10,446%	12,443%
Chile	0,036%	0,004%	1,015%	0,2129	4,8138	-6,047%	8,595%
Hungary	0,091%	0,019%	1,858%	-0,5524	9,7610	-19,402%	12,357%
Czech republic	0,043%	0,019%	1,410%	-0,1632	2,9856	-7,815%	8,620%
Argentina	0,040%	0,000%	2,221%	0,1383	5,2897	-14,740%	16,341%
Egypt	0,082%	0,001%	1,533%	0,3169	4,6662	-9,045%	9,285%
Peru	0,068%	0,000%	1,582%	0,0204	4,7460	-9,035%	10,441%
Philippines	0,014%	0,003%	1,510%	0,6743	10,4707	-9,796%	16,287%
Colombia	0,077%	0,000%	1,380%	0,3657	14,1080	-11,993%	16,703%
Pakistan	0,034%	0,000%	1,911%	-0,3927	6,4690	-15,733%	14,199%
Morocco	0,039%	0,000%	0,724%	0,3049	8,3410	-5,465%	5,681%
Venezuela	0,112%	0,006%	2,170%	0,6651	11,5708	-16,061%	20,787%
Sri Lanka	0,033%	0,000%	1,544%	1,2012	40,0980	-16,050%	27,560%
Average values	0,060%	0,014%	1,779%	0,1475	9,4242	-13,478%	14,472%

This table provides descriptive statistics of the daily log returns of twenty six emerging markets, used in the construction of the portfolios, between 2/1/1995 and 31/8/2006. The number of observation used in calculations for each market is roughly 3100.

Table 3: Descriptive statistics for the nine Portfolios

	Mean	Median	Standard Deviation	Skewness	Kurtosis	Min	Max
Portfolio (I) DM	0,027%	0,059%	0,693%	-0,2835	6,7794	-4,543%	4,220%
Portfolio (II) DM	0,032%	0,063%	0,729%	-0,5177	7,6793	-5,298%	3,872%
Portfolio (III) DM	0,034%	0,064%	0,928%	-0,3395	8,7707	-8,134%	7,938%
Portfolio (IV) DM	0,052%	0,083%	1,415%	-0,1512	6,8399	-9,529%	9,923%
Portfolio (I) EM	0,026%	0,053%	0,756%	-0,5310	9,5260	-6,037%	6,338%
Portfolio (II) EM	0,048%	0,057%	0,838%	-0,3936	7,2431	-6,286%	4,147%
Portfolio (III) EM	0,068%	0,078%	1,144%	-0,0107	9,3105	-9,363%	12,431%
Portfolio (IV) EM	0,102%	0,124%	1,357%	-0,2896	9,7771	-8,953%	8,614%
Portfolio (V) EM	0,141%	0,154%	1,515%	-0,4505	8,5597	-12,020%	7,915%

This table provides descriptive statistics of the daily log returns of four developed markets and five emerging markets portfolios between 2/1/1995 and 31/8/2006. The number of observation used in calculations for each portfolio is roughly 3100.

APPENDIX II: Decomposition of the test of conditional coverage

Table 1: Portfolio (I) Developed Markets

	Tests for 95% Confidence Level					Tests for 99% Confidence Level				
Model	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**
FWA	**0,0078**	31,1146	31,1224	4,968%		31,8487	7,9823	39,8310	2,077%	
	[92,95%]	[0,00%]	[0,00%]			[0,00%]	[0,47%]	[0,00%]		
EWMA	**1,6946**	10,9228	12,6174	5,528%		12,0856	**1,9029**	13,9884	1,641%	
	[19,30%]	[0,09%]	[0,18%]			[0,05%]	**[16,78%]**	[0,09%]		
CC GARCH	**0,6537**	**3,1305**	**3,7842**	5,322%	0,0688	13,5465	**0,0075**	13,5540	1,730%	
	[41,88%]	**[7,68%]**	**[15,08%]**			[0,02%]	**[93,10%]**	[0,11%]		
DCC GARCH	**0,4169**	**2,3636**	**2,7805**	5,256%	**0,0662**	11,4014	**0,0261**	11,4276	1,665%	
	[51,85%]	**[12,42%]**	**[24,90%]**		**3,44**	[0,07%]	**[87,15%]**	[0,33%]		
OGARCH	**0,4169**	4,6763	**5,0932**	5,256%		18,2921	**0,6896**	18,9817	1,861%	
	[51,85%]	[3,06%]	**[7,83%]**			[0,00%]	**[40,63%]**	[0,01%]		
EGARCH	**0,9424**	**0,5236**	**1,4660**	5,387%	0,0701	23,6102	**2,4800**	26,0903	1,992%	
	[33,16%]	**[46,93%]**	**[48,05%]**			[0,00%]	**[11,53%]**	[0,00%]		
HS	**1,5614**	41,1339	42,6953	5,487%		5,9622	4,4691	10,4312	1,457%	
	[21,15%]	[0,00%]	[0,00%]			[1,46%]	[3,45%]	[0,54%]		
WHS	**0,0979**	11,6373	11,7352	5,122%		**1,6149**	**0,4399**	**2,0548**	1,233%	**0,0178**
	[75,44%]	[0,06%]	[0,28%]			**[20,38%]**	**[50,72%]**	**[35,79%]**		**0,83**
FHS	**3,2294**	**0,2866**	**3,5160**	5,700%	0,0677	**2,0056**	**1,0438**	**3,0494**	1,257%	**0,0149**
	[7,23%]	**[59,24%]**	**[17,24%]**			**[15,67%]**	**[30,69%]**	**[21,77%]**		**-0,83**
MC	**0,4557**	**2,6664**	**3,1221**	5,248%	0,0781	34,8707	8,1210	57,5269	2,133%	
	[49,96%]	**[10,25%]**	**[20,99%]**			[0,00%]	[0,44%]	[0,00%]		
MNMC	11,9630	5,6241	17,5870	3,789%		**1,8480**	**1,1007**	**2,9487**	1,235%	0,0187
	[0,05%]	[1,77%]	[0,02%]			**[17,40%]**	**[29,41%]**	**[22,89%]**		
EVT	**0,0008**	**3,6360**	**3,6368**	5,011%	**0,0650**	**3,2661**	**1,1849**	**4,4510**	1,328%	0,0185
	[97,77%]	**[5,65%]**	**[16,23%]**		**-3,44**	**[7,07%]**	**[27,64%]**	**[10,80%]**		

This table provides the components of Christoffersen's test applied to all VaR models, and the best performing model according to the regulatory loss function. The first column gives the name of each model, the next three columns give the values of the estimated likelihood ratio statistics of the test for unconditional coverage, the test of independence and the test for conditional coverage .The estimated likelihood ratio statistics are rejected if they are larger than the critical value of the chi-square distribution with one degree of freedom for the first two tests and with two degrees of freedom for the third test, or if the p-value is smaller that 5%. The likelihood ratio statistics which are significant are boldfaced. The fifth column gives the estimated coverage probability under the hypothesis of an independently distributed failure process and the sixth column reports the average value of the regulatory loss function only for the models that are not rejected. The best performing model minimizes the total loss and its standardized sign statistic is smaller than -1.66. The model selected for 95% confidence level is Extreme Value Theory and for the 99% Weighted and Filtered Historical Simulation are equal.

