Communication Games and the Revelation Principle in Supply Chain Management

by

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To those who never believed in me . . .

Preface

This PhD thesis entitled "Communication Games and the Revelation Principle in Supply Chain Management" has been prepared by Dimitris Zissis during the period September 2010 to September 2015, at Management Science Laboratory, the Department of Management Science and Technology, the Athens University of Economics and Business.

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Contents

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List of Acronyms

Acronym	Description	Page
3PL	Third-Party Logistics Providers	48
AI	Asymmetric Information	34
BE	Bullwhip Effect	8
CPFR	Collaborative Planning Forecasting and Replenishment	26
CI	Complete Information	32
CG	Cooperative Game	3
EOA	Echelon Operation Autonomy	20
EOQ	Economic Order Quantity	4
GT	Game Theory	2
IC	Incentive Compatible	7
IE	Incentive Efficient	13
IR	Individual Rationality	35
QD	Quantity Discount	4
RP	Revelation Principle	4
NCG	Non Cooperative Game	9
SC	Supply Chain	1
SCM	Supply Chain Management	1
VMI	Vendor Managed Inventory	20

Abstract

The aim of this PhD dissertation is the in-depth study of supply chain and how the nodes could coordinate their strategies in a decentralized system. We provide the nodes the opportunity to communicate with each other, without any restrictions. Therefore, we propose a "free" communication system, including all the possible ways of communication among the nodes, and examine how communication leads to the system-wide coordination and, thus, reduces costs, eliminates inefficiencies, and results in better individual profits for all the participants.

We study supply chains with rational nodes, which have to make private decisions in order to maximize their utility functions. These decisions are related to the order quantity, quantity discounts, product prices, inventory levels, etc. Furthermore, these decisions are usually competitive, because every node has different preferences and different information. However, there are cases in which some nodes have incentives to build a coalition; therefore they act as a single entity. As each node is a distinct decision maker and has private information and different preferences, we model the supply chain as a game using tools of Game Theory. Our core objective is to examine how each node decides on his strategy in a decentralized system.

An increasing body of literature in the area of Supply Chain Management addresses the way in which the nodes of a chain can act in a cross-linked mode, in order to reduce both their own costs and the total cost of the chain. Key research work has been published in premier archival journals tackling problems associated with supply chain coordination; however, examination of the recent literature reveals that almost all the papers have restrictive (e.g., sign of contracts) or unrealistic assumptions (e.g., all the nodes possess the same information). Thus, there are many open issues deserving attention. It would be ideal if we could propose ways of coordination without restrictive and unrealistic assumptions to align the individual incentives of the nodes with the incentives of the whole chain.

In this regard, we allow nodes to communicate with each other; with respect to any private information they may possess. Obviously, opportunities for mutual benefits cannot

be found, unless the nodes share their private information. To proceed in sharing private information, nodes should be provided with appropriate incentives. It is worth to investigate how information sharing could be achieved. We consider that all the possibilities for communication are assumed to be entirely controlled by a mediator. The fundamental idea is the framework proposed by Gibbard (1973) and Myerson (1979, 1982), the Revelation Principle. The extended Revelation Principle's framework, by Myerson, Hurwicz, and Maskin, was awarded the 2007 Nobel Prize in economics. Intuitively, the Revelation Principle states that the mediator could design a mechanism to enforce all the nodes to reveal their private information and obey his suggestions about their actions, because it is in their self-interest. Therefore, by using credible mediator, coordination is attainable.

Chapter 1

Introduction

1.1 General Area of Study

A supply chain (SC) is be defined as “the interrelated series of processes within a firm and across different firms that produces a service or product to the satisfaction of customers” (Krajewski et al., 2010). SC is a network of material, service, monetary, and information flows that link a firm’s customer relationship, order fulfilment, and supplier relationship processes to those of its suppliers and customers (Cachon and Terwiesch, 2006). The number of SCs in which a firm participates depends on the mix of services or products it produces. Furthermore, a firm has not always the same position in a SC; i.e., a firm which is a supplier in one SC may not be a supplier in another SC. This happens because the service or product may be different.

The term Supply Chain Management (SCM), refers to the synchronization of a firm’s processes with those of its suppliers and customers to match the flow of services, materials, and information with demand (Cachon and Terwiesch, 2006). SCM provides the theoretical background to managers in developing policies matching resources and customers’ demand to improve the efficiency of SC.

Many firms (nodes) could exist in a SC. Firms make private decisions in order to maximize their utility functions (own goals or profits). These decisions are related to product prices, inventory levels, warehouse locations, transportation means, and many other corporate attributes. Each firm, with its own decisions affects the whole SC, which in turn assimilates the decisions of the other firms and feeds back the influence.

In a globalized society, almost all goods are moved through a SC. Thus, it is important to study and analyze the decisions that nodes have to make and calculate the costs that are incurred in a SC. Reducing even a small portion of these costs may result in significant savings.

The importance of SCM has motivated both private companies and academic researchers to address problems inherent in SC design and operation. Although research approaches tackle simplified versions of the problems faced by SCs, and several real-life SC components or constraints are neglected by academics, the research models typically address the basic properties of the respective systems and thereby provide the core results used in the analysis and implementation of SC design and operations in practical applications.

Most of the times, the nodes of a SC act simultaneously in a competitive and/or cooperative environment. In such a complex framework, we propose the use of Game Theory (GT) to address firm decisions in a SC with respect to the global goals and metrics of the chain. GT is a mathematical approach for studying and analyzing situations (games) in which more than one decision maker (player) is involved and players' success is not based only on their own choices but also depends on the choices of the others. Specifically, it is the study of mathematical models of conflict and cooperation between players, who are rational decision-makers and their unique goal is to maximize their own utility functions (Myerson, 1991). In a game, every decision can affect all players' utility functions and not only the utility function of the node that makes the decision.

The first ideas that can be linked to GT as a scientific discipline appeared in the early 20th century, with the innovative works of Zermelo (1913), Borel (1921) and Von Neumann (1928). The foundation of GT begins with J. Von Neumann and O. Morgenstern, professors at University of Princeton, who presented the book "Theory of Games and Economic Behaviour" in 1944. GT is mainly used in economics, political science, psychology, biology and other academic fields, especially after 1970.

Nowadays, there are many problems from different scientific areas which are analyzed as games, because GT has provided a sound theoretical background for approaching many complex models. A SC is an ideal application area for GT, since distinct nodes make private decisions that have direct impact on both the decisions and the results of other nodes. There is a new trend to study and analyze a SC using GT concepts, a fact that is supported by plethora of scientific publications in this area. Furthermore, GT is becoming a sought-after area in business schools, which include courses of GT in their curriculum (Cachon and Terwiesch, 2006).

Despite its wide applicability, GT has several caveats, the most serious being the fact that there is no more the notion of a solution for the game but that of a solution concept. This happens because the proposed solutions: Nash equilibrium, Bayesian Nash equilibrium, Core, Shapley value and others (Gibbons, 1992) do not have favourable mathematical properties. For instance, sometimes no solution concepts exist, or if a solution concept exists, it is not necessarily unique, or it may not be Pareto optimal, or it may not maximize the sum of the player's pay-offs, and the list continues (Owen, 1995).

Researchers model SC as a game since there are many distinct nodes (players) that both compete and cooperate to produce products or services. Nodes (i.e., firms) are rational players, who make decisions to maximize their objectives (objectives could be expressed as utility functions according to GT). How the players select their actions (strategies) and the impact of these decisions on the entire SC are important research questions with significant actual applications. GT provides the enabling framework to managers for getting answers to SC-associated questions without the actual implementation of strategies and decisions in practice.

Three areas where GT is the enabler for effective SC study are dynamic decision making, coalitions, and private information sharing or preserving, across the nodes of a SC. In the first case, a firm have to make decisions over time, a situation modeled via dynamic games. Stackelberg, repeated, stochastic and differential games are some examples of dynamic games, handling different ways decisions are taken by SC nodes over time (Gibbons, 1992). In the second case, players have incentives to form a coalition, which, when established, acts as a single entity; this can be approached through the GT concept of "Cooperative Games" (CG) (Gibbons, 1992). In the last case, which arises in the majority of practical applications, some of the players, before the game starts, have already (some) private information about the game (e.g., cost structure, better forecast of the demand, etc.), thus would require a "better treatment" in terms of the utility function or payment at the end of the game (Myerson, 1991). Private information is embedded in Bayesian games where every player is equipped with a set of types, and each element of this set includes all the players' private information.

1.2 Motivation

An increasing body of literature in the area of management addresses how the system's participants interact to raise both their own profits as well as the overall system gains (Myerson, 1991). Examples of such systems can be firms (at the enterprise level), SCs with multiple nodes, energy suppliers, and distributors etc. (Tirole, 1988). The participants in such systems make private decisions to maximize solely their own utility functions, without taking into account the global optimum (Cachon and Terwiesch, 2006). Key research work has tackled problems associated SCM (Corbett and de Groote, 2000). They address the way in which the nodes of a chain (players) can act in a cross-linked mode to reduce both their own costs and the total cost of the SC (in terms of inventories, ordering, transportation, etc.). The importance of reducing overall costs instead of just tackling individual node costs is also underlined both by private companies and academic researchers (Cachon and Terwiesch, 2006).

We seek to develop a framework in which the nodes have both private information and the option to coordinate their actions, without being directly constrained when they make their own decisions. If nodes coordinate their strategies, opportunities for mutual benefits may arise. This proposed framework should comply with some standards. At first, it should be capable of modeling all the possible nodes' decisions without any restrictions; these decisions come as the result of the nodes competition for additional profit shares or cooperation for overall profit maximization. Moreover, it is very important that all players are free to choose their own strategies, without resorting to coalitions or contract signing, keeping their unique objectives to achieve personal gains (maximize/minimize their expected profits/costs; rational players).

In the last two decades, key research work has been published in premier archival journals tackling problems associated with SC coordination (e.g., Quantity Discount (QD), or use Economic Order Quantity (EOQ) models); the reader is referred to Weng (1995), Chen et al. (2001), Chen (2005), Corbett and de Groote (2000), Cachon and Fisher (2000), Cachon and Kok (2010). A thorough literature review reveals that almost all contributions make restrictive or unrealistic assumptions, e.g., requiring contracts or assuming that all the players have the same information (Weng (1995); Cachon (2003)). It would be ideal if we could propose ways of coordination without assumptions such as the above, in order to align the individual incentives of the players with the goals of the whole chain and, thus, reduce costs, eliminate inefficiencies, and result in no-worse individual profits for all nodes. Therefore, open research questions and problems still exist, especially with respect to the potential means for coordinating node strategies.

To extend existing knowledge concerning SC node coordination, we propose a novel framework in which the players have the option to communicate with each other regarding any private information they may have. Examining all possible means and ways for communication between the nodes-players might be intractably complex. Thus, we propose a communication system with a mediator to facilitate effective communication achieving immediate effects on the SC coordination.

In mediated communication systems, the Revelation Principle (RP) plays an important role in the analysis of game (here the SC). The RP is a tool of GT, which offers significant insights that allow us to make general statements about the possible communication mechanisms; it was first proposed by Gibbard (1973) and Myerson (1979) and 1982). Myerson, Hurwicz and Maskin, who extended the RP's attributes, were awarded the 2007 Nobel Prize in economics.

Based on the RP, we propose a mediated communication system to coordinate nodes' actions. It is obvious why the problem constitutes a challenge: We intend to use an approach which has been awarded the Nobel Prize, and apply it to coordinate the nodes of a SC, a research that has not been previously pursued. Moreover, in this framework we have the opportunity to study in detail how the nodes of a SC act to increase their individual gains as much as they can, without being constrained while selecting their strategies. Consequently, the total cost of the enterprise network is reduced and at the same time inefficiencies of the SC as a whole are eliminated, a result of paramount importance especially in today's economic crisis.

1.3 Expected Outcomes

The objective of this thesis is the development of an appropriate framework, where the rational nodes of a decentralized supply chain could be coordinated. Our main objective is to achieve perfect coordination without any restrictive assumptions. Therefore, we propose a free communication system. In such systems, all the nodes are free to report everything that they want.

In order to provide the nodes with the opportunity to communicate each other regards to their private information which they possess, we are in need of using a mediator. Then, perfect coordination is attainable and leads to additional profits for all the participants.

1.4 Organization of the Thesis

The thesis is organized along six chapters, which provide all developments related to modeling and coordination of the supply chain, in terms of content, appropriate theoretical background and proposed solutions. The actual contextual description of the chapters beyond this introduction are:

- Chapter 2 is presents the literature review around the specific problems addressed in the dissertation and the basic concepts of Game Theory which will be used.
- Chapter 3 develops a model with two nodes, one of which has private information expressed in a two-level, discrete form. We devise the analytical solution of the associated 2-player game and we provide numerical results offering insights on the

effect of the various model parameters; furthermore, we run a sensitivity analysis for performance evaluation.

- Chapter 4 discusses the extension of the model developed and solved in Chapter 3, in a case where there exist many types of private information. In particular, we formulate the game with three different types (levels) of private information to gain additional insights on a problem that has been characterized as very intrinsic (Tirole, 1988).

- Chapter 5 analyzes a more complex model where both nodes have private information. We develop a game in which each node possesses private information and derive the analytical optimal solution about node decisions. Furthermore, we prove that perfect coordination is attainable, when the reservation levels are not exogenously defined.

- Chapter 6 offers an overall synopsis of the thesis in terms of conclusions, achievements, and directions for future research.

Chapter 2

Literature Review

In this chapter, we present a comprehensive literature review of SC and SCM. We explain the need of an appropriate theoretical background, in order to study and analyze the SC and provide managers with quantitative and qualitative tools, on which they could base decisions order to maximize their firm's profits. We consider as an ideal framework to achieve our goals that of GT, following the work of Cachon, Chen and others that have previously used frameworks based on GT to model and solve SC problems. Subsequently, some preliminaries on GT are presented, mainly on Bayesian Games, Communication Games, RP, mediated communication systems and Incentive Compatible (IC) mediator plans. Finally, we present some specific approaches of key researchers in the areas of SC and SCM, and conclude the literature review by identifying restrictions and limitations of research to-date, facts that motivate our thesis.

2.1 Supply Chain and Supply Chain Management

Economists claim that "prices adjust to match supply with demand". Managers though disagree with this principle. For managers, excess demand means lost revenue while for economists it means that prices rise to match demand with supply; in addition, managers treat excess supply as wasted capital and resources in contrast to economists that just model it by falling prices. It is extremely hard to match supply with demand and of course we are in need of tools on top of simple assumptions related to expected price adjustments emanating from these imbalances. The basic cause of demand and supply mismatches is that demand may vary and supply is not as flexible as to follow the demand variability instantaneously.

For managers it is a major challenge to match supply with demand, since this allows a smooth flow of products and materials. However, a rather small improvement of supply and demand balancing may have a significant effect on a firm's profitability. To demonstrate the importance of this, consider the following example: if British Airways increases its utilization by 0.33 % (this means only one more passenger in each flight), the firm's profits can increase to approximately 65 million dollars (Cachon and Terwiesch, 2006) It is necessary for enterprises to match supply with demand as much as possible, and it is clear that creating only great products/services is not sufficient even for just market survival.

As we have mentioned a SC is the interrelated series of processes within a firm and across different firms that produces a service or product to the satisfaction of customers, while with the term SCM we refer to the synchronization of a firm's processes with those of its suppliers and customers to match the flow of services, materials, and information with demand. SCs performance and efficiency depends on the decisions and actions taken by all the nodes in the SC.

The primary objective of each node in a SC is to maximize its own profit, but this does not result in the optimal overall SC performance (Cachon and Terwiesch, 2006). A characteristic example is the Bullwhip Effect (BE); i.e., the phenomenon in which variability in order quantities is magnified as we move from retailers to manufacturers, a situation that has been observed in the majority of SCs and has negative impact on all SC nodes (more severe in the upstream ones). This reduces the efficiency of a SC leading to excessive inventory, income reduction, cost spikes etc. (Lee et al., 1997). The nodes could reduce the overall system costs and expect to achieve better individual profits, if they could coordinate (Cachon and Terwiesch, 2006).

Moreover, a key factor to achieve coordination in the SC is the incentive conflicts among the SC's independent firms. Each node acts to maximize its own profit; thus, a decision which is optimal for this node, may not necessarily be optimal for another node in the same SC, creating in this way a conflict between these two nodes. A characteristic example is the SC with only two nodes; exemplified by Zamatia Ltd. (an Italian maker of eyewear) and Umbra Visage (retailer of Zamatia in Miami Beach, Florida) (Cachon and Terwiesch, 2006). Umbra Visage makes only one order each season, because Zamatia manufactures its sunglasses in Europe and Asia, thus the replenishment lead time is long and the selling season for sunglasses is short. Obviously, the manufacturer wants to sell as much sunglasses as possible to the retailer; however the retailer does not want to have excessive inventory. Cachon and Terwiesch (2006), with this example, illustrate the following: "Even if every firm in a SC chooses actions to maximize its own expected profit, the total profit earned in the SC may be less than the entire SC's maximum profit".

The literature is full of measures reflecting the performance of a SC, measures that encompass inventories and financial indices (Krajewski et al., 2010). Such measures allow managers to assess the implications of changes in a SC or to (re)design SC, in order to reach better match of supply and demand (more efficient SC). Moreover, there are quantitative models and qualitative strategies in SCM, to help managers use the appropriate tools and make the right decisions.

2.2 Game Theory Framework

GT is a collection of mathematical models formulated to study decision making in situations involving conflict and cooperation (Lucas, 1972). When two or more decision makers (players) participate in such a game, conflict and cooperation naturally arise, according to the players' preferences. In the literature of GT, there are two basic categories of games: the "Non Cooperative Games"(NCG) and the "Cooperative", CG.

In the first category, players choose strategies simultaneously and independently, at the initial stage of the game. This does not imply that players necessarily act at the same instance; it suffices that each one chooses his own action without knowledge of the others' choices and decisions. In the literature many solution approaches for the NCG have been proposed, the prevalent of which is the "Nash equilibrium" (Gibbons, 1992).

In the second category, the players are free to make coalitions with only one restriction: a player can be part of at most one coalition. Thus, the analysis focuses on the coalitions that can be formed, without each player acting separately within the framework of a coalition. The analysis of CG is entirely different from the model which is used in NCG, and it is based on the characteristic function form (for further the reader is referred to Myerson 1991, Chapter 9). Some basic proposed solution concepts for these games are the core and the Shapley value (Myerson, 1991).

Our objective is to study and analyze SCs. To achieve this we develop a framework based on GT to model aspects of a SC as a game. Furthermore, we want the nodes to have as much freedom as they can. Therefore, we focus our analysis on the NCG, since each player acts separately and is free to choose whatever action he desires. To analyze a NCG we need a way to represent it. A simple and useful representation which has been proposed for these games is the "normal form representation" (Myerson, 1991).

Definition 2.2.1 *The normal form representation specifies:*

- *a non-empty set N, which enumerates the players of the game, $N = 1, 2, ..., n$,*
- *a non-empty set of options S^i, available to each player $i, i \in N$,*
- *a pay-off h^i, received from each player $i, i \in N$.*

Remark 2.2.1

- *The set $S^i, i \in N$ is called "strategy set" of player i. We assume that every element of S^i includes the strategy of player i and the player has to choose only one element of his strategy set.*

- *The term "strategy' 'means any rule for determining the players' decision,*

- *We define a "strategy profile" to be a combination of the strategies that could be chosen by the players. Let S denote the set of all possible strategy profiles; i.e., the Cartesian product of all S^i's, so $S = \prod S^i$,*

- *$h^i, i \in N$ is a function from set S into the set of real numbers $\Re$.*

- *A NCG can be described by the form: $< N, S^1, ..., S^n, h^1, ..., h^n >$.*

- *A NCG is finite if the set N, as well as sets $S^i, i = 1, , n$ are finite.*

Definition 2.2.2 *In the NCG $G :=< N, S^1, , S^n, h^1, ..., h^n >$ the strategies $(s_0^1, ..., s_0^{(i-1)}, s_0^i, s_0^{(i+1)}, ..., s_0^n)$ are Nash equilibrium if the following expression is satisfied: $h^i(s_0^1, ..., s_0^{(i-1)}, s_0^i, s_0^{(i+1)}, ..., s_0^n) \geq h^i(s_0^1, ..., s_0^{(i-1)}, s^i, s_0^{(i+1)}, ..., s_0^n), \forall s^i \in S^i, \forall i \in N$.*

Note that, the players may use mixed strategies; i.e., a tagged player i, instead of selecting a specific strategy $s^i \in S^i$, selects a probability distribution over his strategy set S^i, and the actual decision s^i is determined randomly according to this distribution. The set of mixed strategies; i.e., all the probability distributions over S^i, is denoted as $\hat{S}^i$. In the case where the set S^i is finite, the $\hat{S}^i$ is the (S^i) (where (S^i) denotes the set of probability distributions over the set S^i). If the players use mixed strategies and the game is finite, at least one Nash equilibrium always exists (Theorem "Nash", Gibbons (1992)). On the other hand, when the players use such strategies, we have to use the expected values for all problem variables.

2.3 Bayesian and Communication Games

Depending on the information which the players have relevant to the game itself, two game categories exist: games with complete information and games with incomplete information. In the first category, all players are fully informed about the structure of the game, in which they are about to participate; i.e., they are all aware of the form: $< N, S^1, , S^n, h^1, ..., h^n >$ and this knowledge is common to all the players. The second category of games, the games of incomplete information, is more realistic because it models almost all the economic situations where the players have private information about the game. Obviously, the private information of each player is not common knowledge to the remaining players.

In the literature, “Bayesian Games” have been employed to model private information (Gibbons, 1992). In a Bayesian Game, a finite set of types T^i is considered for every player $i,\ i \in N$. Every element of set T^i incorporates the entire i-player’s private information. At each run of the game only one element of T^i is selected; let this be $t^i \in\ T^i$, which is known only to the player i who is subsequently called a player of “type t^i.

