The Interdependence of Financial Risk and Sovereign Risk along the Business Cycle in the Euro Area

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Abstract

We develop a dynamic stochastic general equilibrium model to study the role of the interdependence between financial intermediation and sovereign crises for economic activity and policy. The model is a synthesis of the leading, modern approaches in the literature of financial frictions and sovereign default. In particular, we model financial frictions following Bernanke et al. (1999) and we model the event of sovereign default, using the stochastic fiscal limit concept of Bi and Traum (2012). We find that an increase in capital investment risk, (risk shock), results in a considerably deeper recession, when sovereign risk is also present. The recession strongly depends on the government's countercyclical financial sector rescues policy. An increase in capital investment risk raises government expenditures, which in turn raise debt. The increase in debt increases the probability of government default, which results in higher interest rates on bonds and bank deposits. The higher interest rates, working through the financial accelerator mechanism, further deepen the recession. This result has three policy implications. First, Euro Area policies dealing with failing banks aggravated the recession. Second, although there has been a supranational effort with the creation of the EFSF/ESM to provide loans to sovereigns, as long as there is no direct mechanism for financial sector rescues, Euro Area policies continue to exacerbate the recession. Third, in favor of austerity measures used in the EA, we find that government spending multipliers are smaller in the presence of sovereign risk.

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1. Introduction

The strong interdependence between sovereign debt and financial crisis has been in the forefront of the current European crisis. This interdependence is not limited to the Euro Area (EA). Historically, financial crises tend to be followed by sovereign debt crises (Reinhart and Rogoff (2011)). However, the vicious circle between systemic bank failures and rising sovereign debt is highly profound, recently, in the EA. On the one hand, in the absence of supranational coordination in dealing with failing banks, responsibility for the rescue of national banking systems fell with member states.[1] Given the size and the systemic nature of banks across the EA, fiscal consequences of rescuing banks were overwhelming. This explains how stress in the financial system spread over to sovereigns. On the other hand, domestic banks held on their balance sheets a considerable amount of debt issued by their domestic and other European governments. Government bonds were appealing because they were easily used as collateral by banks and because the Basel regulatory framework allowed for zero risk weighting of bonds issued by EA governments. In addition, governments may have exercised pressure on banks to hold their debt. The large size of government debt holdings by EA banks, explains how concerns about sovereign solvency, immediately affect the stability of the banking system. The resulting two way bank-sovereign link constitutes one of the specific features of the EA that renders it especially vulnerable to shocks. Evidently, channels through which financial shocks, particularly those emanating from sovereign debt markets, propagate from the financial system to the real economy are of great importance. This paper studies the role of interdependence between sovereign debt and financial crises for the propagation of shocks along the business cycle. It should be stated at the outset, that although there is a growing literature on sovereign debt default and a growing literature on financial intermediation and the business cycle, relatively little has been done on the

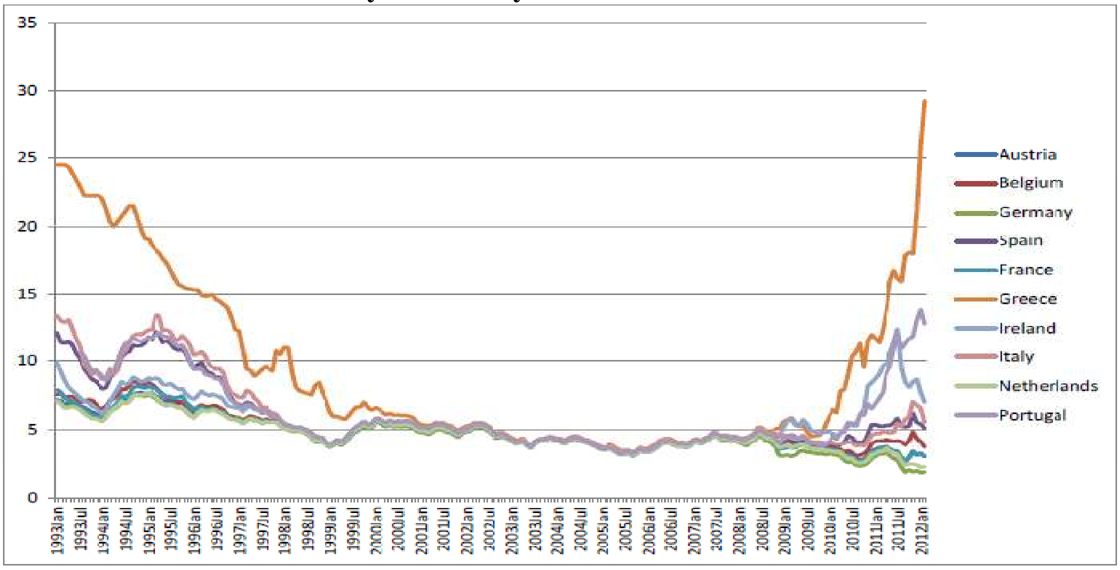
[1] Until recently, there has been no supranational mechanism dealing with failing financial institutions. The European Financial Stability Facility (EFSF) and the European Stability Mechanism (ESM) helped EA sovereign members with loans earmarked for that purpose, but were not entitled to inject capital directly to the respective sovereign's financial system. Further ECB's asset purchases, under the Securities Market Programme, have been limited compared to the Fed's. Recently, there have been efforts towards supranational approaches in dealing with bank solvency, directly. The ESM introduced the direct recapitalization mechanism (DRI), allowing for direct recapitalization of financial institutions and the ECB launched a bond-buying program with €60bn in purchases per month, up to €1.08 trillion.

interdependence of sovereign risk and financial intermediation for the business cycle. The European debt crisis stresses the need to account for this interaction.

1.1 Stylized Facts of the EA Sovereign Debt Crisis

Following the formation of the EMU in 1991 and the introduction of the euro in 1999, sovereign bond yields across the EA converged to those of German Bunds (Figure 1.1). This was due to the elimination of exchange rate and inflation risks, as well as efforts made by European governments to reduce their deficit and debt as shares of GDP, in order to fulfill the Maastricht criteria and join the common currency (Figure 1.2).[2]

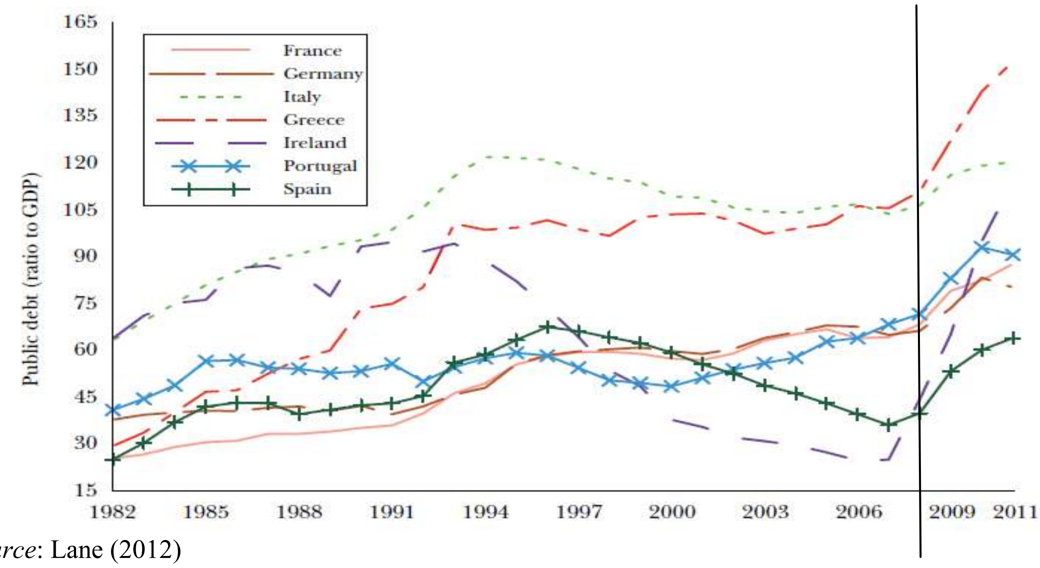
Figure 1.1
10-year bond yields: 1993-2012



Source: Shambaugh (2012)

[2]Ireland, Portugal and Spain each achieved significant reductions in their debt ratios in the late 90's. While the Portuguese ratio began to climb on 2000 onwards, rapid output growth in Spain and Ireland contributed to further rapid reductions to their debt to GDP ratios up to 2007. France and Germany had stable ratios of about 60% in the decade prior to the crisis. Italy and Greece ratios were above 90% in the 90's and although the ratio was declining in the late 90's especially for Italy, these countries never achieved the 60% limit specified by European fiscal rules.

Figure1.2
Public debt as share of GDP: 1982-2011



Source: Lane (2012)

The financial crisis of 2008, that followed the Lehman collapse, resulted in a sudden and disruptive repricing of sovereign risk. Government bond yields started to diverge, sometimes reaching levels that exceeded those of the early 90's (Figure 1.1). The recent IMF paper Mody and Sandri (2012) identifies the rescue of banks in 2008/2009 as the trigger of a regime shift in the dynamics of the euro area countries sovereign spreads. Up to then, focus was on the stability of the banking system with country specific fiscal risks remaining on the background. Following the rescue of Bear Sterns however, factors like the prospects of the financial sector started becoming important in the differentiation of spreads in the EA. The trend was reinforced by the nationalization of Anglo-Irish Bank. These events led financial markets to familiarize with the idea that banks would be bailed out by governments.

The cost of recapitalizing banks was very high especially for countries that were home to banks with significant cross-boarder activities. As documented in ECB (2009), nearly every EA country took some steps to stabilize their financial system which involved fiscal resources. These included direct capital injections to banks, state guarantees of bank liabilities, and asset support measures such as acquisitions of bad assets. By mid 2010 total commitments (capital injections, liability guarantees and asset support) ranged from roughly 20% to 300% across EA countries (Table 1.1). Total commitments for the entire EA amounted to 28% of EA GDP.

Table 1.1
Committed government support to financial institutions (% of 2008 GDP): October 2008-May 2010

Austria	Belgium	Cyprus	Denmark	Spain	Finland	France	Greece
32	47	18	25	24	29	18	18
Ireland	Italy	Lux	Malta	Netherlands	Portugal	Slovenia	Slovakia
319	4	26	0	52	12	32	0

Source: ECB (2009)

Based on IMF estimates (Table 1.2), total direct support to the financial sector by mid-2011 (not including liability guarantees, that may cost or not cost money in the future) was roughly 6% of GDP in countries like Greece and Belgium, 13-14% in the Netherlands and Germany and more than 40% in Ireland.

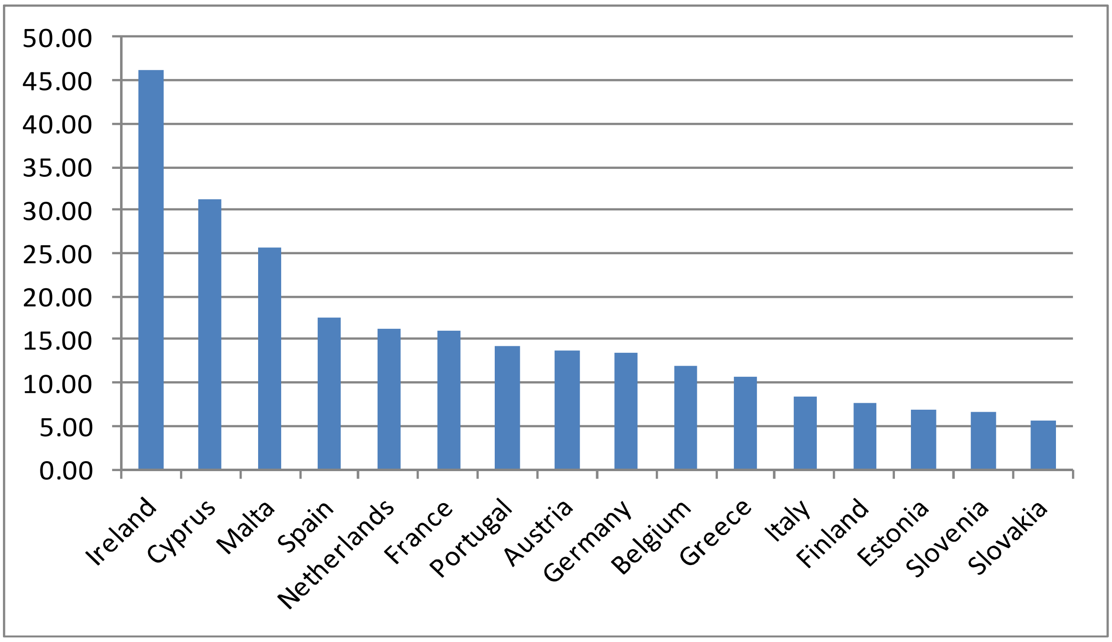
Table 1.2
Realized Financial Sector Support (% of 2011 GDP): 2008-2011

Belgium	Ireland	Germany	Greece	Netherlands	Spain
5.7	40.6	13.2	5.8	14	3

Source: IMF (2011a)

In addition, the degree of leverage reached by EA banks way surpassed the receipts of governments. Still in 2010, total bank assets amounted to 45 times government tax receipts in Ireland and the ratio was high in other countries as well (Figure 1.3). Fiscal expansion to support the financial system thus, resulted in increasing levels of debt to GDP ratios for most euro area countries after 2008 (Figure 1.2), showing the implications of bank rescues on government debt and raising concerns of public finance sustainability.

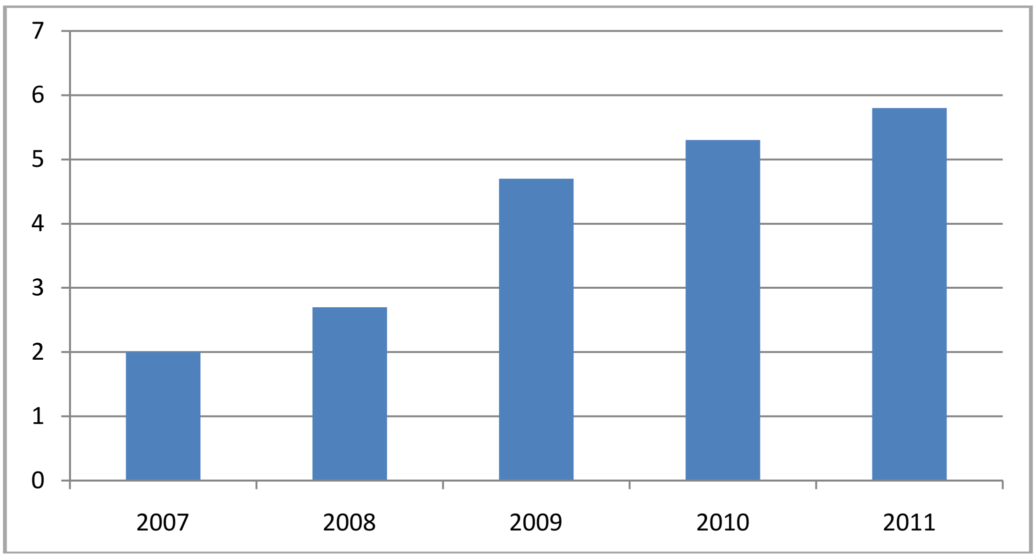
Figure 1.3
Bank assets to government tax receipts: 2010



Source: Pisani-Ferry (2012)

The scale of the recession and rising estimates of prospective banking sector losses on bad loans (Figure 1.4), had thus, a negative impact on government bond values for investors recognized that a deteriorating banking sector imposed fiscal risks.

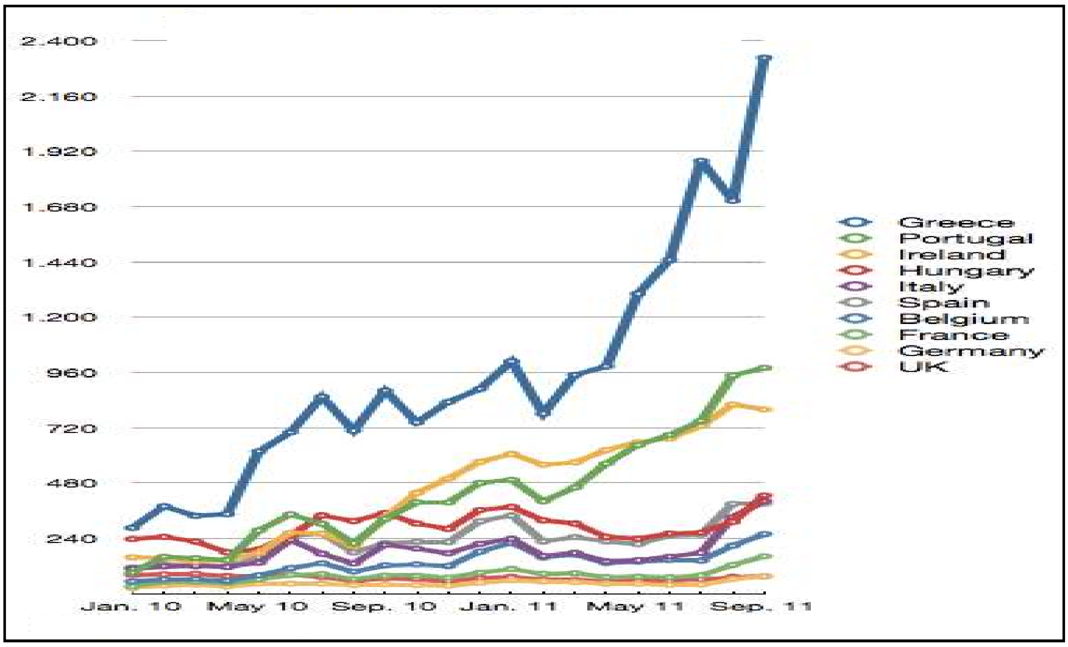
Figure 1.4
Non performing loans as a share of total loans: 2007-2011



Source: World Bank, http://data.worldbank.org

The Greek CDS spread increased rapidly to more than 4% in early 2010, reflecting the concerns of a possible sovereign default or debt restructuring. On April 23, the Greek government requested that the EU/IMF lending mechanism be activated. On April 27, Greek debt was downgraded to junk status by Standard & Poor's, making it ineligible as collateral with the ECB. Portugal's simultaneous downgrade and Spain's subsequent one added to the negative sentiment. CDS spreads on EA countries rose significantly (Figure1.5), with Greek spreads rising to levels never seen before in advanced economies.

<u>Figure1.5</u>
Sovereign CDS: 2010-2011



Source: Bloomberg

Evidently, the financial crisis had resulted into a sovereign crisis, where pressure on the banking system had spilled over to sovereigns through the cost of bank rescues.

The resulting sovereign crisis affected, in turn, EA banks. Most continental EA countries were characterized by the large size of their bank portfolios of domestic government bonds, which were markedly larger than in the UK and the US. Consequently, concern about sovereign solvency was bound to have major consequences for banks, in the entire EA.

As illustrated in Table 1.3, in 2007 just before the outbreak of the financial crisis the share of government debt held by domestic banks was very large. This was especially true for countries that have been subject to great pressure in the sovereign debt market (Greece, Ireland, Italy, Portugal and Spain). More worryingly, these holdings had increased substantially during the crisis, as domestic banks compensated the outflow of scared foreign investors. Further, banks in the EA held considerable volumes of bonds of other European sovereigns. In that way the sovereign crisis could affect the solvency of banks in countries that face no similar debt problems like France and Germany. Thus, stress on sovereign debt markets has translated into pressure on the entire European banking system. The OECD highlights the potential threats of contagion to larger countries. Based on their calculations, OECD (2011), the holdings of Belgian, Italian, and Spanish debt by banks was 188%, 209%, and 171% of Core Tier I capital in Belgium, Italy, and Spain, respectively, and over 50% in France and Germany. The strong increase in the yield spreads of Italian, Spanish and Belgian government bonds over German bonds suggest that these countries could face increasing problems in their future public finances. If the value of some of these assets was set to zero, the capital of these banks would be wiped out, making them insolvent.

Table 1.3

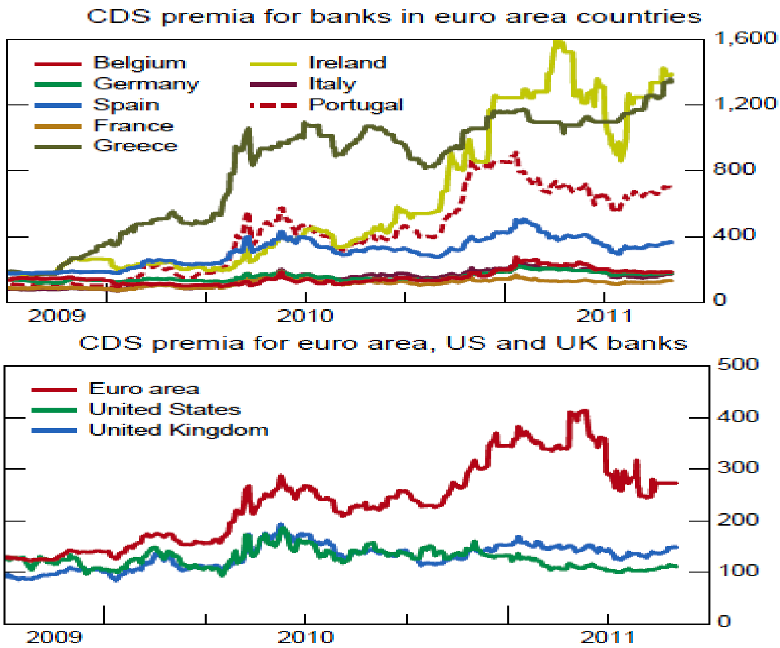
Holdings of marketable sovereign debt by domestic banks as percentage of GDP

	Greece	**Ireland**	**Portugal**	**Italy**	**Spain**	**Germany**	**France**	**UK**	**US**
2007	10.5	0.4	6.2	10.3	7.0	18.8	4.4	-0.6	0.9
2011	16.1	9.6	20.8	16.9	15.9	15.7	6.2	7.5	1.9

Source: Merler and Pisani-Ferry (2012a)

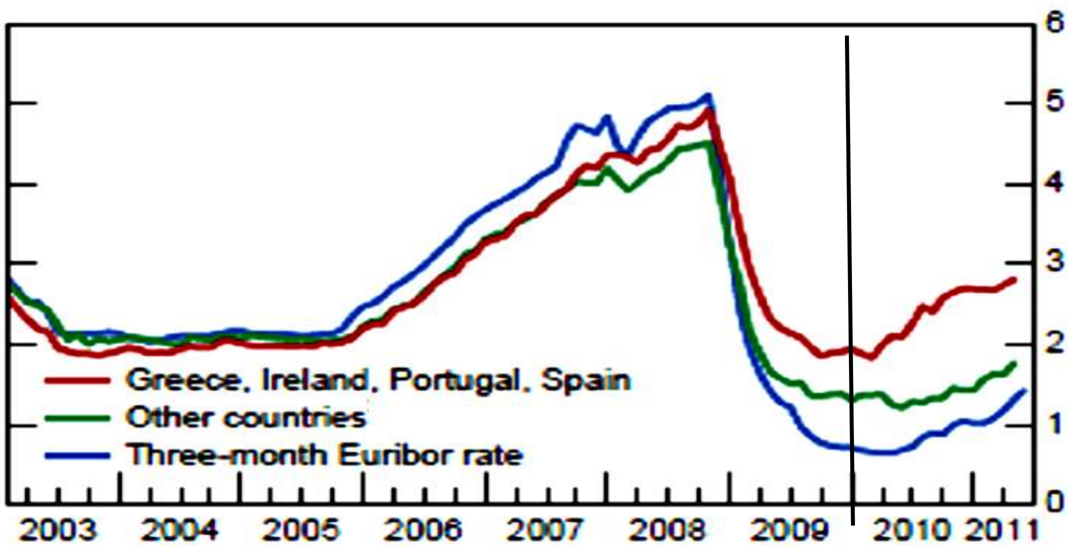
The increased bank holdings of risky government debt have been reflected in the funding conditions of EA banks both via valuation losses and via increases in the perceived risks relating to banks. Sovereign debt concerns have pushed up bank funding costs. As shown in Figure 1.6, CDS premia on European banks had been increasing and were higher than the US and UK rates. Interest rates on bank deposits have been increasing as well (Figure 1.7).

Figure 1.6
Bank funding conditions: 2009-2011



Source: BIS (2011)

Figure1.7
Interest rates on bank term deposits: 2003-2011

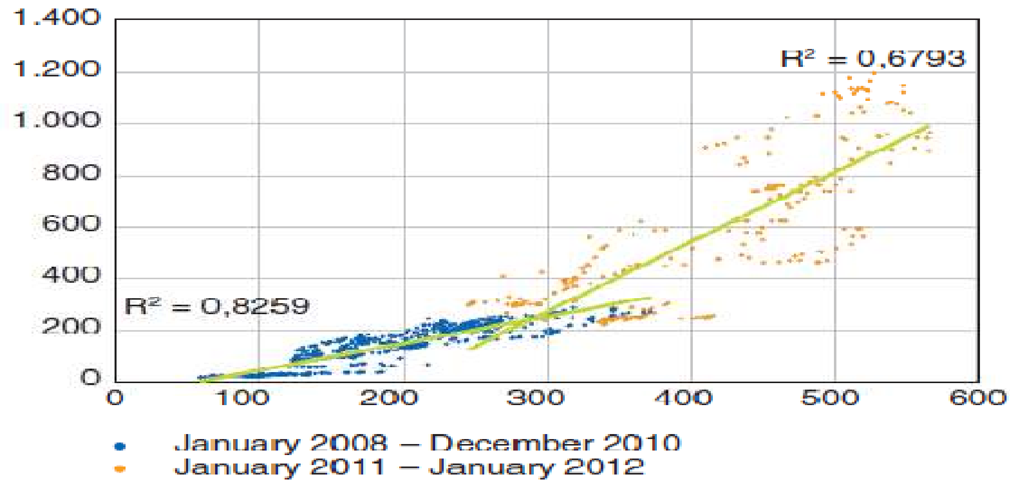


Source: BIS (2011)
Note: Other countries include Austria, Belgium, Germany, Finland, France, Italy and the Netherlands.

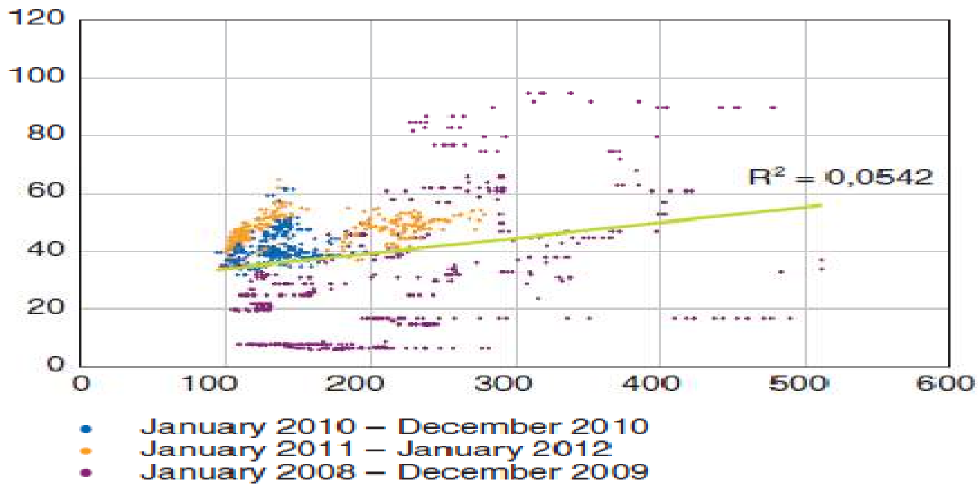
A vicious circle had been created, where financial sector rescues in response to the financial crisis had resulted in a sovereign crisis that was further transmitted to the financial sector through their holdings of government debt. As shown in Figure 1.8, CDS premia on European banks and sovereigns had been increasing and were higher than the US rates.

<u>Figure 1.8</u>
Correlation of sovereign and bank CDS: 2008 – 2012
(x-axis bank CDS 5 years, y-axis sovereign CDS 5 years)

EA



US



Source: Merler and Pisany-Ferry (2012b)

The vicious circle is particularly evident in the case of the Greek debt restructuring and the resulting Greek bank recapitalization. Following Greece's debt restructuring in April 2012 Greece's four biggest banks wrote down about 25 billion Euros in the combined value of their Greek government bond holdings. In May 2012, Greece handed 18 billion Euros those banks through the Hellenic Financial Stability Fund

(HFSF).[3] The HFSF was set up to funnel funds from Greece's bailout programme to recapitalize banks.

1.2 Questions

These facts on the EA debt crisis raise some important economic questions that need to be addressed:

(a) What is the effect of an adverse financial shock on the business cycle when a sovereign debt – financial intermediation channel is present?

(b) What are the implications for EA financial sector rescue policies? How effective was EA financial sector bailout policy? Has the EA been building the right supranational framework for supporting its financial sector?

1.3 How we address them

We aim to construct a model framework that replicates the financial-sovereign interdependence observed in the EA. Using our model we aim to quantitatively access the impact of this interdependence on economic activity and draw implications about EA policy on financial sector bailouts, that triggered this interdependence. In order to examine financial intermediation-sovereign interactions, we need to construct a model framework with a non trivial role for financial intermediaries, where frictions exist in the financial intermediation process and the financial sector has an important role as a source of business cycle fluctuations, as well as a framework that allows for the possibility of sovereign default, where sovereign risk affects business cycle dynamics.

To model financial frictions, we follow the model of Bernanke, Gertler and Gilchrist (1999), (BGG). Standard monetary models are based on the assumption that markets

[3]The HFSF allocated 6.9 billion Euros to National Bank of Greece, 1.9 billion to Alpha Bank, 4.2 billion to Eurobank and 5 billion to Piraeus Bank.

are complete and therefore, there are no financial market frictions. In practice, theborrower-lender relationship is characterised by informational asymmetries that create imperfections and impose agency costs. These asymmetries are modelled in the literature through moral hazard, monitoring costs, liquidity constraints, etc. and are referred to as financial frictions. Until recently, most of the financial frictions literature has focused on the role played by the financial sector in propagating shocks that originate in other sectors of the economy, such as productivity and monetary shocks. Bernanke and Gertler (1989) is one of the earliest studies based on costly monitoring. Kiyotaki and Moore (1997) provide another possible approach for incorporating financial frictions into general equilibrium models, based on credit constraints. In these studies, financial frictions exacerbate the recession, following a negative non-financial shock, but do not cause the recession. Recent economic events starting with the financial crisis of 2008, suggest that the financial sector plays an important role as a source of business cycle fluctuations. The importance of financial shocks – shocks that originate from the financial sector – has started to be explored only recently. Recent papers stressing the role of financial factors for business cycle dynamics include, Gertler and Karadi (2011), Gertler and Kiyotaki (2011), Kiyotaki and Moore (2008), Curdia and Woodford (2009a) and Christiano, Motto and Rostagno (2003, 2010), (CMR). As shown in CMR (2003, 2010), the BGG model is very successful in empirical applications and provides a suitable environment to access the effects of shocks in the financial sector.

BGG (1999) analyse financial frictions in a general equilibrium setup using a costly state verification framework, originally due to Townsend (1979) and Gale and Hellwig (1985). They assume that the borrower has a random return that is not observable to the lender, unless he pays an audit cost. In equilibrium, a financial accelerator arises. The key link involves the inverse relationship between the external finance premium, i.e., the difference between the cost of external funds and the opportunity cost of internal funds, and the borrower's net worth. The inverse relationship arises because when the borrower has little wealth to contribute to the project, he requires a greater loan that comes with a greater spread reflecting, potentially, higher agency costs. To the extent to which net worth varies procyclically, the external finance premium will vary countercyclically, amplifying swings in borrowing and thus investment, spending, and production.

To model sovereign risk, we assume that the government faces a stochastic fiscal limit reflecting uncertainty in political negotiations. Government defaults if the endogenous level of debt surpasses the fiscal limit. We restrict ourselves to the case of no actual default. However, the probability of default arising from the stochastic fiscal limit setup may affect the value of debt even in the case of no actual default. Further, we assume government injects funds, financed through increased government debt, for the bail-out of the financial sector.[4] These funds are an increasing function of financial sector risk as captured by the external finance premium.[5]

This way of modeling sovereign risk relates to the recent literature on stochastic fiscal limits. Bi (2012) and Bi and Traum (2012), assume that government defaults when is bounded by the fiscal limit. Default occurs when outstanding government debt breaches a maximum sustainable debt-output ratio. This ratio can depend upon the state of the economy and/or stochastic fluctuations in political risk.[6] In early literature, Eaton and Gergovitz (1981) and a number of authors including Arellano (2008) and Mendoza and Yue (2012) model sovereign defaults as strategic decisions made by a welfare maximizing government, that balances the gains from foregone debt services against the costs of exclusion from international markets. The above literature can be used to study sovereign default in emerging market economies. However, the predicted level of government debt at which the sovereign default occurs is much lower than the debt level at which the sovereign risk premia are observed in developed countries, making it difficult to use these models for policy making in developed countries. Thus, in our case we model sovereign default using the stochastic fiscal limit approach.

To quantitatively investigate the effect of the financial-sovereign interdependence for economic activity and policy we choose to embed our model in a New Keynesian

[4] Although there are different ways of modeling funds for the bail-out of the financial sector, such as reserves exclusively kept for a potential bail-out, we consider these reserves as cash injections to banks in every period, as this is one of the most common policies used.
[5] CMR (2010) and Gilchrist, Yankov and Zakrajsek (2009) provide support for modeling bank rescue funds as a function of the external finance premium.
[6] Bi (2012) and Bi and Leaper (2010) have constructed models where the distribution of the fiscal limit arises endogenously from the dynamic Laffer curve, which inturn, depends on macroeconomic fundamentals.

