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MSc DISSERTATION

**Portfolio optimization theory: An empirical application on
Athens Stock Exchange**

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Copyright Statement

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Abstract

The present dissertation aims to examine and analyze some important issues relating to the international portfolio management, focusing on portfolio optimization techniques. The theoretical framework of this study is focused on a detailed presentation of the importance of risk in portfolio management, due to the uncertainty of the trust and solvency of individual's actions. In portfolio management, there are numerous risk types, the most important of which are market risk, liquidity risk, credit risk and operational risk. As for the risk measurement, the most important models used here are: Conditional Value-at-Risk (CVaR), which is a special version of Value-at-Risk (VaR), Mean Absolute Deviation (MAD) and Put-Call efficient frontier. Finally, the theoretical discussion of this study closes with a brief presentation of theory of derivatives and hedging strategies.

The empirical framework of this study focuses on the portfolio optimization in the case of Athens Stock Exchange based on 16 companies listed on FTSE 20 index. Applying the optimization techniques as mentioned in the previous paragraph in the period 31/01/2013 - 30/09/2014, portfolio returns of each model were calculated as well as FTSE 20 return. The analysis concluded that the Put-Call portfolio optimization model, is the most efficient from the three compared.

Key Words: Portfolio Management, Risk, Risk Types, Risk Measurement, CVaR, MAD, Put-Call



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1 Introduction

1.1 General information and evidence about risk and portfolio management

In today's complex environment of globalization, of constantly intense competition, of radical economic change and of continual liquidity at the national and international level, among the major problems to be faced by anyone who deals with economic affairs, is the creation and maintenance of an efficient investment portfolio. The investment portfolio is a collection of assets, which are designed to yield profits to the investor, based on a specific economic goal. The idea of investment portfolios is so deeply ingrained in the economic system that it is difficult to imagine a world without their existence. But the fact is not always the case (Schultz Collins Lawson Young, Inc., 2008; Elton, Gruber, Brown, & Goetzmann, 2014).

Until the 1950s, the concept of portfolios, particularly the equity portfolio, was completely different. Regardless of the purpose of each investor, investing in equity securities was a gamble, as there were insufficient economic data available to make an informed decision about the securities market. Investors also focused on opportunities for profit offered by each equity instrument rather than a return-risk ratio. In such an environment, the acquisition of sufficient financial data was the main problem that concerned investors, but not as to draw conclusions as to the correlation of each debt instrument with different investment profiles, for example, finding companies with rising and cheap shares (Capiński & Kopp, 2014).

This situation began to change in 1952 when Markowitz published his work 'Portfolio Selection' and introduced the mathematical sense of the relationship that the risk has a debt with the same performance. Initially, by using Markowitz's mean-variance models and later several other models that tried to expand further the former's study, investors began to create portfolios that favor specific investment style and preferences. As the years passed and loomed the value of economic optimization models for investment decisions, so most researchers involved in this field till nowadays, when the process of creating and managing securities portfolios has now been analyzed and parameterized significantly (Markowitz, 1952; Elton, Gruber, Brown, & Goetzmann, 2014). In the years that followed, economic optimization



models for investment decisions became the norm and parameterized significantly for creating and managing securities portfolios.

The problem of optimizing portfolios continues to exist as international and non-international markets often present a chaotic and unpredictable behavior. As a result, the creation of a model that will completely provide for the course of economic activity is impossible. Moreover, due to the fact that equities are the most vulnerable elements to market fluctuations, the development of models to predict the path that will follow is relatively difficult and has been a major subject of research. Particular emphasis is given to the fact that most optimization models face complex problems on the composition of multiple and often conflicting criteria. Therefore, it is necessary in the process of decision making that the investor has a direct involvement with formulating preferences. Also, during the optimization process, historical data should be used to reach a conclusion as to the optimal composition of the portfolio for each investment style. It is clear that these models cannot fully predict the course of the shares in all cases, but they may provide a satisfactory measure that can be combined with other financial data by the investor to create a portfolio that suits his/her investment policy. (Markowitz, 1952; Elton, Gruber, Brown, & Goetzmann, 2014).

According to Xidonas et al. (2010), an investor's equity portfolio is the most risky investment due to the high volatility that comes with the absence of a potential partial differentiation of risk exposure characteristic of investing in fixed income securities, deposits, or derivative products. Moreover, the very large number of equity instruments relative to other securities classes that are negotiated in stock markets, render it an extremely difficult process to manage an equity portfolio as the simultaneous investigation and evaluation of hundreds or thousands of securities available for investment options are required. (Xidonas, Mavrotas, & Psarras, 2010)

The management of equity portfolios is a particularly complex problem, and focuses successively on three different levels of decision (Xidonas, Mavrotas, & Psarras, 2010; Capiński & Kopp, 2014; Elton, Gruber, Brown, & Goetzmann, 2014):

- a) The selection of equity accruing to the best investment opportunities,
- b) The allocation of funds aiming at the optimum portfolio composition, and



c) Comparative assessment of the portfolios constructed.

The complexity of the problem of managing equity portfolios is linked to three other fundamental parameters which influence any decision making process that are the following (Xidonas, Mavrotas, & Psarras, 2010; Capiński & Kopp, 2014; Elton, Gruber, Brown, & Goetzmann, 2014):

- a) Parameter uncertainty,
- b) The existence of multiple criteria and
- c) Preferences of decision feedback.

Another particularly critical parameter which helps to increase the complexity associated with the problem of managing equity portfolios is the existence of many stakeholders. More specifically, the bodies that make up the environment of the parties involved in the analysis problems can be grouped into four categories (Capiński & Kopp, 2014):

1. Bodies linked to the organization and supervision of the market,
2. Listed stock market companies,
3. Institutional and private investors,
4. Investment services providers.

After this brief analysis, it is now clear that the problem of selecting and holding an equity portfolio shows great complexity and uncertainty. The need for adequate indicators and decision tools that help in this process shows that the old empirical approach of investing needs new methods, such as stochastic analysis, optimization, the usage of genetic/evolutionary algorithms, etc., in order to better address the investment risks that are inevitably associated with managing equity portfolios.



1.2 Research aim and objectives

Taking the above information into account, the focus of this dissertation is to provide a detailed portfolio management theory in the international level so as to determine the optimum level of a specific portfolio. This study will concentrate in the Greek case; analyzing empirically the portfolio optimization of some prominent companies in the Athens Stock Exchange, can be summarized in the following sentence:

"Portfolio optimization theory analysis with empirical application on Athens Stock Exchange"

The research objectives are to demonstrate how each analysis contributes to a specific goal and that together all the analyses provides the conclusion that demonstrates the achievement:

1. Analysis of the risk theory in general, as well as in portfolio management theory, regarding the types of risk and ways of dealing with each of these risks.
2. Detailed analysis of the risk measures, including the optimization models and their mathematical formulas.
3. Discussion of derivatives, including a brief analysis about hedging and relative strategies of them.
4. Description of the methodological tool and approach used in the present study in order to analyze the empirical data.
5. Presentation of the empirical application and the relative research findings.

These objectives are the process that will be followed in this research so as to deduce its empirical application as efficiently as possible. The first three research objectives are related to the literature review whereas the other two consider the methodology and results.



1.3 Structure of the dissertation

The present dissertation consists of five sections. The first section is the introduction, i.e. the present chapter of the study which includes a brief study about risk and portfolio management to identify the research aim and objectives, and its importance.

The second section is the detailed analysis of the literature review related to the topic discussed within. It will include the facts presented in research objectives 1,2 and 3, i.e. types of risks, risk measures and theory of derivatives and hedging.

The third section addresses the research methodology by presenting some brief information about the importance of conducting empirical research, the methodological tools and the adopted strategies

The fourth section includes the findings of the present research by presenting all the necessary facts and results regarding the specific topic examined here.

Finally, the fifth section is that of conclusions and recommendations. findings associated with the empirical study will be addressed, and they will be linked with theories examined in literature review, to determine whether findings are related or not with the theory.

1.4 Importance of the study

The current financial and economic climate is in crisis throughout the world creating new challenges and corporate structures to improve performance in multinational companies. This topic was chosen to demonstrate the importance of this crisis.

The most common concern in the business world today is risk management. Risk management is an integral practice in minimizing risk for corporations to ensure growth and success. The management of risks is an everyday process of identifying, monitoring and evaluating risks to provide a secure decision making process resulting



in success. This strategy can be achieved effectively by using a variety of appropriate methodologies and tools (Kakouri, 2011; Harari, 2015).

It is especially important in the Greek market due to the deterioration of the financial and sovereign default (July 2015), that the practice of measuring risk against corporate and market losses identifies the affects it has had on businesses. These exogenous factors need to be evaluated to assist corporations in their implementation of constructive strategies that will help them overcome this crisis.

Many Greek companies (e.g. banks, oil companies, construction companies and other companies listed on the Athens Stock Exchange) have recently adopted new strategies to overcome the problems inherent with this crisis and have made significant investments in new developments and innovations (Kakouri, 2011).



2 Literature Overview

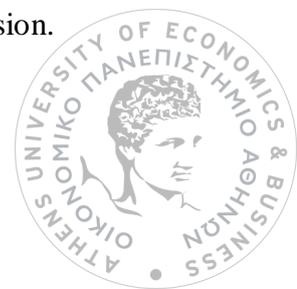
2.1 Classification of financial risks

The theme of "risk assessment" is a vast field of research whereas its theories and applications find response in many sectors of the economy. Risk assessment is a procedure of a particular importance fully realized in recent years by all actors in global economies (e.g. governments, large companies, entrepreneurs, private individuals, etc.). For this purpose, various models of measuring risk assessment and overall risk management have been strongly developed in recent years (Clusif, 2009; Coleman, 2011).

It is also recognized that, eventually, the concept of risk is linked very closely with the logic and the concept of uncertainty. The logic is that, in a financial transaction, one who carries it out takes some risk in that he/she cannot predict the end result. Indeed, in many cases it is possible to be aware of possible outcomes, while in other cases no such information is possible, which increases the risk. Therefore, almost all financial transactions involve risk (Levišauskaite, 2010).

In terms of the risk management, the risk is a concept that is closely related to the concept of uncertainty. The concept of uncertainty is closely related to the concept of variation or variability or volatility. These concepts have to do with the fact that all financial operations are characterized by volatility or instability regarding their possible prices or their likely future developments (Coleman, 2011; Capiński & Kopp, 2014).

For example, the stock of a company determines its value on a daily basis in relation to supply and demand, which are determined by a number of different factors. The stock price is determined at a level which is unknown in advance, in the sense that it is not known exactly how will a stock price close at the end of a trading session.



Therefore, after the stock price is unknown in advance, it is considered that there is uncertainty for this price. Moreover, future prices of stocks are also unknown, reflecting the price's instability in future time periods. Therefore, anyone who attempts transactions with stocks will do it in a climate of uncertainty and inherent risk taking (Coleman, 2011; Capiński & Kopp, 2014).

Another example is the money market characterized by interest rate levels. The interest rate base for various economies (e.g. the European rate, Americans etc.) specified by the relevant central banks is considered unstable, yet the effects are minimized by a not so frequent change rate. Instead, the various interest rates in the interbank market for different maturities change every day in international money markets (Coleman, 2011; Capiński & Kopp, 2014).

Another example is the foreign exchange market. Also, in this case the exchange rate for each currency relative to another varies daily in international currency markets. Thus, in this case, there is instability in price, as in many other examples of markets and assets (Coleman, 2011; Capiński & Kopp, 2014).

However, the risk has not only to do with the pricing of the different assets in different markets, as these assets are bought and sold and what interests most is what profit or what loss an economic entity gains (e.g. a private investor, institutional investor, business, etc.) when it makes a purchase or sale of an asset. The interesting aspect is the outcome of the sale and risk the trader attempted to take (Coleman, 2011; Capiński & Kopp, 2014).

The profit or loss from a transaction (or investment) of an asset is the so-called 'return'. In general, transaction or investment in any asset will have a future effect, i.e. a profit or a loss also unknown in advance. Therefore, it becomes clear that instability and volatility in the prices of different markets cause instability and volatility in returns, i.e. the final result obtained by a transaction of an asset. So, Therefore, the concept of risk is not just about the level of asset prices in different markets but also with returns determined by the transactions that occur under the various assets (Coleman, 2011; Capiński & Kopp, 2014).



Many associate the concept of risk with a level of uncertainty in that it is not known in advance whether there will be profit or loss. *Id est.*, they separate the risk in downside risk, which indicates how likely they are to lose from a trade in an asset and upside risk, which indicates how likely they are to win by this transaction (Elton, Gruber, Brown, & Goetzmann, 2014).

However, others consider the concept of risk only from the negative perspective, taking into account only one dimension (downside risk) and therefore, in the context of risk assessment, they estimate the potential losses from a transaction resulting from volatility and instability in prices as well as the returns on various assets (Elton, Gruber, Brown, & Goetzmann, 2014).

Risk assessment is considered a very important process in the context of risk management. In order, however, to be efficient and correct risk management, two things should take place. One is to understand what types of risks are involved in a particular transaction. The other is to find efficient and effective methods in order to measure these risks (Elton, Gruber, Brown, & Goetzmann, 2014).

The measurement of risk differs depending on the type of risk. It is well known that a risk cannot be measured, nor managed, if we do not know the source or type of the risk. In practice, there are numerous risks arising from similar numerous sources. However, they can be grouped by creating risk "clusters", so that they can be measured in the same way (Elton, Gruber, Brown, & Goetzmann, 2014).

So, clearly, there are various types of risk. Depending on the transaction and the type of an asset, risk on a future outcome may take different forms. A first classification is in Business and Financial risks. Business risks have to do with the ability of each business to operate efficiently and manage to generate significant profits, revenues and cash flows. Each company operates in a sector or market that has many risks, either at market level or at an individual business level. The variability of cash flows is considered a source of the so-called business risk (Andersen, Bollerslev, Christoffersen, & Diebold, 2011).