Apart from the two basic categories, NCG and CG, there are games which combine properties from these two categories. Such are the “Communication Games”, in which the players have the opportunity to communicate with each other before they choose their actions, without any restrictions. In this way, we can model all expressions of communication, even negative ones, such as misinformation. Communication games may be considered as a hybrid case between the two basic game categories. Moreover, it has been proved that the solution concepts for this category have noteworthy properties (Aumann, 1974).

The communication between the players is performed through a mediator. The mediator is a reliable authority, not belonging to the players’ level, having a unique goal: to help the players to communicate with each other, without incurring any additional cost to the system. The possibilities for communication and cooperation are assumed to be entirely controlled by the mediator.

2.4 Incentive - Compatible Mechanism

Myerson (1979) examines IC mechanisms that emanate in games where (some or all) the players have only private information. The author focuses on a Bayesian collective choice problem where all players have private information and must make a single decision based on this information; it can be described by the form: $< N, C, T^1, , T^N, h^1, ..., h^N, P >$, where:

- N the set of the players,
- C the set of choices available to the players,
- T^i the set of private information of player $i,\ i \in N$,
- h^i the utility function of player $i,\ i \in N$,
- P a probability distribution on T.

We denote with T the Cartesian product of all T^i's; i.e., $T = \prod T^i = T^1 \times ... \times T^N$. So, $P(t)$ is the probability that the vector $t,\ t \in N$ where $t = (t^1, ..., t^N)$ is the true vector of types for the players.

The author studies the case of adverse selection incentives within an IC mechanism where $c \in C$ is the single common decision which the players should make based on their private information. Note that, the author allows mixed strategies. The utility function h^i for each player $i,\ i \in N$ depends both on the players' private information T and on the common decision c; i.e., $h^i : C \times T \to \Re$. The adverse selection incentives make sense only in games where players have private information. Under the light of players' preferences coordination and the selection of the common decision, the author suggests their communication via a mediator. The latter is actually the one who makes the decision, which in turn is binding for all players.

The model has the following form: First, the mediator devises a mechanism, called the "mediator plan", which indicates how he will act based on the players' reported types. The mediator plan is a function $m : T \to (C)$, where (C) denotes the set of probability distributions over the set C. The function is a conditional probability distribution over the set C, given a vector of types $t,\ t \in T$. Thus, $m(c|t)$ denotes the conditional probability that the mediator chooses the decision c, when the players report to him the type t. Obviously, $m(c|t)$ must satisfy the probability constraints.

When the plan is finalized, it is announced to the players, and each of them is asked to confidentially and non cooperatively report his type $\hat{t^i}$ to the mediator (the hat is used to differentiate the reported from the actual type, and denotes the reported type). Obviously, $\hat{t^i} \in T^i$ otherwise it is not a plausible type. Every player selects the type $\hat{t^i}$ he will report, independently of the others players, because he does not know anything about what the others have reported. In such a game the strategy of each player coincides with the function linking the type the player reports and his actual type.

In the reporting phase, each player is free to choose what he/she reports to the mediator, as we have assumed that players are rational and try to achieve as much as they can. Moreover, each player is the only one who knows his/her own true type, and no one can prevent him/her from lying about it, since he/she may expect advantage from such a behaviour. The mediator is aware of this, and thus, if he wants all the players to report their real types, he must include appropriate incentives to the plan. The latter are known as "adverse selection incentives".

If a mediator plan includes such incentives, all the players report their types honestly, because in this way they achieve the most gains from the game; i.e., universal honesty is an equilibrium strategy for all players if and only if the mechanism includes adverse selection incentives. In others words, no rational player would expect more gains from being the only player to lie about his type, when the others are planning to honesty report theirs types.

We denote the conditionally expected utility pay-off for the player i, under the mediator plan m, when the player reports to the mediator a type $\hat{t^i}$ and his actual type is t^i, given that the rest players report honestly theirs types as: $H^i_m(\hat{t^i}|t^i)$. Then, the mediator plan m includes adverse selection incentives if and only if:

$$H^i_m(t^i|t^i) \geq H^i_m(\hat{t^i}|t^i), \forall i \in N, \forall \hat{t^i} \in T^i, \forall t^i \in T^i. \tag{2.4.1}$$

The author proves that the mediator plan can be restricted to the IC mechanisms, according to the RP. The RP states that any equilibrium of a coordination mechanism can be replaced by an equivalent IC mechanism; i.e., for any equilibrium in a coordination mechanism there exists an equivalent IC mechanism with identical pay-offs. So, according to the RP, it is feasible to derive general properties about equilibriums of all coordination mechanisms by analyzing only the IC ones. Moreover, the author proves that in such games, the set of IC mechanism is a non-empty, convex and compact subset of the set of all expected utility allocations; this property supports and promotes the use of such mechanisms.

This restriction is significant, in the sense that the set of IC mechanisms is much smaller than the set of all feasible mechanisms. Moreover, this set has “good” mathematical properties, because it is constrained by a finite collection of linear inequalities. Thus, the mediator can restrict his search only to IC mechanisms, when he devises his proposal to the players. Especially in the case of finite games (finiteness being a non-restrictive assumption), the set of IC mechanisms it is easy to analyze because it can be defined by linear inequalities and can be solved by liner programming. In the case of infinite games, the essential ideas still hold, with the exception that the probability vectors must be replaced by measures.

Finally, the author introduces the concept of Incentive Efficient (IE) mechanisms in order to restrict further the feasible mechanisms. Components of the IE set of mechanisms includes IC ones not strictly dominated by other IC mechanisms. Let $H^i(m|t^i) = H^i_m(t^i|t^i)$ denote the pay-off of player i, when he is of type t^i and all other players report honesty their types to the mediator, under the IC mediator plan m. Then the set of IE mechanisms is the set of IC ones, subtracting the IC mediator plan m for which the following holds:

$$H^i(m|t^i) < H^i(m'|t^i), \forall i \in N, \forall t^i \in T^i, \tag{2.4.2}$$

where m' is an IC mediator plan. Intuitively, the author removes some plans from the set of IC mechanisms; because there are IC-plans that lead all the players, for every vector of types t, to better pay-offs (or at least to the same pay-offs).

Myerson (1982) generalizes his previous work (Myerson, 1979) on IC mechanisms. Now, players have both private information and must make individual decisions, extending the original framework (Myerson, 1979) that was based just on private information. The author focuses on a problem which can be described by the form: $< N, C^1, ..., C^N, T^1, ..., T^N, h^1, ..., h^N, P >$, where:

- N the set of the players,
- C^i the set of choices available to the player $i,\ i \in N$,
- T^i the set of private information of player $i,\ i \in N$,
- h^i the utility function of player $i,\ i \in N$, with $h^i : C \times T \to \Re$,
- P a probability distribution on T.

Note that, the utility function h^i for each player $i,\ i \in N$ depends both on the players' private information and their individual private decisions. In order to coordinate the players' decisions and raise their gains, the author suggests their communication via a credible mediator. Thus, a communication game arises. The mediator, apart from the adverse selection incentives that were introduced to force players to report their actual private information, has to also incorporate moral hazard incentives in the mediator plan. These incentives are included to make players respect and implement the mediator's recommendations in their own individual decisions. Thus, an IC mechanism is expanded to include both adverse selection (Myerson, 1979) and moral hazard incentives. The author uses the RP and extends it in situations where the players have to make individual decisions.

The proposed model has the following form: First, the mediator devises his plan, which defines what he proposes to each player, based on the players reported types. The mediator proposes to each player a specific choice $s^i \in S^i$ (or a probability distribution over S^i). Then, the plan is announced to the players, and each player is asked to confidentially and non cooperatively report his type to the mediator. When the mediator receives all the players' reports, he makes his recommendations to each player confidentially, according to the mediator plan. Finally, every player makes his own decision. Note that, both in the report phase and when the players make their own decision, they are free to choose any strategy that maximizes their own utility functions. The mediator is aware of that, and this is the reason that forces him to include proper incentives to the mechanism, if he wants all the players to report the real types and follow his recommendations. In such a game the strategy of each player is what type he reports, given his actual type, and what choice he selects given the recommendation from the mediator; i.e., the strategy for the player i, S^i incorporates two functions, the one from $T^i \to T^i$ and another from $C^i \to C^i$.

If a mediator plan includes adverse selection and moral hazard incentives, all the players report honestly their types and obey to the mediator's recommendations, because in this way they achieve the most gains from the game; i.e., universal honesty and universal obedience is equilibrium for every player if and only if the mechanism includes adverse selection and moral hazard incentives. Because of the utility function maximization, no rational player would expect more gains when he is the only player lying about his type or disobeys the mediator's recommendation, or does both, when all the others players are planning to honesty report theirs types and obey to the mediator's recommendations.

The author proves that the mediator plan in such games can be restricted to the IC mechanisms, according to the RP. Moreover, the author proves the same results as the original framework (Myerson, 1979); i.e., in the case of finite games, the set of IC mechanisms is easy to analyze it, because it can be defined by linear inequalities and can be solved by liner programming, and in the case of infinite games, the essential ideas still hold.

2.5 Game Theory Models for Contract Design in Supply Chains

There have been indications that GT can be used to study and analyze real-life situations in which many decision makers are involved. Lucas (1972) states that GT is the only quantitative approach about such problems and through this theory a change on what many people think about the competitive situations has been observed; as a result, managers are influenced on what rules they should implement to make the right decisions. The interest about GT has arisen due to many and different areas of applications.

Nowadays, there is a plethora of researchers, in the field of SC, who used or developed frameworks based on GT, in order to analyze in detail the SC; the multitude of research papers is an indicator that SC is an ideal application of GT. This happens due to the fact that the players' decisions in a SC are usually competitive, because every player has different preferences and personal information; this contributes to an increase of the overall system's costs. Thus, we can model the SC as a game due to the following facts: i) each player is a distinct decision maker, ii) each player has private information and different preferences, and iii) each decision affects both the decisions of the other players and the profits or costs for all the participants (Cachon and Netessine, 2004). The common goals of most researchers are to determine policies which maximize the combined profits of all nodes and raise the efficiency of the SC as a whole.

Weng (1995) examined the use of QDs as a way to coordinate the SC and achieve more profits for all the nodes. The different preferences of nodes concerning the level of orders lead to an increase of the overall inventory-related cost of the SC. A typical solution offered in the literature is a form of coordination between nodes. The author considers a SC with one supplier and one buyer (or a group of homogeneous buyers), which trade a single product. The supplier produces in a lot-for-lot fashion and does not have the opportunity to stock. A crucial assumption is that the nodes have complete information about the SC. The model is the typical EOQ model, where the buyer chooses the order quantity Q. As the nodes are rational, they make their choices to maximize their profits, without considering the global optimal for the whole SC. Weng (1995) addressed how the nodes could be coordinated and proved that the QDs is a necessary condition for the nodes to achieve maximum joint profit, but is not sufficient and requires additional parameters.

A QD is a function that reduces the per unit product price when larger quantity orders are placed. A survey of QD schemes has been performed by Benton and Park (1996). Moreover, in the last few years there are studies (Kalkanci et al. (2011); Davis et al. (2014)), in which are made behavioural experiments about how the managers uses QDs in practice.

In Weng's (1995) work, an assumption is that all the nodes are aware about the situation they are being involved in (game with complete information). This assumption is particularly restrictive in practice where individual players tend to keep private their cost structures or demand data and not always feasible in a SC. In practice since individual nodes/players tend to keep their cost structures or other interval information private.

Corbett and de Groote (2000) generalized the QDs in case where the players have private information. They consider that the supplier does not have full knowledge of the buyers' unit holding cost (denoted by h_b), but is aware of its distribution. Managing the total inventory-related cost of the SC is the goal proposed in the paper. The problem is formulated as following: first the supplier proposes a discount as a function of h_b, and then the buyer decides upon choosing a specific order or not ordering at all. In this context, with the buyer's choice the supplier receives a signal about the buyers' unit holding cost. The authors allow the buyer to lie about his unit cost (denoted by $\hat{h_b}$, obviously $\hat{h_b} \geq h_b$)

achieving more personal gains. In case of complete information, the supplier offers a single optimal joint QD that the buyer has to accept. The authors use the RP to make the analysis easier and they prove that the optimal order quantity is smaller or equal than the quantity in the case of complete information but larger than the quantity in case there is no coordination or cooperation; this is exactly the benefit of coordination.

Cachon and Fisher (2000) studied the way private information of each SC node affects its policies and how information sharing can reduce the total cost of the enterprise network. In traditional SCs the only information which is shared among the nodes is the quantity of orders in a pair-wise linear fashion. Today, enabled by information technology tools, it is possible to share all private information that the nodes possess, quickly and inexpensively, in order to better handle inventory holding and transferring decisions, thus reducing costs and eliminating inefficiencies. Sharing the private information results in reduced lead times and order quantities, thus decreasing the overall cost of the SC via the effective allocation of the transfer inventory. The authors compare the total cost of the SC in two cases; first in a traditional SC and second in a SC in which the nodes share their private information. For both cases, the authors attempt to find the optimal ordering and stocking policies, using simulation. They show that when all nodes of the SC share their private information, the total inventory-related cost is reduced.

Fiala (2005) underlined the value of information exchange in the SC and the importance of the honest exchange of information among the SC participants for coordination. Corbett et al. (2004) examined how the supplier's decisions can be affected by the retailer's private information, allowing the supplier to refuse to work with some retailers. Ha and Tong (2008) studied information sharing in a model with two competitive SCs, each consisting of one manufacturer and one retailer. Finally, Ozer and Raz (2011) examined how asymmetry of information affects the whole chain in a more complex model with one manufacturer and two competitive suppliers.

Iyer (1999) examined the way demand information affects inventory levels, total cost and stock management policies of a SC's nodes. The author assumes stochastic demand distributed according to a known probability distribution in a SC with one manufacturer and one retailer. The idea introduced by the model is that it allows the retailer to collect data regarding actual sales, and use these data to generate the posterior distribution of the demand. This posterior distribution is then used by the retailer to place his order to the manufacturer, following the classic Bayesian approach. It can be verified that in this model, both the expected quantity in the inventory pipeline and the service level are increased, while the expected quantity left over as stock at the retailer's node is reduced. The major issue is then to determine if the retailer has always some incentive to collect data and to act based on these data. The author proves that such a system is not Pareto improving by itself, and in order to reap the Pareto's gains contractual agreements have to be set in place.

Chen (2005) addressed the incentives which firms should provide to salespeople if firms anticipate from them to reveal the market knowledge (private information) which possess and continue their sales effort intensively. The salespeople's information is vital for firms, because almost all the firm's decisions (forecasts, new product development, production, inventory planning, etc.) are based on market knowledge. Moreover, the firms should provide appropriate incentives to the salespeople, in order to enhance their effort in selling the firm's products. Note that, firms do not have the opportunity to directly observe the market situation and the selling effort of the salespeople. These kinds of problems combine moral hazard and adverse selection incentives. A well-known solution for this problem is the scheme proposed by Gonik (1978), which states that: a firm asks each sales agent to provide a forecast about the sales volumes in his area and his pay-off is a function of two variables: the real sales volumes and the initial forecast, which is quoted to the firm. So, the sales agent has incentive to make accurate forecasts and continue his sales efforts. Another solution is the firm to offer a menu of contracts to sales agents (Kreps, 1990), and the latter to choose one among them. Through the agent's decision the firm receives a signal about the market conditions. Thus, the problem goes back to an appropriate design mechanism by the firm. Chen (2005) studies how the firm can design a menu of contracts to learn the market condition and motivate the agent to continue his work. He proposes two bounds, one upper and one lower, for the expected firm's profits. The upper bound is obtained when the firm observes the market condition and the agent's effort, while the lower bound is obtained when the firm provides only one contract (leaves out the opportunity to understand the market condition). He uses a probabilistic approach and performs optimization to proceed to the analysis, but he also examines several numerical scenarios using a simulation approach. Through the simulation results the author provides some ideas about how specified parameters affect the expected firm's profits. Moreover, the author compares Gonik's solution with the menu of contracts and shows that the Gonik solution is dominated by a menu of linear contracts.

Another restrictive and not realistic assumption of Weng's work (1995) is that he considers only one retailer or a group of homogeneous retailers in the SC; i.e., all the retailers are identical. Chen et al. (2001) extend Weng's work (1995) to the case where the retailers are non-identical, while the problem remains the same; i.e., identification of the mechanisms that should be used to achieve (perfect) coordination among the SC nodes. According to Viswanathan and Wang (2003), coordination is considered to be perfect when the total cost in the decentralized system (a system in which each node makes decision(s) in order to maximize its own pay-off, based on the information it possesses) is equal to the total cost in the centralized one (a system in which there is only a single decision maker). The problem of how the nodes coordinate is vital because it is obvious that each node achieves at least the same gains as those achieved without coordination. The literature offers a variety of solutions and mechanisms for coordination, the preferred one being a decentralized solution in which the pay-offs of the nodes are aligned to the system-wide objectives (Chen et al., 2001).

Chen et al. (2001) find the optimal solution for the centralized system, and then show that this solution can be reached also in a decentralized system under the appropriate coordination mechanism. For the analysis, they use a SC with one supplier who distributes a single product to many non-identical retailers, and the latter sell the product to the market. The supplier does not have the opportunity to sell directly to the consumers. The demand is considered a decreasing function of the retailer price and must be satisfied without backlogging. All nodes are aware of all the demand functions and the cost structures which are both stationary. The authors introduce an addition annual cost for the retailers; this reflects a compensation which is paid from the retailer to supplier, because the latter manages each retailer's inventory needs and transactions. In this model the nodes have to determine the price in which they should sell the product and their own replenishment policy. The coordination can be achieved through contracts. The players are considered rational; i.e., a player accepts a contract only if through it he may achieve at least the same gains he would achieve without it. The authors, with counterexamples, show that the QDs are not sufficient to achieve perfect coordination in a model with non-identical retailers, but it is feasible to achieve perfect coordination via periodically charged, fixed fees and a discount pricing scheme which is based on the retailers' annual sales volumes, order quantity, and order frequency. These parameters are included in the contract which the players are signing. The periodical charges and the fixed fees despite the fact that is essential to make the contract attractive for all the nodes, do not affect the total profits or the nodes' policies; i.e., the pricing and the replenishment strategies. These costs are only in order to achieve a proper allocation to the SC, which is necessary to align the nodes' goals to the system-wide objective. Moreover, the authors investigate the value of coordination though a comparison between profits when the players can or fail to coordinate in two cases: identical or non-identical retailers. The results show that the total profits without coordination are around 30 % smaller compared to the SC system with coordination.

Bernstein et al. (2006) extend the idea of SC's coordination among the suppliers and the retailers trying to achieve coordination through simple pricing schemes. As simple pricing schemes, the authors consider either constant unit wholesale prices or specific volume discounts. The authors examine the conditions which should exist to coordinate nodes' decisions and achieve the lowest overall costs, through simple pricing schemes. In some cases, the pricing schemes are sufficient to achieve node coordination. However, this is not true in general; thus, the authors propose a sufficient condition, named "Echelon Operation Autonomy" (EOA), which states that the supplier's function cost must depend only on the supplier's decisions, while the retailer's function cost may depend both on the supplier's and the retailer's decisions. Under this condition, the authors show that the perfect coordination is feasible through simple pricing schemes. They extent the vendor managed inventory (VMI) context to apply the EOA, distinguishing on which node incurs the carrying costs, either the retailers "VMI-", or the supplier "VMI+". However, one of the most serious drawbacks of VMI, the major investments, still remains a problem, but the authors provide an addition incentive for this investment. In case, where the EOA fails to exist, the VMI plays a significant role to the reduction the overall SC's costs. For the analysis, the authors use two cases, the decentralized and the centralized system, and compare results with and without the EOA condition. The authors model the SC as a Bayesian game, regarding the information each node possess, and show (under the necessary conditions) the existence of Nash equilibrium.

Cachon and Kok (2010) study a SC in which two competitive manufacturers sell their products through a single retailer, for example a supermarket which sells products from competitive firms. This SC is appeared many times in practice, but it is quite common in the literature. The manufacturers compete with each other, while leaving constant the competition with the customers (demand). The authors allow manufacturers to offer three types of contracts to the retailer: a wholesale-price contract, a QD contract, and a two-part tariff (i.e., a per unit price with a fixed fee). The authors refer to the latter two as "sophisticated" contracts. These contracts are proposed in literature to achieve coordination in the SC and better pay-offs for the manufacturers. The game which arises has the following structure: first, the manufacturers simultaneously offer a contract to retailer; then, the retailer decides the price of each product, determining this way the product's demand to achieve more personal gains. The authors make the analysis, derive the optimal strategies (contacts) for the nodes and show the existence (and the uniqueness, wherever it is feasible) of Nash equilibrium points, despite the types of products (i.e., substitute, independent, and complements). They solve the problem using backward induction; i.e., they first analyze the retailer's decision and afterwards the game which arises between the two manufacturers. Moreover, the authors examine several numerical scenarios about different values of the system's parameters using a simulation approach. The authors reach the following results, which are similar to the case where there is only one manufacturer: i) the sophisticated contracts increase the total SC's profits, making possible better profit allocation, and ii) in equilibrium, the manufactures choose the most aggressive contract they can from their available set of potential strategies. On the other hand, there are essential differences in the form of a SC with one manufacturer. These are the following: i) if the retailer decides to sell only one product, the manufacturer whose product is selected must leave to the retailer some profit, in order to induce the retailer to make this selection, ii) both the retailer's reservation profit (security level) and the incremental profit (the maximum profit where each manufacturer add to the whole SC) of each manufacturer are endogenously determined by the system, and iii) the sophisticated contracts are not always preferred neither by the manufacturers nor by the retailer; in case where the products are close substitutes, the retailer is better off with the more sophisticated contracts and the manufacturers' incremental profits are decreased.

