ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ΤΜΗΜΑ ΟΙΚΟΝΟΜΙΚΗΣ ΕΠΙΣΤΗΜΗΣ

PRICING OF STRUCTURED PRODUCTS

ΒΛΑΧΟΥ ΔΗΜΗΤΡΑ

Διατριβή υποβληθείσα προς μερική εκπλήρωση των απαραιτήτων προϋποθέσεων για την απόκτηση του Μεταπτυχιακού Διπλώματος Ειδίκευσης

Αθήνα
Ιανουάριος, 2010

Εγκρίνουμε τη διατριβή της **ΒΛΑΧΟΥ ΔΗΜΗΤΡΑΣ**

Κόρδας Γρηγόριος

Οικονομικό Πανεπιστήμιο Αθηνών ....

Αρβανίτης Στυλιανός

Οικονομικό Πανεπιστήμιο Αθηνών ....

Γάτσιος Κωνσταντίνος

Οικονομικό Πανεπιστήμιο Αθηνών ....

Table of Contents

Abstract

The structured product market has experienced remarkable growth globally in recent years. Greek investment banks offer various structured finance products, which seem an attractive investment alternative for both institutional and individual investors.

Structured products are investment vehicles that combine basic financial instruments to provide investors with packaged solutions of advanced investment strategies on financial markets. They offer features that traditional investments cannot, such as principal protection, diversification, customized payoffs and they suggest a useful hedging vehicle.

The substantial reduction of international interest rates in combination with the high uncertainty of investing in the stock market increased the growth of the structured product market.

This paper examines the Market Index Certificates of Deposit, which constitute a rather recent addition to the growing family of structured products. MICDs are variable-rate certificates of deposit for which the interest rate is contingent upon the performance of a well-known stock market average but with a guaranteed minimum interest rate and they are especially popular within individuals with little or no investment experience, since they provide investors a means for participating in the stock market while at the same time limiting their risk exposure. The lack of experience of investors creates the urgency to find an applicable method for pricing.

Structured products can be evaluated as the combination of their building blocks. In this paper, we present how MICDs can be replicated with a combination of risk-free assets and index options and we derive general pricing formulas for MICDs with embedded vanilla options, binary options and “two-asset cash-or-nothing” options. From the implicit value of the option component we compute implied standard deviations (ISDs) for the market index. We also analyze the issuer's risk exposure and discuss the natural as well as the dynamic hedging strategies.

It is crucial for potential investors of structured products to be able to adequately understand their complicated payoff structure and the implicit fees being charged for these products, in order to avoid serious pitfalls and take advantage of the opportunities offered by these tempting financial instruments.

1. Introduction

One of the most important innovations in modern international financial markets over the last three decades has been the increased understanding and use of financial derivatives. These contracts, including options, futures and swaps, are created by financial institutions for their corporate issuers who seek to lower their cost of capital by legitimately structuring their cash flows and liability claims. Sophisticated institutional investors have long used derivatives to obtain their desired risk exposures and to dynamically manage their existing exposures in a cost – effective manner.

The rapid growth of financial derivatives has been exacerbated by the substantial reduction of international interest rates due to high saving rates in many parts of the world (Solow, 1956). The fall in interest rates caused dissatisfaction to most investors accustomed to obtain higher returns, thus giving birth to a “search for yield”. The desire for higher yields could not be met by traditional investment vehicles nor was it easy for private investors to access non-traditional markets, such as commodities. So it led to arising demand for innovative, and inevitably riskier, financial instruments and for greater leverage, which was easier to obtain. The financial sector responded to the challenge by providing more sophisticated ways (i.e. derivatives) of increasing yields by making more risk.

As derivatives have become more accepted and commonplace in the financial markets, they have also became more competitively priced and therefore profit margins earned by financial institutions dealing on these claims have declined.

At the same time investors started to comprehend that stock markets are characterized of high volatility and great uncertainty in the returns, while the “secure alternative” of bonds offer poor returns even to long term investors.

Responding to the general demand for limited risk and higher returns, financial institutions have begun to engineer, arrange and participate in the distribution of more complex securities, one class of which is known as structured finance products. The later include a wide range of products, such as financial derivatives for market and credit risk, asset-backed and equity backed securities with customized cash flow features and specialized financial conduits that manage basket of purchased assets.

Structured products offer features that traditional investments cannot, such as providing principal protection, diversification and customized payoffs (Troyanovski, 2005).

Structured finance products allow issuers and investors, who prefer a particular pattern of payments over time, to access such a pattern, as well as hedge certain risks. They suggest a useful hedging alternative for an individual with investments in a mutual fund or a stock-based pension fund. Such a hedging vehicle would be particularly attractive to an investor whose tax situation makes it undesirable to liquidate his/her stock position, and who would therefore be willing to pay a premium.

Structured products facilitate the transfer of risk, at a cost, from those who do not want to bear risk to those who are willing to bear it. They also allow investors to more fully diversify their investment portfolios, because investors can asset classes that would not otherwise be available through traditional investments vehicles. The underlying assets can be a mix of different asset classes, indices or baskets of individual equities. For example, structured offerings can take commodities, hedge funds and foreign exchange markets as their underlying assets.

Depository liabilities with interest rates linked to the stock market are a rather recent addition in a long line of new financial instruments. The Market Index Certificates of Deposit are especially popular within individuals with little or no investment experience, since they are designed to provide investors a means for participating in the stock market while at the same time limiting their risk exposure. The MICDs are variable-rate certificates of deposit for which the interest rate is contingent upon the performance of a well-known stock market average (such as the S&P 500 index) but with a guaranteed minimum interest rate.

Fortune magazine chose the MICDs for inclusion in its 1987 list of "Products of the Year," on the grounds that their downside protection makes participation in the stock market attractive to a broader base of investors. In addition, an investor who already has stock holdings in direct form, or through a mutual fund or pension fund, could use a put version of the MICD as a hedge.

The lack of experience of investors and the recent and dynamic appearance of MICDs in the Greek market, create the urgent need to find an applicable method of pricing.

Although they are recently introduced, MICDs can be replicated with existing instruments, like so many other financial products. The construction of a portfolio of instruments that replicate the outcome of the product is a useful and effective method of valuation. MICDs in particular, can be replicated with a combination of risk-free assets and index options, which can be easily priced with analytical or numerical procedures.

Investing in MICDs requires some familiarity with the pricing procedure of the products and individuals who intend to invest in MICDs should demand to be provided with adequate information about them, since transparency is usually limited.

We have to bear in mind that a financial institution offering market index CDs is in the position of an uncovered option writer and clearly, this risk must hedged. Equilibrium pricing of MICDs provides a simple hedging strategy to adopt, since the issuer can completely hedge its exposure to stock market risk by using a small fraction of the principal to purchase offsetting index options. So equilibrium pricing is a desirable goal not only from the investors' point of view but also from the issuers'. However, the more aggressive alternatives of dynamic and partial hedging are also available.

MICDs would not create any new stock-market worries for regulators if all issuers formed an exact hedge at the initiation of each issue, by purchasing option contracts on the underlying index of the MICD in the appropriate quantities. But the reluctance of many issuers to follow this policy raises serious regulatory issues.

The experience with MICDs holds some lessons that will be useful in coping with the newer contingent liabilities that are on the way in the immediate future. The dynamically evolving regulatory environment continues to open new opportunities, while at the same time creating new pitfalls for the unwary or reckless.

2. Structured Products

Structured finance products became widely used in the U.S. during the 1980s and were introduced in Europe in the mid 1990s during years of low interest rates.

Structured products were created by financial institutions in their attempt to respond to the general demand for higher yields and limited risk. They offer features that traditional investments cannot, such as principal protection, diversification and customized payoffs and they suggest a useful hedging alternative for certain risks.

The substantial reduction of international interest rates in combination with the high uncertainty of investing in the stock market increased the growth of the structured product market.

2.1. Common Characteristics and Classification

A structured product has no precise definition, either in a business or a regulatory context. The broad definition used by regulators such as the SEC, NASC and NYSE in the U.S. define a structured product as a security derived from or based on another security (including bonds), basket of securities, index, commodity, or foreign currency. This definition encompasses a wide range of contracts, including equity-linked or commodity-linked dept, collateralized dept obligations, reverse convertibles and credit default swaps. Thus, structured finance products are financial instruments designed to meet specific investor and issuers needs by incorporate special, non-standard features whose values are linked to, or derived from, such underlying assets as stocks, bonds, currencies and commodities. The performance of a structured product is therefore based on the performance of this underlying asset and not the discretion of the product provider.

Structured products include all financial products issued to the public by financial institutions that combine at least two financial investments. They are usually composed of identifiable building blocks. The **debt component** of the product is the equivalent of a bond, and the **equity component** is the equivalent of a long-term option. The option or options included in the note offer a customized payoff to the investor spanning from portfolio insurance to enhanced yields (Braddok, 1997).

A typical structured product would be a zero-coupon or interest-bearing note combined with a derivative whose value is typically realized at the maturity of the note. An example is a gold-linked note that makes a period interest payment of a fixed amount and that pays at maturity the face value of the bond plus an additional amount based on the return of gold over the life of the note.

One form of classification involves dividing up the structured products into two basic categories:

- **growth** products, which provide an element of principal protection. They typically guarantee the repayment of a pre-determined percentage of the nominal and at the same time allow investors to participate in the growth of the underlying instrument.
- **income** products, which provide fixed high income, but with a risk to capital return. They are products without principal protection which let the investor realize enhanced returns without guaranteeing that the capital outlay will be reimbursed.

Further, products can be subdivided on the basis of specific characteristics such as without coupon payments, products with coupon payments and products with exotic features. Products without coupon payments let the investor buy the embedded bond at a discount. Typically, the excess from purchasing the bond at discount is used to buy the embedded option. For products with coupon payments, instead of the discount, a coupon is paid out for the amount of the compounded discount. Nevertheless, these products are the same economically speaking. Products with exotic features exist in many different forms. Adding barrier or Asian options to the product rather that a plain vanilla option are common ways of customizing the payoff for the investor. Products with Asian features can be seen as further developments of the first two categories (Grunbichler et al, 2003).

While there is no legal or technical definition consistency used to describe every possible type of structured finance transaction, U.S. Securities and Exchange Commission (SEC) notes some common characteristics, derived by academic and regulatory studies. Three key characteristics of customized structured products are:

1. cash-based or synthetic linking of pooling of assets
2. de-linking the credit risk of the pooled assets from the originator of the credit, often through transfer to a special purpose vehicle(SPV) or other entity with a finite life

3. splitting the resulting liabilities within the structured product based primarily on risk levels, consequently producing different returns.

Some popular and widely used structured products are presented below.

Asset Swaps

Asset swaps are a common form of structured product written on fixed- rate debt instruments. The end result of an asset swap is to separate the credit and interest rate risks embedded in the fixed- rate instrument. Effectively, one of the parties in an asset swap transfers the interest rate risk in a fixed-rate note or loan to the other party, retaining only the credit risk component. As such, asset swaps are mainly used to create positions that closely mimic the cash flows and risk exposure of floating-rate notes.

Investors who want exposure to credit risk, without having to bother much about interest rate risk, but who cannot find all the floaters they want, can go to the asset swap market to "buy synthetic floaters." Likewise, investor who fund themselves through floating-rate instruments but who hold fixed rate assets might want to transfer the interest rate risk of their assets to someone else by becoming buyers of asset swaps. Banks are significant users of asset swaps and some market participants, such as hedge funds, use the asset swap market to exploit perceived arbitrage opportunities between the cash and derivatives market.

Asset swaps can be used to leverage one's exposure to credit risk. For instance, consider an investor who wants to buy a corporate bond that is trading at a substantial premium over its face value. The investor can either buy the bond in the open market for its full market value or buy it through a par asset swap where the initial cash outlay would be only the bond's face value. Of course, as with other leveraging strategies, buying the asset swap in this case has the implication that the investor could lose more than the initial cash outlay in the event of default by the bond issuer.

To evaluate an asset swap (AP), it is best to think of it as a "package" involving two products: (i) a fixed-rate bond (B), which is bought by the investor from the dealer for par, and (ii) an interest rate swap (IRS) entered between the investor and the dealer. Thus, from the perspective of the asset swap buyer (the investor), the market value of the asset swap can be written as

$$\mathrm{V^{AP}(0,N) = [V^{B}(0,N) - P] + V^{IRS}(0,N)}$$

where P denotes the face value of the bond, and the notation V^Y (.) is used to represent the market value of Y , whatever Y may be. For the interest rate swap, we express its market value to the asset swap buyer; the market value to the asset swap seller would be the negative of this quantity.

In other words, the value of the asset swap to its buyer at its inception is essentially given by the sum of its two components. The buyer paid par for the bond, but its actual market value, $V^B(0,N)$, could well be different from par, and thus the buyer could incur either a profit or a loss if the bond were to be immediately resold in the open market. This potential profit or loss is shown by the term in brackets on the right side of the equation. The second component in the valuation of the asset swap is the market value of the embedded interest rate swap between the buyer and the dealer, denoted above as $V^{IRS}(0,N)$. That swap too may have either positive or negative market value to the buyer.

The market prices of the reference bond and the embedded interest rate swap determine the market value of the asset swap.

Credit Default Swaps

Credit default swaps are the most common type of credit derivative and popular structured products. According to different surveys of market participants, Credit default swaps are by far the main credit derivatives product in terms of notional amount outstanding. Their dominance of the marketplace is even more striking in terms of their share of the total activity in the credit derivatives market. As actively quoted and negotiated single-name instruments, Credit default swaps are important in their own right, but their significance also stems from the fact that they serve as building blocks for many complex multi-name products.

The rising liquidity of the credit default swap market is evidenced by the fact that major dealers now regularly disseminate quotes for such contracts. Furthermore, along with risk spreads in the corporate bond market, Credit default swaps quotes are now commonly relied upon as indicators of investors' perceptions of credit risk regarding individual firms and their willingness to bear this risk. In addition, quotes from the Credit default swap market are reportedly increasingly used as inputs in the pricing of other traditional credit products such as bank loans and corporate bonds, helping promote greater integration of the various segments of the credit market.

At the most basic level, protection sellers use credit default swaps to buy default insurance and protection sellers use them as an additional source of income. In practice, however, market participants' uses of credit default swaps go well beyond this simple insurance analogy.

As we did in the case of Asset swaps, when we priced a simple variant of the asset swap, we will make use of the static replication approach to valuing financial assets. That approach tells us that if we can devise a portfolio made up of simple securities that replicates the cash flows and risk characteristics of the contract we want to price, the price of that contract is, in the absence of arbitrage opportunities, simply the price of setting up the replicating portfolio.

Basket Default Swaps

Basket default swaps are credit derivatives written on a “basket” or portfolio of assets issued by more than one reference entity. In particular, a payment by the protection seller in a basket swap can be triggered by a default of any one of the entities represented in the basket.

Protection buyers find basket swaps attractive because they tend to be less expensive than buying protection on each name in the basket separately through single-name credit default swaps. From the perspective of investors (protection sellers), basket swaps provide an opportunity for yield enhancement with a limited downside risk.