(NK) Dynamic Stochastic General Equilibrium (DSGE) setup along the lines of Christiano, Eichenbaum and Evans (CEE) (2005). The basic structure of their model incorporates sticky wages and prices, variable physical capital utilization, investment adjustment costs and habit persistence in preferences. In recent years, NK DSGE models have become a main tool in business cycle and policy analysis. The idea of using a model as a representation of the actual economy has been around for many years. But it was not until the work of Kydland and Prescott (1972) that models were used to simulate the economy and were judged according to how well the simulated data could replicate various properties of the actual data. As reviewed by Christiano, Trabandt and Walentin (2010) in the Handbook of Monetary Economics, the NK model simulates data of the actual economy quite well. With a combination of price and wage frictions it resolves a classic empirical puzzle about the effects of monetary policy that is, the slow response of inflation to a monetary disturbance. Moreover, the model simultaneously explains the dynamic response of the economy to other shocks. In addition, the NK model rivals a-theoretical statistical models in terms of out of sample forecasting, as demonstrated by Smets and Wouters (2003, 2007). In part because of these successes, a consensus has formed around this particular model structure.

We simulate a financial crisis in our model by graphing impulse response functions (IRF's) to a shock in the financial sector. This approach, of studying how the economy responds to shocks, is in the spirit of a suggestion made by Lucas (1980). We compute model IRF's to a shock in the financial sector. We compare the IRF's to the same financial shock of the following models: a) our baseline model with sovereign-financial interactions; b) a model with the same financial frictions as our baseline model that abstracts from financial sector rescue funds and sovereign debt and risk.[7] Our baseline model is referred to as the Sovereign Risk (SR) model and the latter model as the Financial Accelerator (FA) model. Through this comparison, we can access the effect the additional financial-sovereign channel we have introduced, has for economic activity. We then draw policy implications.

[7]This is the BGG (1999) model incorporated in the NK setup of CEE (2005).

1.4 Findings

Our main findings are as follows:

First, an adverse financial sector shock, simulated by an increase in capital investment risk (risk shock), results in a considerably deeper recession, when sovereign risk-financial intermediation interactions are operating. The drop in net worth is substantial but the drop in loans is not as severe. However, the external finance premium is higher. The result is a substantial drop in investment and output in the order of one and a half times bigger than that obtained under the FA model. The initial drop in consumption is more than double than in the FA model, rebounding as the drop in loans decelerates. To understand the mechanism that leads to this outcome we need to analyze the impact of our shock the FA and SR models. In the FA model a rise in the risk shock results in a rise in the external finance premium. The external finance premium inversely depends on the borrowers net worth, because when entrepreneurs have little net worth to contribute in project financing, agency costs are higher. To the extent which borrowers net worth is procyclical, due to the procyclicality of profits and assets prices, the external finance premium is countercyclical, amplifying swings in borrowing, investment, spending and production. The amplification of swings in the FA model is known as the financial accelerator effect. In the SR model simulations, an increase in financial sector risk, caused by a shock in the financial sector, results in a rise of funds injected to the financial sector as rescue funds. These additional funds injected result in higher government debt. The higher level of debt increases the probability of default of the sovereign, resulting in higher sovereign and financial spreads. Higher spreads result in a higher external finance premium. As a result, the bank-sovereign interdependence reinforces the initial financial accelerator mechanism. The deeper recession in the sovereign risk model strongly depends on the government's countercyclical stand on financial sector rescues. The higher injections result to higher debt causing higher financing costs for banks that are further transmitted to non-financial corporations with a higher premium. Thus, our model replicates the effects of the vicious circle observed in the EA.

Second, we derive policy implications. Our model has three policy implications. The first and main implication of our model for EA policies dealing with failing banks is that they aggravated the recession. The SR model captures the main features of the EA policy in dealing with failing banks, whereby each member state is responsible for the bail out of these banks. The SR model predicts the strong correlation between sovereign and financial spreads, as observed in the EA sovereign debt crisis (Figure 1.8). On the contrary, the simple FA model replicates a hypothetical scenario where the EA countries had not engaged in such financial sector bailouts, thus not causing such a correlation. As discussed above, comparing the SR model to the simple FA model, where financial sector bailouts are not present, the SR model predicts a deeper recession. Thus, our model comparison suggests the EA bank rescue policy has resulted in a deeper recession. Recently, there have been efforts towards supranational approaches in dealing with liquidity and sovereign solvency. However, until recently, there was no supranational mechanism dealing with failing financial institutions. The European Financial Stability Facility (EFSF) and the European Stability Mechanism (ESM) has helped EA sovereign members with loans earmarked for that purpose but they were not entitled to inject capital directly to the respective sovereign's financial system.[8] In addition ECB's asset purchases have been minimal compared to the Fed's. The ECB under the Securities Market Program from May 2010 – February 2012 conducted direct purchases of public and private debt securities, however the size of the program that by the end of 2010 was just over €70 billion and reached about €200 billion, was small compared to Federal Reserve purchases of over $1 trillion in mortgaged backed securities by early 2010.[9] Thus, the second policy implication of our model is that as long as bank solvency remains a national matter, continues to contribute to the recession in the manner identified above. The third policy implication of our model relates to the so called austerity policies. We use our model to access whether such measures can be self-defeating. This is done by comparing government spending multipliers of the FA and SR models. We find that an increase in government spending has a smaller impact on output in the presence of sovereign risk. An increase in government spending increases debt which in turn increases the probability of default of government bonds. This results in higher interest rates

[8]In December 2014, the ESM's direct recapitalization mechanism (DRI), allowing for direct recapitalization of financial institutions under specific circumstances became operational.

[9]In January 2015, ECB announced a new government bond-buying program with €60bn in purchases per month, up to €1.08 trillion.

counteracting the initial positive impact of government spending on output. This suggests that austerity measures are less likely to be self-defeating in the presence of sovereign risk.[10]

In what follows, Chapter 2 presents the related literature, Chapter 3 presents the model framework used is our analysis, Chapter 4 presents the workings of our model, Chapter 5 discusses the various policy implications and Chapter 6 provides a conclusion. In addition, we provide an Appendix presenting the derivation of the model FOC's and the Dynare and Matlab codes used.

[10]This result is in line with recent empirical and theoretical findings (see e.g. Corsetti (2012, 2013)).

2. Related Literature

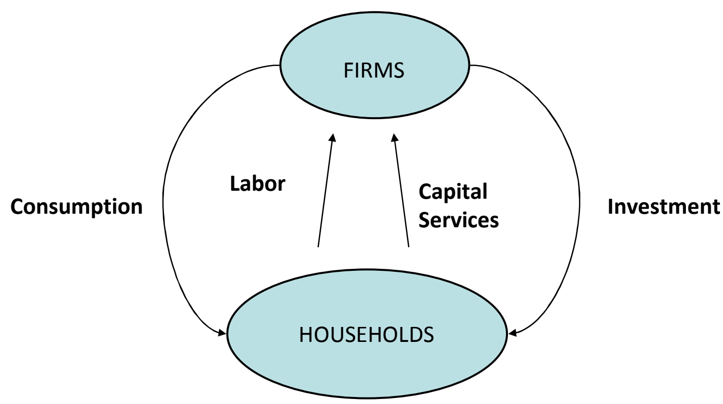
The recent events, of the 2008 financial crisis and the European sovereign debt crisis, have underlined the importance of the financial sector and of sovereign risk for the macroeconomy. Economists try to introduce such features into DSGE modeling. In what follows, we present relevant DSGE models with financial intermediation and DSGE models with sovereign risk.

2.1 DSGE Models with Financial Intermediation

Many economic activities require financing, because one must free up resources in one period by suppressing consumption, and then wait for a period until output occurs. In the neoclassical model, which forms the basis of contemporary business cycle models, finance is required in the construction of physical capital. In the construction of physical capital, resources are allocated in one period and the return does not occur until the next period. As illustrated in Figure 2.1, in the neoclassical model, households provide labor and capital services to firms. Firms combine these services to produce final goods. Final goods are transformed into consumption goods and investment goods. Households allocate the income they earn from capital and labor services to purchase consumption goods, which they consume, and investment goods, which they use to produce new capital goods. In the next period they rent these capital goods to firms. Finance in the neoclassical model does not entail any particular complications since the group suppressing consumption and the one building capital is the same the household.

In standard dynamic general equilibrium models with perfect markets, financial intermediaries are redundant. Lenders and borrowers interact directly between them and given a representative agent setting, net borrowing is zero in equilibrium. In practice, savers and investors are different people and there are reasons to believe that there is a conflict of interest between them. In order to rationalize the existence of financial intermediaries, one needs to consider an economy with heterogeneous agents and informational failures. Indeed such an economy is far more realistic than a simple complete Arrow-Debreu economy.[11] Thus, in order to justify the existence of credit markets from first principles, intermediaries are assumed to have informational advantages in accessing the creditworthiness of agents, have the ability to monitor agents and posses a wider variety of financial assets to match borrowers and lenders needs improving the efficiency of the market. In such a setting, one can distinguish between internal versus external financing.

Figure 2.1
Simple neoclassical model



Introducing credit markets from first principles in business cycle models, gives rise to issues such as heterogeneity among agents, asymmetric information and the effect of net debt in investment decisions. In the early literature, the formal analysis of such issues was difficult. While the role that financial markets had on cyclical fluctuations was stressed by various writers such as Fisher (1933), that attributed the Great Depression mainly to financial distress combined with deflation, any rigorous analysis is absent. Only after the introduction of asymmetric information, first introduced by Akerlof (1970), were economists able to formally model the complicated issues that have to be considered when introducing credit markets into macroeconomic models. It

[11]Such an Arrow-Debreu economy underlies the world of Modigliani and Miller (1958), where financial intermediaries are unnecessary.

is now clear that in a borrower-lender relationship informational asymmetries arise creating imperfections and imposing agency costs. Thus, the Modigliani-Miller Theorem (1958) on financial structure irrelevance fails, since net worth becomes an important variable in the determination of one's creditworthiness. Induced imperfections can be modelled through monitoring costs, credit constraints, moral hazard, etc. These features are referred in the literature as financial frictions.

Financial frictions result in a higher cost of external financing versus internal financing due to the costs of processes associated with obtaining external funding. The data provides such evidence, showing that external funding is almost always more expensive than using internal resources, (De Graeve (2008), Benerjee (2002)). Thus, the external finance premium, defined as the difference in cost of a borrower to raise capital on financial markets (external funding) and the opportunity cost of the borrower's use of internal resources, is almost always positive. The external finance premium reaches different levels for diverse borrowers and depends on net worth, liquidity, the expected stream of income from his investment projects etc. A debtor with a better financial position, high net worth, available liquidity, proven record of realization of successful investment projects, has reduced cost of external financing. The fundamental property of the external finance premium is its countercyclical character (Besley (2008)), because firm's income and worth grows in periods of economic growth and this improves firm's financial position (procyclicality of profits and assets prices). In the event of a positive/adverse shock that propagates in the rest of the economy, the external finance premium will decrease/increase, accelerating the boom/recession a property known as financial accelerator.

Most of the literature on macroeconmic models with financial frictions has focused on the role played by the financial sector in propagating shocks that originate in other sectors of the economy, productivity and monetary shocks. Financial frictions exacerbate the recession but do not cause the recession. A negative shock first happens in the non-financial sector. These shocks would generate a recession even without frictions. With financial frictions, however the magnitude of the recession becomes much bigger. Recent economic events, however starting with the financial crisis of 2008 suggest that the financial sector plays an important role as a source of business cycle fluctuations. A shock which occurs at financial markets propagates to the rest of the economy. The important of financial shocks – shocks that originate from the financial sector – has started to be explored only recently.

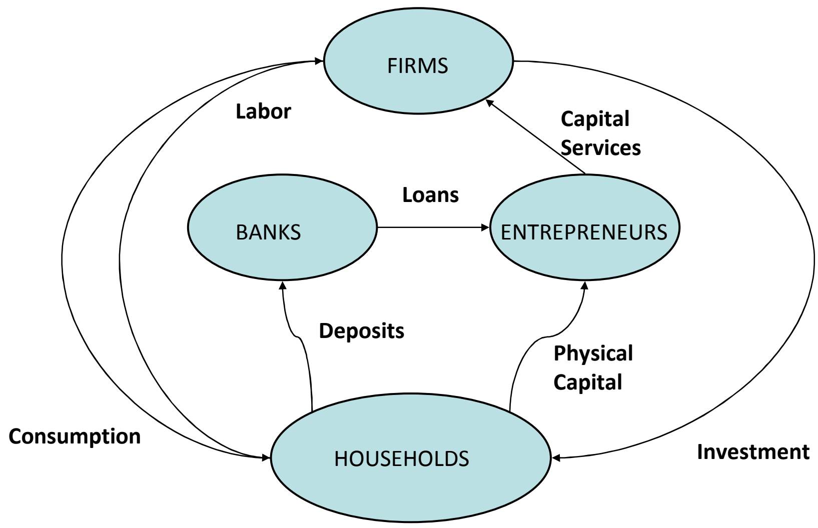
Below we present models widely used in the literature to study financial frictions that stress the importance of financial shocks. In particular, we review costly state verification models (costly monitoring), credit constraint models and moral hazard models. The above models give rise to financial frictions from first principles. Finally we review an important model that stresses financial sector shocks, where financial frictions are introduced in a reduced-form way.

2.1.1 Costly State Verification Models

We introduce the type of agency problems proposed by Robert Townsend (1979) and later implemented in DSGE models in the seminal work of Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1999). Other early contributions include Williamson (1987) and Carstrom and Fuerst (1997). Recent contributions include Christiano, Motto and Rostagno (2010). This type of agency problem is known as costly state verification (CSV). The borrower has a random return that is not observable to the lender unless he pays an audit cost. Townsend shows that under such a setting optimal financial contracts will involve auditing strategies for lenders. The cost of such auditing takes the form of a premium on external financing and is characterised as an agency cost that firms face due to asymmetric information. This agency cost is inversely related to a firms net worth. Thus, for firms with low levels of internal funds, the auditing required by lenders creates a high agency cost making investment projects for such firms unprofitable. This illustrates how the level of investment may depend on the firm's internal sources of finance. The CSV framework allows us to motivate why uncollaterized external finance may be more expensive than internal finance.

Such an agency problem can be introduced in a general equilibrium setup by assuming that the allocation of capital requires a talent that only a special type of agent possesses entrepreneurs. As illustrated in Figure 2.2, the only change from the simple neoclassical model is in the transmission of capital services. Instead of households renting capital services directly to firms, entrepreneurs purchase capital from households, which they subsequently rent to firms. Their investment in capital is characterized by substantial idiosyncratic uncertainty. To finance their capital purchases entrepreneurs combine their own net worth with loans from households that are intermediated through banks.

Figure 2.2
CSV model



To motivate this setup in practice, we can think that households supply raw physical capital, such as land and construction materials, and the entrepreneur transforming that into effective capital such as an office building. In the process of building the office block it may be discovered that the ground of the building may be softer than expected raising the cost of construction. This is a type of idiosyncratic uncertainty in the allocation of capital that is important in practice. Similarly, raw physical capital, such as plastic, glass and computer chips can be allocated into effective physical capital, such as iPods, iPhones and iPads. This allocation of capital can result in a successful product or a market failure. Steve Jobs experienced tremendous success in allocating capital in iPods, iPhones and iPads, but experienced a commercial failure when he allocated capital to the NeXT computer.[12] In the above examples,

[12] See Hammer (2011).

entrepreneurs take the role of firms in the non financial business sector. However, it is possible to interpret them as financial firms that are risky because they hold a non-diversified portfolio of loans to risky non-financial businesses.[13]

The uncertainty experienced by entrepreneurs is modeled by assuming that if an entrepreneur purchases K_{t+1} units of physical capital in period *t*, this transforms into ωK_{t+1} units of effective capital. Here ω is a random variable drawn independently by each entrepreneur, normalized to have unity mean. Entrepreneurs that draw larger than unity ω experience success (iPod, iPhone and iPad), while entrepreneurs that draw close to zero ω experience failure (NeXT computer).

Entrepreneurs purchase capital so they can profit out of their risky investment. Denote the return of effective capital in period *t* by R_{t+1}^{k}. The entrepreneur's return comes from the rental rate he earns on his capital, $r_{t+1}^{k}P_{t+1}$, and any changes in the price of capital, $Q_{K,t+1}$, minus depreciation at a rate δ. For example, an entrepreneur that has invested in an office building, gains from renting the offices to firms as well as from changes in the price of the building minus depreciation. Thus, the gross rate of return on his investment is ωR_{t+1}^{k} , where:

$$R_{t+1}^{k} = \frac{r_{t+1}^{k}P_{t+1} + (1-\delta)Q_{K,t+1}}{Q_{K,t}}$$

The gross return on his investment is therefore $\omega R_{t+1}^{k}K_{t+1}$. Part of the uncertainty in his investment is aggregate, and is reflected in changes in the economy's rental rates and capital prices, and part of it is idiosyncratic.

The entrepreneur purchases capital, $Q_{K,t}K_{t+1}$, using his own net worth, N_{t+1}, and a loan, $L_{t+1} = Q_{K,t}K_{t+1} - N_{t+1}$. It is assumed that his net worth is always less than the amount of assets he is willing to buy and therefore always needs a loan. In the market for loans, the entrepreneur interacts with banks in competitive markets, where standard debt contracts are traded. The flow of funds in financial markets is as

[13] As argued by Christiano and Ikeda (2011), real world banks resemble entrepreneurs in the BGG model. The entrepreneurs in the model have their own net worth, they accept loans (i.e. they take deposits) and they acquire assets.

follows: at the end of production in period *t*, households deposit funds with banks, that offer a deposit rate, R_t^D, and each bank extends loans to entrepreneurs.[14] The realization of ω is not known at the time the entrepreneur receives financing. Depending on the realization of ω, entrepreneurs may be able or not to pay back their loan. When ω is realized, its value is observed by the entrepreneur, but can be observed by the supplier of finance only by undertaking costly monitoring: lenders must pay a fixed auditing cost in order to observe a borrower's realized return.

Under this CSV setup, it has been shown to be optimal for the lender to offer the borrower a standard debt contract.[15] The standard debt contract specifies a loan amount and an interest rate. If the borrower pays the interest rate there is no monitoring. If the borrower declares he or she cannot pay the interest then he is monitored and the bank takes whatever the borrower has. The optimal contract between banks and entrepreneurs implies that the interest on entrepreneurial loans includes a premium over the cost of lending for banks, i.e. the interest rate on deposits, to cover the costs of entrepreneurs that experience low realizations of ω.

The entrepreneur chooses optimally his capital purchases when the return to capital is equated to the marginal cost of external finance:[16]

$$E(R_{t+1}^k) = e\left(\frac{N_t}{Q_{K,t-1}K_t}\right)R_t^D, \; e'(\cdot) < 0$$

Optimal capital purchases depend on the entrepreneurs return on these purchases as well as financial conditions. The interest rate on deposits is the cost of internal finance. The function, *e(.)*, reflects the premium the entrepreneur has to pay over the cost of internal finance in order to have access to external financing. It is referred to as the external finance premium. As shown in the above expression, the external

[14] If we think of entrepreneurs as businesses in the financial sector, then we can think of banks as mutual funds.

[15] Under CSV, it can be shown that when borrowers and lenders are risk neutral, any efficient incentive compatible contract is a standard debt contract. A simple sharing rule contract where the borrower returns a prespecified amount of profits to the lender is not the best arrangement between them. Borrowers would have incentive to hide profits and declare low output. Lenders would have to expend considerable resources monitoring borrowers.

[16] See for example BGG (1999).

finance premium depends negatively on the share of the entrepreneur's capital investment that is financed by his own net worth. Entrepreneurial profits are used to accumulate net worth. Net worth reflects the equity stake that entrepreneurs have in their firms and depends on entrepreneurs earnings net of interest payments. It is a variable associated with the value of the stock market. Unexpected movements in asset prices which are likely to be the largest source of unexpected movements in gross returns, can have substantial effects on the entrepreneurs net worth and thus his financial position.

The CSV model has been widely used to access the implications of financial frictions for productivity and monetary shocks. In one of the first formal papers on financial intermediation, Bernanke and Gentler (1989) incorporate a costly state verification model, into an overlapping generations general equilibrium framework. [17] An adverse productive shock lowers firms cash flows and therefore their ability to finance projects internally increasing the cost of investment. Furthermore, the lower investment spending lowers economic activity and therefore cash flows, propagating the shock into the economy. This creates a financial accelerator effect where an i.i.d. shock in production creates serially correlated movements in output. Two main implications of Bernanke and Gentler (1989) are discussed by Bernanke, Gentler and Gilchrist (1996). First, the dynamics of the cycle are non-linear, that is the financial accelerator effects are stronger the deeper the economy is into a recession. Second, during a recession where agency costs increase, the share of credit towards high net worth borrowers that face lower agency costs increases. This reallocation of credit from low to high net worth borrowers during a recession is referred to by Bernanke, Gentler and Gilchrist (1996) as the 'flight to quality'.

Carlstrom and Fuerst (1997) embed the contracting problem into a real business cycle model. The model generates greater persistence than Bernanke and Gentler (1989), replicating the empirical hump-shaped output response. This behaviour arises because households delay their investment decisions until agency costs are at the lowest-a point in time several periods after the initial shock. Agency costs fall with time because the productivity shock increases the return to internal funds which

[17]Another early paper is that of Williamson (1987).

redistributes wealth from household to entrepreneurs. Both models, however, have difficulties of generating large amplifications in response to productivity shocks.

Bernanke, Gentler and Gilchrist (1999) incorporate the CSV framework in a model with nominal rigidities. They calibrate a log-linear version of their model and find that the effect of the financial accelerator increases the impact of a policy shock. They add adjustment costs in the production of capital goods and find that the financial accelerator could generate sizable amplifications of monetary policy shocks.

Christiano, Motto and Rostagno (2010) use the CSV model to stress the importance of financial shocks in explaining business cycle dynamics. In particular they assume that the standard deviation of the idiosyncratic productivity shock varies with time. In CMR (2010) the dispersion of ω is controlled by a parameter σ_t, which is assumed to be a realization of stochastic process. When σ_t is high the uncertainty of the investment is high. We refer to σ_t as the risk shock. A jump in the risk shock drives investment, consumption, the stock market and credit in the same direction. At the same time changes in σ_t account for the countercyclical nature of the external finance premium accelerating swings along the business cycle. CMR (2010) find that the sharp increase in credit spreads and the accelerated collapse in economic activity that occurred in late 2008 was largely due to increase in the risk shock at the time. The risk shock is thus closely associated with fluctuations in aggregate output (16-19%) and even more closely associated with aggregate financial variables, accounting for a large portion of fluctuations in real equity (64-80%) and very large part of the movements in credit spread (87-97%).

2.1.2 Credit Constraints Models

The above models do not contain limits on the availability of the amount of external funds needed by capital producers. The limiting factor is just the price that is increasing in the amount of the external funding needed from the lender. An alternative approach to model the financial accelerator mechanism is to incorporate a limit to the amount of available funds. It is referred to as a credit constraint. The limit depends on the value of debtor assets that can be used as collateral to secure loans. Also the ability to sell an asset quickly in case the holder suddenly needs cash, may affect the return of such an asset if it is to be held.

Kiyotaki and Moore (1997), provide a model where changes in asset prices, in their case land, are the source of changes in net worth. Their findings are consistent with the financial accelerator effect, where an initial productivity shock is reinforced due to effects on asset prices. There exist patient and impatient households. Impatient farmers have a lower subjective discount factor hence in equilibrium generates an incentive to borrow. Thus credit flows from patient to impatient households. Land is used by farmers to produce output, fruit. A fraction only is tradable and the rest is non tradable and consumed by the farmer. It is assumed that the farmer will never consume more than non tradable output. Thus, all tradable output is used for investment. Farmers technology is idiosyncratic and farmers human capital is inalienable. That is the farmer has always the freedom to withdraw his labor and in that case there would be no fruit output. There would be only land. Creditors therefore do not allow the size of the debt (gross interest) to exceed the value of the collateral. If at date *t* the farmer has land k_t, then he can borrow b_t as long as the repayment does not exceed the market value of his land:

$$R_t^b = q_{t+1} K_t$$

The farmer can expand his scale of production by investing in more land. No gatherer is credit constrained. He also produces fruit from land but with different technology, it does not require any specific skill nor do they produce any non tradable output. In equilibrium borrowing constraint binds. Kiyotaki and Moore (1997) obtain the following equation when linearizing around steady state after an unexpected productivity shock Δ:

$$1 + \frac{1}{h} \hat{K}_t = \Delta + \frac{R}{R\ -1} \hat{q}_t$$

where *h>0* is the elasticity of the residual supply of land to the farmers with respect to the user cost in the steady state.

Two components cause the change in farmer's net worth. The direct effect of productivity shock and the indirect effect of the capital gain arising from the unexpected rise in price q_t. This second effect comes from the collateral constraint which depends on the market price q_t and allows for more capital investment due to relaxation of borrowing constraint. This indirect effect is scaled up by the leverage factor $R/(R-1)$.

The rate of change in asset price is given by:

$$\hat{q}_t = \frac{1}{\eta}\frac{R-1}{R}\frac{1}{1-\frac{1}{R}\frac{\eta}{1+\eta}}\hat{K}_t$$

Combining this equation with the above gives:

$$\hat{q}_t = \frac{1}{\eta}\Delta$$

$$\hat{K}_t = \frac{1}{1+\frac{1}{\eta}}\left(1+\frac{R}{R-1}\frac{1}{\eta}\right)\Delta$$

The indirect effect of q_t at date t is of the same magnitude as the temporary productive shock. The multiplier in the landholding percentage change is greater than one so the effect of the productivity shock on the farmer's landholding is more than one for one. Following a temporary productivity shock that reduces net worth, being unable to borrow more, the credit constrained farmers are forced to cut back on investment in land. This hurts them in the next period, they earn less revenue, their net worth falls and again because of credit constraints, they reduce investment. The amplification effect generated in the Kiyotaki and Moore (1997) model is weak.

The model can be used to highlight the importance of shocks arising in the financial sector by assuming that the constraint factor is stochastic. It affects the borrowing capabilities of entrepreneurs. Changing the tightness of the collateral constraint has an immediate impact on leverage and also affects the price of capital. Thus, the mechanism described for the productivity shock also acts as an amplification mechanism of the financial shock. This channel explored by Jermann and Quadrini (2012). Firms ability to borrow is limited by an enforsement constraint which is subject to random disturbances. With financial shocks only, dynamics of output are quite close to the data. The model also captures dynamics of financial flows. Shocks on firms' ability to borrow play important role in generating business cycle movement when incorporated in the Smets and Wouters (2007) model framework. Financial shocks contribute significantly to the volatility of the growth rate of output (46%), investment (25%), and labor (33%).

In a recent model Kiyotaki and Moore (2008) analyze the effects of financial sector liquidity shocks on aggregate production in an economy where firms are involved in both production and financial intermediation. Their model consists of two kinds of agents, entrepreneurs and workers, homogeneous general output and three assets: fiat money, physical capital, and human capital. Supply of fiat money fixed. Supply of capital changes through investment i_t and depreciation δ_t. Human capital is inalienable, which means he or she cannot borrow against future labor income: in any period the only labor market is the spot market for that period's labor services. There is a commonly available technology for combining labor with capital to produce general output. In each period a fraction of entrepreneurs can invest in producing new capital from general output. Because not all entrepreneurs can invest in each period there is a need to transfer resources from period's savers to period's investors. To acquire general output as input for the production of new capital, investing entrepreneurs sell equity claims to the future returns of newly produced capital. Investing entrepreneur can only issue new equity up to a θ fraction of his investment. That is he faces a borrowing constraint. Because of the borrowing constraint, investing entrepreneur needs to finance the investment partly be selling his holding of money and equity of the other agents which he has acquired in the past. Existing equity cannot be sold as quickly as money. Only a fraction ϕ of his equity can be sold in any given period. The model has two constraints, the borrowing constraint θ and the resaleability constraint ϕ. Both constraints together are referred as liquidity constraints. These constraints inhibit the efficient transfer of resources from savers to investors. The entrepreneur has three kinds of assets in his portfolio: money, equity of the other entrepreneur and unmortgaged capital stock (own capital stock – own equity issued). As it is difficult to analyze the aggregate fluctuations in an economy with three assets, Kiyotaki and Moore (2008) make the simplifying assumption that at every period the entrepreneur can remortgage up to a fraction of ϕ_t of his unmortgaged capital stock. Then, equity of entrepreneurs and unmortgaged capital stock become perfect substitutes as means of saving; they have the same return and the holder can sell up to a fraction ϕ of both. Equity of others is referred to as "outside equity" and unmorgaged capital stock "inside equity". Kiyotaki and Moore (2008) refer to both together as "equity". Let n_t denote the quantity of equity and m_t the quantity of money. Liquidity constraints are expressed as:

$$n_{t+1} \geq (1-\theta)i_t + (1-\phi_t)\delta n_t$$
$$m_{t+1} \geq 0$$

Kiyotaki and Moore (2008) consider a monetary economy, defined as an economy where money holdings are strictly positive, which occurs for low values of θ and average values of ϕ. A necessary feature for a monetary economy is that investment of entrepreneurs is limited by liquidity constraints. The entrepreneur cannot raise the entire cost of investment externally, given that the borrowing constraint binds for the sale of new equity. That is, they he has to make a down payment for each unit of investment from his own internal funds. But in trying to raise funds to make his downpayment, he is constrained by how much of his equity holding can be sold in time: the resaleability constraint binds here. Money is more liquid than equity. In a monetary economy the rate of return of money is very low, less than the return on equity. However, saving entrepreneur chooses to hold money in his portfolio because in the event he has the opportunity to invest in the future he will be liquidity constrained. Gap between return on money and return on equity is a liquidity premium. In a monetary economy there is feedback from asset market to output. Liquidity shock – reduce resaleability of equity persistently – capturing an aspect of the recent financial turmoil where many assets that used to be liquid suddenly have become only partially resaleable. Liquidity shock results in shrinking the amount the investing entrepreneur can use as downpayment. Moreover anticipating a lower resaleability the equity price falls – "flight to liquidity". This raises the size of required downpayment for a unit of investment. Altogether investment suffers from the negative shock to the resaleability of equity. The feedback mechanism causes asset prices and investment to be vulnerable to liquidity shocks.

2.1.3 Moral Hazard Models

Gertler and Karadi (2011) introduce an agency problem between intermediaries and their respective depositors. The agency problem introduces endogenous constraints on intermediary leverage ratios, which have the effect of tieing overall credit flows to the equity capital of the intermediary sector. A deterioration of intermediary capital will disrupt lending and borrowing in a way that raises credit costs. Introduce disturbance to the quality of capital. With frictions in the intermediation process the shock creates a significant capital loss in the financial sector, which inturn induces tightening of credit and a significant downturn.

Households lend funds to competitive financial intermediaries / bankers. Financial intermediaries lend funds obtained from households to non-financial firms. The non-financial firms use the loans to buy capital to produce output. After the production they sell the remaining capital to the open market. It is assumed that there are no frictions in the process of non-financial firms obtaining funding from intermediaries. Frictions exist only in the process of the intermediary obtaining funding from households. These frictions however can affect the supply of funds available to non-financial firms and thus the required rate of return on capital these firms must pay.

Let N_t the net worth of the banker. D_t deposits and S_t the quantity of financial claims on non-financial firms that the banker holds and Q_t the relative price of each claim. The intermediary balance sheet is given by:

$$Q_t S_{jt} = N_{jt} + D_{jt}$$

Household deposits with the intermediary at *t*, pay the non-contingent real gross return R_{t+1} at *t+1*. Intermediary assets earn the stochastic return $R_{k,t+1}$.

Over time, the banker's equity capital evolves as the difference earnings on assets and interest payments on liabilities:

$$N_{jt+1} = R_{kt+1}Q_t S_{jt} - R_{t+1}D_{jt} = (R_{kt+1} - R_{t+1})Q_t S_{jt} + R_{t+1}N_{jt}$$

Let $\beta\Lambda_{t.t+i}$ be the stochastic discount the banker at *t* applies at *t+i*. Since the banker will not fund assets with a discounted return less than the discounted cost of borrowing, for the intermediary to operate the following inequality must apply:

$$E_t \beta\Lambda_{t,t+i}(R_{kt+1+i} - R_{t+1+i}) \geq 0 \,, \forall\, i \geq 0$$

With perfect capital markets, the relation always holds with equality. The risk-adjusted premium is zero. With imperfect capital markets, however, the premium may be positive due to limits on the intermediary's ability to obtain funds.

Gertler and Karadi (2011) introduce the following moral hazard/costly enforcement problem. At the beginning of the period the banker can choose a fraction λ of available funds from the project and transfer them back to the household of which he is a member. The cost of the banker is that the depositors can force the intermediary into bankruptcy and recover the remaining fraction $1-\lambda$ of assets. For the lenders to be willing to supply funds to the banker the following incentive constraint must be satisfied:

$$V_{jt} \geq \lambda Q_t S_{jt}$$

The left side is what banker would lose by diverting a fraction of assets.

$$V_{jt} = \max_{S_{jt}} E_t \sum_i (1-\theta)\theta^i \beta^i \Lambda_{t,t+i}(N_{jt+1+i}) =$$
$$\max_{S_{jt}} E_t \sum_i (1-\theta)\theta^i \beta^i \Lambda_{t,t+i}[(R_{kt+1+i} - R_{t+1+i})Q_{t+i}S_{jt+i} + R_{t+1+i}N_{jt+i}]$$

where $1-\theta$ is the probability that banker exits next period.

