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ADAPTIVE SCHEMES FOR THE MULTIVARIATE CONTROL CHARTS

By

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ΟΙΚΟΝΟΜΙΚΟ
ΠΑΝΕΠΙΣΤΗΜΙΟ
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Θεόδωρος Δ. Περδίκης

ΔΙΑΤΡΙΒΗ

Που υποβλήθηκε στο Τμήμα Στατιστικής

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To everyone who contributed to this effort

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ABSTRACT

Theodoros Perdikis

ADAPTIVE SCHEMES FOR THE MULTIVARIATE CONTROL CHARTS

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Statistical process control deals primarily with control charts. Control charts are used in order to monitor quality variables from a process. Over the past decades, in order to enhance the efficiency and performance of the control charts the use of the adaptive feature in the design parameters has been studied. Our aim is to present, evaluate and compare various multivariate adaptive control charts. This thesis is organized as follows. Firstly, a brief review of the adaptive univariate Shewhart,EWMA and CUSUM charts is presented. Secondly, there will be a detailed presentation of the above charts in the multivariate case and also the adaptive multivariate linear profile monitor schemes will be investigated. Lastly, the VSS,VSI and VSSI control chart will be demonstrated using the R programming language.

ΠΕΡΙΛΗΨΗ

Θεόδωρος Περδίκης

ΠΡΟΣΑΡΜΟΣΜΈΝΑ ΣΥΣΤΉΜΑΤΑ ΓΙΑ ΠΟΛΥΜΕΤΑΒΛΗΤΑ ΔΙΑΓΡΑΜΜΑΤΑ ΕΛΕΓΧΟΥ

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Ο στατιστικός έλεγχος ποιότητας ασχολείται κυρίως με τα διαγράμματα ελέγχου. Τα διαγράμματα αυτά χρησιμοποιούνται για την παρακολούθηση των ποσοτικών μεταβλητών σε μια διαδικασία παραγωγής. Τις τελευταίες δεκαετίες, προκειμένου να ενισχυθεί η αποδοτικότητα και η απόδοση των διαγραμμάτων ελέγχου, έχει μελετηθεί η προσαρμογή των παραμέτρων χαρακτηριστικού στις παραμέτρους σχεδιασμού. Στόχος μας είναι να παρουσιάσουμε, να αξιολογήσουμε και να συγκρίνουμε διάφορα προσαρμοσμένα διαγράμματα ελέγχου. Η διπλωματική αυτή οργανώνεται ως εξής. Αρχικά, παρουσιάζεται μια σύντομη ανασκόπηση των προσαρμοσμένων μονομεταβλητών διαγραμμάτων Shewhart, EWMA και CUSUM. Στη συνέχεια, θα υπάρξει μια λεπτομερής παρουσίαση των παραπάνω διαγραμμάτων στην πολυμεταβλητή περίπτωση και επίσης θα διερευνηθούν τα προσαρμοσμένα συστήματα πολλαπλών γραμμικών παρακολουθήσεων προφίλ. Τέλος, τα διαγράμματα VSS,VSI και VSSI θα παρουσιαστούν γραφικά χρησιμοποιώντας τη γλώσσα προγραμματισμού R

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CHAPTER 1

INTRODUCTION

1.1 Introduction

In the world of business, quality improvement is of high importance for the manufacturing industries. Due to the fact that the competition in the global market is strong, industries are interested in measuring their products' characteristics as a variable using continuous measurements, such as length or content. The quality characteristics should reach a desired value (target value) , relative to specifications. These target values are bounded by a range of values that are believed to be sufficiently close to the target value without affecting the process when the quality characteristic of the product is in this range. Statistical process control (SPC), is a valuable process, using statistica l methods for monitoring and controlling the process performance. SPC techniques are employed to examine specific parts of a process and detect changes in process performance. One of the major tools of the statistical process control is the control chart. Control charts were first introduced by Shewhart[1].They are an on-line process monitoring technique whose purpose is to quickly detect shifts in the mean or in the variance of the process. It is well acknowledged in the literature (see ,e.g. ,Montgomery[2]) that standard control chart usage involves two phases with two different objectives. Phase It is used to establish whether the process is in control. It is an iterative process, and the prediction success of a control chart depends on the success of this analysis. Once the in-control state is established in phase I, the data gathered can be used to estimate any unknown parameters. In phase II, the control chart is used in order to detect shifts in the mean or in the variance by comparing the sample statistic for each successive sample as it I s drawn from the process to the control limits. Over the past decades the efficiency and performance of the control chars on monitoring mean shifts have been enchanted. The adaptive feature has been added in the control charts, giving them the capability of quick detection of a mean shift.

1.2 Univariate Adaptive Control Charts

The conventional Shewhart control charts have their parameters fixed, and even though are simpe in design , it takes a long time to detect small or moderate shifts in the process. As a result, new alternatives have been proposed to improve their performance. One effective modification of control charts is the adaptive control chart that allows one, two, or all the chart parameters to change during production. The first adaptive Shewhart control chart was introduced by Reynolds et al.[7] and it is based on the idea of varying the sampling interval during the process . This chart is called as the VSI chart. Also, there have been many studies focusing on adapting the sample size and the sampling interval and there were firstly introduced by Prabhu et al[8] and Costa[9] independently. Furthermore, Prabhu et al[8] combining the above approaches of changing the sampling interval and the sample size proposed a chart in which both the sampling interval and the sample size are varying during the process. It should be mentioned that in order to design an adaptive control chart, besides the parameters used in the standard control charts (the sample size, sampling interval, control limit coefficient), warning limits should be added, and the warning limit coefficient should be defined. The warning limits, w, split the region between the control limits. Note that depending on the number of the warning limits, we have different state adaptive sampling schemes. The number of states defines the number of values that the design parameters can take. For instance, a two-state control chart consists of one set of warning limits, dividing the area between the control limits into two action areas creating three zones and the design parameters (sample size, sampling interval, coefficient of the control limits/action limit) can take two possible values. It has been shown that the use a two-stated control chart is preferable since, adding a third or fourth state does not significantly increase the performance.(Zimmer et al[10], Jensen et al.[6]) In general, it can be concluded that the fewer the adaptive parameters used, the simpler the chart is. De Magalhães et al.[11] designed and compared all types of adaptive control charts arranging them in a hierarchy of the two-state adaptive $\bar{X}$ chart. Therefore, from the combinations of the design parameters seven adaptive $\bar{X}$ control charts can be designed and two sets of parameters (n_1, t_1, w_1, k_1) and (n_2, t_2, w_2, k_2) are defined. There are three one-parameter variable charts: the variable sampling interval (VSI) chart, where t can take two values and the rest of the parameters are fixed; the variable sample size (VSS) chart, having only n varying; and the variable sample limit (VSL) chart, having the width coefficient of the warning and control limits (w, k) varying. Finally, there are three adaptive control charts having two parameters varying. These are as follows: the variable sample size and sampling interval (VSSI) control chart, having the sample size, n, and the sampling interval, t, varying; the variable sample size and limits (VSSL) chart, having n and (w, k) varying; and the variable sample interval and limits (VSIL) chart, having t and (w, k) varying. Finally, all three design parameters can vary forming the variable parameters (VP) control chart. Morevrer, in order to improve the efficiency and the performance of the above charts many extensions have been proposed. Reynolds[12] presented a VSI chart at fixed times (VSIFT) in which the sampling interval at each sampling point is fixed , but additional samples can been taken between these time points when there is an indication of a process shift. Lin and Chou[13],extended the above approach and presented the variable sample rate with sampling at fixed times control chart (VSRFT) and the variable parameters with sampling at fixed times control chart. These charts are based on the idea that , as long as the sample point is near the center line, the sampling interval is fixed (t_f), but when it is far from the target but inside the control limits additional samples are allowed to be taken between two fixed sampling times. Due to the fact that the frequency of switches in the values of the parameters affects the statistical performance of the chart the used of run rules has been proposed. Mahadik[14] added run rules into the parameters of the variable sample size and sampling interval charts(VSSI chart) in order to reduce the frequency of switches without affecting the statistical performance of the chart. Furthermore, Celano et al.[15] suggested that adding run rules to the VSSI $\bar{X}$ chart as signaling tools of an out-of-control process.

As it was previously stated, even though the conventional standard control charts have the simplicity in the design are not capable in detecting small and moderate shifts. On the other hand, the CUSUM and EWMA charts can detect small to moderate shifts, in the mean or in the variance of the process, faster than the conventional standard control charts duo to the fact that at each sampling point they take into account the previous measurements. Arnold and Reynolds[16] presenting two ways to define the CUSUM control statistic and comparing the effectiveness of the VSI, VSS, and VSSI CUSUM charts. The VSI CUSUM chart was also investigated by Luo et al.,[17] and Wu et al.[18]proposed a CUSUM chart with adaptive reference parameter k and adaptive exponential w. Moreover, the weighted loss CUSUM charts with variable parameters were investigated by Zhang and Wu [19,20] and Wu et al.[21] Furthermore, there have been additional studies on the adaptive CUSUM control charts. For instance, a modification of the CUSUM chart, the ACUSUM chart, was firstly introduced by Sparks[22]. Luo et al[17] and Wu et al[18] extended the above chart ,varying its parameters. Also, Luo et al[17] proposed the upper one-sided VSI ACUSUM chart expecting it to be more efficient and economical in detecting increasing mean shifts.

Furthermore, there have been many studies on the adaptive EWMA charts. Arnold et al.[23] and Reynolds and Arnold.[24] investigated the variable sample and variable sampling interval EWMA charts. These studies were mainly focused on charts monitoring the process mean. Also, Capizzi and Masarotto[25] presented a class of the adaptive EWMA charts(AEWMA chart) , combining a EWMA and a Shewhart chart. These charts are capable of monitoring the process variance or both the process mean and variance The main idea is to adapt the weight of the past observations according the magnitude of the error to detect. Another adaptive chart presented by Castagliola et al[26], for monitoring shifts in the variance based on a three-parameter logarithmic transformation. This chart is called VSS S^2 EWMA chart .Moreover, over the past decade, as more research is been made, new techniques are adopted to improve the performance of the control charts, and new features are added to the so-far known charts. For instance, Shi et al·[27]proposed a new variable sampling scheme at fixed times using the inverse normal transformation for monitoring the process dispersion, and Zhang et al[28] extended the above scheme proposing an EWMA chart with the generalized likelihood test for monitoring both the process mean and variance. Finally, a widely used EWMA chart for monitoring the process variance is the one based on the logarithmic transformation of the subgroup variance, (LEWMA chart).