The main determinants for the premium paid by the protection buyer in a credit basket swap are (i) the number of entities in the reference basket, (ii) the credit quality and expected recovery rate of each basket component, and (iii) the default correlation among the reference entities. Understanding the role of the first two determinants – number and credit quality of the reference entities - is relatively straightforward. In general, other things being equal, the larger the number of entities included in the basket the greater the likelihood that a default event will take place and thus the higher the premium that protection buyers will pay. Likewise, for a given number of names in the basket, the lower the credit quality and recovery rates of those names, the more expensive the cost of protection will be.

Principal- Protected Structures

Principal- protected structures (PPN) are coupon-paying financial products that guarantee the return of one’s initial investment at the maturity of the structure, regardless of the performance of the underlying (reference) assets. The coupon payments themselves are stopped in the event of default by the reference entity. Principal-protected structures can be thought of as a form of a funded credit derivative.

Principal- protected Notes appeal to investors seeking some exposure to credit risk, but who want to protect their initial investment. As such, Principal-protected Notes can be used to make sub- investment- grade debt instruments appealing to conservative investors. Investors may have to give up a substantial portion of the credit spread associated with the reference entity in order to obtain the principal-protection feature. As a result, a Principal-protected Note that references a highly rated entity would have very limited interest to some investors as its yield would be very low. On the other hand, conservative investors might welcome the additional safety that a Principal-protected Note would provide even to investment- grade instruments.

For valuation purposes, it is helpful to decompose a Principal- protected Note into two components, the protected principal and the (unprotected) stream of

PPN = protected principal + stream of coupon payments

For simplicity, we start by assuming that counterparty credit risk is completely dealt with via full collateralization and other credit enhancement mechanisms so that the Principal- protected Note buyer has no credit risk exposure to the Principal- protected Note issuer. This implies that we can think of the protected principal as being akin to a riskless zero- coupon bond that has the same par value and maturity date as the Principal- protected Note.

As for the stream of coupon payments, it can be characterized as a risky annuity that makes payments that are equal to the Principal- protected Note's coupon and on the same dates as the Principal- protected Note. We have thus decomposed the Principal- protected Note into two simpler assets

PPN = Riskless zero-coupon bond + Risky annuity

where the payments made by the annuity are contingent on the reference entity remaining solvent. Indeed, using now familiar terminology, the zero- coupon bond and the annuity constitute the replicating portfolio for this Principal-protected Note, and, as a result, valuing a Principal- protected Note is the same as determining the market prices of the bond and the annuity.

Credit-Linked Notes

Credit- linked notes are essentially securities structured to mimic closely, in funded form, the cash flows of a credit derivative. Credit- linked notes have a dual nature. On the one hand, they are analogous to traditional coupon paying notes and bonds in that they are securities that can be bought and sold in the open market and that promise the return of principal at maturity. On the other hand, they can be thought of as a derivative on a derivative, as a credit- linked note’s cash flow is tied to an underlying derivative contract.

Credit- linked notes play an important role in the credit derivatives market as they have helped expand the range of market participants. In particular, some participants are attracted to the funded nature of a Credit- linked note, either because of their greater familiarity with coupon- bearing notes or because they are prevented from investing in unfunded derivatives contracts by regulatory or internal restrictions.

Credit- linked notes allow the cash flows of derivatives instruments to be “repackaged” into securities that can be bought and sold in the market place. This is especially useful for certain classes of institutional investors, such as some mutual funds, that are precluded from taking sizable positions in unfunded derivatives contracts. Investors who do not have master credit derivatives agreements with dealers are attracted to credit-linked notes because they generally require less documentation and lower setup costs than outright credit derivatives contracts. In addition, credit- linked notes can be tailored to meet specific needs of investors. Lastly, credit- linked notes can be rated at the request of individual institutional investors. Credit- linked notes can help increase the liquidity of certain otherwise illiquid assets or even create a market for assets that would otherwise not exist in tradable form.

As general rule, the single most important risk exposure in a Credit- linked note is, naturally, the credit risk associated with the reference entity. Credit- linked notes spreads are often wider than the spreads associated with the corresponding reference entities, however, although the issuer may reduce the spread paid under the Credit- linked note to cover its administrative costs. The higher Credit- linked note spread reflects the investor’s exposure to the counterparty credit risk associated with the Credit- linked note issuer; the investor would not be exposed to such a risk if buying a note issued directly by the reference entity. Counterparty credit risk is more important for Credit-linked notes issued out of a bank or dealer, rather than from a highly rated special purpose vehicle, which tend to make more widespread use of collateral arrangements, as we shall see in the next chapter.

CDOs

CDOs are instruments that allow one to redistribute the credit risk in a given portfolio into tranches with different risk characteristics and, in the process, meet the risk appetites of different investors.

Collateralized debt obligations are essentially securities with different levels of seniority and with interest and principal payments that are backed by the cash flows of an underlying portfolio of debt instruments. When the debt instruments are loans, the CDO is often called a CLO - a collateralized loan obligation - if they are bonds, the CDO becomes a CBO - a collateralized bond obligation.

CDOs have a wide range of applications in the financial markets and tend to be classified according to the ultimate goals of their sponsors. For instance, from the perspective of a commercial bank, CDOs make it possible to transfer a large portfolio of loans off the bank's balance sheet in a single transaction with a repackaging vehicle. Such CDOs are commonly called balance-sheet CDOs, and, indeed, historically, banks' desire to free up regulatory capital through balance-sheet CDOs was an important driver of CDO market activity in the 1990s. In recent years, a substantial share of CDO issuance has been driven not so much by banks' balance-sheet management needs, but by investor demands for leveraged credit risk exposures. These CDOs are often referred to as arbitrage CDOs in that the institutions behind the issuance are, for instance, attempting to enhance their return on the underlying assets by becoming first-loss (equity) investors in the newly created structures. Most of these arbitrage CDOs are actively managed and thus the CDO investors are exposed both to credit risk and to the particular trading strategy followed by the CDO manager. Insurance companies, asset managers, and some banks are among the main equity investors in arbitrage CDOs.

Over and above their general application to balance-sheet management and return enhancement, CDOs can be used to create some liquidity in what would otherwise be essentially illiquid assets. In addition, as noted, the CDO structure allows, through the tranching process, the creation of new assets with specific profiles that may better match the individual needs and risk tolerances of institutional investors.

Three main factors enter into the pricing of the various tranches of a CDO: the degree of default correlation among the debt instruments in the collateral pool, the credit quality of the individual debt instruments, and the tranching structure of the CDO. In addition, and quite naturally, these are some of the main variables taken into account by the major credit-rating agencies when assessing the risk embedded in individual CDO structures.

Except the traditional CDOs, synthetic collateralized debt obligations are also popular. Synthetic collateralized debt obligations are structured financial products that closely mimic the risk and cash flow characteristics of traditional (cash- funded) collateralized debt obligations. This "mimicking" is done through the use of credit derivatives, such as credit default swaps and portfolio default swaps.

Valuation considerations are similar to those involving traditional CDOs. Default correlation, the credit quality of the individual entities represented in the reference portfolio, and the details of the tranching structure are important factors. In addition, the legal structure of the special- purpose vehicles, as well as the credit quality of the special- purpose vehicle's collateral and of the sponsoring bank may also play a role.

2.2. Trading

Structured products offer investment strategies that private investors may not be able to pursue due to high transaction costs and lack of knowledge. Issuing institutions can in many cases effectively offer these products to the public through their economic scale. However, there is no doubt that the institutions want to be compensated for their work through the pricing of the instruments.

Structured products trade at transaction costs and commissions for private investors that are usually more complex to calculate compared to the corresponding single security trades (buying bonds, selling options). Moreover, most structured products face the risk of trading at prices far from "market norm". Since otherwise there would be no liquid trade in secondary market, issuers are required, to a greater or lesser extend, to act as market-makers for their own products during the exchange trade, although there is no formal obligation for the institution to do so. Alternatively, structured products can be bought / sold over-the-counter from / to the issuing firm. . Since the issuer is the only market maker for the particular instrument one would be able to question whether the issuing institutions are giving private investors fair quotes on the market since transparency and competition are limited. And due to the rather complex valuation, unfavorable prices are even more likely to be quoted.

Issuing institutions have received some international criticism in recent years for not giving adequate information about the product, especially with respect to hidden fees. For the investor it is crucial to be able to judge appropriateness of the pricing of these products.

2.3. Growth in Volume

There is no doubt that the international market for structured products has grown exponentially both in size and complexity in resent years. However, at the absence of a widely accepted definition it has been difficult to estimate the precise amount of aggregate transactions on such products. Nevertheless, recent research has provided some insights.

According to U.S. Securities and Exchange Commission (SEC), it as been estimated that total funded structured finance issuances in 2000 was approximately $400 billion, rising to more than $1 trillion in 2004. The Bank for International Settlements (BIS) reported in November 2005, that credit default swaps bought, rose from $4.46 trillion (notional amounts) in December 2004 to $7.65 trillion in June 2005. BIS also reported that, according to their semiannual survey, notional amounts of OTC derivatives outstanding had risen to $270 trillion.

The Structured Products Association (SPA) estimates $45 to $50 billion worth of products were placed in the United States in the United States in 2005 (J. Bethel, A. Ferrell, 2006). Both registered and unregistered segments of structured products market are experiencing growth. In 2004 issuers sold $12 billion in notional registered structured products in the U.S., up more than 20 percent from 2003 when just under $10 billion was placed. The market has grown 53 percent annually over the last 4 years. In terms of listed registered products, similar growth is reported. The NYSE, for example, reports that by June 2006, it had listed more than double the number of new products than it had listed over the same period in 2005.

This growth in both the unregistered and registered markets is arising in part from new distribution channels including distributions to retail investors. Investment banks are increasingly offering structured products in small denominations to retail investors through their broker networks.

2.4. Potential Investors

The high market growth of structured products has been made possible by financial institutions successfully selling unregistered structured products to new classes of investors.

Financial institutions realized quickly that high net worth investors present an attractive market opportunity beyond their traditional institutional investor client base. High net worth individuals are both willing and able to invest in unregistered structured products.

This kind of products is appealing to high net worth individuals for a number of reasons. To start with, these investors often demand complex financial portfolios. Combinations of long positions in stocks and bonds may not provide the overall risk exposure they desire. Additionally, many of these investors use financial advisors who may be useful in navigating the significant intricacies of structured products. The payout patterns of these securities can be very complex, requiring sophisticated financial models for valuation. Finally, structured products can be tailored to offer high non-linear payout patterns that permit very specialized or state-contingent bets to be made on assets such as currencies, commodities or various baskets of securities.

On the other hand, financial institutions realized that structured products can be registered and sold to the mass retail market. Recent evidence suggests that, in addition to high net worth individuals, traditional retail investors in U.S. and Europe are increasingly buying registered structured products. In the U.S. American Stock Exchange reported an 18 percent increase in the number of structured products issued in 2005 on the exchange over 2004. And if experience with other financial products serves as a guide, structured products will increasingly be sold to the mass market of retail investors.

2.5. Regulatory Risk and Risk Management

The use of structured products has become an increasingly significant means for credit transfer and enhanced risk management. However, this increased efficiency is only effectuated if all parties involved - the credit originator, the creator of the structured product, and the purchaser of the tranched liabilities or other portions of the products, have risk modeling and management capabilities that parallel their own respective risks, and that they conduct their activities consistent with all applicable laws. As a twenty-five year veteran of various regulatory and supervisory regimes and a current official in the U.S. Securities and Exchange Commission's (SEC's) Office of Compliance Inspections and Examinations, my focus for the remainder of my remarks will be on the critical role of risk management for those firms involved in any aspect of the design, offer, purchase or sale of structured finance products - particularly those involving complex structured finance transactions (CSFTs).

Structured finance transactions cover many different products with varying levels of complexity. Standard public mortgage-backed securities transactions, asset-backed commercial paper conduit transactions, or simple derivatives or collateralized loan obligations used for hedging are typically not considered complex. However, there are other more complex variations of structured finance transactions. Risk management may be particularly challenging with respect to such CSFTs because of their complexity. Complexity may arise, for example, from the loss distribution of underlying assets in the pool. In addition, tranching raises analytical complexity as it involves unique covenants for the allocation of principal and interest payments received from the pool, loss distribution, and redirection of cash flows under stress and other scenarios. Other risks involve the interaction of multiple sub-transactions within a CSFT, conflicts of interests, non-performance by third parties, legal risks from bankruptcies, and incomplete documentation and assignments. There is also the risk of being viewed as assisting potential violations by customers who may be using the products for illegal tax, accounting or other purposes.

It is clear that along with continuous, significant and concentrated growth of the structured finance, comes increasing responsibility for the regulatory institutions to properly manage associated risks and to ensure that these complex products are used within the confines of the law.

3. Theoretical Frame of Reference

Structured products can be replicated by a portfolio of existing financial instruments and can be evaluated as the combination of their building blocks. They are usually composed of a **debt component**, which is the equivalent of a bond, and an **equity component,** which is the equivalent of a long-term option.

General concepts of bond and option valuation theory are the theoretical background needed for this approach.

3.1. Valuation of Structured Products

Structured products are valued as the combination of their building blocks. Structures typically involve combinations of bonds and different types of options. Consequently, the price of the structured product can be written as the sum of the bond and the option value as given in Equation (3.1).

$$\textit{Structured Product} = \textit{Bond} + \textit{Option} \tag{3.1}$$

3.2. Valuation of Bonds

The bond portion of the structured product can be estimated using the standard bond valuation model where future cash flows are discounted back to today's date as given in Equation (3.2).

$$Bond\ Value = \sum_{t=1}^{n} \frac{C_t}{1+r_t} + \frac{N}{1+r_n} \tag{3.2}$$

C_t: Coupon payment at time t

N: Nominal value

r_t: Annual default-adjusted yield to maturity

Zero-coupon bonds do not pay periodic interest payments, C_t. The holder is entitled to receive the payment at maturity; instead, they are purchased at deep discount from their nominal value. The value of a zero-coupon bond is thus the nominal value discounted to today's date.

Bond prices are related to the probability of default by the bond issuer. The relative quality of most traded bonds can be judged by bond ratings given by Moody's and Standard and Poor's. Bonds rated Baa or above are referred to as investment-grade bonds. (Brealy et al, 2000)

To value a bond using the appropriate default-adjusted interest rate, one can use bond indexes, which are calculated from discrete credit spreads in the market. Often a surcharge to the risk-free interest rate, typically a Treasury bill rate, is adequate to obtain an accurate approximation.

3.3. Valuation of Options

There are a number of pricing models available to value options. The Black Scholes pricing model is indisputably the most popular model in use today. The reasons behind it are that it is simple, easy to implement due to its analytical closed-form solution and that it provides good estimates of an option's worth under normal circumstances.

There are also a number of numerical procedures that have a large following such as binomial trees, Monte Carlo simulation and finite difference methods. These methods are very useful when no analytical solution is available. Numerical procedures also allow for more flexibility such as valuing options with complex payoffs and more exact handling of parameters. Therefore, in many circumstances they can produce more accurate results for a wider range of option types (Katz et al, 2005).

3.3.1. The Black - Scholes Model

The Black Scholes model, developed by Fischer Black, Myron Scholes and Robert Merton, presented a groundbreaking new way of valuing options when it was introduced in 1973. The model has had a tremendous influence on the way practitioners price and hedge options as well as spurred the growth of financial engineering.