Table 2: Portfolio (II) Developed Markets

	Tests for 95% Confidence Level					Tests for 99% Confidence Level				
Model	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**
FWA	**0,0004**	30,3671	30,3675	4,993%		40,6808	16,0737	56,7545	2,232%	
	[98,47%]	[0,00%]	[0,00%]			[0,00%]	[0,01%]	[0,00%]		
EWMA	**0,1922**	27,4692	27,6614	5,160%		18,1049	7,5324	25,6374	1,785%	
	[66,11%]	[0,00%]	[0,00%]			[0,00%]	[0,61%]	[0,00%]		
CC GARCH	**0,0513**	21,8173	21,8686	5,089%		30,4945	12,2396	42,7340	2,139%	
	[82,08%]	[0,00%]	[0,00%]			[0,00%]	[0,05%]	[0,00%]		
DCC GARCH	**0,0952**	21,3740	21,4692	5,122%		23,1705	4,2691	27,4397	1,977%	
	[75,76%]	[0,00%]	[0,00%]			[0,00%]	[3,88%]	[0,00%]		
OGARCH	**0,0726**	30,3179	30,3905	4,895%		14,3394	9,1872	23,5266	1,750%	
	[78,76%]	[0,00%]	[0,00%]			[0,02%]	[0,24%]	[0,00%]		
EGARCH	**0,0513**	**2,9819**	**3,0333**	5,089%	**0,0657**	11,1075	**0,0284**	11,1359	1,653%	
	[82,08%]	**[8,42%]**	**[21,94%]**		**-0,85**	[0,09%]	**[86,61%]**	[0,38%]		
HS	**1,7960**	38,0858	39,8818	5,526%		**2,9698**	9,5163	12,4861	1,319%	
	[18,02%]	[0,00%]	[0,00%]			**[8,48%]**	[0,20%]	[0,19%]		
WHS	**0,3911**	26,0994	26,4905	5,243%		**1,1297**	**2,9725**	**4,1022**	1,193%	**0,0201**
	[53,17%]	[0,00%]	[0,00%]			**[28,78%]**	**[8,47%]**	**[12,86%]**		**0,12**
FHS	**1,7120**	**3,3777**	**5,0897**	5,502%	**0,0760**	**1,6940**	**0,3875**	**2,0815**	1,233%	**0,0174**
	[19,07%]	**[6,61%]**	**[7,85%]**		**0,85**	**[19,31%]**	**[53,36%]**	**[35,32%]**		**-0,12**
MC	**1,2806**	9,9743	11,2549	5,423%		33,5917	**0,1125**	33,7042	2,123%	
	[25,78%]	[0,16%]	[0,36%]			[0,00%]	**[73,74%]**	[0,00%]		
MNMC	10,2763	15,3590	25,6354	3,877%		**3,7600**	**0,1748**	**3,9348**	1,339%	0,0253
	[0,13%]	[0,01%]	[0,00%]			**[5,25%]**	**[67,59%]**	**[13,98%]**		
EVT	**0,0318**	19,3117	19,3436	5,067%		**3,1220**	**0,2566**	**3,3786**	1,319%	0,0238
	[85,84%]	[0,00%]	[0,01%]			**[7,72%]**	**[61,24%]**	**[18,46%]**		

This table provides the components of Christoffersen's test applied to all VaR models, and the best performing model according to the regulatory loss function. The first column gives the name of each model, the next three columns give the values of the estimated likelihood ratio statistics of the test for unconditional coverage, the test of independence and the test for conditional coverage .The estimated likelihood ratio statistics are rejected if they are larger than the critical value of the chi-square distribution with one degree of freedom for the first two tests and with two degrees of freedom for the third test, or if the p-value is smaller that 5%. The likelihood ratio statistics which are significant are boldfaced. The fifth column gives the estimated coverage probability under the hypothesis of an independently distributed failure process and the sixth column reports the average value of the regulatory loss function only for the models that are not rejected. The best performing model minimizes the total loss and its standardized sign statistic is smaller than -1.66. The models selected for 95% confidence level are Exponential GARCH and Filtered Historical Simulation according to the loss function. Neither model is superior to the other. For the 99% confidence level Weighted and Filtered Historical Simulation are equal.

Table 3: Portfolio (III) Developed Markets

	Tests for 95% Confidence Level					Tests for 99% Confidence Level				
Model	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**
FWA	**0,2619 [60,88%]**	27,2520 [0,00%]	27,5139 [0,00%]	5,188%		44,4920 [0,00%]	18,8549 [0,00%]	63,3469 [0,00%]	2,299%	
EWMA	**2,5551 [10,99%]**	**2,8313 [9,24%]**	**5,3863 [6,77%]**	5,676%	0,1339	19,6110 [0,00%]	14,3383 [0,02%]	33,9493 [0,00%]	1,823%	
CC GARCH	**0,7694 [38,04%]**	**3,4737 [6,24%]**	**4,2431 [11,98%]**	5,349%	0,1205	14,6307 [0,01%]	5,7523 [1,65%]	20,3829 [0,00%]	1,761%	
DCC GARCH	**0,7694 [38,04%]**	**2,0737 [35,46%]**	**2,8431 [24,13%]**	5,349%	**0,1167 2,16**	14,6307 [0,01%]	5,7523 [1,65%]	20,3829 [0,00%]	1,761%	
OGARCH	**0,1281 [72,04%]**	12,9740 [0,03%]	13,1021 [0,14%]	4,860%		18,2398 [0,00%]	11,8292 [0,06%]	30,0690 [0,00%]	1,859%	
EGARCH	4,3135 [3,78%]	9,5940 [0,20%]	13,9075 [0,10%]	5,838%		29,3957 [0,00%]	**1,5070 [21,96%]**	30,9027 [0,00%]	2,120%	
HS	**1,7958 [18,02%]**	32,6055 [0,00%]	34,4013 [0,00%]	5,527%		**1,6008 [20,58%]**	6,2886 [1,22%]	7,8894 [1,94%]	1,232%	
WHS	**0,0006 [98,05%]**	12,9136 [0,03%]	12,9142 [0,16%]	4,991%		**0,0037 [95,17%]**	**0,9332 [33,40%]**	**0,9369 [62,60%]**	1,011%	0,0419
FHS	**1,1583 [28,18%]**	**1,8557 [17,31%]**	**3,0140 [22,16%]**	5,412%	0,1211	**2,1416 [14,34%]**	**2,4752 [11,57%]**	**4,6169 [9,94%]**	1,263%	**0,0309 -1,03**
MC	**1,6456 [19,96%]**	7,9939 [0,47%]	9,6395 [0,81%]	5,482%		40,4048 [0,00%]	**0,0341 [85,36%]**	40,4389 [0,00%]	2,250%	
MNMC	6,5476 [1,05%]	8,7364 [0,31%]	15,2840 [0,05%]	4,094%		**3,3116 [6,88%]**	**0,2028 [65,25%]**	**3,5144 [17,25%]**	1,318%	0,0455
EVT	**0,0003 [98,71%]**	**2,9461 [8,61%]**	**2,9464 [22,92%]**	5,006%	**0,1122 -2,16**	**3,3987 [6,52%]**	**0,6657 [41,45%]**	**4,0644 [13,10%]**	1,330%	**0,0361 1,03**

This table provides the components of Christoffersen's test applied to all VaR models and the best performing model according to the regulatory loss function. The first column gives the name of each model, the next three columns give the values of the estimated likelihood ratio statistics of the test for unconditional coverage, the test of independence and the test for conditional coverage .The estimated likelihood ratio statistics are rejected if they are larger than the critical value of the chi-square distribution with one degree of freedom for the first two tests and with two degrees of freedom for the third test, or if the p-value is smaller that 5%. The likelihood ratio statistics which are significant are boldfaced. The fifth column gives the estimated coverage probability under the hypothesis of an independently distributed failure process and the sixth column reports the average value of the regulatory loss function only for the models that are not rejected. The best performing model minimizes the total loss and its standardized sign statistic is smaller than -1.66. The model selected for 95% confidence level is Extreme Value Theory and for the 99% confidence level Filtered Historical Simulation and Extreme Value Theory are selected according to the loss function. Neither model is superior to the other.