Financial risks have to do with the instability and volatility of different financial markets (e.g. stock markets, money market, foreign exchange, etc.). Such risks affect



financial institutions (e.g. banks, investment companies, insurance companies etc.) and many other companies, organizations or individuals involved with them. Financial risks are considered to have a range of sources depending on the nature of the asset involved in a financial transaction (Andersen, Bollerslev, Christoffersen, & Diebold, 2011; Moles, 2013; Capiński & Kopp, 2014; Elton, Gruber, Brown, & Goetzmann, 2014).

Particularly for the financial risks, the types are the following:

- Market risk,
- Credit risk,
- Operational risk and
- Liquidity risk.

The following subsection describes the financial risk types.



2.1.1 Market risk

Market Risk, which is referred to as "non-diversifiable risk", "risk volatility" or "systematic risk" affects the whole market or a wide area and not just a financial instrument or company, and thus, it cannot be compensated through diversification. Market risk is considered to be the risk of unfavorable changes in the market value of various assets because of the various changes taking place in the market where various assets are traded, during which it is possible to liquidate an asset. The liquidation period is considered very important in the context of the assessment of market risk, as the greater is this period, the more opportunities are existed for great change in the value of an asset (Dowd, 2002; Capiński & Kopp, 2014).

According to several economists, market risk can be addressed either in a simple manner, by liquidating various assets to prevent loss from a possible decline in value, if obviously it can be done, i.e. if the liquidation period is short depending on the nature of an asset, or in a more sophisticated way, i.e. hedging it by using appropriate transactions and derivative financial instruments (Dowd, 2002; Capiński & Kopp, 2014).

Market risk is typically measured using the method of VaR (Value at Risk or VaR). According to this methodology, a possible downside loss is estimated, with 95% or 99% probability of occurrence (Linsmeier & Pearson, 1996). Precisely, the maximum downside loss that occurs at a particular time horizon is estimated and is clearly due to changes in market parameters that affect a specific asset. So, it is evident that this well-known and popular method measures the downside risk. Market risk can be contrasted with non-systemic risk, which measures the risk of decline in value of an asset due to changes in a particular industry or sector, in contrast with the reduction due to a more widespread movement of the market (Linsmeier & Pearson, 1996; Dowd, 2002; Capiński & Kopp, 2014; Elton, Gruber, Brown, & Goetzmann, 2014).

Furthermore, market risk also includes some types of risks that are directly tied to the market. These types are the following (Dowd, 2002):

- Stock market risk,
- Interest rate risk and



- Currency risk.

One source of market risk for an asset are changes in the price index of a stock exchange where stock prices of companies are negotiated. Therefore, regarding the market risk resulting from changes in the general price index, there have been proposed various methodologies of its estimation and management. The portfolio analysis theory indicates that the overall risk of an asset due to changes in the price index of a stock can be distinguished by market or general risk. The specific risk is due to individual factors of an asset which has been assessed with risk factors beyond the market. (Whitelaw, 2000).

Also, regarding interest rate risk, it is considered to be the risk due to interest rate changes in a particular economy. So, it is obvious that the interest rate risk affects assets whose value is directly related to the level of interest that exists in an economy. As the interest rate is considered as the "price of money", it is reasonable for the interest rates as well as commodity prices to be determined by supply and demand curves. So, money supply and demand reflect on the level of interest rates on a daily basis in the operating framework for the financial markets, while the basic interest rate in each economy is determined by decisions taken by the monetary authorities, such as the central banks of various economies or supranational institutions (e.g. European Central Bank) (Basle Committee on Banking Supervision, 1997).

Finally, the currency risk is considered to be the risk of the reduction in the value of an asset due to movements and changes in exchange rates formed on the international currency markets. It is a risk which exists only in assets that are valued and traded in another currency, besides the domestic one of a particular economy. If the foreign assets do not have another type of risk (market, credit, etc.), then they only have currency risk (Papaioannou, 2006).



2.1.2 Credit Risk

Another risk type is credit risk. Credit risk is the risk of loss of any monetary compensation of an investor, due to the inability of a borrower to repay a loan or to fulfill a conventional obligation. Credit risk is closely linked to the expected return on an investment, with bonds to be the most characteristic example. The higher the perceived credit risk, the higher will be the required rates. Investors then hedge the credit risk by requiring payment of interest on the part of the debtor (Van Gestel & Baesens, 2009).

Credit risk includes the following sub-categories of risk (Van Gestel & Baesens, 2009):

- Default risk,
- Exposure risk and
- Recovery risk.

To begin with, Default risk is the probability that the issuer of a bond will be unable to repay the fund and its interest on time. The bonds issued by the government have virtually zero risk (if the government needs money it can print), while commercial bonds are more precarious as the probability of bankruptcy of a company is higher. Lenders and investors, in order to hedge the risk of bankruptcy, require interest, proportionate to the risk of the borrower. Thus, the higher the risk, the higher will be the required interest. Such risks are usually determined by the so-called 'credit ratings' which are mainly done by three major international rating agencies: Standard Poor's Moody's and Fitch (Van Gestel & Baesens, 2009).

Moreover, in general, Exposure risk may be considered as a degree of assessment in which a bank may find itself exposed to a counterparty, such as a borrower, in time and amount of money owed by the contractor at the time of default and is usually called "exposure at the time of default." The calculation of the exposure risk differs if it is achieved with the foundation approach rather than the advanced approach. In the foundation approach, the calculation of risk exposure is governed by regulatory



authorities, whereas under the advanced approach, banks have more flexibility about which method should apply to estimate the risk exposure on the nature of the transactions and its characteristics. The risk of exposure is used mainly in banking sector (Van Gestel & Baesens, 2009).

Last but not least, the Recovery risk describes what part of the amount owed at the time of default the lender is able to recover from the borrower. The percentage *that* they managed to recover in terms of overall debt is called recovery rate, while the percentage they failed to recover in terms of overall debt is called "Loss Given Default» (LGD) (Van Gestel & Baesens, 2009).

2.1.3 Operational Risk

Operational risk is not considered a financial risk. However, it takes place, not only in all businesses operation, but also in all operations within a company, so it should be studied as a type of risk (Girling, 2013).

With the expansion and use of new technologies and innovations in almost all sectors of the economy, operational risk has a special significance in the overall context of risk management, and so more and more businesses are concerned with its nature, measurement procedure and estimation. Operational risk, therefore, is made up of sub operations, especially poor operations of information systems used by an organization, such as various reporting systems, internal risk monitoring systems and any internal processes designed to produce valid and timely results compliant with the rules formed internally on risk management (Girling, 2013).

The general principle of measuring operational risk is to assess the possible emergence of an unpleasant event and the unexpected loss it will cause. This process is not so easy because the classification of sources of operational risk is subjective, and so is the process of collecting the relevant data (Girling, 2013).

2.1.4 Liquidity Risk

Finally, the Liquidity risk plays an important role in forming the market risk. It is believed that liquidity risk further aggravates the negative effects caused by market



risk, which refer to several dimensions such as failure to raise funds at a reasonable cost, asset liquidity risk, market liquidity risk, etc (Basel Committee on Banking Supervision , 2008; Andersen, Bollerslev, Christoffersen, & Diebold, 2011; Elton, Gruber, Brown, & Goetzmann, 2014).

The risk of raising funds depends on how dangerous the market considers an entity that wishes to raise funds. Essentially, one's risk in raising funds has to do, in many cases, with his/her/its creditworthiness. In this case, someone who needs funds, but it has not good credit faces more difficulties to find these funds needed. Thus, the liquidity risk in this case increases the cost of raising capital and thus reduces any imminent future profitability as well as further facilitate in raising additional funds (Basel Committee on Banking Supervision , 2008; Andersen, Bollerslev, Christoffersen, & Diebold, 2011; Elton, Gruber, Brown, & Goetzmann, 2014)..

Furthermore, risk asset liquidity has to do with *how easy is* the ease of which the buying and selling of a specific asset is, regardless of the liquidity that exists in this market that is traded. It is possible that an asset can be difficult to negotiate for purchase or sale. (Basel Committee on Banking Supervision, 2008; Andersen, Bollerslev, Christoffersen, & Diebold, 2011; Elton, Gruber, Brown, & Goetzmann, 2014).

Also, market risk liquidity has to do with whether frequent transactions (buying and selling) are in a particular market. It has been observed that the liquidity risk to that level causes high downside, as well as upside risk, when one counterparty is unwilling, for his/her/its own reasons, to make a transaction. However, it is considered that the risk for raising funds and market risk liquidity are strongly connected and it is considered that one is the substantial cause of the other and vice versa (Basel Committee on Banking Supervision , 2008; Andersen, Bollerslev, Christoffersen, & Diebold, 2011; Elton, Gruber, Brown, & Goetzmann, 2014)..

Thus, liquidity risk is considered a very important risk to be managed as effectively as possible, to eliminate the risk of bankruptcy. Of course, it should be pointed out that this extreme form of liquidity risk is often the result of other risks, albeit a process not very difficult or painful. More specifically, liquidity risk can be managed effectively with the implementation of asset-liability management where, given the liabilities that



exist for some entities in their time structure and liquidity, the most suitable assets are selected in terms of their own time structure and liquidity (Basel Committee on Banking Supervision, 2008; Andersen, Bollerslev, Christoffersen, & Diebold, 2011; Elton, Gruber, Brown, & Goetzmann, 2014).

2.2 Risk measurement

2.2.1 Risk in portfolio management theory

As it was mentioned in the introduction section, "the concept of portfolios, particularly the equity portfolio, was completely different. Regardless of the purpose of each investor, investing in equity securities was a gambling process, due to the unavailability of data to make an informed decision on the securities market (Ortobelli, Rachev, Stoyanov, Fabozzi, & Biglova, 2006; Gambrah & Pirvu, 2014).

But, in 1952, Professor Harry Markowitz formulated the so-called 'Modern Portfolio Theory' in his article "Portfolio Selection" published in the Journal of Finance, proposing the construction of efficiently diversified portfolios for undertaking investment. This was a milestone in the modern financial history. A portfolio is defined as the possession of a set of assets, each of which contributes to the portfolio with a certain ratio. This ratio is determined by the value of any asset below the total value of the portfolio. Markowitz in his publication deals with the selection of the optimal portfolio of assets available depending on their future performance (Markowitz, 1952; Ortobelli, Rachev, Stoyanov, Fabozzi, & Biglova, 2006; Gambrah & Pirvu, 2014).

Thus, the portfolio theory is based on the key assumptions that: any investor can maximize the return on his investment for a given risk level and; all investors are risk averse. Typically, for a given level of expected return, an investor will choose the alternative option with the lowest risk. The risk, or the risk of a portfolio is calculated based on the variance of the expected return (Markowitz, 1952; Ortobelli, Rachev, Stoyanov, Fabozzi, & Biglova, 2006; Gambrah & Pirvu, 2014).



The expected return of an investment for a given period is represented by a probability distribution of all possible outcomes and the corresponding returns. Investors maximize the expected utility of a time period, with curves to present a reduced marginal utility of wealth. Investors then make decisions solely based on the expected return and risk – the differing factors are a function only of the expected return and expected volatility of the return (Markowitz, 1952; Ortobelli, Rachev, Stoyanov, Fabozzi, & Biglova, 2006; Gambrah & Pirvu, 2014).

The theory of Markowitz shows that the appropriate diversification of a portfolio, i.e. the combination of assets with certain statistical properties, can contribute both to risk reduction and to the expected return increase. The basic concept, then, to be introduced here is that of efficient portfolio, defined as follows: "A portfolio is considered efficient if and only if there is no other portfolio that offers a higher expected return with the same or a lower risk, or respectively, lower risk with the same or higher expected return" (Markowitz, 1952; Ortobelli, Rachev, Stoyanov, Fabozzi, & Biglova, 2006; Gambrah & Pirvu, 2014).

However, in 1958, James Tobin's article "Liquidity Preference as a Behavior to Ward Risk" emphasized the importance for the investor to maintain a part of his funds in a form that does not involve risk. In this case, it is noted that the criterion for choosing among portfolios with different expected returns is the attitude of investors toward risk. On this basis, there are three types of investors (Butier, 2003):

- a) One which is neutral to risk and indifferent as to each choice's risk and the utility curve is represented by a straight line,
- b) The conservative investor who believes that the prospects of profits are lower than expected performance of a portfolio. In this case, the marginal utility increases but at a decreasing rate, and
- c) The reckless investor who believes that the potential profits are higher than the expected return of the portfolio. The marginal utility here increases at an increasing rate.



Thus, the importance of Tobin's work is that he introduced a Risk-Free Asset. Combining this asset with a portfolio, which belongs to the boundary of good choices, there are achieved superior returns on portfolios belonging to this border with the same risk rate, or vice versa, lower risk rate for the same returns.

2.2.2 Optimization models

Many tools and models have been developed to calculate the uncertainty on investment as the return it yields is not known in advance and that's why the term 'expected return' is used to measure the degree to which an investor's return expectation is close to the real portfolio return (Capiński & Kopp, 2014; Elton, Gruber, Brown, & Goetzmann, 2014).

A general way to describe the risk is to use the so-called 'scenarios'. Each scenario is a depiction of the future value of all parameters affecting the performance of the test portfolio. All of the scenarios reflect the range of possible variations of parameters that could occur between the current time and the end of the horizon. These uncertainties are essentially related to risk management. Scenarios can record different sources that cause risk and allow the production methods for all types of risk (Capiński & Kopp, 2014; Elton, Gruber, Brown, & Goetzmann, 2014).

But there are specific mathematical models that describe risk in a mathematical formula. The most popular mathematical models used in the present dissertation are the following (Capiński & Kopp, 2014; Elton, Gruber, Brown, & Goetzmann, 2014).

- Value at Risk (VaR)¹,
- Conditional Value at Risk (CVaR),
- Middle Absolute Deviation (MAD) method and
- Put-Call efficient frontier,

¹ VaR was not used in this case but it is an earlier version of CVaR model, so it is worth being analyzed here.



The above models are going to be presented below.