Equations (3.3) and (3.4) illustrate the analytical closed-form solutions to calculate the price of European call and put options on a non-dividend paying stock, developed by Black and Scholes.

$$c = [S_o N(d_1) - Ke^{-rt} N(d_2)] \tag{3.3}$$

$$p = [Ke^{-rt} N(-d_2) - S_o N(-d_1)] \tag{3.4}$$

$$d_1 = \frac{\ln\left({}^{S_0}/_{K}\right) + (r + 0.5\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

S = Current value of the security
K = Exercise price
r = Risk-free rate
σ = Standard deviation of the percentage change in the stock price

The function N(x) is the cumulative probability distribution function for a standardized normal distribution (Hull, 2003).

The following assumptions are used to derive the Black- Scholes pricing model:

1. The price of the underlying instrument St follows a geometric Brownian motion with constant drift μ and volatility σ.
2. It is possible to short sell the underlying stock.
3. There are no transaction costs or taxes. All securities are perfectly divisible.
4. There are no dividends during the life of the derivative.
5. There are no riskless arbitrage opportunities.
6. Security trading is continuous.
7. The risk-free rate of interest, r, is constant and the same for all maturities.

As can be seen in this chapter, a number of variations of the Black Scholes pricing model have been developed to handle violations of assumption such as dividends, stochastic volatility and interest rate in order to use the model in practice.

3.3.2. Index Options and Dividends

In valuing index options the Black- Scholes model can be modified to treat the index as a security paying a known continuous dividend yield.

Equation (3.5) and (3.6) show that by substituting S_0 with S_0e^{-qT} in the Black-Scholes formula the price of European index call and put options can be calculated given that the dividend yield q is known.

$$c = [S_o N(d_1) - Ke^{-rt} N(d_2)] \tag{3.5}$$

$$p = [Ke^{-rt} N(-d_2) - S_o N(-d_1)] \tag{3.6}$$

$$d_1 = \frac{\ln\left({S_0}/{K}\right) + (r - q + 0.5\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

S = Current value of the security

K = Exercise price

r = Risk-free rate

σ = Standard deviation of the percentage change in the stock price

If the absolute amount of the dividend that will be paid out on the stocks underlying index is assumed to be known rather than the dividend yield, the basic Black Scholes formula can be used with the initial stock price being reduced by the present value of the dividends (Hull, 2003).

Dividend adjustment procedures with the Black and Scholes model can have a significant impact upon the calculated option value, as shown in empirical studies by Brenner, Courtadon and Subrahmanyam (1987). The discrete approach is the preferred procedure since it discounts the dividends as they actually are paid. The continuous approach, assumes that dividends are spread out at a continuous rate over the period. It requires less information on specific stock dividends in the index and empirical studies show that it yields a very good approximation for European index options that include a large number of securities (Brenner et al, 1987).

One should note that the term dividend should be regarded as the reduction of share price on the ex-dividend date as a result of the dividends. The stock price may go down less than the actual cash amount caused by tax effects. (Hull, 2003)

3.3.3. Binary Options

Binary option is a type of option where the payoff is either some fixed amount of some asset or nothing at all. One main type of binary option is the cash-or nothing option. The cash-or nothing put for example pays off nothing if the asset ends up above the strike price at time T and pays a fixed amount Q, if it ends up below the strike price. There is an analytical closed form solution available for this type of binary option. Risk-neutral valuation yields that the probability of the asset price being below the strike price at maturity is $N(d_2)$, following the same notation as before. The binary put can thus be valued as shown in Equation (3.7). Equation (3.8) denotes the value of a binary call (Hull, 2003).

$$p = Qe^{-rT}N(-d_2) \tag{3.7}$$

$$c = Qe^{-rT}N(d_2) \tag{3.8}$$

3.3.4. Two-Asset Cash-or-Nothing Options

Four types of two-asset cash-or-nothing options exist:

1. A two-asset cash-or-nothing call (up-up) pays out a fixed cash amount K if asset one, S_1, is above the strike X_1 and asset two, S_2, is above strike X_2 at expiration.
2. A two-asset cash-or-nothing put (down-down) pays out a fixed cash amount K if asset one, S_1, is below the strike X_1 and asset two, S_2, is below strike X_2 at expiration.
3. A two-asset cash-or-nothing call up-down pays out a fixed cash amount K if asset one, S_1, is above the strike X_1 and asset two, S_2, is below strike X_2 at expiration.
4. A two-asset cash-or-nothing put down-up pays out a fixed cash amount K if asset one, S_1, is below the strike X_1 and asset two, S_2, is above strike X_2 at expiration.

The formulas published by Heynen and Kat (1996) can be used to price these binary options:

Two Asset Cash of Nothing call (up-up) :

$$= Ke^{-rT} M\left(d_{1,1}, d_{2,2}; \rho\right) \tag{3.9}$$

Two Asset Cash of Nothing put (down-down) :

$$= Ke^{-rT} M\left(-d_{1,1}, -d_{2,2}; \rho\right) \tag{3.10}$$

Two Asset Cash of Nothing up-down :

$$= Ke^{-rT} M\left(d_{1,1}, -d_{2,2}; -\rho\right) \tag{3.11}$$

Two Asset Cash of Nothing down-up :

$$= Ke^{-rT} M\left(-d_{1,1}, d_{2,2}; -\rho\right) \tag{3.12}$$

where

$$d_{i,j} = \frac{\ln\left({S_i}/{X_j} \right) + (r - {\sigma_i^2}/{2})T}{\sigma_i \sqrt{T}}$$

and where

S_1 = Spot price of underling asset one.

S_2 = Spot price of underling asset two.

X_1 = Strike price of underling asset one.

X_2 = Strike price of underling asset two.

r = Risk free rate

b = Cost of carry

T = Time to maturity

σ_1 = Volatility of underling asset one.

σ_2 = Volatility of underling asset two.

K = Fixed amount of cash

M = Cumulative bivariate normal distribution function.

3.3.5. Interest Rate

The Black Scholes pricing model can handle stochastic interest rates by assuming that r is equal to the zero-coupon risk-free interest rate for a maturity of T. This is theoretically correct providing the stock price at time T is lognormal and the volatility parameter is chosen appropriately (Hull, 2003).

3.3.6. Volatility

The volatility is defined as the standard deviation of the return provided by underlying asset in one year when the return is expressed using continuous compounding.

In order to obtain an estimate of the historical volatility the stock is observed at fixed intervals of time (e.g., every day, week or month). As expressed in Equation (3.13), for each time period, the natural logarithm ratio of the stock price at the end of the time period to the stock price at the beginning of the time period is calculated.

Define:

n+1: Number of observations

S_i: Underlying asset price at end of the i-th (i = 0, 1, …, n) interval

τ : Length of time interval in years

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \qquad for \ \ i = 1, 2, \dots, n \tag{3.13}$$

The standard deviation, s, of the u_i's is given by Equation (3.14) or Equation (3.15).

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \tag{3.14}$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} {u_i}^2 - \frac{1}{n(n-1)}\left(\sum_{i=1}^{n} u_i\right)^2} \tag{3.15}$$

where $\bar{u}$ is the mean of the u_i's

Equation (3.16) shows that the volatility is estimated as the standard deviation divided by the square root of the length of the time period in years. Typically, days when the exchanges are closed are not considered in measuring this time period.

$$\hat{\sigma} = \frac{s}{\sqrt{\tau}} \tag{3.16}$$

Hull (2003) discusses the fact that choosing an appropriate value of n for estimating the historical volatility is not easy. More data generally lead to more accuracy, but σ does change over time and data that are too old may not be relevant for predicting the future.

Hull (2003) suggests that using closing prices from daily data over the most recent 90 to 180 days seems to work reasonably well empirically. An often-used rule of thumb is to set n equal to the number of days to which volatility is to be applied.

3.3.7. Implicit Volatility and Volatility Smiles

When valuing options practically the assumption of constant volatility is in many cases insufficient. In practice, traders usually work with implied volatilities. These are the volatilities implied by option prices observed in the market. Implied volatility is the value of σ that, when substituted in Black- Scholes equation gives the theoretical value of an option equal to the option's market value.

A volatility smile is a pattern that often can be observed when illustrating the implied volatility of an option as a function of its strike price in a graph. If the assumptions of Black-Scholes model were correct, especially the assumption of constant volatility, then the volatility smile would not appear; all options on the index should have the same implied volatility and one should see a horizontal line. Instead, the market's implied volatilities for index options have shown a negative relationship between implied volatilities and strike prices; out-of-the-money puts trade at higher implied volatilities than out-of-the-money calls. Figure 3.1 shows this behavior for European-style options on the S&P 500, as of January 31, 1994. The dotted line is the represents the constant volatility that would prevail if the Black Scholes model were correct in the real world. The data for strikes above (below) spot comes from call (put) prices (Derman et al, 1994).

Form and shapes of the volatility smile varies depending on the underlying asset. In the equity options market, the volatility smile is often negatively skewed, also called “volatility smirk”, whereas in the currency option market the smile is more balanced.

Hull suggests that one explanation for the smile in equity options concerns leverage. As a company’s equity decline in value, the company’s leverage increase. As a result the volatility of its equity increases making even lower stock prices more likely. As a company’s equity increases in value, leverage decreases. As a result the volatility of its equity declines, making higher stock prices likely. His argument shows that one can expect the volatility of its equity to be a decreasing function of price.

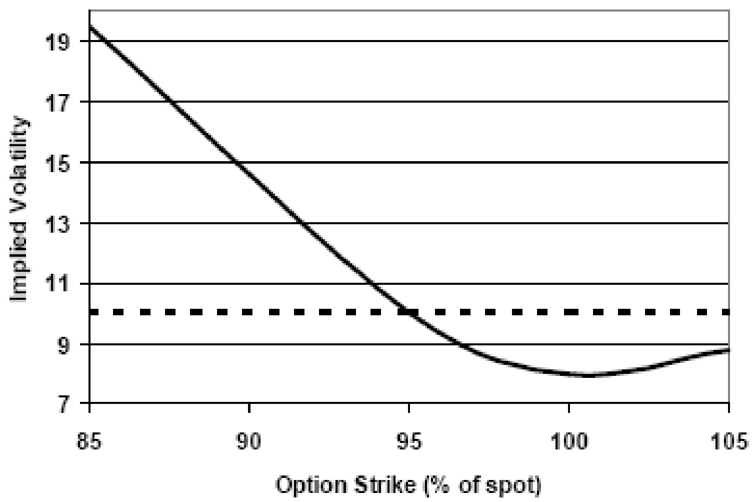


Figure 3.1. Example of Volatility Smirk

A curious fact is that the pattern for equities has existed only since the stock market crash of October 1987. It was first noticed in the academia by Mark Rubinstein. He suggests that one reason for the pattern may be “crashophobia”. In other words, markets are attempting to price in the possibility of catastrophic risk. Rubenstein has some empirical support for these effects; whenever the market declines, there is a tendency for the skew to increase (Hull, 2003).

3.4. Influence of Life Cycle and Moneyness of the Option

Wilkens, Erner and Röder (2005) have found evidence that issuers orient their pricing towards the product lifetime and the incorporated risk of redemption given by the moneyness of the implicit options. They have formulated an econometric model, shown in Equation (3.17) that has proven significant for the German market.

$$\Delta Vi = a + bL_i + cM_i + d\Psi_i + \varepsilon_i \qquad (3.17)$$

ΔV_i = *Average price deviation*

L_i = *Products' relative lifetimes*

M_i = *Moneyness of the implicit option*

Ψ_i = *Quote spreads*

The authors argue that the relative life cycle of the product bears a close relation to the order flow. At issuance, only sales can occur from the issuer's point of view. It is unlikely that investors will resell the products immediately after their issue. The issuer can thus at a time immediately after their structured product issue set prices higher than their fair value without running the risk of increasing their exposure.

As maturity approaches the issuing institution will sell fewer products since the issuance volume is limited and products with only a short time to maturity are demanded less often than those with more of their initial lifetime remaining. In addition, product repurchases will increase, as investors prefer to offset their positions, because of changes in expectations concerning the underlying. In this situation, the issuer can profit by setting prices for the structured products that are lower than the fair value.

Moneyness of the implicit option might also be driving the pricing pattern since it determines the product's current risk of redemption. Investors are more likely to sell their structured products when it is in-the-money in order to realize capital gains. They would typically not sell the product when it is out-of-the money, especially if the product includes a capital guarantee at maturity. The issuer is thus able to benefit by setting prices higher than the fair value in cases where the moneyness of the implicit option is low and set the prices lower than the fair value when moneyness is higher.

Besides the absolute level of mid-quotes, bid-ask spreads serve as another pricing parameter for the issuer. If large surcharges, given by $\Delta\sigma > 0$, are not possible for the issuers (e.g. in the case of numerous expected repurchases), they could widen the spreads as alternative sources of earnings. The spread is defined by Ψ_i= *(Ask quote – Bid quote)/(Mid quote)* (Wilkens et al, 2000).

4. Pricing Methodology of Market Index Certificates of Deposit

Structured products seem to be an attractive investment alternative. Certificates of Deposit are especially popular within individuals with little or no investment experience, since they provide investors a means for participating in the stock market while at the same time they limit their risk exposure. The Market Index Certificates of Deposit (MICDs) are variable-rate certificates of deposit for which the interest rate is contingent upon the performance of a well-known stock market average (such as the S&P 500 index) but with a guaranteed minimum interest rate. The lack of experience of investors creates the urgent need to find an applicable method for pricing the MICDs.

4.1. The Evolution of the MICDs

Depository liabilities with interest rates linked to the stock market are a rather recent addition in a long line of financial instruments.

Chase Manhattan was the first bank to offer a market index product in the form of an insured certificate of deposit, but these Market Index Certificates of Deposit were preceded by several varieties of bonds with interest contingent upon a stock market index. Swedish Export Credit (SEK) pioneered the so-called "bull and bear" bonds in mid-1986. The redemption price of these bonds (which mature in July 1991) is linked to the Nikkei average of 225 stocks on the Tokyo exchange (a detailed description is given in Walmsley, 1988). In August 1986 the Salomon Brothers investment banking firm offered a product called S&P 500 Index Subordinated Notes (SPINs) maturing in 1990 with a 2% annual coupon and a maturity payment linked to the increase in the stock market index. A bear version, Reverse SPINs, was later introduced in September 1987. In October 1987, Merrill Lynch offered an index version of its Liquid Yield Option Notes (LYONS) with zero coupon, four-year maturity, and maturity payment linked to the New York Stock Exchange Composite Index. Then Portfolio Income Notes (PINs) debuted in December 1987, designed to appreciate in direct correlation with declines in the New York Stock Exchange (N YSE) Index, while providing guaranteed return of principal.

Since their introduction, MICDs have been successfully offered by several other commercial banks and thrift institutions as well, worldwide.

4.2. Market Index Certificate of Deposit with Embedded Vanilla Call Option

On March 18, 1987, Chase Manhattan Bank and its affiliate, Chase Lincoln First Bank, introduced a certificate of deposit with the interest rate set at a fixed percentage of the price appreciation of the S&P 500 during the life of the CD with a guaranteed minimum rate. Investors can select different combinations of maturity, guaranteed minimum rate, and percentage participation in the S&P 500.