V_{jt} can be expressed as:

$$V_{jt} = v_t Q_t S_{jt} + n_t N_{jt}$$

where

$$v_t = E_t\left[(1-\theta)\beta\Lambda_{t,t+1}(R_{kt+1} - R_{t+1}) + \beta\Lambda_{t,t+1}\theta\frac{Q_{t+1}S_{jt+1}}{Q_t S_{jt}}v_{t+1}\right]$$

and

$$n_t = E_t\left[(1-\theta)\beta\Lambda_{t,t+1}R_{t+1} + \beta\Lambda_{t,t+1}\theta\frac{N_{jt+1}}{N_{jt}}n_{t+1}\right]$$

The variable v_t has the interpretation of the expected discounted marginal gain to the banker of expanding assets $Q_t S_{jt}$ by a unit, holding net worth constant, and while n_t is the expected discounted value of having of having another unit of N_{jt} holding S_{jt} constant. With frictionless competitive markets, intermediaries will expand borrowing to the point where rates of returns will adjust to ensure v_t is zero. The agency problem introduced, however, may place limits on the arbitrage. When incentive constraint is binding, the intermediary's balance sheet is constrained by its equity capital. The incentive constraint can be expressed in the following way:

$$v_t Q_t S_{jt} + n_t N_{jt} \geq \lambda Q_t S_{jt}$$

If this constraint binds, then the assets the banker can acquire will depend positively on his/her equity capital:

$$Q_t S_{jt} = \frac{n_t}{\lambda - v_t}N_{jt} = \varphi_t N_{jt}$$

φ_t is the ratio of privately intermediated assets to equity, which is referred as the leverage ratio. The constraint limits the leverage ratio to the point where bankers incentive to divert funds is exactly balanced by the cost. The agency problem leads to an endogenous capital constraint on intermediaries ability to acquire assets. A shock that results in the decline in the quality of capital produces a magnified decline in the intermediary's capital. Interest rate spreads increase and output falls.

Gertler and Kiyotaki (2011) extend the model by including an interbank market which is subject to liquidity shocks. To the extend that the agency problem that limits the intermediaries ability to obtain funds from depositors also limits its ability to obtain funds from other financial institutions and to the extent that non financial firms can obtain funds only from a limited set of financial intermediaries, disruptions of inter-bank markets can affect real activity. Intermediaries with deficit funds offer higher rates to non-financial firms than intermediaries with surplus funds. In a crisis, this gap widens. As Gertler and Kiyotaki (2011) show, the inefficient allocation of funds across intermediaries can further depress aggregate activity.

2.1.4 Reduced Form Models

Another recent model, stressing financial factors for the business cycle, is that of Curdia and Woodford (2009a). Financial intermediation exists between households with different discount rates. Half of the households are lenders and half are borrowers. Borrowers have higher marginal utility of consumption than lenders resulting in different interest rates.

They include reduced form frictions. They simply posit an intermediation technology, rather than seeking to provide behavioural justification for the spread. They assume that a spread exists between deposit rates and lending rates. The spread exists because resources are used up in loan origination and because there is a variation of borrowers, good (will repay the loan) and bad (wont repay), and intermediaries can't distinguish between them and thus have to charge the same higher interest rate to both. The above is summarised in the following expressions:

$$1+i_t^b = (1+i_t^d)(1+\omega_t)$$

$$1+\omega_t = \mu_t^b(\mathrm{L}_t)(1+\Xi_t'(\mathrm{L}_t))$$

ω_t is the spread between deposit rates and lending rates. μ_t is a positive time-varying markup in the intermediary sector, assumed to vary as a consequence of total variation in the total volume of lending, or for exogenous reasons. Ξ represents a real resource cost of loan origination and monitoring. $\Xi' \geq 0$. Curdia and Woodford (2009a) use their model to investigate how monetary policy should respond to financial disruptions.

2.2 DSGE Models with Sovereign Risk

In DSGE models sovereign risk is introduced by allowing for the possibility of sovereign default. Following Eaton and Gergovitz (1981), a number of authors including Arellano (2008) and Mendoza and Yue (2012) have modelled sovereign defaults as strategic decisions made by a welfare maximizing government that balances the gains from foregone debt services against the costs of exclusion from international markets. In equilibrium this implies that the probability of default increases in the level of debt. The above literature can be used to study sovereign default in emerging market economies. However, the predicted level of government debt at which the sovereign default occurs is much lower than the debt level at which the sovereign risk premia are observed in developed countries, making it difficult to use those models for policy making in developed countries. A more recent literature, Bi (2012), Bi and Traum (2012) and Bi and Leeper (2010), assumes that government defaults when it is bounded by the fiscal limit. Default occurs when outstanding government debt breaches a maximum sustainable debt-output ratio. This ratio can depend upon the state of the economy and/or stochastic fluctuations in political risk.[18]

Below we review DSGE models of strategic sovereign default as well as DSGE models fiscal limit default models. First we review strategic default models explaining why such models are not appropriate for policy making in developed countries. Then, we review the more recent fiscal limit models.

[18] Bi (2012) and Bi and Leaper (2010) have constructed models where the distribution of the fiscal limit arises endogenously from the dynamic Laffer curve, which depends on macroeconomic fundamentals.

2.2.1 Strategic Default Models

Arellano (2008) extends the approach developed by Eaton and Gersovitz (1981) and analyze how endogenous default probabilities and fluctuations in output are related. Her model accounts for the empirical regularities in emerging markets as an equilibrium outcome of the interaction between risk-neutral creditors and a risk averse borrower that has the option to default. The borrower is a benevolent government in a small open economy who trades bonds with foreign creditors. Bond contracts reflect default probabilities that are endogenous to the borrower's incentive to default. Thus the equilibrium interest rate the economy faces is linked to default. Default entails temporary exclusion from international markets and direct output costs.

The government has access to international financial markets where it can buy one period discount bonds B at price $P_b(\mathrm{B}, \mathrm{y})$. Government decides whether to repay or default to its debt. Bond price function is endogenous to the government's incentive to default and depends on the size of the bond *B* and on the aggregate shock *y* because default probabilities depend on both. A purchase of a discount bond with negative face value for *B* means that the government has entered into a contract, where it receives $-P_b B$ units of period *t* goods and promises to deliver, conditional on not declaring default , *B* units in the following period. Foreign creditors are risk neutral and lend the amount of debt demanded by the government as long as the gross return on bonds equals *1+r*. The equilibrium price accounts for the risk of default creditors face such that the price of the bond equals the risk adjusted opportunity cost:

$$P_b = \frac{1 - p(B, y)}{1 + r}$$

Probability of default is endogenous and depends on governments incentives to repay debt. Letting $v^c(y, B)$ denote the value associated with non defaulting and $v^d(y)$ is the value associated with default. When government defaults, the economy is in temporary financial autarky and income falls and equals consumption. The set of *y* for which default is optimal for a level of debt *B* is expressed as:

$$D(B) = \{y \in Y : \mathrm{v}^c(B, y) < v^d(y)\}$$

With i.d.d. shocks, Arellano shows that default incentives are stronger the lower the endowment. That is for all $y_1 < y_2$ if $y_2 \in D(B)$ then, $y_1 \in D(B)$. In low income times the asset the borrower is giving up is not very valuable and default maybe preferable in recessions. Repayment is more costly when income is low making default a more likely choice. Default happens in recessions when borrower cannot rollover the current debt. After a prolonged recession debt holding can grow so much that the economy experiences net capital outflows. These capital outflows are more costly for a risk averse borrower in times of low shocks, making default more attractive in recessions. This finding is in line with evidence of Yeyati and Panizza (2006), showing that default events coincide with large GDP drops in an event analysis for 39 developing counties covering the 1970-2005 period.

Arellano (2008) calibrates the model for Argentina to study the dynamics observed during its 2001 default episode. The calibrated model can account well for the business cycle statistics in Argentina, as it predicts Argentina's default and can match well multiple features of the data, including the high volatility of consumption relative to income, the negative correlation between output and interest rates and the negative correlation between the trade balance and output. However, the mean debt-GDP ratio implied by the model is small compared to the data.

Mendoza and Yue (2012) try to endogenize the ad-hoc output costs featured in Arellano (2008). They link endogenous default risk with private economic activity via the financing cost of working capital used to pay a subset of imported inputs. Producers of final goods require working capital financing to pay for imports of a subset of intermediate goods. Efficiency loss in final goods production that occurs when the country defaults, because the loss of access to credit for some imported inputs forces firms to substitute into other imported and domestic inputs that are imperfect substitutes. Government can divert private firms repayment when it defaults on its debt.

Firms produce output using labor, intermediate goods M_t and a time-invariant capital stock k. The mix of intermediate goods is determined by a standard CES aggregator combining domestic inputs m_t^d and imported inputs m_t^* with the latter represented by a Dixit-Stiglitz aggregator that combines a continuum of differentiated imported inputs $m_j^*, j \in [0,1]$. Imported inputs are sold in world markets at exogenous time-invariant prices p_j^*. A subset $[0,\theta]$ of the imported inputs needs to be paid in advance using working capital loans. Working capital loans, ψ_t ,are intraperiod loans repaid at the end of the period that are obtained from foreign creditors at the interest rate r_t. The pay-in-advance condition driving demand for working capital is:

$$\frac{\psi_t}{1+r_t} \geq \int_0^\theta p_j^* m_j^* dj$$

The sovereign government trades with foreign lenders one period, zero coupon discount bonds. The face value of these bonds specifies the amount to be repaid next period. The sovereign cannot commit to repay its debt. As in the Eaton and Gersovitz (1981) model, when the country defaults it does not repay at date *t* and the punishement is exclusion from the world credit market in the same period. The country re-enters the credit market with an exogenous probability, and when it does it starts with a fresh record and zero debt. They add to the Eaton and Gersovitz setup an explicit link between default risk and private financing costs. This is done by assuming that a defaulting sovereign can divert the payment of the firm's working capital loans to foreign lenders. Hence, both firms and government default together.

International foreign lenders are risk-neutral and have complete information. They invest in sovereign bonds and working capital loans. Because the sovereign government diverts the repayment of working capital loans when it defaults, foreign lenders assign the same risk of default to private working capital loans as to sovereign debt. No arbitrage between sovereign lending and working capital loans implies:

$$r_t(b_{t+1},\varepsilon_t) = \frac{1}{p_t^b(\mathrm{b}_{t+1},\varepsilon_t)} - 1 \ , \ if \ \psi_t > 0$$

Clearly, sovereign default affecting price of bonds, affect the interest on working capital loans and thus import goods and production. Final good producers cannot operate with the imported input varieties that require credit, substituting them for imperfect substitutes.

The model produces an endogenous output cost of default increasing in the state of productivity. This follows from the efficiency loss induced by the working capital channel. Therefore their model provides foundations for the ad-hoc default cost that Arallano (2008) identifies as important in order to induce default incentives that trigger default in bad states, at non-negligible debt ratios, and at realistic default frequencies. The model is broadly consistent with key stylized facts, the dynamics of macroeconomic aggregates around default events, defaults associated with deep recessions, the negative correlation between interests on sovereign debt and output and high debt-output ratios on average when default takes place. Mean debt-GDP ratios in the Mendoza and Yue (2012) model are 4 times larger than the Arellano (2008) model. However, these ratios are still small when comparing with data from developed counties.

Another model that provides intuition to the large costs imposed to the domestic economy when sovereign default occurs is that of Gennaioli et al. (2011). They build a model where the government's default decision takes into account the link between public default and domestic financial institutions. Gennaiolli et al. (2011) study the link between government default and financial fragility, motivated by the current European debt crisis and other instances where sovereign default cause large losses to the domestic banking system, as domestic banks are heavily invested in public bonds, such as the Russian default (1998) and others. In their setup, domestic banks optimally choose to hold public bonds as a way to store liquidity for financing future investments. Public bonds are useful for this purpose, because government's incentive to repay is highest when investment opportunities are profitable. Given this arrangement, government's decision to default involves a trade-off. On the one hand, default beneficially increases total domestic resources for consumption, as some public bonds are held abroad. On the other hand, a default dries up the liquidity of the domestic banks that also hold a share of public bonds, thereby reducing credit, investment and output. When financial institutions are sufficiently developed, this second effect becomes so strong that government finds it optimal to repay its debt in order to avoid inflicting losses on the domestic banking system. Strong financial institution foster private credit markets by allowing banks to expand their borrowing both domestically and abroad. Their analysis suggests that government default policies depend on the stage of development of domestic markets.

2.2.2 Fiscal Limit Models

Sovereign default has been modeled as an outcome of optimal and strategic decisions by government, which maybe a reasonable assumption for emerging economies. The predicted level of government debt at which sovereign default occurs is much lower than the debt level at which sovereign risk premia are observed in developed countries, making it difficult to use such models for policy making in developed countries. The model of Arellano (2008) predicts a mean debt-output ratio of 6%. The Mendoza and Yue (2012) model predicts a mean debt output ratio of 23%, nearly 4 times larger. However, Argentina's observed mean debt-output ratio was 35%. Argentina defaulted on $100 billion debt which represented 37% of its GDP in 2001. Belgium has AAA rating with debt-GDP ratio of over 70%. Italy's rating was reduced from AA+ to AA only when its debt-GDP ratio reached 100% (Bi (2012)).

To match sovereign risk premia observed in developed countries, Bi (2012) assumes a sovereign default setup where government defaults if its debt surpasses a level called fiscal limit. Bi (2012) allows for an endogenous fiscal limit. The distribution of fiscal limits arises endogenously from a dynamic Laffer curve, which depends on macroeconomic fundamentals. She allows for an endogenous tax policy. Government raises the tax rate when government debt rises, which affects outcome of sovereign default. She solves the model using the monotone mapping method and find that the model can produce substantial risk premia under plausible calibrations. In particular Bi (2012) assumes that every economy faces a fiscal limit, the point at which, for economic or political reasons, taxes and spending can no longer adjust to stabilize the debt. At the fiscal limit the government has no choice but to default on its outstanding debt obligations. That limit depends on the entire economic and political environment: expected fiscal policy behaviour, the distribution of exogenous disturbances, and private agent's behaviour. Government finances lump-sum transfers to household, z_t, and exogenous unproductive purchases, g_t , by collecting tax revenue and issuing one period bonds, b_t. For each unit of bond, the government promises to pay the household one unit of consumption next period. The bond contract is not enforceable. At time *t*, government may partially default on its liability, b_{t-1} , by a fraction $\Delta_t \in [0,1]$. Post default liability follows the transition rule:

$$b_t^d = (1-\Delta_t)b_{t-1}$$

The tax is raised through a time varying tax rate, τ_t , on labor income. Government has a simple tax rule, designed to capture the tax policy in the real world as fiscal authorities tend to increase tax rates when government debt rises:

$$\tau_t - \tau = \gamma(b_t^d - b)\,,\ \gamma > 0$$

where γ is the tax adjustment parameter.

The price of bonds derived from household FOC reflects the household's expectation about the probability and magnitude of sovereign default in the next period:

$$p_t^b = \beta E_t\left((1-\Delta_{t+1})\frac{u_c(t+1)}{u_c(t)}\right)$$

Government may partially default if the debt level reaches the fiscal limit, which is constrained by the economy's dynamic Laffer curve and the countries political willingness to make fiscal adjustments. Dynamic Laffer curves, which arise endogenously from distorting taxes, constraint the government ability to service debt. If the tax rate is on the "slippery" side of the Laffer curve, then the government is unable to raise more taxes even if it is willing to do so. On the other hand, even if it is economically feasible to increase revenues, the government may be unwilling to raise rates. She treats the willingness to pay as a political decision unrelated to economic fundamentals. The distribution of the fiscal limit is a set of maximum levels of debt that government is able and willing to service, given the random disturbances hitting the economy. At each period, an effective fiscal limit is drawn from the endogenous distribution. If the level of government debt surpasses fiscal limit government partially defaults. Households are assumed to know the distribution of the fiscal limit. Using this information, they can decide the quantity of government bonds that they are willing to hold and the price at which they are willing to purchase them.

For a given state of the economy, the maximum primary fiscal surplus the government is willing to raise, $s_t^{\max}$, depends on political risk:

$$s_t^{\max} = \theta_t (T_t^{\max} - g_t - z_t)$$

where $T_t^{\max}$ denotes total tax revenue when the tax rate is at the peak of the Laffer curve and θ_t the political risk parameter.

Defining the fiscal limit as the maximum level of debt that government is willing and able to service we have:

$$B^* = E_0 \sum_{t=0}^{\infty} \Lambda_t^{\max} \theta_t (T_t^{\max} - g_t - z_t)$$

where $\Lambda_t^{\max}$ represents discount rate when the tax rate is at $\tau^{\max}$.

The latter's distribution is computed by Markov Chain Monte Carlo simulations and is approximated as a normal distribution $N(b^*, \sigma_b^2)$. Default rate depends on the distribution of the fiscal limit b_t^* in the following way:

$$\Delta_t = \begin{cases} 0 & if\ b_{t-1} < b_t^* \\ \dfrac{2\sigma_b}{b^*} & if\ b_{t-1} \geq b_t^* \end{cases}$$

The more dispersed the distribution, the higher the default rate becomes, because a more dispersed distribution implies greater uncertainty over the government's ability and willingness to service its debt. If the level of debt equals the mean of the distribution b^*, then government would default enough to bring the level below the lower boundary of the distribution, defined as two standard deviations from the mean, $b^* - 2\sigma_b$.

The predicted distributions of fiscal limits for developed countries are consistent with sovereign downgrades in the data. Higher political risk may reduce the fiscal limit. The default risk premium, reflecting the probability of sovereign default, is a non linear function of the level of government debt.[19]

In another model, Bi and Traum (2012) assume a purely stochastic fiscal limit, reflecting political risk, where its distribution follows the logistical cumulative density function. They use Bayesian methods to estimate probability of sovereign default for Greece and Italy. They find substantial differences in the probability of default across the two countries. For a given level of debt, historically Greece had lower probabilities of default than Italy. The level of debt associated with a given level of default depends on agents expectations about the size of a default. Greece had a small positive probability of default when it joined the EMU in 2001, but fell quickly and remained close to zero until 2009, when it began to rise sharply to the range of 6%-16% by the fourth quarter of 2010. Nevertheless, the surge in real interest rate in Greece in 2011 is generally outside of forecast confidence bands of their rational expectations model. Having estimated the parameters of the model they compute the dynamic Laffer curves for the two countries and calculate the pure economic fiscal limit (with no political risk involved – the maximum level of debt that the government is able to service). Comparing the difference between the estimated fiscal limit distributions and the pure fiscal limit, they find that the Italian government appears to be more willing to service its debt than the Greek government.

[19] Non linearity between sovereign risk premia and government indebtedness has been widely identified in the literature (Alesina et al. (1992)).

Corsetti et al. (2013) develop a NK DSGE model that includes a stochastic fiscal limit, to quantitatively access the effect of fiscal tightening in an environment where sovereign market strains tend to spillover into private credit markets. Through this sovereign risk channel, higher public indebtedness may adversely affect economic activity by raising private sector financing costs. Their model builds on the NK DSGE model with financial frictions of Curdia and Woodford (2009a) with two critical innovations. First, they allow for sovereign risk premia that respond to changes in the fiscal outlook of the country, based on a stochastic fiscal limit concept. Second, private credit spreads rise with sovereign risk because strained public finances raise the cost of financial intermediation. As in Curdia and Woodford (2009a), they postulate the following relationship:

$$(1+i_t^b)=(1+i_t^d)(1+\omega_t)$$

Further, they assume that ω_t depends on sovereign risk. In that way they model the dependence of bank spreads on sovereign risk. This dependence is set ad-hoc and is not an outcome of a particular government policy such as the bailout of the financial sector. Further, although the model captures the transmition channel working from sovereign risk to bank spreads, the opposite channel is not considered. Thus, the interdependence between the two is not explored.

Corsetti et al. (2013) use their model to access fiscal policy in the presence of sovereign risk. They find that the sovereign risk channel amplifies the transmition of shocks to aggregate demand, unless monetary policy manages to offset the spillover from sovereign default risk to private funding costs. Offsetting higher sovereign risk premia would typically require cuts in the policy rate, however their model assumes that monetary policy is constrained by the zero lower bound. When monetary policy is constrained, sovereign risk may give rise to indeterminacy. Expectations may become self-fulfilling, especially when sovereign risk is very high. Fiscal tightening in response to a cyclical fall in tax revenue can help ensure determinacy. Under determinacy they find that the government spending multiplier depends critically on the state of the economy. When central bank is constrained, the sovereign risk channel typically reduces the spending multiplier.

3. Model

In this section we describe the structure of our model. As already mentioned we follow a NK DSGE model based on CEE (2005), incorporating sticky wages and prices, adjustment costs in investment, variable capital utilization and habit persistence in preferences. In this way we introduce some features in our model that are prevalent in conventional quantitative macro models, in order to get a rough sense of the importance of the financial and sovereign debt factors we introduce. We incorporate financial frictions as modeled in BGG (1999). We add the possibility of sovereign default, using a stochastic fiscal limit in the spirit of Bi and Traum (2012).

The model economy is composed of households, firms, banks, entrepreneurs, capital producers, a government and a monetary authority. Households consume, hold bank deposits and supply a differentiated labor input to firms. Banks operate in a perfectly competitive environment. They use household deposits and injected bank rescue funds from the government to fund loans to entrepreneurs and government. Entrepreneurs use their net worth plus bank loans to purchase new installed physical capital. They rent capital services in competitive markets to firms. Agency costs introduce frictions into the entrepreneur-bank relationship. Capital producers combine investment goods with used, depreciated capital, purchased by entrepreneurs at the end of the period, to produce new capital. They face adjustment costs in production. Firms operate in a standard Dixit-Stiglitz framework for final good production. Final good firms are perfectly competitive and use intermediate goods to produce final output. Final output is converted to consumption goods and investment goods. Intermediate firms are monopolists that use labor and capital to produce intermediate goods. They face Calvo type frictions in setting prices. Government issues debt to finance government expenditures. These expenditures include funds for the costs of bank rescues. Government can default and not repay its debt. Monetary authority conducts monetary policy according to a standard Taylor rule, reacting on inflation and output changes.

3.1 FIRMS

There is a competitive, representative final good firm that produces a homogeneous final good, using as inputs a continuum of intermediate goods, according to the technology:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\left(1/\lambda_f\right)} dj \right]^{\lambda_f}, \quad 1 \le \lambda_f < \infty \tag{3.1.1}$$

where Y_t denotes the homogeneous final good, produced by the final good firm. $Y_{j,t}$ denotes an intermediate good, used as an input by the final good firm. It is produced by a continuum of intermediate good firms, indexed by $j \in [0,1]$. λ_f relates to the degree of substitutability between intermediate goods in the production of the final good. A large value of λ_f means that intermediate goods are poor substitutes and consequently the monopoly supplier of intermediate good j has a lot of market power. We assume that final output can be converted into consumption and investment goods one-for-one. The final good firm being competitive takes the price of final good, P_t, and the prices of intermediate goods, $P_{j,t}$, as given. It chooses the amount of inputs that maximize profits. The firm's problem can be written as:

$$\max_{Y_{j,t}} \; P_t Y_t - \int_0^1 P_{j,t} Y_{j,t} dj \tag{3.1.2}$$

s.t. (3.1.1)

The firm's FOC gives us the demand for intermediate good j:

$$Y_{j,t} = \left(\frac{P_t}{P_{j,t}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} Y_t \tag{3.1.3}$$

Furthermore, (3.1.3) implies the following relationship between the prices of intermediate goods and the price of the final good:

$$P_t = \left[\int_0^1 P_{j,t}^{\left(\frac{1}{1-\lambda_f}\right)} \right]^{(1-\lambda_f)} \tag{3.1.4}$$

The j^{th} intermediate good firm, chooses its price subject to Calvo type price setting frictions and is required to satisfy whatever demand materializes as its posted price. Given quantity demanded, the intermediate good firm chooses inputs to minimize costs, subject to the production technology constraint:

$$Y_{j,t} = \begin{cases} K_{j,t}^{\ a}(z_t l_{j,t})^{1-a} - \Phi z_t, & if\ K_{j,t}^{\ a}(z_t l_{j,t})^{1-a} \geq \Phi z_t \\ 0 \qquad , & otherwise \end{cases}, \quad 0 < a < 1 \tag{3.1.5}$$

where $K_{j,t}$ denotes the period t capital services used in the production of the jth intermediate good. $l_{j,t}$ denotes the period t homogeneous labor input. z_t represents the trend growth in technology and has the following time series representation: $z_t = \mu_z z_{t-1}$, $\mu_z \in (0, \infty)$. Φz_t denote fixed costs of production. Fixed costs are used to rule out entry and exit into the production of the intermediate good *j*. In order to ensure this, fixed costs must be specified in such a way that intermediate good production profits are zero in steady state. Thus, fixed costs grow at the same rate as the growth rate of output as specified in the model.

Intermediate good firms rent capital and labor services in perfectly competitive factor markets. Therefore they take the price of capital $P_t r_t^k$ and labor services W_t as given. Note that the above prices are expressed in nominal values. The intermediate good firm's cost minimization problem is given by:

$$S_t(Y) = \min_{K_{j,t}, l_{j,t}} \left\{ P_t r_t^k K_{j,t} + W_t l_{j,t} \,\middle|\, Y_{j,t}\ given\ by\ (3.1.5) \right\} \tag{3.1.6}$$

And, its minimized real marginal cost turns out to be:

$$s_t = \left(\frac{1}{1-a}\right)^{1-a}\left(\frac{1}{a}\right)^{a}\frac{\left(r_t^k\right)^{a}\left(W_t / P_t\right)^{1-a}}{z_t^{1-\alpha}} \tag{3.1.7}$$

Further, the firm's cost minimization problem implies that the following efficiency condition, stating that its real marginal cost must be equal to the cost of renting one unit of capital, divided by the marginal productivity of capital, must hold:

$$s_t = \frac{r_t^k}{\alpha\left(\frac{z_t l_t}{K_t}\right)^{1-\alpha}} \tag{3.1.8}$$

where $K_t = \int_0^1 K_{j,t} dj \;\; and \;\; l_t = \int_0^1 l_{j,t} dj$

Once the firm has solved its cost minimization problem, it chooses its output price so as to maximize profits subject to the demand it faces and its price setting frictions constraint. According to the latter, firms set prices a la Calvo, where a fraction $1-\xi_p$ of intermediate good firms can reoptimize their price in period *t* and a fraction ξ_p sets price as:

$$P_{j,t} = \tilde{\pi}_t P_{j,t-1} \tag{3.1.9}$$

We define the inflation rate as $\pi_t = P_t / P_{t-1}$ and assume that $\tilde{\pi}_t = \pi_{t-1}$.

Now consider firms that can reoptimize in period *t*. These firms take into account the possibility that they might be stuck with this price for several periods in the future and thus solve a dynamic problem. All adjusting firms in period *t* solve the same problem and hence, have the same solution. Therefore we can let $P_{j,t} = \tilde{P}_t$ for all firms that reoptimize. Then, the problem for the *t*-period reoptimizing firm maybe stated as follows:

$$\max_{\tilde{P}_t} E_t \sum_{i=0}^{\infty} (\beta\xi_p)^i \lambda_{t+i} [\tilde{P}_{t+i} Y_{j,t+i} - P_{t+i} s_{t+i} Y_{j,t+i}] \tag{3.1.10}$$

where λ_t is the Lagrange multiplier associated with the household's budget constraint. Since households are the owners of intermediate good firms it is natural that the firms should weight profits according to $\beta^i \lambda_{t+i}$.

Note that real marginal cost, s_t, has already been determined and therefore it is taken as given, in solving the above problem. As it turns out, the FOC associated with this problem is given by:

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t} = \frac{E_t \sum_{i=0}^{\infty} (\beta\xi_p)^i A_{t+i} \lambda_f s_{t+i}}{E_t \sum_{i=0}^{\infty} (\beta\xi_p)^i A_{t+i} X_{t,i}} \equiv \frac{J_{p,t}}{F_{p,t}} \tag{3.1.11}$$

where $A_{t+i} = P_{t+i} \lambda_{t+i} Y_{t+i} X_{t,i}^{(-\frac{\lambda_f}{\lambda_f - 1})}$ and $X_{t,i} = \begin{cases} \dfrac{\tilde{\pi}_{t+i}\tilde{\pi}_{t+i-1}.......\tilde{\pi}_{t+1}}{\pi_{t+i}\pi_{t+i-1}.......\pi_{t+1}} & , i > 0 \\ 1 & , i = 0 \end{cases}$

Expressing $J_{p,t}$ *and* $F_{p,t}$ in recursive form, we have:

$$E_t \left[P_t \lambda_t Y_t + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \beta \xi_p F_{p,t+1} - F_{p,t} \right] = 0 \tag{3.1.12}$$

$$E_t \left[\lambda_f P_t \lambda_t Y_t s_t + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} \beta \xi_p J_{p,t+1} - J_{p,t} \right] = 0 \tag{3.1.13}$$

Turning into the aggregate price index, in view of the proceeding price adjustment structure, this index can be expressed as:

$$P_t = \left[\int_0^1 P_{j,t}^{(\frac{1}{1-\lambda_f})} \right]^{(1-\lambda_f)} = \left[(1-\xi_p)(\tilde{P}_t)^{\frac{1}{1-\lambda_f}} + \xi_p (\tilde{\pi}_t P_{t-1})^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f} \tag{3.1.14}$$

And, dividing by P_t and substituting for $\tilde{p}_t$, we get:

$$\frac{1-\xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}=\left(\frac{J_{p,t}}{F_{p,t}}\right)^{\frac{1}{1-\lambda_f}} \tag{3.1.15}$$

Conditions (3.1.12), (3.1.13) and (3.1.15) fully describe the transition of the average price level.

3.2 Capital Producers

There is a large fixed number of identical competitive capital producers. They are owned by households. The latter receive any profits or losses in terms of lump sum transfers. Capital producers purchase used (depreciated) capital and investment goods to produce new capital according to the following technology:

$$\bar{K}_{t+1}=(1-\delta)\bar{K}_t+(1-v(I_t/I_{t-1}))I_t \tag{3.2.1}$$

where I_t denotes investment in physical capital in period *t* and $\bar{K}_t$ denotes physical capital stock at the beginning of period *t*. As we shall explain in the next section, we make a distinction between capital stock and capital services due to the inclusion of variable capital utilization. The above technology includes investment adjustment costs, $v_t \equiv v(I_t/I_{t-1})$ taken to be a strictly increasing, strictly convex function, such that:

$$v_t=\exp\left[\sqrt{\frac{1}{2}v''}\left(\frac{I_t}{I_{t-1}}-\frac{I}{I_{-1}}\right)\right]+\exp\left[-\sqrt{\frac{1}{2}v''}\left(\frac{I_t}{I_{t-1}}-\frac{I}{I_{-1}}\right)\right]-2 \tag{3.2.2}$$

where I/I_{-1} denotes the steady state growth rate of investment and v'' is a parameter whose value is the second derivative of v_t, evaluated at the steady state I/I_{-1}.

The introduction of adjustment costs introduces curvature in the tradeoff between consumption and additional capital and therefore the price of capital deviates from unity. Investment goods are purchased in the goods market at price P_t. The price of depreciated capital is denoted by $Q_{K,t}$. In addition, note that since the marginal rate of transformation between old and new capital is unity, the price of new capital is also $Q_{K,t}$. Thus, the t-period profits for the capital producer are given by:

$$\Pi_t^k = Q_{K,t}\bar{K}_{t+1} - Q_{K,t}(1-\delta)\bar{K}_t - P_t I_t \tag{3.2.3}$$

The capital producer's problem of profit maximization is dynamic, due the presence of adjustment costs and can be written as:

$$\max_{I_t} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \lambda_{t+i} \Pi_{t+i}^k \right\} \tag{3.2.4}$$

where λ_t is the multiplier on the households budget constraint as in the case of intermediate good firms.

The FOC w.r.t. investment, linking the price of capital to the price of investment goods, is given by:

$$E_t \left[\lambda_t Q_t \left(1 - v_t - v_t' \frac{I_t}{I_{t-1}} \right) - \lambda_t P_t + \beta \lambda_{t+1} Q_{t+1} v_{t+1}' \left(\frac{I_t}{I_{t-1}} \right)^2 \right] = 0 \tag{3.2.5}$$

where v_t' stands for the first derivative of v_t.

3.3 Entrepreneurs

There is a large number of entrepreneurs. An entrepreneur's state at the beginning of period t+1 is determined by its level of net worth, N_{t+1}. Entrepreneurs combine their net worth and a bank loan to buy new installed physical capital, $\bar{K}_{t+1}$, from capital producers. Then, they experience an idiosyncratic shock ω that transforms $\bar{K}_{t+1}$ into

$\omega\bar{K}_{t+1}$ where ω is a log-normally distributed random variable, across all entrepreneurs, with cumulative distribution function $F_t(\omega)$. The mean and variance of $\log\omega$ are μ and σ_t^2 respectively. Moreover, μ is set so that $E(\omega)=1$, when σ_t^2 takes on its steady state value. Stochastic variation in σ_t captures the notion that riskiness of entrepreneurs varies over time and the innovations of the σ_t process are referred to as a risk shock. These assumptions about ω imply that entrepreneur's investment in capital is risky. ω is observed by the entrepreneur but can only be observed by the bank if the bank pays a monitoring cost.