Corbett et al. (2004) examined three different types of contracts in a two-node SC and addressed the value of information. Ha and Tong (2008) studied two types of contracts and proved that the contact type affects the value of information sharing. Feng and Lu (2013) examined contracts in a chain modeled as a Stackelberg game, where the manufacturer is the leader and the retailer the follower. An earlier but comprehensive review of contracts in SC coordination is provided by Cachon (2003).

As we have mentioned a characteristic problem on SCM is the BE (or whiplash or whipsaw effect). In the literature, there are many papers about BE and the reasons behind it, with landmark be considered the work of Lee et al. (1997) who addresses the problem of the BE in a SC, which refers to the amplification of the variance in the order quantities as one moves from downstream (retailer) to upstream (manufacturer) SC nodes. Many companies such as Procter & Gamble, Hewlett-Packard having recognized the BE and attempted to mitigate its implications. In order to counteract the consequences of the BE, the nodes have to understand its causes. All nodes' decisions, e.g., production scheduling, inventory control, etc., relied on the sales orders which they received. Sales orders do not coincide with customers' demand, because each node makes decisions to maximize his profits. Thus the nodes do not make optimal decisions and lead the SC in excessive operational costs and inefficiencies. The nodes are in need of coordination and planning along the overall SC. One common mechanism for coordination is the information exchange between the nodes. The authors propose mathematical models to identify and remedy the causes of BE and reach on four basic reasons which cause it: i) demand forecast updating, ii) order batching, iii) rationing/shortage gaming, and iv) price variation. The key ideas which should be present in any solution according to the authors are: i) integrating new information systems for the nodes to share all the private information which they posses, ii) new organizational relationships among the nodes, and iii) implementing new incentives and measurement systems.

Cachon (1999) addresses the supplier's demand variability, examines how this variability affects the total SC cost. This work is based on the findings of Lee et al. (1997), who proves that the supplier's demand variance depends on batch ordering. Cachon focuses in a SC with one supplier and N retailers who face stochastic consumer demand, a vital assumption, because the QD cannot aid cost reduction in stochastic demand-based SCs. The retailers can order only at fixed intervals and the order quantity equals to some multiple of a fixed batch size. The author considers that the supplier's demand variance is a function of five parameters: i) consumers demand variability, ii) number of retailers, iii) batch size, iv) retailers' order interval, and v) alignment of the retailers' order. He studies the role of scheduled ordering policies in a SC, and specifically how these policies may lead to a reduction in the supplier's demand variance. Assuming balanced orders; i.e., the same number of retailers place orders at each time period, the author demonstrates that it is feasible to reduce further the supplier's demand variance. This can be achieved through either lengthening the order interval or reducing the batch size, but the retailers' holding and backorder costs while and the ordering costs are increased, respectively. The author proposes a flexible quantity strategy; i.e., lengthening the order interval and at the same time reducing the order batch size to improve the performance of SC and reduce the SC costs. The proposed solution is to adjust these two parameters in order to keep constant the retailer's order frequency and simultaneously reduce the supplier's demand variance. The author uses a probabilistic approach to make the analysis, but also examines several numerical scenarios through a simulation approach, and the following conclusions are reached: i) switching from synchronized; i.e., all retailers order at the same time, to balanced ordering, the holding and the backordering SC's costs are reduced; this effect is intensified when the consumers demand variability gets lower, ii) lengthening the order interval, the supplier's demand variance is reduced but the retailers' holding and backorder costs are increased, and iii) the flexible quantity strategy is effective when the customers' demand is low and few retailers exist, this benefit is particularly effective, if in the level of a supplier a high fill rate is required.

Chen et al. (2000) addresses the BE in a SC, and examines how demand forecasting, one of the four causes of BE according to Lee et al. (1997), is involved in generating it. They demonstrate that the information sharing among nodes for the actual retail demand (common proposed solution) can mitigate the BE but not completely eliminate it. Moreover, the authors attempt to quantify the increase in variability from node to node of the SC. The authors derive a lower bound for the variance of the retailer's order and prove that it is a function of three parameters: the lead time L, the number p of observations (periods) which are used for forecasting the demand, and the correlation parameter r with the previous demand. In order to examine the solution of sharing demand data from the retailer to the whole SC, the authors compare the variability in the case that the retailer provides the demand data to the manufacturer to the case that he does not, under the assumption that the nodes have the same inventory policy and forecasting technique, so that factors do not affect the results. They prove that demand sharing significantly reduces the increase in variability.

One of the best illustration of BE is provided by the (well known)"beer game". Through this, we understand that the consequences of BE are stemming from the rational behaviour of nodes, and the SC's structure. Streman (1989) presents the Beer Game, an experiment, which simulates a SC. Through this experiment, the author examines how the decision makers make decisions in real problems and understands the reason why the players except from their personal gains have to be interested on the total gains. Each node should decide about what quantity is ordered and when under the objective to minimize their own costs. An important factor is the level of information which each node possesses while taking decision(s). Only the retailer knows the real demand; the remaining nodes receive a signal about the actual demand through the orders received from their customers; i.e., from their downstream node. First, the demand of the product is constant, and after few periods doubles, in order to examine how the nodes act in this disturbance. It is considered that the nodes do not have the opportunity to communicate with each other or to coordinate their strategies. In the end of game the players calculate their costs and learn the actual demand. The results identify misperceptions of feedback and shows that the players do not act based on decision theory and a direct result is the poor SC's performance. The total costs and the costs at each node are around ten times larger in contrast to the optimal solution. Moreover, it is shown that there is an amplification of variance of the orders' size and the inventory levels are increased, while moving up to the SC, an indicator for existence of BE.

Chen and Samroengraja (2000) have dealt with the BE through a variant of the typical Beer Game, which they called the "Stationary Beer Game". The key difference in this variant is that the demand within each time period is modeled as an independent, identically distributed variable. All the players are aware of both the distribution of the demand and of its parameters, to better approach reality where companies have some knowledge about the demand (through a forecast). The goal of Stationary Beer Game is to minimize the total cost in the entire SC, despite the fact that the players have access only to local inventory status. The authors prove that the optimal strategy for all the players is to order quantities up to the respective installation stock, thus attempting to keep this parameter to a constant target level and not order qualities equal to the mean value of the distribution. Moreover, they prove that the BE in Stationary Beer Game still exists but it affects the SC to a much lesser extent.

Cachon et al. (2007) attempt to examine the strength of the BE in the US industry and identify the way demand seasonality, and therefore production smoothing, affect order amplification. Seasonality has led into the idea of production smoothing. The authors use data from January 1992 to February 2006 were obtained from U.S. Census Bureau and Bureau of Economic Analysis concerning six retailers, eighteen wholesalers and fifty manufactures to study the existence of the BE and how this evolves. The authors propose two measures to address volatility in a SC: the amplification ratio and the amplification difference, defined respectively as (Var(Production))(Var(Demand)) and Var(Production)$-$Var(Demand). Moreover, they compare demand volatility based on these two measures at the different nodes of the SC (retailers, wholesalers and manufactures). They show that seasonality weakens BE. Furthermore, seasonality leads to more variability at the level of wholesalers rather than in the retailers and the manufactures, a fact inconsistent to the typical bullwhip results. A possible explanation is that both retailers and manufactures proceed in smoothing their production or orders. Note that: i) in the data that the authors used, the seasonality ratio is quite large, ii) almost in all the models, where BE is addressed in the literature, stationary demand is assumed, and iii) the results of this work, with adjusted data to compensate for seasonality, are almost identical with those of previous research presented in the BE literature. Moreover, the authors show that the promotion pricing/cost shocks and the correlation between two successive demands (which makes sense only when the data do not have seasonality) amplify the BE. These factors have been studied with the same results by Lee et al. (1997). Finally, they show that the BE is reduced over time and that the size of firm affects the BE, without to be clear how the latter affects the amplification measures.

Shi and Cai (2009) address the BE and suggest ways to remedy its implications based on a GT framework. The authors develop a GT-based model which, when appropriately implemented, has a positive effect on the BE in terms of sales volume. More specifically, they model the SC as a game in which each node sends a message (signal) to the next upstream node. The signal conveys information about the demand of the node that may be honest or dishonest. Thus, every node is continuously updated about the demand of downstream nodes, while incomplete information is the result of true or false signals, a fact that leads to the modeling via a dynamic game with incomplete information. In this framework, the authors restrict the consequences of the BE through "trigger strategies" of the nodes.

2.6 Open Research Topics

The literature review has revealed many proposed solutions about perfect SC's coordination and how this can be achieved, with the work of Weng (1995) being a landmark one. However, almost all contributions make restrictive or unrealistic assumptions, e.g., requiring contracts or assuming that all the players have the same information (Weng (1995) and Cachon (2003)). Many papers, under the same objective (i.e., to coordinate a SC) existed before Weng's work (1995), the earliest being Harris (1913) about the EOQ model. The ultimate solution mechanism for coordination is a decentralized one in which the pay-offs of the nodes are aligned to the system-wide objectives; i.e., all nodes are free to choose their own strategy but the SC in which they participate has an appropriate structure to enforce all of them to select the strategies which maximize the total SC profits. However, such a scheme is almost impossible in practice since via SC nodes' coordination, each player may earn more (individual) profits because the total SC profits are increased while the whole SC is more efficient. The first problem is to specify who could play the role of the unique decision maker and acts to achieve the optimal for the whole SC and not to optimize him/her indivudual objectives. The latter remains an open research question, as Chen et al. (2001) and Bernstein et al. (2006) state. In addition, recent research underlines the need for further work on SC coordination, seeking models free of restrictive assumptions such as the notion of complete information. Corbett and de Groote (2000) and Cachon and Fisher (2000) extended Weng's model (1995) by introducing private information at each node, and showed how private information affects the nodes' decisions.

Cachon and Fisher's model (2000) is built upon a particularly restrictive assumption: the nodes are required to tell the truth about their private information, even if this is in conflict with the individual profit maximization. Furthermore, a limitation of the Cachon and Fisher's (2000) approach is the lack of a policy to enforce true information sharing towards evident individual benefits for all nodes involved in the SC; such a policy would enable all nodes to clearly identify their own benefits and to buy-in global inventory control strategies. In Corbett and de Groote's model (2000), the basic limitation is the alternatives which the players have when they do not accept the discount framework. These alternatives introduce games and potential profits outside the one-to-one relationship between the original nodes, thus shaping coordination mechanisms non-feasible to achieve in every case. Moreover, Corbett and de Groote (2000) study a SC with only two nodes, without extending their results to larger SCs, a topic we plan to address in our work.

Chen et al. (2001) and Cachon and Kok (2010) proposed a solution for SC node coordination through the signing of a contract by all nodes. We underline this assumption, because the players after the signing of a contract are bound by the terms of the contract, a very restrictive mechanism not elegant to enforce in practice. The use of contracts to achieve SC coordination is also a restrictive assumption. Contracts are in principle binding nodes but quite often are broken or non-fully respected in practice due to dynamic realities or changing conditions of the market.

Chen (2005) addressed moral hazard and adverse selection incentives in a SC where the nodes have both private information and private decisions unobservable to the other nodes. The limitations of this work are: i) the author does not use the RP to determine Nash equilibrium for the nodes, and ii) the research is constrained to a simple SC with only two nodes.

In Sterman's work (1989) addressing the Beer game, and in Chen's and Samroengraja's work (2000) addressing the Stationary Beer game, the nodes do not have the opportunity to communicate with each other. However, as some of the latest work on the field supports; e.g., VMI and CPFR (Collaborative Planning, Forecasting and Replenishment) model, communication between the nodes before they chose their strategies may be a huge step towards effective SC coordination. This could also be demonstrated when playing the two versions of the Beer Game; i.e., allowing inter-player communication and observing its consequences on the BE.

Lee et al. (1997) and Chen et al. (2000) studied the causes of the BE, both reaching the conclusion that information exchange between the nodes can remedy the BE. On the other hand, they recognized as an open research question the incentives that should be provided to the SC nodes in order for them to allow access to their inventory status and sales data. It is obvious that a node does not easily allow access to his private information by other nodes. Cachon (1999) extended Lee et al. (1997) studying a specific cause of BE (the scheduled ordering) without allowing communication among the nodes to achieve a better global SC performance. Moreover, Cachon et al. (2007) revisited the BE stating the same restrictions and limitations as in Cachon (1999). Finally, one may say that providing the opportunity to the nodes even to lie about their private information (if such a policy could achieve more individual gains for the node that used it) would constitute an great addition to any proposed model for SC coordination, especially if this is coupled with mechanisms to impose adverse selection implementation.

In summary, the research literature has recognized the value of coordination as means to increase profits in a SC. However, it has also pointed out several difficulties in providing the appropriate incentives to individual agents in order to coordinate their decisions, as well as in ensuring that coordination agreements will be kept. These difficulties mainly arise from the fact that coordinating decisions usually do not coincide with equilibrium strategies in a competitive setting. In this dissertation we propose to develop a rigorous mathematical model that will incorporate the adoption of coordination strategies in a more general game theoretic setting. Specifically:

- Introduction of a mediator in the SC coordination, who receives information and provides directions to SC node decision makers for their actions.
- Incorporation in the models of SC coordination of the nodes' incomplete-private information, as well as allowance of no true-telling (non-passing the correct information to the mediator).
- Application of the results in SCs that are not limited to the typical two node scheme of the literature to-date.
- Disengagement of the approach from restrictive ways of coordination; examples of such restrictive methods are contracts signed before the evolution of the SC, or VMI-schemes often encountered in SCs. An example of non-restrictive way of coordination is communication via the mediator, the approach adopted in this thesis.
- Development of adverse selection and moral hazard incentives models in the context of SC under the RP; this way, SC nodes are free to choose their strategies for maximizing individual profits, while the overall SC operates in more effective levels due to the application of these incentives.

Chapter 3

Supply Chain Coordination under Discrete Information Asymmetries and Quantity Discounts

As already mentioned in Chapter 2, a large body of literature addresses the way in which SC nodes interact to reduce both their own costs and the overall SC cost. SCs involve nodes (players) acting as suppliers, manufacturers, buyers, retailers and customers, who communicate via orders and deliveries (Goyal and Gupta, 1989). The different preferences of the players in regard to the level of orders placed, may lead to an increase in the overall inventory-related cost of the SC. Buyers opt for small orders, in contrast to suppliers who favour large shipments; the latter results in an increase of the annual inventory holding cost but a simultaneous decrease of the annual ordering cost for the buyers and of the annual set-up cost for the suppliers occurs (Monahan, 1984). If the nodes could coordinate their actions, it is evident that they could reduce the global SC costs (Rosenblatt and Lee, 1985). There exist multiple papers addressing SC coordination, a comprehensive review of which is provided by Cachon (2003).

In a typical game-theoretic view of the relationship between suppliers and buyers, each player acts to maximize its own profits without taking into account the global optimal and without entering a coalition. Thus, decentralized solutions are promoted; among them, the most preferable ones are those in which the pay-offs of the players are aligned with system-wide objectives (Chen et al., 2001).

The supplier may seek chain coordination if in this case he achieves higher individual gains. Therefore, he offers an incentive to the retailer to influence the quantity the latter orders. Such an incentive is a QD; i.e., reduced per unit product price when larger orders are placed. A survey of QD schemes has been performed by (Benton and Park, 1996).

We adopt QDs as the means for node coordination, since they are widely used in practice (Mansini et al., 2012), can be easily implemented, and require no additional information or physical flow between the two players beyond the initial transaction (Burnetas et al., 2007), in contrast to other coordination mechanisms (e.g. returns policies, back-up agreements, quantity flexibility, etc.). Many firms, such as H. J. Heinz Company use QDs to reduce their own costs (Altintas et al., 2008). Economies of scale are achieved through QDs, yielding higher profits for several or even all the players, while allowing each of them to make its own decisions (Cachon and Terwiesch, 2006).

In this chapter, we study a two-node SC through which a single product is manufactured and forwarded to the market. We assume that both the retail price and the demand are constant and exogenously defined, a common assumption in the literature (Corbett, 2001). Our goal is to examine node coordination and the resulting players' benefits, in terms of operational costs. The retailer has an ordering and a holding cost and needs to decide on the order quantity (lot size) to place to the supplier, satisfying demand and minimizing his own cost. The supplier produces under a lot for lot policy; i.e., quantities equal to the retailer's orders. There exists a set-up cost for the supplier; thus he prefers large order quantities from the retailer. To force the retailer's orders to a higher level and achieve larger profits, the supplier uses QDs.

Similar SC have been studied by Corbett and de Groote (2000) who considered a continuously distributed holding cost for the retailer, and Ha (2001) for the case of an expanded newsvendor model. Our framework differs from the aforementioned ones in two ways: i) we consider reservation levels that depend upon the retailer's private knowledge, in contrast to previous works, where the reservation levels are exogenous, and ii) we assume discrete asymmetric information; i.e., two possible values for the retailer's holding cost. In practice both our assumptions are more realistic: reservation levels depend upon business relationships that are indeed affected by information that partners keep for themselves. Furthermore, continuous asymmetries are not very realistic in applications compared to discrete asymmetries (Lovejoy, 2006). For example, a retailer importing goods from a manufacturer may store inventory at privately owned warehouses (low cost) or at the customs location (high cost) - the latter in case duty is paid only when the product is delivered to the end customer. This discrete treatment of the holding cost's values leads to a different solution approach compared to the one proposed by Corbett and de Groote (2000), thus justifying our research endeavour.

Finally, it is worth noting that a discrete treatment of information asymmetry has been proposed by Cakanyildirim et al. (2012). The first study considered a supplier-manufacturer chain, in which the manufacturer has private information about production cost. The second study addressed a SC similar to our model, but employed a reverse information asymmetry; i.e., production costs at the supplier level take two potential values. In both cases, the authors derived closed form solutions of the underlying optimization problems and proved that even with asymmetry of information perfect coordination is feasible.

The contribution of our work lies in the analytical derivation of QDs offered by a manufacturer to a retailer that enable the establishment of the business relationship and allow reduced operational costs for both players, without the existence of bilateral contracts and under discrete information asymmetry emanating from the retailers' storage options.

The remainder of the chapter is organized as follows: Section 3.1 provides the mathematical model for a two-node SC and the GT perspective of the players' interaction via orders and discounts. Section 3.2 develops the analytical solution of the game, proving the joint EOQ result for the case of complete information and devising exact values for orders and discounts based on global optimization for the case of asymmetric information. Section 3.3 provides numerical results for sample data sets concerning inventory holding cost and set-up cost relationships, offering insights on the effect of the various parameters and providing sensitivity analysis for performance evaluation. Section 3.4 summarizes the conclusions of our work.

3.1 Model Description

Let us consider a two node SC, with S denoting the supplier or the manufacturer (referred to as he) and R denoting the retailer or the buyer (she), interacting via orders for a single product. The market demand D is constant, exogenously defined, and known to both parties. Shortages or backorders are not allowed. Both players are rational and risk neutral; hence, they choose their strategies to minimize their own expected cost function.

The retailer has an ordering and a holding cost denoted by K_R and H_R, respectively, and decides on the order quantity $Q > 0$ that she will place to the supplier, satisfying demand and minimizing her own cost. The retailer's cost is a function of her order quantity Q and can be expressed as $C_R(Q) = K_R D/Q + H_R Q/2$. There exists a set-up cost in the production phase, included in the supplier cost function and denoted by K_S. The supplier produces under a lot for lot policy; i.e., a quantity equal to the retailer's order Q. As a result, the supplier is not a decision maker and his cost is a function of the retailer's order quantity, expressed as $C_S(Q) = K_S D/Q$ and not influenced by any of his potential actions. It is obvious that if the supplier could decide about the order quantity he would favour huge quantities because in this way he would reduce his own total costs (the supplier's cost function is a decreasing function of the order quantity Q). Consequently, the total SC (or joint) cost can be expressed as $C_J(Q)$ and is equal to the sum of the retailer's and supplier's cost, i.e.:

$$C_J(Q) = C_S(Q) + C_R(Q) = (K_R + K_S)D/Q + H_R Q/2 \tag{3.1.1}$$

The retailer selects the order quantity to minimize her own cost function. The optimal value can be directly derived by taking the first order derivative of the cost function, setting it equal to zero and solving with respect to Q, giving $Q_R^* = \sqrt{2K_R D/H_R}$. This results in the following costs:

Retailer's cost: $C_R(Q_R^*) = K_R D/Q_R^* + H_R Q_R^*/2 = \sqrt{2K_R D H_R}$.
Supplier's cost: $C_S(Q_R^*) = K_S D/Q_R^* = K_S\sqrt{DH_R/2K_R}$.
Joint cost: $C_J(Q_R^*) = C_S(Q_R^*) + C_R(Q_R^*) = (2K_R + K_S)\sqrt{DH_R/2K_R}$.

Note that, the optimal cost for the whole SC is the minimum of the function $C_J(Q)$. This is achieved when the order quantity is $Q_J^* = \sqrt{2(K_R + K_S)D/H_R}$ and we observe that $Q_J^* > Q_R^*$. For the overall SC costs the following inequality holds:

$$C_J(Q_J^*) < C_J(Q_R^*) \tag{3.1.2}$$

Thus, a higher than Q_R^* order quantity is preferable to reduce the total costs. However, this is reached at the expense of increased retailer's cost, rendering her negative to a potential cooperation. Therefore, to raise the retailer's order level (preferred case for the supplier) and achieve reduced costs, the supplier should offer her an incentive when selecting the order quantity. We allow the supplier to provide QDs to the retailer, in order to affect the order quantity placed by the latter. As already mentioned, the supplier is rational and risk neutral so she acts to minimize her own expected cost. When the supplier provides a QD $P(Q)$, the players' cost functions become:

$$C_S^T(P(\cdot), Q) = K_S D/Q + P(Q) \tag{3.1.3}$$

$$C_R^T(P(\cdot), Q) = K_R D/Q + H_R Q/2 - P(Q) \tag{3.1.4}$$

$$C_J^T(P(\cdot), Q) = C_S^T(P(\cdot), Q) + C_R^T(P(\cdot), Q) = C_S(Q) + C_R(Q) = C_J(Q) \tag{3.1.5}$$

We observe that, the joint cost remains the same, because the discount $P(Q)$ affects only the allocation of the cost among the two nodes via the increase in the retailer's order levels.