Table 4: Portfolio (IV) Developed Markets

	Tests for 95% Confidence Level					Tests for 99% Confidence Level				
Model	LR_{un}	LR_{ind}	LR_{CC}	π	RLF	LR_{un}	LR_{ind}	LR_{CC}	π	RLF
FWA	**0,5389**	8,7978	9,3367	4,734%		16,3581	**2,4007**	18,7588	1,747%	
	[46,29%]	[0,30%]	[0,94%]			[0,01%]	**[12,13%]**	[0,01%]		
EWMA	**0,1218**	**1,4407**	**1,5625**	5,128%	0,2849	13,1155	**0,8474**	13,9628	1,662%	
	[72,71%]	**[23,00%]**	**[45,78%]**			[0,03%]	**[35,73%]**	[0,09%]		
CC GARCH	**0,2103**	**0,9783**	**1,1886**	5,182%	0,2967	17,2869	**0,0008**	17,2878	1,837%	
	[64,65%]	**[32,26%]**	**[55,19%]**			[0,00%]	**[97,68%]**	[0,02%]		
DCC GARCH	**0,3876**	**1,5383**	**1,9259**	5,248%	**0,2785**	12,6535	**0,0143**	12,6678	1,705%	
	[53,36%]	**[21,49%]**	**[38,18%]**		**2,72**	[0,04%]	**[90,49%]**	[0,18%]		
OGARCH	**0,0021**	9,6293	9,6314	5,018%		10,5688	**3,6190**	14,1878	1,640%	
	[96,36%]	[0,19%]	[0,81%]			[0,12%]	**[5,71%]**	[0,08%]		
EGARCH	**1,6184**	**2,3883**	**4,0067**	5,510%	0,2940	22,5029	**0,0304**	22,5334	1,968%	
	[20,33%]	**[12,22%]**	**[13,49%]**			[0,00%]	**[86,15%]**	[0,00%]		
HS	**1,1933**	22,8683	24,0616	5,425%		**2,2029**	**2,5313**	**4,7343**	1,271%	0,0777
	[27,47%]	[0,00%]	[0,00%]			**[13,77%]**	**[11,16%]**	**[9,37%]**		
WHS	**0,6027**	9,5247	10,1274	5,301%		**2,2029**	**2,5313**	**4,7343**	1,271%	**0,0688**
	[43,75%]	[0,20%]	[0,63%]			**[13,77%]**	**[11,16%]**	**[9,37%]**		**0,59**
FHS	**0,8457**	**0,7951**	**1,6408**	5,355%	0,2832	**0,8951**	**0,5270**	**1,4221**	1,170%	**0,0681**
	[35,78%]	**[37,26%]**	**[44,03%]**			**[34,41%]**	**[46,79%]**	**[49,11%]**		**-0,59**
MC	**0,1809**	**0,7175**	**0,8984**	5,156%	0,3278	30,6783	**0,1564**	30,8348	2,057%	
	[67,06%]	**[39,70%]**	**[63,81%]**			[0,00%]	**[69,25%]**	[0,00%]		
MNMC	13,3958	**1,7838**	15,1796	3,719%		**0,0074**	**0,7676**	**0,7750**	1,014%	0,0840
	[0,03%]	**[18,17%]**	[0,05%]			**[93,16%]**	**[38,10%]**	**[67,88%]**		
EVT	**0,5896**	**1,0748**	**1,6644**	4,709%	**0,2756**	**2,5741**	**0,3173**	**2,8914**	1,293%	0,0832
	[44,26%]	**[29,99%]**	**[43,51%]**		**-2,72**	**[10,86%]**	**[57,32%]**	**[23,56%]**		

This table provides the components of Christoffersen's test applied to all VaR models and the best performing model according to the regulatory loss function. The first column gives the name of each model, the next three columns give the values of the estimated likelihood ratio statistics of the test for unconditional coverage, the test of independence and the test for conditional coverage .The estimated likelihood ratio statistics are rejected if they are larger than the critical value of the chi-square distribution with one degree of freedom for the first two tests and with two degrees of freedom for the third test, or if the p-value is smaller that 5%. The likelihood ratio statistics which are significant are boldfaced. The fifth column gives the estimated coverage probability under the hypothesis of an independently distributed failure process and the sixth column reports the average value of the regulatory loss function only for the models that are not rejected. The best performing model minimizes the total loss and its standardized sign statistic is smaller than -1.66. The model selected for 95% confidence level is Extreme Value Theory and for the 99% confidence level Weighted and Filtered Historical Simulation are selected according to the loss function. Neither model is superior to the other.

Table 5: Portfolio (I) Emerging Markets

	Tests for 95% Confidence Level					Tests for 99% Confidence Level				
Model	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**
FWA	**0,5026**	49,0351	49,5377	4,735%		16,1852	30,2356	46,4208	1,768%	
	[47,83%]	[0,00%]	[0,00%]			[0,01%]	[0,00%]	[0,00%]		
EWMA	**0,3947**	44,8718	45,2665	4,765%		18,5642	19,4165	37,9806	1,828%	
	[52,98%]	[0,00%]	[0,00%]			[0,00%]	[0,00%]	[0,00%]		
CC GARCH	**0,0002**	29,1438	29,1440	5,005%		21,1749	15,3730	36,5479	1,974%	
	[98,97%]	[0,00%]	[0,00%]			[0,00%]	[0,01%]	[0,00%]		
DCC GARCH	**0,0002**	26,2220	26,2222	5,005%		19,8134	15,8419	35,6553	1,939%	
	[98,97%]	[0,00%]	[0,00%]			[0,00%]	[0,01%]	[0,00%]		
OGARCH	**0,2573**	57,8242	58,0816	4,794%		15,9542	27,0202	42,9744	1,833%	
	[61,20%]	[0,00%]	[0,00%]			[0,01%]	[0,00%]	[0,00%]		
EGARCH	**0,1765**	17,8016	17,9781	4,829%		8,3584	4,2828	12,6411	1,586%	
	[67,44%]	[0,00%]	[0,01%]			[0,38%]	[3,85%]	[0,18%]		
HS	**2,2745**	61,3760	63,6505	5,618%		**0,0897**	25,8100	25,8997	1,055%	
	[13,15%]	[0,00%]	[0,00%]			**[76,45%]**	[0,00%]	[0,00%]		
WHS	**1,0316**	45,1286	46,1602	5,414%		**0,7017**	7,3891	8,0908	1,158%	
	[30,98%]	[0,00%]	[0,00%]			**[40,22%]**	[0,66%]	[1,75%]		
FHS	**1,0316**	**3,2329**	**4,2645**	5,414%	**0,0771**	**0,2312**	**3,7598**	**4,0909**	1,090%	**0,0149**
	[30,98%]	**[7,22%]**	**[11,86%]**			**[63,07%]**	**[5,25%]**	**[12,93%]**		**-0,32**
MC	**0,0083**	24,7398	24,7481	5,034%		9,8961	6,5434	16,4396	1,588%	
	[92,73%]	[0,00%]	[0,00%]			[0,17%]	[1,05%]	[0,03%]		
MNMC	**12,6668**	28,7261	41,3929	3,716%		**0,3558**	4,7590	**5,1149**	0,899%	
	[0,04%]	[0,00%]	[0,00%]			**[55,08%]**	[2,91%]	**[7,75%]**		
EVT	**3,5748**	25,3476	28,9224	4,276%		**0,0413**	8,3394	8,3807	1,037%	**0,0310**
	[5,87%]	[0,00%]	[0,00%]			**[83,90%]**	[0,39%]	[1,51%]		**0,32**