The entrepreneur rents out capital he buys, from capital producers, to firms. Variable capital utilization is introduced in the entrepreneur's decision on renting out capital. By introducing capital utilization, we have a distinction between capital services and capital stock. The relationship between the utilization of capital, capital services and physical capital stock is: $K = u\bar{K}$. Therefore, at the beginning of period t+1 entrepreneurs will rent out capital services $K_{t+1} = u_{t+1}\omega\bar{K}_{t+1}$. Entrepreneurs choose their capital utilization rate in any period *t+1*, u_{t+1}, so as to maximize profits:

$$[u_{t+1}r_{t+1}^k - a(u_{t+1}-1)]\omega\bar{K}_{t+1}P_{t+1} \tag{3.3.1}$$

where $a(u_{t+1}-1)\omega\bar{K}_{t+1}$ is the adjustment cost from deviating from the "natural" utilization rate of 100%, defined as:

$$a(u_t-1) = \frac{1}{2}\sigma_a(u_{t+1}-1)^2 + r^k(u_{t+1}-1); \quad \sigma_a \geq 2r^k \tag{3.3.2}$$

where, r^k stands for the steady state level of r_t^k. Clearly, this specification implies that capital utilization costs are positive and rise at the margin with any deviation of u_{t+1} from 1.

The introduction of variable capital utilization is motivated by a desire to explain the slow response of inflation to a monetary policy shock. In standard models, prices are heavily influenced by costs and these in turn are influenced by the elasticity of factors of production. If factors of production can be expanded with a small rise in cost, then inflation will not rise so much after a positive monetary policy shock. Allowing for variable capital utilization is a way to make capital services more elastic.

The FOC associated with the entrepreneur's capital utilization decision problem is given by:

$$r_t^k - a'(u_t) = 0 \tag{3.3.3}$$

At the end of each period, entrepreneurs sell the undepreciated fraction $1-\delta$ of their capital to capital producers at price $Q_{K,t+1}$. Entrepreneurs' earnings therefore, come from renting out capital and from changes in the value of capital they have purchased. Total earnings in period *t+1* in currency units received by an entrepreneur with idiosyncratic shock ω are:

$$\left\{[(u_{t+1}r_{t+1}^k - a(u_{t+1})]P_{t+1} + (1-\delta)Q_{\bar{K},t+1}\right\} \omega\bar{K}_{t+1} \tag{3.3.4}$$

The entrepreneur's payoff from buying capital can be written as:

$$(1+R_{t+1}^k)Q_{K,t}\omega_{t+1}\bar{K}_{t+1} \tag{3.3.5}$$

where, $$1+R_{t+1}^k \equiv \frac{(u_{t+1}r_{t+1}^k - a(u_{t+1}))P_{t+1} + (1-\delta)Q_{K,t+1}}{Q_{K,t}} \tag{3.3.6}$$

is the average rate of return on capital.

We assume that entrepreneurs have net worth $N_{t+1} < Q_{K,t}\bar{K}_{t+1}$ and therefore, need a loan to finance capital purchases equal to:

$$L_{t+1} = Q_{K,t}\bar{K}_{t+1} - N_{t+1} \tag{3.3.7}$$

Depending on the realization of ω, entrepreneurs may or may not be able to pay back their loan. Further, as already mentioned, we assume that lenders must pay a fixed auditing cost in order to observe a borrower's realized return. This setup, first analyzed by Townsend (1979), is known as costly state verification. Under this setup, it has been shown to be optimal for the lender to offer the borrower a standard debt contract.[20] The standard debt contract specifies a loan amount and an interest rate. If the borrower pays the interest rate there is no monitoring. If the borrower declares he or she cannot pay the interest then he or she is monitored and the bank takes whatever the borrower has.

We define the endogenously determined cutoff point $\bar{\omega}_{t+1}$, for which entrepreneurs can just pay back their loan, as:

$$\bar{\omega}_{t+1}(1+R_{t+1}^{k})Q_{K,t}\bar{K}_{t+1} = (1+R_{t+1}^{L})L_{t+1} \tag{3.3.8}$$

Entrepreneurs with ω above $\bar{\omega}$ pay interest R_{t+1}^{L} on their bank loan. Entrepreneurs with $\omega < \bar{\omega}$ cannot pay this interest and must give everything they have to the bank. Bankrupt entrepreneurs are monitored by the bank with a monitoring cost μ.

The funds loaned to entrepreneurs by the bank in period *t* are obtained by banks from households. We suppose that banks can secure these funds by accepting deposits that pay a nominal interest rate R_{t+1}^{D}. Following CMR (2010), it is assumed that this interest rate is not state contingent on the realization of *t+1* shocks. This assumption allows the model to inorporate Fisher's (1933) "debt-deflation" hypothesis. An unexpected change in the price level during the period of a loan contract, reallocates wealth between entrepreneur and lenders.

Banks operate in perfectly competitive credit markets and therefore have zero profits from intermediating funds between households and entrepreneurs. The bank's zero profit condition is given by:

$$[1-F_t(\bar{\omega}_{t+1})](1+R_{t+1}^{L})L_{t+1} + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega\, dF_t(\omega)(1+R_{t+1}^{k})Q_{K,t}\bar{K}_{t+1} = (1+R_{t+1}^{D})L_{t+1} \tag{3.3.9}$$

The first term on the LHS of the above expression is the number of non bankrupt entrepreneurs times the interest and principal paid by each one. The second term on the LHS corresponds to the funds received by the bank from bankrupt entrepreneurs

[20] Under a costly state verification framework, it can be shown that when borrowers and lenders are risk neutral, any efficient incentive compatible contract is a standard debt contract.

net of monitoring costs. The term on the RHS is the quantity of funds the bank must pay to households.

The entrepreneur's expected payoff is expressed as:

$$\int_{\bar{\omega}_{t+1}}^{\infty} [\omega(1+R^k_{t+1})Q_{K,t}\bar{K}_{t+1} - (1+R^L_{t+1})L_{t+1}]dF_t(\omega) \tag{3.3.10}$$

The entrepreneur chooses the parameters of the standard debt contract, that is the loan amount and the interest rate, so as to maximize his expected payoff, subject to the bank's zero-profit condition. In view of the definition of the cutoff point $\bar{\omega}_{t+1}$, and the entrepreneur's balance sheet, both the loan amount and interest rate can be substituted out for $\bar{\omega}_{t+1}$ and the entrepreneur's leverage, $\frac{Q_t\bar{K}_{t+1}}{N_{t+1}}$. Then, the bank's zero profit condition can be re-stated as[21]:

$$\max E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R^k_{t+1}}{1+R^D_{t+1}}\frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}}\right\} \tag{3.3.11}$$

s.t.

$$[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})]\frac{1+R^k_{t+1}}{1+R^D_{t+1}} = \left(1-\frac{N_{t+1}}{Q_{K,t}\bar{K}_{t+1}}\right) \tag{3.3.12}$$

where, $\Gamma_t(\bar{\omega}_{t+1}) = G_t(\bar{\omega}_{t+1}) - \bar{\omega}_{t+1}[1-F_t(\bar{\omega}_{t+1})]$, $G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)$

The FOC associated with the entrepreneur's problem, then, is given by:

$$E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R^k_{t+1}}{1+R^D_{t+1}} + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})}\left[\frac{1+R^k_{t+1}}{1+R^D_{t+1}}\left(\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})\right) - 1\right]\right\} = 0 \tag{3.3.13}$$

As it turns out, under some regularity conditions the above FOC gives a unique solution to the contracting problem.[22] The above FOC implies that every entrepreneur,

[21] This transformation is done to overcome the technical problem of the non-convex representation of the entrepreneur's preferences over the *Z-L* space.
[22] Details can be found in BGG (1999).

regardless of his net worth, receives a loan contract with the same rate of interest and with a loan amount that is the same fraction of his net worth.[23]

After the entrepreneur has settled his debt with the bank in period *t+1* and the entrepreneurs capital has been sold to capital producers, the entrepreneur's *t+1* level of net worth is determined. At this point the entrepreneur exits the economy with probability $1-\gamma$ and survives for another period with probability γ. Each period new entrepreneurs enter in such a way so that the number of entrepreneurs remains constant. New entrepreneurs entering in period *t+1* receive a transfer W^e. This exit and entry process ensures that the entrepreneurs never accumulate enough net worth in order to become independent of external financing. The evolution of entrepreneurs aggregate net worth is as follows:

$$N_{t+1} = \gamma \left\{ (1+R_t^k)Q_{K,t-1}\bar{K}_t - \left[1 + R_t^D + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF_t(\omega)(1+R_t^k)Q_{K,t-1}\bar{K}_t}{Q_{K,t-1}\bar{K}_t - N_t} \right] (Q_{K,t-1}\bar{K}_t - N_t) \right\} + W^e$$

(3.3.14)

The term in brackets represents total receipts of entrepreneurs minus total payments to banks. The term in square brackets is the unit cost of borrowing where

$\dfrac{\mu \int_0^{\bar{\omega}_t} \omega dF_t(\omega)(1+R_t^k)Q_{K,t-1}\bar{K}_t}{Q_{K,t-1}\bar{K}_t - N_t}$ is the external finance premium.

3.4 Banks

There is a continuum of identical competitive banks. Banks intermediate funds between households and entrepreneurs and finance government debt. Let total loans

[23] This is very convenient, for we avoid aggregation issues over entrepreneurs with different levels of net worth.

made by the representative bank to entrepreneurs and the government in period t are denoted by L_{t+1} and B_{t+1} respectively. The representative bank finances its loans by issuing deposits, D_t. Further, the bank gets government injections Ξ_t.[24] Thus, the bank's balance sheet equation is:

$$B_{t+1} + L_{t+1} = D_t + \Xi_t \tag{3.4.1}$$

The bank's profit maximization problem is given by:

$$\begin{aligned} &\max_{D_t, B_{t+1}} R_{t+1}^B B_{t+1} + R_{t+1}^D L_{t+1} - R_{t+1}^D D_t \\ &s.\text{t}.\,(3.4.1) \end{aligned} \tag{3.4.2}$$

where R_t^B denotes the return on bonds, $R_t^D L_t$ is the bank's return from its period t loans to entrepreneurs as discussed in the previous section and R_t^D denotes the return on deposits. In view of the above, the bank's FOC implies that the return on bonds must be equal to the return on deposits:

$$R_t^B = R_t^D \tag{3.4.3}$$

3.5 Households

There is a continuum $j \in (0,1)$ of households, that consume, provide deposits and supply a differentiated labor input $h_{j,t}$. We assume that households face Calvo-type frictions in setting their wages. Since uncertainty is idiosyncratic, different households will work different amounts and earn different wage rates. So, in principle, each household should be heterogeneous with respect to consumption and asset holdings. We assume however that there exist state-contingent securities and as shown in Erceg,

[24] It should be pointed out that the particular way of treating funds devoted to the rescue of banks as cash injections in every period mitigates the effects of the risk shock compared to a setup where funds are kept as reserves (in the ECB of EFSF) for potential bank bailouts. In the latter setup these funds would not appear in the balance sheet of the banks. A more general setup would include a dynamic equation for ξ_t , where part of the rescue funds are kept as reserves and accumulate over time.

Henderson and Levin (2000), this establishes that in equilibrium households are homogeneous with respect to consumption and asset holdings.

The preferences of the j^{th} household are given by:

$$E_t \sum_{l=0}^{\infty} \beta^l \left\{ \log(C_{t+l} - b_c C_{t+l-1}) - \psi_L \frac{h_{j,t+l}^{1+\sigma_L}}{1+\sigma_L} \right\}, 0 \leq \beta \leq 1, \sigma_L \geq 0 \tag{3.5.1}$$

where C_t denotes period t consumption and $1/\sigma_L$ can be interpreted as the Frisch labor supply elasticity. For $b_c > 0$ the model allows for habit persistence in consumption. This is motivated by VAR-based evidence according to which a positive monetary policy triggers a persistent reduction in the interest rate and a hump-shaped response in consumption. For $b_c = 0$ a fall in the interest rate would cause households to rearrange consumption intertemporally, so consumption is highest immediately after the shock and lower later, by allowing however $b_c > 0$, we get the desired hump-shaped response.

The household uses its funds to acquire consumption goods $P_t C_t$. In addition, it can acquire deposits D_t which pay a return R_{t+1}^D at the end of time-$t+1$, known at time-t. Furthermore, households receive profits, Π_t, from capital producers, banks and intermediate good firms. They, also, receive an amount $A_{j,t}$ which is the net payoff on the state contingent securities the household purchases to insure against uncertainty arising from wage optimization. Finally, households pay a lump sum taxes W^e and T_t to finance the transfer payments made to the γ entrepreneurs that survive and $1-\gamma$ entrepreneurs that enter in each and every period and part of government expenditures respectively.

The household's budget constraint is therefore given by:

$$W_{j,t} h_{j,t} + (1 + R_t^D) D_{t-1} + \left[\Pi_t + A_{j,t} - W^e - T_t \right] \geq D_t + P_t C_t \tag{3.5.2}$$

Next, we assume that employment services used by intermediate good producers, l_t, are related to the differentiated labor services of households according to the technology:

$$l_t = \left[\int_0^1 (h_j)^{1/\lambda_w} dj \right]^{\lambda_w}, \ 1 \leq \lambda_w < \infty. \tag{3.5.3}$$

Thus, household j faces the following demand for its labor:

$$h_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_t \tag{3.5.4}$$

where $W_t = \left[\int_0^1 (W_{j,t})^{1/(1-\lambda_w)} dj \right]^{(1-\lambda_w)}$ is the average wage rate in period t (i.e., the wage rate associated with l_t).

Finally, we assume that households face wage setting frictions a la Calvo, whereby, a household chooses its wage optimally with probability $1-\xi_w$ and with probability ξ_w sets its wage rate according to:

$$W_{j,t} = \pi_{t-1} \mu_z W_{j,t-1} \tag{3.5.5}$$

Wage frictions help slow the response of inflation to a monetary policy shock. Sticky wages make labor supply highly elastic and thus a positive monetary shock leads to a big increase in employment and output and a small increase in cost and hence inflation.

Then, note that since each household that re-optimizes sets the same wage rate, we can denote this wage rate by $\tilde{W}_t$. The demand for labor faced by the j^{th} household and the average wage rate can be restated as:

$$h_{j,t} = \left(\frac{\tilde{W}_t}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_t \tag{3.5.6}$$

and

$$W_t = [(1-\xi_w)\tilde{W}_t^{1/(1-\lambda_w)} + \xi_w(\pi_{t-1}\mu_z W_{t-1})^{1/(1-\lambda_w)}]^{(1-\lambda_w)} \tag{3.5.7}$$

Now, the household's problem is to maximize its expected utility subject to its budget constraint, the demand for its labor and the Calvo wage-setting frictions. Let $w_t = \dfrac{W_t}{P_t z_t}$, $\pi_{w,t} = \dfrac{W_t}{W_{t-1}}$ and $\tilde{\pi}_{w,t} = \pi_{t-1}$ .The first order conditions associated with the j^{th} household's problem can be stated as follows:

(FOC w.r.t. consumption):

$$\frac{1}{C_t - b_c C_{t-1}} + b\beta E_t \frac{1}{C_{t+1} - b_c C_t} = P_t \lambda_t \tag{3.5.8}$$

(FOC w.r.t. deposits):

$$\lambda_t = \beta E_t \lambda_{t+1} R_{t+1}^D \tag{3.5.9}$$

(FOC w.r.t wage setting and employment decisions):

$$J_{w,t} = w_t F_{w,\mathrm{t}} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mu_z \right)^{1/(1-\lambda_w)}}{1-\xi_w} \right]^{1-\lambda_w(1+\sigma_L)} \tag{3.5.10}$$

$$E_t \left\{ l_t \frac{\lambda_z}{\lambda_w} + \beta \xi_w \mu_z^{\frac{1}{1-\lambda_w}-1} \left(\frac{1}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} - F_{w,t} \right\} = 0 \tag{3.5.11}$$

$$E_t \left\{ l_t^{1+\sigma_L} + \beta \xi_w \left(\mu_z \frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} J_{w,t+1} - J_{w,t} \right\} = 0 \tag{3.5.12}$$

Equations (3.5.10)-(3.5.12) completely determine the transition of wages.

3.6 Government and Sovereign Risk

Government spends resources on the acquisition of government consumption goods, $P_t G_t$, and repayment of debt services, $(1+R_t^B)B_t$. In addition, government spends resources in acquiring funds for the bailout of the banking sector, Ξ_t. Further, we assume that government obtains resources from lump sum taxes to households, T_t and debt issuance, B_{t+1}. Government's budget constraint, defining that its total resources must equal its total expenditure, is given by:

$$B_{t+1} + T_t = (1+R_t^B)B_t + P_t G_t + \Xi_t \tag{3.6.1}$$

Following Bohn (1998), we assume that the government follows the following fiscal rule for its tax policy:

$$T_t = \phi_\tau B_t \tag{3.6.2}$$

Taxes are raised in a lump sum way as a fraction of outstanding debt, ensuring a no-Ponzi scheme. To ensure a balanced growth path, we model government expenditures for the bailout of banks as increasing at the rate of technological progress. These funds are assumed to be a positive and strictly increasing function of banking risk as measured by the external finance premium:

$$\Xi_t = \xi_t P_t z_t \tag{3.6.3}$$

where $\xi_t = \xi(e_t)$, $\xi'(\cdot) > 0$ and

$$e_t = \bar{\omega}_t (1+R_t^k) \frac{Q_{K,t-1} K_t}{Q_{K,t-1} K_t - N_t} - (1+R_t^D) \tag{3.6.4}$$

The rationale behind this formulation is to capture the notion that governments face increasing burdens due to costs of financial sector rescues. Risk in the financial sector is reflected in the external finance premium entrepreneurs end up paying on loans. Further, as proved by BGG (1999), the external finance premium depends negatively on the share of the entrepreneur's capital investment that is financed by his own net worth. Therefore, our formulation of government provisions for the banking sector reflects the fact that bail-out funds rise in the presence of a weak financial sector, where entrepreneurs' net worth is low.[25]

In standard NK models, the return on government bonds, R_t^B, would be set equal to the safe rate, R_t, set by the monetary authority. In the case where there is sovereign risk, however, the return on government bonds may differ from the safe return. Consider the following setting based on the recent growing literature of fiscal limits. For each bond the government promises to pay the household/bank one unit of consumption next period. However, the bond contract is not enforceable. At each period a stochastic fiscal limit, b_t^*, is drawn from a given probability distribution function. The fiscal limit denotes a level of debt that if government surpasses will go bankrupt. We assume that this limit is determined by political negotiations. We therefore model the fiscal limit as a random variable reflecting uncertainty in political negotiations.[26] If government debt surpasses its fiscal limit the government defaults.

[25]Related literature provides support for modeling bank rescue funds as a function of the external finance premium. As shown by CMR (2010), the external finance premium is a good proxy for the risk shock: 87% of the fluctuations in the external finance premium in the EA and 97% in the US are due to fluctuations in the risk shock, when modeled with a news shock structure. As discussed by Gilchrist, Yankov and Zakrajsek (2009), research on the role of financial asset prices in cyclical fluctuations stresses the information content of credit spreads for the state of the economy. Information content of credit spreads likely reflects disruption in the supply of credit stemming from the worsening of the quality of borrowers' balance sheets and the deterioration in the soundness of financial intermediaries. Disruptions in credit markets have important consequences for macroeconomic outcomes. Note that we model financial sector rescues replicating government cash injections policies following 2008. We do not attempt to take into account macro-prudential policies for modeling bank rescue funds. This could be done by assuming rescue funds are a positive function of the risk premium as well as the gap between output and steady state output.

[26]Bi (2012) models the fiscal limit as a function of both the Laffer curve, which is endogenously determined and political uncertainty. He assumes that the government pays back its debt unless it hits the peak of the Laffer curve. In addition the government might be constrained by some political limit that is much lower than the peak of the Laffer curve. For simplicity we consider the fiscal limit as being purely stochastic. This simplification is in line with Bi and Traum (2012), where the estimate the fiscal limit for Greece. They argue that political considerations may make the government unwilling and unable to achieve the fiscal limit, due to features such as political inability to raise taxes. In the case of Greece, the protests against austerity measures suggest the relevance of such political considerations. They therefore take the fiscal limit distribution as exogenously given.

The default decision can be summarized by:

$$\delta_t = \begin{cases} 0 \; if \; b_{t-1} < b_t^* \\ 1 \; if \; b_{t-1} \geq b_t^* \end{cases}$$

where we have expressed government debt as a stationary real variable $b_t = \dfrac{B_t}{z_t P_t}$.

Note that the level of debt is endogenous in our model and is a function of bank risk. So the default decision depends upon the risk in the banking sector of the economy. The higher the bank risk the higher the debt and the higher the probability of government default.

In the above setting, the economy switches endogenously between default and non-default regimes. Thus, the model cannot be solved using a first order approximation.[27] In order to be able to solve the model using first order approximations, we focus on the case where there is no actual default in the current period. The probability of default however, might affect the value of the bond. This modification is in line with evidence reported by Yeyati and Panizza (2011), who find that the output costs of default materialize in the run-up to defaults rather than the time when the default actually takes place.

We construct the price of the bond under the above setup, in the case where there is no actual default, as follows:

First, let $s = (\mathrm{b}, d, b^*)$ denote the aggregate state at the beginning of any given period. It describes the endogenous level of debt b, the default history *d* and some exogenous variable b^*. We assume that b^* follows a Markov process and that all decisions are described in terms of the state *s*. The probability measure describing the transition from b^* to $b^{*\prime}$ shall be denoted with $\mu(db^{*\prime} \,|\, b^*)$. We assume debt is valued in financial markets where risk neutral traders discount future debt repayments at some return *R*. Given the level of debt and no past defaults, *d=0*, let $D(\mathrm{b}) = \left\{ b^* \,\middle|\, \delta(s) = 1 \; for \; s = (\mathrm{b}, 0, b^*) \right\}$ be the default set, and let

[27] Such models can be solved using the monotone mapping method, Coleman (1991) and Davig (2004).

$A(\mathrm{b}) = \left\{ b^* \middle| \delta(s) = 0 \ for \ s = (B, 0, b^*) \right\}$ be the set of all b^*, such that government will not default and instead, continue to honor its debt obligations.

Then, the market price of debt, denote by $\overline{q}$ in case of no current default is:

$$\overline{q}(\mathrm{b}'; s) = \frac{1}{R} \int_{b^{*\prime} \in A(\mathrm{b})} \mu(db^{*\prime} \,|\, b^*)$$

This follows from risk neutral discounting and the hypothesis of no default in the current period. Note that over the set *A(b)* the price of the bond is continuous. Thus, by restricting ourselves to the non default set, we can solve the model using first order approximations. Further, defining the probability of a continuation next period as $P(B'; s) = \Pr ob(b^{*\prime} \in A(B') \,|\, s)$,we can rewrite the above expression as:

$$\overline{q}(B'; s) = \frac{1}{R} P(B'; s)$$

It is clear from the above expression that sovereign bond prices are reflecting expected default risk, even when there is no current default.

In the case of sovereign risk we, therefore, have:

$$R_t^B = \frac{R_t}{P(b_{t+1})} \tag{3.6.5}$$

The return on bonds inversely depends on the probability of default of the government. Sovereign risk affects the interest rate on bonds and thus other interest rates in the economy and vice versa.

3.7 Monetary Policy

Monetary policy is set according to the Taylor rule:

$$\hat{R}_{t+1} = \rho_i \hat{R}_t + (1 - \rho_i) \frac{a_y}{4R} (\hat{y}_t - \hat{y}_{t-1}) + (1 - \rho_i) a_{d\pi} \frac{\pi}{R} (\hat{\pi}_t - \hat{\pi}_{t-1}) \tag{3.7.1}$$

where the “^” above a variable denotes its percentage deviation from its steady state value. As the monetary authority is not explicitly optimizing we do not consider any interactions between cash injections and the Taylor rule.[28]

3.8 Resource Constraint

To derive the aggregate resource constraint for this economy, note that the aggregate production function can be written as:[29]

$$Y_t = K_t^{a}(z_t l_t)^{1-a} - \Phi z_t \tag{3.8.1}$$

Market clearing for final goods implies:

$$\mu \int_0^{\bar{\omega}_t} \omega\, dF(\omega)(1+R_t^k)\frac{Q_{K,t-1}\bar{K}_t}{P_t} + a(\mathrm{u}_t)\bar{K}_t + C_t + I_t + G_t \leq Y_t \tag{3.8.2}$$

where the first term on the LHS of the above expression represents final output used up in bank monitoring and the last term on LHS represents resources used for government expenditures. Combining the above two expressions we get the aggregate resource constraint of the economy:

$$\mu \int_0^{\bar{\omega}_t} \omega\, dF(\omega)(1+R_t^k)\frac{Q_{K,t-1}\bar{K}_t}{P_t} + a(u_t)\bar{K}_t + C_t + I_t + G_t \leq K_t^{a}(z_t l_t)^{1-a} - \Phi z_t \tag{3.8.3}$$

3.9 Equilibrium Conditions

We adopt a sequence-of-markets equilibrium defined by a set of allocations and rate of returns and prices which have the property that optimality for households, good producing firms, entrepreneurs and banks are satisfied and goods and loan markets clear.

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[28] Such interactions could have been taken into account in way similar to the one suggested in Footnote 36.

[29] Where following Yun (1996), we have ignored Yun’s distortion as it does not matter for first order linear approximations.

We let the following transformations of variables:

$$w_t = \frac{W_t}{P_t z_t} \ , \ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_{t+1}} \ , \ \lambda_{z,t} = \lambda_t P_t z_t \ , \ q_t = \frac{Q_{K,t}}{P_t} \ , \ n_{t+1} = \frac{N_{t+1}}{z_{t+1} P_{t+1}} \ , \ c_t = \frac{C_t}{z_t} \ , \ i_t = \frac{I_t}{z_t} \ ,$$
$$y_t = \frac{Y_t}{z_t} \ , \ b_t = \frac{B_t}{P_t z_t} \ , \ g_t = \frac{G_t}{z_t} \ , \ \xi_t = \frac{\Xi_t}{P_t z_t}$$

We adopt the convention that variables decided in period *t* appear with a time subscript of *t* in period *t*. So variables that are already decided in period *t* have a subscript of *t-1* in period *t*.

The equilibrium is summarized by the following equilibrium conditions:

<u>Firms</u>

Real MC:

$$s_t = \left(\frac{1}{1-a}\right)^{1-a} \left(\frac{1}{a}\right)^{a} \left(r_t^k\right)^a \left(w_t\right)^{1-a} \tag{3.9.1}$$

Efficiency condition:

$$s_t = \frac{r_t^k}{\alpha \left(\dfrac{\mu_z l_t}{u_t \bar{k}_{t-1}}\right)^{1-\alpha}} \tag{3.9.2}$$

Price setting equations:

$$E_t \left[\lambda_{z,t} y_t + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_f}} \beta \xi_p F_{p,t+1} - F_{p,t} \right] = 0 \tag{3.9.3}$$

$$E_t \left[\lambda_f \lambda_{z,t} y_t s_t + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_f}{1-\lambda_f}} \beta \xi_p J_{p,t+1} - J_{p,t} \right] = 0 \tag{3.9.4}$$

$$J_{p,t} = F_{p,t}\left(\frac{1-\xi_p\left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p}\right)^{1-\lambda_f} \tag{3.9.5}$$

Capital Producers

Law of motion of capital:

$$\bar{k}_t = \frac{(1-\delta)\bar{k}_{t-1}}{\mu_z} + \left(1 - S\left(\frac{i_t}{i_{t-1}}\mu_z\right)\right)i_t \tag{3.9.6}$$

Investment FOC:

$$E_t\left[\lambda_{z,t}q_t\left(1-\nu_t(\bullet)-\nu_t'(\bullet)\mu_z\frac{i_t}{i_{t-1}}\right)-\lambda_{z,t}+\beta\frac{\lambda_{z,t+1}q_{t+1}}{\mu_z}\nu_{t+1}'(\bullet)\left(\mu_z\frac{i_{t+1}}{i_t}\right)^2\right]=0 \tag{3.9.7}$$

Entrepreneurs

FOC w.r.t utilization:

$$r_t^k - a'(u_t) = 0 \tag{3.9.8}$$

Rate of return to capital:

$$R_t^k = \frac{u_t r_t^k - a(u_t) + (1-\delta)q_t}{q_{t-1}}\pi_t - 1 \tag{3.9.9}$$

FOC's determining the two parameters of optimal debt contract:

$$E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R_{t+1}^k}{1+R_t^D} + \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1})-\mu G_t'(\bar{\omega}_{t+1})}\left[\frac{1+R_{t+1}^k}{1+R_t^D}\left(\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})\right)-1\right]\right\}=0 \tag{3.9.10}$$

$$[\Gamma_t(\bar{\omega}_t)-\mu G_t(\bar{\omega}_t)]\frac{1+R_t^k}{1+R_{t-1}^D} = \left(1-\frac{n_{t-1}}{q_{t-1}\bar{k}_{t-1}}\right) \tag{3.9.11}$$

Law of motion of aggregate net worth:

$$n_t = \frac{\gamma_t}{\pi_t \mu_z} q_{t-1}\bar{k}_{t-1}(R_t^k - R_{t-1}^D - \mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) R_t^k) + w^e + \frac{\gamma_t}{\pi_t \mu_z}(1 + R_{t-1}^D) n_{t-1} \tag{3.9.12}$$

Banks

Interest on deposits:

$$R_t^B = R_t^D \tag{3.9.13}$$

Households

Consumption FOC:

$$\frac{\mu_z}{c_t \mu_z - b_c c_{t-1}} + b\beta E_t \frac{1}{c_{t+1}\mu_z - b_c c_t} = \lambda_{z,t} \tag{3.9.14}$$

Deposits FOC:

$$\lambda_{z,t} = \beta E_t \frac{\lambda_{z,t+1}}{\mu_z} \frac{1 + R_t^D}{\pi_{t+1}} \tag{3.9.15}$$

Labor FOC:

$$J_{w,t} = \frac{1}{\psi_L} \left[\frac{1 - \xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mu_z \right)^{\frac{1}{1-\lambda_w}}}{1 - \xi_w} \right]^{1-\lambda_w(1+\sigma_L)} w_t F_{w,t} \tag{3.9.16}$$

$$E_t \left\{ l_t \frac{\lambda_z}{\lambda_w} + \beta \xi_w \mu_z^{\frac{1}{1-\lambda_w}-1} \left(\frac{1}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} - F_{w,t} \right\} = 0 \tag{3.9.17}$$

$$E_t\left\{ l_t^{1+\sigma_L} + \beta\xi_w \left(\mu_z \frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} J_{w,t+1} - J_{w,t} \right\} = 0 \tag{3.9.18}$$

Government

Government debt:

$$b_t = (1+R^B_{t-1})b_{t-1} / \pi_t\mu_z + g_t + \xi_t - \phi_\tau b_{t-1} / \pi_t\mu_z \tag{3.9.19}$$

Interest on government debt:

$$R_t^B = \frac{R_t}{P(b_t)} \tag{3.9.20}$$

Monetary Policy

$$\hat{R}_t = \rho_i\hat{R}_{t-1} + (1-\rho_i)\frac{a_y}{4R}(\hat{y}_t - \hat{y}_{t-1}) + (1-\rho_i)a_{d\pi}\frac{\pi}{R}(\hat{\pi}_t - \hat{\pi}_{t-1}) \tag{3.9.21}$$

Resource constraint

Aggregate production function:

$$y_t = \left(\frac{u_t\bar{k}_{t-1}}{\mu_z} \right)^a l_t^{1-a} - \Phi \tag{3.9.22}$$

Aggregate resource constraint:

$$\mu\int_0^{\bar{\omega}_t} \omega dF(\omega)R_t^k \frac{q_{t-1}\bar{k}_{t-1}}{\mu_z\pi_t} + a(u_t)\frac{\bar{k}_{t-1}}{\mu_z} + c_t + i_t + g_t \le y_t \tag{3.9.23}$$

We therefore have 23 equations to be solved for the following 23 endogenous variables, defined by the vector Z_t:

$$Z_t = \left(s_t, F_{p,t}, J_{p,t}, \pi_t, \lambda_{z,t}, R_t^k, i_t, r_t^k, l_t, w_t, q_t, \bar{\omega}_t, n_t, y_t, c_t, \bar{k}_t, R_t^D, b_t, R_t^B, R_t, u_t, F_{w,t}, J_{w,t} \right)'$$

In the next chapter, we describe an algorithm to obtain this solution.

4. Workings of the Model

In the previous chapter we have presented the model's equilibrium conditions in terms of stationary variables, detrending appropriate variables by the growth rate.[30] The economy evolves around a stochastic growth path. In section 4.1 we find the non-stochastic steady state of the detrended system. The model solution involves a log linear approximation around the models non-stochastic steady state and then solving the resulting linear system of stochastic difference equations. We calibrate the parameters of our model for the EA using values from the literature. We compute model IRF's to a shock in the financial sector. We compare the impulse response functions of our baseline, SR model to a financial shock to those of the FA model to that shock. The FA model includes the same financial frictions as the SR model but abstracts from bank bailout funds and sovereign debt and risk. In that way, we can access the effect the additional financial-sovereign channel we have introduced has for economic activity, compared to the FA model.

4.1 Steady State

In this section we develop an algorithm for computing the steady state of the model, $Z=\left(s,F_p,J_p,\pi_t,\lambda_z,R^k,i,r^k,l,w,q,\varpi,n,y,c,\bar{k},R^D,b,R^B,R,u,F_w,J_w\right)'$, such that $Z_t=Z,\quad \forall t\in\mathbb{N}_+$. The existence of this algorithm proves that an interior steady state exists. However, the uniqueness of this steady state in not proved in this paper.

The algorithm used in computing the steady state values has as follows:

<u>Step 1 (Inflation):</u>
Since in the steady state, $\tilde{\pi}=\pi$, it follows from the Taylor Rule, (3.9.21), that π is set equal to the target inflation rate of the Rule.