The problem is formulated as a Stackelberg game, a common way of modeling node interaction in SC (Chen et al., 2012). The supplier is the leader and sets the QD and the retailer is the follower and selects the order quantity. Let $Q^*_R(P(\cdot))$ be the retailer's optimal response function; the supplier's problem then is:

$$C^*_{S,CI} = \min_{P(\cdot)}\{C^T_S(P(\cdot)), Q^*_R(P(\cdot))\} \tag{3.1.6}$$

Moreover, we consider the case where H_R is known to the retailer but partially known to the supplier. Assume that the supplier knows that H_R is either h_L or h_H (a common way to model information asymmetry - see Desiraju and Moorthy (1997)) and has a prior probability assessment; i.e., the supplier considers H_R as a discrete random variable such that $P(H_R = h_L) = p = 1 - P(H_R = h_H)$. We adopt a Bayesian game approach (Gibbons, 1992). The supplier selects a discount to minimize his expected cost under the prior distribution, assuming that the retailer will respond optimally and taking into account the true value of the holding cost known to her. Thus, in the problem with asymmetric information the formulation of the retailer's response is a function of the true holding cost value ($Q^*_R(P(\cdot); H_R)$) and the supplier's problem becomes:

$$C^*_{S,AI}(h_L, h_H, p) = \min_{P(\cdot)}\{pC^T_S(P(\cdot), Q^*_R(P(\cdot); h_L)) + (1-p)C^T_S(P(\cdot), Q^*_R(P(\cdot); h_H))\} \tag{3.1.7}$$

3.2 Analytical Solution

In this section, we develop the analytical solution of the game, devising exact values for the decision variables (i.e., order quantity for the retailer and QD for the supplier) in two cases: the complete and the asymmetric information. In the second case, the retailer has private information and the supplier uses a QD mechanism in order to reduce his individual costs. Moreover, the discount can act as a screening device to induce the retailer to reveal her private information (Kolay et al., 2004), while the cost for both players is reduced.

3.2.1 The Case of Complete Information (CI)

In this case, we assume that all the parameters of the game: D, K_R, K_S and H_R are constant and known by both players (complete information). Given this knowledge and according to the design mechanism theory (Fudenberg and Tirole (1991), p. 254-256) it is sufficient to consider discount policies with a quantity-price pair (X, Y), such that $P(Q) = Y$ if $Q = X$ and zero otherwise. This means that the discount is valid if and only if the order quantity is equal to X (and not when $Q > X$). Obviously, the supplier will only consider pairs such that $X \geq Q^*_R$.

Due to the Stackelberg game formulation, first we have to determine the retailer's strategy. The retailer can achieve as minimum cost to be $C_R(Q^*_R)$ without any QDs. Therefore, she cannot accept any solution in which her cost exceeds $C_R(Q^*_R)$. Hence,

$C_R(Q_R^*)$ is the reservation (or security/safety) level for the retailer, typically denoted by C_R^+. The retailer has two options: order quantity Q_R^* (optimal order quantity without receiving any discount) or order quantity X to ensure the discount Y. The retailer's choice is based on her cost function since she is a rational player; thus, in order to select the order quantity X a pay-off at least C_R^+ should accrue for her; i.e., $C_R^T(\cdot) \leq C_R^+$. This means that the retailer accepts the discount only if $Y \geq Y_0(X)$, where:

$$Y_0(X) = K_R D/X + H_R X/2 - \sqrt{2K_R D H_R}.$$

Note that, $Y_0(Q_R^*) = 0$ as expected. Thus, the retailer's optimal order quantity becomes:

$$Q = \begin{cases} Q_R^*, & \text{if } Y < Y_0(X) \\ X, & \text{if } Y \geq Y_0(X). \end{cases}$$

Subsequently, we should determine the supplier's strategy for minimizing his own cost given the retailer's strategy. We distinguish two cases according to the retailer's selection, because the latter is affected by the value of the discount:

Case CI(I): $Y \geq Y_0(X)$, thus the retailer's order is equal to X. The supplier has to minimize his cost given by equation (3.1.6), i.e.

$$\begin{aligned} C_{S,CI}^* &= \min_{X,Y} C_S^T((X,Y),Q) = \min_{X,Y}\{K_S D/X + Y\} = \min_X\{\min_Y\{K_S D/X + Y\}\} \\ &= \min_X\{K_S D/X + Y^o\} = \min_X\{K_S D/X + K_R D/X + H_R X/2 - \sqrt{2K_R D H_R}\} \end{aligned} \tag{3.2.1}$$

The optimal value can be directly derived by taking the first order derivative of the cost function, setting it equal to zero and solving with respect to x^*, giving:

$$X^* = \sqrt{2(K_R + K_S)D/H_R} = Q_J^* \text{ and } Y^* = K_R D/X^* + H_R X^*/2 - \sqrt{2K_R D H_R}.$$

Case CI(II): $Y < Y_0(X)$, thus the retailer's order is equal to Q_R^*. The retailer does not accept the QD leaving the supplier with a cost of C_S^+. So, the supplier has to select the lower cost between $C_{S,CI}^*$ and C_S^+, for which the following holds:

$$C_{S,CI}^* = \sqrt{2DH_R}(\sqrt{K_S + K_R} - \sqrt{K_R}) < C_S^+ \tag{3.2.2}$$

Therefore, the supplier minimizes his own costs by choosing to provide the retailer with the discount policy (Q_J^*, Y^*) to alter the retailer's order quantity. Note that, in this case it is feasible to achieve perfect coordination for the supply chain via the discount policy (Q_J^*, Y^*). To minimize his costs the supplier provides the discount $Y^* = Y_0(Q_J^*)$ if the order quantity is Q_J^*. The result is that the retailer selects the discount to minimize her own costs as well. This means that the individual incentives of the players are aligned with the incentives of the whole supply chain. Thus, we have a decentralized solution (each player is a decision maker), in which the player's cost functions are aligned to the system-wide objectives. The order quantity is Q_J^*, with the retailer keeping her costs equal to her reservation level C_R^+, while the supplier's costs are: $C_{S,CI}^* = C_J(Q_J^*) - C_R^+$. This means that the supplier capitalizes on his knowledge of the retailer's data and secures all the gains from coordination for himself by paying the supplier just enough to induce her to order the higher quantity Q_J^* (Corbett, 2001).

3.2.2 The Case of Asymmetric Information (AI)

In this case, we still assume that the parameters: D, K_R and K_S are constant and known by the two players. However, for the retailer's holding cost H_R, we assume that there exist two possible values: a low one h_L, which occurs with probability p, and a high one h_H ($h_H > h_L$), which occurs with probability $1-p$. In terms of the Bayesian formulation, we refer to these two holding cost values as retailer of type-L and type-H, respectively. We also assume that the retailer identifies her true holding cost as soon as the game starts, while the supplier considers that type-L (h_L) occurs with probability p and type-H (h_H) with probability $1-p$.

When the retailer is type-L, her cost function and reservation level can be written as $C_{R,L}(Q) = K_R D/Q + h_L Q/2$ and $C_{R,L}^+ = \sqrt{2K_R D h_L}$, and when she is type-H as $C_{R,H}(Q) = K_R D/Q + h_H Q/2$ and $C_{R,H}^+ = \sqrt{2K_R D h_H}$. The retailer's reservation levels are reached when she selects the order quantity to minimize her own cost function and the supplier does not provide any discount. Consequently, the retailer's order quantity is $Q_{R,L}^* = \sqrt{2K_R D/h_L}$ for the type-L retailer, and $Q_{R,H}^* = \sqrt{2K_R D/h_H}$ for the type-H. The supplier is not aware of the retailer's actual holding cost and he assumes a two-value distribution for it. Since the supplier is rational and risk neutral, his goal is to provide a discount policy to the retailer in order to minimize his own expected cost accounting for the holding cost distribution.

The analysis of a Bayesian game is based on the RP according to which it is sufficient to consider discount mechanisms or policies such that: i) the supplier sets one pair (X, Y) of order quantity and discount amount respectively for each retailer type, and ii) the values of (X_L, Y_L) and (X_H, Y_H) are such that it is optimal for the retailer to select the option (X_L, Y_L) if she is type-L and (X_H, Y_H) if she is type-H. Therefore, the supplier should design a mechanism under which the retailer will act according to her actual type. Without loss of generality, we can restrict the set of supplier's strategies to those that satisfy the RP and have attractive properties (Myerson, 1979).

Note that, in contrast to the case in section 3.2.1, the retailer now has three options concerning her order quantity: i) to order without discounts, ii) to order according to her type e.g., if she is type-L to order a quantity equal to X_L and receive the discount of Y_L, and iii) to order a quantity that would allow her to get the discount he would if she was of the other type. The retailer selects the option that minimizes her own expected cost (rational and risk neutral player).

If the supplier anticipates the retailer to select the order quantity according to her actual type, he should include the appropriate incentives in the QD mechanism. Specifically, in order for the retailer to prefer ordering the quantity with the discount instead of her own optimal solution without discount, the following holds:

$$C_{R,L}(X_L) - Y_L \leq C^{+}_{R,L} \tag{3.2.3}$$

$$C_{R,H}(X_H) - Y_H \leq C^{+}_{R,H} \tag{3.2.4}$$

Moreover, to ensure that the retailer selects the discount pair designed for her real type, the two inequalities below should be enforced:

$$C_{R,L}(X_L) - Y_L \leq C_{R,L}(X_H) - Y_H \tag{3.2.5}$$

$$C_{R,H}(X_H) - Y_H \leq C_{R,H}(X_L) - Y_L \tag{3.2.6}$$

Inequalities (3.2.3) - (3.2.6) become constraints in the supplier's optimization problem (RP, Myerson (1979)). In the literature of mechanism design, inequalities (3.2.3) and (3.2.4) are referred as individual rationality (IR) or participation constraints, while inequalities (3.2.5) and (3.2.6) are known as IC constraints (Fudenberg and Tirole, 1991). So, the supplier solves the following optimization problem:

(S.1)

$$C^{*}_{S,AI} = \min_{X_L, Y_L, X_H, Y_H} \{p\{C_S(X_L) - Y_L\} + (1-p)\{C_S(X_H) - Y_H\}\}$$

$$\text{s.t.} \quad \begin{aligned} Y_L &\geq C_{R,L}(X_L) - C^{+}_{R,L} \\ Y_H &\geq C_{R,H}(X_H) - C^{+}_{R,H} \\ Y_L - Y_H &\geq C_{R,L}(X_L) - C_{R,L}(X_H) \\ Y_L - Y_H &\leq C_{R,H}(X_L) - C_{R,H}(X_H) \end{aligned}$$

As mentioned before, the supplier prefers larger orders because they reduce his costs; this preference is reflected in Proposition 3.2.1 below that provides properties for order quantities X_L and X_H, for which the supplier offers a discount policy.

Proposition 3.2.1 *In any optimal solution of the supplier's expected cost, we have:*

A) $X_L \geq Q^{*}_{R,L}$ *and* $X_H \geq Q^{*}_{R,H}$, *B)* $X_L \geq X_H$.

Proof:

A. We prove the first part of the proposition via contradiction. Let (X_L, Y_L) be a quantity-price pair with $X_L < Q^*_{R,L} = \sqrt{2K_R D/h_L}$. Since $C_{R,L}(X)$ is a convex function minimized at $Q^*_{R,L}$, there exists $X^{'}_L > Q^*_{R,L}$ such that $C_{R,L}(X^{'}) = C_{R,L}(X)$. Furthermore, $C_S(X^{'}) < C_S(X)$. Therefore, the pair $(X^{'}_L, Y_L)$ is a feasible solution of problem (S.1) and has a lower objective value than that of (X_L, Y_L); thus, (X_L, Y_L) cannot be optimal. Similarly, we can prove that $X_H \geq Q^*_{R,H} = \sqrt{2K_R D/h_H}$.

B. From the IC constraints (3.2.5) and (3.2.6) we can derive the following inequality: $C_{R,L}(X_L) - C_{R,L}(X_H) \leq Y_L - Y_H \leq C_{R,H}(X_L) - C_{R,H}(X_H)$. After some algebra it follows that X_L and X_H must satisfy $(h_H - h_L)(X_L - X_H)/2 \geq 0$, from which $X_L \geq X_H$. $\diamondsuit$

We analyze problem (S.1) in two stages. Specifically:

$$C^*_{S,AI} = \min_{X_L, X_H} \{pC_S(X_L) + (1-p)C_S(X_H) + \tilde{f}(X_L, X_H)\} \tag{3.2.7}$$

$$\text{s.t.} \quad \begin{aligned} X_L &\geq X_H \\ X_L &\geq Q^*_{R,L} \\ X_H &\geq Q^*_{R,H} \end{aligned}$$

where $\tilde{f}(X_L, X_H)$ is the subproblem of optimal discounts given quantities (X_L, X_H):

$$\tilde{f}(X_L, X_H) = \min_{Y_L, Y_H} \{pY_L + (1-p)Y_H\} \tag{3.2.8}$$

$$\text{s.t.} \quad \begin{aligned} Y_L &\geq C_{R,L}(X_L) - C^+_{R,L} && (IR_L) \\ Y_H &\geq C_{R,H}(X_H) - C^+_{R,H} && (IR_H) \\ Y_L - Y_H &\geq C_{R,L}(X_L) - C_{R,L}(X_H) && (IC_L) \\ Y_L - Y_H &\leq C_{R,H}(X_L) - C_{R,H}(X_H) && (IC_H) \end{aligned}$$

The IR constraints reflect the fact that the retailer has the option to select the order quantity which is optimal without discount ($Q^*_{R,L}$ or $Q^*_{R,H}$, according to her actual type), if through such a policy he achieves lower costs. That means that the retailer cannot accept any solution in which her cost exceeds $C^+_{R,L}$ or $C^+_{R,H}$, when she is type-L or type-H, respectively. The IC constraints ensure that the retailer incurs the lowest possible cost when selecting the discount pair designed for her actual type. In other words, the retailer cannot achieve better individual gains if she behaves like to be the other type. For brevity, we use $a_L = C_{R,L}(X_L) - C^+_{R,L}$, $a_H = C_{R,H}(X_H) - C^+_{R,H}$, $b_L = C_{R,L}(X_L) - C_{R,L}(X_H)$ and

$b_H = C_{R,H}(X_L) - C_{R,H}(X_H)$, without explicitly noting the dependence of (X_L, X_H). From the two IC constraints, (3.2.5) and (3.2.6), we have the inequality: $b_L \leq Y_L - Y_H \leq b_H$. If $b_L > b_H$ the solution space of the optimization problem (S.1) is empty; thus the interesting case is to assume that $b_L \leq b_H$. Note that, $\tilde{f}(X_L, X_H)$ is a Linear Programming problem. It is obvious that the feasible region is unbounded and can take one of the three forms depicted in Figure 5.1, depending on whether point $A = (a_L, a_H)$ lies inside, above or below the feasible region, respectively.

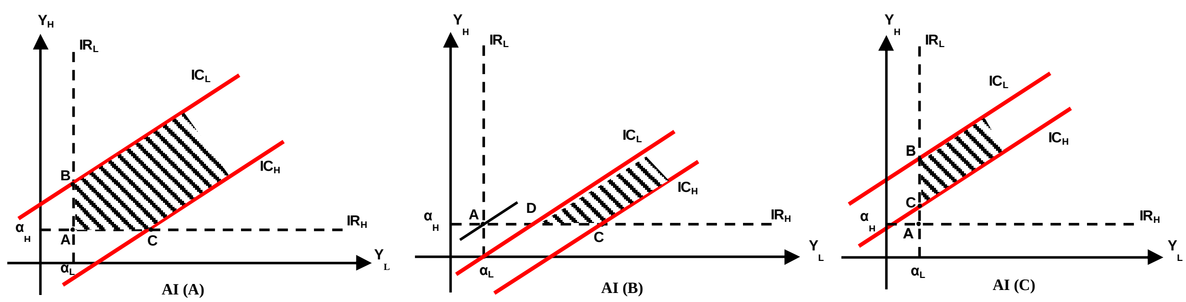

Figure 3.1: The three cases of $\tilde{f}(X_L, X_H)$'s feasible region

For each case in Figure 5.1, we identify the set of values of (X_L, X_H) so that the case holds and we derive the corresponding optimal solution for subproblem (3.2.8).

Case AI(A): Point A lies inside the feasible region, thus $b_L \leq a_L - a_H \leq b_H$. After some algebra, this reduces to $X_H \leq T \leq X_L$, where $T = \frac{2\sqrt{2K_R D}}{\sqrt{h_H}+\sqrt{h_L}}$. Note that, $Q^*_{R,H} \leq T \leq Q^*_{R,L}$. Therefore, taking into account the constraints on X_L and X_H in the first stage problem, we conclude that this case arises when $Q^*_{R,H} \leq X_H \leq T$ and $X_L \geq Q^*_{R,L}$. Regarding the optimal solution, it is easily shown that since $0 \leq p \leq 1$ the function is minimized at extreme point A of the feasible region; i.e., $Y_L = a_L$ and $Y_H = a_H$ and $\tilde{f}(X_L, X_H) = pa_L + (1-p)a_H$.

Case AI(B): Point A lies above the feasible region, thus $a_L - a_H < b_L$. After some algebra, we obtain $X_H > T$ and $X_L \geq Q^*_{R,L}$ and it is obvious that the optimal solution corresponds to the extreme point $D = (Y_L, Y_H) = (b_L + a_H, a_H)$, with $\tilde{f}(X_L, X_H) = p(b_L + a_H) + (1-p)a_H$.

Case AI(C): Point A lies below the feasible region, thus $a_L - a_H > b_H$. This reduces to $X_L < T$. From Proposition 3.2.1, X_L must satisfy: $X_L \geq Q^*_{R,L}$. Since $Q^*_{R,H} \leq T \leq Q^*_{R,L}$ this case is not feasible.

Note that, in case AI(A) the supplier always provides the retailer with the appropriate discount to lead her to the reservation level. Consequently, the supplier ensures that he keeps all gains from coordination. In case AI(B), the supplier leads (through the discount) the retailer to the latter's reservation level only if the retailer is type-H. For type-L, the retailer achieves some gains from coordination. This happens due to the fact that the retailer has private information and the supplier should pay information rent in order to acquire the retailer's knowledge.

Given the solution to subproblem (3.2.8) we can proceed in solving the primary problem (S.1). We have to study two cases according to the two solutions of subproblem (3.2.8). Note that, in both cases we have $Q^*_{R,L} \leq X_L$ and the difference between the two cases is the constraint for quantity X_H. Therefore, the objective function of problem (S.1) can be written as:

$$F(X_L, X_H) =$$

$$\begin{cases} p[C_S(X_L) + C_{R,L}(X_L) - C^+_{R,L}] + (1-p)[C_S(X_H) + C_{R,H}(X_H) - C^+_{R,H}], Q^*_{R,H} \leq X_H \leq T, \\ p[C_S(X_L) + C_{R,L}(X_L) - C_{R,L}(X_H)] + (1-p)C_S(X_H) + C_{R,H}(X_H) - C^+_{R,H}, T < X_H \leq X_L. \end{cases}$$

We approach the problem for each branch of the objective function. Let $Q^*_{J,L} = \sqrt{2(K_R + K_S)D/h_L}$, $Q^*_{J,H} = \sqrt{2(K_R + K_S)D/h_H}$, $f_1 = \sqrt{1 + \frac{K_S}{K_R}}(1 + \sqrt{\frac{h_L}{h_H}})$, $f_2 = \sqrt{1-p}\sqrt{1 + \frac{K_S}{K_R}\frac{\sqrt{h_L}+\sqrt{h_H}}{\sqrt{h_H - p h_L}}}$ and $W = \sqrt{\frac{2(1-p)(K_R+K_S)D}{h_H - p h_L}}$. Proposition 3.2.2 provides the minimum of function $F(X_L, X_H)$, obtaining the optimal quantity for which the supplier offers a discount.

Proposition 3.2.2 *The minimum of function $F(X_L, X_H)$ is achieved at:*

$$(X_L, X_H) = \begin{cases} (Q^*_{J,L}, Q^*_{J,H}), f_1 \leq 2 \\ (Q^*_{J,L}, T), f_1 > 2, \end{cases} \quad \textit{for case AI(A)},$$

$$(X_L, X_H) = \begin{cases} (Q^*_{J,L}, T), f_2 \leq 2 \\ (Q^*_{J,L}, W), f_2 > 2, \end{cases} \quad \textit{for case AI(B)}.$$

Proof:

First we solve the unconstrained problem with the objective function formed for case AI(A); i.e., $Q^*_{R,H} \leq X_H \leq T$. The unconstrained minimum is $X_L = Q^*_{J,L}$ and $X_H = Q^*_{J,H}$. To check when this is feasible, note that $Q^*_{J,L} > Q^*_{R,L}$. Furthermore, since the objective function is convex in X_H it is minimized either at $X_H = Q^*_{J,H}$, when $Q^*_{R,H} \leq Q^*_{J,H} \leq T$, or at one of the endpoints $Q^*_{R,H}$ or T. It is always true that $Q^*_{J,H} \geq Q^*_{R,H}$. On the other hand, it follows that $Q^*_{J,H} \leq T$ when $f_1 \leq 2$. Combining the above, we conclude that the optimal solution on the upper branch is achieved at:

$$(X_L, X_H) = \begin{cases} (Q^*_{J,L}, Q^*_{J,H}), f_1 \leq 2 \\ (Q^*_{J,L}, T), f_1 > 2. \end{cases}$$

Similarly, we solve the second branch of the objective function (case AI(B)) corresponding to $T < X_H \leq X_L$, in which the unconstrained problem has the optimal solution $X_L = Q^*_{J,L}$ and $X_H = W$. Since the objective function is convex in X_H, it is minimized either at $X_H = W$ or at one of the endpoints T or $Q^*_{J,L}$. It is easily proved that:

$$\frac{W}{Q^*_{J,L}} = \frac{\sqrt{(1-p)h_L}}{\sqrt{h_H - ph_L}} < 1.$$

Therefore, $Q^*_{J,L} \geq W$. On the other hand, after some algebra we get that $W > T$ when $f_2 > 2$. Thus, the optimal solution of the constrained problem on the lower branch is achieved at:

$$(X_L, X_H) = \begin{cases} (Q^*_{J,L}, T), f_2 \leq 2 \\ (Q^*_{J,L}, W), f_2 > 2. \quad \diamondsuit \end{cases}$$

Before we proceed to the deviation of the supplier's optimal policy, we observe that:

$$\frac{f_2}{f_1} = \frac{\sqrt{1-p}}{\sqrt{1-p\frac{h_L}{h_H}}}.$$

Since $h_L < h_H$, it follows that: $\frac{f_2}{f_1} < 1$.