This table provides the components of Christoffersen's test applied to all VaR models and the best performing model according to the regulatory loss function. The first column gives the name of each model, the next three columns give the values of the estimated likelihood ratio statistics of the test for unconditional coverage, the test of independence and the test for conditional coverage .The estimated likelihood ratio statistics are rejected if they are larger than the critical value of the chi-square distribution with one degree of freedom for the first two tests and with two degrees of freedom for the third test, or if the p-value is smaller that 5%. The likelihood ratio statistics which are significant are boldfaced. The fifth column gives the estimated coverage probability under the hypothesis of an independently distributed failure process and the sixth column reports the average value of the regulatory loss function only for the models that are not rejected. The best performing model minimizes the total loss and its standardized sign statistic is smaller than -1.66. The model selected for 95% confidence level is Filtered Historical Simulation and for the 99% confidence level Filtered Historical Simulation and Extreme Value Theory are selected according to the loss function. Neither model is superior to the other.

Table 6: Portfolio (II) Emerging Markets

	Tests for 95% Confidence Level					Tests for 99% Confidence Level				
Model	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**
FWA	**1,3196**	27,6331	28,9527	4,536%		16,2091	17,2565	33,4656	1,843%	
	[25,07%]	[0,00%]	[0,00%]			[0,01%]	[0,00%]	[0,00%]		
EWMA	**2,5499**	10,9224	13,4723	4,359%		10,5175	**3,8431**	14,3606	1,665%	
	[11,03%]	[0,10%]	[0,12%]			[0,12%]	**[5,00%]**	[0,08%]		
CC GARCH	**0,4662**	12,0102	12,4763	4,694%		14,8759	14,6467	29,5226	1,895%	
	[49,48%]	[0,05%]	[0,20%]			[0,01%]	[0,01%]	[0,00%]		
DCC GARCH	**0,9515**	13,1667	14,1182	4,565%		14,8759	14,6467	29,5226	1,895%	
	[32,93%]	[0,03%]	[0,09%]			[0,01%]	[0,01%]	[0,00%]		
OGARCH	**2,1582**	12,6016	14,7598	4,350%		7,9707	8,7662	16,7369	1,637%	
	[14,18%]	[0,04%]	[0,06%]			[0,48%]	[0,31%]	[0,02%]		
EGARCH	**0,5541**	**0,8644**	**1,4185**	5,340%	**0,1092**	20,4220	**2,7969**	23,2189	2,067%	
	[45,67%]	**[35,25%]**	**[49,20%]**		**3,99**	[0,00%]	**[9,44%]**	[0,00%]		
HS	**5,4745**	18,6874	24,1619	6,069%		**3,5443**	15,7294	19,2737	1,404%	
	[1,93%]	[0,00%]	[0,00%]			**[5,98%]**	[0,01%]	[0,01%]		
WHS	**1,4003**	17,3976	18,7979	5,533%		**3,5443**	**2,8259**	6,3702	1,404%	
	[23,67%]	[0,00%]	[0,01%]			**[5,98%]**	**[9,28%]**	[4,14%]		
FHS	7,8909	**2,6297**	10,5206	6,265%		**3,6616**	**0,2699**	**3,9316**	1,397%	**0,0301**
	[0,50%]	**[10,49%]**	[0,52%]			**[5,57%]**	**[60,34%]**	**[14,00%]**		**3,33**
MC	**0,0001**	**2,1316**	**2,1317**	4,996%	0,1139	20,1008	**2,4582**	22,5590	1,949%	
	[99,31%]	**[14,43%]**	**[34,44%]**			[0,00%]	**[11,69%]**	[0,00%]		
MNMC	17,8257	10,2430	28,0688	3,366%		**2,5133**	**2,7635**	**5,2767**	1,311%	0,0389
	[0,00%]	[0,14%]	[0,00%]			**[11,29%]**	**[9,64%]**	**[7,15%]**		
EVT	**2,4536**	**2,2383**	**4,6920**	4,328%	**0,0972**	**0,3139**	**1,7127**	**2,0266**	0,890%	**0,0291**
	[11,73%]	**[13,46%]**	**[9,58%]**		**-3,99**	**[57,53%]**	**[19,06%]**	**[36,30%]**		**-3,33**

This table provides the components of Christoffersen's test applied to all VaR models and the best performing model according to the regulatory loss function. The first column gives the name of each model, the next three columns give the values of the estimated likelihood ratio statistics of the test for unconditional coverage, the test of independence and the test for conditional coverage .The estimated likelihood ratio statistics are rejected if they are larger than the critical value of the chi-square distribution with one degree of freedom for the first two tests and with two degrees of freedom for the third test, or if the p-value is smaller that 5%. The likelihood ratio statistics which are significant are boldfaced. The fifth column gives the estimated coverage probability under the hypothesis of an independently distributed failure process and the sixth column reports the average value of the regulatory loss function only for the models that are not rejected. The best performing model minimizes the total loss and its standardized sign statistic is smaller than -1.66. The model selected according to the loss function for both 95% and 99% confidence level is Extreme Value Theory.