1.3 Performance determination

When selecting control charts, practitioners should be interested in how effective they are. The most widely used measure of performance of a control chart is the average run length (ARL), defined as the average plotted points on the chart until an out-of-control condition is signaled. Specifically, ARL is the average number of points that must be plotted before an out-of-control signal occurs. Note that, ARL is divided into two cases, the average run length when the process is in control (ARL_0),and the average run length when the process is out-of-control (ARL_1). In general an efficient control chart must have small values of ARL_1 and large values of ARL_0.Generally the value of ARL_0 for $a = 0.005$. (error type I) is set to 200. Another convenient measure to express the performance of a control chart in terms of its average time to signal (ATS), that is, the expected time from the start of the process to the time when the chart signals. If samples are taken at fixed intervals of time that are h hours apart, then ATS = ARL$\times$ h. However, the ARL can be only used when the sampling intervals remain constant and equal for the compared schemes. As a result, the ARL can be inappropriate for evaluating the performance of the adaptive charts because in most of the cases the sampling interval and the sample size are not fixed. The widely used performance metrics for adaptive control charts include (psarakis[3], Tagaras[4], Seifa et al[5]):

ANSS – the average number of samples to signal, which is defined as the expected value of the number of samples taken from the start of the process to the time when the chart indicates an out-of-control signal.

$$\text{ANSS} = \frac{1}{1-\beta}$$

where β is the probability that the sample statistic falls between the control limits when actually the process is out of control.

ATC= the average time from the start of production until the first signal after an assignable cause arrives.We assume the shift distribution is exponential with parameter λ.

ATS- the average time to signal, the expected time from the start of the process to the time when the chart signals. If samples are taken at fixed intervals of time that are h hours apart, then ATS = ARL x h

AATS – the adjusted average time to signal, which is defined as the expected value of the time from the occurrence of an assignable cause to the time when the chart indicates an out-of-control signal.

$$\text{AATS} = \text{ATC} - 1/\lambda$$

SSATS- It is assumed that these statistics have reached their steady-state distributions by the time the shift occurs. Under this assumption the expected time from the shift to the signal is called the steady-state ATS

ANOS – the average number of observations to signal, which is defined as the expected number of individual observations from the start of the process to the time when the chart indicates an out-of-control signal.

ANSW – the average number of switches, which is defined as the expected value of the number of switches between different values of the adaptive parameters from the start of the process until the chart signals.

ASWR – the average switching rate, which is defined as the expected number of switches between different values of the adaptive parameters from the start of the process until the chart signals, divided by the average number of samples to signal.

In order to compare the adaptive control charts, their performance measurements need to be calculated and compared. A powerful way to evaluate these properties is to represent the chart as a finite Markov chain. Jensen et al.[6] presented a general Markov chain model that can be used for any finite number of sample sizes or sample intervals. For instance, for the adaptive Shewhart or Hotelling's control charts with one warning limit the in-control and out-of control area is divided into three zones where the absorbing state of the Markov chain corresponds to a signal is denoted as zone 3,$S_3=[CL,\infty)$ while zone 1 , $S_1=[0,w]$, and zone 2, $S_2=[w,CL]$, corresponding to different states of the Markov chain

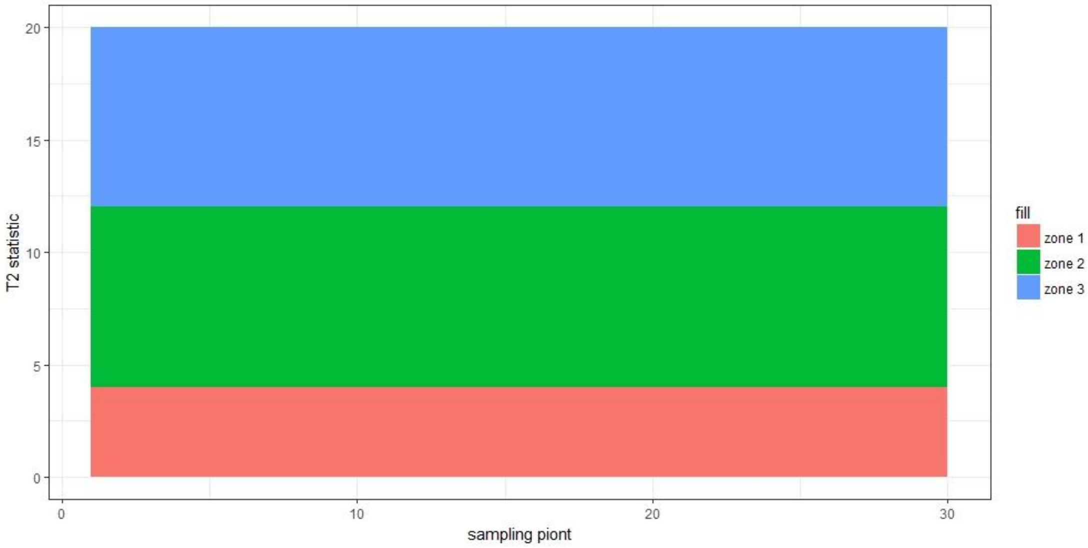

Figure 1: States of the sample at each sampling point

CHAPTER 2

MULTIVARIATE ADAPTIVE HOTELLING'S T^2 CONTROL CHARTS

2.1 Introduction

Traditional T^2 charts are based on the idea that the design parameters, for example the sample size and the sampling interval are fixed values. These charts are called FSR(Fixed Sample Ratio) charts. FSR charts due to the fact that are simple in design, have been widely used for monitoring mean shifts in a process. However, the FSR charts do not perform well in detecting small to moderate shifts. In order to improve the charts' performance in detecting small mean shifts quickly, the VSR(Variable Sample Ratio charts have been proposed. As it was previously stated, a standard FSR Hotelling's T^2 chart uses a fixed sample size (n_0) every t_0 time units. On the other hand, in the VSR charts the adaptive feature is added on the design parameters. If one of the above parameters in not fixed but a function of the position of the current sample point, the chart is called adaptive. Moreover, on adaptive control charts the in-control region is divided in two areas warning and safety, with w the line between the warning and safety region. In this section, some of the most commonly used adaptive T^2 charts are investigated. At each chart, the design parameters will be presented and the chart's performance in detecting mean shifts will be examined. Note that, in order to compare the adaptive control charts, their performance measurements need to be calculated and compared. A powerful way to evaluate these properties is to represent the chart as a finite Markov chain.

2.2 FSR Hotelling's T^2 Control Charts

On a conventional Hotelling's multivariate control chart, it is assumed that a sequence of $p \times 1$ random vectors $\bar{X} = (\bar{X}_1, \bar{X}_2, \bar{X}_3, \,.\, \bar{X}_p)$ is collected, each representing a sample mean vector of p related quality characteristics vectors with sample size of n, observed over time. The p related quality characteristics are assumed jointly distributed as p-variate normal distribution with mean vector $\boldsymbol{\mu}_0$ and covariance matrix $\boldsymbol{\Sigma}_0$. When the process mean, $\boldsymbol{\mu}_0$, and the covariance matrix, $\boldsymbol{\Sigma}_0$, are known, the Hotelling's multivariate control chart signals that a process mean shift has occurred as soon as $T_i^2 = \mathrm{n}(\bar{X}_\imath - \mu_0)' \, \Sigma_0^{-1} (\bar{X}_\imath - \mu_0) > k$, where

$k = C(m.n.p)F_{p,v,a}$ *and* $C(m.n.p) = \frac{p(m+1)(n-1)}{mn-m-p+1}$ with $v = mn - m - p + 1$ and $F_{p,v,a}$ is the upper percentage point of F distribution with p and m degrees of freedom if sample size n > 1.Moreover $C(m.n.p) = \frac{p(m+1)(n-1)}{m^2-mp}$ with $v = mn - m - p + 1$ if sample size n = 1.Note that for n,m→∞ k=$X_{p,a}^2$.Finally the non-centrality parameter $\Delta = n\sqrt{(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})' \Sigma^{-1} (\boldsymbol{\mu_1} - \boldsymbol{\mu_0})}$ is used to measure a change in the process mean vector, and $\boldsymbol{\mu_1}$ is the p characteristics mean vector, when a change in at least one of the means exists.

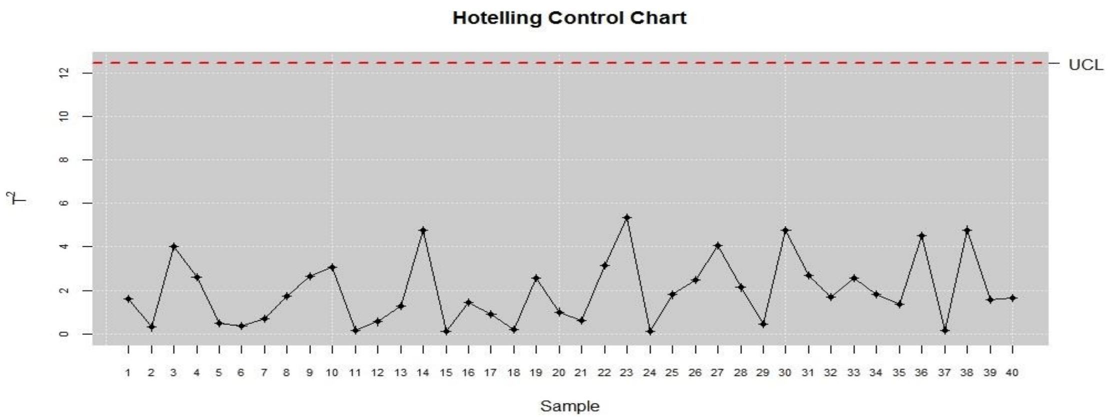

Figure 2: Hotelling's T^2 Control Chart

2.3 Variable sample size T^2 char(VSS T^2 chart)

The VSS chart was first introduced by Burr [29],Daudin[30] ,Prabhu et al [8] and Costa[9].They proposed a new adaptive Shewhart chart for monitoring the process mean where there are two possible sample sizes at each sample point. Aparissi [31] extended the VSS chart into the multivariate case ,presenting the VSS T^2 chart for monitoring the mean

2.3.1 Design of the VSS _T^2 chart

Let n_1, n_2 be the two possible sample sizes of the j^{th} subgroup at sampling point j, where $n_1<n_0<n_2$ and n_0 a fixed average value of the sample size of a subgroup. Finally, let *w* be the warning limit, which identifies when to change the sample size, where $0 < w < CL$ .The choice of the value of the sample size used at the next sample point depends on the value of the statistic T^2 at the current sample point, where $T_i^2 = n(i)(\overline{X}_i - \mu_0)' \Sigma_0^{-1}(\overline{X}_i - \mu_0)$ .At each sampling point the decision for the value of the sample size is based on the following rule:

$$n(i), = n_1 \text{ , if } 0 < T_{i-1}^2 \leq w$$

or

$$n(i), = n_2 \text{ , if } w < T_{i-1}^2 < CL$$

Also if $T_{i-1}^2 > X_{P,a}^2$ an out-of-control signal will be given.