2.2.2.1 Value at Risk (VaR)

The method of Value-at-Risk (VaR) is a percentage measure which is the basis for risk measurement purposes. It is defined usually as the maximum expected loss/damage which an investor may suffer in a given period, for a selected level of confidence (certainty) $\alpha \cdot 100\%$. In finance, VaR, in terms of returns, is the worst return of a portfolio for a predetermined confidence level $\alpha \cdot 100\%$. Thus, the mathematical formula of VaR is as follows:

$$\mathbf{VaR(x, \alpha) = \min\{u: F(x, u) \geq 1 - \alpha\} = \min\{u: P\{R(x, \tilde{r}) \leq u\} \geq 1 - \alpha\}} \text{ Equation 1}$$

Where VaR (x, α) the $(1-\alpha) \cdot 100\%$ percentile of the distribution of portfolio returns and r is the random variable of asset returns which are unknown in the selection of the portfolio and R is the performance of the portfolio, which is a function of the portfolio x and the random variables of portfolio returns \tilde{r} (Consiglio, Nielsen, & Zenios, 2009).

Despite its popular use as a risk measure, VaR is not used in mathematical models to select optimal portfolio. While the calculation for a given x portfolio reveals that the performance of the portfolio will be under VaR (x, α) with probability $(1-\alpha) \cdot 100\%$, it does not provide information about the extent of the tail of the distribution, which can be big enough. In such cases, the performance of the portfolio may take significantly lower prices than the VaR and lead to substantial losses (Zenios & Markowitz, 2008).

VaR as a risk measure has a theoretical capacity of coherent risk measures, namely, sub-additivity. Furthermore, VaR is difficult to be optimized; when returns of the assets are specified in the terms of scenarios function, VaR function is non-smooth and not convex in contrast to the portfolio X and displays multiple local extremes. Thus, effective algorithms for solving problems with corresponding objective functions do not exist (Zenios & Markowitz, 2008).



2.2.2.2 Conditional Value at Risk (CVaR)

A similar, more effective and most frequently used mathematical model to measure risk is the Conditional Value at Risk model, or simply, CVaR. CVaR is a relative measure of risk. It is usually defined as the conditional expectation of losses exceeding the VaR in a certain level of confidence (In this case, VaR is also defined as a percentile of a function loss). In other words, CVaR is the conditional expectation of the returns of the portfolio to be lower than the performance of VaR (Zenios & Markowitz, 2008). For continuous distributions, the CVaR is defined as in the following formula: (Zenios & Markowitz, 2008)

$$\text{CVaR}(\mathbf{x}, \alpha) = E[\mathbf{R}(\mathbf{x}, \tilde{r}) | \mathbf{R}(\mathbf{x}, \tilde{r}) \leq \text{VaR}(\mathbf{x}, \alpha)] \text{ Equation 2}$$

Therefore, the definition of CVaR that is valid for continuous distributions, measures the expected value of $(1-\alpha)*100\%$ lower returns for the portfolio x (i.e., the conditional expectation of portfolio returns under VaR (x, α)) (Zenios & Markowitz, 2008).

Excluding matters relating to calculations, there is a debate among academics and practitioners about whether the appropriate measure of risk is the VaR or CVaR. VaR is an industry standard for measuring risk. On the other hand, CVaR is a popular measure of risk in the insurance sector and is gradually gaining acceptance by the financial industry. Its popularity as a risk measure is not based only on the theoretical properties, but also to the ease of its application to optimization portfolio problems and the ability to reduce the tail of the distribution, thereby controlling risk management (Zenios & Markowitz, 2008).

Essentially, the risk measure (conditional value at risk) was created as an extension of the earlier VaR. VaR manages to minimize the damage magnitude but not in all cases. In some types of risk VaR is effective but in other types is not so much effective. This is because the scope is limited and the distribution being irregularly distributed fails to measure significant loss size (Consiglio, Nielsen, & Zenios, 2009).

In other words, although the traditional measure quantifying risk is VaR, there are many cases requiring the analysis of losses that exceed the point defined by the VaR. This calculation requires the measure of quantifying risk with CVaR which



determines the average value of losses that exceed the VaR (Zenios & Markowitz, 2008).

Finally, it is worth mentioning that the case where the two measures are not significantly different, it means that losses would not exceed the boundary identified by VaR, but in the opposite case where the CVaR is significantly higher, the plans derived from the VaR have to be reversed (Zenios & Markowitz, 2008).

2.2.2.3 Mean Absolute Deviation (MAD)

In MAD model, risk is defined as the mean absolute deviation of the real portfolio return from its expected return. The mathematical formula of this model is as follows (Konno & Yamazaki, 1991):

$$\mathbf{MAD}(\mathbf{x}) = \mathbf{E} [|\mathbf{R}(\mathbf{x}, \tilde{\mathbf{r}}) - \mathbf{R}(\mathbf{x}, \bar{\mathbf{r}})|] \text{ Equation 3}$$

where

$\tilde{\mathbf{r}} = (r_1, r_2, \dots, r_n)^T$ are the assets generate returns,

$\bar{\mathbf{r}} = \mathbf{E}(\mathbf{r}) = (r_1, r_2, \dots, r_n)^T$ their average return and

$\mathbf{X} = (X_1, X_2, \dots, X_n)^T$ the percentage we place in our portfolio from each asset (e.g. stock, bond etc.)

Due to the fact that assets returns in period t are unknown, we make forecasting by estimating their returns in the next period using probabilities (p). Therefore (Konno & Yamazaki, 1991):

$$\mathbf{MAD}(\mathbf{x}) = \sum_{s=1}^S p_s |\mathbf{x}^T \mathbf{r}_s - \mathbf{x}^T \bar{\mathbf{r}}| \text{ Equation 4}$$

Thus, MAD model minimizes the following sum formula:

$$\sum_{s=1}^S p_s |\mathbf{x}^T \mathbf{r}_s - \mathbf{x}^T \bar{\mathbf{r}}| \text{ Equation 5}$$

Subjected to the following limitations:

1. $\mathbf{x} \in \mathbf{X}$



2. $y_s \geq x^T(r_s - r)$ for $s = 1, 2, \dots, S$
3. $y_s \geq x^T(r - r_s)$ for $s = 1, 2, \dots, S$
4. $y_s \geq 0$, for $s = 1, 2, \dots, S$

Where y = random variable used to linearize the expression of absolute return.

2.2.2.4 Put-Call Efficient Frontier

Investors, generally, put their money in portfolios consisting of one or more assets. However, no portfolio yields the same return, while the risk is also different. Therefore, the goal here is to find portfolios that increase the wealth of the investor, and the risk it faces is the least possible one, something that is known in the literature as the efficient portfolio frontier (EPF) (IBM, 2013).

The EPF consists of investments with higher return and lower volatility. The most "effective" investments are those that no other investment has outperformed them for the same level of standard deviation. Putting that differently, there is no investment with the lowest standard deviation for the same expected return (IBM, 2013).

A model that is based on the above facts is the so-called 'Put-Call Efficient Frontier' model. This is a model which is characterized by the portfolio's disadvantage, i.e. the risk against the benefit of the portfolio, which is its reward, taking into account the random target (benchmark: g) that is set (IBM, 2013).

The upside margin for profits has identical profits with those of call option, for the future return of the portfolio compared with the target-benchmark. In this case (IBM, 2013):

- i. When the return of the portfolio is lower than the target, the upside margin is zero as the Call option is «out-of-the-money».
- ii. When the performance of the portfolio exceeds this goal, the upside margin is zero as the Call option is «in-the-money».

Similarly, the downside margin for risk (loss) has identical effects with those of a Put option, for the future return of the portfolio compared with the target-benchmark.



The «Call Value» portfolio is the expected margin of up siding profits, and «Put Value» portfolio is the expected margin to reduce risk. The portfolios that achieve the biggest «call» for «put» data are named as "efficiently put / call portfolios" (IBM, 2013).

Finally, deviations of the performance of the portfolio from the random target-benchmark are expressed using random auxiliary variables, y^+ and y^- , where the former is a measure of the growth margin for the portfolio profits in order to overcome the target-benchmark, i.e.

$$\hat{y}^+ = \max [0, R(\mathbf{x}, \mathbf{r}) - \mathbf{g}] \text{ Equation 6}$$

Whereas the latter is the measure of the margin reduction of loss in the portfolio, i.e.

$$\hat{y}^- = \max [0, \mathbf{g} - R(\mathbf{x}, \mathbf{r})] \text{ Equation 7}$$

2.2.2.5 *Other models*

In the literature there are many other methods that can be used to solve portfolio optimization problems. Some of these are (Capiński & Kopp, 2014):

- Tracking Models,
- Regret Models,
- Expected Maximization of Utility Models,
- Lattice models,
- Optimization Models of a Scenario,
- Safety first models and
- Models of Stochastic Dominance.



2.3 Derivatives

Much has been said about the derivatives market in several academic writings and economic researches. The truth is that many are trying to assess the new market in a very short time and without having the appropriate knowledge in these products. The reason that derivatives made their appearance was the hedging of unexpected risks in portfolio investments. Many investors nowadays tend to link derivatives with profit speculations. Their great utility in the financial world is the alternative option offered as investment 'vehicles' to be used either to leverage offered either to insure the portfolio investments may include not only shares but also goods (Stulz, 2002; Chance & Brooks, 2015).

Within this framework, derivatives are financial instruments whose value depends on an underlying asset. The underlying asset can be financial, commercial or agricultural, such as stocks, bonds, currencies, precious metals (gold, silver, platinum, etc.), minerals (copper, iron, tin, etc.), oil and gas, agricultural and livestock products such as wheat, corn etc (Stulz, 2002; Chance & Brooks, 2015).

Derivatives were created to reduce the risks faced by investors in the markets. Risks such as the decrease or increase in the price of a product can be controlled and confined largely to the use of derivatives. For example, the risk of increase in the price of raw materials, for an industry is a big "loss" because it destroys the entire production planning. Furthermore, an increase in the oil price in a petroleum industry, the metals in an automobile etc. can generate many important problems. Also, the change in currency exchange rates can generate a major problem in an exporting or importing company with catastrophic consequences for its survival (Stulz, 2002; Chance & Brooks, 2015).

Therefore, another practice that most investors use in derivatives is the so-called 'hedging', i.e. the use of hedge funds. Hedge funds are those fund that can incorporate a broader range of investment and financial products in relation to other investment models. But these funds are only available to certain investors such as Pension Funds, Foundations, university endowments and investors with very large sums to invest. The investor's permitted type is determined by the regulatory authorities of each country. As hedge funds are usually open, ie the investor can invest additional funds



or to withdraw part of the fund at specific intervals. In Hedge funds, the risk can be compensated from the volatility of interest rates, exchange rates, stock prices, bonds and commodities. The compensation is usually done by using forward transactions or financial future contracts and options (Travers, 2012).

The value of an investment in a hedge fund is calculated as a percentage of the net asset value of the fund, which means that increases and decreases in the value of assets of the fund are directly reflected in the fees that an investor may later withdraw. Managers of hedge funds invest their own funds in the portfolio they manage in order to align their interests with other investors. The investment manager of a hedge fund usually receives an annual management fee (management fee), which is calculated as a percentage of the total value of annually invested capital and a success fee, if the net value of invested capital increases (Travers, 2012).

Most investment strategies of hedge funds aim to achieve a positive return regardless of the rise or fall in markets. Hedge funds involve a wide range of financial products and investment tools that differentiate the investment, but they also trade liquid securities on the open market values. They also use a wide variety of investment strategies, and use techniques such as leverage and short selling. The main strategies used are (Travers, 2012):

- **Global Macroeconomic Strategy:** hedge funds that follow a global macroeconomic strategy invest significant funds in stocks, bonds or foreign exchange markets in anticipation of global macroeconomic events in order to generate returns on weighted risk. The managers of such investments use macroeconomic analysis and are based on global events and market trends to identify investment opportunities that will benefit from the expected price developments. While this strategy has been a large degree of flexibility as the diversified investment across multiple markets, the timing of the implementation of the strategy is particularly important to generate high returns on weighted risk.
- **Directional Strategy:** The directional investment strategy uses market movements, trends or inconsistencies when stocks in diversified markets are selected. This strategy uses either computer models or fund managers identify



and select the right investments. These types of strategy have greater exposure to fluctuations in the overall market than the market-neutral strategies. The directional strategies include long / short hedge funds, where the long stock positions are hedged with short sales of shares.

- **Event-driven strategy:** Such strategies are related to situations in which the investment opportunity and the associated risk are linked with an event. The event-driven strategies include investment opportunities in corporate trade events, such as mergers, acquisitions, recapitalizations, bankruptcies and liquidations. Fund managers use such a strategy in order to exploit inconsistencies in evaluating the market before or after such events and take initiatives at the planned movement of the aforementioned securities. This strategy is usually adopted by large institutional investors, because they have the expertise and resources to analyze corporate events transactions for investment opportunities.
- **Relative Value Strategy:** The relative value strategies are benefitted from the relative price differences between securities. The price difference can be caused by poor estimation of a security compared with similar securities, the underlying asset or the overall market. The manager can use various analysis methods to identify variations in prices of securities, including mathematical, technical and fundamental techniques. The strategy in this category usually has very little or no directional (directional) exposure on the market as a whole.

In summary, due to the fact that investments in hedge funds diversify the portfolio of investments, investors can use them as a tool to reduce the total exposure of the portfolio to specific risks. Managers of hedge funds use specific trading strategies and tools specifically designed to reduce market risk and generate returns on a weighted risk chosen by investors. Ideally, hedge funds generate returns that are not directly related to market indicators. While the "hedging" may be a way of reducing an investment risk, the hedge funds, like all other types of investment, are not immune from danger.



The best known derivatives with hedging strategies are the following (Stulz, 2002; Chance & Brooks, 2015):

a) Forward and Future Contracts and

c) Options.