Table 7: Portfolio (III) Emerging Markets

	Tests for 95% Confidence Level					Tests for 99% Confidence Level				
Model	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**	**LR_{un}**	**LR_{ind}**	**LR_{CC}**	**π**	**RLF**
FWA	**0,0133** **[90,81%]**	59,8659 [0,00%]	59,8792 [0,00%]	5,044%		22,8701 [0,00%]	26,8588 [0,00%]	49,7289 [0,00%]	1,933%	
EWMA	**0,6809** **[40,93%]**	49,6036 [0,00%]	50,2846 [0,00%]	5,316%		13,2380 [0,03%]	9,1462 [0,25%]	22,3843 [0,00%]	1,691%	
CC GARCH	**0,5583** **[45,49%]**	45,3055 [0,00%]	45,8638 [0,00%]	4,696%		12,8353 [0,03%]	6,4816 [1,09%]	19,3169 [0,01%]	1,743%	
DCC GARCH	**0,3262** **[56,79%]**	40,2670 [0,00%]	40,5931 [0,00%]	4,767%		9,6467 [0,19%]	7,3471 [0,67%]	16,9938 [0,02%]	1,636%	
OGARCH	**3,3206** **[6,84%]**	63,6922 [0,00%]	67,0128 [0,00%]	4,269%		6,0192 [1,42%]	23,1038 [0,00%]	29,1230 [0,00%]	1,494%	
EGARCH	**0,0181** **[89,31%]**	30,5131 [0,00%]	30,5312 [0,00%]	4,945%		5,2304 [2,22%]	**2,1233** **[14,51%]**	7,3538 [2,53%]	1,459%	
HS	**0,1512** **[69,74%]**	75,3594 [0,00%]	75,5106 [0,00%]	4,844%		**1,1307** **[28,76%]**	16,8659 [0,00%]	17,9966 [0,01%]	1,202%	
WHS	**0,0022** **[96,27%]**	44,5121 [0,00%]	44,5143 [0,00%]	4,981%		**0,6156** **[43,27%]**	10,9037 [0,10%]	11,5193 [0,32%]	0,859%	
FHS	**0,0279** **[86,74%]**	10,6273 [0,11%]	10,6551 [0,49%]	5,068%		**0,1963** **[65,77%]**	**0,8824** **[34,76%]**	**1,0787** **[58,31%]**	1,084%	**0,0430** **-1,62**
MC	**0,6809** **[40,93%]**	20,9623 [0,00%]	21,6433 [0,00%]	5,316%		16,5939 [0,00%]	0,0027 [95,88%]	16,5965 [0,02%]	1,782%	
MNMC	12,5891 [0,04%]	32,4688 [0,00%]	45,0580 [0,00%]	3,715%		**0,1070** **[74,36%]**	**0,7514** **[38,60%]**	**0,8584** **[65,10%]**	1,057%	0,0899
EVT	4,8322 [2,79%]	31,5398 [0,00%]	36,3719 [0,00%]	4,151%		**0,0004** **[98,39%]**	**1,0461** **[30,64%]**	**1,0465** **[59,26%]**	0,996%	**0,0653** **1,62**

This table provides the components of Christoffersen's test applied to all VaR models and the best performing model according to the regulatory loss function. The first column gives the name of each model, the next three columns give the values of the estimated likelihood ratio statistics of the test for unconditional coverage, the test of independence and the test for conditional coverage .The estimated likelihood ratio statistics are rejected if they are larger than the critical value of the chi-square distribution with one degree of freedom for the first two tests and with two degrees of freedom for the third test, or if the p-value is smaller that 5%. The likelihood ratio statistics which are significant are boldfaced. The fifth column gives the estimated coverage probability under the hypothesis of an independently distributed failure process and the sixth column reports the average value of the regulatory loss function only for the models that are not rejected. The best performing model minimizes the total loss and its standardized sign statistic is smaller than -1.66. In the 95% confidence level all models are rejected. In the 99% confidence level Filtered Historical Simulation and Extreme Value Theory are selected according to the loss function. Neither model is superior to the other.

Table 8: Portfolio (IV) Emerging Markets

	Tests for 95% Confidence Level					Tests for 99% Confidence Level				
Model	**LR$_{un}$**	**LR$_{ind}$**	**LR$_{CC}$**	**π**	**RLF**	**LR$_{un}$**	**LR$_{ind}$**	**LR$_{CC}$**	**π**	**RLF**
FWA	**3,1841**	53,3524	56,5365	5,687%		41,9793	24,0663	66,0456	2,305%	
	[7,44%]	[0,00%]	[0,00%]			[0,00%]	[0,00%]	[0,00%]		
EWMA	**3,4617**	11,8065	15,2681	5,717%		18,4971	**2,2928**	20,7899	1,826%	
	[6,28%]	[0,06%]	[0,05%]			[0,00%]	**[13,00%]**	[0,00%]		
CC GARCH	**0,6195**	**3,3423**	**3,9618**	4,681%	**0,2110**	7,4020	**1,3849**	8,7869	1,549%	
	[43,12%]	**[6,75%]**	**[13,79%]**		**3,05**	[0,65%]	**[23,93%]**	[1,24%]		
DCC GARCH	**1,3004**	4,0208	**5,3211**	4,541%		8,3110	**1,4491**	9,7600	1,584%	
	[25,41%]	[4,49%]	**[6,99%]**			[0,39%]	**[22,87%]**	[0,76%]		
OGARCH	**1,5109**	11,6681	13,1790	4,505%		14,6805	**3,1283**	17,8088	1,795%	
	[21,90%]	[0,06%]	[0,14%]			[0,01%]	**[7,69%]**	[0,01%]		
EGARCH	**0,2589**	4,7064	**4,9653**	5,209%		13,5143	**1,7922**	15,3065	1,760%	
	[61,08%]	[3,01%]	**[8,35%]**			[0,02%]	**[18,07%]**	[0,05%]		
HS	**2,7347**	39,7504	42,4851	5,678%		4,1097	5,3891	9,4988	1,394%	
	[9,82%]	[0,00%]	[0,00%]			[4,26%]	[2,03%]	[0,87%]		
WHS	**0,1736**	15,2435	15,4171	5,168%		**0,4258**	**0,7517**	**1,1775**	1,122%	**0,0642**
	[67,70%]	[0,01%]	[0,04%]			**[51,40%]**	**[38,59%]**	**[55,50%]**		**2,46**
FHS	**1,3820**	**0,8494**	**2,2314**	5,467%	**0,1907**	**3,6138**	**1,1575**	**4,7713**	1,359%	0,0735
	[23,98%]	**[35,67%]**	**[32,77%]**		**-3,05**	**[5,73%]**	**[28,20%]**	**[9,20%]**		
MC	**1,0379**	**2,8829**	**3,9208**	5,390%	0,2180	31,2234	**3,1183**	34,3417	2,108%	
	[30,83%]	**[8,95%]**	**[14,08%]**			[0,00%]	**[7,74%]**	[0,00%]		
MNMC	13,8632	**1,5572**	15,4204	3,641%		**1,6710**	**1,0000**	**2,6710**	1,234%	0,0680
	[0,02%]	**[21,21%]**	[0,04%]			**[19,61%]**	**[31,73%]**	**[26,30%]**		
EVT	**0,0292**	**2,9051**	**2,9343**	4,933%	0,2120	**0,1982**	**0,5206**	**0,7188**	0,921%	**0,0622**
	[86,43%]	**[8,83%]**	**[23,06%]**			**[65,62%]**	**[47,06%]**	**[69,81%]**		**-2,46**

This table provides the components of Christoffersen's test applied to all VaR models and the best performing model according to the regulatory loss function. The first column gives the name of each model, the next three columns give the values of the estimated likelihood ratio statistics of the test for unconditional coverage, the test of independence and the test for conditional coverage .The estimated likelihood ratio statistics are rejected if they are larger than the critical value of the chi-square distribution with one degree of freedom for the first two tests and with two degrees of freedom for the third test, or if the p-value is smaller that 5%. The likelihood ratio statistics which are significant are boldfaced. The fifth column gives the estimated coverage probability under the hypothesis of an independently distributed failure process and the sixth column reports the average value of the regulatory loss function only for the models that are not rejected. The best performing model minimizes the total loss and its standardized sign statistic is smaller than -1.66. The model selected for 95% confidence level is Filtered Historical Simulation and for the 99% confidence Extreme Value Theory.