2.4 Variable sampling interval T^2 chart (VSI T^2 chart)

The VSI chart was first introduced by Reynolds et al[23] in the univariate case .This adaptive T^2 chart is based on the idea that the sampling interval at each sampling point is varied .Note that the conventional T^2 chart supposes that the sample size and the sampling interval are fixed values. Reynolds and Arnold[32] and Runger and Pignatiello [33] mentioned that using two potential values for the sampling interval at each sampling improves the capability of the chart in detecting small mean shifts. Aparasi and Haro[34] extended the VSI chart in the multivariate case and presented the VSI T^2 chart

2.4.1 Design of the VSI T^2 chart

Let at_0 and bt_0, be the two possible the sample intervals at sampling point j, a >1, 0 <b<1 (at_0 and bt_0 are consider to be the long and short sampling interval respectively) .Note that the sample size at each sampling point is considered to be fixed. Finally, let w be the warning limit, which identifies when to change the sampling interval, where $0 < w < CL$. The value of the sample size used at the next sample point depends on the value of the statistic T^2 at the current sample point where

$T_i^2 = n(\overline{X}_i - \mu_0)' \Sigma_0^{-1}(\overline{X}_i - \mu_0)$.Consequently, at each sampling point the decision for the value of the sampling interval is based on the following rule:

$$t(i) = bt_0 \quad \text{,if } w < T_{i-1}^2 < CL$$

$$\text{or}$$

$$t(i) = at_0 \quad \text{, if } 0 < T_{i-1}^2 \leq w$$

Also when $T_{i-1}^2 > CL = X_{P,a}^2$ an out-of-control signal is given.

2.5 Variable sampling size and sampling interval T^2 chart (VSSI T^2 chart)

The VSSI T^2 chart was firstly introduced by Aparasi and Haro[35]. This adaptive chart is a combination of a VSS and a VSI chart. It is based on the idea that at each sampling point the values of the sample size and sampling interval are varied.

2.5.1 Design of the VSSI chart

Let n_1, n_2 be the two possible values of the sample size of the j^{th} subgroup at sampling point j, where $n_1<n_0< n_2$ and n_0 the fixed sample size of a subgroup. Also let at_0 and bt_0 ,be the two possible values of the sample interval at sampling point j, where at_0 and bt_0 are consider to be the long and short sampling interval respectively (a >1, 0 <b<1) . Finally, let w be the warning limit, where $0 < w < CL$ .Moreover, the value of the sample size used at the next sample point depends on the value of the statistic, T^2, at the current sample point where $T_i^2 = n(i)(\bar{X}_i - \mu_0)' \Sigma_0^{-1}(\bar{X}_i - \mu_0)$.

At each sampling point , the value of the sampling interval is based on the following rule:

$$(\mathrm{n(i)}, \mathrm{t(i)}) = (n_2, bt_0) \quad \text{if } \mathrm{w} < \mathrm{T}_{\mathrm{i-1}}^2 < CL$$

or

$$\mathrm{n(i)}, \mathrm{t(i)} = (n_1, at_o)\,, \text{ if } 0 < \mathrm{T}_{\mathrm{i-1}}^2 \leq w$$

When $T_{\mathrm{i-1}}^2 > CL = X_{P,a}^2$ an out-of-control signal is given

2.5.2 Performance of the VSSI chart

Aparasi and Haro[35] compared the performance of the VSSI chart with the VSS,VSI and MEWMA charts using the metrics ARL and ATS. The results showed that for shifts equal to one(d=1) VSSI and VSS have similar performance. On the other hand for shifts larger than one(d>1) the use of VSS is recommended. Moreover comparing the VSSI with the VSI chart the authors mentions that for small shifts(0.75 to 1.25),the VSSI chart outperforms the VSI chart .However, for large shifts the VSI chart is preferable. Finally, compaired the performance of the above charts with the

MEWMA chart Aparasi and Haro(2003) saw that for small shifts (d=0.25) the MEWMA chart is superior to the others in detecting quickly mean shifts .For moderate shifts (d=0.5) the VSI,VSSI and MEWMA charts have similar performance and for large shifts all the above charts have similar performance .Finally, for shifts larger than one(d>1) and taking three observations at each subgroup, VSS has the best performance.

2.6 Double sampling chart (DP chart)

The double sampling $\bar{X}\, chart$ was first introduced by Croasdale[36] in the univariate case. Also He[37], He and He et al[38] extended the DS chart using triple sampling. DS chart is based on the idea of using two samples on each sampling point. If the first sample does not indicate any potential shifts in the mean a second sample is taken and combined with the first sample in order to obtain more information. Furthermore, there have been additional studies on DS charts. He[37] developed a multivariate DS char and Champ and Aparissi[39] studied two DS charts in order to monitor the mean. Also, Reynolds and Kim[40] studied the multiple sampling T^2 as a type of MEWMA chart. Is should be mentioned that DS chart is a special type of multiple T^2 chart.

2.6.1 DS combined T^2 and DS T^2 chart

Champ and Aparissi[39] ,proposed two forms of DS charts , DS combined T^2 and DS T^2 chart.Each of the two *DS* schemes first plots the Hotelling's T^2 statistic $T^2_{k,1} = n_1\left(\overline{X_{k,1}} - \mu_0\right)' \Sigma_0^{-1}\left(\overline{X_{k,1}} - \mu_0\right)$ versus the sample number k,. where $X_{k,1}$ is the mean of the first sample collected at sampling stage k. The decision of taking second sample is determined by the following rules:

If $T^2_{k,1} \geq h_1$ means that the process might be out of control. Then if $0 < T^2_{k,1} < w_1 < h_1$ the practitioner should wait until the next sample stage to study the process further. Finally if $w_1 \leq T^2_{k,1} < h_1$ then the practitioner should take the second sample and combine the information with the first. The main difference between these two chart is in the statistic each chart uses in order to summarize the information in the combined sample .

2.6.1.1 DS combined T^2 chart

The DS combined T^2 chart summarizes the information in the combined sample using the statistic :

$Q^2_k = \frac{1}{n}\left(n_1 T^2_{K,1} + n_2 T^2_{K,2}\right)$ with $T^2_{k,1} = n_2\left(\overline{X_{k,2}} - \mu_0\right)' \Sigma_0^{-1}\left(\overline{X_{k,2}} - \mu_0\right)$ where $n=n_1+n_2$,giving signal when $Q^2_k \geq h$ and Q^2_k is a weighted average of T^2 statistic. This chart is called DS combined T^2 chart.

2.6.1.2 DS T^2 chart

The second chart uses the statistic

$T^2_k = n(\overline{X_k} - \mu_0)' \Sigma_0^{-1}(\overline{X_k} - \mu_0)$ which is a weighted average of the sample mean vectors giving signal when $T^2_k \geq h$, where $\overline{X_k} = \frac{1}{n}(n_1\overline{X_{k,1}} + n_2\overline{X_{k,2}})$, $n= n_1 + n_2$

The main difference between these two chart is that the *DS* combined T^2 control chart, uses a weighted average of the T^2 statistics associated with each sample. The other, the *DS* T^2 control chart, uses the Hotelling's T^2 statistic based on the pooled sample.

2.6.2 Performance of the DS combined T^2 and the DS T^2 chart

Champ and Aparissi[39] using simulation examined the performance of the above charts comparing them with MEWMA and VSS charts based on their ARL values .At first, it should be mentioned that the DS T^2 chart has better performance than DS combined T^2 chart. As a result the use of the DS T^2 chart is preferable(Champ and Aparissi([39] Also even though a conventional T^2 chart(T^2_{fixed})is simpler , DS T^2 outperforms the T^2_{fixed} chart in ARL performance especially for small to moderate shifts in the mean. On the other hand, it should be mentioned that for large shifts the T^2_{fixed} and DS T^2 charts have similar results .Furthermore comparing the performance between the DS T^2 chart and the Mewma chart the DS T^2 chart is always better than the MEWMA chart except small shift magnitudes(d=0.5).Finally, the performance of the VSS chart in detecting mean shifts is inferior to the DS T^2 chart for every shift magnitude.

2.7 2scaled variable sample size and control limit chart (2VSSC T^2 chart)

It has been shown that VSI T^2 charts improve the performance of ATS on moderate shifts and VSSI T^2 charts have quite good performance in detecting small process shifts. Furthermore, ,comparing the VSSI T^2 chart with the multivariate multiple sampling (MMS) control chart proposed by He and Grigoryan[41] it can be seen that VSSI T^2 chart is better in detecting small shifts. On the other hand, it should be mentioned that MMS charts with sample size more than 3 observations have better performance but it is not easy to work with in practice. Chen and Hsieh[42] presented a two-scaled variable sample size and control limit called as the 2VSSC T^2 chart, which is based on an optimization problem which aims to minimizes the ATS ,giving the average sample size and false alarm rate constraints when the process is in control

2.7.1 Design of the 2VSSC T^2 chart

This chart sets the sampling interval as a constant(t_0), defining two possible values for the sample size where n_1 is the minimum and n_2 the maximum sample size .Moreover, there two possible values for the warning limits and for the action/control limits. If the prior sample is in the safe region [o,w_j] ,

n_1 is used .On the other hand, if it falls into the warning region $(w_j,k_j]$, n_2 is used. Lastly, if the prior sample size falls in the action region (k_j,∞) means that the process is out-of-control, where j=1 if the prior sample is n_1 and j=2 ohterwise. ($w_1>w_2$, $k_1>k_o>k_2$). In summary:

$$n(i), w(i), k(i) = (n_2, w_2, k_2)\ if\ w\ (i-1) < T_{i-1}^2 \leq k(i-1)$$

or

$$n(i), w(i), k(i) = (n_1, w_1, k_1)\ \ if\ \ 0 \leq T_{i-1}^2 \leq w(i-1)$$

2.7.2 Performance of the 2VSSC T^2 chart

Chen and Hsieh[42]compared the performance of FSR charts with the 2VSSC for different values of n_0 ,m(number of subgroups) and mean shifts and mentioned that the 2VSSC chart have quite good performance for small shifts(d>1) . Also in order to examine the effect of the subgroups size,m ,on the optimal design parameters they applied a sensitive analysis. It was found that only maximum action and warning limits (w_2, k_2) are sensitive to the m values. In fact, they are decreasing as the m increases. However m does not have any effect beyond Nedumaran and Pignatiello's lower bound. Furthermore, comparing the 2VSSC chart with the VSI,VSS,VSSI, and the 2VP chart the authors showed that for very small mean shifts the VSSC chart has better performance than the above charts. Also, the 2VSSC and the 2VP chart, have similar performances according to their ATS values. In conclusion, the 2VSSC chart has better performance than the FSR charts. However for d>1 there is no significant difference between theses charts but it should be mentioned that for large shifts the ATS is already small.

2.8 2scaled Variable parameters chart(2VP chart)

This chart was first introduced by Cocta[43] in the univariate case. VP $\overline{X}$ chart is based on the idea that all the parameters (sample size, sampling interval, and action limits) are not considered fixed but varied. Chen[44] extended the above chart to a multivariate process.