M. Chance & B. Broughton (1988) present an interesting approach to pricing products of that kind.

4.2.1. Pricing the MICD

Let today be denoted as time zero. A $1 market index CD issued today matures at time T; thus, its original maturity is T-0 = T. The market index when the CD is issued is S_0. The deposit pays interest at the rate of γ percent of the return on the market index over the life of the CD. We shall refer to γ as the participation percentage. The CD guarantees the holder a minimum return of i percent continuously compounded.

Thus, at maturity the CD is worth

$$
\begin{array}{ll}
e^{iT} & if \quad 1+\left[\left(\frac{S_T}{S_0}\right)-1\right]\gamma \leq e^{iT} \\
1+\left[\left(\frac{S_T}{S_0}\right)-1\right]\gamma & if \quad 1+\left[\left(\frac{S_T}{S_0}\right)-1\right]\gamma > e^{iT}
\end{array}
$$

where S_T is the index at time T (we assume that the CD will not be redeemed prior to maturity). It will be helpful to restate these conditions as

$$
\begin{array}{ll}
\lambda & if \quad S_T \leq S_0\left[\frac{\lambda-1}{\gamma}+1\right] \\
1+\left[\left(\frac{S_T}{S_0}\right)-1\right]\gamma & if \quad S_T > S_0\left[\frac{\lambda-1}{\gamma}+1\right]
\end{array}
$$

where $\lambda = e^{iT}$.

4.2.2. Replicating the MICD

Now suppose we wish to determine the value of the CD at some time t during the life of the CD; thus, t is prior to T. Let D_t represent the value of the CD at time t. One way to value the CD is to find another portfolio of instruments that replicates the outcome of the CD. One such portfolio is

a risk-free pure discount bond with a face value of λ and a maturity of $\tau = T\text{-}t$, and

φ European call options on the market index expiring at T with exercise price of X.

Let $\varphi = (\gamma/S_0)$, $X = S_0[(\lambda-1)/\gamma + 1]$, and the call price be $c(S_t, \tau, X, r, \sigma^2)$. The notation for the option price expresses the call as a function of the current market index, S_t, the time to expiration, τ, the exercise price, X, the risk-free rate, r, and the variance of the continuously compounded return, σ^2. The payoffs from the portfolio are shown in Table 4.1.

We use $X = S_0\left[\frac{\lambda-1}{\gamma} + 1\right]$, $\varphi = \gamma/S_0$ and $\lambda = e^{iT}$.

Instrument	**Current value**	$\mathbf{S_T \leq X}$	$\mathbf{S_T} > X$
Bond	$\lambda e^{-r\tau}$	λ	λ
Calls	$\varphi\, c(S_t, \tau, X, r, \sigma^2)$	0	$(\gamma/S_0)(S_T-X)$
MICD		λ	$1+[(S_T/S_0)- 1]\gamma$

Table 4.1. Payoffs at expiration from portfolio identical to market index CD

The bond-option portfolio replicates the payoffs of the CD. Thus, the CD must be priced at the current value of the bond-option portfolio:

$$\mathbf{D_t = \lambda e^{-r\tau} + \varphi\, c(S_t, \tau, X, r, \sigma^2)} \qquad (4.1)$$

We shall refer to the components of this portfolio as the bond and the implicit call option. The exercise price X will be called the implicit exercise price. Intuitively, the implicit exercise price is the lower boundary of the market index at which the CD is "in-the-money," and thus pays interest at a rate in excess of the guaranteed rate. Note that a 0% guaranteed rate gives an implicit exercise price equal to the index value at the initiation of the CD. Such CDs could be viewed as "at-the money" at the onset.

Equation (4.1) is a preference-free formula for valuing a market index CD. We only assume that investors will exploit and eliminate available arbitrage opportunities. Consequently, the guaranteed rate i, must always be less than the risk-free rate, r.

4.2.3. Input Combinations that Make the MICD Correctly Priced

At August 14th, 1987, Chase Manhattan's one-year market index CD offered a return of 45% of the change in the S&P 500 with a 4% guaranteed minimum or 70% of the change in the S&P 500 with a 0% minimum.(We shall refer to these as the 4%-45% and 0%-70% CDs).

The CD can be priced using the Black-Scholes model. Since the CD is valued by constructing a replicating portfolio, the appropriate risk-free rate is the rate relevant to investors in the bond and option markets and not the bank's borrowing cost. The best proxy would be the Treasury bill rate.

The estimated risk-free rate is 5.49%, the standard deviation of the S&P 500 return is about 21%, the yield on the S&P 500 is 2.3% and the S&P itself is 333.99. The implicit exercise prices are 364.28 for the 4%-45% CD and 333.99, the original S&P 500 value, for the 0%-70% CD. Assuming a $1 initial deposit, the 4%-45% CD is worth $1.012 and the 0%-70% CD is worth $1.014. The implication of these figures is that both CDs are underpriced.

This conclusion may seem strange but a call option price is extremely sensitive to the volatility of the underlying security. Thus, the bank could be estimating a volatility lower than 21%.

It is also important to note that Equation (4.1) is a preference-free formula for valuing a market index CD. We assume that investors will exploit and eliminate available arbitrage opportunities. Consequently, the guaranteed rate i, must always be less than the risk-free rate, r, so $i<5.49\%$.

Figure 4.1 shows the value of the CD at the initial date as a function of the participation percentage, γ, for both 0% and 4% minimum guaranteed rates. The horizontal line indicates the equilibrium value of \$1. The graph shows that a correctly priced 0% guaranteed CD should have a participation percentage of about 55%. A correctly priced 4% guaranteed CD should have a participation percentage of about 30%. The value of the CD will, of course, change with the passage of time and changes in the market index.

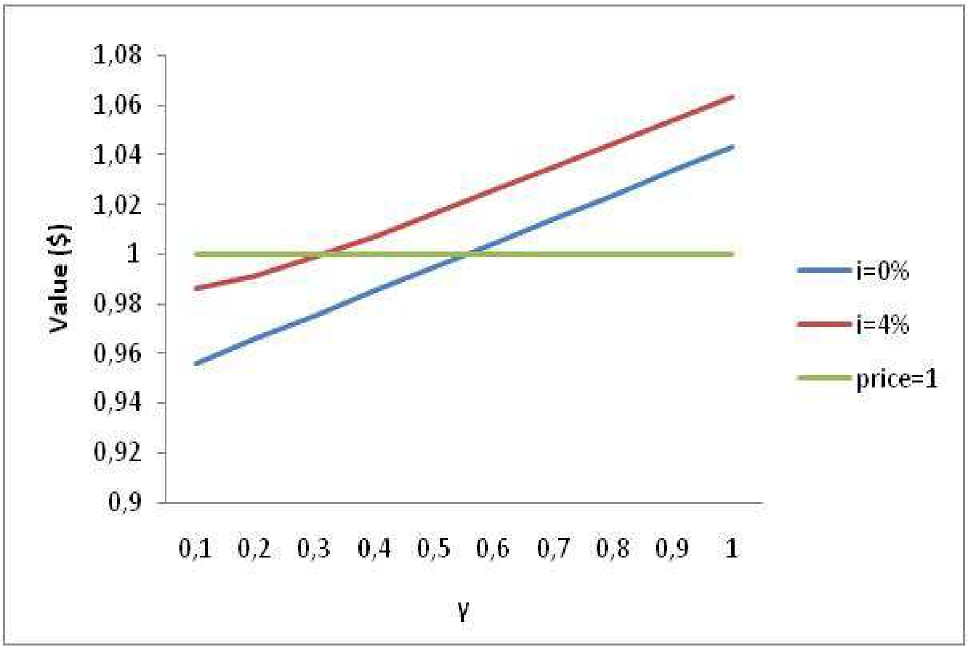


Figure 4.1. Value of the MICD with embedded vanilla call option

Figure 4.2 shows the combinations of participation percentage, γ and guaranteed rate, i, that make the CD correctly priced at the initial date. Assuming a \$1 initial deposit, the CD has to be worth \$1 to be priced correctly. In other words, the desirable combinations of γ and i verify the equation:

$$\lambda e^{-r\tau} + \varphi\, c(S_t, \tau, X, r, \sigma^2) = 1 \qquad (4.2)$$

When the guaranteed minimum interest rate increases, the risk of investing in the CD decreases, so its equilibrium participation percentage is reduced.

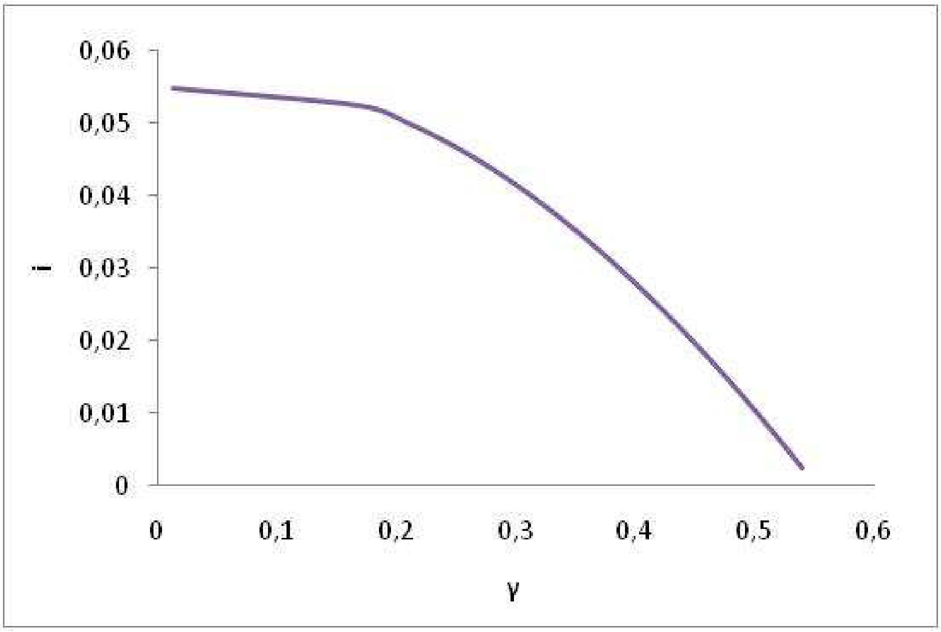


Figure 4.2. Combinations of i and γ that make the MICD correctly priced (price=$1)

Illustrations such as Figure 4.1 and Figure 4.2 can be used by investors and analysts to evaluate the CD and by banks to determine the terms that will price the CD at its correct initial value of $1.

4.2.4. Some Related Topics in Valuation of the MICD

4.2.4.1. A Boundary Condition

During the life of the CD, its value will change with t and S_t. However, the minimum value at any time t can be established by constructing the following portfolio: buy one market index CD, sell short φ shares of the market index, buy risk-free bonds with face value of X, and sell one risk-free bond with face value of λ. The payoffs at expiration are presented in Table 4.2.

Since the payoff is always nonnegative, the following must hold:

$$D_t - \varphi(St - Xe^{-r\tau}) - \lambda e^{-r\tau} \geq 0. \qquad (4.3)$$

Thus Equation (4.3) establishes the minimum bound on the CD value. Since the option component can not have negative value, (4.3) can be rewritten as

$D_t \geq \text{Max}\,(\lambda e^{-r\tau}, \lambda e^{-r\tau} + \varphi(S_t - Xe^{-r\tau})),$ (4.4)

which can be considered the CD's intrinsic or parity value.

Instrument	Current value	$S_T \leq X$	$S_T > X$
Buy CD	D_t	λ	$1+[(S_T/S_0)- 1]\gamma$
Short φ shares	$-\varphi S_t$	$-\varphi S_t$	$-\varphi S_t$
Buy φ bonds	$\varphi\, Xe^{-r\tau}$	φX	φX
Short bond	$-\lambda\, Xe^{-r\tau}$	$-\lambda$	$-\lambda$
		$-\varphi(S_T-X)$	0

Table 4.2. Payoffs at expiration of the minimum value portfolio

Notes: $X = S_0 \left[\frac{\lambda-1}{\gamma} + 1\right]$, $\varphi = \gamma/S_0$, $\lambda = e^{iT}$, τ=T-t .

4.2.4.2. Put Option MICDs and Put-Call Parity

Put-style CDs can also be easily created. At maturity, the CD would be worth Max(λ, 1 - [(S_T/S_0) - 1]γ). Following the same approach as before and letting D_{pt} represent the current value of the put-style deposit, we have

$D_{pt} = \varphi\, p(S_t, \tau, X_p, r, \sigma^2) + \lambda e^{-rt}$ (4.5)

where $X_p = S_0((1-\lambda)/\gamma+1)$ and $p(S_t, \tau, X_p, r, \sigma^2)$ is the value of a European put on the market index.

Now let D_{ct} denote the value of the call-style deposit where Xc is the implicit exercise price. Since $D_{ct} = \varphi\, c(S_t, \tau, X_c, r, \sigma^2) + \lambda e^{-rt}$ and $D_{pt} = \varphi\, p(S_t, \tau, X_p, r, \sigma^2) + \lambda e^{-rt}$, then

$D_{ct} - D_{pt} = \varphi\, c(S_t, \tau, X_c, r, \sigma^2) - \varphi\, p(S_t, \tau, X_p, r, \sigma^2)$ (4.6) which can be viewed as a form of put-call parity. Equation (4.6) relates the values of the call and put CDs originated simultaneously to the values of their implicit call and put components. These underlying calls and puts do not necessarily have the same exercise prices ($X_p < X_c$). However, Equation (4.6) does provide a no-arbitrage condition that must hold in equilibrium.

4.2.4.3. The Effect of the Underlying Variables

The value of the CD is affected by the index price, the risk-free rate, the remaining time to maturity, the volatility, and the implicit exercise price. The effect of each variable can be seen by differentiating D_t with respect to the given variable. A brief explanation of these effects is presented below.

The level of stock prices. The value of the CD will vary directly with the current level of the index St. This statement should be obvious because the CD contains an implicit call option on the index. The original stock price is directly related to the implicit exercise price. Thus, the value of the CD is inversely related to the original stock price.

The level of interest rates. Even though the value of the implicit call option varies directly with the risk-free rate, the value of the CD varies inversely with the risk-free rate. This is because the bond component of the CD varies inversely with the risk-free rate and dominates the call option effect. Thus, ceteris paribus, the CD, like all zero-coupon bonds, will be more attractive in falling interest rate environments.

Maturity. Unlike ordinary options, both the remaining time to maturity and the original maturity affect the value of the CD. Thus, there are two maturity variables in the model, and their effects are not equivalent.

Although the value of the implicit call option varies directly with the remaining time to maturity, the value of the bond component varies inversely with the remaining time to maturity. The overall effect can be either positive or negative.

This is significant in that it indicates that a longer-term deposit will not necessarily have a greater value.

The effect of the original maturity, like the maturity at any interim time point, cannot be signed. The original maturity has an inverse effect on the bond component and an ambiguous effect on the implicit call. This ambiguity arises because the original maturity directly affects both the call's remaining time to maturity and the implicit exercise price. The former has a direct effect on the call's value, while the latter has an inverse effect.