Table 9: Portfolio (V) Emerging Markets

	Tests for 95% Confidence Level					Tests for 99% Confidence Level				
Model	LR_{un}	LR_{ind}	LR_{CC}	π	RLF	LR_{un}	LR_{ind}	LR_{CC}	π	RLF
FWA	5,7872	19,5387	25,3259	4,044%		11,6154	**3,6420**	15,2574	1,703%	
	[1,61%]	[0,00%]	[0,00%]			[0,07%]	**[5,63%]**	[0,05%]		
EWMA	**2,2035**	15,0835	17,2870	4,405%		**3,5946**	**2,4520**	6,0466	1,374%	
	[13,77%]	[0,01%]	[0,02%]			**[5,80%]**	**[11,74%]**	[4,86%]		
CC GARCH	**2,7383**	19,0462	21,7845	4,269%		13,6546	6,9975	20,6522	**1,854%**	
	[9,80%]	[0,00%]	[0,00%]			[0,02%]	[0,82%]	[0,00%]		
DCC GARCH	**3,8284**	14,6231	18,4516	4,140%		12,4262	7,3247	19,7509	**1,811%**	
	[5,04%]	[0,01%]	[0,01%]			[0,04%]	[0,68%]	[0,01%]		
OGARCH	**3,4436**	17,0379	20,4815	4,183%		11,2463	21,2746	32,5209	1,768%	
	[6,35%]	[0,00%]	[0,00%]			[0,08%]	[0,00%]	[0,00%]		
EGARCH	**2,7383**	19,0462	21,7845	4,269%		6,1171	5,5453	11,6623	1,552%	
	[9,80%]	[0,00%]	[0,00%]			[1,34%]	[1,85%]	[0,29%]		
HS	**0,2191**	36,1461	36,3652	5,209%		**0,0601**	10,8593	10,9194	0,951%	
	[63,97%]	[0,00%]	[0,00%]			**[80,63%]**	[0,10%]	[0,43%]		
WHS	**0,0070**	13,1950	13,2020	4,963%		**0,2748**	14,4261	14,7009	1,107%	
	[93,33%]	[0,03%]	[0,14%]			**[60,01%]**	[0,01%]	[0,06%]		
FHS	**0,3014**	**0,6422**	**0,9436**	5,240%	**0,3330**	**0,8743**	**0,7235**	**1,5978**	1,191%	**0,1132**
	[58,30%]	**[42,29%]**	**[62,39%]**		**4,98**	**[34,98%]**	**[39,50%]**	**[44,98%]**		**2,12**
MC	**1,0963**	**0,7415**	**1,8378**	4,576%	0,3360	13,8565	**0,0143**	13,8708	1,774%	
	[29,51%]	**[38,92%]**	**[39,90%]**			[0,02%]	**[90,48%]**	[0,10%]		
MNMC	23,0624	**1,4952**	24,5576	3,157%		**0,2741**	**0,8624**	**1,1364**	1,100%	0,1291
	[0,00%]	**[22,14%]**	[0,00%]			**[60,06%]**	**[35,31%]**	**[56,65%]**		
EVT	**2,2350**	**3,4653**	**5,7003**	4,497%	**0,2962**	**0,0283**	**0,5022**	**0,5304**	1,035%	**0,0901**
	[13,49%]	**[6,27%]**	**[5,78%]**		**-4,98**	**[86,65%]**	**[47,85%]**	**[76,70%]**		**-2,12**

This table provides the components of Christoffersen's test applied to all VaR models and the best performing model according to the regulatory loss function. The first column gives the name of each model, the next three columns give the values of the estimated likelihood ratio statistics of the test for unconditional coverage, the test of independence and the test for conditional coverage .The estimated likelihood ratio statistics are rejected if they are larger than the critical value of the chi-square distribution with one degree of freedom for the first two tests and with two degrees of freedom for the third test, or if the p-value is smaller that 5%. The likelihood ratio statistics which are significant are boldfaced. The fifth column gives the estimated coverage probability under the hypothesis of an independently distributed failure process and the sixth column reports the average value of the regulatory loss function only for the models that are not rejected. The best performing model minimizes the total loss and its standardized sign statistic is smaller than -1.66. The model selected according to the loss function for both 95% and 99% confidence level is Extreme Value Theory.

Table 10: How Value at Risk estimates could be used by a practitioner

Available Portfolios	Total Market Value at 22/8/2006	Total Market Value at 23/8/2006	Actual Profit/Loss	95% VaR forecast at 22/8/2006	99% VaR forecast at 22/8/2006
Portfolio (I) DM	100.000.000 €	100.435.157 €	435.157 €	-839.753 €	-1.304.591 €
Portfolio (II) DM	100.000.000 €	99.596.468 €	-403.532 €	-729.182 €	-1.046.240 €
Portfolio (III) DM	100.000.000 €	99.239.440 €	-760.560 €	-933.832 €	-1.496.664 €
Portfolio (IV) DM	100.000.000 €	100.659.966 €	659.966 €	-1.329.373 €	-2.194.383 €
Portfolio (I) EM	100.000.000 €	98.938.457 €	-1.061.543 €	-1.153.036 €	-1.804.463 €
Portfolio (II) EM	100.000.000 €	101.837.572 €	1.837.572 €	-1.573.398 €	-2.663.089 €
Portfolio (III) EM	100.000.000 €	98.363.657 €	-1.636.342,79	-1.681.941 €	-2.478.317 €
Portfolio (IV) EM	100.000.000 €	98.299.787 €	-355.548 €	-1.415.158 €	-2.023.590 €
Portfolio (V) EM	100.000.000 €	100.355.548 €	-1.700.213 €	-1.706.291 €	-2.271.765 €

This table presents an example of how Value at Risk estimates could be used in practice. Assume that at 22/8/2006 there are nine different mutual funds with total market value of 100.000.000 €. Each of the mutual funds has invested in one of the developed and emerging markets portfolios described in the previous sections. The manager of each mutual fund uses the best VaR model, which correspond to each portfolio, in order to make one day forecasts of the maximum potential loss of his portfolio's market value with probability 5% and 1% that the actual loss will be greater than the estimated. The first column gives the name of each portfolio, the next two columns give the market value of each portfolio at 22/8/2006 and at 23/8/2006 respectively. The fourth column gives the actual profit or loss between these two days. Finally the next two columns give the VaR forecast of the maximum potential loss made at 22/8/2006 for the next day for the 95% and the 99% confidence level. For example, assume that one of the mutual funds has invested in the fifth emerging markets portfolio (V). The best model selected with the regulatory loss function for this portfolio is extreme value theory for both 95% and 99% confidence levels. The total market value of the portfolio at 22/8/2006 is 100.000.000 €. The 95% and the 99% extreme value theory forecasts of the maximum potential loss for the next day are 1.706.291 € and 2.271.765 € respectively. The actual loss is 1.700.213 €, which indicates that the extreme value theory model has correctly forecasted the magnitude of the loss.