<u>Step 2 (Starting value):</u>
Fix the value of the real rental cost of capital services, r^k.

[30] The equilibrium conditions presented in chapter 4 correspond to our baseline, SR model.

<u>Step 3 (Financial variables):</u>

From (3.9.15), (3.9.7), and (3.9.8):

$$1+R^D=\frac{\pi\mu_z}{\beta} \tag{4.1.1}$$

$$q=1 \tag{4.1.2}$$

and

$$u=1, \tag{4.1.3}$$

respectively.

Then, (3.9.9), gives:

$$1+R^k=(1+r^k-\delta)\pi \tag{4.1.4}$$

From (3.9.10),

$$[1-\Gamma(\varpi)]\frac{1+R^k}{1+R^D}+\frac{\Gamma'(\varpi)}{\Gamma'(\varpi)-\mu G'(\varpi)}\left[\frac{1+R^k}{1+R^D}\left(\Gamma(\varpi)-\mu G(\varpi)\right)-1\right]=0 \tag{4.1.5}$$

But with $1+R^D$ and $1+R^k$ determined as in (4.1.1) and (4.1.4), (4.1.5) can be solved for ϖ. Moreover, (3.9.11) gives:

$$\frac{n}{k}=1-\frac{1+R^k}{1+R^D}[\Gamma(\varpi)-\mu G(\varpi)] \tag{4.1.6}$$

which, in view of (4.1.1), (4.1.4) and the solution of (4.1.5), allows us to compute n/k.

Furthermore, (3.9.12) gives:

$$n = \frac{\gamma}{\pi\mu_z}(R^k - R^D - \mu\int_0^{\bar{\omega}} \omega dF(\omega) R^k)\frac{\bar{k}}{n} n + w^e + \frac{\gamma}{\pi\mu_z}\frac{(1+R^D)}{} n.$$

The latter can be solved for n, to get:

$$n = \frac{w^e}{1 - \frac{\gamma}{\pi\mu_z}(R^k - R^D - \mu\int_0^{\bar{\omega}} \omega dF(\omega) R^k)\frac{\bar{k}}{n} - \frac{\gamma\,(1+R^D)}{\pi\mu_z}} \tag{4.1.7}$$

In view of (4.1.1), (4.1.6), (4.1.7) allows us to compute entrepreneurs steady state net worth, n.

Next, from the public debt transition equation, (3.9.19), we have:

$$b = \frac{(g+\xi)\pi\mu_z}{\pi\mu_z - (1+R^B) + \phi_\tau} \tag{4.1.8}$$

Since (3.9.13) implies that $R^B = R^D$, (4.1.8) allows us to compute steady state government bonds, b. Therefore, (3.9.20) implies that the steady state safe rate of return can be computed from:

$$R = R^D P(b), \tag{4.1.9}$$

where $P(b)$ is the probability of government default at the steady state level, b.

Step 4 (Real economy variables):

In view of (4.1.6) and (4.1.7), steady state capital, $\bar{k}$, can be computed from:

$$\bar{k} = \left(\frac{\bar{k}}{n}\right)n \tag{4.1.10}$$

And, in view of (4.1.10), steady state capital investment, i, can be computed from the capital transition equation, (3.9.6), to get:

$$i = \left(1 - \frac{(1-\delta)}{\mu_z}\right)\bar{k} \tag{4.1.11}$$

Next, we compute steady state marginal cost of aggregate output, the real wage rate, the capital – labor ratio and eventually, labor demand. Since, $\tilde{\pi} = \pi$ in the steady state, it follows from (3.9.3)-(3.9.5) that:

$$s = \frac{1}{\lambda_f} \tag{4.1.12}$$

Then, in view of (3.9.1) and (3.9.2), the real wage rate and the capital - labor ratio can be computed from:

$$w = s(1-\alpha)\left(\frac{k}{\mu_z l}\right)^{\alpha} \tag{4.1.13}$$

and

$$\frac{k}{l} = \mu_z\left(\frac{\alpha s}{r^k}\right)^{\frac{1}{1-\alpha}} \tag{4.1.14}$$

But, since (4.1.3) implies that $\bar{k} = k$ and in view of (4.1.10), labor demand can be computed from:

$$l = \left(\frac{l}{k}\right)k \tag{4.1.15}$$

Finally, we turn to the aggregate real economic activity variables. First, we compute Φ to guarantee that profits of intermediate good producers are zero in steady state. Write sales of final good firms as $y-\Phi$. Also, real marginal cost is constant in steady state thus total costs of the firm are sy. The zero profit condition implies: $sy = y-\Phi$. Hence, in view of the aggregate production function (3.9.22):

$$y = \left(\frac{k}{\mu_z}\right)^a l^{1-a} - \Phi \tag{4.1.16}$$

Φ is given by:

$$\Phi = \left(\frac{k}{\mu_z}\right)^\alpha l^{1-\alpha}\left(1-\frac{1}{\lambda_f}\right) \tag{4.1.17}$$

Substituting Φ in (4.1.16), by the left hand side of (4.1.17), we have aggregate output in the steady state:

$$y = \frac{1}{\lambda_f}\left(\frac{k}{\mu_z}\right)^a l^{1-a} \tag{4.1.18}$$

Next, in view of the economy's resource constraint, (3.9.23), we have:

$$\mu\int_0^{\varpi} \omega\, dF(\omega) R^k \frac{k}{\mu_z \pi} + c + i + g = y \tag{4.1.19}$$

Then, (4.1.19) allows us to compute *c*. And, in turn, since the household Euler condition (3.9.14), implies:

$$\lambda_z = \frac{1}{c}, \tag{4.1.20}$$

we can compute the steady state value of the Lagrange multiplier associated with the household's budget constraint from (4.1.20).

<u>Step 5 (Labor market equilibrium and the adjustment of the the real rental cost of capital services, r^k):</u>

A simple count of the equilibrium conditions (i.e., (3.9.1)-(3.9.23)) makes clear that we have taken into account all of them except the labor supply conditions (3.9.16) – (3.9.18). Since in the steady state, $\tilde{\pi}_w = \pi$, (3.9.16)-(3.9.18) imply that, labor supply in the steady state is given by:

$$h = \left(\frac{\lambda_z w}{\lambda_w \psi_L} \right)^{1/\sigma_L} \tag{4.1.21}$$

in equilibrium it must be that, labor demand, l, computed in (4.1.15) is equal to labor supply, h, computed in (4.1.21). Then, since l (h) is increasing (decreasing) in r^k, if for the value of r^k we started with in Step1: $l > (< h)$, we try an arbitrarily small decrease (increase) in this value and repeat Steps 3-4, until we find a value of r^k such that: $l = h$. (Obviously, this is a trial and error process and we are not certain for the uniqueness and the global asymptotic stability of the steady state obtained with such an algorithm.)

4.2 Fundamental Shocks

This section we describe the stochastic nature of the model. Our model includes the following vector of shocks:

$$\Upsilon_t = (\sigma_t, g_t)'$$

These are exogenous variable in our model and follow a Markov process. The first shock, risk shock, is associated with financial sector of the model. The second shock, government spending shock, is associated with government consumption. We model shocks, in percentage deviations from steady state, with the following first order autoregressive representation:

$$\hat{\sigma}_t = \rho_\sigma \hat{\sigma}_{t-1} + u_t^\sigma \ , u_t^\sigma \sim iid \tag{4.2.1}$$

$$\hat{g}_t = \rho_\gamma \hat{g}_{t-1} + u_t^g \ , u_t^g \sim iid \tag{4.2.2}$$

We study the response of the simulated economy to a financial shock by a positive jump in the risk shock. The risk shock is an important shock when fed in a NK DSGE model for, it generates responses that resemble the business cycle.[31] By triggering such a shock in the financial sector we mimic the recession that was brought about by the financial crisis of 2008.[32] The government spending shock is used to access the implication of financial intermediation sovereign risk interdependence on the government spending multiplier.

4.3 Model Solution

The model's equilibrium conditions in terms of stationary variables presented in the previous section can be written in the following form:

$$E_t f(Z_{t+1}^+, Z_t, Z_{t-1}^-, \Upsilon_t) = 0$$

where Z_t is the vector of endogenous variables, Z_t^+ is the subset of variables of Z_t that appear with a lead and Z_t^- is the subset of variables of Z_t that appear with a lag and Υ_t is the vector of exogenous variables. The model solution involves a log linear approximation around the models non-stochastic steady state, Z, and then solving the resulting linear system of stochastic difference equations. The solution method follows Villemot (2011), where the procedure for solving the stochastic dynamic model in the form above is outlined.

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31 CMR (2010).

32 CMR (2013) provide evidence that the accelerated collapse in economic activity that occurred in late 2008 was largely due to increase in risk at the time.

4.4 Calibration

We calibrate the model for the EA. For all preference, technology and financial frictions parameters we use conventional values from the literature. In particular we use the parameters from CMR (2010). They divide the parameters into two sets. The one take standard values from the literature and reproduce key sample averages in the data. The second is estimated using Bayesian methods. The values of these parameters are listed in Table 4.1.

Table 4.1
Model Parameters

Firms		
a	Share on capital	0.36
μ_z	Growth rate (APR)	1.5
β	Discount rate	0.999
ξ_p	Calvo prices	0.719
λ_f	Intermediate good firms markup	1.20
Capital Producers		
v''	Investment adjustment cost	39.149
δ	Depreciation rate	0.02
Entrepreneurs		
σ_a	Capacity utilization	26.730
μ	Financial sector inefficiency	0.1
γ	Percent of entrepreneurs who survive	97.80
$F(\bar{\varpi})$	Percent of businesses that go bankrupt	2.60
σ	Variance of log of idiosyncratic uncertainty	0.12
W	Transfer to entrepreneurs	0.03
Households		
b_c	Habit persistence	0.56
λ_w	Supply of labor markup	1.05
ξ_w	Calvo wages	0.747
ψ_L	Labor functional form parameter	6
σ_L	Curvature of disutility of labor	1
Monetary Policy		
ρ	Coeff. on lagged interest rate	0.871
α_π	Weight on inflation on Taylor rule	1.824
α_y	Weight on output growth in Taylor rule	0.251

Shocks		
ρ_σ	AutoCorr. Coef. Riskiness shock	0.958
ρ_g	AutoCorr. Coef Government spending shock	0.988
σ_σ	Var. Riskiness shock	0.0458
σ_g	Var. Government spending shock	0.0121

For the additional model parameters we introduce, relating to government debt and financial sector rescue funds, we proceed as follows. We set ϕ_τ at 12.5%, so that most of government expenditures are financed by debt.[33] We model the government bank rescue function, ξ_t , as a linear function, parameterized so that an increase in the risk shock results in an increase in funds at 11% of steady state GDP.[34] Finally, we model the cumulative density function of the fiscal limit distribution as a logistic function following Bi and Traum (2012). In that way we capture the strong non-linearity of the endogenous distribution of the fiscal limit derived by Bi (2012) – once the default probability begins to rise it does so rapidly. In addition the logistic function approximates the endogenous distribution implied by models of strategic default (Gordon and Guerron-Quintana (2013)). The logistic function is parameterized so that the resulting increase in debt, following the risk shock, increases the probability of default by 0.12 from its steady state value. Such an increase has been observed by various EA countries such as Greece, Portugal and Ireland since 2008.[35]

We calibrate parameters common to both the FA and SR models to the same values. Since this is a comparative models exercise, our results turn out to be robust to these common values. Parameters that relate only to the SR model can affect the numerical magnitude of our outcomes. For example, using the total committed funds for EA bailouts instead of actual would result in a higher probability of default and a deeper recession.

[33] Following the ability of EA countries tax income to bail-out their respective banks as calculated by Pisany-Ferry (2012).
[34] Following the actual total funds being injected by various EA countries as estimated by IMF (2011a) for Belgium, Ireland, Germany, Greece, Netherlands and Spain.
[35] Lucas et al. (2013).

4.5 Impulse Response Functions to Financial Risk

In this section we investigate the effect that financial intermediation-sovereign interaction has on economic activity. We compare the impulse response functions of the SR model to a financial shock to those of the FA model to that shock.

The approach of studying the model's reaction to shocks is in the spirit of a suggestion made by Lucas (1980). He argues that economists

> "…need to test them (models) as useful limitations of reality by subjecting them to shocks for which we are fairly certain how actual economies or parts of economies would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder question."

The advantage of this econometric approach is transparency and focus. The transparency reflects that the estimation strategy has a simple graphical representation, involving objects – impulse response functions – about which economists have strong intuition. The advantage of focus comes from the possibility of studying the empirical properties of the model without having to specify a full set of shocks.

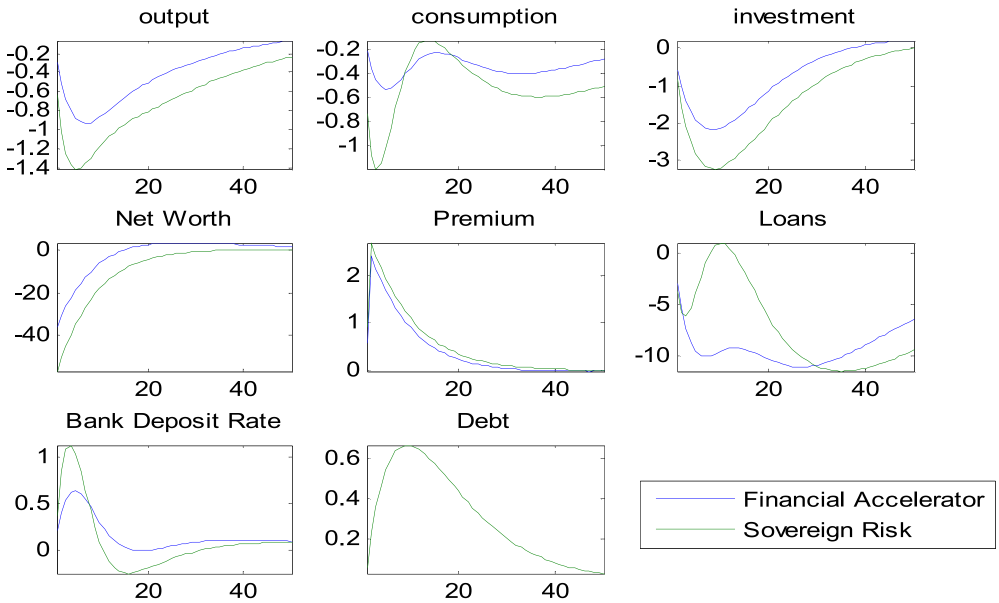
4.5.1 Financial Accelerator Model

In the FA model, a rise in the risk shock, σ_t, results in a rise in the external finance premium. When the risk is high, the credit spread is high and the credit extended to entrepreneurs is low. Entrepreneurs then acquire less physical capital. Since investment is a key input in the production of capital, it follows that investment falls. With this decline output falls as well as consumption. Furthermore, the net worth of entrepreneurs, an object that can be identified with the stock market, falls. This is due to the fact that rental income earned by entrepreneurs on their capital falls with the reduction in economic activity. In addition, the fall in the production of capital results in a fall in the price of capital which results in capital losses for entrepreneurs. The risk shock, therefore, predicts countercyclical interest rate premium and procyclical investment, consumption, the stock market and credit, amplifying swings in borrowing, investment, spending and production. The amplification of swings in the FA model is known as the financial accelerator effect.

4.5.2 Sovereign Risk Model

In the SR simulations, an increase in financial sector risk increases the external finance premium which in turn increases government debt. The higher level of debt increases the probability of default of the sovereign resulting in higher interest rates on bonds and bank deposits. The higher interest rates on bonds and bank deposits result in an even higher external finance premium, than in the FA model. As a result the bank-sovereign interdependence reinforces the initial financial accelerator mechanism. We find that an increase in the risk shock results in a considerably deeper recession when financial intermediation-sovereign interactions are present. Output, consumption, investment and net worth all decline substantially. As shown in Figure 4.1, debt increases as a result of government injections to banks. The drop in loan is not as severe owing to these injections. However, the premium is higher. The fall of investment, net worth, and output is in one and a half times bigger than the one obtained under the FA model. And, the initial drop in consumption is more than double than that of the FA model.

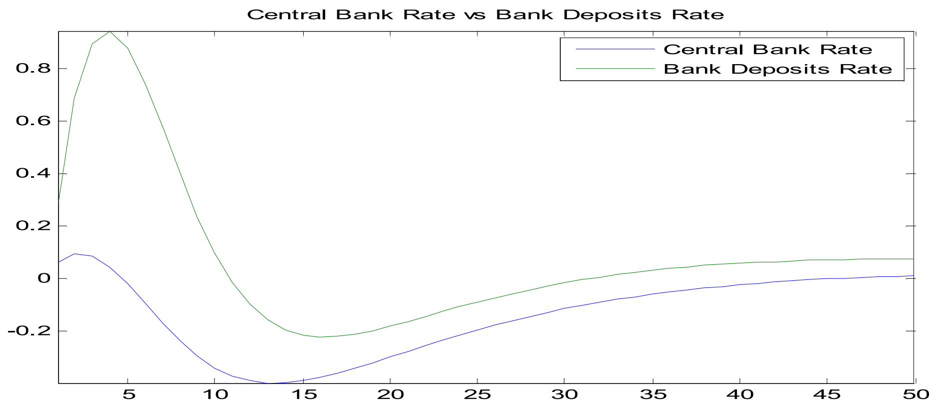
Figure 4.1
Response to a positive risk shock



The deeper recession in the sovereign risk model strongly depends on government's countercyclical stand on required costs for financial sector rescues. These cause the higher financing costs for banks that are further transmitted to non-financial corporations with a higher premium. What is important for the determination of the conditions of direct financing in financial markets and deposit funding for banks are developments in benchmark interest rates. These include, the key ECB interest rates, money market rates and government bond yields, with the latter containing the term structure of risk free interest rates, sovereign risk and liquidity premia. In tranquil times these interest rates co-move and by controlling the ECB rate the central bank can affect the funding cost of the economy. When there is sovereign risk, however, the funding cost of the economy no longer depends solely on the ECB key interest rates but also on the sensitivity of the euro-zone crisis. Increases in sovereign credit risk can lead directly to higher financing costs for the private sector via capital markets as well as bank lending rates. In our model government bond yields function as the benchmark interest rate, particularly in that they contain sovereign credit risk. An increase in sovereign credit risk can lead directly to higher financing costs for the private sector via bank lending rates. As seen in Figure 4.2, while in response to a risk shock the interest set by the monetary authority remains low, the interest rate on government bonds, rises by almost 1%.

Figure 4.2

Economy's interest rates



This implies that the monetary authority has limited control over the benchmark interest rate and therefore on the financing conditions of the economy.[36] Our model results in a positive co-movement between sovereign spreads, financial spreads and private lending rates. The higher interest rates on bank deposits are transmitted to non-financial corporations with a higher premium. As a result our model predicts a recession similar to the one experienced by many EA countries today. In that way our model captures the effects of the vicious circle created in the EA. In the EA banks play a key intermediary role. Bank based financing is the predominant source of external debt financing for the non financial private sector. The increased cost of financing for banks is transmitted, through increased bank lending rates, to the financing conditions of firms and households affecting economic activity and the business cycle. [37]

[36]Note that our model does not include a zero lower bound to capture the inability of the monetary authority to counteract the crisis by cutting interest rates. However, the simple Taylor rule used is not aggressive enough to offset the recession brought about by the increase in the risk shock. A more aggressive monetary policy cut, following a modified Taylor rule, reacting to financial risk, could counteract the recession.

[37]Harjes (2011) finds, for a sample of 11 EA countries in 2008-2011 that sovereign credit costs are closely related to private funding costs.

5. Policy Implications

Our model has three policy implications. The first and main implication of our model for EA policies dealing with failing banks is that they aggravated the recession. The SR model captures the main features of the EA policy in dealing with failing banks, whereby each member state is responsible for the bail out of these banks. On the contrary, the simple FA model replicates a hypothetical scenario where the EA countries had not engaged in financial sector bailouts, in response to the 2008 financial crisis. As discussed in the previous chapter, comparing the SR model to the simple FA model, where financial sector bailouts are not present, the SR model predicts a deeper recession. Thus, our model comparison suggests the EA bank rescue policy has resulted in a deeper recession. The outcome of our analysis is supported in practice when comparing the EA with the US. In the US no interdependence is observed (Figure 1.8). The reliance on the specific government where the bank is located to fund the bailouts is in contrast to policy within the US, where the specific state location is not the determinant of who bears costs. Both bank support (through the FDIC and in the case of capital injections through the TARP) and bank regulation takes place at the US currency union level. One can imagine that if the State of Washington had to bear the fiscal burden when Washington Mutual collapsed with nearly \$200bn in deposits, there is no way its fiscal resources could have borne the cost. Instead, FDIC was able to broker a deal with JP Morgan and avoid any fiscal cost. In the EU setup, a currency union wide fund may have loaned the State of Washington the financing necessary, but state tax payers would be responsible for whatever funding they provided. The mutualization of bank losses across states can only reasonably be achieved if there is area wide supervision and area wide funding resources, such as the FDIC. Financial institutions in Europe were operating across borders, and with the same currency and lender of last resort, but with different supervision and without any mutual bank support across countries.

In response to the additional funding problems at banks, following the sovereign crisis, a supranational approach was employed to solve liquidity, but not solvency, problems in the financial sector. The liquidity problem has been a supranational issue countered by the ECB, where the ECB provided liquidity to the financial sector through Long Term Refinancing Operations (LTROs). However, the ECB did not increased liquidity by buying assets to expand its balance sheet.[38] Instead it loaned to banks for terms up to three years. In doing so the ECB filled the liquidity needs for banks, but relative to a policy of purchasing assets from the market, it has left any credit risk to the balance sheets of banks. In that sense LRTOs are notably different from the quantitative easing policies followed by the Federal Reserve where the Fed purchased assets outright rather than help banks ability to purchase them. Further, the LRTOs were used to fund more sovereign debt purchases, strengthening the connection between banks and sovereigns. These findings lead to question the effectiveness of ECB provision of liquidity to banks as a means to alleviate the sovereign crisis.

Realizing the problem, on May 9, 2010, the European Financial Stabilization Fund (EFSF) was created. The EFSF has acted to stave off insolvency of sovereigns. The EFSF was created as a temporary rescue mechanism. On September 27, 2012, the EA member states created a permanent rescue mechanism, the European Stability Mechanism (ESM). The ESM can provide instant access to financial assistance programs for the EA member states in financial difficulty, with a maximum lending capacity of €500 billion.

The EFSF in concert with the IMF and EFSM has provided funds to Greece, Ireland and Portugal to prevent disorderly default. The ESM has provided funds to Spain for the recapitalization of the country's banking sector and Cyprus to address financial, fiscal and structural challenges. As shown in Table 5.1, the economic adjustment programme for Ireland included a joint financing package of €85 billion for the period 2010-2013, where €67.5 billion was external support and €17.5 billion was contribution from Ireland (Treasury and National Pension Reserve Fund). The first economic adjustment program for Greece involved bilateral loans pooled by the European Commission (so-called "Greek Loan Facility" – GLF) for a total amount of €80 billion to be disbursed over the period May 2010 through June 2013. (This amount was eventually reduced by €2.7 billion, because Slovakia decided not to

[38] The ECB under the Securities Market Program from May 2010 – February 2012 conducted direct purchases of public and private debt securities, however the size of the program that by the end of 2010 was just over €70 billion and reached about €200 billion, was small compared to Federal Reserve purchases of over $1 trillion in mortgaged backed securities by early 2010.

participate in the Greek Loan Facility Agreement while Ireland and Portugal stepped down from the facility as they requested financial assistance themselves). The financial assistance agreed by EA member states was part of a joint package, with the IMF committing additional €30 billion under a stand-by arrangement (SBA). The second programme for Greece foresees financial assistance of €164.5 billion until the end of 2014. Of this amount, the euro area commitment amounts to €144.7 billion to be provided via the EFSF, while the IMF contributes €19.8 billion. (This is part of a four-year €28 billion arrangement under the Extended Fund Facility for Greece that the IMF approved in March 2012). When launching the second programme it was agreed that there should be private sector involvement (PSI) to improve the sustainability of Greece's debt. The economic adjustment programme for Portugal included a joint financing package of €78 billion with the EFSM, EFSF and IMF contributing €26 billion each. It contained reforms to promote growth and jobs, fiscal measures to reduce the public debt and deficit, and measures to ensure the stability of the country's financial sector. Spain's program was agreed by the Eurogroup in July 2012 for a period of 18 months and provided an external financing by the EA Member States of up to €100 billion. Eventually, Spain used only close to €38.9 billion for bank recapitalisation, under restructuring and resolution plans approved by the European Commission (EC) under State-aid rules, and around €2.5 billion for capitalising Sareb (the Spanish asset management company). Spain exited successfully the financial assistance programme for the recapitalization of financial institutions in January 2014. In April 2013 an Economic Adjustmnet Programme was agreed for Cyprus covering the period 2013-2016. The ESM will provide up to €9 billion and has currently disbursed €4.5 billion and the IMF €1 billion.

Table 5.1
Committed Funds for Sovereign Rescues (billion Euros)

	IMF	GLF	EFSM	EFSF	ESM	**Total**
Ireland	22.5		22.5	22.5		**67.5**
Greece	48.1	52.7		144.7		**245.5**
Portugal	26		26	26		**78**
Spain					100	**100**
Cyprus	1				9	**10**
Total	**97.6**	**52.7**	**48.5**	**193.2**	**109**	**501**

Source: EC, http://ec.europa.eu/economy_finance/assistance_eu_ms/index_en.htm

Although part of the funds has been used to recapitalize financial sectors of member countries, the EFSF/ESM has not mutualized bank losses in a way that would break the sovereign-bank link. The EFSF/ESM has been unable / unwilling to provide capital directly to banks, instead it provided loans to countries with solvency issues which in turn provided funds to their banks. Thus, the second policy implication of our model is that as long as bank solvency remains a national matter, continues to contribute to the recession in the manner identified above. Our finding, suggests that recent efforts for dealing directly with bank solvency with the implementation of the ESM's DRI and the ECB's €60bn a month bond buying program up to €1.08 trillion are in the right direction.

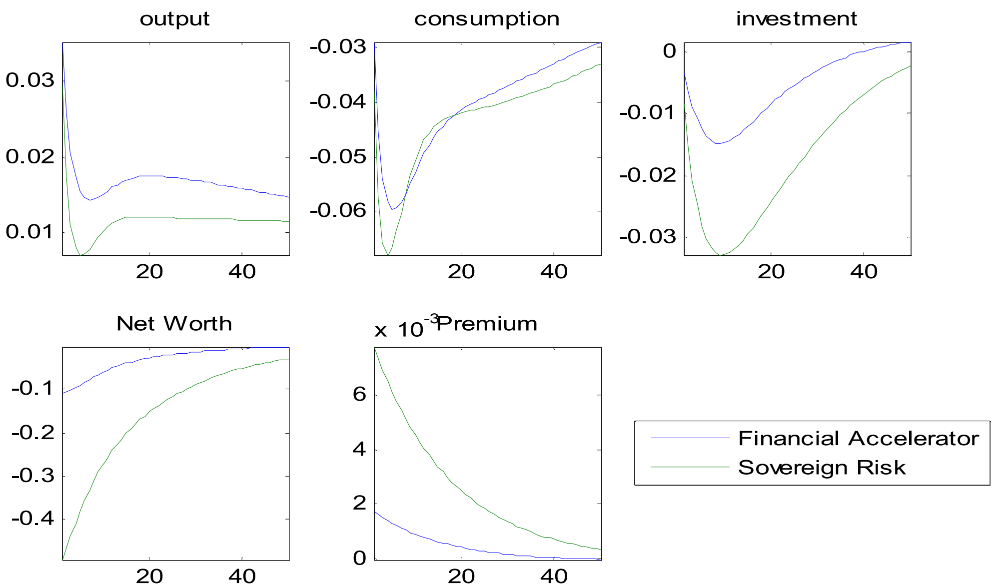
The third policy implication of our model relates to the so called austerity policies. These policies have been implemented as a response to rising concerns about sovereign solvency. However, such measure can be self-defeating, as the success of austerity depends on the fiscal multiplier. Ceteris paribus, fiscal austerity is more likely to be self-defeating if the multiplier is large.

There is a large literature on the size of fiscal policy multiplier. In her survey of this literature, Ramey (2011) lists studies where the government spending multiplier ranges from -0.3 to 3.6. Fiscal multipliers change with the economic environment, monetary policy, exchange rate regime, unemployment, health of banking and financial system, credibility of fiscal policy, fiscal stress, expansion vs contraction, openness of trade and capital. Recently, this has been recognized by various authors such as Perotti (1999), Ilzetzki, Mendoza and Vegh (2012), Corsetti et al. (2012).

We study the effect of a government spending shock in both the SR and FA models. The effects of a government spending shock are discussed based on the impulse response functions of the endogenous variables generated by the log-linearized version of the model. We apply a 1% shock to government spending Comparing the FA model with the SR model, we can provide insights about the impact of austerity measures. A smaller multiplier in the case of sovereign risk would provide evidence in favor of the above mentioned austerity measures employed in various EA countries.

In the FA model output increases on impact, but strong crowding out effects on both household consumption and investment are present. Inflation increases since an increase in aggregate demand raises marginal production costs. Monetary authority raises nominal interest rate to counteract upward pressure on inflation and this drives up the real interest rate. The rise in government spending alleviates risk premium in credit markets as increase in demand drives up the net worth of entrepreneurs. In the SR model an increase in the probability of default following a rise in deficit financed public expenditures leads to higher financing costs. As seen in Figure 5.1, the premium in the case of sovereign risk is higher. Although the value of the multiplier is close to zero in both models, there is a comparably big drop in net worth and investment as well as a drop in consumption, following the shock. The impact of an increase in government spending has a smaller effect on output in the case of sovereign risk. The sovereign risk channel therefore, presents an alternative fiscal transmission mechanism through which increase in deficit financed government spending and associated increases in risk premia crowd out household and firm consumption and investment through higher private credit risk.

Figure 5.1
Response to an increase in government spending



Thus, the multiplier in the sovereign risk model is smaller than the FA models. In related empirical literature, Corsetti et al. (2012) find that the fiscal multiplier increases markedly during times of financial crises being 2.3 at impact and 2.9 at the peak. They also find that fiscal strains may take the multiplier into negative territory. Coenen et al (2012) show that monetary policy stance is important. When central banks follow a Taylor rule fiscal multiplier is small. If monetary policy is accommodative – interest rate kept constant – multiplier is greater. Recent studies based on modern business cycle models have made clear that the multiplier is likely to be large if monetary policy is constrained by the zero lower bound on nominal interest rates (Eggertsson (2011) and Christiano et al. (2009)). In related theoretical literature, Canzoneri et al. (2015) find that government spending multipliers in recessions are large exceeding 2. Corsetti et al. (2013), analyze the effects of fiscal retrenchment in a NK model where sovereign risk affects private sector interest rate spreads, but not the other way around. They find that when monetary policy is constrained and unable to offset changes in the sovereign risk premium, the multiplier becomes negative for a sufficient degree of sovereign risk. We conclude that for countries with high public debt, a fiscal stimulus introduces deterioration of public finances and hence increases in sovereign risk premium, which in turn reduces the size of the multiplier, favoring austerity measures used in EA countries.

6. Concluding Remarks

The events of the past years in the EA make it clear that quantitative equilibrium models must be expanded to account for financial intermediation – sovereign interactions. These interactions have contributed to a deep recession for many EA countries. Our thesis aims at constructing a quantitative model with sovereign-financial interdependence. The model is then solved using first order approximations and predicts the recession seen in EA countries. The model is finally used to provide policy implications.

Chapter 1 presents the stylized facts of the EA sovereign debt crisis. The data indicate the strong interdependence between sovereign and financial risk for the EA over that period.

Chapter 2 presents related literature to our model. Our model stresses the importance of the interdependence of financial risk and sovereign risk for the business cycle. To study the effect of this interdependence for the business cycle we need to construct a DSGE model with such features. We, therefore, present related DSGE models that stress financial risk and DSGE models that stress sovereign risk. First, we present models of financial risk such as CMR (2010), Gertler and Karadi (2011), Gertler and Kiyotaki (2011), Kiyotaki and Moore (2008) and Curdia and Woodford (2009a). The models imply that financial risk modeled through shocks to the financial sector is important for the business cycle. We then present models of sovereign risk stressing the recent literature of the stochastic fiscal limit, Bi (2012) and Bi and Traum (2012). In these models sovereign default depends on a probability distribution function that reflects political risk.

Chapter 3 presents the model. We construct a model framework with financial intermediation – sovereign risk interactions that can be used to study the effect of such interactions for economic activity and policy. Interactions in our model arise from the government's financial sector rescue policy. Such a policy increases government debt, which affects the interest on bonds and deposits raising bank funding conditions. Higher bank funding costs result in higher financial spreads and a further need for government rescues, creating a vicious circle. This policy replicates EA response to the crisis were each member state was responsible for rescuing its financial system.

Chapter 4 presents the workings of our model. We find the steady state of our model, linearize around the steady state and solve the model. Then, we compute our model's IRF's to a financial shock. We find that an increase in capital investment risk, (risk shock), originating in the financial sector, results in a considerably deeper recession when financial intermediation-sovereign interactions are also present. The recession strongly depends on the government's countercyclical policy on required funds for financial sector rescues. This policy results in positive co-movement between sovereign and financial spreads. Thus, our model replicates the effects of the vicious circle created in the EA.