Theorem 3.2.1 *The supplier's optimal policy is:*

i) *if* $f_2 < f_1 \leq 2$, *then* $X^*_L = Q^*_{J,L}, Y^*_L = C_{R,L}(Q^*_{J,L}) - C^+_{R,L}$, $X^*_H = Q^*_{J,H}, Y^*_H = C_{R,H}(Q^*_{J,H}) - C^+_{R,H}$,

ii) *if* $f_2 \leq 2 < f_1$, *then* $X^*_L = Q^*_{J,L}, Y^*_L = C_{R,L}(Q^*_{J,L}) - C^+_{R,L}$, $X^*_H = T, Y^*_H = C_{R,H}(T) - C^+_{R,H}$,

iii) *if* $2 < f_2 < f_1$, *then* $X^*_L = Q^*_{J,L}, Y^*_L = C_{R,L}(Q^*_{J,L}) - C_{R,L}(W) + C_{R,H}(W) - C^+_{R,H}$, $X^*_H = W, Y^*_H = C_{R,H}(W) - C^+_{R,H}$.

Proof:

The objective function $F(X_L, X_H)$ is convex in X_H for every X_L, in both branches. We consider three cases regarding the ordering of f_1, f_2.

Case AI(I): $f_2 < f_1 \leq 2$.

In this case, the minimum is achieved in the upper branch. Thus, when $X_H < T$, the optimal solution is:

$(X^*_L, X^*_H) = (Q^*_{J,L}, Q^*_{J,H})$ with

$(Y^*_L, Y^*_H) = (C_{R,L}(Q^*_{J,L}) - C^+_{R,L}, C_{R,H}(Q^*_{J,H}) - C^+_{R,H})$.

Case AI(II): $f_2 \leq 2 < f_1$.

The minimum is achieved at $X_H = T$, so we have:
$(X_L^*, X_H^*) = (Q_{J,L}^*, T)$ with
$(Y_L^*, Y_H^*) = (C_{R,L}(Q_{J,L}^*) - C_{R,L}^+, C_{R,H}(T) - C_{R,H}^+)$.

Case AI(III): $2 < f_2 < f_1$.

In this case, the minimum is achieved in the lower branch. Thus, when $X_H > T$, the solution is:
$(X_L^*, X_H^*) = (Q_{J,L}^*, W)$ with
$(Y_L^*, Y_H^*) = (C_{R,L}(Q_{J,L}^*) - C_{R,L}(W) + C_{R,H}(W) - C_{R,H}^+, C_{R,H}(W) - C_{R,H}^+)$. ◊

From the expressions of the optimal solution in Theorem 3.2.1 we can make the following observations:

1. Perfect coordination is achieved if the retailer's holding cost is low ($X_L = Q_{J,L}^*$).
2. For case AI(I), we have shown that it is attainable to achieve perfect coordination for the supply chain regardless of the retailer's type, under some specific values of the model's parameters ($f_1 \leq 2$). This happens because it is optimal for the supplier to provide a discount at quantities $Q_{J,L}^*$ and $Q_{J,H}^*$, for type-L and type-H retailer, respectively. This result is particularly interesting since it comes against several intuitive statements of previous researchers Corbett and Tang (1999); Ha (2001) and Ozer and Raz (2011), who claim non-existence of perfect coordination under information asymmetry. It is in agreement though with the results in Cakanyildirim et al. (2012), who identified cases where perfect coordination is feasible under asymmetric information.
3. In cases AI(I) and AI(II), the retailer is constrained to her reservation level irrespective of her type; this happens because the supplier provides an appropriate QD based on the expectation of the retailer's holding cost and assimilates all the gains from coordination. In case AI(III), the supplier does not have the power to lead the retailer to her reservation level as he pays information rent. This is due to the disadvantage of the supplier because of the retailer's private information.
4. It is easily shown that all order quantities, fixed payments and costs of the optimal solution are proportional to $\sqrt{D}$. Therefore, the demand level does not affect the structure of the optimal policy, except from scaling by a factor $\sqrt{D}$.

In order to achieve perfect coordination in our model, the value of $f_1 = \sqrt{1 + \frac{K_S}{K_R}}(1+\sqrt{\frac{h_L}{h_H}})$ should be less than 2; f_1 is the product of two terms relative to set-up and holding costs. Therefore, we have to examine the possible values and joint product implications of these two terms. Regarding holding costs, we have assumed that $h_L < h_H$ and should separate only two cases with respect to the holding cost values:

- First, if $h_L \simeq h_H$, which can be interpreted as elimination of the information asymmetry, then the second term of f_1 tends to the value of 2 and the inequality is not satisfied irrespective of the other term ($\sqrt{1+\frac{K_S}{K_R}}$). Consequently, the retailer's private information becomes almost irrelevant and perfect coordination cannot be reached since utility costs are minimal for the supplier and he does not seek to distinguish high and low cases for his discount offering.
- Second, if the low holding cost value is much smaller than the high one ($h_L << h_H$); i.e., asymmetry is a dominant factor for the decisions along the supply chain, then the second term of inequality ($1+\sqrt{\frac{h_L}{h_H}}$) tends to 1 and we have to examine the other term. When the supplier's and retailer's set-up costs are almost equal ($K_R \simeq K_S$), perfect coordination is achieved since the two players have almost identical set-up costs and it is profitable for them to jointly establish decisions (perfect coordination). On the other hand, when K_S is small, the supplier does not have any cost function relevant to the node interactions and all such costs are linked to the retailer; consequently, the costs of the latter are aligned with the costs of the whole supply chain and, thus, the joint optimal quantity is selected leading to perfect coordination. Furthermore, if $K_R << K_S$, it is not possible to achieve perfect coordination, since the first term of f_1 tends to infinity. This can be interpreted as follows: although the supplier wants to ship extremely large quantities through the chain because of the high set-up cost, the true decision maker is the retailer who has to be provided with an enormous incentive from the supplier in order to accept placing such large orders. As a side effect, the retailer's private information is very important in establishing the business and may instigate no truth-telling policies without the appropriate incentive (RP).

In conclusion, we have derived analytical expressions of the optimal QDs and resulting costs for both players and we can directly implement them in any numerical case. The powerful aspect of the approach is the use of concepts from the RP to reduce the search space for optimality; this has rendered our method tractable in obtaining closed form equations for all problem variables in the final solution.

3.3 Numerical Experiments

To evaluate the proposed QD based mechanism we consider numerical examples that cover all potential dimensions of the various cost parameters involved in our model. Furthermore, we investigate the sensitivity of the optimal players' strategies with respect to changes in the values of these parameters. The goal is to assess the overall benefits emanating from the "intelligent" discount that the supplier provides to the retailer, a policy that elevates the efficiency of the whole supply chain as is theoretically expected. Note that, we have a model that involves six independent parameters: D, K_R, K_S, h_L, h_H and p. As already mentioned, the contribution of the annual demand D in the strategies of the two players is straightforward. Thus, the independent parameters are reduced to five.

The approach we use to evaluate the benefits and the efficiency of the proposed solution is as follows: initially, we calculate the results incurred for a problem instance without any discount offered by the supplier (when the supplier does not provide any discount, his expected cost is equal to $pC_S(Q^*_{R,L}) + (1-p)C_S(Q^*_{R,H})$) and the results of the centralized solution that leads to perfect coordination (ideal case). Subsequently, we perform a comparison of the costs (supplier, retailer and joint) in the previous two solutions with the respective costs reached by solving our proposed discount model. The first comparison shows the level of improvement achieved via our quantity discount-based policy, whereas the second comparison provides the gap between our solution and the solution derived under perfect coordination.

To reduce the complexity of comparisons we proceed in normalizing the values of set-up and holding costs, by considering ratios instead of actual values. Although one can argue that individual set-up and holding cost values are important when deriving actual supply chain costs, what really defines the direction of any managerial decision is the relative value of these costs. In this sense, we assume that:

$$\text{Ratio of the fixed (or set-up) costs: } R_f = K_S/K_R,$$
$$\text{Ratio of the holding costs: } R_h = h_H/h_L.$$

Therefore, the evaluation parameters now become: R_f, R_h and p, while the values of K_R and h_L are set to 1.

We consider a numerical experiment where the three parameters (R_f, R_h and p) take 200 values each in the following ranges: $R_h \in (1,5]$, $R_f \in (0,10]$, $p \in [0,1]$. Thus, we examine 8×10^6 scenarios, which are programmed and run using MatLab (Davis and Sigmon, 2005). Note that, the ratios of holding and fixed costs are bounded by numerical values, while for probability p all range values are examined. In this experiment, our solution's joint cost is at most 45% and on average 26% better than the cost of the solution without discounts. Furthermore, the maximum percentage efficiency loss (i.e., the divergence from the whole supply chain cost under perfect coordination) is just above 11% with average less than 1.7%; the latter is a particularly encouraging result since it provides an indication of an attractive upper bound on the difference between our solution and that of the perfect coordination.

It is important to note that perfect coordination is the ideal scenario but requires either a single decision maker or a single owner of all the nodes, a fact that is extremely restrictive for practical applications. Nevertheless, our approach, which leaves players alone to decide on entering the relationship or not and achieves results that are so close (on the average) to the ideal case, justifies its application to real life supply chain interactions.

To further examine the outcome of our method under more realistic cases, we consider the same experiment with the ratio of holding costs (R_h) taking values only in the range $(1, 2]$ (see Becerril-Arreola et al. (2013)). In this case, the maximum percentage of supply chain efficiency loss is around 5.4% (average 0.7%), while the percentage improvement in the joint cost remain the same. This result is consistent with our intuition, because the reduction of the maximum value of R_h is interpreted as elimination of the information asymmetry. As already mentioned, in the case of complete information it is always feasible to achieve perfect coordination.

Furthermore, it is useful to compare the minimum percentage improvement in the joint cost of our solution and the same parameter when no discount is provided. Until now, we have included results for small values of the suppliers' set-up costs (i.e., $K_S \simeq 0$) which is not reasonable in practice. Therefore, we limit the ratio of fixed costs (R_f) by raising the lower bound. Thus, we take again 200 values of each of the three parameters (R_f, R_h and p) in the following ranges: $R_h \in (1, 2],\ R_f \in (4, 10],\ p \in [0, 1]$. In this case, the minimum improvement in the joint cost is 22.1%. The mean value of the percentage improvement is less than 36% and the mean value of the percentage efficiency loss is less than 0.9%. Thus, as the problem parameters approach real life values (i.e., $R_h < 2$ and $R_f > 4$), our method better supports node interaction.

Subsequently, we present some graphs that capture the sensitivity of the optimal players' strategies with respect to changes in the values of the model parameters. In Figure 3.2, we show how the ratio of fixed costs (R_f) affects costs; the horizontal axis corresponds to R_f and the vertical axis reflects the values of:

- the joint cost under perfect coordination, represented by the thick solid line,
- the joint cost under our solution, represented by the dotted line,
- the supplier's cost under our solution, represented by the dashed line (to identify the part of the total cost attributed to this player), and
- the joint cost without discount, represented by the solid thin line.

In Figure 3.3, the horizontal axis corresponds again to R_f, while the vertical axis represents cost divergence or improvement from a baseline case. The figure depicts two distinct lines: the dashed line, which represents the percentage improvement in the joint cost achieved in our solution, and the continuous line representing the percentage loss for the joint cost from the ideal solution.

As mentioned before, it is more realistic to consider $R_f > 3$. So, we focus in the rele-

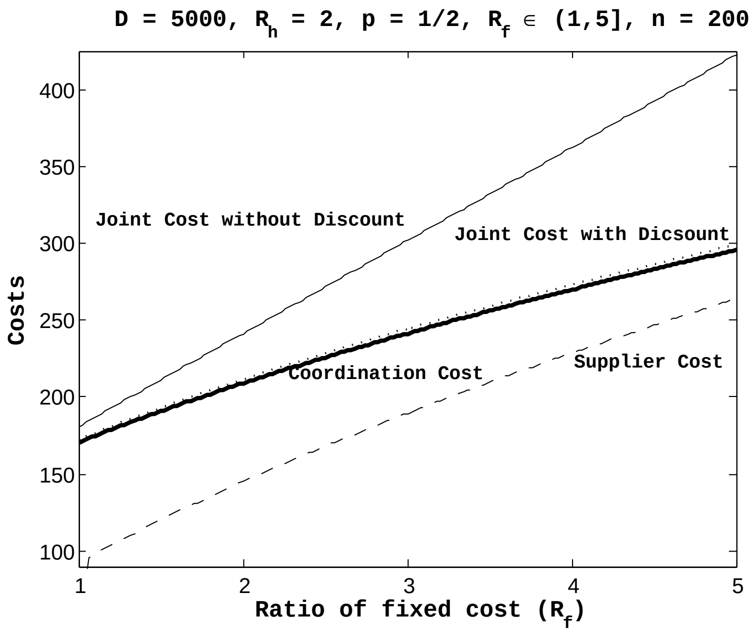

Figure 3.2: Cost evolution in R_f

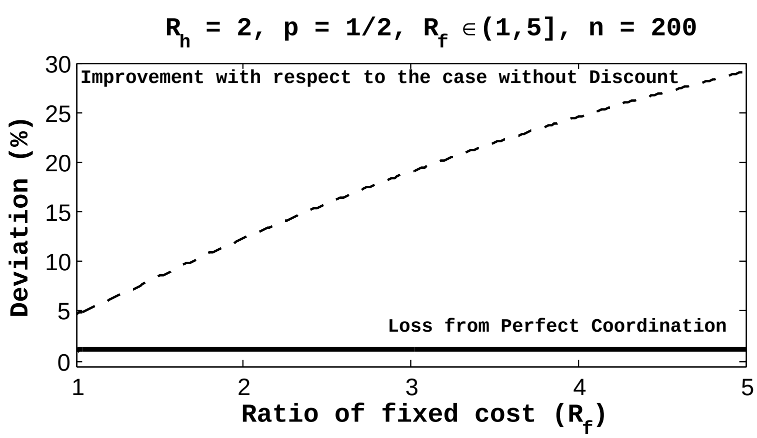

Figure 3.3: Joint cost difference evolution in R_f

vant areas of Figures 3.2 and 3.3. We observe that the joint cost under our solution almost matches the cost of the solution under perfect coordination, without actually enforcing by any means such coordination. For small values of R_f, the percentage improvement in the joint cost is small although not negligible (above 5%); as the values of R_f increase, the percentage improvement increases significantly. This result is a strong indication that our solution reduces inefficiencies and provides much better results than the case without any discount. This is consistent with our intuition, because small values of R_f mean that the supplier does not bear a significant proportion of the total cost, so he has not a powerful incentive to provide the discount.

Subsequently, we keep the ratio of fixed cost constant at $R_f = 4$ and consider 200 different values of the ratio of holding cost in the range $(1, 2]$ for three values of the probability of the retailer being type-L (p equal to: 0.25, 0.5 and 0.75). The objective is to examine how the parameters p and R_h affect the percentage improvement in the joint cost with regards to the solution without any discount and the percentage efficiency loss from the ideal case. The results are illustrated in Figures 3.4 and 3.5, where the horizontal axis corresponds to R_h. The vertical axis in Figure 3.4 represents the percentage efficiency loss from coordination, while in Figure 3.5 represents the percentage cost improvement in the joint cost. Three distinct lines are depicted in each figure:

- the solid thin line represents the retailer being type-L (p) with probability 0.75
- the thick solid line represents the case where $p = 0.5$
- the dashed line represents the case where $p = 0.25$

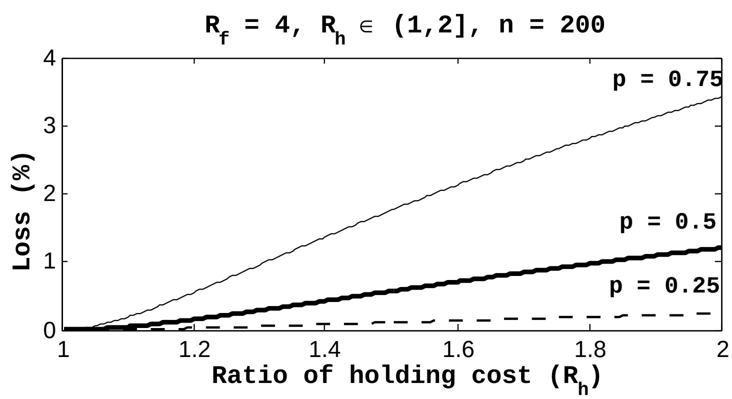

Figure 3.4: Efficiency loss from perfect coordination

From Figure 3.4, we can conclude that as R_h increases, the relative divergence of the proposed QD from the ideal case increases as well, but does not exceed 3.5%, when $p = 0.75$, 1.5% when $p = 0.5$ and 0.4% when $p = 0.25$. This is not surprising because $h_L \simeq h_H$ can be interpreted as elimination of the information asymmetry. Therefore, we have an indication that the proposed QD does not diverge from the ideal case. Moreover, we observe that as the probability of the retailer being type-L increases, so does the gap between our solution and that of the perfect coordination.

In Figure 3.5, we observe that as R_h increases, the percent improvement in the joint cost achieved by our QD approach decreases compared to the optimal joint cost; however it remains at a significant level (over 22.5%), which is also affected in a negative way by the probability of the retailer being type-L. The latter is expected since a type-L retailer is associated with a smaller value of the holding cost.

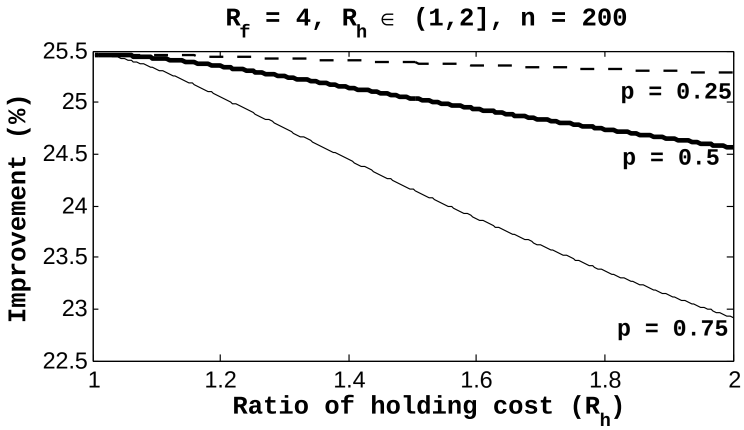

Figure 3.5: Improvement in the joint cost

Figure 3.6: Joint cost difference evolution in R_f for three probability values

3.4 Findings

In this chapter, we considered a two-node SC with one manufacturer producing a single product in a lot-for-lot fashion and one retailer who orders and stores the same product in fixed quantities. We modeled the problem of node interaction as a game and for the case of a unique and known inventory holding cost we proved the joint EOQ result. For the case of a two-level holding cost (distribution to high and low), we formulated a Stackelberg game and reached closed form expressions of the QDs that the manufacturer should offer to minimize his costs while enabling the establishment of the business. We also proved that even with information asymmetry, perfect coordination is attainable under specific conditions. The results were evaluated using numerical experiments that suggested superiority of our approach compared to cases with no discounts, thus justifying the potential for application to real-life business ventures.

Chapter 4

Quantity Discounts in Supply Chain Coordination under Multi-level Information Asymmetry

This chapter is an extension of the previous one. We assume that there are three possible choices about the retailer's holding cost. We propose a model with QDs as a way to capture the retailer's private information, while our objective is to coordinate the SC and achieve a better profit allocation for all the participants. The problem is formulated as a Stackelberg game, where the supplier is the leader and the retailer the follower. We make some conjectures on how results we have obtained for the case of two possible holding cost values may or may not be extended in this setting.

4.1 Model description

We consider a SC with two distinct nodes, where the nodes are legally obliged to interact each other. One node can be thought of as a supplier or a manufacturer (referred to as he) that produces a single product in a lot-for-lot fashion. The other node can be thought of as a retailer or a buyer (she), ordering items from the supplier to satisfy market demand. The retailer has to decide on the order quantity (lot size) to place to the manufacturer, satisfying demand and minimizing her own cost; shortages or backorders are not allowed. The manufacturer does not own a warehouse facility, nor can it accommodate inventory at other premises; thus, completed lots are directly forwarded to the retailer.

We assume that the retailer has private information about the actual holding cost;

but this time where exists three possible choices about the concerning stock warehousing: a) store inventory at owned warehouse, a fact that provides the minimum per unit holding cost; b) allocate inventory holding at a 3PL company, a medium-cost value; or c) rent storage facilities particularly for the retailer, a choice that leads to maximum holding cost.

Both nodes are rational, risk neutral and base their decisions on sound utility functions. They have set-up costs (production-related for the manufacturer and order-related for the retailer) and interact via order quantities. We assume that both the retail price and the market demand D are constant, exogenously defined and known to the nodes, a common assumption in the literature (Corbett, 2001). We make this assumption, due to the fact that our goal is to examine node coordination (without affect the market demand) and the resulting players' benefits, in terms of operational costs.