Stock maturity volatility. The greater the volatility, the higher the value of the CD. This result should be obvious, since volatility has a direct effect on the value of the implicit call option. This suggests that the CDs will be more attractive during turbulent market periods.

Guaranteed rate and participation percentage. The value of the CD varies directly with the guaranteed rate. The guaranteed rate has two effects on the deposit value. An increase in the guaranteed rate raises the implicit exercise price, which lowers the value of the implicit call option but raises the value of the bond component. The effect on the bond component is the more dominant effect. The participation percentage has a direct effect on the value of the CD. An increase in the participation percentage lowers the value of the implicit exercise price, which raises the value of the implicit call option. In intuitive terms, the higher the participation percentage, the smaller the change in the index necessary for the return to exceed the guaranteed rate.

4.2.4.4. The Early Redemption of MICDs

Some banks permit their market index CDs to be redeemed prior to maturity. In many cases, the banks offer a return of the principal or an interest penalty that is a function of the remaining time to maturity.

The option to redeem the CD early imbues the CD with the characteristics of an American option. However, we emphasize that even though the CD has the early redemption feature, the option underlying its valuation is <u>not</u> an American option. (An American call can be exercised any time prior to expiration, but it is well known that it is only exercised early an instant before an ex-dividend date. However, early redemption of the CD could occur even if there were no dividends on the underlying stock.)

From Equation (4.4), when $S_t \to 0$, the minimum value of the CD is $\lambda e^{-r\tau}$. Suppose the CD permits early redemption at a forfeiture of some or all of the guaranteed interest and possibly a portion of the principal. If the redemption value exceeds the minimum value, it may be optimal to redeem the CD early. However, if the penalty is greater than $(1 - e^{-r\tau})\lambda$, the redemption value will be less than the minimum value of $\lambda e^{-r\tau}$. Under this condition, the CD is worth more alive. Thus, if the bank discounts the guaranteed return at a rate in excess of the risk-free rate, the CD will not be redeemed early.

Let us assume, however, that the CD is redeemable at a constant amount of α dollars at any time during its life and that α exceeds $\lambda e^{-r\tau}$ for all τ. Let the stock price, S_t, follow the standard lognormal diffusion process,

$$\frac{dS_t}{S_t} = \mu dt + \sigma dz \tag{4.7}$$

where μ is the instantaneous mean return, σ is the instantaneous standard deviation and dz is the Gauss-Weiner process.

Using Ito's Lemma on D_t, and assuming continuous, costless trading, an investor can form a riskless hedge with the CD and risk-free bonds, giving the well-known partial differential equation

$$rS_t \ \frac{\partial D_t}{\partial S_t} - \ rD_t + \frac{\partial D_t}{\partial t} + \frac{1}{2}\frac{\partial^2 D_t}{\partial S_t^2 \sigma^2 S_t^2} = 0 \tag{4.8}$$

which is, of course, the same as the Black-Scholes problem. With no early redemption, the boundary condition at expiration is (4.3), and the solution is Equation (4.1) with c (S_t, τ, X, r, σ^2) given by the Black-Scholes formula. If early redemption is permitted, we must impose the additional boundary condition, $D_t \geq \alpha$ for all $t < T$. Unfortunately, there is no solution to the differential equation. However, the effect of early redemption can be examined by pricing the CD with a numerical technique. One simple numerical procedure is the binomial model of Cox-Ross-Rubinstein (1979). At each node in the binomial tree, we evaluate whether the value of the CD is greater alive or redeemed for α. By working from the expiration backwards, we can derive a value of the redemption privilege for specific numerical inputs.

4.2.5. Implied Standard Deviation (ISD)

To gain more insight into the pricing policies, we calculated the implied standard deviation (ISD) for the stock market index from the combinations of participation percentage γ and the guaranteed rate of return i. The Implied Standard Deviation or implied volatility is the volatility value that would make the theoretical value (in this case the Black-Scholes Model) equals to the given market price. For the ISD, we solve the equilibrium equation (4.2) for the given participation percentage and guaranteed rate of each issue of the MICD, using the Black-Scholes Option Pricing Model.

We calculate the ISD for the two issues of the Chase Manhattan's one-year MICD we studied before, using the market conditions on August 14, 1987. The combination of participation percentage of 45% and 4% guaranteed minimum rate results to an ISD of 14.03%, while the combination of 70% participation percentage and 0% guaranteed rate of return results to an ISD of 15.58%. Considering that the volatility of S&P 500 on August 14, 1987 was about 21%, the issuer does not appear to be concerned about maintaining an equilibrium pricing policy.

A. Chen and J. Kensinger (1990) calculated the implied standard deviation (ISD) for the stock market index from participation percentages quoted in January 1988 by a large issuer (Chase Manhattan Bank) and a small issuer (Murray Savings Association of Dallas), for both call (bull) and put (bear) option CDs. The results are shown in Table 4.3, ranked from lowest to highest ISD. The range of values is wide, reflecting significant inconsistencies.

For comparison, the ISD from the S&P 500 call options quoted by the Chicago Board Options Exchange on Friday, on 8^{th} January 1988, maturing March 1988, was calculated and an ISD of 0.40 was found. Although the terms of two of the Chase Manhattan call MICDs were consistent with this calculation, Chase's participation percentages were smaller than the equilibrium values for the most part, while Murray Savings MICDs were closer to being priced in harmony with historical norms.

Issuer	ISD	Type	Minimum rate	Participation Percentage	Maturity
Murray Savings	0. 19	Bull	0%	45%	6 mos.
	0.19	Bull	0%	60%	1 year
	0.24	Bear	0%	100%	6mos.
	0.31	Bear	0%	45%	6mos.
	0.33	Bear	0%	100%	1 year
	0.37	Bear	0%	60%	1 year
Chase Manhattan	0.38	Bull	0%	37%	1 year
	0.39	Bull	4%	24%	1 year
	0.40	Bull	4%	10%	3 mos.
	0.45	Bull	0%	15%	3 mos.
	0.45	Bear	4%	11%	3 mos.
	0.47	Bear	0%	45%	1 year
	0.47	Bear	4%	32%	1 year
	0.50	Bear	0%	16%	3 mos.
	0.52	Bull	4%	13%	6 mos
	0.52	Bull	0%	20%	6 mos.
	0.56	Bear	0%	23%	6 mos.
	0.59	Bear	4%	15%	6 mos.

Table 4.3. Implied standard deviations in market-index certificates of deposit. January 1988.

To see if the anomalies persisted as MICDs became a seasoned product, A. Chen and J. Kensinger also calculated ISDs from the terms offered by Chase Manhattan and Murray Savings in January 1989 (Table 4.4).

With the increase in Treasury Bill rates and the lack of significant change in the terms of the MICDs from the previous year, the ISDs were even higher in 1989 than they were in 1988. This is surprising, since the volatility of the index had by then returned to the low end of the range of historic norms.

Issuer	ISD	Type	Minimum rate	Participation Percentage	Maturity
Murray Savings	0.21	Bull	0%	65%	1 year
	0.25	Bull	0%	45%	6 mos.
	0.40	Bear	0%	45%	6 mos.
	0.43	Bear	0%	65%	1 year
	0.45	Bear	0%	100%	6 mos.
	0.50	Bear	0%	100%	1 year
Chase Manhattan	0.43	Bull	0%	40%	1 year
	0.46	Bull	0%	33%	9 mos.
	0.48	Bull	2%	32%	1 year
	0.50	Bull	0%	18%	3 mos.
	0.50	Bull	2%	27%	9 mos.
	0.52	Bull	0%	25%	6 mos.
	0.52	Bull	2%	15%	3 mos.
	0.52	Bull	4%	25%	1 year
	0.57	Bull	2%	20%	6 mos.
	0.58	Bull	4%	20%	9 mos.
	0.59	Bear	0%	45%	1 year
	0.65	Bear	A%	32%	1 year
	0,65	Bull	4%	10%	3 mos.
	0.67	Bear	0%	16%	3 mos.
	0.73	Bear	0%	23%	6 mos.
	0.73	Bear	4%	11%	3 mos.
	0.75	Bull	4%	13%	6 mos.
	0.88	Bear	4%	15%	6 mos.

Table 4.4. Implied standard deviations in market-index certificates of deposit, January 1989.

One potential explanation is that a big issuer like Chase Manhattan is engaging in arbitrage, taking advantage of market imperfections that prevent customers from creating better homemade alternatives. Although this may provide part of the explanation, however, it does not fully explain why the best terms tend to be associated with longer term bull versions of the MICD. Pricing policies aimed at encouraging customers to choose longer maturities suggest that the issuer is more concerned with obtaining a committed funding source than with earning arbitrage profits.

4.3. Market Index Certificate of Deposit with Embedded Vanilla Call Option - Capped Version

Since their introduction, MICDs have been offered by several commercial banks and thrift institutions as well, including an innovative "capped" version. The capped version places an upper bound on the interest rate the issuer is liable to pay, making it possible to offer a higher participation percentage than would otherwise be feasible.

Murray Savings Association of Dallas, for example, offered MICDs in both capped and uncapped versions. In January 1989, the participation percentages on its uncapped Wall Street CDs were 45 percent for the six-month maturity and 65 percent for the one-year maturity, for both the call and put versions, with guaranteed return of principal. In exchange for a cap of 20 percent on the interest rate, however, depositors could get participation percentages of 100 percent in both the call and put versions.

A. Chen and W. Kensinger (1990) present a pricing formula of the "capped" version of the MICDs.

4.3.1. Pricing the MICD

Let us consider an MICD with principal of one dollar and maturity on day T. For easy reference, all the symbols used in the analysis are summarized in Table 4.5.

λ = the guaranteed minimum payment at maturity for an MICD with principal of one dollar
κ = the guaranteed maximum payment at maturity for an MICD with principal of one dollar
γ_c = the participation percentage for the call version of the MICD, stated as a decimal fraction
γ_p = the participation percentage for the put version of the MICD, stated as a decimal fraction
St = the ratio of the raw S&P 500 index on day t divided by the raw index on day 0 (where day 0 is the issue date of the CD)

Table 4.5. List of symbols.

Let $\lambda(\kappa)$ be the guaranteed minimum (maximum) payment at maturity for an MICD with principal of one dollar. Let γ_c (γ_p) be the participation percentage for the call (put) version of the MICD, stated as a decimal fraction. Since the uncertainty about the maturity payment arises from the volatility of the stock market, let us define a scaled index, S_t which is the ratio of the raw S&P 500 index on day / divided by the raw index on day U. Thus on the initial date of the contract, this scaled index is defined to be 1(i.e., $S_0 = 1$). For an MICD with a principal of one dollar, then, the payment at maturity is defined as follows:

Maturity Payment (call version) = min {κ, max [λ, 1 + $\gamma_c(S_T - 1)$]}.

Maturity Payment (put version) = min {κ, max [λ, 1 + $\gamma_p(1 - S_T)$]}.

The price of an MICD is not revised continuously on the floor of an exchange, but instead is manifested implicitly through the terms offered by the issuer at the beginning of the MICD's life (i.e., the participation percentage, as well as the minimum and maximum maturity payment). Since these parameters remain fixed over the life of the specific contract, and there is typically a substantial early withdrawal penalty that binds the deposit until maturity, the appropriate focus of attention is on the fairness of the terms at the time the contract is initiated.

Let us therefore begin by breaking down the MICDs into an equivalent package of Treasury securities and index options.

4.3.2. Replicating the MICDs

A call version of the MICD with a principal of one dollar can be replicated on the issue date by a portfolio containing:

1. a risk-free pure discount bond that matures on day T with maturity value X
2. γ_c call options on the scaled market index that expire on day T with exercise price X_c and
3. a short position in γ_c call options on the scaled market index that expire on day T with exercise price x_c

where X_c and x_c are defined as follows:

$$X_c = \frac{\lambda - 1}{\gamma_c} + 1$$

$$x_c = \frac{\kappa - 1}{\gamma_c} + 1$$

Suppose, for example, that λ = 1.04, κ = 1.15 and γ_c = 0.80. Then the scaled index would have to rise above 1.05 before the maturity payment on the MICD would exceed the guaranteed minimum. The combination of the bond and the first option in the replicating portfolio duplicates not only the guaranteed minimum but also the additional return which would occur if the scaled index exceeded 1.05 at the maturity date. If the scaled index exceeded 1.1875 at the maturity date, however, the cap would take effect, and the third component of the replicating portfolio duplicates this aspect of the MICD.

Likewise, the put version of the MICD with a principal of one dollar can be replicated at time 0 by a portfolio containing:

1. a risk-free pure discount bond that matures on day T with maturity value λ
2. γ_c put options on the scaled market index that expire on day T with exercise price X_p and
3. a short position in γ_p put options on the scaled market index that expire on day with exercise price x_p,

where X_p, and x_p are defined as follows:

$$X_p = \frac{1 - \lambda}{\gamma_p} + 1$$

$$x_p = \frac{1 - \kappa}{\gamma_p} + 1$$

The portfolio replicates the payoffs of the CD. Let $c(S_t, \tau, X, r, \sigma^2)$ represent the value of a call option on the scaled index with time to maturity of T and exercise price X, when r is the risk-free interest rate and σ^2 is the instantaneous variance of the raw index. Thus, the call version of the CD must be priced at the current value of the portfolio:

$$D_t = \lambda e^{-r\tau} + \gamma_c\, c(S_t, \tau, X_c, r, \sigma^2) - \gamma_c\, c(S_t, \tau, x_c, r, \sigma^2) \quad (4.9)$$

And likewise, the put version of the MICD:

$$D_t = \lambda e^{-r\tau} + \gamma_p\, c(S_t, \tau, X_p, r, \sigma^2) - \gamma_p\, c(S_t, \tau, x_p, r, \sigma^2) \qquad (4.10)$$

Since MICDs cannot be redeemed early under favorable terms, there is no viable early exercise opportunity. Thus the options involved in these contracts are European-type, so we can use the Black-Scholes option pricing model in order to determine fair pricing policies. To be in equilibrium at the time of issue (assuming the CD has principal of one dollar), the participation percentage for the call version must be set so that:

$$\lambda e^{-r\tau} + \gamma_c\, c(1, \tau, X_c, r, \sigma^2) - \gamma_c\, c(1, \tau, x_c, r, \sigma^2) = 1 \qquad (4.11)$$

Respectively for the put version, the participation percentage must be set so that:

$$\lambda e^{-r\tau} + \gamma_p\, c(1, \tau, X_p, r, \sigma^2) - \gamma_p\, c(1, \tau, x_p, r, \sigma^2) = 1 \qquad (4.12)$$

The provision of a cap, comparing with the simple version of the CD which offers no cap, reduces the value of the CD to the investor, requiring an upward adjustment of the participation percentage in order to restore equilibrium.

4.3.3. Implied Standard Deviation (ISD)

A. Chen and J. Kensinger (1990) calculated the implied standard deviation (ISD) for the capped call versions of the Murray Savings Association's MICDs and another kind of anomaly was detected. Both the six-month and one-year maturities offered guaranteed return of principal, a participation percentage of 100 percent and a cap of 20 percent. Solving Equation (4.11) reveals that if the standard deviation of the index were 0.20, the 1988 equilibrium participation percentage would be 72 percent for the one-year CDs—and if the standard deviation were 0.40, it would be 65 percent. The participation percentage of 100 percent that was actually offered, therefore, renders the call version of this CD more favorable to the depositor than any homemade replica that could be constructed, distinguishing it among the various MICDs offered to the general public. In order to explain this, an official of Murray Savings indicated that an effort was made to offer an attractive call version of this capped CD in order to build market share.