2.3.1 Forward and Future Contracts

To begin with, a Forward contract is a current agreement and obligation for a transaction at a predetermined future time (maturity) and at a predetermined price (delivery price). The agreement is usually closed among financial institutions or between financial institutions and their customers. The product in a transaction can be a commodity, oil, indices or currencies. In parallel, the same contract obliges the second contractor to sell (Short position) the subject of the contract under the terms of this (Sales Item). At the conclusion of the Agreement no payment is required, but the two contracting parties have to conclude the Agreement at the it's expiry date (Stulz, 2002; Chance & Brooks, 2015).

Like a forward contract, so a future contract is an agreement between two parties to buy or sell a commodity at an agreed future date and price. Unlike forward contracts, futures are standardized, traded in organized or regulated markets, subject to a daily valuation process to the method marking to market and there is guarantee of Derivatives exchange for their fulfillment. This means that the investor (buyer - seller of a futures contract) is obliged to maintain a margin account. In this account the investor deposits a percentage (5% to 10%) of the nominal value of the contract as maintenance margin which is defined by the Derivatives. This amount is the safety in case the investor cannot meet its obligations resulting from the daily settlement. The amount required to make a transaction is called initial margin. The required margin is altered by changes in the price of the underlying and the expectations of investors (Hull, 2014).

When developing a new contract, an exchange must specify in some detail the exact nature of the agreement between the two parties. It is the party with the short position that chooses between these alternatives. Specifically, there must be specified the following (Hull, 2014).



1) **Asset:** When the asset is a good or a commodity, there may be a variation in the quality of what is available in the marketplace. The financial assets in futures contracts are generally well defined.

2) **Contract Size:** The contract size is the amount of the asset that has to be delivered under one contract. If the contract size is too large, perhaps the majority of traders who wish to implement hedging strategies for relatively small exposures or who wish to take relatively small speculative positions will have not the ability to use the exchange. But if the contract size is too small, the trade exchange may be expensive as there is a transaction cost with each contract traded.

3) **Delivery Arrangements:** The delivery arrangements are important in understanding the relationship between the futures and the spot price of an asset (e.g. commodity, good etc.).

4) **Delivery Months:** An exchange must also specify the exact time period during the month when delivery is made. For a vast amount of futures contracts, the delivery period is the entire month.

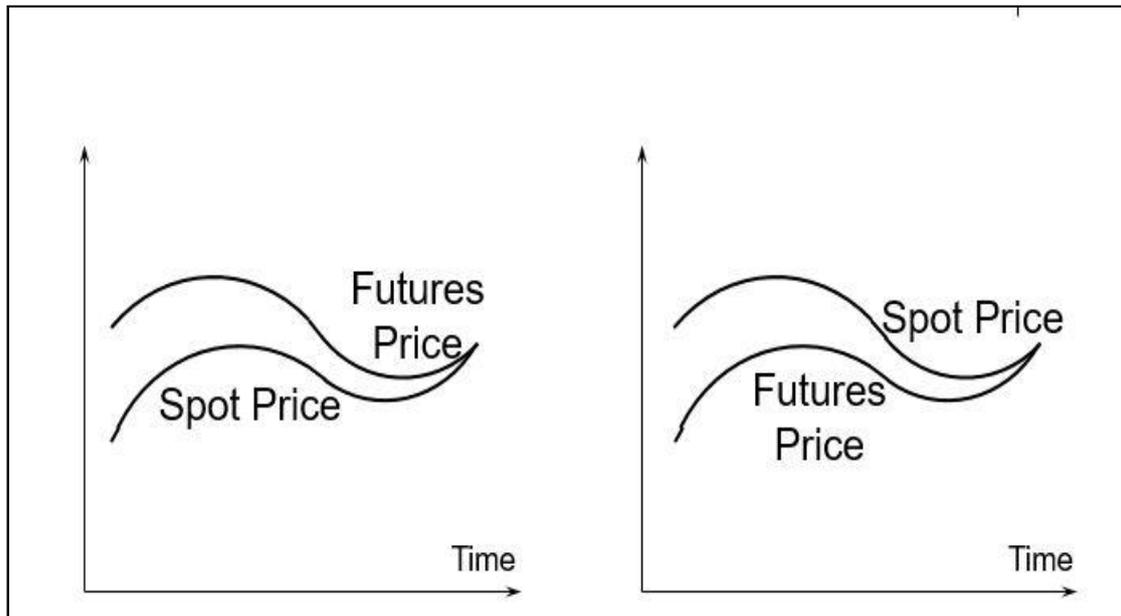
5) **Price Quotes:** An exchange should clarify that futures price is quoted in an convenient and easy way for someone to understand it.

6) **Daily Price Movement Limits:** daily price movement limits are another aspect of futures that must be specified by the exchange. The aim is to prevent large price movements from happening due to speculative excesses.

As the delivery month of a futures contract is approached, the futures price converges to the spot price of the underlying asset (Hull, 2014). When the delivery period is reached, the futures price equals (or is approximately the same) to the spot price as it is presented in the following figure.

Figure 1: Futures and Spot Price through delivery time period





Source: <http://www.slideshare.net/iipmff2/derivatives-lecture3-4-futures>

Furthermore, in the following figure are presented the main differences between futures and forward prices.

Figure 2: Forwards vs Futures Prices

FORWARDS	FUTURES
Private contract between 2 parties	Exchange traded
Non-standard contract	Standard contract
Usually 1 specified delivery date	Range of delivery dates
Settled at end of contract	Settled daily
Delivery or final cash settlement usually occurs	Contract usually closed out prior to maturity
Some credit risk	Virtually no credit risk

Source: <http://slideplayer.com/slide/1571197/>

Within this framework, a company that knows it is about to sell an asset at a specific time in the future can implement the so-called 'hedging strategies' (hedging will be analytically presented in section 2.3.3.), sometimes by taking a short futures position.



This are known as short hedging strategies. In this case, if a price of the asset falls, the company does not fare well on the sale of the asset but makes gain on the short futures position. On the other hand, if the price of an asset rises, the company gains from the sale of the asset but takes a loss on the futures position. Similar to this practice, a company that knows that it is due to buy an asset at a specific time in the future can hedge by taking a long futures position, i.e. by a long hedge (Scheidler, 2013).

The basis in a hedging situation is defined as follows (Scheidler, 2013):

Basis = spot price of asset to be hedged – futures price of contract used equation 8

If the asset to be hedged is the same to the asset underlying the futures contract, the basis may be zero at the futures contract expiration. But prior to expiration, the basis should has either positive or negative value. Furthermore, if the spot price increases by more than the futures price, then the basis increases as well, referring to a 'strengthening of the basis'. Conversely, if the spot prices decreases by more than the futures prices, then the basis decreases as well. This is generally called a 'weakening of the basis' (Scheidler, 2013).

Future prices are often observed at stock indices. A stock index is an investment asset that pays dividends to a company's shareholders. The asset in this case is the portfolio of stocks underlying the index, and the dividends are those dividends that would in the future be received by the holder of the specific portfolio. Moreover, there are many stocks underlying the index which provide dividends at different times. In that case, the index can be considered as an asset that provides a continuous dividend yield. These strategies are known as index arbitrage. Assuming that q is the dividend yield rate, S_0 the spot price today, T the time until delivery date and r the risk-free interest rate for maturity T , the futures price (F_0) is calculated by the following formula (Scheidler, 2013):

$$F_0 = S_0 e^{(r-q) \cdot T} \text{ equation 9}$$



In case of $F_0 > S_0e^{(r-q)*T}$, profits can be made by buying the stocks underlying the index and shorting futures contracts, whereas, when $F_0 < S_0e^{(r-q)*T}$, profits can be made by selling the stocks underlying the index and taking a long positions in futures contracts (Scheidler, 2013).

So, futures at stock indices can be used for hedging the risk in a well-diversified portfolio of stocks. As usual, the relationship between the expected return on a portfolio of stocks and the return on the market is described by the beta coefficient. If beta equals one, the return on the portfolio tends to reflect on the market return, whereas when beta equals two, the excess return on the portfolio tends to be twice as great as the excess return on the market (Scheidler, 2013).

Finally, the optimum number of contracts to short in order to hedge the risk in the portfolio is calculated by the following formula (Scheidler, 2013):

$$\beta * P / A \text{ equation 10}$$

Where P: the value of the portfolio and A: the value of the future contract. The basic assumption of the above formula is that the maturity of the futures contract is almost the same to the maturity of the hedge while ignoring the daily settlement of the futures contract.

2.3.2 Options

The option contracts are one of the most popular categories of derivatives. They are similar with futures contracts with the difference that the former give the buyer the right, but not the obligation, in return for compensation to buy or sell one underlying security at a specified price (called strike price) to a specified date. On the other hand, the option seller is obligated, for a price recovery, to take or deliver the underlying asset to the buyer if he exercises his right. The option contracts are traded both on regulated markets and Over-The-Counter (Hull, 2014).

The main elements of the options are the following (Hull, 2014):

1) **Underlying Asset:** The underlying asset can be a bond, a stock index or a commodity under which the right is concluded. In other words, it is the product which the holder of the call option has the right to buy and the holder of the put option has the right to sell it.



2) **Contract size:** The size of the option contracts include the number of shares covered by each option.

3) **Expiration Date (Maturity):** is the time period within which an option will be exercised, i.e. is the time to maturity.

4) **Strike/Exercise Price:** is the unchanged preset price at which the holder of an option can buy or sell the asset.

5) **Premium Option Price:** is the monetary value to be paid by the option buyer to the seller's right regardless of whether the right will be executed or not in return for the concession of the right to buy or sell the underlying security. This amount is determined by supply and demand in the market traded.

6) **Call/Put Option**

Options are categorized according to their exercise date. When the ability to exercise the option at any time exists until the expiration date is called American-style option, and when the right is only exercised at the end of the transaction is called European-style option. The categorization of options is dependent on the underlying asset and, so there are the following option types (Hull, 2014):

- Commodity Options
- Currency Options
- Index Options
- Interest Rate Cap Options
- Interest Rate Floor Options
- Interest Rate Options and
- Stock Options

Also, there are two types of options contracts (Hull, 2014):

- 1) Options that give the right to buy (call option) an underlying asset at a specified future time and for a predetermined price,



- 2) Options that give the right to sell (put option) an underlying asset at a specified future time and for a predetermined price.

In this case, four basic option positions are available (Hull, 2014):

- A long position in a call option
- A long position in a put option
- A short position in a call option
- A short position in a put option

Associating options with hedging practices, there are certain strategies that concern the above combinations of options. The most popular hedging strategies in options are the Straddle, the Strip & Strap, the Strangle and Bears spread strategy. All of these strategies involve combinations of long or short positions in call and put options (Elton, Gruber, Brown, & Goetzmann, 2014; Hull, 2014).

First of all, the Straddle Strategy refers to the simultaneous buy of a Call Option and Put Option with the same exercise price and the same expiration date on the same underlying asset. This strategy is usually applied when the stock has high volatility (fluctuation). In this case, there are long and short straddle strategies. A long straddle comprises long positions in one call and one put option on the same underlying asset, strike price and expiration date. So, an investor enters a long straddle position when he expects an increase in volatility but is not sure about the movement direction, so he wants to be covered in case of steep changes in the price of the underlying asset in either direction. As for short straddle strategies, they involve simultaneously selling a put and the call option of the same underlying asset, the same strike price and the same expiration date. The profit of an investor in this case is limited to the premium received from put and call sale (Natenberg, 2014).

Furthermore, the Strip Strategy is a purchase of a call option and two put options, i.e. more put options with the same exercise price and the same maturity. The investor expects a significant change in the price of the underlying security with greater likelihood of this change relate to price reduction. In contrast to strip, Strap strategy involves a purchase of two call options and one put option, i.e. more call options with



the same exercise price and the same maturity. The investor expects a significant change in the price of the underlying asset with greater likelihood of this change relate to the price increases (Natenberg, 2014).

The Strangle strategy is carried out by simultaneously buying a call option and a put option on the same underlying asset with the same expiration date. The strangle strategy differs from the straddle strategy only in that the strike price is different to the purchased call option than the purchased put option, unlike the straddle strategy where the two strike prices are identical. The sale of a strangle strategy is formed with the simultaneous sale of a call option and a put option, which have similar characteristics other than the strike price. More specifically, they have the same date and are related to the same underlying asset, but the strike price of the two options differ (Natenberg, 2014).

Finally, the Bearsread strategy is formed so as to contain low risk, and generate only low profits in a market with declining trends. It is formed by buying a call option in combination with selling a call option with a lower price than the purchase option. The most popular strategies of this type is the vertical bull spread and vertical bear spread with call and put options (Natenberg, 2014). On one hand, vertical bull spread reduces the risk of the investor, both in anode and cathode of the price of the underlying asset and is mainly used in case of a slightly upward market movement. It involves low risk and generates low profits in market upswings. It can also be created with either call or put options (Natenberg, 2014). On the other hand, The vertical bear spread is used in the case of slightly downward movement in the price of the underlying asset. If sharp downward movement is expected, the investor would buy a put option. Since, however, it is usually expected little market decline, he is unwilling to pay the entire premium of the put option, as he is going to exploit the opportunity to make profit offered by the put in a strongly declining market. The buy or sell positions therefore, can be created with either call or put options (Natenberg, 2014).

Before the analysis proceeds to the literature review of the general theory of portfolio optimization and asset allocation, it is worth noting that pricing options is very important in order to determine its volatility and movement in a specific time period. This time period can be either discrete or continuous. When time period is discrete,



Binomial Trees are used so as to determine different possible paths that might be followed by the asset price over the life of the option. But when time period is continuous, it is often used the so-called '**Black-Scholes**' model. This model assumes that a price of an asset (e.g. stock) follows a Geometric Brownian Motion as in the following formula (Natenberg, 2014):

$$dS = \mu S dt + \sigma S dW \text{ equation 11}$$

Where S is the (stock) price, μ the mean, σ the standard deviation, W is a standard Wiener process and d the rate of change.