According to our assumptions, the retailer's cost C_R is a function of her order quantity Q and can be expressed as: $C_R(Q) = K_R D/Q + h_R Q/2$, where denote with K_R the retailer's set-up cost and with h_R the retailer's holding cost. The manufacturer's cost C_M is solely a function of the retailer's decision Q and can be expressed as: $C_M(Q) = K_M D/Q$, where K_M is the manufacturer's set-up cost. Note that, the manufacturer is not yet a decision maker. The total SC cost can be expressed as $C_J(Q)$ and is equal to the sum of the retailer's and manufacturer's cost, i.e.:

$$C_J(Q) = C_M(Q) + C_R(Q). \tag{4.1.1}$$

The retailer selects the order quantity to minimize her own cost function (rational). The retailer's cost corresponds to an EOQ-type cost, thus the optimal lot size is: $Q_R^* = \sqrt{2K_R D/h_R}$. When the retailer's order is equal to Q_R^* the costs are:

Retailer's cost: $C_R(Q_R^*) = K_R D/Q_R^* + h_R Q_R^*/2 = \sqrt{2K_R D h_R}$.
Manufacturer's cost: $C_M(Q_R^*) = K_M D/Q_R^* = K_M \sqrt{Dh_R/2K_R}$.
Joint cost: $C_J(Q_R^*) = C_M(Q_R^*) + C_R(Q_R^*) = (2K_R + K_M)\sqrt{Dh_R/2K_R}$.

Thus, both nodes are able to accept costs until these values $C_R(Q_R^*)$ and $C_M(Q_R^*)$ and we have an indication about the maximum join cost. The values $C_R(Q_R^*)$ and $C_M(Q_R^*)$ are known to the literature as retailer's and manufacturer's reservation levels, respectively. The reservation level is defined as the cost when the player determines his strategy under the worst case scenario for him (Gibbons, 1992). Thus, the players' costs cannot exceed their reservation levels under any proposed solution.

The manufacturer due to the fact that he has set-up cost, prefers the largest possible orders from the retailer, since this would reduce his operational costs. The retailer on the other hand is responsible for the quantity ordered, and should consider both storage and set-up costs when he determines the preferred quantity levels, in addition to all other problem parameters. Moreover, we observe that the optimal cost for the whole chain is the minimum of the function $C_J(Q)$. Due to the fact that total SC cost $C_J(Q)$ corresponds to an EOQ-type cost, the minimum of function $C_J(Q)$ is achieved when the order quantity is:

$$Q^J = \sqrt{2(K_R + K_S)D/H_R P_S}. \tag{4.1.2}$$

We observe that $Q_J^* > Q_R^*$, so we have $C_J(Q_J^*) < C_J(Q_R^*)$. Therefore, a higher than Q_R^* order quantity is preferable to reduce the total costs. The difference $C_J(Q_J^*) - C_J(Q_R^*)$ denotes the maximum benefits that coordination of the two nodes can incur. Thus, we have an indication about the profits which arise from node coordination. However, this is reached at the expense of the retailer's increased cost, rendering her negative to a potential cooperation. Therefore, to raise the retailer's order level (preferred case for the manufacturer) and achieve reduced costs, the manufacturer must offer her an incentive when selecting the order quantity.

We allow the manufacturer to provide QD to the retailer, in order to force the latter to increase order levels (retailer's decision). We observe that when the manufacturer uses QDs; the joint cost C_J remains the same, because the discount affects only the allocation of the cost between the two nodes, via the increase in the retailer's order levels. The aim of this work is to determine the appropriate QD, which the manufacturer will provide to the retailer, in order to coordinate the chain. The problem is formulated as a Stackelberg game, a common way of modeling node interaction in SCM, in which the manufacturer is the leader and the retailer is the follower. Moreover, we examine the role of QDs as a way to capture retailer's private information and coordinate the supply chain. The latter leads to achieve the maximum profit for the whole chain with result more profits to be available for the participants.

The asymmetry information reflects the three levels of warehousing cost that the retailer knows when opting for it, while the manufacturer assumes a probability function for these values. We model the information asymmetry assuming that the retailer's holding cost h_R is discrete random variable, which could take three different values (a high, a medium or a low one). We assume that retailer's holding cost takes the low value h_L with probability p, the medium value h_M with probability q, while the high value h_H happens with probability $1 - p - q$. The retailer learns the real value of h_R before making her own decisions, while the manufacturer considers h_R as a discrete random variable such that:

$$P(h_R = h_L) = p, P(h_R = h_M) = q, P(h_R = h_H) = 1 - p - q. \tag{4.1.3}$$

The interaction between the two nodes can be modeled via a Bayesian game. In terms of the Bayesian formulation, we refer to these three holding cost values as retailer of type-L, type-M and type-H, respectively.

The manufacturer (leader) sets the QD and the retailer (follower) and selects the order quantity. In this model and according to the design mechanism theory (Fudenberg and Tirole, 1991) it is sufficient to consider discount policies with quantity-price pair (X, Y). This means that the discount is valid if and only if the order quantity is equal to X, and then the retailer achieves discount equal to Y. Due to the assumption that the nodes are rational, the manufacturer selects a discount to minimize his expected cost under the prior distribution, assuming that the retailer will respond optimally and taking into account the true value of the holding cost known to him.

4.2 Mathematical framework

In this section, we provide the appropriate analysis and the necessary conditions about the QD, describing all the necessary constraints about the two decision variables of the problem; i.e., order quantity for the retailer and QD for the manufacturer. The QD can act as a screening device to induce the retailer to reveal her private information about the holding cost (Kolay et al., 2004). The analysis is based on the RP.

As already mentioned, we assume that the retailer identifies her true holding cost as soon as the game starts, while the manufacturer considers that type-L (h_L) occurs with probability p, type-M (h_M) happens with probability q and type-H (h_H) with probability $1 - p - q$. According to the retailer's type, the retailer's cost function and the retailer's reservation level can be written as:

$$\text{Retailer's cost}\begin{cases} C_{R,L}(Q) = K_R D/Q + h_L Q/2 & \text{if he is type-L} \\ C_{R,M}(Q) = K_R D/Q + h_M Q/2 & \text{if he is type-M} \\ C_{R,H}(Q) = K_R D/Q + h_H Q/2 & \text{if he is type-H,} \end{cases} \tag{4.2.1}$$

$$\text{Retailer's reservation levels}\begin{cases} C^+_{R,L}(Q) = \sqrt{2K_R D h_L} & \text{if he is type-L} \\ C^+_{R,M}(Q) = \sqrt{2K_R D h_M} & \text{if he is type-M} \\ C^+_{R,H}(Q) = \sqrt{2K_R D h_H} & \text{if he is type-H.} \end{cases} \tag{4.2.2}$$

The retailer's reservation levels are reached when she selects the order quantity to minimize her own cost function and the manufacturer does not provide any discount.

Thus, the retailer's order quantity is:

$$\text{Optimal retailers order quantity} \begin{cases} C^*_{R,L}(Q) = \sqrt{2K_R D/h_L} & \text{if he is type-L} \\ C^*_{R,M}(Q) = \sqrt{2K_R D/h_M} & \text{if he is type-M} \\ C^*_{R,H}(Q) = \sqrt{2K_R D/h_H} & \text{if he is type-H.} \end{cases} \tag{4.2.3}$$

The analysis is based on the RP according to which it is sufficient to consider QDs such that: i) the manufacturer sets one pair (X, Y) of order quantity and discount amount respectively for each retailer type, and ii) the values of (X_i, Y_i) are such that it is optimal for the retailer to select the option (X_i, Y_i) if he is type-i, where i=L,M,H. Therefore, the manufacturer should design a mechanism under which the retailer will act according to her actual type. Without loss of generality, we can restrict the set of supplier's strategies to those that satisfy the RP and have attractive properties (Myerson, 1979).

The retailer has three options concerning his order quantity: i) to order without discounts, ii) to order according to her actual type and achieve the corresponding discount e.g., if she is type-L to order a quantity equal to X_L and receive the discount of Y_L, and iii) to order a quantity that would allow her to get a discount she would if she was of an another type. The retailer selects the option that minimizes her own expected cost (rational and risk neutral player). According to the RP it is sufficient to the manufacturer to include the appropriate incentives in the QD, in order the retailer to select the order quantity according to her actual type; i.e., the QD should include IC constraints (Fudenberg and Tirole, 1991). The IC constraints ensure that the retailer cannot incur lower costs if she acts in order to achieve different type of discount; i.e., the retailer will be truthful because it is in her self-interest. Thus, the following should hold:

$$\begin{gathered} C_{R,L}(X_L) - Y_L \leq C_{R,L}(X_M) - Y_M \\ C_{R,L}(X_L) - Y_L \leq C_{R,L}(X_H) - Y_H \\ C_{R,M}(X_M) - Y_M \leq C_{R,M}(X_L) - Y_L \\ C_{R,M}(X_M) - Y_M \leq C_{R,M}(X_H) - Y_H \\ C_{R,H}(X_H) - Y_H \leq C_{R,H}(X_L) - Y_L \\ C_{R,H}(X_H) - Y_H \leq C_{R,H}(X_M) - Y_M. \end{gathered} \tag{4.2.4}$$

Due to the fact that the retailer could deny the QD (from the manufacturer) and acts alone; i.e., to order without receive any discounts, base to RP the manufacturer when he designs the QD he should include IR or participation constraints. These constraints exist in order the retailer to prefer ordering the quantity with the discount instead of her own optimal solution without discount. Thus, the following should hold:

$$\begin{aligned} C_{R,L}(X_L) - Y_L &\leq C^+_{R,l} \\ C_{R,M}(X_M) - Y_M &\leq C^+_{R,M} \\ C_{R,H}(X_H) - Y_H &\leq C^+_{R,H}. \end{aligned} \tag{4.2.5}$$

The system of inequalities (4.2.4) and (4.2.5) become constraints in the manufacturer's optimization problem (RP, Myerson (1979)). Consequently, the manufacturer solves the following optimization problem:

$$\min_{(X_L,Y_L,X_M,Y_M,X_H,Y_H)} p\{C_M(X_L)+Y_L\} + q\{C_M(X_M)+Y_M\} + (1-p-q)\{C_M(X_H)+Y_H\}$$

$$\begin{array}{lll} \text{s.t.} \quad Y_L & \geqslant & C_{R,L}(X_L) - C^+_{R,L} \\ Y_M & \geqslant & C_{R,M}(X_M) - C^+_{R,H} \\ Y_H & \geqslant & C_{R,H}(X_H) - C^+_{R,H} \\ Y_L - Y_M & \geqslant & C_{R,L}(X_L) - C_{R,L}(X_M) \\ Y_L - Y_H & \geqslant & C_{R,L}(X_L) - C_{R,L}(X_H) \\ Y_M - Y_L & \geqslant & C_{R,M}(X_M) - C_{R,M}(X_L) \\ Y_M - Y_H & \geqslant & C_{R,M}(X_M) - C_{R,M}(X_H) \\ Y_H - Y_L & \geqslant & C_{R,H}(X_H) - C_{R,H}(X_L) \\ Y_H - Y_M & \geqslant & C_{R,H}(X_H) - C_{R,H}(X_M). \end{array}$$

In our model, coordination is achieved when there is only one decision maker who controls both nodes and makes all the decisions under the objective to minimize the joint supply chain costs. This means that the order quantities from the retailer to the manufacturer are equal to:

$$Q_i^* = \sqrt{(2(K_R+K_M)D/h_i)} \quad \text{for } i\text{=L,M,H.} \tag{4.2.6}$$

The first question which arises is if the coordination of the chain is always attainable under the appropriate QD; i.e., if the manufacturer could design a QD where X_L,X,X_H are equal to Q_L^*,Q^*,Q^*. It is easy to show that perfect coordination is not always possible, which is rational according to the results of (Zissis et al., 2015). Thus, the remarkable question is to find under which specific parameters' conditions the chain could be coordinated and to give the managerial explanation.

A second question is to find the appropriate indexes which evaluate the improvement that is achieved through our proposed way of coordination, comparatively to the solution without discounts. A final question is about the information rent, which the retailer can achieve due to the fact that she possesses private information. It is well known to the literature that if the manufacturer knows the actual retailer's holding cost (retailer's private information) he has the power to lead the latter to the reservation level. In this case the manufacturer secures all the gains from coordination for himself by paying the retailer just enough to induce her to alter the order quantity.

4.3 Findings

In this chapter, we considered a two-node SC with one manufacturer producing a single product in a lot-for-lot fashion and one retailer who orders and stores the same product in fixed quantities. We modeled the SC as a Bayesian game, due to the fact that the retailer had private information about her holding cost. We assumed that the retailer's holding cost was discrete random variable, which could take three different values (a high, a medium or a low one). We formulated a Stackelberg game and wrote down all the necessary constraints of the QDs that the manufacturer should offer in order to minimize his costs while enabling the establishment of the business. Moreover, we discussed some conjectures on the feasibility of coordination in the information asymmetry setting.

Chapter 5

Supply Chain Coordination via Mediator

In this chapter, we address a two-stage SC with two distinct rational nodes (supplier - buyer) which interact, in a decentralized manner. Both nodes have discrete private information that affects both their reservation levels and the way in which they decide their actions. In order to achieve the alignment of individual nodes and overall system objectives, we provide the players with the opportunity to communicate concerning any private information they may possess, through a credible mediator (third trusted party). The latter designs a mechanism to minimize the total supply costs.

The nodes' communication takes place before players decide on their actions, without any restrictions (modeling of misinformation is also accepted). Obviously, opportunities for mutual benefits cannot be found unless the players share honestly their private information (Fiala, 2005). We assume that all the possibilities for communication are entirely controlled by a credible mediator and use the RP as the technical approach that allows the derivation of statements about what rules are feasible in the communication system. Using the RP, we are able to capture the nodes' private information and prove that perfect coordination is attainable under bilateral information asymmetries.

In the proposed work the role of mediator can be assumed by auditing firms (Klein, 2002), especially if they are common between the nodes, or by Supervising Authorities. To proceed with sharing private information, players should be provided with appropriate incentives. The mediator includes incentives in his proposal mechanism to the nodes, so that they report honestly their private information because it is in their self interest; such incentives can be expressed via QD schemes. Therefore, we examine how the communication system, through the mediator, leads to the system-wide coordination, and then measure the resulting players' benefits.

The contribution of this chapter is twofold: i) we introduce the notation of a mediator as a means of coordination in SCM, by developing a mediator mechanism that is based on the RP, and ii) we prove that perfect coordination under discrete information asymmetries on both nodes is possible, when the nodes' reservation levels are not exogenously defined. To the best of our knowledge there are not any prior works in the literature which study and analyse how the nodes could be coordinated under 2-way information asymmetries.

The chapter is organized as follows: Section 5.1 provides the mathematical formulation about our proposed model and the GT perspective of the players' interaction via communication. In Section 5.2 we develop the analytical solution of the game and we prove that coordination is attainable. Section 5.3 provides insights about the coordination benefits and mediator's flexibility. Section 5.4 summarizes the conclusions of our work.

5.1 Model description

We consider a SC with two nodes which trade a single product. One node can be thought of as a supplier (or producer) denoted by S (referred to as he). The supplier is producing a single product in a lot-for-lot fashion since he does not own a warehouse facility, nor can he accommodates inventory at other premises. Completed lots are then directly forwarded to the other node, who acts as a buyer (she) and is denoted by R. The buyer has the market power to determine the order quantity (lot size), denoted by Q ($Q > 0$), to place to the supplier, satisfying market demand and minimizing her own cost, without taking into account the global optimum. We assume that shortages or backorders are not allowed, which are standard assumptions in the literature (Li and Wang, 2007). The nodes are forced to interact with each other; no alternatives for external interactions are allowed. Moreover, they are rational, risk neutral and interact exclusively via order quantities. It is assumed that market demand D ($D > 0$) is constant, exogenously defined and known to nodes. We make this assumption since our objective is to examine node coordination and the resulting benefits, in terms of operational costs.

The buyer who decides on the order quantity Q, has both an ordering and a holding cost, denoted by K_R ($K_R > 0$) and H_R ($H_R > 0$), respectively. For the holding cost, it is assumed that it is a percentage of the production cost P_S (Krajewski et al., 2010). Therefore, the buyer's cost function and can be expressed as:

$$TC_R(Q) = K_R D/Q + H_R P_S Q/2. \tag{5.1.1}$$

Obviously, the buyer's cost is a function of her decision Q. As the buyer is a rational player, she selects the lot size Q, that minimizes her own costs. $TC_R(Q)$ corresponds to an EOQ-type cost, thus the optimal lot size is $Q^R = \sqrt{2K_RD/H_RP_S}$ and the minimum cost is $TC_R(Q^R) = \sqrt{2K_RDH_RP_S}$. The supplier has a setup and a per unit production cost, denoted by K_S ($K_S > 0$) and P_S ($P_S > 0$), respectively, his cost is solely a function of the buyer's decision Q, and it can be expressed as:

$$TC_S(Q) = K_SD/Q + P_SD. \tag{5.1.2}$$

Note that the supplier is not yet a decision maker. If the supplier could decide about the order quantity, he would favor large quantities because in this way he would reduce his own operational costs. The optimal buyer's lot size is Q^R, which leads to a supplier cost $TC_S(Q^R) = K_S\sqrt{DH_RP_S/2K_R} + P_SD$. The total channel cost is denoted by $C_J(Q)$ and is equal to the sum of supplier's and buyer's cost:

$$C_J(Q) = (K_R + K_S)D/Q + H_RP_SQ/2 + P_SD. \tag{5.1.3}$$

We observe that $C_J(Q)$ corresponds to an EOQ-type cost, with setup being the sum of K_R and K_S and the optimal joint lot size is $Q^J = \sqrt{2(K_R + K_S)D/H_RP_S}$. In our model, perfect coordination (ideal scenario) exists when the buyer decides on the joint optimal lot size Q^J and imposes it to the supplier. Obviously, $Q^J > Q^R$, thus $C_J(Q^J) < C_J(Q^R)$. It is clear that a higher order quantity is preferable to reduce the total costs. The difference $C_J(Q^R) - C_J(Q^J)$ denotes the maximum benefits that coordination of the two nodes can incur. Thus, we have an indication about the profits which arise from node coordination. However, this is achieved at the expense of increased buyer's cost, rendering the latter negative to such an option. In order to raise the buyer's order level and achieve reduced overall costs, the buyer should be provided with the appropriate incentives. In our model, we focus on employing QD policies for achieving this. Furthermore, the QDs affect only the cost allocation among the participants and not the total chain cost.

The case which we consider is a decentralized model under incomplete information on both sides. In this case the nodes decide their actions based only on individual criteria. We model the information asymmetry assuming that the supplier's production cost and the buyer's holding cost are both discrete random variables, which could take a high or a low value. According to Yu et al. (2009) a dual sourcing strategy is a common approach, after the March 2000 fire at Philips microchip plant in Albuquerque, that led two of the cell-phone giants (Nokia and Ericsson) to chaos. Thus, we assume that the supplier has alternative choices about production which induce different production cost. After the production phase, the single product is forwarded to the buyer who is responsible to store it. The buyer is not aware of the exact cost when the deal is made with the supplier since this is a function of the production plant and the limited capacities prohibit the a priori assumption of "lower price selection" by the supplier. Concerning the buyer's holding cost, it is assumed that there are discrete choices which incur different costs. For example, the buyer could store inventory at owned warehouse (low cost) or/and to rent storage facilities or at the customs location (high cost). Hence, our model is developed based on the fact that both nodes have discrete private information. The assumption of discrete private information is more realistic for practical applications, where the cost or the prices could take some discrete values; according to Lovejoy (2006) continuous asymmetries cannot be directly applicable, while discrete ones can.

We assume that the production cost P_S takes the low value P_d, with probability q and the high value P_u ($P_u > P_d$) with probability $1-q$, while the holding cost H_R takes the low value H_l, with probability p and the high value H_h ($H_h > H_l$) with probability $1-p$. The supplier learns the real value of P_S before making his own decision, while the buyer considers P_S as a discrete random variable such that $P(P_S = P_d) = q = 1 - P(P_S = P_u)$. Similarly, H_R is known to the buyer but partially known to the supplier, who considers it as a discrete random variable with $P(H_R = H_l) = p = 1 - P(H_R = H_h)$. According to the Bayesian formulation (Gibbons, 1992), the supplier can be of type-d or type-u and the buyer can be of type-l or type-h. Thus, four different combinations of the players' types arise: l/d, l/u, h/d,and h/u. Since production and holding costs are independent, probabilities about the cases l/d, l/u, h/d, and h/u are $pq, p(1-q), (1-p)q$ and $(1-p)(1-q)$, respectively.

The cost of each node is a function of the order quantity Q and depends on the node type, because each node is aware of his/her own parameter value. Therefore, the costs of the two nodes are:

$$\text{Supplier's Cost} \begin{cases} TC_{S,d}(Q) = K_S D/Q + P_d D & \text{if he is type-d} \\ TC_{S,u}(Q) = K_S D/Q + P_u D & \text{if he is type-u,} \end{cases} \tag{5.1.4}$$

$$\text{Buyer's Cost} \begin{cases} TC_{R,l}(Q) = K_R D/Q + H_l P_S Q/2 & \text{if she is type-l} \\ TC_{R,h}(Q) = K_R D/Q + H_h P_S Q/2 & \text{if she is type-h.} \end{cases} \tag{5.1.5}$$

In this context of incomplete information, information sharing between the nodes is a critical factor for achieving coordination and it should be incorporated in the player's strategies. In this chapter we propose that both nodes are able to communicate concerning any private information they possess through a mediator. We assume that the mediator proposes QD schemes as incentives to coordinate the nodes' decisions. Therefore, a communication game arises, which can be viewed as a hybrid game between the two basic game categories: the non-cooperative and the cooperative games. This holds because the communication game combines properties from both categories (Myerson, 2007). The possibilities for communication are assumed to be entirely controlled by the mediator, who: i) is considered to be a credible authority, ii) does not coincide with any of the nodes, and iii) serves a unique purpose of optimizing the chain. The existence of a mediator is only to facilitate nodes to communicate without incurring any additional cost to them and to the whole chain.