4.4. Market Index Certificate of Deposit with Embedded Binary Call Option

ABN AMRO has introduced a variety of simple and more complicated structured products, through the years. Among them, there are many certificates of deposit with performance dependent on market indexes or on a basket of stocks and guarantee the return of the principal or even offer a minimum guaranteed return.

Because of the appeal this kind of products has to the investors, in this paragraph we will deal with two Market Index Certificates of Deposit that share similar characteristics and are based on the following product introduced by ABN AMRO.

ABN AMRO's description of the MICD:

"The investor pays a principal of 10,000 Euros and buys a 5-year MICD on the indexes SPX, NKY and SX5E. At the end of each year, he gets a coupon of 1.5% (150 Euros) for sure. If none of the previous indexes is under 90% of its initial price, at the maturity of the CD, the investor gets a coupon of 8% (800 Euros), not 1.5%. At the end of the year the principal of 10.000 is given back."

4.4.1. MICD with Embedded Cash-or-Nothing Option

The first analyzed product is a certificate of deposit with a guaranteed minimum rate of return, plus a fixed additional rate, if a specific index is not under a predetermined percentage of its initial price S_0, at the maturity of the CD.

Investors can select different combinations of maturity, guaranteed minimum rate, additional rate and percentage (<100%) of S_0 that determines when the additional rate is paid.

<u>MICD Example:</u>

The investor pays 10.000 Euros and buys a MICD on SPX. At the end of the year, he gets a coupon of 1.5% (150 Euros) for sure. If SPX is not under 90% of its initial price S_0, at the maturity of the CD, the investor gets a coupon of 8% (800 Euros), not 1.5%. In this case the additional rate over the guaranteed minimum is 6.5%. At the end of the year the principal of 10.000 is given back.

4.4.1.1. Pricing the MICD

Let today be denoted as time zero. A 1 euro market index CD issued today matures at time T; thus, its original maturity is T-0=T. The market index when the CD is issued is S_0. The CD guarantees the holder a minimum return of i percent continuously compounded. If the market index is not under φ S_0 at the maturity of the CD, the CD pays an additional rate of γ percent. So the percentage φ of the initial price of the index is the minimum price that the index may get at maturity and the CD still pays the additional rate.

Thus, at maturity the CD is worth

$$
\begin{array}{ll}
e^{iT} & if \quad S_T < \varphi S_0 \\
e^{(i+\gamma)T} & if \quad S_T > \varphi S_0
\end{array}
$$

where S_T is the index at time T (we assume that the CD will not be redeemed prior to maturity).

4.4.1.2. Replicating the MICD

Suppose we wish to determine the value of the CD at some time t during the life of the CD; thus, t is prior to T. Let D_t represent the value of the CD at time t. One way to value the CD is to find another portfolio of instruments that replicates the outcome of the CD. One such portfolio is

1. a risk-free pure discount bond with a face value of e^{iT} and a maturity of τ = T-t, and
2. European binary call option on the market index expiring at T with exercise price of X.

Let X= φS_0 and the option price be c(St, τ, X, r, σ^2). The notation for the option price expresses the binary call option as a function of the current market index, S_t, the time to expiration, τ, the exercise price, X, the risk-free rate, r, and the variance of the continuously compounded return, σ^2. The binary call option can be priced using the Black-Scholes formula. The payoffs from the portfolio are shown in Table 4.6.

Instrument	Current value	$S_T \leq X$	$S_T > X$
Bond	$e^{iT}\, e^{-r\tau}$	e^{iT}	e^{iT}
Option	$c(S_t, \tau, X, r, \sigma^2)$	0	$e^{iT}(e^{\gamma T} - 1)$
MICD		e^{iT}	$e^{(i+\gamma)T}$

Table 4.6. Payoffs at expiration from portfolio identical to market index CD with embedded binary option

The bond-option portfolio replicates the payoffs of the CD. Thus, the CD must be priced at the current value of the bond-option portfolio:

$$\mathbf{D_t = e^{-r\tau}e^{iT} + c(S_t, \tau, X, r, \sigma^2)}$$

And using the Black-Scholes formula for binary call options:

$$\mathbf{D_t = e^{-r\tau}e^{iT} + e^{-rt}\, e^{iT}(e^{\gamma T} - 1)\Phi(d_2)}$$
$$\mathbf{= e^{-r\tau}\, e^{iT}\, [1 + (e^{\gamma T} - 1)\Phi(d_2)]}$$

4.4.1.3. Input Combinations that Make the MICD Correctly Priced

We shall valuate the MICD with embedded cash-or-nothing option with the characteristics stated at the example above. The CD offers a guaranteed return of 1.5% and if SPX is not under 90% of its initial price at the maturity of the MICD, it offers an additional rate of return of 6.5%.

We will make some logical assumptions for the variables of the Black-Scholes Model.

The appropriate risk-free rate is the rate relevant to investors in the bond and option markets and not the bank's borrowing cost, since the CD is valued by constructing a replicating portfolio. A suitable proxy would be the Treasury bill rate.

We will use a risk-free rate of 4%, 40% volatility for the index and that the index itself is 1000. Assuming a 1Euro initial deposit, the CD is worth 1.0123 Euros, which means that the CD is underpriced. However, we have to keep in mind that the option price is sensitive to the volatility of the underlying asset and the risk-free rate. Thus, the issuer could be estimating different prices for volatility and risk-free rate.

Figure 4.3 shows the value of the CD at the initial date as a function of the additional rate, γ, for 0.5%, 1.5% and 2.5% guaranteed rates. The horizontal line indicates the equilibrium value of 1 Euro. The graph shows that a correctly priced 0.5% guaranteed CD should have an additional rate of about 6%, a correctly priced 1.5% guaranteed CD should have an additional rate of about 4.5% and a correctly priced 2.5% guaranteed CD should have an additional rate of about 2.5%.

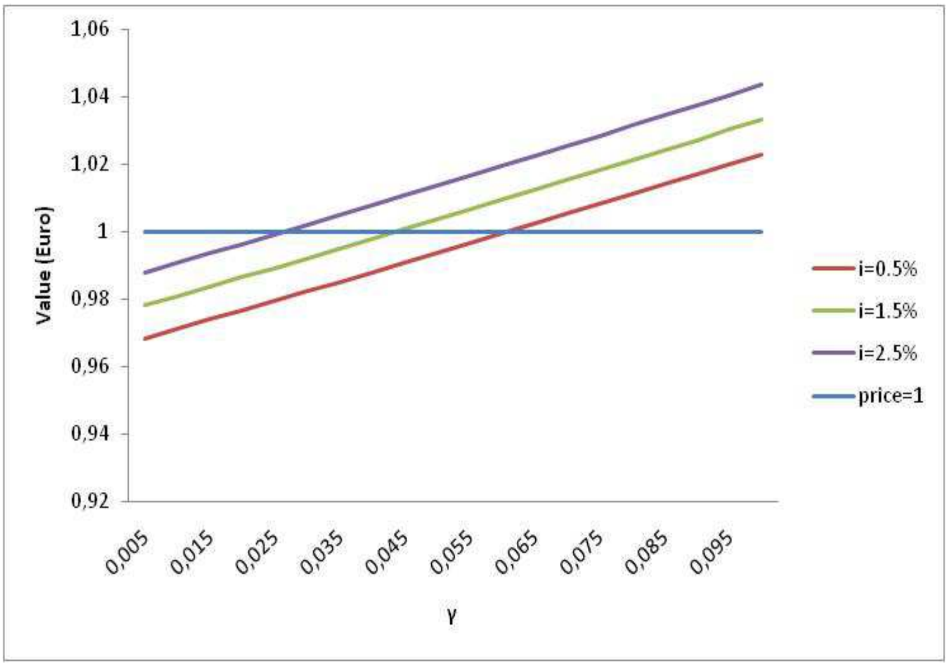


Figure 4.3. Value of the MICD with embedded cash-or-nothing option

Figure 4.4 shows the combinations of additional rate, γ and guaranteed rate, i, that make the CD correctly priced at the initial date. Assuming a 1 Euro initial deposit, the desirable combinations of γ and i verify the equilibrium equation:

$$\mathbf{e^{-r\tau}e^{iT} + c(S_t, \tau, X, r, \sigma^2)} = 1$$

It is obvious that the guaranteed rate i, must always be less than the risk-free rate (i<4%), so that the equilibrium equation gives positive prices for the option.

When the guaranteed minimum interest rate increases, the risk of investing in the CD decreases, thus its equilibrium additional rate is reduced.

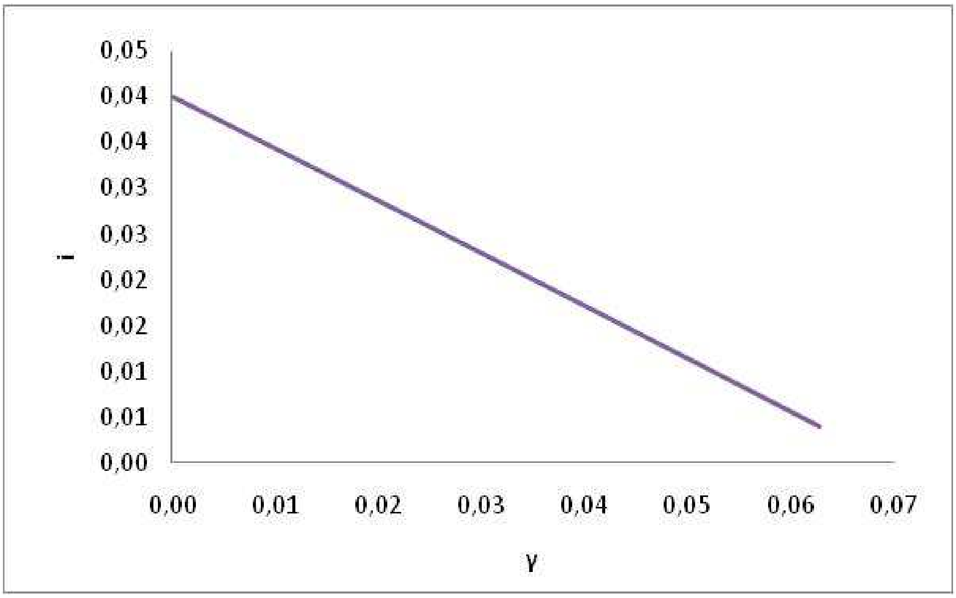


Figure 4.4. Combinations of i and γ that make the MICD correctly priced (price=1Euro)

4.4.1.4. Implied Standard Deviation (ISD)

The Implied Standard Deviation or implied volatility is the volatility value that would make the theoretical value (in this case the Black-Scholes Model) equals to the given market price. We calculated ISD for the stock market index from the combination of the 1.5% guaranteed rate of return, i and 6.5% the additional rate, γ, which resulted to an ISD of 93.73%. Considering that 93.73% is not realistic price for volatility, the issuer does not appear to be concerned about maintaining an equilibrium pricing policy.

It is important to note that as volatility changes, binary options exhibit different theoretical valuation behavior than call options.

Vanilla options respond to increases and decreases in the volatility of the underlying asset in a fairly predictable manner. In comparison, the effect of underlying volatility on the price of a binary option is more exponential, depending on whether the option is deep out of the money, near the money, or deep in the money. This aligns with the fact that a binary has very high Gamma as it nears and crosses the money threshold.

In situations where both the binary and vanilla options are in the money, volatility can have the opposite effect on value for each. An increase in volatility will increase the value of a vanilla option, as there is greater probability that the option will expire further in the money and produce a greater payoff. In comparison, for a binary, any increase in volatility will increase the probability that the underlying instrument might move out of the money at expiration. Therefore, the in the money binary option value will respond by falling, not increasing as the vanilla option value would.

4.4.2. MICD with Embedded "Two-Asset Cash-or-Nothing" Option

The second analyzed product is a certificate of deposit with a guaranteed rate of return, plus a fixed additional rate, if two indexes are both not under a predetermined percentage of their initial prices $S_{x,0}$, at the maturity of the CD.

Investors can select different combinations of maturity, guaranteed minimum rate, additional rate and percentage (<100%) of $S_{x,0}$ that determines when the additional rate is paid.

<u>MICD Example:</u>

The investor pays 10.000 Euros and buys a MICD on the indexes SPX and NKY. At the end of the year, he gets a coupon of 1.5% (150 Euros) for sur. If none of the previous indexes is under 90% of its initial price, at the maturity of the CD, the investor gets a coupon of 8% (800 Euros), not 1.5%. In this case the additional rate over the guaranteed minimum is 6.5%. At the end of the year the principal of 10.000 is given back.

4.4.2.1. Pricing the MICD

To evaluate the MICD with embedded "two-asset cash-or-nothing" option, we use the same approach as in the case of simple binary option in the previous paragraph.

Let today be denoted as time zero. A 1 euro market index CD issued today matures at time T; thus, its original maturity is T-0=T. The market indexes when the CD is issued are $S_{1,0}$ and $S_{2,0}$. The CD guarantees the holder a minimum return of i percent continuously compounded. If market index S_1 is not under $\varphi S_{1,0}$ and market index S_2 is not under φ $S_{2,0}$ at the maturity of the CD, the CD pays an additional rate of γ percent.

Thus, at maturity the CD is worth

$$e^{iT} \qquad\qquad if\ (S_{1,T} > \varphi S_{1,0}\ \cap\ S_{2,T} > \varphi S_{2,0})'$$

$$e^{(i+\gamma)T} \qquad\qquad if\quad S_{1,T} > \varphi S_{1,0}\ \cap\ S_{2,T} > \varphi S_{2,0}$$

where $S_{1,T}$, $S_{2,T}$ are the indexes at time T (we assume that the CD will not be redeemed prior to maturity).

4.4.2.2. Replicating the MICD

Let D_t represent the value of the CD at time t. The replicating portfolio is similar to the portfolio used for the CD with simple binary option:

1. a risk-free pure discount bond with a face value of e^{iT} and a maturity of τ = T-t, and
2. European binary call option on the two market indexes expiring at T

Let $X_1 = \varphi S_{1,0}$, $X_2 = \varphi S_{2,0}$ and the option price be c(St, τ, X_1, X_2, r, σ^2, ρ).

The notation for the option price expresses the call as a function of the current market index, S_t, the time to expiration, τ, the exercise prices, X_1 and X_2, the risk-free rate, r, the variances of the continuously compounded return of the indexes, σ_1^2 and σ_2^2 and the correlation of the indexes, ρ.

The binary call option can be priced using the Black-Scholes formula. The payoffs from the portfolio are shown in Table 4.7.