2.8.1 Design of the 2VP control scheme

Suppose that (n_1,t_1), ($n_{2,}t_2$) are the pairs of minimum sample size with the longest sampling interval and maximum sample size with the shortest sampling interval respectively($n_1 < n_0 < n_2$, $t_2 < t_o < t_1$). If the prior sample falls into the tightening region then in the current sample the pair ($n_{2,}t_2$) should be used and if the prior sample falls into the relaxing region then in the current sample the pair (n_1,t_1) should be used . Note that, the relaxing and tightening regions are given by the warning limit w_j and the action limit $k_j = C\left(m_j, n_j p,\right) F_{p,v_j,a_j}$,where relaxing-safe region=[o,w_j] and tightening- warning region =[w_j,$k_{j]}$]. Also,j = 1 means that the prior sample point comes from the small sample, and j = 2 that the prior sample point comes from the large sample. Moreover, it is assumed that $w_1 > w_2$ and $k_1 > k_0 > k_2$.In summary at each sampling point the decision for the values of the parameters is based on the following rule:

$$\mathrm{n(i), t(i) w(i), k(i)} = \left(\mathrm{n_2, t_{2,} w_2, k_2}\right)$$
$$\text{if } \mathrm{w\ (i-1)} < \mathrm{T^2_{i-1}} < k\mathrm{(i-1)}$$
$$\text{or}$$
$$\mathrm{n(i), t(i), w(i), k(i)} = \mathrm{(n_1, t_1 w_1, k_1)}$$
$$\text{if } 0 \leq \mathrm{T^2_{i-1}} \leq w\mathrm{(i-1)}$$

In order to choose the initial values for the sampling interval and the sample size at the start of the process or after a false alarm it is proposed the use of the following rule. Small size and long sampling interval are selected with probability p_0 ,while large size and short sampling interval are used with probability(*1*- p_0) Moreover, for the sake of simplicity we set w_{j} as follows $p_0 = Pr\{T_i^2 < w_1 | T_i^2 < k_1\} = Pr\{T_i^2 < w_2 \big| T_i^2 < k_2\}$. Since warning limits and action regions are not fixed but are depended on the sampling size ,there have been proposed several ways to plot and construct the charts. A possible way is to create two charts one for small and another for large samples or constructing a chart with two scales. The observation from small sample can be plotted according to the left scale, and the one from large sample can be plotted according to the right scale. However, breaking the left scale and plotting these sample points anywhere inside the right region, regardless of the right position is the best choice. In this way, the effort to monitor a process with the VP control chart or with the FSR control chart is almost the same (Costa [43]).

2.8.2 Performance of the 2VP chart

In order to examine the performance of the VP chart the metrics ATS and AATS are used in order to measure the efficiency of the chart. The 2VP chart has lower ATS and AATS values than the VSR charts. Furthermore, similar result are obtained when the 2VP chart is compared with the VSI,VSS and VSSI charts. The values of ATS and AATS are always smaller for different mean shifts.(0.2<d<1.5).

2.8.3 The effect of the sample size

It should be mentioned that on the 2VP chart, the design parameters are not affected by the value of m, except the maximum action limit which is decreasing as the value of m is increased. However it remains constant as long as m value goes beyond Nedumaran and Pignatiello's lower bound[45].

2.9 VP chart with one warning limit

VP chart was first introduced by Costa [43] in the univaritate case. Chen [44] extended the VP $\bar{X}$ chart to the multivariate case and proposed the 2scaled VP chart .He mentioned that the 2VP chart outperforms the VSI,VSS and the VSSI chart on small mean shifts. Seifa et al [5] presented an alternative sampling scheme for the variable parameter chart, called VP chart. The main difference between the 2VP and the VP chart is that the VP chart uses a fixed warning limit .On the other hand, the 2VP chart uses the adaptive feature in order to choose the value of the warning limit which depends on the statistic T^2 at each sampling point.

2.9.1 Design of the VP chart with one warning limit

Suppose that that (n_1,t_1,k_1) and (n_2,t_2.k_2) are the pairs of minimum sample size with the longest sampling interval and maximum sample size with the shortest sampling interval respectively($n_1 < n_0 < n_2$, $t_2 < t_o < t_1$, $0 < k_2 < k_1$).If at the i-1th sampling point the sample falls into the safe region then the pair (n_1,t_1,k_1) is used .Otherwise if the i-1th sample falls into the warning region the pair (n_2,t_2.k_2) is used. Finally if the sample falls into the action region the sample is considered out-of-control where safe region=[o,w) ,warning region =[w,k_j] , action region=(k_j,∞) and $k_j=C\left(m_j, n_j p,\right) F_{p,v_j,a_j}$ .In summary the choice of the parameters is based on the following rule:

$$\mathrm{t(i)k(i)} = (\mathrm{n_1}, \mathrm{t_1}, \mathrm{k_1}) \text{ if } 0 \leq \mathrm{T}^2_{\mathrm{i}-1} < w$$

or

$$\mathrm{t(i)}, \mathrm{k(i)} = (\mathrm{n_2}, \mathrm{t_2}, \mathrm{k_2}) \text{ if } \mathrm{w} \leq \mathrm{T}^2_{\mathrm{i}-1} < k(\mathrm{i}-1)$$

As it was mentioned before, the main difference between a 2scaled VP chart and the proposed VP chart is that a 2VP chart require the user to construct a T^2 chart with two different measuring scales (w_1,w_2), one on the left-hand side and the other on the right-hand side.Moreover**,** Seifet al [5] defined the VSICL T^2 chart with one waning limit, as a special case of the VP scheme. This chart is obtained by letting $n_1 = n_2 = n_0$. In this case, the expression of w is the same on the scaled VP scheme. When $t_1 = t_2 = t_0$, the VP T^2 chart is called the VSSCL T^2 chart. Other special cases of the VP scheme are VSSI, VSS, and VSI schemes which are obtained by letting $k_1 = k_2 = k_0$; $k_1 = k_2 = k_0$ and $t_1 = t_2 = t_0$ and $k_1 = k_2 = k_0$ and $n_1 = n_2 = n_0$, respectively (see Faraz and Parsian[46], Faraz and Moghadam[47], Faraz et al[48]).

2.9.2 Performance of the VP,VSSCL,VSICL T^2 chart

Seifa et al [5] in order to measure the performance of the above charts they used the AATS metric where AATS=ATC – 1/λ .(ATC= the average time from the start of production until the first signal after an assignable cause arrives (assuming the shift distribution is exponential with parameter λ) and the ATC is computed by using the MC approach. Also he used a genetic algorithm, in order to compute the appropriate values of the parameters and derives the optimal AATS for each chart. The author mentions that the proposed VP chart has better performance than the 2VP chart and also is simpler due to the fact that has fewer parameter than the 2VP chart and has one scaled warning limit. Minor differences exist between the two schemes even for small shifts. In the case where sampling interval is not practical to be used, the VSSCL chart is recommended and if the variable sample size is not to be desired VSICL chart is suitable as an alternative of the VP chart. Comparison results for mean shifts less than 0.5 showed that the VSSCL scheme has better performance than the VP,VSICL,VSSI,VSS, and the VSI scheme. On the other hand, the VP chart is superior to the others in detecting moderate shifts ($0.5 < d < 1.5$). Finally, shifts greater than 1.5, the use of the VSICL and the VSI scheme is preferable.

<u>2.10 Variable sampling interval with fixed times chart</u>

(VSIFT chart)

Lee[49] presented the VSIFT T^2 control chart (variable sampling interval with fixed times on the multivariate case. This chart is an extension of the scheme proposed by Reynolds on the univariate case.

2.10.1 Design of the VSIFT chart

The VSIFT T^2 Hotteling's chart is a modification of the VSI charts using the fixed sample interval t_f which have the capability of taking additional samples between two fixed times equally spaced time points when there is an indication for mean shift. Assume that a sample (or subgroup) of size n is taken at every sampling point, and let $\overline{X_\iota}$ be the average vector for the sample (rational subgroup) in which the sample size is n. Then the sample statistic is defined as $T_{\mathrm{i}}^2 = \mathrm{n}(\overline{X_\mathrm{1}} - \mu_0)' \, \Sigma_0^{-1}(\overline{X_\iota} - \mu_0)$.Samples are taken at fixed time t_f, $2t_f$, $3t_f$, $4t_f$, …Nevertheless, if the sample point is far from the target but there is an indication for mean shift additional sample can be taken between two fixed times. The sample interval can be divided into m subintervals of length t_s. So the possible sampling times within the interval t_f are t_s, $2t_s$, $3t_s$, (m-1)h_s. (mt_s=t_f) . If the current sample is from a fixed time ("fixed time"= t_f, $2t_f$, $3t_f$, $4t_f$, …) and there is no indication for mean shift the next sample is taken after t_f time units ("next time"= "fixed time" + t_f) .Finally, the current sample is taken at ct_s time units after a fixed time ,c=1,2,….m-1then the next sample will be taken after (m-c)t_s time units. So if $T_i^2 \leq w$ (safety region) then the next sampling interval, would be the next fixed time. If w<T_i^2 ≤CL (warning region) the next sampling interval would be at the next t_s time units.

2.10.2 Performance of the VSIFT chart

In order to evaluate the performance of VSIFT Lee(2010) using the metrics ATS (zero state ATS) and AATS, compared the result of these values between the FSR Hotelling's T^2 chart and the VSIFT T^2 chart for different values of d_0(probability of a sampling point falling in the safety region given that does not fall in the action region) .He showed that a VSIFT chart in general for different values for d_0 gives smaller ATS and AATS values than the FSR chart. Lee(2010) also mentioned that due to the fact that small values on d_o give small ATS_0 and large values give large values on $AATS_1$, a VSIFT chart with small d_0 Is appropriate on detecting mean shifts but gives a large false alarm rate. Also , comparing VSIFT for various values for m (number of possible subintervals).Lee saw that a chart with large values on m is more efficient on detecting mean shifts. It should be mentioned that for m>5 the results are similar. Furthermore, Lee compared the AATS of a VSIFT chart with FSR,VSI,VSS,VP and VSSC charts with the same ATS (for p=2) .The results clearly point out that VSIFT T^2 chart is better than the FSR charts except for large shifts. Also in comparison with the VSI charts a VSIFT chart is preferable since haw better performance and is easier in administration. Furthermore, comparing VSS and VSIFT charts Lee mentions that VSIFT performs better for moderate and large mean shifts (d>1,5) but for small shifts (d<1) VSS has better performance. Generally, VP and VSSI charts are preferable on detecting small shift but the adaptive sampling scheme may cause potential problems because the sampling interval between two sampling points cannot be predicted. On the other hand on the VSIFT charts ,the sampling interval between two sampling points is fixed. Finally comparing VSSC and VSIFT charts the results showed that VSIFT chart has better performance on moderate and large shifts.

2.11 A special variable sample size and sampling interval T^2 chart (SVSSI T^2 chart)

A it was previously stated , the VSSI T^2 chart is the quickest chart to detect small shifts while the VSI chart is preferable in detecting large shifts. On moderate shifts VSI and VSSI charts give similar results. Also a VSSI chart is quicker than a VSS chart in detecting large shifts. However in some cases a VSSI chart is even inferior to a conventional Hotellinig's T^2 chart in detecting large shifts. Mahadik & Shirke [50] proposed new type of VSSI

T^2 chart (SVSSI T^2 chart) in order to improve the chart's performance on large shifts. This chart has been design in a way to be similar to a VSSI chart for small shifts and similar to a VSI chart for large shifts

2..11.1 Design of the SVSSI T^2 chart

The SVSSI chart uses the statistic $T_i^2 = n(i)(\overline{X_\iota} - \mu_0)' \Sigma_0^{-1}(\overline{X_\iota} - \mu_0)$ which gives signal if $T_i^2 > L$,where L is the control limit. This chart is based on the idea of using two sampling intervals t_1,t_2 where $t_{max} \geq t_1 \geq t_2 \geq t_{min}$, three sample sizes n_1,n_2,n_3 . $n_{min} \leq n_1 \leq n_2 \leq n_3 \leq n_{max}$ and two threshold-warning limits w_1,w_2 , $0 < w_1 \leq w_2 < L$ which divide the in control area into three regions.I_1=[0,w_1],I_2=(w_1,w_2) and I_3=(w_2,L).The values of the parameters are chosen according to the following function:

$$(n(i), t(i)) = (n_1, t_1) if\ T_{i-1}^2 \in I_1$$
$$\big(n(i), t(i)\big) = (n_2, t_2) if\ T_{i-1}^2 \in I_2$$
$$\big(n(i), t(i)\big) = (n_3, t_2) if\ T_{i-1}^2 \in I_3$$

where n(i) and is the sample size and the sampling interval between the i-1th and the ith sample point.