Also, a variable has lognormal distribution if the natural logarithm of the variable is normally distributed, i.e.

$$d \ln S = (\mu - \sigma^2/2) dt + \sigma dW \text{ equation 12}$$

That is, the change in $\ln S$ between time 0 and time T is normally distributed, so that

$$\ln S_T - \ln S_0 \sim N[(\mu - \sigma^2/2)T, \sigma(T)^{1/2}] \text{ equation 13}$$

Or

$$\ln (S_T/S_0) \sim N[(\mu - \sigma^2/2)T, \sigma(T)^{1/2}] \text{ equation 14}$$

And

$$\ln S_T \sim N[\ln S_0 + (\mu - \sigma^2/2)T, \sigma(T)^{1/2}] \text{ equation 15}$$

Where S_T is the stock price at a future time T, S_0 is the stock price at time 0, and $N(m,s)$ denotes a normal distribution with mean m and standard deviation s. This shows that $\ln S_T$ is normally distributed so that S_T has a lognormal distribution.



2.4 Literature Review on Portfolio Optimization and Asset Allocation

From the analysis already presented above, it is clear that nowadays, the most popular risk measure is that of variance, which has attracted widely the interest of academics in many finance and other studies, while it is also the most popular risk measure used by investors in order to determine the amount of profits and/or losses (Evans, 2004). Nevertheless, variance is not the only risk measure used because there are several other important alternatives that can be used as risk measures, such as Lower Partial Moment (downside risk), MAD, Minimax and Maximum Drawdown (Evans, 2004).

Unfortunately, it is very difficult for academics to determine the best risk measure that offers the best return while risk trade-off may be a vain exercise unless a common risk measure is identified for this purpose (Byrne and Lee, 2004). This difficulty is also pointed out by the study of Cheng and Wolverton (2001) and Cheng (2001) who conclude that (downside) risk and variance are not directly comparable. Therefore, it is almost impossible to conclude which risk measure is the best one of all the others. In addition, there are other empirical evidence suggesting that different risk measures often provide different asset allocation for portfolios as well as different returns for an asset (Biglova et al., 2004; Byrne & Lee, 2004) These findings are applied to investors to whom the choice of the risk measure depends on their risk attitudes and investment goals. Therefore, there are both advantages and drawbacks of each different risk measure, which are inevitable for investors. Given the problems of each risk measure, they can select, according to their investment objectives and risk attitudes, the most appropriate risk measure.

Within this framework, Ahn et al. (1999) used a Value-at-Risk (VaR) with options in order to study the problem of covering the market risk of a given exposure in a risky stock. From this model, they found that the optimal hedge comprises an option position whose strike price is dependent on the asset exposure distribution, the time period of the hedge and the desired protection level rather than the acceptable cost level for hedging. Based on this study, Annaert et al. (2007) focused on determining the strike price of a bond put option to hedge a position in a given bond by using an affine term structure model. The conclusion of this study was that hedging is optimal



when it minimizes, either the Value-at-Risk (VaR) or the Conditional Value at- Risk (CVaR) model.

An earlier study of Carr et al. (2001) considered optimal investment in a risky asset, as well as in derivatives on this asset, whereas Aliprantis et al. (2000) concluded that there is a minimum-cost insurance at arbitrage-free security prices when holding a put option in combination with a specific portfolio. Also, Liu and Pan (2003) studied optimal investment strategies under the assumption that investor access is not only to bonds and stocks but also to derivatives. By assessing the optimum portfolio of the S&P 500 index and options markets, the general conclusion of this study was that when derivatives are included in the portfolio, its performance is dramatically improved. The same outcomes were derived by the study of Muck (2010) who investigated potential portfolio improvements when investors had either partial or full access to derivatives on the given assets.

However, the first efforts to create an optimization model were in the 1970s when Hodges (1976) used a model based on Markowitz's portfolio theory. This study suggested that the effect of errors in forecasts of expected returns may be reduced if these forecasts are used in order to modify a prior distribution of securities which leads to minimal trading activity. Effective differentiation between industrial groups often hampered by difficulties in predicting fluctuations. Roll (1992) also worked on the model of Markowitz, comparing the curve related to the variation of price performance that exceeds a particular index to the curve related to variation and performance in the classical mean-variance model. He also attempted to combine the model of Markowitz with models of factors, introducing a limitation in the beta factor of the portfolio that follows the index, resulting in an improved performance. Adcock and Meade (1994) investigated the problem of adjustment of a passive portfolio (based on optimization strategy) over time, taking into account the factor of transaction costs at each adjustment. These costs include transaction, using a weighting factor, the objective function of the model presented, but there is no restriction on the percentage of available capital consumed in them.

Furthermore, Worzel et al. (1994) presented the approach, based on the performance of multiple portfolios scriptable, when monitoring a particular index (mortgage



index). The modeling of the problem used, which took into account historical data of a single year, was designed in such a way that maximized performance while limiting, in all scenarios, the possibility that the performance of the portfolio falls below performance index.

The same approach was used by Consiglio and Zenios (2001) to monitor a bond index. Also, Zenios et al. (1998) presented a model that was to follow over several periods a pointer. This model was used a multistage stochastic programming in combination with the Monte Carlo descriptive simulation models. Also, there were carried out extensive experiments in order to validate the effectiveness of the model against the uncertainty and to evaluate the performance of the model compared to a period of time. The results presented showed that the performance of this model was significantly superior to its competitors. Also, Ammann and Zimmermann (2001) investigated the relationship between statistical measures of surveying error and the existence of restrictions on securities participation rates in passive portfolios. Specifically, they dealt with identifying imprinting errors that would occur if the composition of the portfolio was deviated from the recommendation of the respective indicator to measure the percentage held by each of the main categories of securities. Measurements were made in portfolios that had the lowest correlation to the index, according to the above measure.

Also, Fang and Wang (2005) considered the problem of tracking the index as a programming problem with two objects. One is the mean absolute deviation of prices that underperform the corresponding index. The second is the performance that surpasses that of the index. They proposed a theoretical fuzzy decision model which led to a mathematical problem that can be solved in a linear fashion. Based on this study, Gaivoronoski et al. (2005) investigated the problem of determining the difference between the return of a portfolio that follows that of an index, and the performance of the index. They also dealt with several ways of revaluation of such portfolio. They concluded that the selection of stocks is based on, initially, the solution of a problem optimization without restricting their number, and then the classification of stocks based on their appearance in the portfolio generated by the solution of this problem. They gave results for a simulation made in 65 stocks from the stock exchange in Oslo, but the execution times of the model were not provided.



In using hedging approaches, the study of Filatov and Rappoport (1992) was the first that suggested a selective hedge approach where hedge ratios may vary across currencies. They concluded that by using selective hedging approaches, they can yield different optimal hedge ratios for each currency. Empirically, they found that the optimal selective hedging approach differs among investors with different reference currencies. Also, Beltratti et al. (2004) included selective hedging approaches within a single-stage portfolio optimization model; they concluded that selective hedging is more efficient than unitary hedging. In particular, Beltratti et al. (2004) used a mean-absolute deviation model, as well as historical data bootstrapping for scenario generation. This model was extended in the study by Topaloglou et al. (2002) for international asset allocation, where adopted a more suitable scenario generation process and risk measures that are suitable for asymmetric distributions presented in international asset returns and exchange rates.

There are also other studies that focus on analyzing the derivatives role in improving trading opportunities as well as the combination of many optimality and return dynamics criteria in order to determine the performance of a portfolio under certain conditions (Brennan and Cao, 1996; Haugh and Lo, 2001). Also, Blomvall and Lindberg (2003) examined actively managed portfolios which comprised a stock index, call options on the index and a risk-free asset. Their empirical results showed that options can, under certain conditions, be used to create a portfolio outperforming the index, achieving a higher return using options at a given level of risk. Moreover, Dimson and Mussavian (1999) examined asset and derivative pricing approaches in theoretical perspective, referring to their use in various hedging contexts.

Finally, more recent studies in derivatives (Ogryczak and Ruszczyński, 2002, Dentcheva and Ruszczyński, 2006) investigated the relationship between mean-risk models and second-order stochastic dominance conditions in portfolio optimization processes, while other group of studies examined the relationship between CVaR models and uncertainty. This stream of literature also establishes the consistency of choices based on stochastic dominance with preferences associated with concave, non-decreasing utility functions. Another recent stream of research (Natarajan et al., 2009; Chen et al., 2010) examines the connection between CVaR models and uncertainty sets in the robust optimization setting (Miller and Ruszczyński, 2008).



3 Empirical Application

Having presented the purpose of the dissertation as well as the mathematical model that is used to identify efficient portfolios for the period 31/01/13 - 3/09/14, at this point is presented the full methodological framework which is followed in this research. The first step was the calculation of stock prices from the Datastream base, for the period from 31/12/1999 up to 30/09/14, of companies listed on FTSE ATHEX Large Cap. These companies are: ALPHA BANK, NATIONAL BANK OF GREECE, BANK OF PIRAEUS, EUROBANK ERGASIAS, HELLENIC TELECOM.ORG., OPAP, FOLLI FOLLIE, HELLENIC PETROLEUM, PUBLIC POWER, TITAN CEMENT CR, ATH.WT.SUPPLY & SEWAGE, GRIVALIA PROPERTIES REIC, JUMBO, MOTOR OIL, MYTILINEOS HOLDINGS, ELLAKTOR, GEK TERNA, HELLENIC EXCHANGES HDG., INTRALOT INTGRTD.SYSV., MARFIN INV.GP.HDG., METKA, PIRAEUS PORT, TERNA ENERGY, VIOHALCO, COCA COLA and HBC. Of these 25 companies were selected as many as remained in the index throughout the period being examined. The final sample of enterprises amounted to 16 companies, which are: ALPHA BANK, NATIONAL BK.OF GREECE, BANK OF PIRAEUS, EUROBANK ERGASIAS,, HELLENIC TELECOM.ORG., FOLLI FOLLIE, HELLENIC PETROLEUM, TITAN CEMENT, JUMBO, MYTILINEOS, ELLAKTOR, GEK TERNA, INTRALOT, MARFIN INV.GP.HDG., METKA and COCA COLA.

The first step in data analysis is the conversion of shares prices at returns. Having share prices for the period 31/12/ 99 - 30 /09/14, the returns of the aforementioned shares were calculated for the period 31/01/00 - 30/09/14. Initially, the first return figure that was used is the monthly stock returns for the period 31/01/00 to 12/31/12 for the extraction of CVaR, MAD and Put-Call efficient frontier for that period. Then, there were set up 21 tables, for carrying out the backtests, which also relate in share returns for the period 29/02 /00 - 31 /01/13, 31/03 /00 - 29 / 02/13, 30/04/00 - 03.31.13, and so on up to 30/09/14. Based on CVaR, MAD and Put-Call, the scenarios are the months, which are up to 156, both for the period 31/01/00 to 12/31/12 and, but for the individual intervals. Having exported efficient portfolios of CVaR, MAD and Put-Call, the next step in the analysis is to create the actual return of



the portfolio, based on the shares of stock, and the stock returns observed during the period 31/01 / 13-30 / 09/14. Because equity returns were negative, it was not possible to put a restriction on the portfolio performance target and therefore this restriction was not used.

Besides of the maximization problems of the aforementioned models, it is also maximized the overall performance of the portfolio, both for the period 31/01/00 - 31/12/12 and for the individual intervals presented above, through the backtest. The stock returns obtained were used to calculate real returns, with stock returns over the period 31/01/13 - 30/09/14, as also mentioned above.

The returns of the portfolios resulted from the maximization problems above are placed in geometric performance chart for each month, along with the returns of the index FTSE ATHEX Large Cap, for the period 31/01/13 - 30/09/14. The aim is to show how an investor would move this time, if he/she indeed chooses to follow both the CVaR, MAD and Put-Call and the maximization of the overall portfolio return. The analysis was performed with the GAMS program.

The present section presents the main results derived from the analysis followed as mentioned in the methodology section. Firstly, there were calculated the descriptive statistics for each company's stock price as well as return. Then, the second thing was to calculate the stock weights from the companies selected. Furthermore, each model was utilized in order to determine the portfolios returns and the overall return, relative to the returns of the FTSE ATHEX Large Cap index. Finally, from each of the models used in the present study are determined the geometric portfolio returns and the overall return relative to FTSE ATHEX Large Cap index.

3.1 Descriptive Statistics

In the present sub section are presented the descriptive statistics from each stock price and return in the sample companies selected. So, the descriptive statistics for stock prices (including FTSE 20 index) for the 178 observations are presented in the following figure:



Figure 3: Descriptive Statistics for stock prices

	n	Minimum	Maximum	Mean	Std. Deviation
ALPHABANK	178	181,10	7907,30	3405,7112	2370,48832
NATIONALBANKOFGRE ECE	178	52,40	4109,80	1568,6781	1117,94800
BANKOFPIRAEUS	178	63,80	8299,30	2690,1433	2271,04015
EUROBANK	178	1,70	1457,70	558,1404	411,53670
HELLENICTELECOM	178	11,50	272,00	115,6629	56,05938
FOLLIFOLLIE	178	24,50	370,20	139,5348	65,88833
HELLENICPETROLEUM	178	90,70	297,30	154,1461	42,70006
TITANCEMENT	178	4207,80	18340,90	9106,7843	3373,53165
JUMBO	178	297,70	5179,80	1782,6174	1304,70907
MYTILINEOS	178	414,40	9969,40	3193,0803	2207,72247
ELLAKTOR	178	47,80	955,20	354,8713	186,51963
GEKTERNA	178	1488,30	60365,70	13787,0978	9106,48646
INTRALOT	178	45,30	1015,30	337,3169	241,61787
MARFIN	178	30,70	2562,70	531,6197	466,57058
METKA	178	4487,30	29951,30	13194,4236	5605,06302
COCACOLA	178	379,20	1621,70	877,7517	311,16617
FTSE20	178	188,84	2910,10	1296,9452	761,53421

From the above figure it seems that, on average, the highest stock prices of the selected sample were those of GEK TERNA (13787,0978) and METKA (13194,4236) and the lowest ones in FOLLI FOLLIE, HELLENIC TELECOM and HELLENIC PETROLEUM. Comparing to the price of FTSE 20 index, only 8 out of 16 companies presented a stock price below the general stock index whereas the remaining 8 companies' stock prices were higher than that of the market. As for the standard deviations, the highest ones are also observed in GEK TERNA and METKA, indicating that the variation level through the whole time period was significant in the said companies.