Instrument	Current value	$(S_{1,T} > X_1 \cap S_{2,T} > X_2)'$	$S_{1,T} > X_1 \cap S_{2,T} > X_2$
Bond	$e^{iT}\, e^{-r\tau}$	e^{iT}	e^{iT}
Option	$c(S_t, \tau, X, r, \sigma^2, \rho)$	0	$e^{iT}(e^{\gamma T} - 1)$
MICD		e^{iT}	$e^{(i+\gamma)T}$

Table 4.7. Payoffs at expiration from portfolio identical to market index CD with embedded "two-asset cash-or-nothing" option

The bond-option portfolio replicates the payoffs of the CD. Thus, the CD must be priced at the current value of the bond-option portfolio:

$$D_t = \mathbf{e^{-r\tau} e^{iT}} + c(S_t, \tau, X, r, \sigma^2)$$

And using the Black-Scholes formula for "two-asset cash-or-nothing" call options published by Heynen & Kat (1996):

$$D_t = \mathbf{e^{-r\tau} e^{iT}} + \mathbf{e^{-rt}}\, e^{iT}(e^{\gamma T} - 1)\, \mathbf{M}(\mathbf{d}_{1,1}, \mathbf{d}_{2,2}; \boldsymbol{\rho})$$

$$= \mathbf{e^{-r\tau}\, e^{iT}}\, [\mathbf{1} + (\mathbf{e^{\gamma T}} - \mathbf{1})\, \mathbf{M}(\mathbf{d}_{1,1}, \mathbf{d}_{2,2}; \boldsymbol{\rho})]$$

where $$d_{i,j} = \frac{\ln\left(S_i/X_j\right) + (r - \sigma_i^2/2)T}{\sigma_i\sqrt{T}}$$

In the previous Black-Scholes formula, the cumulative bivariate normal distribution function $\mathbf{M}(\mathbf{d}_{1,1}, \mathbf{d}_{2,2}; \boldsymbol{\rho})$ is calculated using Mathematica, as seen in the Appendix.

4.4.2.3. Input Combinations that Make the MICD Correctly Priced

We shall valuate the MICD with embedded "two-asset cash-or-nothing" option with the characteristics stated at the example above. The MICD offers a guaranteed return of 1.5% and if both SPX and NKY are not under 90% of their initial prices at the maturity of the MICD, it offers an additional rate of return of 6.5%.

We will use the same logical assumptions for the variables of the Black-Scholes Model that we used in the case of the simple cash-or-nothing option. We will use a risk-free rate of 4%, 40% volatility for the first index and 35% for the second index, 0.2 correlation of the indexes and that the indexes are 1000 each. Assuming a 1Euro initial deposit, the CD is worth 1,008609 Euros, which means that the CD is underpriced.

Figure 4.5 shows the value of the CD at the initial date as a function of the additional rate, γ, for 1.5% and 2.5% guaranteed rates. The horizontal line indicates the equilibrium value of 1 Euro. The graph shows that a correctly priced 0.5% guaranteed CD should have an additional rate of about 7%, a correctly priced 1.5% guaranteed CD should have an additional rate of about 4% and a correctly priced 2.5% guaranteed CD should have an additional rate of about 1.5%.

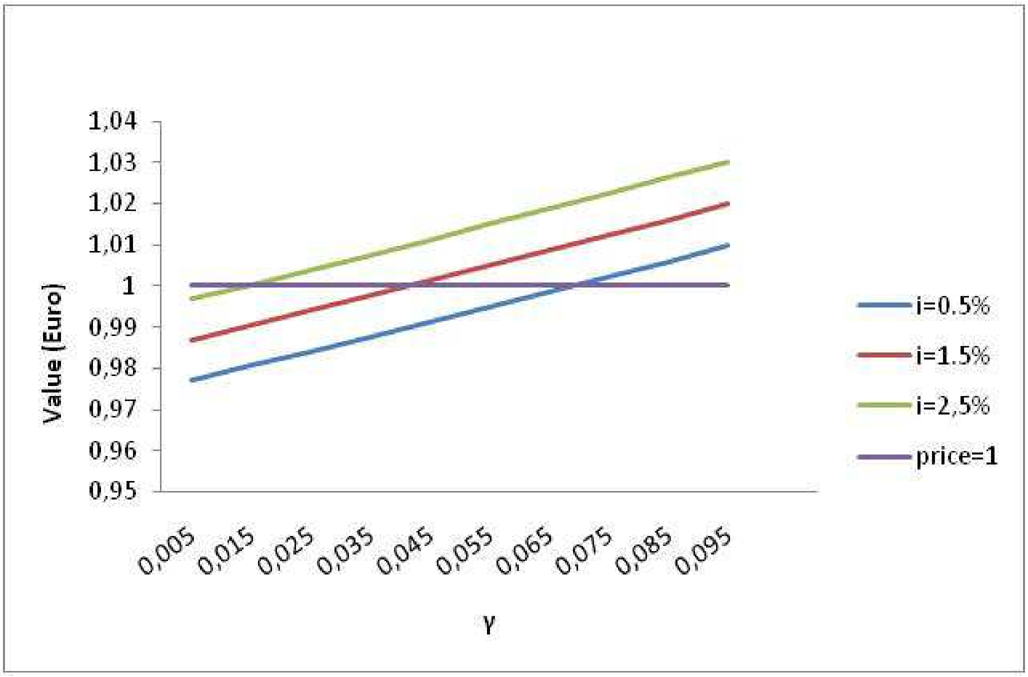


Figure 4.5. Value of the MICD with embedded "two-asset cash-or-nothing" option

4.5. A Realistic Approach to the Replicating Portfolios

In a world of perfect markets, the CD is no more than a combination of instruments that already exist. Investors could costlessly replicate market index CDs with index options and risk-free bonds. However, the existence of banks is, itself, inconsistent with perfect markets, and in such an environment the CDs may offer new investment opportunities.

For example, the replication of the CD with options and bonds will incur transaction costs. Thus, if banks are willing to bear the cost of transacting in order to attract or retain deposits, consumers will find the CD more attractive than the replication strategy. Also, since the bank can almost surely obtain lower transaction costs than consumers, they can pass those savings on to individuals. However we have to bear in mind that banks incur a cost of hedging.

In addition to the transaction cost problem, there are several other difficulties in replicating the deposits. One is that index options with the exercise price equal to the implicit exercise price are probably not available. For example, if the implicit exercise price is 333.99 and a consumer chose to replicate the CD with index options, the closest exercise price would be 335. Although the range of exercise prices available on index options would mean that consumers could construct a broad range of combinations of i and γ, they could not precisely replicate the bank's offerings, except in rare, coincidental cases. Another related problem is that the bank offers new CDs every day. To replicate a CD would require the availability of index options expiring every day. Index options, of course, expire only at monthly intervals. Similar problems exist with the Treasury-bill component of the CD.

Another problem in replicating the CDs is the unevenness of the dollar or euro amounts. A bank generally requires that the CD be established in amounts that are even multiples of $1,000. While this creates problems for investors who wish to deposit odd amounts, it is even more difficult for investors who replicate the deposits with index options and bonds, given the denomination constraints on these instruments.

Consequently, it is clear that although the replicating portfolios is an effective approach to evaluate the CDs, the exact replication of the payoffs of the CDs is practically feasible only in the ideal world of perfect markets and not in the real world.

4.6. Possible Reasons for MICDs' Mispricing

Clearly, no one can determine precisely what terms a bank would offer under a given set of market conditions.

Banks may intentionally misprice the CDs to attract customers or to recover some of their hedging costs.

One potential explanation is that a big issuer like Chase Manhattan is engaging in arbitrage, taking advantage of market imperfections that prevent customers from creating better homemade alternatives. If imperfections such as minimum contract size and high transactions costs for individuals make it possible for the issuer to receive $ 1 for a package that theoretically is worth, say, $ 0.99, the issuer is enabled to borrow the proceeds for less than the Treasury security rate.

Although this may provide part of the explanation, however, it does not fully explain why the best terms tend to be associated with longer term bull versions of the MICD. Pricing policies aimed at encouraging customers to choose longer maturities suggest that the issuer is more concerned with obtaining a committed funding source than with earning arbitrage profits. In order to explain this, an official of Murray Savings indicated that an effort was made to offer an attractive call version of the CD in order to build market share.

Chase Manhattan, moreover, offered attractive "loss-leader"' terms on some of its MICDs when they were first introduced, and later lowered its participation percentages. Even when MICDs are priced as loss leaders, however, they can still provide a source of funds that is competitive with traditional CDs. For example, if the issuer chooses a pricing policy which results in receiving $ 1.00 for a package theoretically worth, say $1.01, it could hedge away stock market risk and still receive the net proceeds at a cost about 1 percent above the Treasury security rate.

Of course, regulators must be vigilant for a less laudable motive; a troubled institution may offer loss-leader terms on its MICDs as an alternative to offering high interest rates on conventional CDs. Since the price paid for funds is implicit rather than explicit in the case of MICDs, such an institution may issue them in an attempt to disguise the truth of its situation.

The distortions are most intense for the put versions of the MICDs. Not only were the put versions of the stock MICDs out of equilibrium, they were also introduced later than the call variety and were not as actively marketed as the call version of the MICD. An executive of one of the major issuers, who was asked why the put-type CDs were treated this way, readily acknowledged that higher participation percentages were possible in the case of the put MICDs, but observed that there were no competitive pressures to offer better terms—and, moreover, there were concerns among top management that such an offer might be interpreted negatively by regulators and potential customers. Because the interest rates on put-type MICDs would be higher in the wake of a stock market downturn, top management feared that high participation percentages would be interpreted as an indication of increased vulnerability in an economic slump. Rightly or wrongly, management worried that the general public would not understand the hedging used to resolve such risks.

Given our observations of the anomalies in pricing MICDs during the introductory period of a new issuer or a new version, as well as the anomalous pricing of the put version, it appears that issuers have been concerned more about marketing-related issues (e.g. market share and an innovative image) than about achieving equilibrium pricing.

5. Risk and Hedging Strategies for Issuers

A bank offering market index CDs is in the position of an uncovered option writer. If the bank is only offering call-style CDs, its risk is that a bull market will occur. If the bank also offers put-style CDs, it faces the risk of both bull and bear markets. It, thus, has the position of a short straddle.

Clearly, the bank must hedge the risk. King and Remolona (1987) provide an excellent description of the processes and problems in hedging this risk. They discuss the types of hedging instruments available and the risks and costs of hedging, particularly when parameter estimates are imperfect. It should be added that a dynamic hedge with stock index futures, as described in their article, is similar to portfolio insurance. There is evidence that portfolio insurance may not have functioned well in crashes in the past. Thus, while there may be risk to the banking system if Market Index CDs become widely used, the risk is associated with the institutional problems of hedging and not with any factors associated with market index CDs per se.

5.1. Equilibrium Pricing and Hedging

Although the stock-market-related aspects may tend to dominate one's first impression of MICDs, in reality the Treasury-security component makes up most of the replicating portfolio when the terms of the CDs are set at equilibrium. From equilibrium equation (shown in the previous chapter)

$$\lambda e^{-r\tau} + \varphi\, c(S_t, \tau, X, r, \sigma^2) = 1$$

one can see that when the conditions for equilibrium are met, the fraction of the replicating portfolio represented by Treasury securities is $\lambda e^{-r\tau}$, which is generally close to l. That is, the cost of buying a complete hedge for an MICD series on the issue date is a small fraction of the principal.

In effect the bulk of the principal an issuer raises through an equilibrium-priced MICD series represents proceeds borrowed at the same rate as the U.S. Treasury, and the issuer can completely hedge its exposure to stock market risk by using the remainder of the principal to purchase offsetting index options.

For example, with a maturity of one year, an interest rate of 7 percent on U.S, Treasury securities, and a guaranteed minimum maturity payment of $ 1.04 for every dollar of principal, the Treasury securities represent 97 percent of the replicating portfolio. (Thus from every dollar of principal the issuer could set aside 3 cents for hedging, leaving 97 cents in proceeds.) The issuer could break even on this MICD series simply by investing the proceeds at the Treasury security rate.

Equilibrium pricing, therefore, seems to be a desirable goal not only from the investors' point of view but also from the issuers'.

5.2. Natural Partial Hedging and Dynamic Hedging

It is possible for the issuer of the MICDs to hedge the stock market risk completely by purchasing the options in the replicating portfolios for each series of MICDs it issues. Other degrees of hedging, of course, are possible.

To begin assessing such alternatives let us consider the extreme of no offsetting option hedges at all. An issuer with both put and call MICDs outstanding has a natural position that straddles the index, so there is already a built-in cushion against stock market fluctuations.

To analyze the risks involved in an unhedged position, let n_c represent the number of call CDs and n_p the number of put CDs, both with principal of one dollar in a given issue, for a combined total number of call and put MICDs equal to n_c plus n_p. Let ρ represent the uncertain rate of return the issuer earns from investing the deposited funds and i the rate of return the issuer guarantees to the investor.

At maturity the issuer will have loan repayments equal to $e^{\rho\tau} (n_c + n_p)$ available to make the required payment to depositors. If the value of the index falls between X_p and X_c at maturity, neither the call MICDs nor the put MICDs will earn more than the guaranteed minimum interest rate, so the issuer will keep $(n_c + n_p)(e^{\rho\tau} - \lambda)$, where $\lambda=e^{i\tau}$.

The payoff function in Figure 5.1 is therefore flat in the region between X_p and X_c.

The best-case boundary is equal to $(n_c + n_p)(e^{\rho\tau} - \lambda)$. If the index rises above X_c the worst-case boundary is $(n_c + n_p)e^{\rho\tau} - \lambda n_p - \kappa n_c$; and if the index falls below X_p, the worst-case boundary is $(n_c + n_p)e^{\rho\tau} - \lambda n_c - \kappa n_p$.

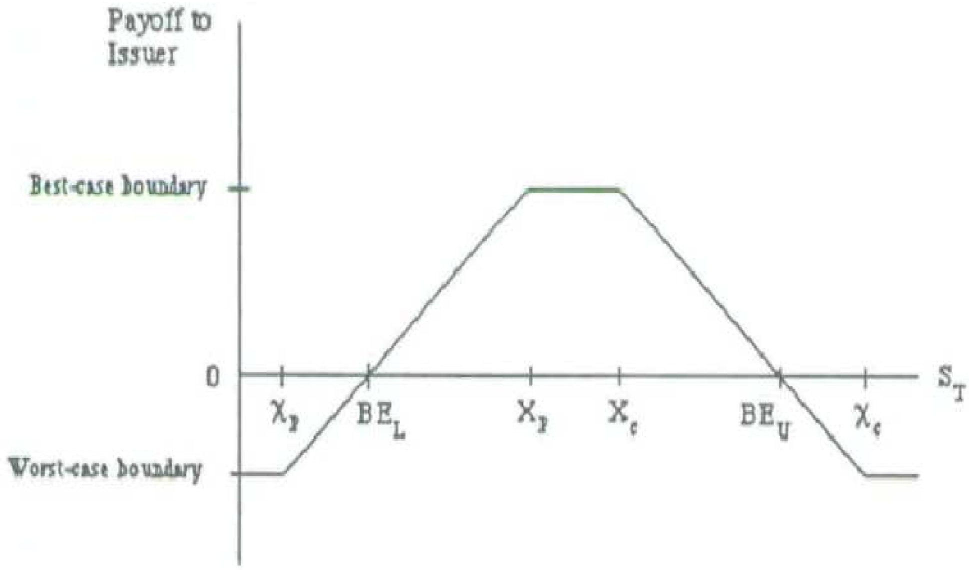


Figure 5.1. Illustration of issuer's exposure.