It should be mentioned, if w_1=w_2 then the SVSSI chart is a VSSI chart , if sample size is fixed then is a VSI chart and finally if sampling interval is fixed and w_1=w_2 is a VSS chart.

2.11.2 Performance of the SVSSI T^2 chart

Mahadik &. Shirke (2010) in order to evaluate the performance of the SVSSI chart used the metrics SSAS,ANSS,ANOS for different shifts and cases. They showed that in each case, the SSATS of the SVSSI chart is the smallest or close to that for any shift comparing with the VSI and VSSI chart. Moreover, it is closer to the SSATS of the VSSI chart for small shifts and closer to the SSATS of the VSI chart for large shifts. Finally, when it is needed to detect both small and large shifts as quickly as possible then an SVSSI T^2 chart is a better alternative to the VSI and VSSI T^2 charts

2.12. VSI T^2 Chart with Runs Rules (VSIRRS chart)

As it was previously stated,a significant disadvantage of using adaptive charts is the frequent switches of the values on the adaptive parameters. It has been showed that using runs rules on VSI charts reduce the frequency of switches on the parameters can be reduced and the chart performance can be improved in detecting small to moderate mean shifts. Admin and Letsinger[51] proposed the VSI .$\bar{X}$ charts with runs rules for switching between the sampling interval lengths.Mahadik[52] extended the above chart into the multivariate case presenting the VSIRRS T^2 Chart.

2.12.1 Design of the VSIRRS T^2 chart

Let t_1, t_2 be the length of the two possible sampling intervals between the (i-1)th and the ith trial where $t_{max} \geq t_1 \geq t_2 \geq t_{min}$ .Furthermore, let L_1=(0,w) ,L_2= (w,L) be the regions which compose the in-control area. Also the sample size at each sampling point is considered to be fixed . This chart uses the statistic $T_i^2 = n(\bar{X}_\iota - \mu_0)' \Sigma^{-1} \ (\bar{X}_\iota - \mu_0)$ giving an out of control signal when >L ,L=X_a^2 In order to choose the value of the sampling interval at each sampling point . Mahadik(2012) proposed the use of the following run rule: Let Ni be the numbers of sample points falling in l_j, j=2,3 in the successive m (≥1) trials ending at the ith trial, where l_3=[L,∞) and k be an integer less than or equal to m. For the trial, i=m+1, m+2,. if in the successive m trials before the i^{th} trial, no out-of-control signal is received and the number of sample points falling in l_2 is less than k,the long sampling interval t_1 should be chosen .On the other hand if in the successive m trials before the s^{th} trial, no out-of-control signal is received and the number of sample points falling in I_2 is greater than or equal to k, the short sampling interval t_2 should d be chosen .This runs rule is called k out of m runs rule and is abbreviated as runs rule (k,m).

2.12.2 Performance of the VSIRRS T^2 chart

In order to evaluate the performance of the VSIRRS chart and compared it with the VSI chart Shashibhushan B. Mahadik(2012) uses the metric SSATS, ANSS and ANSW. It should be mentioned that all the rules reduce the ANSW for small to moderate mean shifts. Generally, the way that the VSI chart has been designed leads to small values of ANSW. However, VSIRRS chart.About the choice of the rule that gives the best performance of the chart ,Mahadik(2012) mentions that the optimal rule is the rule(1,3) where k=1 and m=3.Seting the above values of k and m to the chart gives the smallest value for ANSW. Moreover, the second choice could be the rule (3,5) as ANSW has small values for small shifts than the other rules except the rule (1,3) . However, the SSATS values for the moderate shifts with this rule are larger than those with all the rules except rule (3, 3).Finally, rule (3, 3) is preferable if the reduction in ANSW is more important than the reduction in SSATS; otherwise, rule (2, 3) is preferable.

2.13 VSSI T^2 chart with variable warning limit (VSSIWL T^2 chart)

It is well known, that in the adaptive control charts a main disadvantage is the frequent switches on the design parameters during the process monitoring. In order to reduce the frequency of switches on the design parameters, there have been several studies proposing the use of run rules and variable warning limits. For instance, Amin and Letsinger [51] proposed the use runs rules for switching between sampling interval lengths of VSI charts in the univariate case. Mahadik [52,53] studied the exact properties of VSI charts with such runs rules in the multivariate case. Alternatively, Mahadik [54,55] suggested the use of variable warning limits to reduce frequency of switches between the values of adaptive design parameters of VSI and VSSI $\bar{X}$ charts Also, Mahadik[56] proposed a new T^2 VSSI chart with variable warning limits (VSSIWL chart)

2.13.1 Design of the VSSIWL T^2 chart

This chart uses the statistic $T_i^2 = n(i)(\bar{X}_\iota - \mu_0)'\Sigma^{-1}\ (\bar{X}_\iota - \mu_0)'$ giving an out of control signal when >L ,L=X_a^2 .Let t(i) be the length of the sampling interval between the (i-1)th and the ith trial, w(i) the warning limits and n(i) the sample size on the ith trial. There are two possible values for the parameters n(i), t(i) and w(i)) ,(n_1, t_1, w_1) and(n_2, t_2, w_2) where $n_{min} \le n_1 \le n_2 \le n_{max}$,

$$t_{max} \ge t_1 \ge t_2 \ge t_{min}\ , L > w_1 > w_2 > 0\ ,$$

Note that n_{min} and n_{max} are the smallest and largest possible sampling size and t_{max} and t_{min} the shortest and longest possible sampling intervals.. The warning limit on the (i-1)th trial divides the in control area (0,L) into two regions I_1=[0,W(i-1)] and I_2=(w(i-1),L) .In summary, the values of the parameters on the ith trial are chosen according to the following rule

$$n(i), t(i), w(i) = (n_1, t_1, w_1)\ if\ \ T_{i-1}^2\ \in I_1$$

or

$$n(i), t(i), w(i) = (n_2, t_2, w_2)\ if\ \ T_{i-1}^2 \in I_2$$

It should be mentioned that the above rule is quite logical since falling into region I_1 means that the process is safe so it is reasonable the usage of long sampling interval, small sampling size and wide warning limit. .However, if the statistic falls into the I_2 region ,it is an indication of potential shift in the mean , so it is reasonable to use short sampling interval, large sample size and narrow warning limits. Furthermore, for the first trial it is recommended to use the pair of (n_2, t_2, w_2).

2.13.2 Performance of the VSSIWL T^2 chart

Mahadik(2014) in order to evaluate the performance of the chart uses the metrics SSATS(the expected value of the time between a shift that occurs at some random time after the process starts and the time the chart signals) , ANSS and ANOS (are the expected values of the number of samples and the number of observations, respectively, taken from the time of a shift to the time the chart signals) and in order to measure the administrative performance used the metric the ANWS(the expected value of the number of switches between the triplets of values of sample size, sampling interval length, and warning limit from a shift to the signal.) It was observed that there are no significant differences among the statistical performances of the three charts. A VSSIWL chart can be set to be dramatically superior to the VSSI charts and also even significantly better than the VSSI chart with run rules (1, 3) .Note that the statistical performance of the VSSIWL chart is almost similar to that of VSSI and VSSI (1, 3) charts. Thus, using the adaptive feature in the warning limits is very much efficient in reducing the inconvenience associated with the implementation of a VSSI chart caused by the frequent switches between the values of its adaptive design parameters without affecting its statistical performance.

CHAPTER 3

MULTIVARIATE ADAPTIVE EXPONENTIALL WEIGHTED MOVING AVERAGECONTROLCHARTS

3.1 Introduction

Multivariate adaptive exponentially weighted moving average control chart(MEWMA chart) was firsttly introdused by lowry et al [57] Let $\mathbf{X_1}$, $\mathbf{X_2}$,… be a sequence of p × 1 random vectors taken at regular time intervals, each representing the p quality characteristics to be monitored. We assume that x_i, i = 1,2….., are independent, identically distributed (i.i.d.) multivariate normal random vectors with known mean vector $\boldsymbol{\mu}$ and known constant covariance matrix ,$\boldsymbol{\Sigma}$. The random vector $\mathbf{x}_i$ can be either the sample mean vector or individual observation vector at time i.The MEWMA chart uses the statistic $T_i^2 = \boldsymbol{Z}_i' \Sigma_{z_i} \boldsymbol{Z}_i$, where $\boldsymbol{Z}_i = \lambda \boldsymbol{X}_i - (1-\lambda)\boldsymbol{Z}_{i-1}$ and non -centrality parameter,$\delta = \sqrt{\boldsymbol{\mu}' \Sigma^{-1} \boldsymbol{\mu}}$. Note that the sample mean is considered to be zero. A signal is given when T_i^2 >H, where the control limit H satisfies a pre-specified in-control ARL. In order to enhance the efficiency of the MEWMA chart, in detecting shifts in the process mean the use of the adaptive feature in the design parameters has been studied. It has been showed that MEWMA charts with small values of the smoothing parameter can quickly detect small changes in the process mean vector. However this technique may affect the performance of the chart. For instance , a sudden large shifts , might cannot be detected since EWMA and MEWMA chart can be designed to detect either small or large shifts but not both.(giving small values in the smoothing parameter means that the observation it is not close to the target . As a result, it will not detect an out-of-control signal immediately) .Furthermore another problem could be the inertia problem(Yashchin [58,59], and Lowry et al.[57]). Specifically , before an upward shift in the process the chart appears a trend with downward direction.Therefore , the value of the statistic is close to the lower control limit when the upward shift occurs. As a result the chart cannot detect a signal as fast as it could be able to, if the statistic value was close to the centerline for example. Woodall and mahmoud[60] in order to measure the interia problem defined the signal resistance(SR) as the largest standardized distance of the sample mean vector from the target vector in any direction not leading to an immediate out-of-control signal where $SR = max_{x_i}\sqrt{X'_i\Sigma^{-1}X_i}$. They showed that SR is inversely proportional to the smoothing parameter . Because of the previous potential problems it has been proposed to use together MEWMA and shewhart charts (S-shewhart).In that case , if one of those two charts signals means that there is a shift in the mean.(Lowry and Montgomery[61]). In this cection, the AMEWMA and $\text{MEWMA}_{\text{VSS}}$ charts will investigated. Also, combination a MEWMA and shewhart charts will be presented using the adaptive feature in the design parametrs.