The mediator announces a plan which describes his role and defines his potential actions (referred to as the "mediator plan"). Then, both nodes report confidentially their private information to the mediator. The latter cannot compel truthful behavior by the nodes and anticipates that either of them may lie to him in an attempt to manipulate the mediator plan. Note that each node is the only one who knows his/her own true type, and no one can prevent the nodes from lying about it, since the nodes may expect advantage from such a behavior. Hence, the nodes may or may not lie about their types, and thus, the real types and the reported types may not coincide. The mediator is aware of that and if he anticipates both nodes to report their real types, he should must include appropriate adverse selection incentives in the mediator plan. According to the RP, all the rational players will then report honestly their types, because in this way they achieve the largest individual gains. No rational player would expect higher individual gains from being the only player to lie about his type, when the others are planning to honesty report their types.

After receiving the reports from both nodes, the mediator specifies actions for them, according to the preannounced mediator plan. The latter incorporates any rule that emanates from the nodes' reports and enables the specification of actions. In this work, the mediator plan is a quantity discount pair (X, Y) that depends on the reported types; i.e., given the types that nodes report to the mediator, he recommends a quantity X that the buyer should order and a discount Y that the supplier should provide to the buyer, if the buyer indeed follows his recommendations. Nodes could either accept the recommendation according to the reported type or refuse the plan (nodes cannot alter the specific quantity-price pair). According to the RP, it is sufficient to consider discounts such that the mediator sets one quantity-price pair (X, Y) for each feasible combination of the nodes' types, in order to be separated the different type combinations. Thus, in our model it is sufficient to determine four quantity-price pairs.

Before the application of the RP, it is necessary to define the nodes' reservation levels; i.e., the costs when the nodes select their strategies under the worst case scenario for them (Gibbons, 1992). These are required, because the nodes are free to decide whether to participate in the mediator plan or act alone. Therefore, the mediator's plan should include constraints that ensure voluntary participation of the nodes. Specifically, the worst case scenario for the buyer is when the supplier's production cost is high and the supplier does not provide any discount for opting for it. Because the buyer is able to distinguish the low value H_l and the high value H_h of her holding cost, the buyer's cost function under the worst case scenario is either $K_R D/Q + H_l P_u Q/2$ when she is type-l, or $K_R D/Q + H_h P_u Q/2$ when she is type-h. To minimize her EOQ-type cost (rational player), she orders quantity equal to $Q_l^R = \sqrt{2K_R D/H_l P_u}$, or $Q_h^R = \sqrt{2K_R D/H_h P_u}$, respectively. This results in the following costs, which are defined as the buyer's reservation levels, depending on her type: $C_{R,l}^+ = \sqrt{2K_R D H_l P_u}$, or $C_{R,h}^+ = \sqrt{2K_R D H_h P_u}$, respectively. To calculate the supplier's reservation levels we consider the worst case scenario for him, which occurs when the buyer's order is equal to Q_h^R (minimum quantity order since the supplier's cost is a decreasing function of the order quantity). Thus, supplier's reservation levels are: $C_{S,d}^+ = K_S\sqrt{DH_h P_u/2K_R} + P_d D$, if he is type-d and $C_{S,u}^+ = K_S\sqrt{DH_h P_u/2K_R} + P_u D$, if he is type-u.

The mediator designs a mediator plan m as follows:

$$m = \{(X_{lu}, Y_{lu}), (X_{hu}, Y_{hu}), (X_{ld}, Y_{ld}), (X_{hd}, Y_{hd})\} \tag{5.1.6}$$

which determines the quantity-price pair for each combination of node types, using the prior probability distributions of production and holding costs: $P(P_S = P_d) = q = 1 - P(P_S = P_u)$ and $P(H_R = H_l) = p = 1 - P(H_R = H_h)$, because he is not aware of the real values of them. The mediator's objective is to minimize the expected value of the total chain cost $E(C_J(Q))$ which is equal to:

$$E(C_J(Q)) = p(1-q)C_J(X_{lu}) + pqC_J(X_{ld}) + (1-p)(1-q)C_J(X_{hu}) + (1-p)qC_J(X_{hd}). \tag{5.1.7}$$

Recall that mediator plans are constrained by the participation and adverse selection constraints.

5.2 Analytical Solution

In this section, we prove that perfect coordination when both nodes have discrete private information is attainable through a mediator. Furthermore, we devise exact values for the decision variables; i.e., the order quantity Q for the buyer and the quantity-price pair (X, Y) for the supplier, which achieve coordination.

It is crucial for the nodes to reveal voluntarily their private information, if we want to reach coordination. The RP asserts that any equilibrium of a communication game can be reached by an appropriate mechanism (Myerson, 1979). In our model, such a mechanism is a mediator plan, in which the mediator includes incentives of adverse selection to force nodes to report information honestly. Thus, nodes reveal their private information, because it is in their self interest. A mechanism is appropriate for reaching an equilibrium if, under the hypothesis that the other players are honest, no player could reduce his cost by reporting incorrect information to mediator. The benefit from the RP is that it guarantees that it is sufficient to consider only such mechanisms when devising the mediator plan. This restriction is significant, in the sense that this class is much smaller than the set of all feasible mechanisms and in general can be characterized by a finite number of inequalities, when there is a finite number of type combinations (Myerson, 1979).

Therefore, by the RP, it is sufficient to consider mediator plans consisting of four quantity-price pairs (X, Y), one for each feasible combination of nodes' types. Both nodes report that they are of a certain type, but they are free to report whatever type they desire. Their objective when they report their type is to minimize their individual expected cost, according to their prior distribution and conditional on their own known type (rational and risk neutral players). Consequently, each node's cost is a function of the mediator plan and of the reported and real types. For example, the supplier's expected cost under mediator plan m when he reports type-d, given that he is type-u is:

$$\begin{aligned} C_S(m,d|u) &= p(TC_{S,u}(X_{ld}) + Y_{ld}) + (1-p)(TC_{S,u}(X_{hd}) + Y_{hd}) \\ &= p(K_S D/X_{ld} + Y_{ld}) + (1-p)(K_S D/X_{hd} + Y_{hd}) + P_u D, \end{aligned} \tag{5.2.1}$$

while buyer's expected cost under mediator plan m when she reports type-h, given that she is type-l is:

$$\begin{aligned} C_R(m,h|l) &= q(TC_{R,l}(X_{hd}) - Y_{hd}) + (1-q)(TC_{R,l}(X_{hu}) - Y_{hu}) \\ &= q(K_R D/X_{hd} + H_l P_d X_{hd}/2 - Y_{hd}) + (1-q)(K_R D/X_{hu} + H_l P_u X_{hu}/2 - Y_{hu}). \end{aligned} \tag{5.2.2}$$

The other expected costs; i.e., $C_S(m,u|u)$, $C_S(m,d|d)$, $C_S(m,u|d)$, $C_R(m,l|l)$, $C_R(m,h|h)$ and $C_R(m,l|h)$ are similarly defined.

Since the nodes could deny the mediator plan and act alone, their cost under any plan cannot exceed their reservation levels $(C^+_{R,l}, C^+_{R,h}, C^+_{S,d}, C^+_{S,u})$. Both players prefer the solution under the mediator plan m when the following inequalities hold:

$$C_R(m,l|l) \leq C^+_{R,l}$$

$$C_R(m,h|h) \leq C^+_{R,h}$$

$$C_S(m, d|d) \leq C^+_{S,d} \tag{5.2.3}$$
$$C_S(m, u|u) \leq C^+_{S,u}.$$

Moreover, to ensure that both nodes report their private information honestly, because it is in their self interest, the mediator should include adverse selection incentives in his mediator plan m, expressed as:

$$\begin{aligned} C_R(m, l|l) &\leq C_R(m, h|l) \\ C_R(m, h|h) &\leq C_R(m, l|h) \\ C_S(m, d|d) &\leq C_S(m, u|d) \\ C_S(m, u|u) &\leq C_S(m, d|u). \end{aligned} \tag{5.2.4}$$

According to the RP, (5.2.3) and (5.2.4) become constraints when the mediator designs a plan. In the literature of mechanism design, inequalities (5.2.3) are referred to as IR or participation constraints and inequalities (5.2.4) are known as IC constraints (Fudenberg and Tirole, 1991).

The mediator designs the plan to minimize the expected value of the channel cost $E\left(C_J(Q)\right)$ (5.1.7); i.e., he solves an optimization problem with the objective function $E(C_J(Q))$ under constraints (5.2.3) and (5.2.4). Therefore, the mediator solves the following nonlinear optimization problem:

(M)

$$C^*_J = \min_{\{(X_{rs} \geq 0, Y_{rs} \geq 0), r=l,h,s=d,u\}} E(C_J(Q))$$

$$\begin{array}{llll} \text{s.t.} & C_R(m, l|l) & \leq & C^+_{R,l} \\ & C_R(m, h|h) & \leq & C^+_{R,h} \\ & C_S(m, d|d) & \leq & C^+_{S,d} \\ & C_S(m, u|u) & \leq & C^+_{S,u} \\ & C_R(m, l|l) & \leq & C_R(m, h|l) \\ & C_R(m, h|h) & \leq & C_R(m, l|h) \\ & C_S(m, d|d) & \leq & C_S(m, u|d) \\ & C_S(m, u|u) & \leq & C_S(m, d|u) \end{array}$$

In problem (M) the objective function coincides with the total channel cost in a centralized model where a single decision maker controls both nodes. Therefore, the minimum total channel cost under the centralized solution is a lower bound on the optimal solution of the mediator's problem (M). If we prove that there exists a feasible solution $\{(X_{rs}, Y_{rs}), r = l, h$ and $s = d, u\}$ to (M) such that X_{rs} are equal to the coordination quantities; i.e., $Q^J_{r,s} = \sqrt{2(K_R + K_S)D/H_r P_s}$, $r = l, h$ and $s = d, u$, then this solution is optimal for problem (M), and furthermore it allows the mediator to achieve perfect channel coordination. The main result of the chapter is that coordination is indeed attainable, as this is stated in the following Theorem.

Theorem 5.2.1 *There exists an optimal solution of problem (M), in which:*

$$X_{rs} = \sqrt{2(K_R + K_S)D/H_r P_s}, r = l, h \text{ and } s = d, u.$$

The proof of the Theorem follows directly from the intermediate properties in Lemma 5.2.1, and Proposition 5.2.1 below. The main steps are outlined as follows: First, by setting $X_{rs} = Q^J_{r,s}$, the constraints of (M) become a system of linear inequalities in Y_{rs}. By appropriate changes of variables we transform this system into an equivalent one, with two variables only. In Proposition 5.2.1; first, we establish a necessary and sufficient condition so that the last system is feasible and then, we show that this condition is always true. In the remainder of the section we present the steps of the proof in details.

Setting $X_{rs} = Q^J_{r,s}$, and after some simplifications the constraints of (M) are expressed as:

$$\begin{aligned}
qY_{ld} + (1-q)Y_{lu} &\geq G\sqrt{R_H}\left((1+F)\zeta - 2\sqrt{F}\right)\\
qY_{hd} + (1-q)Y_{hu} &\geq G\left((1+F)\zeta - 2\sqrt{F}\right)\\
pY_{ld} + (1-p)Y_{hd} &\leq G\frac{1-F}{\sqrt{F}}(1-\sqrt{R_P}\sqrt{F}\theta)\\
pY_{lu} + (1-p)Y_{hu} &\leq G\frac{1-F}{\sqrt{F}}(1-\sqrt{F}\theta) \qquad (5.2.5)\\
(qY_{hd} + (1-q)Y_{hu}) - (qY_{ld} + (1-q)Y_{lu}) &\leq G\zeta(1-\sqrt{R_H})(F-\sqrt{R_H})\\
(qY_{hd} + (1-q)Y_{hu}) - (qY_{ld} + (1-q)Y_{lu}) &\geq G\zeta(1-\sqrt{R_H})(F-1/\sqrt{R_H})\\
(pY_{lu} + (1-p)Y_{hu}) - (pY_{ld} + (1-p)Y_{hd}) &\geq G\theta Z\\
(pY_{lu} + (1-p)Y_{hu}) - (pY_{ld} + (1-p)Y_{hd}) &\leq G\theta Z
\end{aligned}$$

where:
$$\begin{aligned}
G &= \sqrt{DP_u H_h(K_R+K_S)}/\sqrt{2}\\
F &= K_R/(K_R+K_S)\\
R_H &= H_l/H_h\\
R_P &= P_d/P_u\\
\zeta &= q\sqrt{R_P} + 1 - q\\
\theta &= p\sqrt{R_H} + 1 - p\\
Z &= (1-F)(\sqrt{R_P}-1).
\end{aligned}$$

Under this reparametrization, it is true that F, R_H, R_P, ζ and θ take values in the range $(0,1)$, while $Z < 0$. System (5.2.5) can be rewritten using the following variable transformations:

$$\begin{aligned} y_l &= qY_{ld} + (1-q)Y_{lu} \\ y_h &= qY_{hd} + (1-q)Y_{hu} \\ y_d &= pY_{ld} + (1-p)Y_{hd} \\ y_u &= pY_{lu} + (1-p)Y_{hu}. \end{aligned} \tag{5.2.6}$$

Variables y_l, y_h denote the expected discount to a buyer of type-l or type-h, respectively. Similarly y_d, y_u represent the expected discount paid to the buyer by the supplier of type-d or type-u, respectively. However, finding $y_l, y_h, y_d, y_u \geq 0$ that satisfy (5.2.6) does not necessarily mean that there exist feasible discounts in problem (5.2.5). Lemma 1 shows that this is true if and only if the new variables satisfy a linear relationship.

Lemma 5.2.1 *For every non negative numbers y_l, y_h, y_u, y_d there exist $Y_{rs} \geq 0, r = l, h$ and $s = d, u$ satisfying* (5.2.6) *if and only if*

$$py_l + (1-p)y_h = qy_d + (1-q)y_u. \tag{5.2.7}$$

Proof:
Suppose (5.2.6) has a non negative solution in Y_{rs}. Multiplying the first two equations by $p, 1-p$ and the last two by $q, 1-q$, respectively and adding, it follows that:

$$py_l + (1-p)y_h = pqY_{ld} + p(1-q)Y_{lu} + (1-p)qY_{hd} + (1-p)(1-q)Y_{hu} = qy_d + (1-q)y_u \equiv w.$$

Therefore, (5.2.7) is necessary for (5.2.6) to have a solution. To show that it is also sufficient, if y_l, y_h, y_u, y_d satisfy (5.2.7), then one solution of (5.2.6) is:

$$Y_{lu} = y_l y_u / w, Y_{hu} = y_h y_u / w, Y_{ld} = y_l y_d / w, Y_{hd} = y_h y_d / w. \quad \diamondsuit$$

Note that if (5.2.7) holds, then (5.2.6) admits an infinite number of solutions Y_{rs} ($r = l, h$ and $s = d, u$) since the four linear equations are dependent; i.e., there are infinite choices of discounts for every set of values of y_l, y_h, y_u, y_d. This provides flexibility to the mediator when he designs the plan and he can propose a range for each of the four discounts. The minimum values of discounts are preferable by the supplier because he pays them to the buyer, where the buyer prefers the maximum values of discounts.

Based on Lemma 5.2.1, coordination is attainable if and only if there exist non negative numbers y_l, y_h, y_u and y_d that satisfy the equivalent system of constraints (5.2.5) and equation (5.2.7). From the last two inequalities of (5.2.5), we have that $y_u = G\theta Z + y_d$.

Substituting into (5.2.7) we obtain:

$$y_d = py_l + (1-p)y_h - (1-q)G\theta Z \tag{5.2.8}$$

$$y_u = py_l + (1-p)y_h + qG\theta Z. \tag{5.2.9}$$

Therefore y_d and y_u are uniquely determined by y_l and y_h, which reduces the numbers of variables by two. Thus, it suffices to substitute y_d, y_u in (5.2.5) from (5.2.8) and (5.2.9), and seek solution $y_l, y_h \geq 0$, that satisfy (5.2.5) and also result in $y_d, y_u \geq 0$. By doing this we obtain the following necessary and sufficient inequalities:

$$\begin{aligned}
y_l \geq & max\{0, G\sqrt{R_H}\left((1+F)\zeta - 2\sqrt{F}\right)\} \\
y_h \geq & max\{0, G\left((1+F)\zeta - 2\sqrt{F}\right)\} \\
\zeta(1-\sqrt{R_H})(F - \frac{1}{\sqrt{R_H}})G \leqslant & y_h - y_l \leqslant \zeta(1-\sqrt{R_H})(F-\sqrt{R_H})G \\
max\{0, -qG\theta Z\} \leqslant & py_l + (1-p)y_h \\
& \leqslant min\{G\frac{1-F}{\sqrt{F}}\left(1-\sqrt{R_P F\theta}\right) + (1-q)G\theta Z, \\
& G\frac{1-F}{\sqrt{F}}\left(1-\sqrt{F}\theta\right) - qG\theta Z\}.
\end{aligned} \tag{5.2.10}$$

In (5.2.10) we have that: $-qG\theta Z > 0$ and the two terms inside the minimum are equal, which results in further simplification. In summary, to find a feasible solution to problem (M), it is necessary and sufficient to find non negative values of y_l and y_h such that:

$$\begin{aligned}
y_l &\geqslant a^+ G\sqrt{R_H} \\
y_h &\geqslant a^+ G \\
d_1 G \leqslant y_h &- y_l \leqslant d_2 G \\
-qG\theta Z \leqslant py_l &+ (1-p)y_h \leqslant eG
\end{aligned} \tag{5.2.11}$$

where:
$$\begin{aligned}
a &= (1+F)\zeta - 2\sqrt{F} \\
a^+ &= max\{a, 0\} \\
e &= \tfrac{1-F}{\sqrt{F}}(1-\sqrt{F}\theta) - q\theta Z = \tfrac{1-F}{\sqrt{F}}(1-\sqrt{F}\theta\zeta) \\
d_1 &= \zeta(1-\sqrt{R_H})(F - 1/\sqrt{R_H}) \\
d_2 &= \zeta(1-\sqrt{R_H})(F-\sqrt{R_H}).
\end{aligned}$$

We have finally reduced the problem of finding a coordinating discount plan to a system of linear inequalities in y_l and y_h. In Proposition 3.2.1 we show that this system is always feasible.

Proposition 5.2.1 *i) A necessary and sufficient condition for the system of constraints* (5.2.11) *to have a solution is that:*

$$a^{+} - p\min\{d_2, a^{+}(1-\sqrt{R_H})\} \leqslant e. \tag{5.2.12}$$

ii) Condition (5.2.12) *is always true.*

Proof: I) Note that (5.2.11) corresponds to six linear inequality constraints. The first four define an unbounded polyhedron K in the non negative quadrant of the (y_l, y_h) plane. We consider two cases for the form of this set and in each case examine when K has non empty intersection with the last two constrains. First, we observe $a^+G\sqrt{R_H} \leqslant a^+G$ and $d_1 < 0$, since $F < 1 < 1/\sqrt{R_H}$. Therefore, the point $(a^+G\sqrt{R_H}, a^+G)$ always satisfies $y_h - y_l = a^+G(1-\sqrt{R_H}) \geqslant 0 > d_1G$ and may or may not satisfy $y_h - y_l \leqslant d_2G$. We thus consider two cases:

Case A: $a^+(1-\sqrt{R_H}) \leqslant d_2$.

In this case, the point $(a^+G\sqrt{R_H}, a^+G) \in K$ (Figure 5.1). Furthermore condition (5.2.12) becomes $a^+ - pa^+(1-\sqrt{R_H}) \leqslant e$. Suppose this is not satisfied; then, for any $(y_l, y_h) \in K$ we have that $y_l \geqslant a^+G\sqrt{R_H}$, $y_h \geqslant a^+G$. Therefore $py_l + (1-p)y_h \geqslant G(a^+ - pa^+(1-\sqrt{R_H})) > Ge$; thus, the sixth constraint in (5.2.11) is violated; i.e., the system (5.2.11) is not feasible. On the other hand, if $a^+ - pa^+(1-\sqrt{R_H}) \leqslant e$, we can find a point $(y_l, y_h) \in K$ that satisfies the last constraint of (5.2.11). To do this, let $y_l = a^+G\sqrt{R_H} + \delta, y_h = a^+G + \delta$, with $\delta > 0$. Then, $y_h - y_l = a^+G(1-\sqrt{R_H}) \leqslant d_2G$ and $py_l + (1-p)y_h = a^+G\theta + \delta$. If we set $\delta = G(e - a^+\theta) > 0$, then the last inequality of (5.2.11) is satisfied with equality, then the fifth inequality is also satisfied.

Case B: $a^+(1-\sqrt{R_H}) > d_2$.

In this case, $(a^+G\sqrt{R_H}, a^+G) \notin K$, but $(a^+G - d_2G, a^+G) \in K$. For any $(y_l, y_h) \in K$ it is true that $y_l \geqslant a^+G - d_2G$ and $y_h \geqslant a^+G$. Then, by following an analogous reasoning as in Case A, we can find a solution that satisfies the last two constraints of (5.2.11) if and only if holds $a^+ - pd_2 \leqslant e$.

II) To show that condition (5.2.12) is always true, we distinguish four separate cases, according to the value of $min\{d_2, a^+(1-\sqrt{R_H})\}$ and the sign of a.

Case 1: $a^+(1-\sqrt{R_H}) \leqslant d_2$.

In this case condition (5.2.12) can be written as $a^+\theta \leqslant e$.

Case 1a: $a \leq 0$. This means that $a^+ = 0$. Then (5.2.12) holds, since $e > 0$.