Also, regarding the stock returns for the specific period, the descriptive statistics are presented below:



Figure 4: Descriptive Statistics for Stock Returns

	n	Minimum	Maximum	Mean	Std. Deviation
ALPHABANKR	177	-,41149	1,82551	,0015736	,20957222
NATIONALBANKOFGRE ECER	177	-,52799	,69788	-,0094179	,16177966
BANKOFPIRAEUSR	177	-,55298	1,68831	-,0068413	,19954975
EUROBANKR	177	-,69444	1,42381	-,0162305	,21179247
HELLENICTELECOMR	177	-,47248	,53913	,0030429	,12444288
FOLLIFOLLIER	177	-,37340	,46243	,0047932	,13476442
HELLENICPETROLEUR	177	-,27168	,39854	-,0018287	,09313861
TITANCEMENTR	177	-,42672	,39249	,0026055	,09649363
JUMBOR	177	-,31060	,35830	,0168246	,11299127
MYTILINEOSR	177	-,38210	,48191	,0060083	,14996201
ELLAKTORR	177	-,45183	,63180	,0007151	,13434025
GEKTERNAR	177	-,38552	,71511	,0039193	,17338111
INTRALOTR	177	-,36994	,42698	,0023377	,13209728
MARFINR	177	-,37344	,66089	-,0071927	,17404735
METKAR	177	-,29348	,32286	,0043964	,12551201
COCACOLAR	177	-,40792	,20335	,0060391	,08665402
FTSE20R	177	-,29487	,27779	-,0073727	,09489704

From this figure it seems that, on average, the most highly increased stock return for the selected time period was that of JUMBO (16,82%) whereas EUROBANK presented the most highly decreased average stock return (-1,6%). Comparing stock return with FTSE 20 index, it seems that 2 out of 16 companies stock returns decreased for the selected period more than FTSE 20 (-0,73%), which are EUROBANK (-1,6%) and NATIONAL BANK OF GREECE (-0,941%). Regarding standard deviations, it seems that EUROBANK and ALPHA BANK presented the highest deviations from the average stock return and, comparing them with the standard deviation of FTSE 20, only HELLENIC PETROLEUM (0,9313) and COCA COLA (0,8665) had values lower than that of FTSE 20 (0,9489), indicating that these stock returns were the most relatively stable in a constant level through this period whereas the others were more volatile.



3.2 CVaR

Here are presented the results derived from the maximization problem of CVaR and maximizing the total return. For the period 2000-2012, the CVaR efficient frontier revealed the following results:

Table 1: Stock weights and the efficient frontier for FTSE Large Cap index

Stock	Weight
ALPHA BANK	0,000000
PIRAEUS BANK	0,000000
COCA COLA	0,052906
EUROBANK	0,000000
FOLLI FOLLIE	0,101217
ELLAKTOR	0,092967
GEK TERNA	0,000000
HELLENIC PETROLEUM	0,321473
HELLENIC TELECOM.ORG.	0,330785
INTRALOT	0,000000
JUMBO	0,058790
MARFIN	0,000000
METKA	0,000000
MYTILINEOS	0,000000
NATIONAL BK.OF GREECE	0,000000
TITAN	0,041863

As it seems from the figure above, the CVaR efficient frontier shows that the efficient portfolio for the period 2000-2012 would include approximately 5.3% of Coca-Cola's stock, 10% of Folli Follie's stock, 9.3% of the stock of Ellaktor, 32,1% the Greek Petroleum, 33% of OTE, 5.9% of Jumbo and 4.18% of Titan.

From the backtest, according to the maximization problem of CVaR, 21 effective frontiers were found, which calculated the stock shares that make up the effective portfolios. On this basis, returns of the portfolio were calculated according to the stock shares that resulted from each period. It was the same problem for maximizing the total return of the portfolio. Multiplying the shares of stock to their actual returns for the period 31/01/13 - 30 /09/14, the following results were obtained as presented in the table:



Table 2: Portfolio Returns based on maximization problem CVaR and the total return of portfolio, compared with the FTSE 20 Return

Date	Portfolio Return (max CVAR)	Portfolio Return (max total return)	FTSE 20 Return
31/1/2013	12,66%	7,20%	7,26%
28/2/2013	2,80%	14,06%	0,70%
29/3/2013	-16,16%	-23,83%	-16,23%
30/4/2013	23,87%	28,59%	15,04%
31/5/2013	-0,30%	8,25%	7,35%
28/6/2013	-7,82%	-1,81%	-18,08%
31/7/2013	8,46%	5,26%	5,90%
30/8/2013	-2,67%	-0,13%	2,50%
30/9/2013	13,96%	15,27%	11,61%
31/10/2013	19,28%	7,49%	14,89%
29/11/2013	-1,44%	15,76%	0,17%
31/12/2013	-2,79%	1,22%	-2,62%
31/1/2014	2,42%	8,62%	0,11%
28/2/2014	8,65%	8,97%	9,82%
31/3/2014	0,72%	-1,01%	1,71%
30/4/2014	-8,60%	-10,39%	-7,84%
30/5/2014	-5,24%	-6,09%	-0,16%
30/6/2014	3,90%	9,32%	-0,99%
31/7/2014	-1,36%	-6,36%	-4,41%
29/8/2014	-0,95%	0,00%	0,51%
30/9/2014	-6,11%	-10,62%	-8,50%
Pearson's R	0,891836868	0,750053964	
Beta	0,927012098	0,917533866	

The results from the table above are that, in general, based on the CVaR, the portfolio an investor would have if he trusted the results of this measure, would give a portfolio which would, during this time period, have significant correlation with index returns (Pearson's R = 0,891). Similarly, for total portfolio returns, the correlation is positive and medium degree (Pearson's R = 0.75). Calculating also portfolio betas (covariance portfolio returns with the index to the variability of returns of the index) to be received by the investor who invest according to the CVaR to the total return of the portfolio as well, in the first case, he would receive a portfolio risk equal to 0,92, while in the second case, he would receive a portfolio risk amount equal to 0.91.



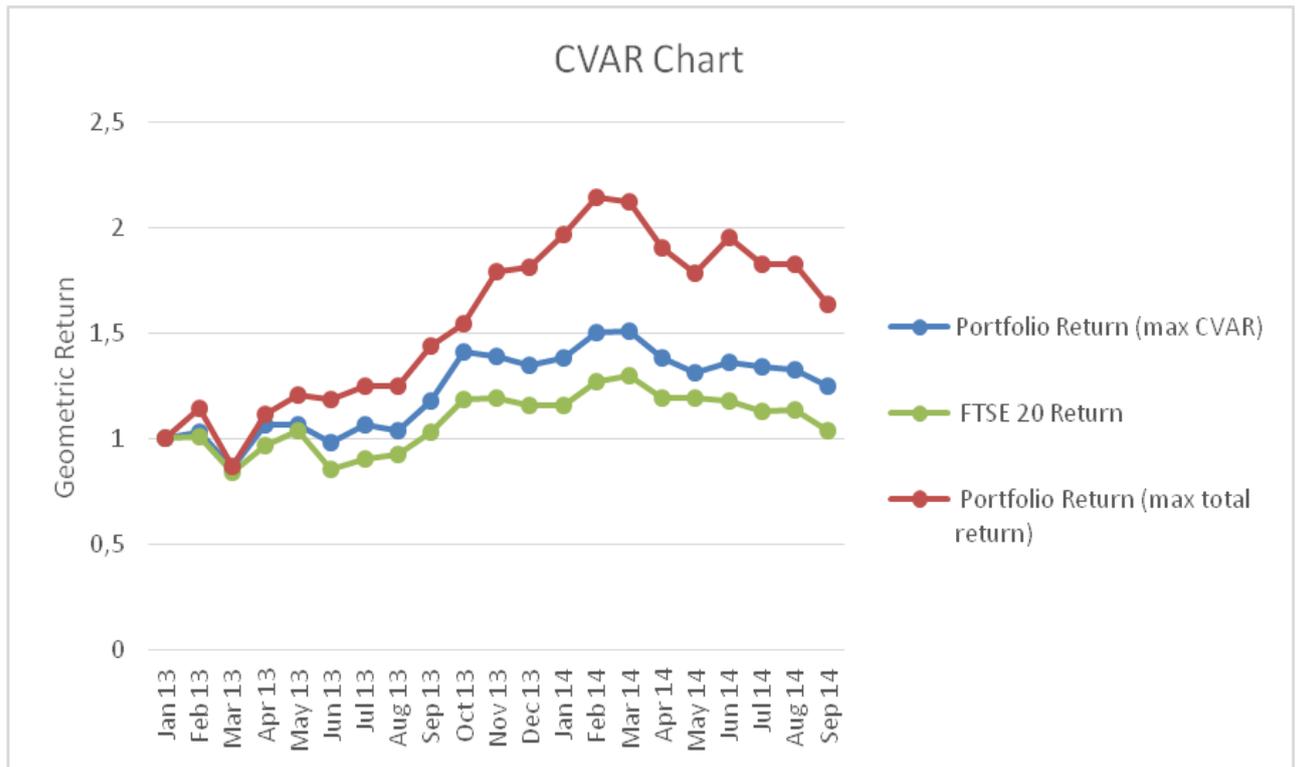
Since in both cases the portfolio betas are less than one, portfolios are characterized as defensive.

Finally, calculating for the same series their geometric returns, a diagram is created linking the returns of the portfolio based on maximizing CVaR, maximize total return, with the returns of the index FTSE ATHEX Large Cap. The table as well as the related diagram are shown below:

Table 3: Geometric returns CVaR maximization problem, total portfolio return index and FTSE ATHEX Large Cap.

Date	Geometric Portfolio Return (max CVAR)	Geometric Portfolio Return (max total return)	Geometric FTSE 20 Return
31/1/2013	1	1	1
28/2/2013	1,027965952	1,140608822	1,006984796
29/3/2013	0,861797544	0,868751294	0,843534548
30/4/2013	1,067519886	1,117167115	0,970435044
31/5/2013	1,064296847	1,209360116	1,041758242
28/6/2013	0,981088524	1,187492234	0,853409604
31/7/2013	1,06412161	1,249989646	0,9037182
30/8/2013	1,035731591	1,248415821	0,926328466
30/9/2013	1,18036224	1,439055705	1,033840132
31/10/2013	1,407944126	1,546862704	1,187776607
29/11/2013	1,387621764	1,790598468	1,189823875
31/12/2013	1,348924204	1,812507766	1,158633148
31/1/2014	1,381589762	1,968730586	1,159927743
28/2/2014	1,501162953	2,14528888	1,273852175
31/3/2014	1,511964327	2,123586664	1,295679663
30/4/2014	1,381986519	1,903044109	1,194099052
30/5/2014	1,309571255	1,78707807	1,192172211
30/6/2014	1,360639983	1,953696417	1,180340208
31/7/2014	1,342189653	1,829529923	1,128285413
29/8/2014	1,329376594	1,829529923	1,13400572
30/9/2014	1,248150805	1,63516256	1,037603492





Graph 1: Graphical presentation of geometric returns CVaR maximization problem, total portfolio return index and FTSE ATHEX Large Cap

The results of the above graph indicate that up to April 2013, it is noted that the portfolio return (max CVaR), portfolio return (max total return) are quite close to the index returns. But since then, it seems that the portfolio return (max total return) will follow a path with an upward trend, while based on CVaR, would follow a trend almost similar to the FTSE 20 Return.

3.3 MAD

The same process is followed to the MAD model as with CVaR. For the period 2000-2012, stock weights for the selected companies are presented below:

Table 4: Stock Weights and the optimal effective portfolio

Stock	Weight
ALPHA BANK	0,000000
BANK OF PIRAEUS	0,000000
COCA COLA	0,294950



EUROBANK	0,000000
FOLLI FOLLIE	0,033090
ELLAKTOR	0,020570
GEK TERNA	0,000000
HELLENIC PETROLEUM	0,141630
HELLENIC TELECOM.ORG.	0,095480
INTRALOT	0,000000
INTGRTD.SYSV.	0,000000
JUMBO	0,002990
MARFIN INV.GP.HDG.	0,000000
METKA	0,000000
MYTILINEOS HOLDINGS	0,000000
NATIONAL BK.OF GREECE	0,000000
TITAN CEMENT CR	0,411280

By using MAD model, the situation differs in stock weights from those extracted by the CVaR maximization problem. In this case, the efficient portfolio for the period 2000-2012 would include approximately 29.5% of Coca-Cola's share, 33,1% for Folli Follie's, 20% of the stock of Ellaktor, 14,15% for Greek Petroleum's, 9,55% for OTE's, 0,3% for Jumbo and 41,13% for Titan. Comparing to CVaR results, the stock weights of Coca Cola, Hellenic Petroleum, OTE and Jumbo decreased while Titan presents the most weighted stock of all the others.

Furthermore, the Portfolio Return compared with the FTSE 20 Return are presented in the following table:

Table 5: the total return of portfolio, compared with the FTSE 20 return

Date	Portfolio Return	FTSE 20 Return
31/1/2013	4,17%	7,26%
28/2/2013	7,95%	0,70%
29/3/2013	-9,42%	-16,23%
30/4/2013	8,27%	15,04%
31/5/2013	-1,16%	7,35%
28/6/2013	-6,36%	-18,08%



31/7/2013	7,22%	5,90%
30/8/2013	2,79%	2,50%
30/9/2013	16,37%	11,61%
31/10/2013	9,41%	14,89%
29/11/2013	1,88%	0,17%
31/12/2013	-5,02%	-2,62%
31/1/2014	-1,39%	0,11%
28/2/2014	7,13%	9,82%
31/3/2014	3,59%	1,71%
30/4/2014	-7,06%	-7,84%
30/5/2014	-4,62%	-0,16%
30/6/2014	3,78%	-0,99%
31/7/2014	-0,44%	-4,41%
29/8/2014	-4,82%	0,51%
30/9/2014	-3,89%	-8,50%
Pearson's R	0,80984976	
Beta	0,57699263	

This table shows that total portfolio returns are highly correlated with the FTSE 20 Return (Pearson's R = 0.81). Furthermore, the calculation of portfolio beta shows that an investor would receive a portfolio amount of risk equal to 0,57, Again, given that the portfolio beta is less than one, portfolio is also characterized as defensive as in the case of CVaR.