In chapter 5 we present policy implications of our model. The findings of chapter 4 imply that EA national policy measures adopted in response to crisis have not been effective. Further, despite the strong degree of monetary integration reached in the EA, states remained individually responsible for rescuing banks in their jurisdiction. The EFSF/ESM, could help them with loans earmarked for that purpose but it was not yet entitled to inject capital directly to the banking system. Only recently such a mechanism has become operational. The cost of recapitalizing banks remained with individual states and it was very high especially for countries that were home to large banks with significant cross boarder activities. EA policy where the EFSF/ESM provides assistance to sovereigns coupled with austerity measures does not break the sovereign-bank link. Although our model provides evidence in favor of austerity, as long as the cost of recapitalizing banks remains with individual states the resulting vicious circle will continue to drag the economy.

A main current issue is the existence of a supra-national system for failing banks. Area wide policies to help recapitalize banks (through the EFSF/ESM and the ECB) might take pressure of the currently stressed sovereigns by removing the question of whether they will need to extend further bailouts as well as restarting lending in the euro are countries. Our model framework creates a platform for such additional policy issues to be addressed.

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Appendix

A.1 Deriving First Order Conditions

In this section we derive non-trivial FOC's presented in our model.

FIRMS

Firm's FOC for the demand for intermediate good j (3.1.3):

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{(-\frac{\lambda_f}{\lambda_f - 1})} Y_t$$

Proof:

Replacing Y_t from (3.1.1) in the firm's maximization problem (3.1.2) we have:

$$\max_{Y_{j,t}} \; P_t \left[\int_0^1 Y_{t,j}{}^{(1/\lambda_f)} dj\right]^{\lambda_f} - \int_0^1 P_{j,t} Y_{j,t} dj$$

Differentiating w.r.t $Y_{j,t}$ and setting equal to zero we have:

$$P_t \lambda_f \left[\int_0^1 Y_{t,j}{}^{(1/\lambda_f)} dj\right]^{\lambda_f - 1} \frac{1}{\lambda_f} Y_{j,t}{}^{(1/\lambda_f - 1)} - P_{j,t} = 0$$

$$\Leftrightarrow \; P_t \left[\int_0^1 Y_{t,j}{}^{(1/\lambda_f)} dj\right]^{\lambda_{f,t}\left(\frac{\lambda_f - 1}{\lambda_f}\right)} Y_{j,t}{}^{(1/\lambda_f - 1)} - P_{j,t} = 0$$

$$\Leftrightarrow \; P_t Y_t{}^{(\frac{\lambda_f - 1}{\lambda_f})} Y_{j,t}{}^{(1/\lambda_f - 1)} - P_{j,t} = 0$$

$$\Leftrightarrow \; Y_{j,t} = Y_t \left(\frac{P_{j,t}}{P_t}\right)^{(-\frac{\lambda_f}{\lambda_f - 1})}$$

Relationship between the price of intermediate good $P_{j,t}$ and the price of final good P_t (3.1.4):

$$P_t = \left[\int_0^1 P_{\mathrm{j},t}^{\left(\frac{1}{1-\lambda_f}\right)} \right]^{(1-\lambda_f)}$$

<u>Proof:</u>

Rewrite equation (3.1.3) as

$$\left(\frac{Y_{j,t}}{Y_t}\right)^{\frac{1}{\lambda_f}} = \left(\frac{P_t}{P_{j,t}}\right)^{\frac{1}{\lambda_f - 1}}$$

Integrating both sides w.r.t. j we have

$$\int_0^1 \left(\frac{Y_{j,t}}{Y_t}\right)^{\frac{1}{\lambda_f}} dj = \int_0^1 \left(\frac{P_t}{P_{j,t}}\right)^{\frac{1}{\lambda_f - 1}} dj$$

$$\Leftrightarrow \left(\frac{1}{Y_t}\right)^{\frac{1}{\lambda_f}} \int_0^1 Y_{j,t}^{\frac{1}{\lambda_f}} dj = P_t^{\frac{1}{\lambda_f - 1}} \int_0^1 \left(\frac{1}{P_{j,t}}\right)^{\frac{1}{\lambda_f - 1}} dj$$

$$\Leftrightarrow \left(\frac{1}{Y_t}\right)^{\frac{1}{\lambda_f}} \left(\int_0^1 Y_{j,t}^{\frac{1}{\lambda_f}} dj \right)^{\lambda_{f,t} \frac{1}{\lambda_{f,t}}} = P_t^{\frac{1}{\lambda_f - 1}} \int_0^1 \left(\frac{1}{P_{j,t}}\right)^{\frac{1}{\lambda_f - 1}} dj$$

$$\Leftrightarrow \left(\frac{1}{Y_t}\right)^{\frac{1}{\lambda_f}} Y_t^{\frac{1}{\lambda_f}} = P_t^{\frac{1}{\lambda_f - 1}} \int_0^1 \left(\frac{1}{P_{j,t}}\right)^{\frac{1}{\lambda_f - 1}} dj$$

$$\Leftrightarrow 1 = P_t^{\frac{1}{\lambda_f - 1}} \int_0^1 \left(\frac{1}{P_{j,t}}\right)^{\frac{1}{\lambda_f - 1}} dj$$

$$\Leftrightarrow P_t = \left[\int_0^1 \left(\frac{1}{P_{j,t}}\right)^{\frac{1}{\lambda_f - 1}} dj \right]^{(1-\lambda_f)}$$

Firms minimized real marginal cost (3.1.7):

$$s_t = \left(\frac{1}{1-a}\right)^{1-a} \left(\frac{1}{a}\right)^{a} \frac{\left(r_t^k\right)^a \left(w_t\right)^{1-a}}{z_t^{1-\alpha}}$$

and efficiency condition (3.1.8):

$$s_t = \frac{r_t^k}{\alpha\left(\frac{z_t l_t}{K_t}\right)^{1-\alpha}}$$

Proof:

The Lagrangean for the minimization problem is:

$$L = r_t^k K_t + w_t l_t + \lambda_t \left(Y_t - K_t^\alpha (z_t l_t)^{1-\alpha} + z_t \Phi\right)$$

Differentiating w.r.t K_t, l_t and the Lagrange multiplier λ_t we have:

$$r_t^k - \lambda_t \alpha_t K_t^{\alpha-1} (z_t l_t)^{1-\alpha} = 0$$

$$w_t - \lambda_t (1-\alpha) K_t^\alpha z_t^{1-\alpha} l_t^{-\alpha} = 0$$

$$Y_t - K_t^\alpha (z_t l_t)^{1-\alpha} + z_t \Phi = 0$$

Solving the above system of equations we get the following optimal choices for capital and labor:

$$l_t^* = \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \left(\frac{w_t}{r_t^k}\right)^{-\alpha} z_t^{\alpha-1} (Y_t + z_t \Phi) \text{ , } K_t^* = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{w_t}{r_t^k}\right)^{1-\alpha} z_t^{\alpha-1} (Y_t + z_t \Phi)$$

Plugging the above into the cost function we get:

$$S(Y) = \left[r_t^k \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left(\frac{w_t}{r_t^k}\right)^{1-\alpha} + w_t \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \left(\frac{w_t}{r_t^k}\right)^{-\alpha} \right] z_t^{\alpha-1} (Y_t + z_t \Phi)$$

Differentiating w.r.t. Y_t and rearranging we get:

$$s_t = \left(\frac{1}{1-a}\right)^{1-a} \left(\frac{1}{a}\right)^a \frac{\left(r_t^k\right)^a \left(w_t\right)^{1-a}}{z_t^{1-\alpha}}$$

Furthermore using the envelope theorem and the expression of the derivative of the Lagrangean w.r.t K_t we have:

$$s_t = \lambda_t = \frac{r_t^k}{\alpha\left(\frac{z_t l_t}{K_t}\right)^{1-\alpha}}$$

FOC associated with price setting (3.1.11):

$$\tilde{p}_t = \frac{E_t \sum_{i=0}^{\infty} (\beta\xi_p)^i A_{t+i} \lambda_f s_{t+i}}{E_t \sum_{i=0}^{\infty} (\beta\xi_p)^i A_{t+i} X_{t,i}}$$

where $A_{t+i} = \lambda_{t+i} P_{t+i} Y_{t+i} X_{t+i}^{(-\frac{\lambda_f}{\lambda_f - 1})}$ and $X_{t,i} = \begin{cases} \frac{\tilde{\pi}_{t+i}\tilde{\pi}_{t+i-1}.......\tilde{\pi}_{t+1}}{\pi_{t+i}\pi_{t+i-1}.......\pi_{t+1}}, & i > 0 \\ 1 & ,i = 0 \end{cases}$

<u>Proof:</u>

Replacing equation (3.1.3) into the firm's price optimization problem (3.1.10) and reordering we have:

$$\max_{\tilde{p}_t} E_t \sum_{i=0}^{\infty} (\beta\xi_p)^i \lambda_{t+i} P_{t+i} Y_{t+i} [\left(\tilde{p}_t X_{t,i}\right)^{(1-\frac{\lambda_f}{\lambda_f - 1})} - s_{t+i}\left(\tilde{p}_t X_{t,i}\right)^{(-\frac{\lambda_f}{\lambda_f - 1})}]$$

where $\tilde{p}_t = \dfrac{\tilde{P}_t}{P_t}$

The FOC is:

$$E_t \sum_{i=0}^{\infty} (\beta\xi_p)^i \lambda_{t+i} P_{t+i} Y_{t+i} X_{t+i}^{(-\frac{\lambda_f}{\lambda_f - 1})} \tilde{p}_t^{(-\frac{\lambda_f}{\lambda_f - 1} - 1)} [\tilde{p}_t X_{t,i} - \lambda_f s_{t+i}] = 0$$

Solving for $\tilde{p}_t$ we have

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j A_{t+i} \lambda_f s_{t+i}}{E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j A_{t+i} X_{t+i}}$$

The aggregate price level (3.1.14) evolves as:

$$P_t = \left[(1-\xi_p)(\tilde{P}_t)^{\frac{1}{1-\lambda_f}} + \xi_p(\tilde{\pi}_t P_{t-1})^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f}$$

Proof:

$$P_t = \left[\int_0^1 P_{j,t}^{\frac{1}{1-\lambda_f}} dj \right]^{1-\lambda_{f,t}} = \left[\int_0^{\xi_p} P_{j,t}^{\frac{1}{1-\lambda_f}} dj + \int_{\xi_p}^1 P_{j,t}^{\frac{1}{1-\lambda_f}} dj \right]^{1-\lambda_f} =$$

$$= \left[\int_0^{\xi_p} (\tilde{\pi}_t P_{t-1})^{\frac{1}{1-\lambda_f}} dj + \int_{\xi_p}^1 (\tilde{P}_t)^{\frac{1}{1-\lambda_f}} dj \right]^{1-\lambda_{f,t}} = \left[(\tilde{\pi}_t P_{t-1})^{\frac{1}{1-\lambda_f}} \int_0^{\xi_p} dj + (\tilde{P}_t)^{\frac{1}{1-\lambda_f}} \int_{\xi_p}^1 dj \right]^{1-\lambda_f} =$$

$$= \left[(1-\xi_p)(\tilde{P}_t)^{\frac{1}{1-\lambda_f}} + \xi_p(\tilde{\pi}_t P_{t-1})^{\frac{1}{1-\lambda_f}} \right]^{1-\lambda_f}$$

Entrepreneurs

Rewriting entrepreneurs expected payoff (3.3.10) in terms of the cutoff point $\bar{\omega}_{t+1}$ and leverage $\frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}}$:

$$[1-\Gamma_t(\bar{\omega}_{t+1})] \frac{(1+R_{t+1}^k)}{(1+R_{t+1}^D)} \frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}}$$

where $\Gamma_t(\bar{\omega}_{t+1}) = G_t(\bar{\omega}_{t+1}) - \bar{\omega}_{t+1}[1-F_t(\bar{\omega}_{t+1})]$, $G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega)$

Proof:

Using (3.3.8) rewrite (3.3.10) as:

$$\frac{\int_{\bar{\omega}_{t+1}}^{\infty} [\omega(1+R_{t+1}^k)Q_{K,t}\bar{K}_{t+1} - (1+R_{t+1}^k)\bar{\omega}_{t+1}Q_{K,t}\bar{K}_{t+1}] dF_t(\omega)}{N_{t+1}(1+R_{t+1}^D)} =$$

$$= \int_{\bar{\omega}_{t+1}}^{\infty} [\omega - \bar{\omega}_{t+1}] dF_t(\omega) \frac{(1+R_{t+1}^k)}{(1+R_{t+1}^e)} \frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}} =$$

$$=\left\{\int_{\bar{\omega}_{t+1}}^{\infty}\omega dF_t(\omega)-\bar{\omega}_{t+1}\int_{\bar{\omega}_{t+1}}^{\infty}dF_t(\omega)\right\}\frac{(1+R_{t+1}^k)}{(1+R_{t+1}^D)}\frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}}=$$

$$=\left\{\left[1-\int_{0}^{\bar{\omega}_{t+1}}\omega dF_t(\omega)\right]-\bar{\omega}_{t+1}\left[1-\int_{0}^{\bar{\omega}_{t+1}}dF_t(\omega)\right]\right\}\frac{(1+R_{t+1}^k)}{(1+R_{t+1}^D)}\frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}}=$$

$$=\left\{1-G_t(\bar{\omega}_{t+1})-\bar{\omega}_{t+1}[1-F_t(\bar{\omega}_{t+1})]\right\}\frac{(1+R_{t+1}^k)}{(1+R_{t+1}^D)}\frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}}=$$

$$=[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{(1+R_{t+1}^k)}{(1+R_{t+1}^D)}\frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}}$$

Rewriting the banks zero profit condition (3.3.9) in terms of $\bar{\omega}_{t+1}$ and $\frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}}$:

$$[\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{(1+R_{t+1}^k)}{(1+R_{t+1}^D)}=1-\frac{N_{t+1}}{Q_{K,t}\bar{K}_{t+1}}$$

Proof:

$$[1-F_t(\bar{\omega}_{t+1})]\bar{\omega}_{t+1}(1+R_{t+1}^k)Q_{K,t}\bar{K}_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega\, dF_t(\omega)(1+R_{t+1}^k)Q_{K,t}\bar{K}_{t+1}=(1+R_{t+1}^D)\,\mathrm{L}_{t+1}$$

$$\Leftrightarrow\left[[1-F_t(\bar{\omega}_{t+1})]\bar{\omega}_{t+1}+(1-\mu)\int_0^{\bar{\omega}_{t+1}}\omega\, dF_t(\omega)\right]\frac{(1+R_{t+1}^k)}{(1+R_{t+1}^D)}\frac{Q_{\bar{K},t}\bar{K}_{t+1}}{L_{t+1}}=1$$

$$\Leftrightarrow\left[[1-F_t(\bar{\omega}_{t+1})]\bar{\omega}_{t+1}+\int_0^{\bar{\omega}_{t+1}}\omega\, dF_t(\omega)-\mu\int_0^{\bar{\omega}_{t+1}}\omega\, dF_t(\omega)\right]\frac{(1+R_{t+1}^k)}{(1+R_{t+1}^D)}\frac{Q_{K,t}\bar{K}_{t+1}}{Q_{K,t}\bar{K}_{t+1}-N_{t+1}}=1$$

$$\Leftrightarrow\left[\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})\right]\frac{(1+R_{t+1}^k)}{(1+R_{t+1}^D)}=\left(1-\frac{N_{t+1}}{Q_{\bar{K},t}\bar{K}_{t+1}}\right)$$

Due to constant returns to scale note that

$\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})$ is the share of entrepreneurial earnings received by the bank net of monitoring costs.

$1-\Gamma_t(\bar{\omega}_{t+1})$ denotes the share of gross entrepreneurial earnings retained by entrepreneurs.

The entrepreneur's problem of choosing an optimal debt contract therefore can be written as:

$$\max E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})}\frac{Q_{K,t}\bar{K}_{t+1}}{N_{t+1}}\right\}$$

$$\text{s.t.}\quad [\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})}=1-\frac{N_{t+1}}{Q_{K,t}\bar{K}_{t+1}}$$

The FOC associated with this problem are the zero profit condition and the following:

$$E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R^k_{t+1}}{1+R^D_{t+1}}+\frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1})-\mu G'_t(\bar{\omega}_{t+1})}\left[\frac{1+R^k_{t+1}}{1+R^D_{t+1}}\left(\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})\right)-1\right]\right\}=0 \tag{3.9}$$

Proof:

Under some regularity conditions[39] the lenders expected return achieves a maximum for some value of ω, say ω^*. For $\omega<\omega^*$ the function is increasing and concave. For $\omega>\omega^*$ expected return decreases due to the increased likelihood of default. If the lenders opportunity cost is so large that there does not exist an ω that generates the required expected return then the borrower is rationed from the market. We consider equilibria without rationing where ω lies below the maximum. Over that region the lenders expected return is an increasing function of ω. Replacing for entrepreneur's leverage from (3.3.12) into (3.3.11) we have:

$$\max_{\bar{\omega}_{t+1}} E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})}\left[\frac{1}{1-[\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})}}\right]\right\}$$

[39] $\dfrac{\partial\left(\omega\dfrac{dF(\omega)}{1-F(\omega)}\right)}{\partial\omega}>0$

This is equivalent to:

$$\max_{\bar{\omega}_{t+1}} E_t \left\{ \log[1-\Gamma_t(\bar{\omega}_{t+1})] + \log\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})} - \log\left(1-[\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})}\right)\right\}$$

The FOC is:

$$E_t\left\{\frac{1}{1-\Gamma_t(\bar{\omega}_{t+1})}\left(-\Gamma'_t(\bar{\omega}_{t+1})\right) - \frac{1}{1-[\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})}}[\Gamma'_t(\bar{\omega}_{t+1})-\mu G'_t(\bar{\omega}_{t+1})]\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})}\right\} = 0$$

$$\Leftrightarrow E_t\left\{\left(1-\Gamma_t(\bar{\omega}_{t+1})\right)\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})} + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1})-\mu G'_t(\bar{\omega}_{t+1})}\left[[\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{(1+R^k_{t+1})}{(1+R^D_{t+1})}-1\right]\right\} = 0$$

Note using the Leibnitz rule we have:

$$G'_t(\bar{\omega}_{t+1}) = \frac{\partial}{\partial\bar{\omega}_{t+1}}\int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega) = \bar{\omega}_{t+1}F'_t(\bar{\omega}_{t+1})$$

The law of motion of aggregate net worth (3.3.14) is:

$$N_{t+1} = \gamma\left\{(1+R^k_t)Q_{K,t-1}\bar{K}_t - \left[1+R^D_t+\frac{\mu\int_0^{\bar{\omega}_t}\omega dF_t(\omega)(1+R^k_t)Q_{K,t-1}\bar{K}_t}{Q_{K,t-1}\bar{K}_t-N_t}\right](Q_{K,t-1}\bar{K}_t-N_t)\right\}+W^e$$

<u>Proof:</u>

Note

$$N_{t+1} = \gamma V_t + W^e$$

where

$$
\begin{aligned}
V_t &= [1-\Gamma_t(\bar{\omega}_t)](1+R_t^k)Q_{K,t-1}\bar{K}_t = \\
&= (1+R_t^k)Q_{K,t-1}\bar{K}_t - (1+R_t^k)Q_{K,t-1}\bar{K}_t\Gamma_t(\bar{\omega}_t) = \\
&= (1+R_t^k)Q_{K,t-1}\bar{K}_t - (1+R_t^k)Q_{K,t-1}\bar{K}_t\left[\bar{\omega}_t\left(1-F_t(\bar{\omega}_t)\right)+\int_0^{\bar{\omega}_t}\omega dF(\omega)\right] = \\
&= (1+R_t^k)Q_{K,t-1}\bar{K}_t - (1+R_t^k)Q_{K,t-1}\bar{K}_t\left[\bar{\omega}_t\left(1-F_t(\bar{\omega}_t)\right)+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF(\omega)+\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)\right] = \\
&= (1+R_t^k)Q_{K,t-1}\bar{K}_t - \left\{\left[\bar{\omega}_t\left(1-F_t(\bar{\omega}_t)\right)+(1-\mu)\int_0^{\bar{\omega}_t}\omega dF(\omega)\right](1+R_t^k)Q_{K,t-1}\bar{K}_t+\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)(1+R_t^k)Q_{K,t-1}\bar{K}_t\right\} = \\
&= (1+R_t^k)Q_{K,t-1}\bar{K}_t - \left\{(1+R_t^e)B_t+\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)(1+R_t^k)Q_{K,t-1}\bar{K}_t\right\} = \\
&= (1+R_t^k)Q_{K,t-1}\bar{K}_t - \left\{(1+R_t^e)(Q_{\bar{K},t-1}\bar{K}_t-N_t)+\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)(1+R_t^k)Q_{K,t-1}\bar{K}_t\right\} = \\
&= (1+R_t^k)Q_{K,t-1}\bar{K}_t - \left\{(1+R_t^e)+\frac{\mu\int_0^{\bar{\omega}_t}\omega dF(\omega)(1+R_t^k)Q_{K,t-1}\bar{K}_t}{(Q_{\bar{K},t-1}\bar{K}_t-N_t)}\right\}(Q_{\bar{K},t-1}\bar{K}_t-N_t)
\end{aligned}
$$

where we have used the banks zero profit condition.

Households

We first consider the FOC's associated with non wage decisions C_t, D_t.

The Lagrangian for the problem ignoring constants and equations associated with wage decisions is:

$$
L = E_t\sum_{l=0}^{\infty}\beta^l\left\{\begin{array}{l}\log(C_{t+l}-bC_{t+l-1})-\psi_L\dfrac{h_{j,t+l}^{1+\sigma_L}}{1+\sigma_L}+ \\ +\lambda_{t+l}\left[W_{j,t+l}h_{j,t+l}+[\Pi_{t+l}+A_{j,t+l}-W^e-\mathrm{T}_t]+(1+R_{t+l}^D)D_{t+l-1}-D_{t+l}-P_{t+l}C_{t+l}\right]\end{array}\right\}
$$

where λ_t is the household multiplier.

It is easy to show that we obtain FOC's (3.5.8) and (3.5.9).

Turning to wage decisions we suppose the j^{th} household can reoptimize its wage at time t. Since each household that is able to reoptimize sets the same wage we can denote this wage rate by $\tilde{W}_t$. In choosing $\tilde{W}_t$ the household considers the discounted utility of future periods for which it cannot reoptimize. Neglecting terms that are not associated with the wage decision problem this is given by:

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left\{ -\zeta_{t+i} \psi_L \frac{h_{j,t+i}^{1+\sigma_L}}{1+\sigma_L} + \lambda_{t+i} \left[W_{j,t+i} h_{j,t+i} \right] \right\}$$

The demand for the j^{th} household's labor services conditional on it having optimized in period t and not again since, is:

$$h_{j,t+i} = \left(\frac{\tilde{W}_{t+i}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+i}$$

Note $\dfrac{\tilde{W}_{t+i}}{W_{t+i}} = \dfrac{\tilde{W}_t \pi_{t+i-1} \mu_z .... \pi_t \mu_z}{W_{t+i}} = \dfrac{\tilde{w}_t W_t}{W_{t+i}} X_{t,i}^w = \dfrac{\tilde{w}_t w_t}{w_{t+i}} X_{t,i}^w$

where $\tilde{w}_t = \dfrac{\tilde{W}_t}{W_t}$

and

$$X_{t,i}^w = \begin{cases} \dfrac{\tilde{\pi}_{t+i} .... \tilde{\pi}_{t+1}}{\pi_{t+i} .... \pi_{t+1}} \mu_z^i \, , i > 0 \\ 1 \; , \; i = 0 \end{cases}$$

Also

$$\frac{W_{j,t+i}}{P_{t+i} z_{t+i}} = X_{t,i}^w \frac{W_{j,t}}{P_t z_t} = X_{t,i}^w \frac{\tilde{W}_t}{W_t} \frac{W_t}{P_t z_t} = X_{t,i}^w \tilde{w}_t w_t$$

Substituting for $h_{j,t}$ from (3.5.6) and using the above notation the maximization problem neglecting non wage decisions is written as:[40]

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left\{ \psi_L \frac{\left(\left(\frac{\tilde{w}_t w_t}{w_{t+i}} X^w{}_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} + \lambda_{t+i} \left[w_{t+i} \left(\frac{\tilde{w}_t w_t}{w_{t+i}} X^w{}_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}+1} l_{t+i} \right] \right\}$$

Differentiating w.r.t $\tilde{w}_t$ and rearranging we obtain the necessary FOC for household optimization of $\tilde{w}_t$.

$$E_t \sum_{i=0}^{\infty} (\beta\xi_w)^i \left(\frac{w_t}{w_{t+i}} X^w{}_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+i} \left\{ \frac{\lambda_{z,t+i}}{\lambda_w} \tilde{w}_t w_t X_{t,i} - \psi_L \left(\left(\frac{\tilde{w}_t w_t}{w_{t+i}} X^w{}_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} l_{t+i} \right)^{\sigma_L} \right\} = 0$$

We can rewrite the FOC as:

$$\tilde{w}_t = \left[\frac{\psi_L}{w_t} \frac{J_{w,t}}{F_{w,t}} \right]^{\frac{\lambda_w - 1}{\lambda_w (1+\sigma_L) - 1}}$$

where $J_{w,t}$ *and* $F_{w,t}$ are expressed in recursive form as follows:

$$E_t \left\{ l_t \frac{\lambda_z}{\lambda_w} + \beta\xi_w \mu_z^{\frac{1}{1-\lambda_w}-1} \left(\frac{1}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \frac{\tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} F_{w,t+1} - F_{w,t} \right\} = 0$$

$$E_t \left\{ l_t^{1+\sigma_L} + \beta\xi_w \left(\mu_z \frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} J_{w,t+1} - J_{w,t} \right\} = 0$$

[40] l_t denotes the unweighted labor integral and we have assumed, based on Yun (1996), that

$$w_t^* = \left[\int_0^1 W_t(j)^{\frac{\lambda_w}{1-\lambda_w}} dj \right]^{\frac{1-\lambda_w}{\lambda_w}} \Big/ W_t = 1$$

The aggregate wage index is given by:

$$W_t = \left[(1-\xi_w)(\tilde{W}_t)^{\frac{1}{1-\lambda_w}} + \xi_w(\pi_{t-1}\mu_z W_{t-1})^{\frac{1}{1-\lambda_w}} \right]^{1-\lambda_w}$$

Dividing both sides by W_t and rearranging:

$$\tilde{w}_t = \left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mu_z\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{1-\lambda_w}$$

where $\pi_{w,t} \equiv \frac{W_t}{W_{t-1}} = \frac{w_t z_t P_t}{w_{t-1} z_{t-1} P_{t-1}} = \frac{w_t \mu_z \pi_t}{w_{t-1}}$

Substituting the above expression of the aggregate wage index into the households FOC for wage optimization we have:

$$J_{w,t} = \frac{1}{\psi_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mu_z\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right]^{1-\lambda_w(1+\sigma_L)} w_t F_{w,t}$$