Case 1b: $a > 0$. This means that $a^+ = a$; thus, we must show that $a - pa(1 - \sqrt{R_H}) \leqslant e$; i.e., $a\theta \leqslant e$, which after some algebra reduces to: $\theta(\zeta - \sqrt{F}) \leqslant (1 - F)/(2\sqrt{F})$. If $\zeta - \sqrt{F} \leqslant 0$ then it is immediate. If $\zeta - \sqrt{F} > 0$ then $\theta(\zeta - \sqrt{F}) < \zeta - \sqrt{F} < 1 - \sqrt{F}$, since F, θ and $\zeta \in (0,1)$. In addition, $(1-F)/(2\sqrt{F}) = (1+\sqrt{F})(1-\sqrt{F})/(\sqrt{F}+\sqrt{F}) > 1 - \sqrt{F}$. Therefore, the inequality holds.

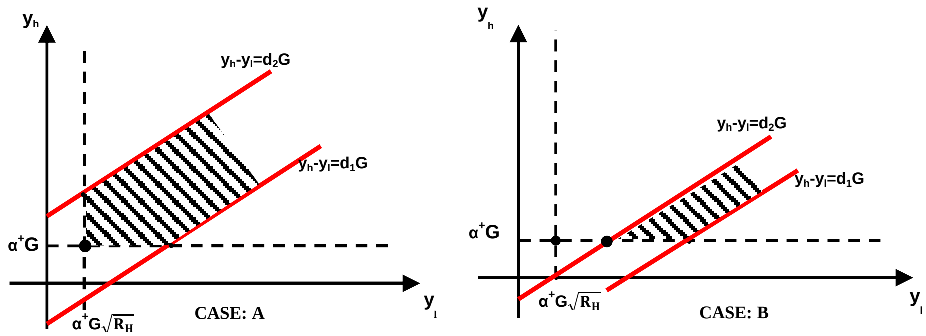

Figure 5.1: The two cases of condition (5.2.12).

Case 2: $a^+(1-\sqrt{R_H}) > d_2$.

In this case condition (5.2.12) can be written as $a^+ - pd_2 \leqslant e$.

Case 2a: $a \leq 0$. We must show that $-pd_2 \leqslant e$. After substitutions and some algebra, the inequality becomes: $\zeta(1-\theta)(\sqrt{R_H}-F) \leq (1-F)(1/\sqrt{F}-\theta\zeta)$. For the left hand size, we have: $\zeta(1-\theta)(\sqrt{R_H}-F) \leq (\zeta-\zeta\theta)(1-F) \leq (1-\zeta\theta)(1-F)$, while for the right hand size, we have: $(1-F)(\frac{1}{\sqrt{F}}-\theta\zeta) \geqslant (1-F)(1-\theta\zeta)$. Thus, the inequality holds.

Case 2b: $a > 0$. We must show that $a - pd_2 \leqslant e$. After some algebra, the inequality becomes: $\zeta\left(1+\theta-(1-\theta)\sqrt{R_H}\right) \leq (1+F)/\sqrt{F}$. Since F, R_H, θ and $\zeta \in (0,1)$, it is easy to show that: $\zeta\left(1+\theta-(1-\theta)\sqrt{R_H}\right) \leq 2 \leq (1+F)/\sqrt{F}$. $\diamondsuit$

We prove Theorem 5.2.1; i.e., the mediator always could design an appropriate plan to coordinate the whole chain. This means that there exists a feasible plan in which the individual objectives are aligned with the incentives of the chain. Thus, there exists (non-negative) discounts $Y_{rs}, r = l, h$ and $s = d, u$, which should be provided by the supplier to the buyer to induce the latter to order the joint optimal lot size, because these discounts optimize the individual costs of both nodes.

The inclusion of the mediator is crucial because it completely eliminates the asymmetry of information, making possible the centralized solution under private information in a decentralized model. By assuming a different definition of reservation levels, we prove that perfect coordination is always attainable under two-way information asymmetry; a particularly interesting finding, since it is in contrast to several statements of previous researchers who claim non-existence of coordination under information asymmetry (Ha, 2001; Ozer and Raz, 2011). In our model the reservation levels of the nodes are not exogenously defined and depend on the private information that the nodes possess. Furthermore, our work extends the approaches of Cakanyildirim et al. (2012) and Zissis et al. (2015) who showed the feasibility of perfect coordination under information asymmetry for some cases.

The nodes decide to report honestly their private information and follow the mediator plan since this is in their self interest; i.e., the node's individual cost functions are minimized under the mediator plan m. The difference $C_J(Q^R) - C_J(Q^J)$ is equal to the cost savings that can be achieved by the mediator mechanism, compared to the case where the buyer acts alone without receiving any discounts. It represents the mediation benefit which will be shared among the participants. As mentioned, the mediator does not increase the system cost; he will facilitate the communication between the nodes and help them to coordinate their decisions, reaching the chain's optimal level of overall costs. It is beyond the scope of this work to define the relative power of the nodes and allocate the mediation benefit between them. However, in Section 5.3 we examine the two extreme cases; when the supplier minimizes his expected cost under the coordination (minimum values of discounts) and the case in which the buyer minimizes her expected cost under the coordination (maximum values of discounts). Hence, all the intermediate cases are possible according the relative power of the nodes.

5.3 Mediator's Flexibility

The preceding analysis leads to several interesting questions regarding to the QDs that coordinate the chain. We have shown that the coordination is always attainable via a credible mediator. In this section, we conduct computational experiments which offer insights about the coordination benefits and the sensitivity of the mediator plans that coordinate the chain, with respect to various model parameters. The main aim of the computational experiments is to assess the flexibility that the mediator has when he designs the mechanism under which the SC will be coordinated.

Recall that the model involves the following nine independent parameters: D, K_R, K_S, H_l, H_h, P_d, P_u, p, and q, while the solution is the coordination of the chain via the mediator plan $m = \{(X_{rs}, Y_{rs}), r = l, h \text{ and } s = d, u\}$ (Theorem 5.2.1). The mediator is always able to design a mechanism that coordinates the channel; however the discounts $Y_{rs}, r = l, h$ and $s = d, u$ are not unique (Lemma 5.2.1). The existence of multiple feasible solutions to the coordination problem is a beneficial feature, since it provides the mediator with the flexibility to take into account secondary objectives in the design of the discount plan. The discounts represent net payments from the supplier to the buyer; so it is reasonable that the supplier prefers as small discounts as possible and the buyer the opposite. We consider the difference between the minimum and the maximum values of the expected discounts as the mediator's flexibility in designing a coordination mechanism that ensures minimum total channel cost. The experiments we perform provide us with insights about the mediator flexibility and how this affects the payoffs of the nodes, indicating the feasible range of the acceptable payoffs. According to the nodes' relative power, each of them may be able to enforce a mediator plan that optimizes its individual costs (as a secondary objective) given the system's coordination (primary objective).

In the experiments we calculate the maximum and the minimum percentage of the overall system costs that the buyer bears under the coordinating mechanism m. The maximum and the minimum buyer's cost is presented as a function of the ratio of setup, holding and production costs, keeping all other parameters constant in each case. Although one can argue that individual values of the costs are important when deriving actual supply chain costs, what really defines the direction of any managerial decision is the relative value of these costs. Since in our model there are only two nodes, the remaining cost is paid by the supplier. Therefore, depending on which of the nodes has higher bargaining power, the actual plan that will be implemented will enforce a cost allocation between the two extreme cases. The lower surface corresponds to the case in which the buyer has higher bargaining power (i.e., implementation of the maximum values of discounts), and similarly the higher one for the supplier (i.e., implementation of the minimum values of discounts). All the experiments have been performed under a large range of parameter values; although we only present specific cases, the observations and insights we discuss are quite robust.

In the first experiment it is examined how the ratio of the setup costs (K_S/K_R) affects the cost allocation between the nodes, showing also mediator's flexibility (Figure 5.2). We assume that there is no prior knowledge about the private information that the nodes possess, thus in the Bayesian game formulation we use a non informative prior about the low and the high value of both holding and production costs (i.e., $p = q = 1/2$).

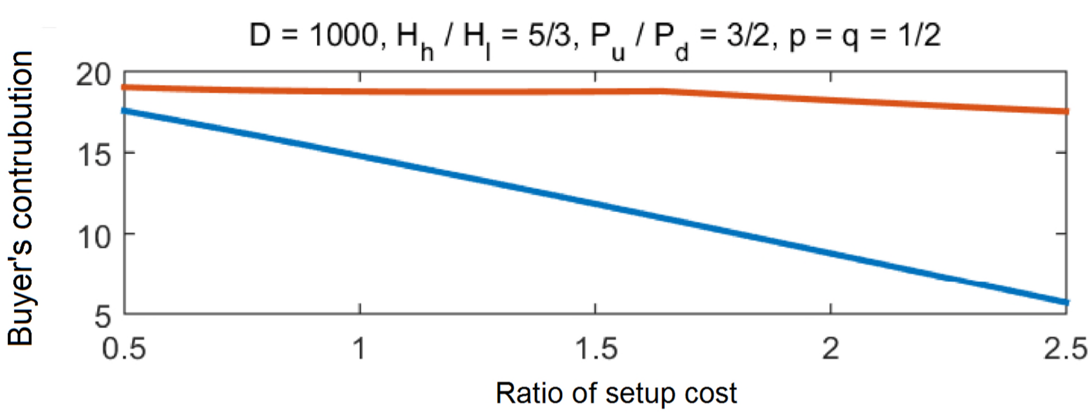

Figure 5.2: Range of buyer's cost percentage as a function of the ratio of the setup costs.

In Figure 5.2 we observe that when the supplier has the relative power to enforce his preferable mediator plan, he can keep his percentage contribution to the total cost fixed as the ratio of setup costs increases. If the buyer can enforce the plan which optimizes her individual cost, she can reduce her percentage contribution making the supplier to get hurt more significantly as ratio of setup costs increases. We observe that the mediator's flexibility is increasing for larger values of the ratio K_S/K_R, making the mediator more powerful. A particularly interesting observation that arises from Figure 5.2 is that as the ratio of the setup costs decreases the mediator's flexibility is decreasing. The actual decision maker about the order quantity is the buyer and, when $K_R >> K_S$, her individual objective is almost aligned with the chain (and thus the mediator's) objective; i.e., to minimize the total channel cost (5.1.3). A side effect is that in this case the presence of a mediator is not crucial for channel coordination.

In the next two experiments we investigate the impact of private information of each participant both on the cost allocation between the nodes and on the mediator's flexibility. Figure 5.3 shows how the buyer's degree of information asymmetry as depicted by the ratio of holding costs, as well as the corresponding prior probability, affect the coordination mechanism m. More specifically, in Figure 5.3:

- the x-axis corresponds to the ratio of the holding costs that represents a measure of the buyer's information asymmetry,
- the y-axis corresponds to the probability $p = P(H_R = H_l)$; i.e., the probability of the holding cost to take the low value,
- the z-axis corresponds to the percentage of the overall system costs that the buyer bears under the mediator's coordinating mechanism m.

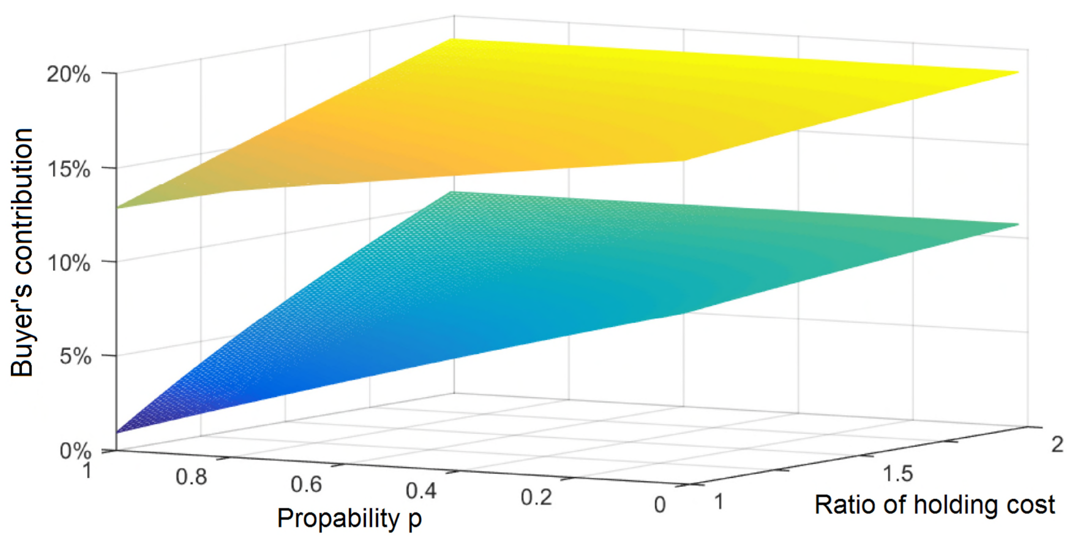

Figure 5.3: Range of buyer's cost percentage as a function of the ratio of holding cost and the probability of low value of it.

Similarly, Figure 5.4 shows how the cost allocation and mediator flexibility are affected by the supplier's private information on the production cost and the probability q. The x-axis corresponds to the ratio of the production costs, while the y-axis corresponds the probability of the production cost P_S takes the low value P_u. In both experiments, the two surfaces correspond to the minimum and maximum buyer's percentage contribution to the total channel cost, under the coordinating plan m.

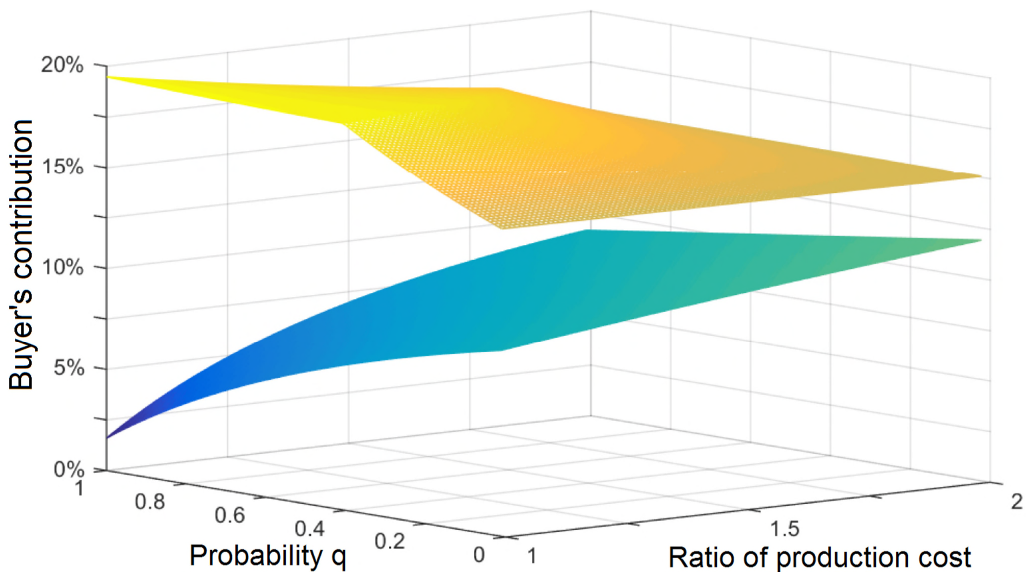

Figure 5.4: Range of buyer's cost percentage as a function of the ratio of the production cost and the probability of low value of it.

From Figures 5.3 - 5.4 as the asymmetry of either node decreases; i.e., the ratio of the holding costs (buyer's private information) and the ratio of the production costs (supplier's private information) take values close to 1, the mediator's flexibility also decreases. This is reasonable, because the elimination of two-way information asymmetries moves one of the nodes to have perfect information on the other, restricting the mediator's flexibility. For example, in Figure 5.3, the elimination of information asymmetry means that the supplier is able to reduce his contribution to the total channel cost even when the buyer has the power to enforce her preferable mediator plan. The new insight we obtain from these figures is that as the asymmetry, and as a result the lack of information, increases, the significance of relative bargaining power is substantially increased.

In the last experiment we examine how the information asymmetry affects the cost allocation and the mediator's flexibility (Figure 5.5). We study the cost allocation between the two nodes, as a function of both players' private information, using a non informative prior about the low and the high value of the production and holding cost (i.e., $p = q = 1/2$). The mediator is only aware about the prior probability distributions of production and holding costs. Hence, the mediator is faced with the information asymmetry of H_l/H_h and P_d/P_u, when he designs the coordinating plan. In our experiment the range of both ratios is from 1 up to 2. Note that the case of complete information corresponds to ratio equal to one.

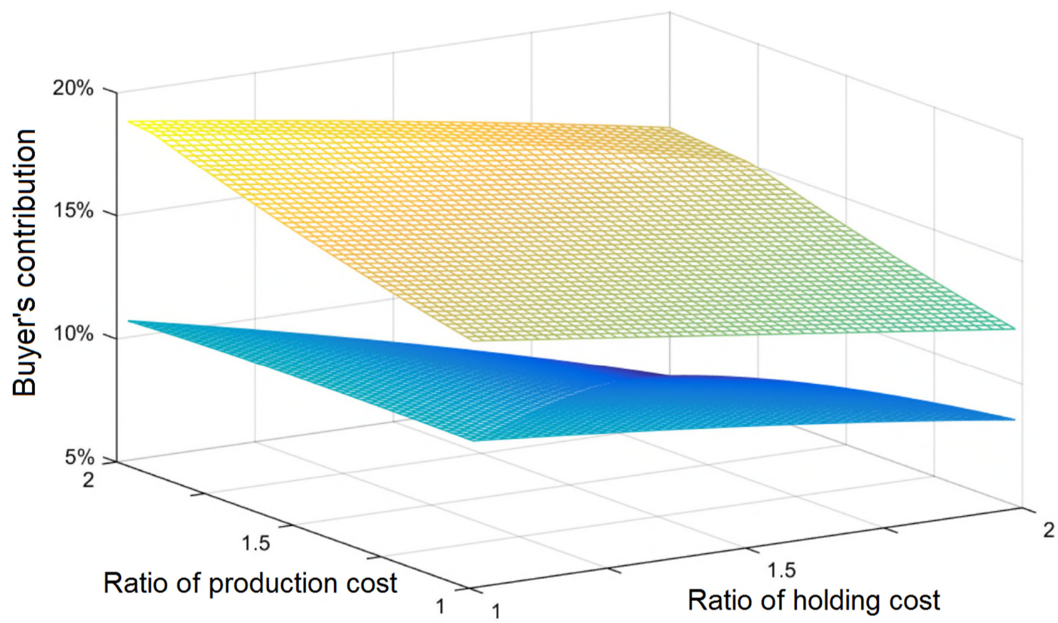

Figure 5.5: Range of buyer's cost percentage as a function of the information asymmetry.

We observe from Figure 5.5 that as the system information asymmetry increases, the mediator's flexibility is reduced. Consistently to the literature, in the case of complete information the relative node power becomes crucial for the cost allocation, since we know that the node who has the greater power is able to coordinate the channel and absorb all the benefits for itself (Corbett, 2001; Ha, 2001).

5.4 Findings

In this chapter we considered a two-stage SC, one supplier producing a single product in a lot-for-lot fashion, and one buyer that orders and stores the same product in fixed quantities. Both nodes have private information that affects their decisions and their reservation levels. Our model provides fundamentally new insights into the nature of information that nodes possess and how this information affects both the total channel cost and the cost allocation between the nodes. We have modeled the problem of node interaction via GT and used the RP to coordinate the nodes' decisions through communication. The nodes are free to communicate any private information they may possess, exclusively through a credible mediator. Misinformation and deception are allowed and both nodes choose if they will join to the mediator mechanism or not. Our results indicate that both nodes reveal their private information because it is in their self interest and perfect coordination even with asymmetry of information is always attainable via the mediator. This result is in sharp contrast to previous SCM research, where coordination is not always possible. Finally, we have reached closed form expressions of the QDs that the supplier should provide to the buyer in order for the latter to decide on the joint optimal lot size and achieve coordination of the whole chain.

Chapter 6

Conclusions and Recommendations for Future Research

6.1 Synopsis of Research Contributions

The work presented in this thesis provided several contributions in the way in which we can model and study the communication within Supply Chains and how the nodes of such systems could be coordinated. We aimed at developing a framework in which the nodes have both private information and the option to coordinate their actions, without being directly constrained when they make their own decisions. Below we summarize the key contributions of the dissertation.

6.1.1 Coordination

The core problem studied in this thesis was the coordination of the nodes decisions in modern Supply Chains. We proposed an innovative way to achieve such a smooth coworking of the chain without restricting the nodes' freedom to make decisions, since we wanted to address decentralized systems (such as those encountered in almost all real-life situations), in which all the nodes are independent decision makers. The nodes make their decisions in order to maximize their own utilities functions and our proposed approach managed to offer a way in order to align individual node incentives with the incentives of the supply chain as a whole.

6.1.2 Information Asymmetry Modeling and Node Communication

We studied and analysed two node supply chains based on game theory. In the second part of the dissertation, we provided the opportunity to the nodes to communicate with each other through a mediator. The latter allows the honest sharing of private information in order to enable perfect coordination of the chain. Through this perfect coordination, without contracts or constraints, the individual profits for all the participants are raised and better overall solutions are reached.

6.2 Future Research Directions

There is ample room for potential advancements regarding the proposed framework of the nodes' communication to converge their decisions (strategies) without any contracts. A worth pursing research direction is the investigation of SCs with one manufacturer and many retailers or one retailer procuring the same product from many manufacturer; in this case we should consider the notion of competition which alters the properties of the game. Another very promising line of research is the study of SCs with more than two distinct nodes. For example, to study a SC with manufacturer, distributor and retailer. Finally, advanced coordination policies (expect of QDs) could be examined for SCs that exhibit additional complexities (e.g., multi-echelon inventory systems, multiple cost functions, capacities etc.).

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