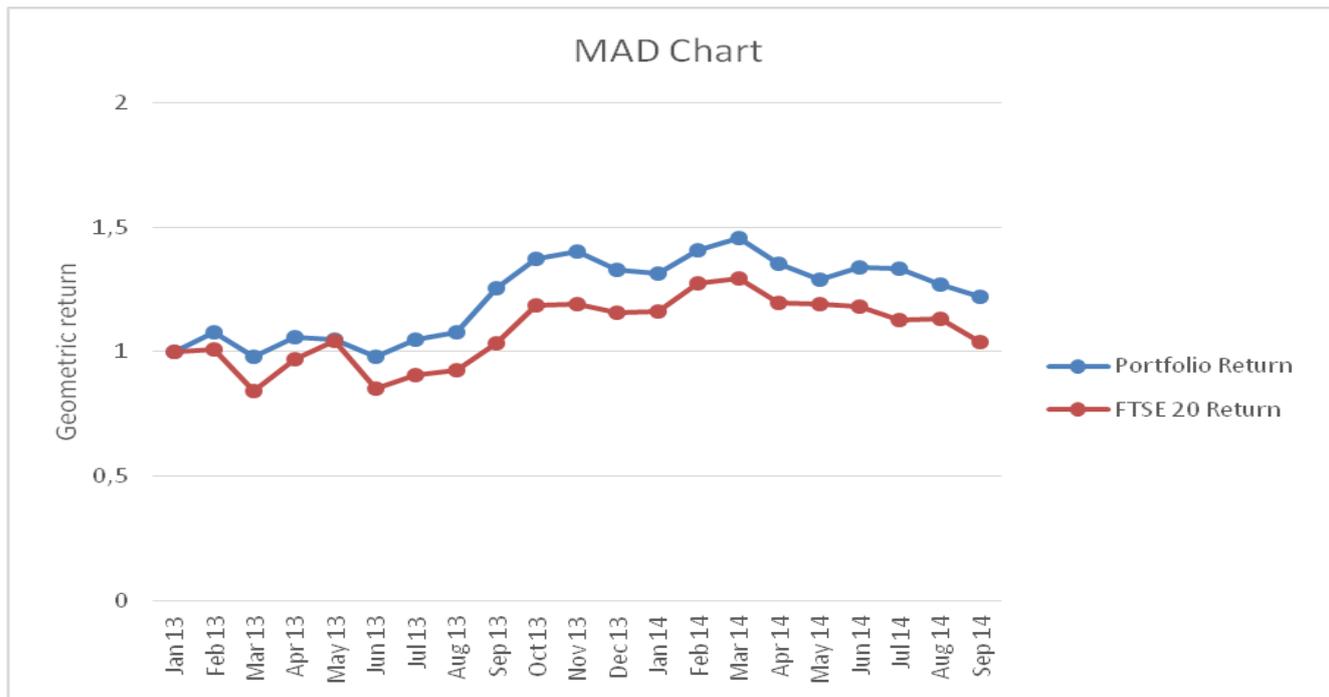
Finally, the graphical presentation of total returns and FTSE 20 returns is presented below, along with the related table from which the graph was derived:

Table 6: Geometric portfolio return and geometric FTSE 20 Return

Date	Geometric Portfolio Return	Geometric FTSE 20 Return
31/1/2013	1	1
28/2/2013	1,079536493	1,006984796
29/3/2013	0,977842128	0,843534548
30/4/2013	1,058677312	0,970435044
31/5/2013	1,046395498	1,041758242
28/6/2013	0,979810992	0,853409604



31/7/2013	1,050592331	0,9037182
30/8/2013	1,079946908	0,926328466
30/9/2013	1,256708616	1,033840132
31/10/2013	1,37493055	1,187776607
29/11/2013	1,400793934	1,189823875
31/12/2013	1,330412291	1,158633148
31/1/2014	1,311922934	1,159927743
28/2/2014	1,40551803	1,273852175
31/3/2014	1,455952725	1,295679663
30/4/2014	1,35317137	1,194099052
30/5/2014	1,290714177	1,192172211
30/6/2014	1,3394737	1,180340208
31/7/2014	1,333642289	1,128285413
29/8/2014	1,269375054	1,13400572
30/9/2014	1,219975863	1,037603492



Graph 2: Diagrammatic presentation of geometric portfolio return and geometric FTSE 20 return

From the above graph it seems that portfolio returns were very close to the FTSE 20 returns till May 2013, but since then, portfolio returns were higher than those of FTSE 20.



3.4 Put-Call

Finally, by using the Put-Call efficient frontier model, stock weights are as in the following table:

Table 7: Stock weights and optimum effective portfolio return

Stock	Weight
ALPHA BANK	0,000000
PIRAEUS BANK	0,000000
COCA COLA	0,000000
EUROBANK	0,000000
FOLLI FOLLIE	0,000000
ELLAKTOR	0,000000
GEK TERNA	0,000000
HELLENIC PETROLEUM	0,000000
HELLENIC TELECOM.ORG.	0,000000
INTRALOT	0,000000
JUMBO	1,000000
MARFIN	0,000000
METKA	0,000000
MYTILINEOS	0,000000
NATIONAL BK.OF GREECE	0,000000
TITAN	0,000000

The above table shows that an optimum portfolio return would include only the stock prices of Jumbo, which means that an investor should invest only in Jumbo stocks. All the other companies for the current period are not considered as effective as Jumbo.

Moreover, comparing the total portfolio return with the index return on their correlation and the degree of portfolio return's aggressiveness, the following table presents these information:



Table 8: Portfolio return and FTSE 20 Return; Correlation and beta

Date	Portfolio Return	FTSE 20 Return
31/1/2013	7,20%	7,26%
28/2/2013	14,06%	0,70%
29/3/2013	-23,83%	-16,23%
30/4/2013	28,59%	15,04%
31/5/2013	8,25%	7,35%
28/6/2013	-1,81%	-18,08%
31/7/2013	5,26%	5,90%
30/8/2013	-0,13%	2,50%
30/9/2013	15,27%	11,61%
31/10/2013	7,49%	14,89%
29/11/2013	15,76%	0,17%
31/12/2013	1,22%	-2,62%
31/1/2014	8,62%	0,11%
28/2/2014	8,97%	9,82%
31/3/2014	-1,01%	1,71%
30/4/2014	-10,39%	-7,84%
30/5/2014	-6,09%	-0,16%
30/6/2014	9,32%	-0,99%
31/7/2014	-6,36%	-4,41%
29/8/2014	0,00%	0,51%
30/9/2014	-10,62%	-8,50%
Pearson's R	0,750054	
Beta	0,917534	

This table indicates that portfolio return is relatively highly correlated with FTSE 20 returns (Pearson's $R=0,75$) while the beta value (0,917) indicates that an investor would receive a portfolio risk equal to 0,917. This also indicates that portfolio is defensive, i.e. stock's systematic risk is lower than market risk, something that makes sense due to the environment of uncertainty in the Greek environment.

Finally, the geometric portfolio return, compared with geometric FTSE 20 return are presented in the following table and graph:

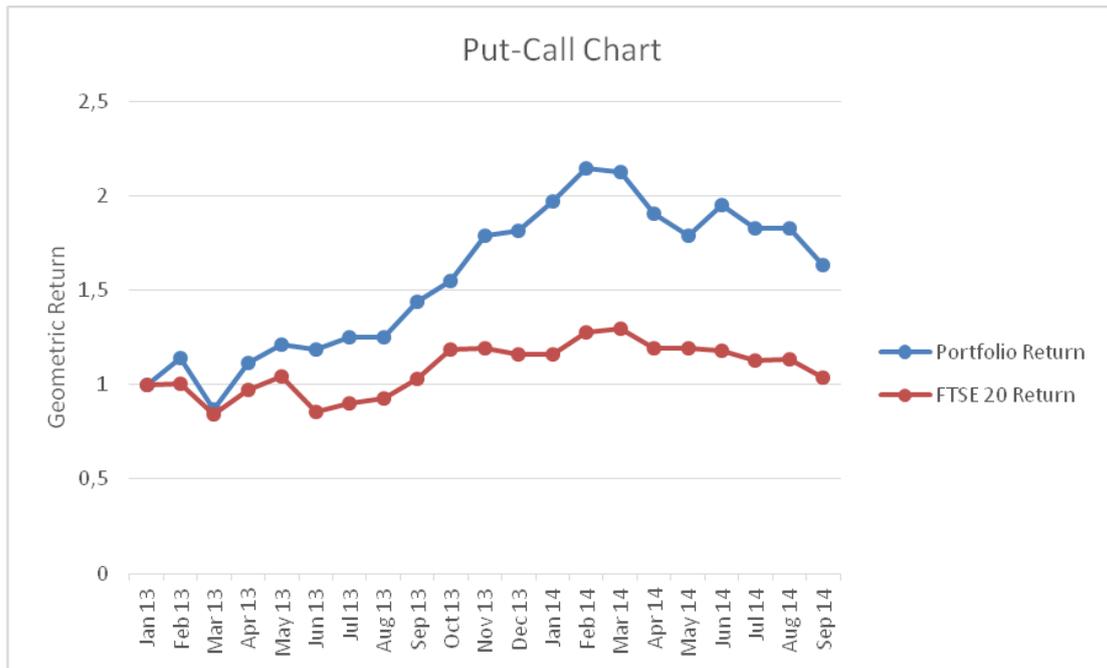


Table 9: Geometric Portfolio Return and Geometric FTSE 20 Return

Date	Geometric Portfolio Return	Geometric FTSE 20 Return
31/1/2013	1	1
28/2/2013	1,140608822	1,006984796
29/3/2013	0,868751294	0,843534548
30/4/2013	1,117167115	0,970435044
31/5/2013	1,209360116	1,041758242
28/6/2013	1,187492234	0,853409604
31/7/2013	1,249989646	0,9037182
30/8/2013	1,248415821	0,926328466
30/9/2013	1,439055705	1,033840132
31/10/2013	1,546862704	1,187776607
29/11/2013	1,790598468	1,189823875
31/12/2013	1,812507766	1,158633148
31/1/2014	1,968730586	1,159927743
28/2/2014	2,14528888	1,273852175
31/3/2014	2,123586664	1,295679663
30/4/2014	1,903044109	1,194099052
30/5/2014	1,78707807	1,192172211
30/6/2014	1,953696417	1,180340208
31/7/2014	1,829529923	1,128285413
29/8/2014	1,829529923	1,13400572
30/9/2014	1,63516256	1,037603492

From the table above, as well as the graph below, it seems that portfolio returns are almost the same with those of FTSE 20 till March 2013. After this period, portfolio returns are growing more than FTSE 20 returns and remain in high levels till 30/09/2014.





Graph 3: Diagrammatic presentation of Geometric portfolio return and FTSE 20 return (Put-Call method)

3.5 Comparing CVaR, MAD and Put-Call Returns with FTSE 20 Return

Having presented the geometric return of each portfolio returns, i.e. by using different models such as CVaR, MAD and Put-Call, it is now essential to put all these into one graph in order to clearly present what model presents the higher portfolio returns than FTSE 20. The related data are as in the following table:

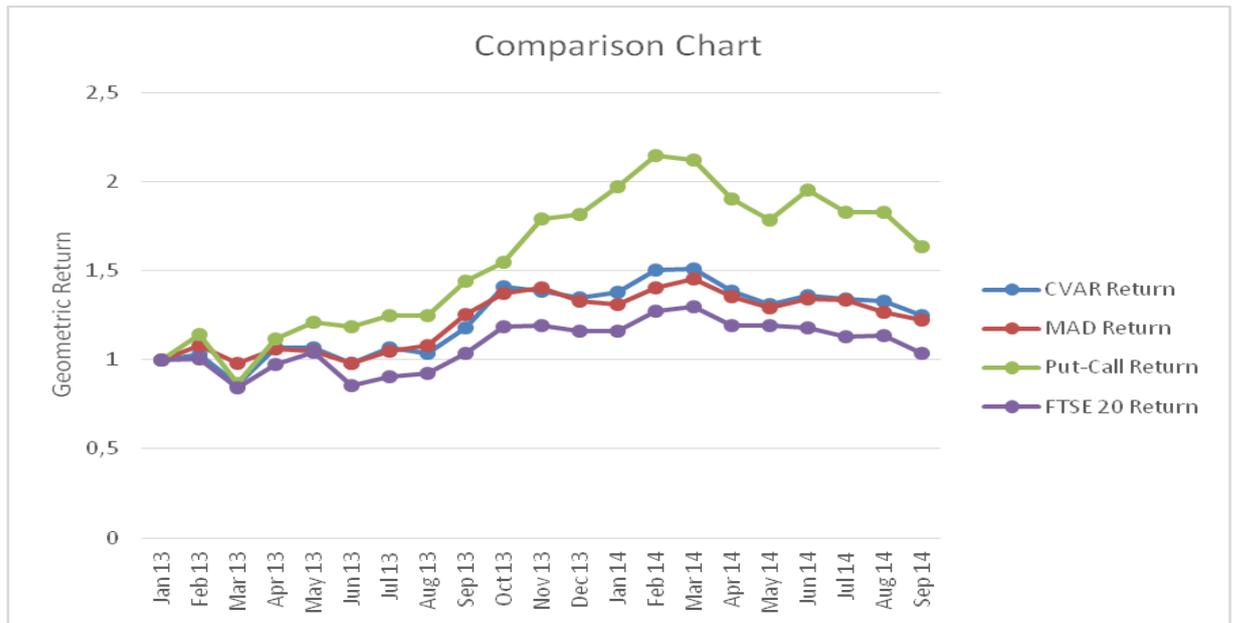


Table 10: Portfolio returns of each model, compared with FTSE 20 Return

Date	CVAR Return	MAD Return	Put-Call Return	FTSE 20 Return
Jan 13	1	1	1	1
Feb 13	1,027966	1,079536	1,140609	1,006985
Mar 13	0,861798	0,977842	0,868751	0,843535
Apr 13	1,06752	1,058677	1,117167	0,970435
May 13	1,064297	1,046395	1,20936	1,041758
Jun 13	0,981089	0,979811	1,187492	0,85341
Jul 13	1,064122	1,050592	1,24999	0,903718
Aug 13	1,035732	1,079947	1,248416	0,926328
Sep 13	1,180362	1,256709	1,439056	1,03384
Oct 13	1,407944	1,374931	1,546863	1,187777
Nov 13	1,387622	1,400794	1,790598	1,189824
Dec 13	1,348924	1,330412	1,812508	1,158633
Jan 14	1,38159	1,311923	1,968731	1,159928
Feb 14	1,501163	1,405518	2,145289	1,273852
Mar 14	1,511964	1,455953	2,123587	1,29568
Apr 14	1,381987	1,353171	1,903044	1,194099
May 14	1,309571	1,290714	1,787078	1,192172
Jun 14	1,36064	1,339474	1,953696	1,18034
Jul 14	1,34219	1,333642	1,82953	1,128285
Aug 14	1,329377	1,269375	1,82953	1,134006
Sep 14	1,248151	1,219976	1,635163	1,037603

From this table, a graph is derived as presented below. This graph shows that all the returns are almost equal with each other as well as with FTSE 20 return till April 2013. Since then, Put-Call return is becoming higher than all the other returns and till the end of the specific time period, Put-Call has the highest portfolio return of all. It must also be pointed out that CVaR and MAD returns are very close together during all the time period and differ significantly from FTSE 20 index after May 2013.