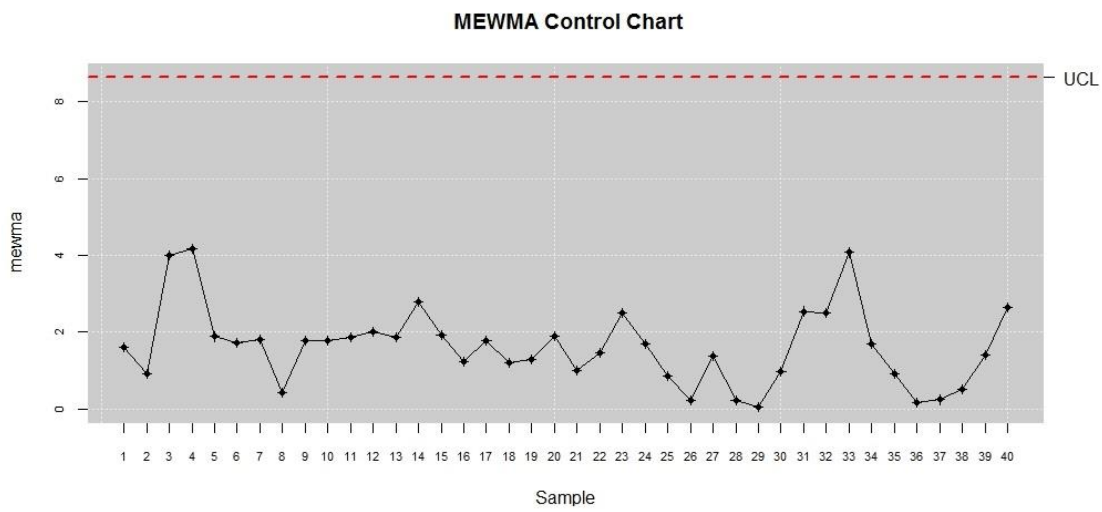

Figure 2: MEWMA control chart

3.2 AMEWMA Chart

This charts was firstly introduced by Capizzi and Masarotto[25] in the univariate case, as a combination of a Shewhart and EWMA chart, based on the idea to adjust the smoothing parameter λ according to the deviation of the observation from the most recent value of the AEWMA statistic using a score function. Mahmoud and Zahran [62] extended the above method to the multivariate case and proposed the MAEWMA chart.

3.2.1 Design of the AMEWMA chart

As it was previously stated, Mahmoud and Zahran [62] extended the AEWMA chart to the multivariate case and proposed the MAEWMA chart. Note that, in this chart the adaptive feature is added in the score function. This chart uses the chart statistic:

$\boldsymbol{y_i} = \boldsymbol{y_{i-1}} + \omega(e_i)(\boldsymbol{x_i} - \boldsymbol{y_{i-1}})$ where $\omega(e_i) = \frac{\Phi(e_i)}{e_i}$, $\Phi(°)$ is a score function, $e_i = \|(\mathbf{x}_\mathrm{i} - \boldsymbol{y}_{i-1})'(\boldsymbol{x}_i - \boldsymbol{y}_{i-1})\|$ the error term, $\mathbf{y_0}=\mathbf{0}$ and $\Sigma_Y = \Sigma\ c_t$, where Σ the covariance matrix of the observations. $\frac{\lambda}{2-\lambda} < c_t < 1$. Note that many functions could be used. In this scheme, the the score function $\Phi(e_i)$ is defined as:

$$\Phi(e_i) = e - (1-\lambda)k, if\ e > k$$

$$or$$

$$\Phi(e_i) = \lambda e, if\ e \leq k$$

The MAEWMA chart signals when $\boldsymbol{y}_i' \Sigma_Y \boldsymbol{y}_i > H_1$ or $\boldsymbol{y}_i' \Sigma \boldsymbol{y}_i > H^*$,$H^* = H_1 E(c_i)$. The values of the control limits H_1, H^* and k, satisfies a pre-specified in-control ARL and can be obtained by means of simulation. It should be mentioned that this modification in the design of the charts statistic, improves the capability of the chart in detecting mean shifts of different sizes and also diminishes the inertia problem which was discussed before.

3.2.2 Design of the optimal parametrs

The values of the parametes λ,k,H*are abtained according to the methodology of Capizzi and Masarotto[25] which is a two-stage method. For a chosen in-control ARL value, say ARL∗, and small and large non centrality parameter values, that are to be detected as soon as possible, say d_1 and d_2, respectively, the practitioner has to perform the following steps. First the chart parameter vector $\boldsymbol{\theta}= (\lambda, k, H^*)$ that minimizes the ARL at δ_2, say θ^* should be found. Then using a suitable level of significance γ, we find θ^{**}, the optimal parameter vector of the chart, that minimizes the ARL at δ_1 among those for which the ARL at δ_2 is nearly minimum.

3.2.3 Performance of the AMEWMA chart

Mahmoud and Zahran [62] using simulation obtained the ARL_1 with fixed in-control ARL approximately in 200,between MAWEMA,MEWMA and S-MEWMA charts for p=2,3,4(MEWMA and S- MEWMA have the same values in the parameters λ and h_1,h_2). They observed that $\log ARL_1$ is similar in all three charts for small shifts. On the other hand, for large shifts(δ>1.5) MAEWMA has better performance. About the standard deviation of the run length (SDRL) in MAEWMA chart is smaller than that of S-MEWMA chart only for very large shifts. Both MEWMA and S-MEWMA charts have slightly smaller SDRL than the MAEWMA chart for δ< 4. Furthermore, they compared the performance of the above three charts according to the worst case ARL. It has been shown that MAEWMA gives the best performance, comparing with the other two charts. What is more, even though the performance of the MAEWMA chart is similar to S-MEWMA chart, MAWMA charts use only one statistic .On the other hand, S-MEWMA chart uses 2 statistics and despite the fact that it can be designed in one chart with two control limit is it is not as simple as the MAEWMA chart. Finally, another advantage of using MAEWMA charts is the capability of creating design tables in order to give values in the chart parameters and ARL_1.

Moreover, Mahmoud and Zahra[62] evaluated the signal resistance off the three charts as it has been defined by Woodall and Mahmoud [60]: $SR_{(mewma)} = \sqrt{\lambda/(2-\lambda)}\ [\sqrt{h_2} + (1-\lambda)w/\lambda$, w=$\sqrt{T_i^2}$

$SR_{S-mewma} = \min[\sqrt{h_1}, SR_{(mewma)}]$,

$SR_{(maewma)} = \min\left[\sqrt{H^*}\ + (1-\lambda)\mathrm{k}, \frac{\sqrt{H^*} + (1-\lambda)w}{r}\right]$

Finally Mahmoud and Zahran [62] mentioned that MAEWMA chart has better worst-case signal resistance performance than the other charts. As it was previously stated , MAEWMA charts can be designed in order to detect at the same time small and large shifts in the mean while MEWMA charts cannot .Also AEWMA and S-MEWMA charts are simple and they have better performance in ARL.

3.3 MEWMA Control Chart with Adaptive Sample Sizes (MEWMA$_{vss}$ chart)

M.l LEE [63] in order to improve the MEWMA chart performance on mean shift detection, presented a new MEWMA chart where the sample size at each sampling point is not fixed, but varied. This chart is called as MEWMA$_{vss}$ chart.

3.3.1 Design of the MEWMA$_{vss}$ chart

Let n_1 ,n_2 and n_0 be the small and large sample size at each sampling point respectively n_0 is the fixed sample if the value of the sample size was fixed . Due to the fact that there are two possible values for the sample size ,a warning limit, w, needs to be is defined, which is referred to the change between n_1,n_2 . . ($0<w<H$, $n_1<n_0<n_2$). Note that, at the start of the procedure,

the choice of the initial sample size can be selected at random or the large sample size could be used (n_2) in order to avoid potential problems. In general, at the i^{th} sampling point the value of the sample size, is given according to the following rule :

The sample size at the i^{th} step is n_1 if $T^2_i \leq w$ or n_2 if $w < T^2_t < H$, using the statistic $T_i^2 = \boldsymbol{Z}_i' \Sigma_{z_i} \boldsymbol{Z}_i$ giving an out of control signal if $T_i^2 > H$. The value of the parameter H is chosen according to the desirable ARL

3.3.2 Performance of the $\mathbf{MEWMA_{vss}}$ chart

Comparing the optimal values of ARL_1 for several mean shifts between a MEWMA chart with fixed values($MEWMA_{fixed}$ chart) and a MEWMA chart with adaptive sample size ($MEWMA_{vss}$ chart), Lee [63] stated that the $MEWMA_{vss}$ chart is superior to the $MEWMA_{fixed}$ chart in most cases except for large shifts(δ>3) . Also, Lee [63] mentioned that the optimal value of the smoothing parameter.λ, for the $MEWMA_{vss}$ chart is larger than $MEWMA_{fixed}$ chart. Since, large values of λ give more weight to the recent data, whereas small values of λ give more weight to the past data, means that using the $MEWMA_{vss}$ chart , the weight of the past data can be reduced .

3.4 VSI-SZ charts

A modification in the statistic of the conventional Hotteling's T^2 control chart can be the use of the standardized mean for each variable. So, assuming that $\boldsymbol{\Sigma}_0$ is known, then the conventional Hotteling's T^2 control chart is equivalent to a control chart based on the statistic:

$S_k^z = (Z_{k1}, Z_{k2}, \ldots\ldots, Z_{kp}) \Sigma_{z_o}^{-1} (Z_{k1}, Z_{k2}, \ldots\ldots, Z_{kp})'$ where $Z_{ki} = \frac{\sqrt{n}\,(\bar{X}_{ki} - \mu_{0i})}{\sigma_{0\iota}}$ and $\bar{X}_{ki} = \sum_i^n X_{kij}/n$ the standardized mean for variable i at sampling point k. This chart signals when $S_k^z > h$, where h is the upper CL. Also, note that

$S_k^z \sim X_p^2$. Reynolds and Cho (2011) extended the previous chart in the case where sapling interval is not a fixed value and presented the VSI-SZ chart. Suppose that $t_{2,}$ and t_1 are the pairs of longest and the shortest sampling interval respectively($t_1 < t_o < t_2$).The longest sampling interval will be used if $S_k^z \leq g$ and shortest sampling interval if $g < S_k^z \leq h$ where $0 \leq g \leq h$**.** Since $S_k^z \sim X_p^2$ the values g,h can easily be determined using the quantiles of the chi-squared distribution.

3.5 VSI- MZ chart

Reynolds and Cho [64] proposed a new adaptive MEWMA chart defined as a quadratic form of the EWMA statistics using variable sampling interval and standardized observations at each sampling point. They use a combination of a MEWMA and shewhart chart in order to improve the perfomance of the MEWMA chart in detecting mean shifts in the process.