APPENDIX III: Time series plots of daily equity portfolio returns

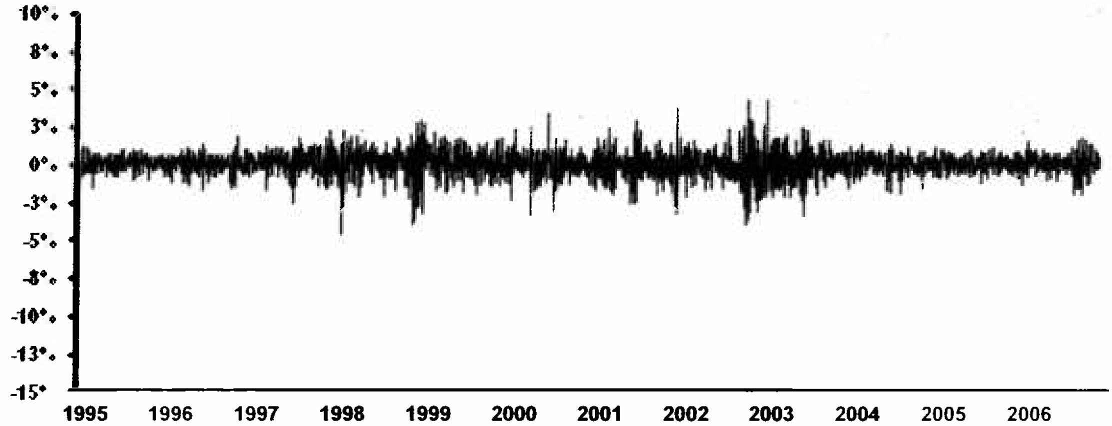

Figure 1: Portfolio (I) DM

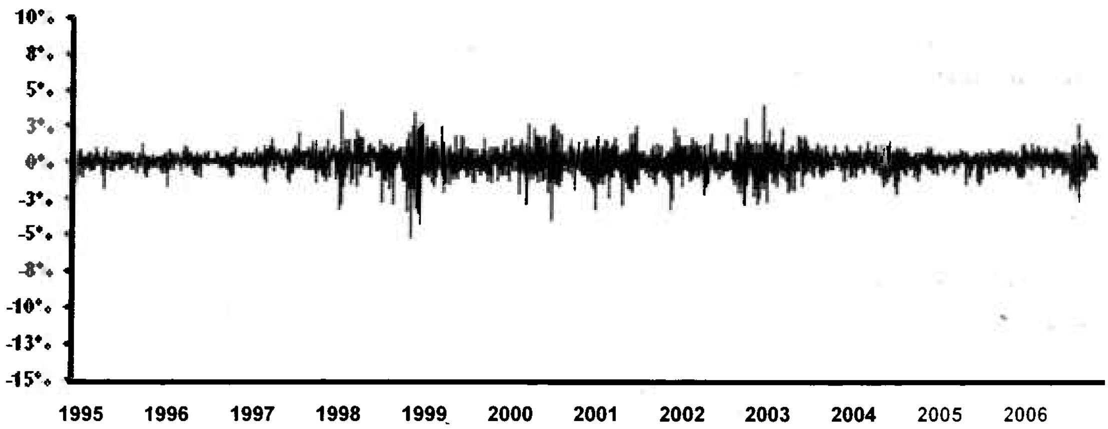

Figure 2: Portfolio (II) DM

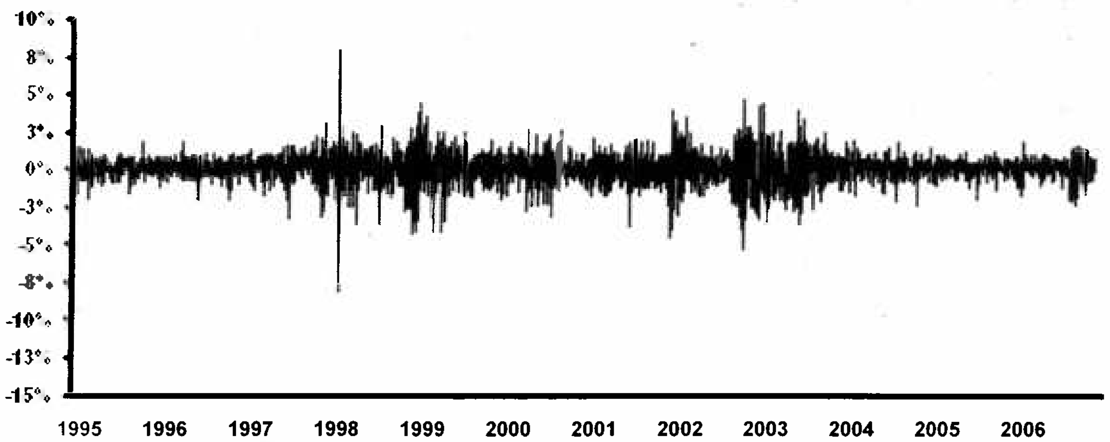

Figure 3: Portfolio (III) DM

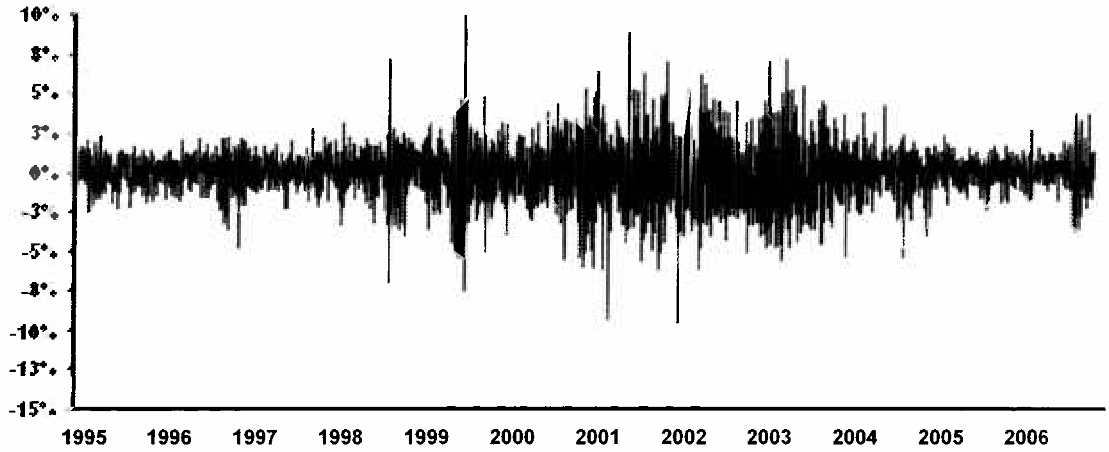

Figure 4: Portfolio (IV) DM