A.2 Dynare and Matlab Code

```
// Dynare Code for Financial Accelerator Model.
// Note that the code below includes a larger number of variables
and shocks than needed.
// Notation: UU - parameter, U - variable, if there are uppercase
letters, then (in case the code is run in Dynare 3 that does not
recognize uppercase) there is an X

dbstop if error

var piU sU rkU iU RkXU qU RXU uzcU cU wU lhU kbarU pstarU wstarU
FpXU KpXU FwXU KwXU wplusU yzU
lambdafU dly dli dlc uU gdU gndU Util Welf serialmps epsilU
zetaiU zetacU pitargetU
     signalstate1 signalstate2 signalstate3 signalstate4
signalstate5 signalstate6 signalstate7
signalstate8 muupU dlgd dlpi dlh linfl SXU SpXU lambdazU muzstarU
ygdpU zetaU tauoU pitildewU piwU FXU
// variables associated with entrepreneurs
loansU premU omegabarU sigmaU nU dU gammaU FprimeU GXU FomegaXU
GAMMAXU riskyRXU;


varexo tau0x taulx taukx lambdafx mps epsilx zetaix zetacx sig1
sig2 sig3 sig5 sig6 sig7 sig8 sig4 gdx gndx pitargetx muupx
muzstarx zetax sigmax gammax;

//PARAMETERS
parameters rkUU, nUU, kUU, hUU, iUU, wUU, dUU, cUU,lambdazUU,
yzUU, sigmaUU, omegabarUU, phiUU, mcUU, pitildUU, pstarUU,
pitildwUU, wstarUU, wplusUU, RkXUU, RXUU, FpXUU, KpXUU, FwXUU,
KwXUU, uczUU, qUU, GammaXUU, Gam_muGXUU,
lambdafUU, piUU, muzstarUU, betaUU, deltaUU, FomegabarXUU, muUU,
alphaUU, gammaUU, sigmaLXUU,
psiLXUU, lambdawUU, weUU, bigthetaUU, bUU, muupUU, taukUU,
upsilUU, iota1UU, iota2UU,
pibarUU, xipUU, iotaw1UU, iotaw2UU, xiwUU, epsilUU, gdUU, gndUU,
zetaiUU, tauoUU, zetaUU, psikUU, psilUU, rhoUU, zetacUU,
xUU, pitargetUU, nbeta,piwUU,
rhotilUU,aptilUU,aytilUU,taucUU,SdouprXUU,sigmaaUU, iotamuUU,
taulUU, actilUU,
        rho_lambdafUU, rho_zetaiUU, rho_zetacUU,
rho_gammaUU,rho_sigmaUU,rho_muupUU,rho_muzstarUU,rho_zetaUU,
rho_tauoUU,
lambdawsUU,lambdafsUU,
        rho_mps,
respnd,rho1d,rho2d,rho3d,rho1nd,rho2nd,rho3nd,rho_pitargetUU,
        rho1_taul, rho2_taul, rho3_taul, rho1_tauk, rho2_tauk,
rho3_tauk, dtaul, dtauk, resptaul, resptauk, ygdpUU,
taulU,taukU;

params_mode;
tauoUU=tauoUUst;
zetaiUU=zetaiUUst;
```

```
zetaUU=zetaUUst;
zetacUU=zetacUUst;
nbeta=nbetast;
taukUU=0;
taulUU=0;
taucUU=0;
psikUU=0;
psilUU=0;
iotamuUU=0;
taulU=taulUU;
taukU=taukUU;

// get parameter values and steady states...
[xgdUU,xgndUU,xrkUU, xnUU, xkUU, xhUU, xiUU, xwUU, xdUU, xcUU,
xlambdazUU, xyzUU, xsigmaUU, xomegabarUU, xphiUU, xmcUU,
xpitildUU, xpstarUU, xpiwUU,xpitildwUU, xwstarUU, xwplusUU,
xRkXUU, xRXUU, xFpXUU, xKpXUU, xFwXUU, xKwXUU, xuczUU, xqUU,
xGammaXUU, xGam_muGXUU,xlambdafUU, xpiUU, xmuzstarUU, xbetaUU,
xdeltaUU, xFomegabarXUU, xmuUU, xalphaUU, xgammaUU,
xsigmaLXUU,xpsiLXUU, xlambdawUU, xweUU, xbigthetaUU, xbUU,
xmuupUU, xtaukUU, xupsilUU, xiota1UU, xiota2UU,xpibarUU, xxipUU,
xiotaw1UU, xiotaw2UU, xxiwUU,
xepsilUU,xxUU,xrhotilUU,xaptilUU,xaytilUU,xactilUU,xSdouprXUU,xsi
gmaaUU,xlambdawsUU,xlambdafsUU] = getsteadyexogm;

// assign values computed in sstate to variables declared as
parameters in the parameter statement
gdUU=xgdUU;gndUU=xgndUU;rkUU=xrkUU; nUU=xnUU; kUU=xkUU; hUU=xhUU;
iUU=xiUU; wUU=xwUU;
dUU=xdUU; cUU=xcUU; lambdazUU=xlambdazUU; yzUU=xyzUU;
sigmaUU=xsigmaUU;
omegabarUU=xomegabarUU; phiUU=xphiUU; mcUU=xmcUU;
pitildUU=xpitildUU;
pstarUU=xpstarUU; piwUU=xpiwUU;pitildwUU=xpitildwUU;
wstarUU=xwstarUU;
wplusUU=xwplusUU; RkXUU=xRkXUU; RXUU=xRXUU; FpXUU=xFpXUU;
KpXUU=xKpXUU;
FwXUU=xFwXUU; KwXUU=xKwXUU; uczUU=xuczUU; qUU=xqUU;
GammaXUU=xGammaXUU;
Gam_muGXUU=xGam_muGXUU;lambdafUU=xlambdafUU; piUU=xpiUU;
muzstarUU=xmuzstarUU;
betaUU=xbetaUU; deltaUU=xdeltaUU; FomegabarXUU=xFomegabarXUU;
muUU=xmuUU;
alphaUU=xalphaUU; gammaUU=xgammaUU;
sigmaLXUU=xsigmaLXUU;psiLXUU=xpsiLXUU;
lambdawUU=xlambdawUU; weUU=xweUU; bigthetaUU=xbigthetaUU;
bUU=xbUU; muupUU=xmuupUU;
taukUU=xtaukUU; upsilUU=xupsilUU; iota1UU=xiota1UU;
iota2UU=xiota2UU;pibarUU=xpibarUU;
xipUU=xxipUU; iotaw1UU=xiotaw1UU; iotaw2UU=xiotaw2UU;
xiwUU=xxiwUU; epsilUU=xepsilUU;
xUU=xxUU;rhotilUU=xrhotilUU;aptilUU=xaptilUU;aytilUU=xaytilUU;act
ilUU=xactilUU;
SdouprXUU=xSdouprXUU;sigmaaUU=xsigmaaUU;lambdawsUU=xlambdawsUU;la
mbdafsUU=xlambdafsUU;

// get values for the stochastic parameters of the model
paramsshocks
```

```
// assign values to the names of the stochastic parameters
rho_lambdafUU=rho_lambdafUUst;rho_zetaiUU=rho_zetaiUUst;
rho_zetaUU=rho_zetaUUst;rho_zetacUU=rho_zetacUUst;
rho_gammaUU=rho_gammaUUst;rho_sigmaUU=rho_sigmaUUst;
rho_muupUU=rho_muupUUst;rho_muzstarUU=rho_muzstarUUst;
rho_mps=rho_mpsst;rho_tauoUU=rho_tauoUUst;rho_pitargetUU=rho_pita
rgetUUst;
rho1nd=rho1ndst;rho2nd=rho2ndst;rho3nd=rho3ndst;respnd=respndst;
rhoUU=rhoUUst;rho1d=rho1dst;rho2d=rho2dst;rho3d=rho3dst;
rho1_taul=rho1_taulst;rho2_taul=rho2_taulst;
rho3_taul=rho3_taulst;rho1_tauk=rho1_taukst;
rho2_tauk=rho2_taukst;rho3_tauk=rho3_taukst;
resptaul=resptaulst;resptauk=resptaukst;
dtaul=dtaulst;dtauk=dtaukst;

pitargetUU=(1+xUU)/muzstarUU;

UtilUU = zetacUU*(log(cUU*muzstarUU-bUU*cUU)-
zetaUU*psiLXUU/(1+sigmaLXUU)*(wstarUU/wplusUU)^((1+sigmaLXUU)*lam
bdawUU/(lambdawUU-1))*hUU^(1+sigmaLXUU));
WelfUU=UtilUU/(1-nbeta);
xUtilUU=UtilUU;xWelfUU=WelfUU;
ygdpUU = gdUU + gndUU + cUU + iUU/muupUU;
ixx=1;
pitildewUU=(pitargetUU^iotaw1UU)*(piUU^iotaw2UU)*(pibarUU^(1-
iotaw1UU-iotaw2UU));
if abs(pitildewUU-pitildwUU) > .1e-10
error('fatal (model_financial) two different ss values for
pitilde')
end

model;

// period utility
Util = zetacU*(log(cU*muzstarU-bUU*cU(-1))-
zetaU*psiLXUU/(1+sigmaLXUU)*(wstarU/wplusU)^((1+sigmaLXUU)*lambda
wUU/(lambdawUU-1))*exp(lhU)^(1+sigmaLXUU));

// welfare
Welf = Util + nbeta*Welf(+1);

// Monetary Policy Rule
log(1+RXU) = (1-rhotilUU)*log(1+RXUU) + rhotilUU*log(1+RXU(-1)) +
(1/(1+RXUU))*(1-rhotilUU)*aptilUU*piUU*(log(piU(+1)/pitargetU))
   + (1-rhotilUU)*aytilUU*(1/(4*(1+RXUU)))*log(ygdpU/ygdpUU)-
0*(1-rhotilUU)*0.01*(1/(4*(1+RXUU)))*log(premU) + serialmps;


//marginal cost in terms of both factor costs
-sU+((1-alphaUU)^(alphaUU-1)*alphaUU^(-
alphaUU))*((rkU*(1+psikUU*RXU))^alphaUU)*((wU*(1+psilUU*RXU))^(1-
alphaUU))/epsilU;

//marginal cost in terms of rental rate of capital
rkU*(1+psikUU*RXU)/( alphaUU*epsilU*( (upsilUU*muzstarU*(
exp(lhU)*(wstarU ^ (lambdawUU/(lambdawUU-1)) ) )/(uU*kbarU(-
1)))^(1-alphaUU) ) )-sU;
```

```
// multiplier on household budget constraint is zetacU*lambdazU
lambdazU=uzcU/(zetacU*(1+taucUU));

//adjustment cost function
SXU= exp( ( sqrt(SdouprXUU/2) )*((
zetaiU*iU*muzstarU*upsilUU/iU(-1) )-muzstarUU*upsilUU))
  +  exp(-( sqrt(SdouprXUU/2) )*((
zetaiU*iU*muzstarU*upsilUU/iU(-1) )-muzstarUU*upsilUU)) - 2 ;

//derivative of adjustment cost function
SpXU=  sqrt(SdouprXUU/2) * ( exp( ( sqrt(SdouprXUU/2) )*((
zetaiU*iU*muzstarU*upsilUU/iU(-1) )-muzstarUU*upsilUU))
                           - exp(-( sqrt(SdouprXUU/2) )*((
zetaiU*iU*muzstarU*upsilUU/iU(-1) )-muzstarUU*upsilUU)) )  ;

//I fonc
zetacU*lambdazU/muupU = zetacU*qU*lambdazU*(1-SXU-
SpXU*zetaiU*upsilUU*muzstarU*iU/iU(-1))
  +
betaUU*(zetacU(+1)*zetaiU(+1)*qU(+1)*lambdazU(+1)/(upsilUU*muzsta
rU(+1)))*SpXU(+1)*(upsilUU*muzstarU(+1)*iU(+1)/iU)^2;

//this is the first order condition associated with utilization
//rkU - tauoU*rkUU*exp(sigmaaUU*(uU-1));
rkU-sigmaaUU*(uU-1)-rkUU;

// definition of rate of return on capital
RkXU - ( ( (1-taukU)*(uU*rkU-tauoU*( rkUU*(exp(sigmaaUU*(uU-1))-
1)/sigmaaUU ))+(1-deltaUU)*qU)*piU/(upsilUU*qU(-1)) +
taukU*deltaUU - 1);

// marginal utility of consumption
uzcU - (muzstarU*zetacU/(cU*muzstarU-bUU*cU(-1))) +
bUU*betaUU*zetacU(+1)/(cU(+1)*muzstarU(+1)-bUU*cU);

// definition of nominal rate of interest
(betaUU*zetacU(+1)* lambdazU(+1)/(piU(+1)*muzstarU(+1)))*(1+RXU)
- zetacU*lambdazU ;

// ex ante real rate of interest
(betaUU*zetacU(+1)*lambdazU(+1)/(muzstarU(+1)))*FXU -
zetacU*lambdazU;

// output
yzU - ((pstarU^(lambdafU/(lambdafU-1)))*(epsilU*((uU*kbarU(-
1)/(muzstarU*upsilUU))^alphaUU )
             *(( exp(lhU)*(wstarU ^ (lambdawUU/(lambdawUU-1)) )
)^(1-alphaUU) ) - phiUU ) );

// 6-created variables for estimation
dly   =  log((ygdpU*muzstarU)/ygdpU(-1))-log(muzstarUU);
dli   =  log(iU)-log(iU(-1))+log(muzstarU)-log(muzstarUU);
dlc   =  log(cU*muzstarU/cU(-1))-log(muzstarUU);
dlgd  =  log(gdU*muzstarU/gdU(-1))-log(muzstarUU);
dlpi  =  -(log(muupU)-log(muupU(-1)));
dlh   =  (lhU-lhU(-1));
linfl =  log(piU)-log(piUU);
```

```
// national income identity:
ygdpU = gdU + gndU + cU + iU/muupU;

// capital law of motion
kbarU - (1-deltaUU)*(kbarU(-1)/(muzstarU*upsilUU)) - (1-SXU)*iU;

// sticky price/wage equations....

// p-star equation
pstarU = ((1-xipUU)*(KpXU/FpXU)^(lambdafU/(1-
lambdafU))+xipUU*(piU(-1)*pstarU(-1)/piU)^(lambdafU/(1-
lambdafU)))^((1-lambdafU)/lambdafU);

// w-star equation
wstarU = ( (1-xiwUU) * ((psiLXUU*KwXU/(wU*FwXU))^(lambdawUU/(1-
lambdawUU*(1+sigmaLXUU))))
  + xiwUU * (( (pitargetU^iotaw1UU) * (piU(-1)^iotaw2UU) *
(pibarUU^(1-iotaw1UU-iotaw2UU)) )/( piU*muzstarU*wU/wU(-1) )
    * (muzstarUU^(1-iotamuUU)) * (muzstarU^iotamuUU) * wstarU(-
1))^(lambdafUU/(1-lambdafUU)) ) ^ ((1-lambdawUU)/lambdawUU);

// Fp equation
-FpXU + zetacU*lambdazU*yzU
 + betaUU * xipUU * ( (( (pitargetU(+1)^iota1UU) * (piU^iota2UU)
* pibarUU^(1-iota1UU-iota2UU) )/piU(+1)) ^ (1/(1-lambdafU(+1))) )
* FpXU(+1);

// Kp equation
-KpXU + zetacU*(lambdafU/lambdafsUU)*lambdazU*yzU*sU
 + betaUU * xipUU * ( (( (pitargetU(+1)^iota1UU) * (piU^iota2UU)
* pibarUU^(1-iota1UU-iota2UU) )/piU(+1)) ^ (lambdafU(+1)/(1-
lambdafU(+1))) ) * KpXU(+1);

//
KpXU=FpXU * (( (1 - xipUU*(( (pitargetU^iota1UU) * (piU(-
1)^iota2UU) * pibarUU^(1-iota1UU-iota2UU) )/piU)^(1/(1-lambdafU))
)/ (1-xipUU) )^(1-lambdafU));


//wage-updating equation
pitildewU=(pitargetU^iotaw1UU)*(piU(-1)^iotaw2UU)*(pibarUU^(1-
iotaw1UU-iotaw2UU));

//nominal wage inflation
piwU=piU*muzstarU*wU/wU(-1);

// Fw equation
lambdawsUU*zetacU*((wstarU^(lambdawUU/(lambdawUU-
1)))*exp(lhU)*(1-taulU)*lambdazU/lambdawUU)
  + betaUU * xiwUU * (muzstarUU^((1-iotamuUU)/(1-lambdawUU))) *(
(pitildewU(+1)^(1/(1-lambdawUU))) / piU(+1) )
 *( (1/piwU(+1)) ^ (lambdawUU/(1-lambdawUU)) ) *
(muzstarU(+1)^(iotamuUU/(1-lambdawUU)-1)) * FwXU(+1) - FwXU;
```

```
// Kw equation

-KwXU + zetacU*zetaU*((wstarU^(lambdawUU/(lambdawUU-
1))*exp(lhU))^(1+sigmaLXUU))
 + betaUU * xiwUU * (( (pitildewU(+1)/piwU(+1)) * (muzstarUU^(1-
iotamuUU)) * (muzstarU(+1)^iotamuUU) )
 ^ (lambdawUU*(1+sigmaLXUU)/(1-lambdawUU))) * KwXU(+1);

KwXU =  ( ( (1 - xiwUU * ( ( (pitildewU/piwU)
 * (muzstarUU^(1-iotamuUU)) * (muzstarU^iotamuUU) ) ^ (1/(1-
lambdawUU)) ) ) / (1-xiwUU) ) ^ (1-lambdawUU*(1+sigmaLXUU)) ) *
wU * FwXU/psiLXUU;

// w-plus equation
wplusU - ( ((1-xiwUU)*( ( (1 - xiwUU * ( ( (pitildewU/piwU) *
(muzstarUU^(1-iotamuUU)) * (muzstarU^iotamuUU) ) ^ (1/(1-
lambdawUU)) ) ) / (1-xiwUU) )^(lambdawUU*(1+sigmaLXUU)) ) +
xiwUU*( (( (( (pitargetU^iotaw1UU) * (piU(-1)^iotaw2UU) *
(pibarUU^(1-iotaw1UU-iotaw2UU)) )/( piU*muzstarU*wU/wU(-1) )) *
(muzstarUU^(1-iotamuUU)) * (muzstarU^iotamuUU) )*wplusU(-1))^(
lambdawUU*(1+sigmaLXUU)/(1-lambdawUU) ) ) )^(1/(
lambdawUU*(1+sigmaLXUU)/(1-lambdawUU) )) );

// equations pertaining to the entrepreneur
// entrepreneur
loansU=(qU*kbarU-nU);

// period t premium = interest paid by entrepreneurs in period t
- interest earned by households in period t.
premU=(qU(-1)*kbarU(-1)/(qU(-1)*kbarU(-1)-nU(-
1)))*(1+RkXU)*omegabarU-(1+RXU(-1));

riskyRXU=(qU(-1)*kbarU(-1)/(qU(-1)*kbarU(-1)-nU(-
1)))*(1+RkXU)*omegabarU;

// law of motion for aggregate net worth
-nU+(gammaU/(piU*muzstarU))*(RkXU-RXU(-1) -
muUU*GXU*(1+RkXU))*kbarU(-1)*qU(-1)+weUU+gammaU*(1+RXU(-1))*nU(-
1)/(piU*muzstarU);

// zero profit condition on banks
(qU(-1)*kbarU(-1)*(1+RkXU)*( GAMMAXU - muUU*GXU )/(nU(-
1)*(1+RXU(-1))))-(qU(-1)*kbarU(-1)/nU(-1))+1;



// efficiency condition on standard debt contract
(1-GAMMAXU(+1))*( (1+RkXU(+1))/(1+RXU) ) + ( (1 -
FomegaXU(+1))/(1- FomegaXU(+1) - muUU*omegabarU(+1)*FprimeU(+1))
)
   * ( ( (1+RkXU(+1))/(1+RXU) )*( GAMMAXU(+1)-muUU*GXU(+1)) - 1
);


FprimeU=(1/sqrt(2*3.14159265358979))*exp(-
.5*((log(omegabarU)+sigmaU(-1)^2/2)/sigmaU(-
1))^2)/(omegabarU*sigmaU(-1));
```

```
GXU=normcdf(( (log(omegabarU)+sigmaU(-1)^2/2)/sigmaU(-1) )-
sigmaU(-1),0,1);

FomegaXU=normcdf(( (log(omegabarU)+sigmaU(-1)^2/2)/sigmaU(-1)
),0,1);

GAMMAXU=omegabarU*(1-FomegaXU)+GXU;

// resources used up in monitoring
dU = muUU*GXU*(1+RkXU)*qU(-1)*kbarU(-1)/(muzstarU*piU);

log(sigmaU/sigmaUU) = rho_sigmaUU*log(sigmaU(-1)/sigmaUU) +
sigmax;
log(gammaU/gammaUU) = rho_gammaUU*log(gammaU(-1)/gammaUU) +
gammax;

// resource constraint
yzU - gndU - gdU  - cU - iU/muupU - dU - bigthetaUU*((1-
gammaU)/gammaU)*(nU-weUU) -
tauoU*(rkUU/sigmaaUU)*(exp(sigmaaUU*(uU-1))-1);



// exogenous shocks

log(muzstarU/muzstarUU) = rho_muzstarUU*log(muzstarU(-
1)/muzstarUU)+muzstarx;
log(tauoU/tauoUU) = rho_tauoUU*log(tauoU(-1)/tauoUU) + tau0x;
log(epsilU) = rhoUU*log(epsilU(-1))+epsilx;
log(lambdafU/lambdafUU) = rho_lambdafUU*log(lambdafU(-
1)/lambdafUU)+lambdafx;
log(muupU/muupUU) = rho_muupUU*log(muupU(-1)/muupUU)+muupx;
log(zetacU/zetacUU) = rho_zetacUU*log(zetacU(-1)/zetacUU)+zetacx;
log(zetaiU/zetaiUU) = rho_zetaiUU*log(zetaiU(-1)/zetaiUU)+zetaix;
log(zetaU/zetaUU) = rho_zetaUU*log(zetaU(-1)/zetaUU)+zetax;
serialmps = rho_mps*serialmps(-1) + mps ;
log(gdU/gdUU) = rho1d*0+0.988*log(gdU(-1)/gdUU) +
0*rho2d*log(gdU(-2)/gdUU) + 0*rho3d*log(gdU(-3)/gdUU) + gdx +
signalstate1(-1);
//log(taulU/taulUU) = rho1_taul*log(taulU(-1)/taulUU) +
rho2_taul*log(taulU(-2)/taulUU) + rho3_taul*log(taulU(-3)/taulUU)
//            + taulx + resptaul*log(ygdpU(-1)/yzUU)+
dtaul*log(gdU(-1)/gdUU);
//log(taukU/taukUU) = rho1_tauk*log(taukU(-1)/taukUU) +
rho2_tauk*log(taukU(-2)/taukUU) + rho3_tauk*log(taukU(-3)/taukUU)
//            + taukx + resptauk*log(ygdpU(-1)/yzUU) +
dtauk*log(gdU(-1)/gdUU);
log(gndU/gndUU) = rho1nd*log(gndU(-1)/gndUU) + rho2nd*log(gndU(-
2)/gndUU) + rho3nd*log(gndU(-3)/gndUU) + gndx
                   + respnd*log(ygdpU(-1)/yzUU);
log(pitargetU/pitargetUU)=  rho_pitargetUU * ( log(pitargetU(-
1)/pitargetUU) ) + pitargetx;

signalstate8=sig8;
signalstate7=signalstate8(-1)+sig7;
signalstate6=signalstate7(-1)+sig6;
signalstate5=signalstate6(-1)+sig5;
signalstate4=signalstate5(-1)+sig4;
```

```
signalstate3=signalstate4(-1)+sig3;
signalstate2=signalstate3(-1)+sig2;
signalstate1=signalstate2(-1)+sig1;

end;

initval;
riskyRXU=(kUU/(kUU-nUU))*(1+RkXUU)*omegabarUU;
GXU=normcdf(( (log(omegabarUU)+sigmaUU^2/2)/sigmaUU )-
sigmaUU,0,1);
FomegaXU=normcdf(( (log(omegabarUU)+sigmaUU^2/2)/sigmaUU ),0,1);
GAMMAXU=omegabarUU*(1-normcdf((
(log(omegabarUU)+sigmaUU^2/2)/sigmaUU ),0,1))+normcdf((
(log(omegabarUU)+sigmaUU^2/2)/sigmaUU )-sigmaUU,0,1);
FprimeU=(1/sqrt(2*3.14159265358979))*exp(-
.5*((log(omegabarUU)+sigmaUU^2/2)/sigmaUU)^2)/(omegabarUU*sigmaUU
);
gammaU=gammaUU;
omegabarU=omegabarUU;
loansU=(kUU-nUU);
premU=(kUU/(kUU-nUU))*(1+RkXUU)*omegabarUU-(1+RXUU);
sigmaU=sigmaUU;
dU=dUU;
nU=nUU;
pitildewU=pitildewUU;
piwU=piUU*muzstarUU;
tauoU=tauoUU;
zetaU=1;
KpXU=KpXUU;
KwXU=KwXUU;
ygdpU=gdUU + gndUU + cUU + iUU/muupUU;
muzstarU=muzstarUU;
SXU=0;
SpXU=0;
lambdazU=lambdazUU;
linfl=0;
dlh=0;
pitargetU=pitargetUU;
muupU=muupUU;
FXU=muzstarUU/betaUU;
yzU=yzUU;
qU=qUU;
piU=piUU;
RkXU=RkXUU;
rkU=rkUU;
RXU=RXUU;
pstarU=pstarUU;
wstarU=wstarUU;
wplusU=wplusUU;
kbarU=kUU;
lhU=log(hUU);
cU=cUU;
iU=iUU;
FpXU=FpXUU;
FwXU=FwXUU;
uzcU=lambdazUU*zetacUU*(1+taucUU);
sU=mcUU;
wU=wUU;
```

```
uU=1;
Util=UtilUU;
Welf=WelfUU;
lambdafU=lambdafUU;
epsilU=1;
zetaiU=zetaiUU;
serialmps=0;
dly = 0;
dli = 0;
dlc = 0;
dlgd = 0;
dlpi = 0;
gdU=gdUU;
gndU=gndUU;
zetacU=zetacUU;
signalstate1=0;
signalstate2=0;
signalstate3=0;
signalstate4=0;
signalstate5=0;
signalstate6=0;
signalstate7=0;
signalstate8=0;

end;

steady;

// following are the shocks....the ones with zero variance are in
effect not activated.
shocks;
var sigmax;
stderr 1;
var gammax;
stderr 0;
var muupx;
stderr 0;
var zetax;
stderr 0;
var pitargetx;
stderr 0;
var taulx;
stderr 0;
var taukx;
stderr 0;
var gdx;
stderr 0.01;
var gndx;
stderr 0.0;
var epsilx;
stderr 0;
var zetacx;
stderr 0;
var zetaix;
stderr 0;
var mps;
stderr 0;
var lambdafx;
```

```
stderr 0;
var sig1;
stderr 0;
var sig2;
stderr 0;
var sig3;
stderr 0;
var sig4;
stderr 0;
var sig5;
stderr 0;
var sig6;
stderr 0;
var sig7;
stderr 0;
var sig8;
stderr 0;
var muzstarx;
stderr 0;

end;

stoch_simul(order=1,irf=200,nograph) kbarU riskyRXU FXU wU sU cU
iU ygdpU RkXU RXU epsilU piU nU qU rkU lhU premU loansU yzU dU
FomegaXU debt;



cons_base_risk=100*cU_sigmax/cUU;
inv_base_risk=100*iU_sigmax/(iUU/muupUU);
gdp_base_risk=100*ygdpU_sigmax/(gdUU + gndUU + cUU + iUU/muupUU);
networth_base_risk=100*nU_sigmax/nUU;
premium_base_risk=premU_sigmax;
loans_base_risk=100*loansU_sigmax/(gdUU + gndUU + cUU +
iUU/muupUU);
riskf_base_risk=RXU_sigmax;

cons_base_gov=100*cU_gdx/cUU;
inv_base_gov=100*iU_gdx/(iUU/muupUU);
gdp_base_gov=100*ygdpU_gdx/(gdUU + gndUU + cUU + iUU/muupUU);
networth_base_gov=100*nU_gdx/nUU;
premium_base_gov=premU_gdx;
loans_base_gov=100*loansU_gdx/(gdUU + gndUU + cUU + iUU/muupUU);
debt_base_gov=debt_gdx;
//pdebt_debt_gov=pdebt_gdx;
rloans_base_gov=RXU_gdx;

save CEEplusBGG cons_base_risk inv_base_risk gdp_base_risk
networth_base_risk premium_base_risk loans_base_risk
riskf_base_risk cons_base_gov inv_base_gov gdp_base_gov
networth_base_gov premium_base_gov loans_base_gov rloans_base_gov
debt_base_gov;
```

```
// Dynare Code for Sovereign Risk Model.
// Note that the code below includes a larger number of variables
and shocks than needed.
// Notation: UU - parameter, U - variable, if there are uppercase
letters, then (in case the code is run in Dynare 3 that does not
recognize uppercase) there is an X

dbstop if error

var piU sU rkU iU RkXU qU RXU uzcU cU wU lhU kbarU pstarU wstarU
FpXU KpXU FwXU KwXU wplusU yzU
lambdafU dly dli dlc uU gdU gndU Util Welf serialmps epsilU
zetaiU zetacU pitargetU
     signalstate1 signalstate2 signalstate3 signalstate4
signalstate5 signalstate6 signalstate7
signalstate8 muupU dlgd dlpi dlh linfl SXU SpXU lambdazU muzstarU
ygdpU zetaU tauoU pitildewU piwU FXU RXXU
// variables associated with entrepreneurs
loansU premU omegabarU sigmaU nU dU gammaU FprimeU GXU FomegaXU
GAMMAXU riskyRXU
//variables associated with risky debt
debt pdebt tax;

varexo tau0x taulx taukx lambdafx mps epsilx zetaix zetacx sig1
sig2 sig3 sig5 sig6 sig7 sig8 sig4 gdx gndx pitargetx muupx
muzstarx zetax sigmax gammax;

//PARAMETERS
parameters rkUU, nUU, kUU, hUU, iUU, wUU, dUU, cUU,lambdazUU,
yzUU, sigmaUU, omegabarUU, phiUU, mcUU, pitildUU, pstarUU,
pitildwUU, wstarUU, wplusUU, RkXUU, RXUU, FpXUU, KpXUU, FwXUU,
KwXUU, uczUU, qUU, GammaXUU, Gam_muGXUU,
lambdafUU, piUU, muzstarUU, betaUU, deltaUU, FomegabarXUU, muUU,
alphaUU, gammaUU, sigmaLXUU,
psiLXUU, lambdawUU, weUU, bigthetaUU, bUU, muupUU, taukUU,
upsilUU, iota1UU, iota2UU,
pibarUU, xipUU, iotaw1UU, iotaw2UU, xiwUU, epsilUU, gdUU, gndUU,
zetaiUU, tauoUU, zetaUU, psikUU, psilUU, rhoUU, zetacUU,
xUU, pitargetUU, nbeta,piwUU,
        //RXXUU
rhotilUU,aptilUU,aytilUU,taucUU,SdouprXUU,sigmaaUU, iotamuUU,
taulUU, actilUU,
        rho_lambdafUU, rho_zetaiUU, rho_zetacUU,
rho_gammaUU,rho_sigmaUU,rho_muupUU,rho_muzstarUU,rho_zetaUU,
rho_tauoUU,
lambdawsUU,lambdafsUU,rholUU,rho_muuplUU,rho_lambdaflUU,rho_muzst
arlUU,rho_zetaclUU,rho_zetailUU,
        rho_mps, rho_gU,
respnd,rho1d,rho2d,rho3d,rho1nd,rho2nd,rho3nd,rho_pitargetUU,
        rho1_taul, rho2_taul, rho3_taul, rho1_tauk, rho2_tauk,
rho3_tauk, dtaul, dtauk, resptaul, resptauk, ygdpUU,
taulU,taukU;

params_mode;
tauoUU=tauoUUst;
zetaiUU=zetaiUUst;
zetaUU=zetaUUst;
zetacUU=zetacUUst;
```

```
nbeta=nbetast;
taukUU=0;
taulUU=0;
taucUU=0;
psikUU=0;
psilUU=0;
iotamuUU=0;
taulU=taulUU;
taukU=taukUU;

// get parameter values and steady states...
[xgdUU,xgndUU,xrkUU, xnUU, xkUU, xhUU, xiUU, xwUU, xdUU, xcUU,
xlambdazUU, xyzUU, xsigmaUU, xomegabarUU, xphiUU, xmcUU,
xpitildUU, xpstarUU, xpiwUU,xpitildwUU, xwstarUU, xwplusUU,
xRkXUU, xRXUU, xFpXUU, xKpXUU, xFwXUU, xKwXUU, xuczUU, xqUU,
xGammaXUU, xGam_muGXUU,xlambdafUU, xpiUU, xmuzstarUU, xbetaUU,
xdeltaUU, xFomegabarXUU, xmuUU, xalphaUU, xgammaUU,
xsigmaLXUU,xpsiLXUU, xlambdawUU, xweUU, xbigthetaUU, xbUU,
xmuupUU, xtaukUU, xupsilUU, xiota1UU, xiota2UU,xpibarUU, xxipUU,
xiotaw1UU, xiotaw2UU, xxiwUU,
xepsilUU,xxUU,xrhotilUU,xaptilUU,xaytilUU,xactilUU,xSdouprXUU,xsi
gmaaUU,xlambdawsUU,xlambdafsUU] = getsteadyexogm;

// assign values computed in sstate to variables declared as
parameters in the parameter statement
gdUU=xgdUU;gndUU=xgndUU;rkUU=xrkUU; nUU=xnUU; kUU=xkUU; hUU=xhUU;
iUU=xiUU; wUU=xwUU;
dUU=xdUU; cUU=xcUU; lambdazUU=xlambdazUU; yzUU=xyzUU;
sigmaUU=xsigmaUU;
omegabarUU=xomegabarUU; phiUU=xphiUU; mcUU=xmcUU;
pitildUU=xpitildUU;
pstarUU=xpstarUU; piwUU=xpiwUU;pitildwUU=xpitildwUU;
wstarUU=xwstarUU;
wplusUU=xwplusUU; RkXUU=xRkXUU; RXUU=xRXUU; FpXUU=xFpXUU;
KpXUU=xKpXUU;
FwXUU=xFwXUU; KwXUU=xKwXUU; uczUU=xuczUU; qUU=xqUU;
GammaXUU=xGammaXUU;
Gam_muGXUU=xGam_muGXUU;lambdafUU=xlambdafUU; piUU=xpiUU;
muzstarUU=xmuzstarUU;
betaUU=xbetaUU; deltaUU=xdeltaUU; FomegabarXUU=xFomegabarXUU;
muUU=xmuUU;
alphaUU=xalphaUU; gammaUU=xgammaUU;
sigmaLXUU=xsigmaLXUU;psiLXUU=xpsiLXUU;
lambdawUU=xlambdawUU; weUU=xweUU; bigthetaUU=xbigthetaUU;
bUU=xbUU; muupUU=xmuupUU;
taukUU=xtaukUU; upsilUU=xupsilUU; iota1UU=xiota1UU;
iota2UU=xiota2UU;pibarUU=xpibarUU;
xipUU=xxipUU; iotaw1UU=xiotaw1UU; iotaw2UU=xiotaw2UU;
xiwUU=xxiwUU; epsilUU=xepsilUU;
xUU=xxUU;rhotilUU=xrhotilUU;aptilUU=xaptilUU;aytilUU=xaytilUU;act
ilUU=xactilUU;
SdouprXUU=xSdouprXUU;sigmaaUU=xsigmaaUU;lambdawsUU=xlambdawsUU;la
mbdafsUU=xlambdafsUU;

// get values for the stochastic parameters of the model
paramsshocks

// assign values to the names of the stochastic parameters
```

```
rho_lambdafUU=rho_lambdafUUst;rho_zetaiUU=rho_zetaiUUst;
rho_zetaUU=rho_zetaUUst;rho_zetacUU=rho_zetacUUst;
rho_gammaUU=rho_gammaUUst;rho_sigmaUU=rho_sigmaUUst;
rho_muupUU=rho_muupUUst;rho_muzstarUU=rho_muzstarUUst;
rho_mps=rho_mpsst;rho_tauoUU=rho_tauoUUst;rho_pitargetUU=rho_pita
rgetUUst;
rho1nd=rho1ndst;rho2nd=rho2ndst;rho3nd=rho3ndst;respnd=respndst;
rhoUU=rhoUUst;rho1d=rho1dst;rho2d=rho2dst;rho3d=rho3dst;
rho1_taul=rho1_taulst;rho2_taul=rho2_taulst;
rho3_taul=rho3_taulst;rho1_tauk=rho1_taukst;
rho2_tauk=rho2_taukst;rho3_tauk=rho3_taukst;