3.5.1 Design of the VSI- MZ chart

This chart uses the statistic $M_k^z = c_\kappa^{-1}\left(E_{k1}^z, E_{k2}^z, \ldots . E_{kp}^z\right)\Sigma_{z_0}^{-1}\left(E_{k1}^z, E_{k2}^z, \ldots . E_{kp}^z\right)'$ where E_{ki}^z=$(1-\lambda)\, E_{k-1,i}^z + \lambda Z_{ki}$, $E_{0i} = 1$ and Σ_{z_0} be the in-control value of Σ_z, $c_k = \frac{\lambda\left[1-(1-\lambda)^{2\kappa}\right]}{2-\lambda}$. when k→∞ $c_\infty^{-1} = \left(\frac{\lambda}{2-\lambda}\right)^{-1}$, Suppose that $t_{2,}$ and t_1 are the pairs of longest and the shortest sampling interval respectively ($t_1 < t_o < t_2$).Therefore, the longest sampling interval will be used if $M_k^z \leq g$ and shortest if $g \leq M_k^z \leq h$ where $0 \leq g \leq h$ .This chart signals when $M_k^z > h$, where h is the upper CL.

3.6 VSI- M_2RZ^2 chart

It should be mentioned that the above charts are mainly focused on monitoring the mean shift in a process. However in many cases the variability of the process also needs to be monitored. In order to achieve that, a combination of two EWMA charts has been proposed. In the univariate setting is proposed the use of a standard EWMA chart for monitoring the mean and an EWMA chart of squared deviations from target for monitoring variability(Reynolds and Stoumbos[65,66,67,68], Stoumbos and Reynolds[69]). This type of combination is very effective for detecting small or large changes in the mean or variability of a process. Reynolds and Cho [64], Reynolds and Kim [40] and Reynolds and Stoumbos [71,72] investigated multivariate extensions of this combination to combinations consisting of a standard MEWMA chart for monitoring $\boldsymbol{\mu}$ and an MEWMA-type chart based on squared deviations from target for monitoring $\boldsymbol{\Sigma}$. These charts are called as M_2RZ^2 charts

3.6.1 Design of the VSI- M_2RZ^2 chart

As it was mentioned before, several ways have been proposed in order to define an MEWMA-type chart based on squared deviations from target. Reynolds and Cho [64] and Reynolds and Stoumbos [71] presented two types of quadratic forms of squared deviations from target, where one type has the in-control expectation subtracted, and the second type does not. For the problem of detecting increases and decreases in variability, Reynolds and Kim [40] and Reynolds and Stoumbos [71,72] investigated squared deviation charts in which a reset is used in the EWMA statistics. It appears that using EWMA statistics with a reset in the second type of quadratic form generally gives the best performance, so this approach will be used here.

This chart uses the statistic

$$M_{2k}^{RZ^2} = n(2c_{\infty})^{-1}\left(E_{k1}^{RZ^2}, E_{k2}^{RZ^2}, \ldots . E_{kp}^{RZ^2}\right)\left(\Sigma_{z_0}^{(2)}\right)^{-1}\left(E_{k1}^{RZ^2}, E_{k2}^{RZ^2}, \ldots . E_{kp}^{RZ^2}\right)'$$

where $E_{ki}^{RZ^2} = (1-\lambda)\max\left[E_{k-1,i}^{RZ^2}, 1\right] + \lambda(\sum_i^n \frac{z_{kij}^2}{n})$ the statistic of squared standardized deviations from target for variable i at sampling point k and $E_{0i}^{RZ^2}$=1. The subscript "2" is used to indicate the second type of quadratic form studied by Reynolds and Cho[64] and $\Sigma_{z_0}^{(2)}$ represents the matrix with elements that are the squares of the corresponding elements of Z_0. Note that the in-control covariance matrix of the above EWMA statistic without the reset is $c_k\Sigma_{z_0}^{(2)}/n$.With the reset, $c_k\Sigma_{z_0}^{(2)}/n$ is the approximate in-control covariance matrix. In the above statistic the approximate in-control covariance matrix is being used. Moreover, Reynolds Jr. & Cho[64] based on the above statistic presented the M_2RZ^2 chart with variable sampling interval (VSI- M_2RZ^2 chart) where the longest sampling interval will be used if $M_{2k}^{RZ^2} \leq g$ and the shortest if $g < M_{2k}^{RZ^2} \leq h$ where $0 \leq g \leq h$.This chart signals when $M_{2k}^{RZ^2} > h$, where h is the upper CL.As it was previously stated, for detecting shifts in both $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ at the same time the use of a combination of two types of MEWMA charts is proposed. When two VSI charts, such as the VSI MZ chart and the VSI M_2 RZ^2 chart, are used together in combination, the short sampling interval is used if either of the two charts indicates that the short sampling interval should be used. Otherwise, the long sampling interval is used.

3.7 Special cases of the M^2 RZ^2 chart

3.7.1 VSI SZ^2 -SZ charts

In general , the SZ chart, is designed to monitor large shifts in $\boldsymbol{\mu}$, Moreover, it should be noted that at each sampling point when individual observations are used ,the SZ chart can also detect large increases in $\boldsymbol{\sigma}$ .However, when n > 1 the SZ chart loses its capability in detecting increases in $\boldsymbol{\sigma}$. Reynolds Jr. & Cho [64] presented the SZ^2 chart which is practically a special case of the

M_2 RZ^2 chart by setting the smoothing parameter at one (λ=1). Note that in the univariate case , an EWMA chart with λ=1 is basically a typical Shewhart chart. Finally at the SZ^2 chart and SZ charts the VSI feature can be applied and a combination of these two chart can be used in order to detect shifts in the mean and variability at the same time. (Reynolds Jr. & Cho ,[64]).These charts are called as VSI SZ^2 -SZ charts.

3.7.2 MZ - M_2 RZ^2 with sequential sampling(MZ - M_2 RZ^2_{SS})

In general, the sequential sampling method in the univariate case is based on applying a sequential probability ratio test (SPRT) at each sampling point (Stoumbos and Reynolds, 1996, 1997, 2001), but this approach can not be applied in the multivariate setting unless there is only one shift direction that is of interest. Reynolds Jr. & Cho [68] presented a MZ - M_2 RZ^2 chart combination base on sequential sampling

3.7.2.1 Design of the MZ - M_2 RZ^2_{SS} chart

Firstly, it should be noted, that a sequential sampling scheme can be considered as a special case of a VSI scheme. In fact, the sequential sampling scheme is practically a VSI scheme with n = 1, a short sampling interval of t_1= 0, and a long sampling interval of t_2=t. Note that at this section the definition of sequential sampling will be explained first in terms of the MZ chart, and then extended to the sequential sampling version of the M_2.RZ^2 chart .Suppose that at each sampling point observations are taken one by one , samples are taken using a fixed sampling interval t. Let MZ_{kj} be the value of the MZ_k statistic after observation j at sampling point k, j = 1,2… . After observation j at sampling point k, a signal is given if $MZ_{kj} > h$. If $g < MZ_{kj} \leq h$ the practitioner will continue sampling at sampling point k by taking observation j + 1. If $MZ_{kj} \leq g$ sampling must be stopped at sampling point k and the practitioner must wait t time units (until we get to sampling point k + 1) to sample again. It is assumed that the time required to obtain the individual observations within a sample is short enough that it can be neglected relative to the time between samples, so the time between these individual observations is assumed to be 0.finally, The same procedure is applied to the sequential sampling version of the M_2RZ^2 chart.

3.8 Checking the performance of the above charts

Reynolds Jr. & Cho, [64] using simulation examined the performance of the above charts and compared them with counterpart FSR charts. In order to evaluate the performance of the charts they used the average time to signal(ATS). Since a shift in a process parameter may occur at some random time after process monitoring has started, the appropriate measure of detection time for sustained shifts is the expected length of time from the point of the shift to the time that the control scheme signals(ATS).However, in some cases the statistics used in a control chart may not be at their starting values when a shift occurs .It is assumed that these statistics have reached their steady-state distributions by the time the shift occurs. Under this assumption the expected time from the shift to the signal is called the steady-state ATS (SSATS).

3.8.1 VSI-Shewhart

In general, using the VSI feature on the Shewhart charts instead of a typical FSR chart improves the ability in detecting small shifts in the mean or in the variance(in case where the variability is increased) . Also, using the VSI SZ^2 -SZ combination the expected time to detect the shifts is reduces significantly. However, using the VSI feature can give slightly worse performance for very large shifts.

3.8.2 VSI MZ Chart

Comparing the performance of VSI MZ chart with the FSR MZ chart it can been seen that the MZ chart is preferable in cases when when the smoothing parameter is large and the mean in the shift is small.

3.8.3 The MZ - M_2 RZ^2 chart combination

Comparing the performance of the FSR MZ - M_2 RZ^2 chart combination with the adaptive combination of the MZ chart and M_2 RZ^2 chart, Reynolds Jr. & Cho, [64] mention that using the VSI feature, the performance of the MZ - M_2 RZ^2 chart combination is significant improved in detecting small mean shifts and increases in the variability. However for quite large mean shifts the FSR SZ chart performs slightly better

3.8.4 VSI versus Sequential Sampling

Suppose that, for charts based on sequential sampling at each sampling point individual observations are taken, and the sampling interval is t.Furthermore, if the average sampling rate is to be one observation per unit time when the process is in in control, then the expected sample size at each sampling point must be t. Reynolds Jr. & Cho, [64] pointed out that for sequential sampling schemes the smaller the value of t is the lower SSATS is achieved for intermediate and large shifts, whereas larger values of t is are preferable for small shifts. However, it should be mentioned that control charts with sequential sampling can be applied when it is desirable to sample at fixed sampling intervals, where the sample size at each sampling point can depend on the process data, .As a result, it might be expected that sequential sampling would typically be used when the sampling interval is not extremely small. In fact, if t = 1 is used, then exactly one observation would have to be taken at each sampling point to maintain the sampling rate of one per unit time, so the sequential sampling in this case would reduce to a FSR scheme

3.9 The optimal values of the sample size and sample interval

In order to construct a chart which would have the best performance in detecting shifts the optimal choice of the sample size and the sampling interval is essential. For the chars using the adaptive feature in the sampling interval Reynolds Jr. & Cho[64], mentioned than the best choice for the long sampling interval (t_2) is between 0.25t and 1.90t, where t is a fixed sampling intervals it was mentioned before.. Furthermore, in VSI charts the value of the short sampling interval (t_1) , it is recommended to be as small as it is possible., The optimal choice of the sample size it has been shown that depends on the type of the chart(Hawkins and Olwell (1998) , Reynolds and Stoumbos[66,67],Reynolds and Cho[67a]Reynolds and Kim[67b]). For instance, CUSUM,EWMA, and MEWMA charts are not affected by the choice of the sample size. Overall, using individual observations at each sampling point is the best choice for these charts. On the other hand, Shewhart charts are sensitive to the choice of the sample size. Note that, using small values of n , Shewhart charts have good performance in detecting large shifts. On the other hand, for small shifts, a large value of n it is preferable. Finally, Reynolds Jr. & Cho[64], mentioned that if it is desirable for a chart to be able to detect both small and large shifts, then some intermediate value of n, such as n = 4, would be a reasonable compromise.