Every point by which the value of the index at maturity exceeds X_c will cost the issuer $\gamma_c n_c$ dollars, and likewise every point by which the value of the index at maturity falls below X_p will cost the issuer $\gamma_p n_p$ dollars, where γ_c is the participation percentage for the call version of the MICD and γ_p is the participation percentage for the put version of the MICD.

The issuer earns a profit, however, so long as the value of the index at maturity falls within the bounds of the upper and lower break-even points represented by BE_U and BE_L in Figure 5.1, which are defined as follows:

Upper break-even point = $BE_U = X_c + (e^{\rho\tau} - \lambda)\frac{(n_c+n_p)}{\gamma_c n_c}$

Lower break-even point = $BE_L = X_p - (e^{\rho\tau} - \lambda)\frac{(n_c+n_p)}{\gamma_p n_p}$

The more the issuer is able to earn from its investments, the wider the gap between the lower and upper break-even points. The issuer's risk exposure depends not only upon the variability of the rate of change in the stock market index but also upon its correlation with the rate of return on the loan portfolio. To the extent that the performance of the stock market index reflects overall economic performance, it would be reasonable to anticipate a positive correlation with loan portfolio performance. The more positive the correlation (i.e. the more situations that dictate paying high interest on the call MICDs tend to be associated with better-than-expected loan portfolio performance), the less the issuer's risk exposure.

With a high ratio of call- to put-type CDs, furthermore, the lower break-even is nonexistent, making a hedge against a falling market unnecessary. Thus a high correlation between the stock market index and the performance of the loan portfolio motivates the issuer to seek a high call/put ratio, which may help explain the biases observed in the pricing and marketing of the put-type CDs.

When the MICD specifies a cap, the issuer has a modified butterfly spread on the index, allowing the potential losses to be contained within the coverage of the issuer's capital reserves. So long as sufficient reserves are set aside to carry the maximum burden of potential loss, no further hedging against the index is required. When the dollar amount of sales of call MICDs is identical to that for put MICDs the issuer has a classic butterfly spread on the index. (If the issue is not balanced, the spread becomes lopsided.) Even though Figure 5.1 is drawn to show a negative worst-case payoff to the issuer, it could be positive if the cap were sufficiently small.

Since the MICD product was designed to provide a means for the issuer to compete with stock market investments for deposits, it can be argued that containing the issuer's exposure to stock market fluctuations within the bounds of capital coverage is a reasonable alternative to eliminating such risk altogether.

In the absence of a cap, however, a partial hedge can be used to limit the issuer's exposure to stock-market risk. The issuer could form a partial hedge by purchasing $\gamma_c n_c$ call options with exercise price X_U and $\gamma_p n_p$ put options with exercise price X_L. To define X_U and X_L, let R be the dollar reserve against losses on this issue of MICDs and H be the cost of the options purchased for hedging.

$$H = \gamma_c n_c \, c(1, \tau, X_U, r, \sigma^2) + \gamma_p n_p p(1, \tau, X_L, r, \sigma^2)$$

Then for any given level of reserves, X_U and X_L can be found by simultaneously solving the following expressions:

$$X_U = \mathrm{BE_U} + \frac{\mathrm{R - H}}{\gamma_c n_c}$$

$$X_L = \mathrm{BE_L} - \frac{\mathrm{R - H}}{\gamma_p n_p}$$

Although the cost of a partial hedge depends on the exercise prices chosen, the solutions can be found by numerical techniques.

The cost of hedging, of course, would reduce the amount available for investment in the issuer's loan portfolio, which in turn reduces the investment earnings and alters the old break-even points accordingly. The numerical solution technique readily adapts to this requirement, however.

5.3. Regulatory Problems Associated with Partial Hedging and Dynamic Hedging

MICDs would not create any new stock-market worries for regulators if all issuers formed an exact hedge at the initiation of each issue, by purchasing option contracts on the underlying index of the MICD in the quantities specified in the equations before.

An issuer may argue that its newly created exposure to stock market risk is hedged by the correlation between returns on the stock market and the performance of its loan portfolio, however. With both call and put MICDs outstanding, furthermore, the issuer has a straddle on the stock market index, and it is possible for a partial hedge to contain potential losses within bounds the issuer may contend are covered by its capital reserves. In the case of capped CDs, moreover, such a partial hedge is already in place. By increasing the probability of insolvency, issuers which seek permission for no hedging or partial hedging create new regulatory oversight burdens.

Adding to the regulators' cause for concern about MICDs, some issuers (including Chase Manhattan) have announced the intention to manage their exposure to stock market risk by means of "dynamic hedging" - applying portfolio insurance techniques using futures contracts on the underlying index of the MICDs. The essence of dynamic hedging is to create synthetic options for substitution into the hedges described above, by means of a frequently adjusted position in index futures contracts. So the distinction between dynamic hedging and hedging with options contracts, therefore, reflects a different method rather than a different objective. Such dynamic hedging strategies offer valuable advantages for issuers but create costly problems for regulators. For a large issuer, dynamic hedging offers savings in transactions costs. Another obvious advantage of dynamic hedging is that synthetic options can be created with maturities or exercise prices that are not available from listed options — hedging with option contracts, which offer a finite set of exercise prices and maturities, is less flexible than dynamic hedging, which theoretically can be accomplished for any exercise price or maturity.

In addition, a dynamic hedging strategy allows the flexibility for dealing with the risk exposure created by aging issues through the pricing policies associated with new issues. A rising market, for example, creates the need to add to the issuer's long position in index futures as a hedge against aging call MICDs that are outstanding. As an alternative, however, the outstanding call MICDs could be offset in the issuer's aggregate position by establishing a pricing policy that encourages a preponderance of put MICDs to be sold. The newly issued put versions of the MICD will then generate a profit for the issuer if the stock market index continues to rise, which will partially offset the losses generated by seasoned call-type CDs. Thus, apparent anomalies in pricing policy may simply reflect hedging activity.

Issuers who use dynamic hedging will, therefore, have a competitive advantage relative to issuers who hedge with options contracts. A major stock market fluctuation, however, could hurt weak banks that move heavily into MICD products, engage in aggressive pricing policies, and hedge inadequately. Conservative regulatory policies, on the other hand, result in less attractive terms for consumers (i.e., lower guaranteed minimum interest rates, lower participation percentages, or lower caps) and reduce the issuers' ability to compete against mutual funds.

6. Conclusion

The Market Index Certificates of Deposit have been rather recently introduced and they are especially popular within individuals with little or no investment experience, since they provide investors a means for participating in the stock market while at the same time they limit their risk exposure.

MICDs have the potential to expand significantly the investment opportunities available to the general public, while offering financial institutions a committed funding source.

More specifically, MICDs offer some interesting alternative investment opportunities, which seem highly appealing after the recent reduction of international interest rates. Furthermore, a long-term put version of the MICD suggests a useful hedging alternative for an individual with investments in a mutual fund or a stock-based pension fund. Such a hedging vehicle would be particularly attractive to an investor whose tax situation makes it undesirable to liquidate his/her stock position, and who would therefore be willing to pay a premium.

Their multiple uses, the lack of experience of investors, as well as their recent and dynamic introduction in the Greek market, create the urgency to find an applicable method of pricing.

We have presented how a MICD can be replicated with a combination of risk-free assets and index options and we have derived general pricing formulas for MICDs with embedded vanilla call and put options, binary options and "two-asset cash-or-nothing" options.

From the implicit value of the option component, we computed implied standard deviation (ISD) for the market index, which can be proven a useful tool for potential investors in order to judge if the structured product is correctly priced. We have also examined the pricing policies and hedging strategies for the issuers.

It is crucial for individuals who intend to invest in MICDs, to be able to judge the appropriateness of the pricing of the products and to demand to be provided with adequate information about them, since transparency is usually limited.

In an environment of greater homogeneity of financial institutions, these instruments will present important challenges to consumers, banks, securities industry, and regulators. Depository liabilities with interest rates linked to the stock market are a rather recent addition in a long line of new financial instruments. Like so many other financial products, however, they can be replicated with existing instruments. The process of replication provides useful insights into the valuation of these instruments, their comparability to other instruments, and their uniqueness. It can also provide guidelines for investment decisions and regulatory actions.

References

Abken, P., 1989, “A survey and analysis of index-linked certificates of deposit”, Federal Reserve Bank of Atlanta Working Paper 89-1.

Albanese, C., Campolieti, G., 2006, “ Advanced Derivatives, Pricing and Risk Management”, Elsevier Academic Press.

Bethel, J., Ferrell, A., 2006, “Policy Issues Raised by Structured Products”, John M. Olin Center for Law Economics and Business, Harvard Law School, Discussion Paper No. 560, October 2006.

Braddok, J., 1997, “Derivatives Demystified- Using Structured Financial Products”, John Wiley & Sons,

Brealey, R., Myers, S., 2000, “Principles of corporate finance”, Irwin McGraw-Hill, 6th Edition.

Brenner, M., Courtadon, G., Subrahmanyam, M., 1987, “The Valuation of Stock Index Options”, Salomon Brothers Center for the Study of Financial Institutions, Graduate School of Business Administration, New York University.

Burth, S., Kraus, T., Wohlwend H., 2001, “The Pricing of Structured Products in the Swiss Market”, The Journal of Derivatives, Fall 2001.

Chance, D. M., Broughton, J. B., 1988, “Market Index Depository Liabilities: Analysis, Interpretation, and Performance”, Journal of Financial Services Research, 1: 335-352.

Chen, A.H., Chen, K.C., 1995, “An Anatomy of Elks”, The Journal of Financial Engineering. Vol. 4. No.4.

Chen, A. H., Kensinger, J. W., 1990, “An Analysis of Market-Index Certificates of Deposit”, Journal of Financial Services Research, July 1990.

Choudhry, Moorad, 2004, “Corporate Bonds and Structured Financial Products”, Elsevier Academic Press.

Cox, John C., Ross, Stephen A., Rubinstein, Mark, 1979, “Option Pricing: A Simplified Approach”, Journal of Financial Economics, 7, September 1979.

Derman, E., Kani, I., 1994, “Quantitative Strategies Research Notes: The Volatility Smile and Its Implied Tree”, Goldman Sachs.

Edwards, Michelle, Swidler, Steve, 2005, “Do equity-linked certificates of deposit have equitylike returns?”, Financial Services Review 14.

Espen Gaarder Haug, 1997, “The Complete Guide to Option Pricing Formulas”, McGraw Hill.

Fabozzi, Frank J., 2007, “Fixed Income Analysis”, John Wiley & Sons.

Grunbichler, A., Wohlwend, H., 2003, “The Valuation of Structured Products: Empirical Findings for the Swiss Market”, Financial Markets and Portfolio Management, December 2005.

Heynen, R. C., Kat, H. M., 1996, “Brick by brick”, Risk Magazine, 9(6).

Hull, J., 2003, “Options, Futures and other derivatives”, 5th Edition, Prentice Hall.

Hull, J., White, A., 1990, “Pricing interest Rate Derivative Securities”, Review of Financial Studies, 3(4).

Kat, H., 2001, “Structured Equity Derivatives: The Definitive Guide to Exotic Options and Structured Notes”, John Wiley & Sons.

Katz, J. O., McCormick, D. L., 2005, “Advanced Option Pricing Models”, McGraw-Hill.

King, S.R., Remolona, E.M., 1987, “The Pricing and Hedging of Market Index Deposits”, Federal Reserve Bank of New York Quarterly Review 12, Summer 1987.

McDonald, Robert L., 2006, “Derivatives Markets”, 2nd Edition, Addison Wesley.

Solow, Robert, M., 1956, “A Contribution to the Theory of Economic Growth", Quarterly Journal of Economics, 70 (1), February.

Stoimenov, Pavel A., Wilkens, Sascha, 2005, “Are structured products ‘fairly’ priced? An analysis of the German market for equity-linked instruments”, Journal of Banking & Finance 29.

Troyanovski, S., 2005, “Lots of upside, Limited Risk”, On Wall Street, September.

Walmsley, J., 1988, “The New Financial Instruments”, John Wiley & Sons.

Wasserfallen, W., Schenk, C., 1996, “Portfolio Insurance for the Small Investor in Switzerland”, The Journal of Derivatives, Spring 1996.

Wilkens, S., Erner, C., Röder, K., 2000, “The Pricing of Structured Products in Germany”, The Journal of Derivatives, Fall 2003.

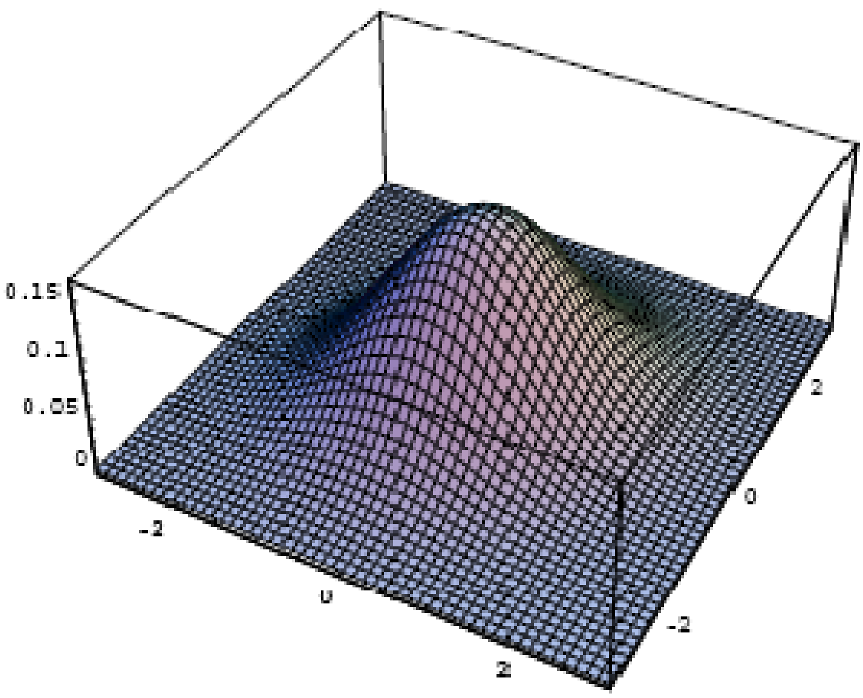
Appendix: The cumulative bivariate normal distribution function $M(d_{1,1}, d_{2,2}; \rho)$ calculated in Mathematica

$$f[x\_, y\_] = \frac{1}{2\pi\sqrt{1-\rho^2}}\,\mathrm{Exp}\left[-\frac{x^2+y^2-2\rho x y}{2\left(1-\rho^2\right)}\right]$$

$$\frac{e^{-\frac{x^2+y^2-2\rho x y}{2\left(1-\rho^2\right)}}}{2\pi\sqrt{1-\rho^2}}$$

```
ρ = 0.2
Plot3D[f[x, y], {x,  3, 3}, {y,  3, 3}, PlotPoints → 50]
```

0.2



- SurfaceGraphics -

```
ρ := 0.2
d11 := 0.1384
d22 := 0.2117
M = NIntegrate[f[x, y], {x, -100, d11}, {y, -100, d22}]
```

0.355165