resptaul=resptaulst;resptauk=resptaukst;
dtaul=dtaulst;dtauk=dtaukst;

pitargetUU=(1+xUU)/muzstarUU;

UtilUU = zetacUU*(log(cUU*muzstarUU-bUU*cUU)-
zetaUU*psiLXUU/(1+sigmaLXUU)*(wstarUU/wplusUU)^((1+sigmaLXUU)*lam
bdawUU/(lambdawUU-1))*hUU^(1+sigmaLXUU));
WelfUU=UtilUU/(1-nbeta);
xUtilUU=UtilUU;xWelfUU=WelfUU;
ygdpUU = gdUU + gndUU + cUU + iUU/muupUU;
ixx=1;
pitildewUU=(pitargetUU^iotaw1UU)*(piUU^iotaw2UU)*(pibarUU^(1-
iotaw1UU-iotaw2UU));
if abs(pitildewUU-pitildwUU) > .1e-10
error('fatal (model_financial) two different ss values for
pitilde')
end

model;

// period utility
Util = zetacU*(log(cU*muzstarU-bUU*cU(-1))-
zetaU*psiLXUU/(1+sigmaLXUU)*(wstarU/wplusU)^((1+sigmaLXUU)*lambda
wUU/(lambdawUU-1))*exp(lhU)^(1+sigmaLXUU));

// welfare
Welf = Util + nbeta*Welf(+1);

// Monetary Policy Rule
log(1+RXXU) = (1-rhotilUU)*log(1+RXUU) + rhotilUU*log(1+RXXU(-1))
+ (1/(1+RXUU))*(1-rhotilUU)*aptilUU*piUU*(log(piU(+1)/pitargetU))
   + (1-rhotilUU)*aytilUU*(1/(4*(1+RXUU)))*log(ygdpU/ygdpUU) -
0*(1-rhotilUU)*0.01*(1/(4*(1+RXUU)))*log(premU)  - serialmps;


//Sovereign Debt
debt=(30*premU+gdU+(1+RXU(-1))*debt(-1)-tax)/(muzstarU(-1)*piU(-
1));
tax=0.125*debt;
pdebt=1/(1+exp(debt));
RXXU=RXU*pdebt;
```

```
//marginal cost in terms of both factor costs
-sU+((1-alphaUU)^(alphaUU-1)*alphaUU^(-
alphaUU))*((rkU*(1+psikUU*RXU))^alphaUU)*((wU*(1+psilUU*RXU))^(1-
alphaUU))/epsilU;


//marginal cost in terms of rental rate of capital
rkU*(1+psikUU*RXU)/( alphaUU*epsilU*( (upsilUU*muzstarU*(
exp(lhU)*(wstarU ^ (lambdawUU/(lambdawUU-1)) ) )/(uU*kbarU(-
1)))^(1-alphaUU) ) )-sU;

// multiplier on household budget constraint is zetacU*lambdazU
lambdazU=uzcU/(zetacU*(1+taucUU));

//adjustment cost function
SXU= exp( ( sqrt(SdouprXUU/2) )*((
zetaiU*iU*muzstarU*upsilUU/iU(-1) )-muzstarUU*upsilUU))
  +  exp(-( sqrt(SdouprXUU/2) )*((
zetaiU*iU*muzstarU*upsilUU/iU(-1) )-muzstarUU*upsilUU)) - 2 ;

//derivative of adjustment cost function
SpXU=  sqrt(SdouprXUU/2) * ( exp( ( sqrt(SdouprXUU/2) )*((
zetaiU*iU*muzstarU*upsilUU/iU(-1) )-muzstarUU*upsilUU))
                            - exp(-( sqrt(SdouprXUU/2) )*((
zetaiU*iU*muzstarU*upsilUU/iU(-1) )-muzstarUU*upsilUU)) )  ;

//I fonc
zetacU*lambdazU/muupU = zetacU*qU*lambdazU*(1-SXU-
SpXU*zetaiU*upsilUU*muzstarU*iU/iU(-1))
  +
betaUU*(zetacU(+1)*zetaiU(+1)*qU(+1)*lambdazU(+1)/(upsilUU*muzsta
rU(+1)))*SpXU(+1)*(upsilUU*muzstarU(+1)*iU(+1)/iU)^2;

//this is the first order condition associated with utilization
//rkU - tauoU*rkUU*exp(sigmaaUU*(uU-1));
rkU - sigmaaUU*(uU-1)-rkUU;

// definition of rate of return on capital
RkXU - ( ( (1-taukU)*(uU*rkU-tauoU*( rkUU*(exp(sigmaaUU*(uU-1))-
1)/sigmaaUU ))+(1-deltaUU)*qU)*piU/(upsilUU*qU(-1)) +
taukU*deltaUU - 1);

// marginal utility of consumption
uzcU - (muzstarU*zetacU/(cU*muzstarU-bUU*cU(-1))) +
bUU*betaUU*zetacU(+1)/(cU(+1)*muzstarU(+1)-bUU*cU);

// definition of nominal rate of interest
(betaUU*zetacU(+1)* lambdazU(+1)/(piU(+1)*muzstarU(+1)))*(1+RXU)
- zetacU*lambdazU ;

// ex ante real rate of interest
(betaUU*zetacU(+1)*lambdazU(+1)/(muzstarU(+1)))*FXU -
zetacU*lambdazU;

// output
yzU - ((pstarU^(lambdafU/(lambdafU-1)))*(epsilU*((uU*kbarU(-
1)/(muzstarU*upsilUU))^alphaUU )
```

```
             *(( exp(lhU)*(wstarU ^ (lambdawUU/(lambdawUU-1)) )
)^(1-alphaUU) ) - phiUU ) );

// 6-created variables for estimation
dly   =  log((ygdpU*muzstarU)/ygdpU(-1))-log(muzstarUU);
dli   =  log(iU)-log(iU(-1))+log(muzstarU)-log(muzstarUU);
dlc   =  log(cU*muzstarU/cU(-1))-log(muzstarUU);
dlgd  =  log(gdU*muzstarU/gdU(-1))-log(muzstarUU);
dlpi  =  -(log(muupU)-log(muupU(-1)));
dlh   =  (lhU-lhU(-1));
linfl =  log(piU)-log(piUU);

// national income identity:
ygdpU = gdU + gndU + cU + iU/muupU;

// capital law of motion
kbarU - (1-deltaUU)*(kbarU(-1)/(muzstarU*upsilUU)) - (1-SXU)*iU;

// sticky price/wage equations....

// p-star equation
pstarU = ((1-xipUU)*(KpXU/FpXU)^(lambdafU/(1-
lambdafU))+xipUU*(piU(-1)*pstarU(-1)/piU)^(lambdafU/(1-
lambdafU)))^((1-lambdafU)/lambdafU);

// w-star equation
wstarU = ( (1-xiwUU) * ((psiLXUU*KwXU/(wU*FwXU))^(lambdawUU/(1-
lambdawUU*(1+sigmaLXUU))))
  + xiwUU * (( (pitargetU^iotaw1UU) * (piU(-1)^iotaw2UU) *
(pibarUU^(1-iotaw1UU-iotaw2UU)) )/( piU*muzstarU*wU/wU(-1) )
    * (muzstarUU^(1-iotamuUU)) * (muzstarU^iotamuUU) * wstarU(-
1))^(lambdafUU/(1-lambdafUU)) ) ^ ((1-lambdawUU)/lambdawUU);

// Fp equation
-FpXU + zetacU*lambdazU*yzU
 + betaUU * xipUU * ( (( (pitargetU(+1)^iota1UU) * (piU^iota2UU)
* pibarUU^(1-iota1UU-iota2UU) )/piU(+1)) ^ (1/(1-lambdafU(+1))) )
* FpXU(+1);

// Kp equation
-KpXU + zetacU*(lambdafU/lambdafsUU)*lambdazU*yzU*sU
 + betaUU * xipUU * ( (( (pitargetU(+1)^iota1UU) * (piU^iota2UU)
* pibarUU^(1-iota1UU-iota2UU) )/piU(+1)) ^ (lambdafU(+1)/(1-
lambdafU(+1))) ) * KpXU(+1);

//
KpXU=FpXU * (( (1 - xipUU*(( (pitargetU^iota1UU) * (piU(-
1)^iota2UU) * pibarUU^(1-iota1UU-iota2UU) )/piU)^(1/(1-lambdafU))
)/ (1-xipUU) )^(1-lambdafU));


//wage-updating equation
pitildewU=(pitargetU^iotaw1UU)*(piU(-1)^iotaw2UU)*(pibarUU^(1-
iotaw1UU-iotaw2UU));

//nominal wage inflation
piwU=piU*muzstarU*wU/wU(-1);
```

```
// Fw equation
lambdawsUU*zetacU*((wstarU^(lambdawUU/(lambdawUU-
1)))*exp(lhU)*(1-taulU)*lambdazU/lambdawUU)
  + betaUU * xiwUU * (muzstarUU^((1-iotamuUU)/(1-lambdawUU))) *(
(pitildewU(+1)^(1/(1-lambdawUU))) / piU(+1) )
 *( (1/piwU(+1)) ^ (lambdawUU/(1-lambdawUU)) ) *
(muzstarU(+1)^(iotamuUU/(1-lambdawUU)-1)) * FwXU(+1) - FwXU;

// Kw equation

-KwXU + zetacU*zetaU*((wstarU^(lambdawUU/(lambdawUU-
1))*exp(lhU))^(1+sigmaLXUU))
 + betaUU * xiwUU * (( (pitildewU(+1)/piwU(+1)) * (muzstarUU^(1-
iotamuUU)) * (muzstarU(+1)^iotamuUU) )
 ^ (lambdawUU*(1+sigmaLXUU)/(1-lambdawUU))) * KwXU(+1);

KwXU =  ( ( (1 - xiwUU * ( ( (pitildewU/piwU)
 * (muzstarUU^(1-iotamuUU)) * (muzstarU^iotamuUU) ) ^ (1/(1-
lambdawUU)) ) ) / (1-xiwUU) ) ^ (1-lambdawUU*(1+sigmaLXUU)) ) *
wU * FwXU/psiLXUU;

// w-plus equation
wplusU - ( ((1-xiwUU)*( ( (1 - xiwUU * ( ( (pitildewU/piwU) *
(muzstarUU^(1-iotamuUU)) * (muzstarU^iotamuUU) ) ^ (1/(1-
lambdawUU)) ) ) / (1-xiwUU) )^(lambdawUU*(1+sigmaLXUU)) ) +
xiwUU*( (( (( (pitargetU^iotaw1UU) * (piU(-1)^iotaw2UU) *
(pibarUU^(1-iotaw1UU-iotaw2UU)) )/( piU*muzstarU*wU/wU(-1) )) *
(muzstarUU^(1-iotamuUU)) * (muzstarU^iotamuUU) )*wplusU(-1))^(
lambdawUU*(1+sigmaLXUU)/(1-lambdawUU) ) ) )^(1/(
lambdawUU*(1+sigmaLXUU)/(1-lambdawUU) )) );

// equations pertaining to the entrepreneur
// entrepreneur
loansU=(qU*kbarU-nU);

// period t premium = interest paid by entrepreneurs in period t
- interest earned by households in period t.
premU=(qU(-1)*kbarU(-1)/(qU(-1)*kbarU(-1)-nU(-
1)))*(1+RkXU)*omegabarU-(1+RXU(-1));

riskyRXU=(qU(-1)*kbarU(-1)/(qU(-1)*kbarU(-1)-nU(-
1)))*(1+RkXU)*omegabarU;

// law of motion for aggregate net worth
-nU+(gammaU/(piU*muzstarU))*(RkXU-RXU(-1) -
muUU*GXU*(1+RkXU))*kbarU(-1)*qU(-1)+weUU+gammaU*(1+RXU(-1))*nU(-
1)/(piU*muzstarU);

// zero profit condition on banks
(qU(-1)*kbarU(-1)*(1+RkXU)*( GAMMAXU - muUU*GXU )/(nU(-
1)*(1+RXU(-1))))-(qU(-1)*kbarU(-1)/nU(-1))+1;



// efficiency condition on standard debt contract
(1-GAMMAXU(+1))*( (1+RkXU(+1))/(1+RXU) ) + ( (1 -
FomegaXU(+1))/(1- FomegaXU(+1) - muUU*omegabarU(+1)*FprimeU(+1))
)
```

```
   * ( ( (1+RkXU(+1))/(1+RXU) )*( GAMMAXU(+1)-muUU*GXU(+1)) - 1
);


FprimeU=(1/sqrt(2*3.14159265358979))*exp(-
.5*((log(omegabarU)+sigmaU(-1)^2/2)/sigmaU(-
1))^2)/(omegabarU*sigmaU(-1));

GXU=normcdf(( (log(omegabarU)+sigmaU(-1)^2/2)/sigmaU(-1) )-
sigmaU(-1),0,1);

FomegaXU=normcdf(( (log(omegabarU)+sigmaU(-1)^2/2)/sigmaU(-1)
),0,1);

GAMMAXU=omegabarU*(1-FomegaXU)+GXU;

// resources used up in monitoring
dU = muUU*GXU*(1+RkXU)*qU(-1)*kbarU(-1)/(muzstarU*piU);

log(sigmaU/sigmaUU) = rho_sigmaUU*log(sigmaU(-1)/sigmaUU) +
sigmax;
log(gammaU/gammaUU) = rho_gammaUU*log(gammaU(-1)/gammaUU) +
gammax;

// resource constraint
yzU-gndU - gdU  - cU - iU/muupU - dU - bigthetaUU*((1-
gammaU)/gammaU)*(nU-weUU) -
tauoU*(rkUU/sigmaaUU)*(exp(sigmaaUU*(uU-1))-1);



// exogenous shocks

log(muzstarU/muzstarUU) = rho_muzstarUU*log(muzstarU(-
1)/muzstarUU)+muzstarx;
log(tauoU/tauoUU) = rho_tauoUU*log(tauoU(-1)/tauoUU) + tau0x;
log(epsilU) = rhoUU*log(epsilU(-1))+epsilx;
log(lambdafU/lambdafUU) = rho_lambdafUU*log(lambdafU(-
1)/lambdafUU)+lambdafx;
log(muupU/muupUU) = rho_muupUU*log(muupU(-1)/muupUU)+muupx;
log(zetacU/zetacUU) = rho_zetacUU*log(zetacU(-1)/zetacUU)+zetacx;
log(zetaiU/zetaiUU) = rho_zetaiUU*log(zetaiU(-1)/zetaiUU)+zetaix;
log(zetaU/zetaUU) = rho_zetaUU*log(zetaU(-1)/zetaUU)+zetax;
serialmps = rho_mps*serialmps(-1) + mps ;
log(gdU/gdUU) = rho1d*0+0.988*log(gdU(-1)/gdUU) +
0*rho2d*log(gdU(-2)/gdUU) + 0*rho3d*log(gdU(-3)/gdUU) + gdx +
signalstate1(-1);
//log(taulU/taulUU) = rho1_taul*log(taulU(-1)/taulUU) +
rho2_taul*log(taulU(-2)/taulUU) + rho3_taul*log(taulU(-3)/taulUU)
//            + taulx + resptaul*log(ygdpU(-1)/yzUU)+
dtaul*log(gdU(-1)/gdUU);
//log(taukU/taukUU) = rho1_tauk*log(taukU(-1)/taukUU) +
rho2_tauk*log(taukU(-2)/taukUU) + rho3_tauk*log(taukU(-3)/taukUU)
//            + taukx + resptauk*log(ygdpU(-1)/yzUU) +
dtauk*log(gdU(-1)/gdUU);
log(gndU/gndUU) = rho1nd*log(gndU(-1)/gndUU) + rho2nd*log(gndU(-
2)/gndUU) + rho3nd*log(gndU(-3)/gndUU) + gndx
                    + respnd*log(ygdpU(-1)/yzUU);
```

```
log(pitargetU/pitargetUU)=  rho_pitargetUU * ( log(pitargetU(-
1)/pitargetUU) ) + pitargetx;



signalstate8=sig8;
signalstate7=signalstate8(-1)+sig7;
signalstate6=signalstate7(-1)+sig6;
signalstate5=signalstate6(-1)+sig5;
signalstate4=signalstate5(-1)+sig4;
signalstate3=signalstate4(-1)+sig3;
signalstate2=signalstate3(-1)+sig2;
signalstate1=signalstate2(-1)+sig1;

end;

initval;
riskyRXU=(kUU/(kUU-nUU))*(1+RkXUU)*omegabarUU;
GXU=normcdf(( (log(omegabarUU)+sigmaUU^2/2)/sigmaUU )-
sigmaUU,0,1);
FomegaXU=normcdf(( (log(omegabarUU)+sigmaUU^2/2)/sigmaUU ),0,1);
GAMMAXU=omegabarUU*(1-normcdf((
(log(omegabarUU)+sigmaUU^2/2)/sigmaUU ),0,1))+normcdf((
(log(omegabarUU)+sigmaUU^2/2)/sigmaUU )-sigmaUU,0,1);
FprimeU=(1/sqrt(2*3.14159265358979))*exp(-
.5*((log(omegabarUU)+sigmaUU^2/2)/sigmaUU)^2)/(omegabarUU*sigmaUU
);
gammaU=gammaUU;
omegabarU=omegabarUU;
loansU=(kUU-nUU);
premU=(kUU/(kUU-nUU))*(1+RkXUU)*omegabarUU-(1+RXUU);
sigmaU=sigmaUU;
dU=dUU;
nU=nUU;
pitildewU=pitildewUU;
piwU=piUU*muzstarUU;
tauoU=tauoUU;
zetaU=1;
KpXU=KpXUU;
KwXU=KwXUU;
ygdpU=gdUU + gndUU + cUU + iUU/muupUU;
muzstarU=muzstarUU;
SXU=0;
SpXU=0;
lambdazU=lambdazUU;
linfl=0;
dlh=0;
pitargetU=pitargetUU;
muupU=muupUU;
FXU=muzstarUU/betaUU;
yzU=yzUU;
qU=qUU;
piU=piUU;
RkXU=RkXUU;
rkU=rkUU;
RXU=RXUU;
RXXU=RXUU;
pstarU=pstarUU;
```

```
wstarU=wstarUU;
wplusU=wplusUU;
kbarU=kUU;
lhU=log(hUU);
cU=cUU;
iU=iUU;
FpXU=FpXUU;
FwXU=FwXUU;
uzcU=lambdazUU*zetacUU*(1+taucUU);
sU=mcUU;
wU=wUU;
uU=1;
Util=UtilUU;
Welf=WelfUU;
lambdafU=lambdafUU;
epsilU=1;
zetaiU=zetaiUU;
serialmps=0;
dly = 0;
dli = 0;
dlc = 0;
dlgd = 0;
dlpi = 0;
gdU=gdUU;
gndU=gndUU;
zetacU=zetacUU;
signalstate1=0;
signalstate2=0;
signalstate3=0;
signalstate4=0;
signalstate5=0;
signalstate6=0;
signalstate7=0;
signalstate8=0;
tax=0;

end;

steady;

// following are the shocks....the ones with zero variance are in
effect not activated.
shocks;
var sigmax;
stderr 1;
var gammax;
stderr 0;
var muupx;
stderr 0;
var zetax;
stderr 0;
var pitargetx;
stderr 0;
var taulx;
stderr 0;
var taukx;
stderr 0;
var gdx;
```

```
stderr 0.01;
var gndx;
stderr 0;
var epsilx;
stderr 0;
var zetacx;
stderr 0;
var zetaix;
stderr 0;
var mps;
stderr 0;
var lambdafx;
stderr 0;
var sig1;
stderr 0;
var sig2;
stderr 0;
var sig3;
stderr 0;
var sig4;
stderr 0;
var sig5;
stderr 0;
var sig6;
stderr 0;
var sig7;
stderr 0;
var sig8;
stderr 0;
var muzstarx;
stderr 0;


end;

stoch_simul(order=1,irf=200,nograph) kbarU riskyRXU FXU wU sU cU
iU ygdpU RkXU RXU epsilU piU nU qU rkU lhU premU loansU yzU dU
FomegaXU debt pdebt RXXU;




cons_debt_risk=100*cU_sigmax/cUU;
inv_debt_risk=100*iU_sigmax/(iUU/muupUU);
gdp_debt_risk=100*ygdpU_sigmax/(gdUU + gndUU + cUU + iUU/muupUU);
networth_debt_risk=100*nU_sigmax/nUU;
premium_debt_risk=premU_sigmax;
loans_debt_risk=100*loansU_sigmax/(gdUU + gndUU + cUU +
iUU/muupUU);
debt_debt_risk=debt_sigmax;
pdebt_debt_risk=pdebt_sigmax;
rloans_debt_risk=RXU_sigmax;
riskf_debt_risk=RXXU_sigmax;
infl_debt_risk=piU_sigmax;

cons_debt_gov=100*cU_gdx/cUU;
inv_debt_gov=100*iU_gdx/(iUU/muupUU);
gdp_debt_gov=100*ygdpU_gdx/(gdUU + gndUU + cUU + iUU/muupUU);
```

```
networth_debt_gov=100*nU_gdx/nUU;
premium_debt_gov=premU_gdx;
loans_debt_gov=100*loansU_gdx/(gdUU + gndUU + cUU + iUU/muupUU);
debt_debt_gov=debt_gdx;
pdebt_debt_gov=pdebt_gdx;
rloans_debt_gov=RXU_gdx;


save CEEplusBGGplusDEBT cons_debt_risk inv_debt_risk
gdp_debt_risk networth_debt_risk premium_debt_risk
loans_debt_risk debt_debt_risk pdebt_debt_risk rloans_debt_risk
gdp_debt_gov riskf_debt_risk infl_debt_risk cons_debt_gov
inv_debt_gov gdp_debt_gov networth_debt_gov premium_debt_gov
loans_debt_gov debt_debt_gov pdebt_debt_gov rloans_debt_gov



%Matlab Code for Steady State

%

function [gdUU,gndUU,rkUU, nUU, kUU, hUU, iUU, wUU, dUU, cUU,
lambdazUU, yzUU, sigmaUU, omegabarUU, phiUU, mcUU, pitildUU,
pstarUU, piwUU,  pitildwUU, wstarUU, wplusUU, RkXUU, RXUU, FpXUU,
KpXUU, FwXUU, KwXUU, uczUU, qUU, GammaXUU, Gam_muGXUU,
lambdafUU, inflUU, muzstarUU, betaUU, deltaUU, FomegabarXUU,
muUU, alphaUU, gammaUU, sigmaLXUU,  psiLXUU, lambdawUU, weUU,
bigthetaUU, bUU, muupUU, taukUU, upsilUU, iota1UU, iota2UU,
pibarUU, xipUU, iotaw1UU, iotaw2UU, xiwUU,
epsilUU,xUU,rhotilUU,aptilUU,aytilUU,actilUU,SdouprXUU,sigmaaUU,
lambdawsUU,lambdafsUU] = getsteadyexogm

    params_mode
    %the parameters in params_mode need to be translated...

lambdawUU=lambdawUUst;sigmaLXUU=sigmaLXUUst;zetaUU=zetaUUst;betaU
U=betaUUst;nbeta=nbetast;betaUU=betaUUst;bUU=bUUst;psiLXUU=psiLXU
Ust;zetacUU=zetacUUst;iotaw1UU=iotaw1UUst;iotaw2UU=iotaw2UUst;iot
amuUU=iotamuUUst;tauoUU=tauoUUst;xiwUU=xiwUUst;SdouprXUU=SdouprXU
Ust;sigmaaUU=sigmaaUUst;muupUU=muupUUst;zetaiUU=zetaiUUst;Fomegab
arXUU=FomegabarXUUst;muUU=muUUst;gammaUU=gammaUUst;weUU=weUUst;bi
gthetaUU=bigthetaUUst;tauoUU=tauoUUst;muzstarUU=muzstarUUst;upsil
UU=upsilUUst;lambdafUU=lambdafUUst;alphaUU=alphaUUst;psikUU=psikU
Ust;psilUU=psilUUst;deltaUU=deltaUUst;epsilUU=epsilUUst;xipUU=xip
UUst;iota1UU=iota1UUst;iota2UU=iota2UUst;etagUU=etagUUst;etadgUU=
etadgUUst;taukUU=taukUUst;taulUU=taulUUst;taucUU=taucUUst;lambdaw
sUU=lambdawsUUst;lambdafsUU=lambdafsUUst;phi=phist;xUU=xUUst;piba
rUU=pibarUUst;aptilUU=aptilUUst;aytilUU=aytilUUst;rhotilUU=rhotil
UUst;actilUU=actilUUst;pitargetUU=pitargetUUst;zetaiUU=zetaiUUst;
zetaUU=zetaUUst;zetacUU=zetacUUst;
infl=pibarUU;

    %we need to first compute phi. This is set to a value which
implies
    %steady state profits are zero in the case, pibar=steady
state
    %inflation (this is why on input, infl=[]).
```

```
    %if phi=0, you trigger the steadystate.m into computing phi
so that
    %profits are zero in steady state.

[g,rk,n,k,h,I,w,d,c,lambdaz,yz,sigma,omegabar,phi,mc,pitild,pstar
,piw,pitildw,wstar,wplus,Rk,R,Fp,Kp,Fw,Kw,ucz,q,Gamma,Gam_muG] =
steadystate(lambdafUU,infl,muzstarUU,betaUU,deltaUU,FomegabarXUU,
muUU,alphaUU,gammaUU,sigmaLXUU,psiLXUU,lambdawUU,weUU,bigthetaUU,
bUU,phi,muupUU,taukUU,upsilUU,iota1UU,iota2UU,pibarUU,xipUU,iotaw
1UU,iotaw2UU,xiwUU,epsilUU,lambdawsUU,lambdafsUU,etagUU,
zetacUU,zetaUU,taulUU,psikUU,psilUU,taucUU,iotamuUU);
gd=etadgUU*(c+I+g);
gnd=g-gd;
gUU=g;
infl=(1+xUU)/muzstarUU;

gdUU=gd;gndUU=gnd;rkUU=rk;nUU=n;kUU=k;hUU=h;iUU=I;wUU=w;dUU=d;cUU
=c;lambdazUU=lambdaz;yzUU=yz;sigmaUU=sigma;omegabarUU=omegabar;ph
iUU=phi;mcUU=mc;pitildUU=pitild;pstarUU=pstar;piwUU=piw;pitildwUU
=pitildw;wstarUU=wstar;wplusUU=wplus;RkXUU=Rk;RXUU=R;FpXUU=Fp;KpX
UU=Kp;FwXUU=Fw;KwXUU=Kw;uczUU=ucz;qUU=q;GammaXUU=Gamma;Gam_muGXUU
=Gam_muG;
end

%

function
[g,rk,n,k,h,I,w,d,c,lambdaz,yz,sigma,omegabar,phi,mc,pitild,pstar
,piw,pitildw,wstar,wplus,Rk,R,Fp,Kp,Fw,Kw,ucz,q,Gamma,Gam_muG] =
steadystate(lambdaf,infl,muz,beta,delta,Fomegabar,mu,alpha,gam,si
gmaL,psiL,lambdaw,we,bigtheta,b,phi,muup,tauk,upsil,iota1,iota2,p
ibar,xip,iotaw1,iotaw2,xiw,epsil,lambdaws,lambdafs,etag,zetac,zet
a,taul,psik,psil,tauc,iotamu)


R=infl*muz/beta-1;
rkk=[muz/beta-(1-delta)+0.0000000001:.001:0.05];
    %rkk=[.0333:.0000001:.0334];

rk=rkk(1);

    [ffold] =
ssteadystate(rk,phi,Fomegabar,mu,gam,delta,infl,R,muz,lambdaf,big
theta,we,beta,psiL,sigmaL,lambdaw,alpha,b,muup,tauk,upsil,iota1,i
ota2,pibar,xip,iotaw1,iotaw2,xiw,epsil,sw,lambdaws,lambdafs,etag,
psik,psil);
ix=0;
fx(1)=ffold;
for ii = 2:length(rkk)
rk=rkk(ii);

[ff]=ssteadystate(rk,phi,Fomegabar,mu,gam,delta,infl,R,muz,lambda
f,bigtheta,we,beta,psiL,
sigmaL,lambdaw,alpha,b,muup,tauk,upsil,iota1,iota2,pibar,xip,iota
w1,iotaw2,xiw,epsil,sw,lambdaws,lambdafs,etag,psik,psil);

if ff > 0 & ffold < 0
```

```
ix=ix+1;
I(ix)=ii;
break

end
ffold=ff;
fx(ii)=ff;
end

if ix > 1
error('fatal (steadystate) found more than one steady state')
end
if ix == 0

error('fatal (steadystate) failed to bracket a steady state')
end
    rk1=rkk(I(ix)-1);
    rk2=rkk(I(ix));

opt=optimset('diagnostics','on','LevenbergMarquardt','on','TolX',
1e-16,'TolFun',1e-16);
    [rkopt,fval,exitflag,output] =
fzero(@ssteadystate,[rk1,rk2],opt,phi,Fomegabar,mu,gam,delta,infl
,R,muz,lambdaf,bigtheta,we,beta,psiL,sigmaL,lambdaw,alpha,b,muup,
tauk,upsil,iota1,iota2,pibar,xip,iotaw1,iotaw2,xiw,epsil,sw,lambd
aws,lambdafs,etag,psik,psil);
if abs(fval ) > .1e-9 | abs(imag(rkopt))>.1e-10 | exitflag <= 0
error('fatal (steadystate) failed to find steady state')
end

rk=rkopt;


[ff,g,n,k,h,I,w,d,c,lambdaz,yz,sigma,omegabar,phi,mc,pitild,pstar
,piw,pitildw,wstar,wplus,Rk,q,Gamma,Gam_muG] =
steadystate(rk,phi,Fomegabar,mu,gam,delta,infl,R,muz,lambdaf,bigt
heta,we,beta,psiL,
sigmaL,lambdaw,alpha,b,muup,tauk,upsil,iota1,iota2,pibar,xip,iota
w1,iotaw2,xiw,epsil,sw,lambdaws,lambdafs,etag,psik,psil);

end

x=infl*muz-1;
ucz=(muz*zetac/(c*muz-b*c)) - b*beta*zetac/(c*muz-b*c);

%

function
[ff,g,n,k,h,I,w,d,c,lambdaz,yz,sigma,omegabar,phi,mc,pitild,pstar
,piw,pitildw,wstar,wplus,Rk,q,Gamma,Gam_muG] =
ssteadystate(rk,phi,Fomegabar,mu,gam,delta,infl,R,muz,lambdaf,big
theta,we,beta,psiL,
sigmaL,lambdaw,alpha,b,muup,tauk,upsil,iota1,iota2,pibar,xip,iota
w1,iotaw2,xiw,epsil,sw,lambdaws,lambdafs,etag,psik,psil)
if nargin < 33
    g=0;
end
```

```
infl=pibar;%when sw=1, then compute steady state for the case,
infl=pibar
pitild=infl;
pstar=1;
piw=muz*infl;
pitildw=infl;
wstar=1;
wplus=1;
mc=lambdafs/lambdaf;

end

wwstar=(wstar^(lambdaw/(lambdaw-1)));
hk=(1/muz)*(1/wwstar)*(rk*(1+psik*R)/(mc*alpha))^(1/(1-alpha));
Rk=(rk+(1-delta))*infl-1;
s=(1+Rk)/(1+R);
[sigma,omegabar,Gamma,Gam_muG,ix] = getomega(s,Fomegabar,mu);
q=1;

if ix == 0
kn=1/(1-s*Gam_muG);
    G=Gamma-omegabar*(1-Fomegabar);
    n=we/( 1 - (gam/(infl*muz))*( Rk - R - mu*G*(1+Rk) )*kn -
gam*((1+R)/(infl*muz)) );
    k=kn*n;
    h=hk*k;
    I=k*(1-(1-delta)/muz);
    w = mc * (1-alpha) * ((wwstar*hk*muz)^(-alpha))/(1+psil*R) ;
    d=mu*G*(1+Rk)*k/(infl*muz);
if sw == 1
phi=((k/muz)^alpha)*(h^(1-alpha))*(1-1/lambdaf);
end
yz=(pstar^(lambdaf/(lambdaf-1))) * ((k/muz)^alpha) *
((wwstar*h)^(1-alpha)) - phi;
    c = (1-etag)*(yz - bigtheta*((1-gam)/gam)*(n-we) - d)-I;
    g = (etag/(1-etag))*(c+I);
lambdaz=(muz-b*beta)/(c*(muz-b));
    f0=1-xiw*((pitildw/infl)^(1/(1-lambdaw)));
ffnum=1-beta*xiw*((pitildw/infl)^(1/(1-lambdaw)));
ffden=1-beta*xiw*((pitildw/infl)^((1+sigmaL)*lambdaw/(1-
lambdaw)));
    f1=(f0/(1-xiw))^(lambdaw*(1+sigmaL)-1);
    W=(wwstar^sigmaL)*f1*ffnum/ffden;
hh=(lambdaz*w*lambdaws/(W*lambdaw*psiL))^(1/sigmaL);
if n > 0 & k > 0 & h > 0 & I > 0 & w > 0 & c > 0 & hh > 0
ff=h-hh;
else
ff=10000;
end
else
ff=10000;
end

%

function [sigma,omegabar,Gamma,Gam_muG,ixx] =
getomega(s,Fomegabar,mu)
```

```
ixx=0;

oom=[.0000001:.1:.99];

ij=0;
omega=oom(1);
[ff(1),ix] = findomega(omega,s,Fomegabar,mu);
for ii = 2:length(oom)
omega=oom(ii);
    [ff(ii),ix] = findomega(omega,s,Fomegabar,mu);

if ff(ii)*ff(ii-1) < 0 & ix == 0
ij=ij+1;
II(ij)=ii;
end

end

if ij > 1
ixx=2;
    %disp('(getomega) multiple solutions)
sigma=[];
omegabar=[];
    Gamma=[];
    Gam_muG=[];
return
end
if ij < 1
ixx=1;
    %disp('(getomega) no solution)
sigma=[];
omegabar=[];
    Gamma=[];
    Gam_muG=[];
return
end
omega1=oom(II(1)-1);
omega2=oom(II(1));

opt=optimset('diagnostics','on','LevenbergMarquardt','on','TolX',
1e-16,'TolFun',1e-16);
[omegabar,fval,exitflag,output] =
fzero(@findomega,[omega1,omega2],opt,s,Fomegabar,mu);
if abs(fval ) > .1e-9 | abs(imag(omega))>.1e-10 | exitflag <= 0
ix(4)=1;
error('fatal (getomega) failed to find omega')
end
[fff,ix,Gamma,Gam_muG] = findomega(omegabar,s,Fomegabar,mu);
[sigma] = ffsigma(omegabar,Fomegabar);
if ix ~= 0
ixx==3;
end

%

function [ff,ix,Gamma,Gam_muG] = findomega(omega,s,Fomegabar,mu)
if s<=1,error('(findomega) s must be larger than 1'),end
sigma   =   ffsigma(omega,Fomegabar);
```

```
[ff,ix,Gamma,Gam_muG] = ffindomega(omega,sigma,mu,Fomegabar,s);

%

function [sigma,ix] = ffsigma(omega,Fomega)
% This routine finds the value of sigma
%for the lognormal distribution with mean
%forced to equal zero, such that prob < omega = Fomega.
%For Fomega = .03, this program requires
%omega to live in the interval
%[0.000000001,0.999];
ix=0;
sigma1=.00000001;
[ff1] = findsigma(sigma1,omega,Fomega);
sigma2=5;
[ff2] = findsigma(sigma2,omega,Fomega);
if ff1*ff2 > 0
ix=1;
sigma=[];
return
end
x0=[sigma1 sigma2];
opt=optimset('diagnostics','on','LevenbergMarquardt','on','TolX',
1e-12,'TolFun',1e-12);
[sigma,fval,exitflag,output] =   fzero(@findsigma,x0,opt, ...
omega,Fomega);
if abs(fval ) > .1e-9 | abs(imag(sigma))>.1e-10 | sigma < 0 |
exitflag <= 0
ix=2;
sigma=[];
return
end

%

function [ff] = findsigma(sigma,omega,Fomega)
ff=logncdf(omega,-sigma^2/2,sigma)-Fomega;

%

function [ff,ix,Gamma,Gam_muG] =
ffindomega(omega,sigma,mu,Fomegabar,s)
z       =   (log(omega)+sigma^2/2)/sigma;
Gamma   =   normcdf(z-sigma)+omega*(1-normcdf(z));
Gam_muG =   (1-mu)*normcdf(z-sigma)+omega*(1-normcdf(z));

ff  =   (1-Gamma)*s+(1-Fomegabar)/(1-Fomegabar-
mu*omega*lognpdf(omega,-sigma^2/2,sigma))*(s*Gam_muG-1);
ix=0;
if max(abs(imag(ff))) > .1e-8 | (1-Fomegabar)/(1-Fomegabar-
mu*omega*lognpdf(omega,-sigma^2/2,sigma)) < 0
ix=1;
end
```