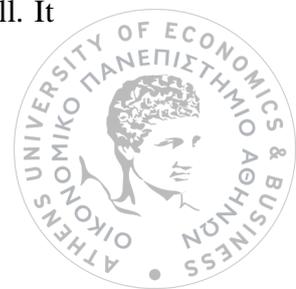


Graph 4: Comparing CVar, MAD and Put-Call returns with FTSE 20 return

4 Conclusions and Recommendations

In this dissertation, I developed a simulation and optimization approach for managing portfolios of financial assets through extensive experiments using real market data from the Greek Stock Exchange. We used General Algebraic Modeling System (GAMS) in order to optimize the portfolio contained of the FTSE 20 stocks. For portfolio optimization problem we examined the maximum portfolio return, using the following three models: CVar, MAD and Put-Call. In addition to these models' maximization problems, the overall performance of the portfolio is also maximized for the periods 31/01/00 through 31/12/12 and for the individual intervals presented above through the backtest. The stock returns obtained were used to calculate real returns, with stock returns over the period 31/01/13 - 30/09/14.

The implementation of the models controls the portfolio's maximum total return, and compared with the FTSE 20's return, at the same period (January 2013- September 2014). We observed that Put-Call return is higher than all the other returns and within the end of the specific time period, Put-Call has the highest portfolio return of all. It



has to be mentioned that CVaR and MAD returns are very close together during all the time period and differ significantly from FTSE 20 index after May 2013. Despite, the positive outcome for Put-Call model, it is necessary to underline that optimum portfolio return would include only the stock prices of Jumbo, which concludes that an investor should invest only in Jumbo stocks. All the other companies for the current period are not considered as effective as Jumbo.

In conclusion, an ideal situation would be to develop extensive and dynamic research experiments on various stock indexes in the international Stock Exchange to portray and accurate and quantifiable estimation of a models ability to contribute to the portfolio optimization problem.



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Appendix

GAMS CODE

CVAR

```
TITLE Conditional Value at Risk models

* CVaR.gms: Conditional Value at Risk models.
* Consiglio, Nielsen and Zenios.
* PRACTICAL FINANCIAL OPTIMIZATION: A Library of GAMS Models, Section 5.5
* Last modified: Apr 2008.

* Uncomment one of the following lines to include a data file

* $INCLUDE "Corporate.inc"
$INCLUDE "WorldIndices.inc"

SCALARS
    Budget          Nominal investment budget
    alpha           Confidence level
    MU_TARGET       Target portfolio return
    MU_STEP         Target return step
    MIN_MU          Minimum return in universe
    MAX_MU          Maximum return in universe
    RISK_TARGET     Bound on CVaR (risk)
    LossFlag        Flag selecting the type of loss;

Budget = 100.0;
alpha = 0.99;

PARAMETERS
    pr(l)           Scenario probability
    P(i,l)          Final values
    EP(i)           Expected final values;

pr(l) = 1.0 / CARD(l);

P(i,l) = 1 + AssetReturns ( i, l );

EP(i) = SUM(l, pr(l) * P(i,l));

MIN_MU = SMIN(i, EP(i));
MAX_MU = SMAX(i, EP(i));

* Assume we want 20 portfolios in the frontier

MU_STEP = (MAX_MU - MIN_MU) / 20;

PARAMETER
    TargetIndex(l) Target index returns;

* To test the model with a market index, uncomment the following two lines.
* Note that, this index can be used only with WorldIndexes.inc.

*$INCLUDE "Index.inc";

*TargetIndex(l) = Index(l);
```



```

POSITIVE VARIABLES
    x(i)                Holdings of assets in monetary units (not
proportions)
    VaRDev(l)          Measures of the deviations from the VaR;

VARIABLES
    VaR                Value-at-Risk
    z                  Objective function value
    Losses(l)          Measures of the losses;

EQUATIONS
    BudgetCon          Equation defining the budget constraint
    ReturnCon          Equation defining the portfolio return constraint
    CVaRCon            Equation defining the CVaR allowed
    ObjDefCVaR         Objective function definition for CVaR minimization
    ObjDefReturn       Objective function definition for return
maximization
    LossDef(l)         Equations defining the losses
    VaRDevCon(l)       Equations defining the VaR deviation constraints;

BudgetCon ..          SUM(i, x(i)) =E= Budget;

ReturnCon ..          SUM(i, EP(i) * x(i)) =G= MU_TARGET * Budget;

CVaRCon ..            VaR + SUM(l, pr(l) * VaRDev(l)) / (1 - alpha) =L=
RISK_TARGET;

VaRDevCon(l) ..      VaRDev(l) =G= Losses(l) - VaR;

LossDef(l)..          Losses(l) =E= (Budget - SUM(i, P(i,l) *
x(i)))$(LossFlag = 1) +
                                (TargetIndex(l) * Budget - SUM(i, P(i,l) *
x(i)))$(LossFlag = 2) +
                                (SUM(i, EP(i) * x(i)) - SUM(i, P(i,l) *
x(i)))$(LossFlag = 3);

ObjDefCVaR ..         z =E= VaR + SUM(l, pr(l) * VaRDev(l)) / (1 - alpha);

ObjDefReturn ..       z =E= SUM(i, EP(i) * x(i));

MODEL MinCVaR 'PFO Model 5.5.1' /BudgetCon, ReturnCon, LossDef, VaRDevCon,
ObjDefCVaR/;

MODEL MaxReturn 'PFO Model 5.5.2' /BudgetCon, CVaRCon, LossDef, VaRDevCon,
ObjDefReturn/;

FILE FrontierHandle /"CVaRFrontiers.csv"/;

FrontierHandle.pc = 5;
FrontierHandle.pw = 1048;

PUT FrontierHandle;

PUT "Status","VaR","CVaR","Mean";

LOOP(i, PUT i.tl);

PUT /;

LossFlag = 2;

* Comment the following line if you want to
* track the market index.

TargetIndex(l) = 1.01;

```



```

FOR (MU_TARGET = MIN_MU TO MAX_MU BY MU_STEP,
    SOLVE MinCVaR MINIMIZING z USING LP;
    PUT MinCVaR.MODELSTAT:0:0,VaR.l:6:5,z.l:6:5,(MU_TARGET *
Budget):8:3;
    LOOP (i, PUT x.l(i):6:2);
    PUT /;
);
$OFFSYMXREF
$OFFSYMLIST
OPTION LIMCOL=0;
OPTION LIMROW=0;
OPTION SOLPRINT=OFF;
OPTION RESLIM=1000000;
OPTION ITERLIM=1000000;
* select optimization solver
OPTION LP = GAMSCHK;

*$offlisting;

OPTION LIMROW = 100;
OPTION LIMCOL = 5;
OPTION SOLPRINT=ON;

* define the set of asset classes

SET class the investment asset classes / s1*s16 /;

SET n / n1*n156 /;
$INCLUDE 21.txt

parameter Prob(n)          Scenario probabilities;
loop(n,
Prob(n)=1/177);

Variables
    cvar          objective function
    z             value at risk
    tot_return    expected final value of portfolio
    f_return(n)  portfolio return at leaf node n
    hold(class)  proportions of asset classes invested at root node
    y(n)         loss shortfall beyond var;

Positive Variables hold(class)
                  y(n);

Scalar    Rho          target expected return /0.05/
          a             critical percentile for var and cvar /0.95/;

equations
    ObjDefCVar          objective function for cvar maximisation
*Target_return          equations defining a minimum target over the
expected return
    Final_return(n)    equation defining the final portfolio return
under scenario n
    Expected_return    equation defining the expected return constraint
init_balance          asset balance constraint at the root
VarCon(n)             equation defining the VaR deviation constraint;

```



```

Final_return(n)..
  f_return(n) =E= sum(class, hold(class)* Ret(n,class));

  Expected_return.. tot_return =E= sum(n, f_return(n)*Prob(n));

init_balance..
  sum(class, hold(class)) =E= 1.0;
ObjDefCVar..
  cvar =E= z - sum(n, Prob(n) * y(n)/(1-a));
*Target_return..tot_return =G= Rho;
VarCon(n)..
  y(n) =G= z-f_return(n);

model portfolio / all /;

file results;
results.ap=1;

Solve portfolio using lp maximizing tot_return;

put results;
results.nd=6;
put tot_return.l /;
put Expected_return.l /;
put CVaR.l /;
put z.l /;
put /;
loop(class, put CLASS.tl, hold.l(class) /;
);

```

MAD

```

$TITLE Testing for first order stochastic dominance

OPTION LIMCOL = 0;
OPTION LIMROW = 0;
OPTION SOLPRINT = OFF;

* select optimization solver
$offlisting;
OPTION ITERLIM=100000;
OPTION RESLIM=100000;
OPTION LP=GUROBI;

set asset /asset1*asset16/;
set s /s1*s156/;
$include Book156.txt

parameter Prob(s)
Loop (s,Prob(s)= 1/177);

Scalar Rho;

variables
mad
f_return(s)
tot_return
y(s)

```



```

tar_return
hold(asset);
positive variables
y,hold;

Equations
ObjDefMad
DevMean1(s)
DevMean2(s)
Targetreturn
FinalReturn(s)
Init_balance
Totalreturn;

objDefMad.. mad =E= sum(s,Prob(s)*y(s));
DevMean1(s).. y(s) =G= tot_return - f_return(s);
DevMean2(s).. y(s) =G= f_return(s) - tot_return;
Targetreturn.. tar_return =E=
sum(s,Prob(s)*(sum(asset,hold(asset)*Ret(s,asset))));
FinalReturn(s).. f_return(s) =E= sum(asset,hold(asset)*Ret(s,asset));
TotalReturn.. tot_return =E= sum(s,Prob(s)*f_return(s));
init_balance.. sum(asset,hold(asset))=E=1;

Model Portfolio /all/;
file resultsmad;
solve Portfolio using Lp minimazing mad;

put resultsmad;
resultsmad.nd=5;
put 'mad=' , mad.l;
put /;
loop(asset, put asset.tl, hold.l(asset)
put /;
);

```

PUT CALL

```

OPTION LIMROW=144;
OPTION LIMCOL=16;
OPTION SOLVELINK=1;
OPTION SOLPRINT=OFF;
$offlisting;
OPTION ITERLIM=100000;
OPTION RESLIM=100000;
OPTION LP=GUROBI;
*OPTION LP=OSL;

Set stocks /s1*s16/;
Set dates /n1*n156/;

Table Ret(dates,stocks)
$ondelim
$INCLUDE ret.csv
$offdelim

scalar Omega portfolio's return target /-0.001/;
parameter pr(dates);
parameter P(dates,stocks);

loop(dates,pr(dates)=1/177);

Table Index(dates,stocks)
$ondelim

```



```
$INCLUDE g.csv
$offdelim
```

```
Positive Variables yPos(dates),yNeg(dates),x(stocks);
```

```
VARIABLES
```

```
zFunction          Objective function value
x(stocks)          Holdings of assets in monetary units (not
proportions);
```

```
EQUATIONS
```

```
z                Objective function definition for MAD
TargetDevDef(dates)  Equations defining the positive and negative
deviations
PutCon           Constraint to bound the expected value of the negative
deviations
totalxi         equation defining the summary of xi ;
```

```
z..              SUM(dates,pr(dates)*yPos(dates))=E=zFunction ;
```

```
totalxi..       SUM(stocks,x(stocks))=E=1.0;
```

```
TargetDevDef(dates).. SUM(stocks,(P(dates,stocks)-
Index(dates,stocks))*x(stocks))=E= yPos(dates)- yNeg(dates);
```

```
PutCon..        SUM(dates,pr(dates)*yNeg(dates))=L=Omega;
```

```
*MODEL UnConPutCallModel 'Model PFO 5.7.1' / PutCon, TargetDevDef, ObjDef /;
model portfolio /all/;
*SOLVE UnConPutCallModel MAXIMIZING z USING LP;
solve portfolio maximizing zFunction USING LP;
```

```
file resultscvar
put resultscvar;
resultscvar.nd=17
put "cvar" z.l/;
put /;
loop(class,put stocks.tl,hold.l(stocks));
```