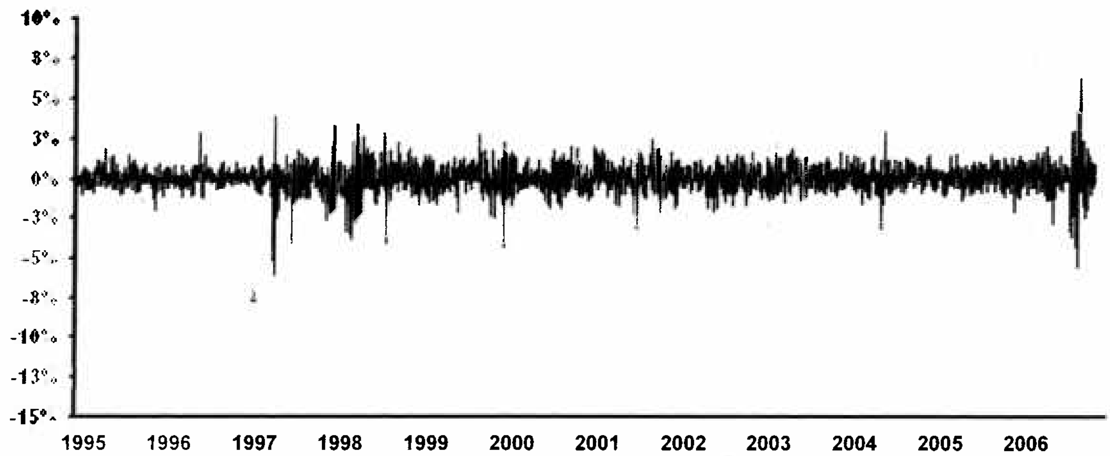

Figure 5: Portfolio (I) EM

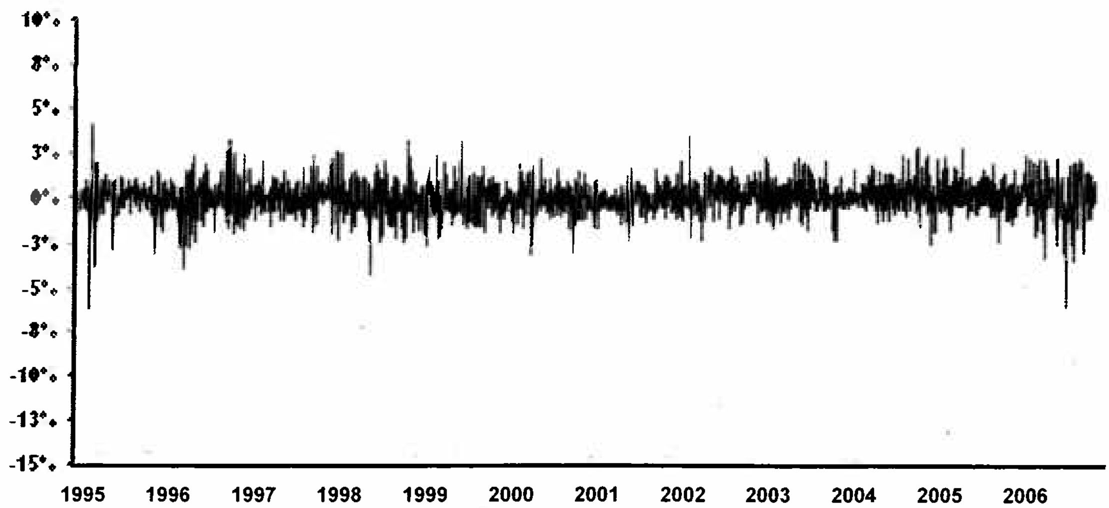

Figure 6: Portfolio (II) EM

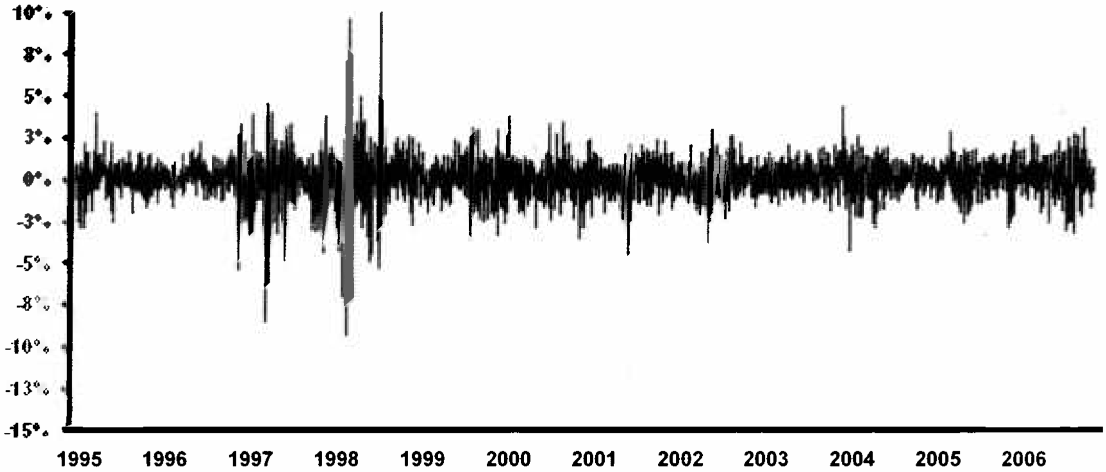

Figure 7: Portfolio (III) EM

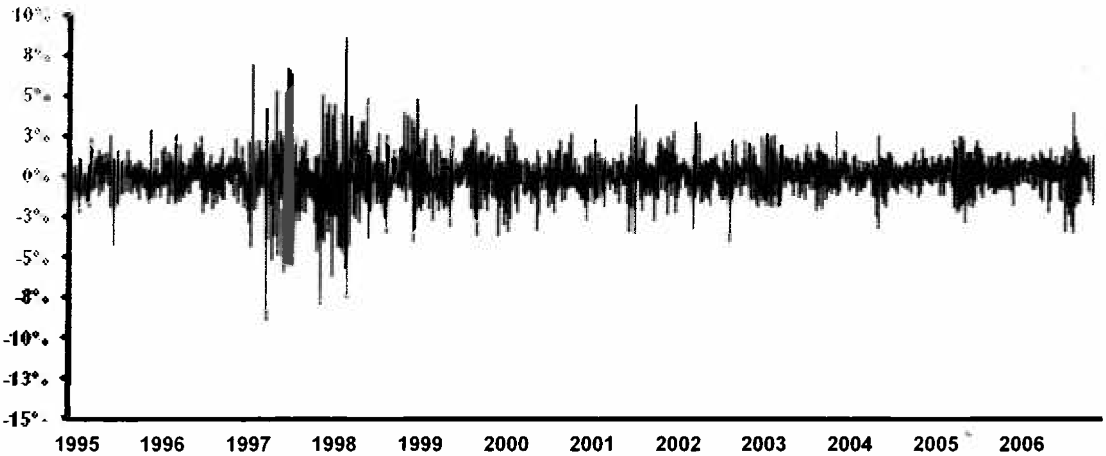

Figure 8: Portfolio (IV) EM

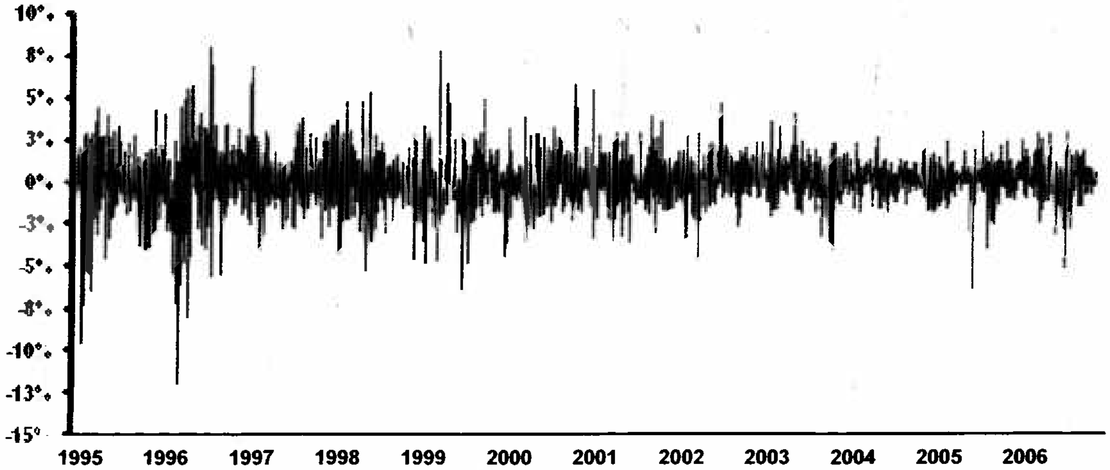

Figure 9: Portfolio (V) EM