CHAPTER 4

ADAPTIVE MULTIVARIATE CUMULATIVE SUM CONTROL CHARTS

4.1Introduction

As it was previously stated, one of the major tools of SPC is the Hotelling's T^2 control chart, due to its simplicity in design and also it performs well in detecting relatively large shifts (generally when the non-central parameter is larger than three), However,, Hotelling's T^2 control charts are not effective in detecting small or moderate mean shifts. Sullivan and Woodall [73] improved the sensitivity of Hotelling's T^2 control chart by considering other estimators of the variance-covariance matrix. However, the inherent problem with Shewhart's chart or the multivariate Hotelling's T^2 control chart is that it uses only the current data for statistical testing. Some alternative control charts, including multivariate adaptive cumulative sum control charts (CUSUM charts) have been proposed in order to enhance the capability of the charts in detecting small or moderate shifts. These charts are based on the idea of taking into account not only the current but also the previous measurements .Many studies have shown that adding a variable sampling parameter feature make these charts even more efficient in detecting shifts faster. In this section the design of these adaptive multivariate CUSUM schemes will be described, and comparisons with other schemes will be made to verify their superiority.

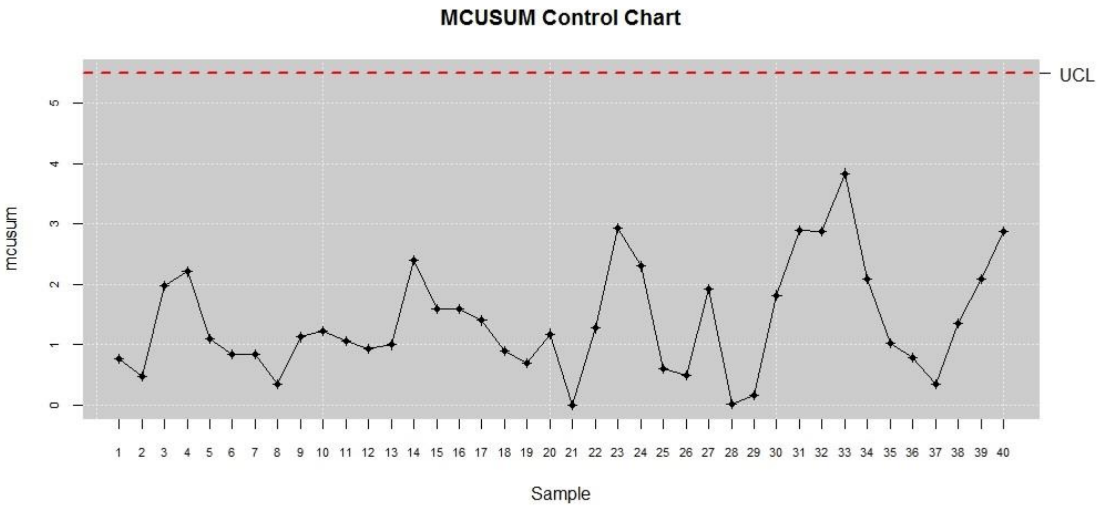

Figure 4:Multivariate Cusum control chart

4.2 Adaptive Crosier's MCUSUM chart (AC-MCUSUM)

C-MCUSUM chart was firstly introduced by Crosier[74] and is a method to design an multivariate CUSUM control chart. C-MCUSUM chart at each sampling point uses the statistic :

$C_t = \sqrt{(\boldsymbol{S}_{t-1} + \boldsymbol{X}_t - \boldsymbol{\mu}_0)'\Sigma^{-1}(\boldsymbol{S}_{t-1} + \boldsymbol{X}_t - \boldsymbol{\mu}_0)}$ where $\boldsymbol{S}_t$=0 if $c_{\,t} < k$ *or* $\boldsymbol{S}_t$= $(\boldsymbol{S}_{t-1} + \boldsymbol{X}_t - \boldsymbol{\mu}_0)(1 - k/c_{\,t})$ otherwise. Note that $\boldsymbol{\mu}_0 = 0$. The parameter k is called reference value. This chart signals when $y_t = \sqrt{\boldsymbol{S}'_t\Sigma^{-1}\boldsymbol{S}_t} > h(k)$ where $h(k)$ is a function of k. It can be seen that the value of the control limit, depends on the reference value. If the shift is known then the optimal value of k giving the minimal ARL_1 is δ/2 where δ is the non -centrality parameter(Healy[75]) .Furthermore if δ is unknown but it is expected to be between $(\delta_{min},\delta_{max})$ a fixed reference value k is the optimal choice only when the shift size happens to be $2k$.There have been proposed many approaches for the M-CUSUM charts. For instance, Lucas [75a] suggested the combined Shewhart-CUSUM control charts that have a good overall ARL performance for shifts in a range. Another approach by Lorden [76] uses the C-MCUSUM statistic together with a few pre-determined different reference values. Since

C-MCUSUM accumulates observations more than k multivariate size from the target, several MCUSUM statistics with different k_i values can be used to retain several levels of memory of past observations. This approach was also discussed in Sparks [22].Moreover, some other literature, such as Jiang et al[77] Shu and Jiang [78] and Shu et al.[79] discussed the adaptive CUSUM statistic in the univariate scenario.

4.2.1 Design of the AC-MCUSUM chart

Dai et al. [80] used the adaptive feature in the reference value to the Crosier's MCUSUM method and proposed the adaptive C- MCUSUM chart. (AC- MCUSUM chart) .Note that, the performance of the MCUSUM chart depends on a pre-knowledge about the process shift. MCUSUM charts can only signal either small or large shift quickly once the corresponding reference value k is chosen appropriately to that shift. On the other hand the On the other hand , AC- MCUSUM not only operates without any pre-knowledge about the process shift but also operates with the existence of previous knowledge. Let $C_t = \sqrt{(\boldsymbol{S}_{t-1} + \boldsymbol{X}_t - \boldsymbol{\mu}_0)'\Sigma^{-1}(\boldsymbol{S}_{\boldsymbol{t-1}} + \boldsymbol{X}_t - \boldsymbol{\mu}_0)}$ where:

$$\boldsymbol{S_t}=0 \text{ if } C_t \leq k_t$$

$$or$$

$$\boldsymbol{S}_t = (\boldsymbol{S}_{\boldsymbol{t-1}} + \boldsymbol{X_t} - \boldsymbol{\mu}_0)(1 - k_t/C_t) \text{ otherwise.}$$

Note that $\boldsymbol{\mu}_0 = 0$. The parameter k is called reference value. This chart signals $when\, y_t = \sqrt{\boldsymbol{S}'_t\Sigma^{-1}\boldsymbol{S}_t} > h(k_t)$ where $h(k_t)$ is a function of k_t, $k_t = \frac{\delta_t}{2}$ if the shift is known. and δ_t is the estimation of δ at time t. Moreover $h(k_t)$ is the control limit at time t. Due to the fact that $h(k_t)$ is changing over time the statistic y_t is not convenient .Also is difficult for the practitioner to have an equal false alarm rate at every step. So, in order to monitor the detection sensitivity for both small and large shifts Dai et al. (2011) proposed the use of the statistic $Y_t^* = \sqrt{\frac{\boldsymbol{S}_t'\Sigma^{-1}\boldsymbol{S}_t}{h(k_t)}} > H^*$ $,0 < H^* < 1$, where H^* is the control limit to obtain a pre-defined ARL_0. As it was previously stated, in practice the shift at every step is unknown but it is expected to be between $(\delta_{min},\delta_{max})$. Moreover, in real practice, process products may perform fairly well for small shifts in the process characteristics. Therefore, there is often a minimum magnitude of process shifts with high importance for early detection, denoted by δ_{min}>0. So, it is essential for a control procedure to designed, in order to be more efficient at signaling the shifts $\delta \geq \delta$min, but it may reduce the efficiency in signaling any shifts $0<\delta<\delta$min. So, an estimation of the shift at time t, could be $\delta_t^* = \max(\delta_{min}, \delta_t)$. However, the most commonly used way to estimate the shift magnitude at every step is the use of a exponentially weighted moving average control scheme, (Roberts [81] and Saccucci[82],)In conclusion, the AC-MCUSUM chart using the EWMA algorithm for the estimation of the shift at every steps takes the following form

$C_t{}^* = \sqrt{(\boldsymbol{S}_{t-1}{}^* + \boldsymbol{X}_t - \boldsymbol{\mu}_0)' \Sigma^{-1} (\boldsymbol{S}_{t-1}{}^* + \boldsymbol{X}_t - \boldsymbol{\mu}_0)}$ where

$$\boldsymbol{S}_t = 0 \text{ if } C\,_t{}^* < k_t^*$$

or

$$\boldsymbol{S}_t{}^* = (\boldsymbol{S}_{t-1}{}^* + \boldsymbol{X}_t - \boldsymbol{\mu}_0)(1 - k_t{}^*/C\,_t{}^*) \text{ otherwise.}$$

Note that, $\boldsymbol{\mu}_0 = 0$. This chart signals when $Y_t^* = \sqrt{\frac{\boldsymbol{S}_t'{}^* \Sigma^{-1} \boldsymbol{S}_t{}^*}{h(k_t{}^*)}} > H^*$,$0 < H^* < 1$, where H^* a threshold to maintain a pre-defined ARL_0. The (*) means that the these parameters are estimated at each sampling point. In order to design the AC-MCUSUM chart, the practitioner needs to set and estimate the initial values vales of the parameters $h(k)$, δ_{min} and the smoothing parameter,λ. Dai et al. (2011), proposed an operating model of $h(k)$ to standardize the control limit over time for easy implementation. Furthermore, as it discussed above, the value of δ_{min} improves the detection performance of AMCUSUM charts for shifts $\delta \geq \delta_{min}$, but reduces the efficiency for shifts $\delta < \delta_{min}$, Dai et al. (2011) mentioned that the optimal value for δ_{min} is set to be at 0.5 giving the capability of balancing both large and small shifts. Furthermore, about the choice of the smoothing parameter larger values are preferable for large shifts and small otherwise. In other words, the larger the value of λ, the more sensitive the chart is to large shifts. However, since that the differences to the performance of the chart for different values of λ there are minor Dai et al. (2011) , suggested that setting λ=0.2 is the optimal value of the smoothing parameter in detecting both small and large shifts. Finally, the optimal initial value of δ_0^* is given as $\delta_0 = \frac{\delta_{min}+\delta_{max}}{2}$. In conclusion, in order to design a AMCUSUM control chart the practitioner should apply the following steps

(a) Select the detecting range of interest ($\delta_{\min}$,$\delta_{\max}$) based on preliminary investigation.

(b) Choose $\delta_0 = \frac{\delta_{min}+\delta_{max}}{2}$ *to* balance the performance of the AMCUSUM control chart at all points in the range ($\delta_{\min}$,$\delta_{\max}$).

(c) Choose $\lambda \in (0.05,0.25)$ based on the rule-of-thumb. Here, in practice, $\lambda=0.2$ is recommended .

(d) Select an appropriate control limit $H*$ to achieve the desired ARL_0.

(e) Run the AMCUSUM control chart. If $y_t^* > H*$, then a signal is issued.

4.2.2 Performance of the AC-MCUSUM chart

Using simulation Dai et al. (2011) calculated the values of the metric IRARL(Zhao et al.,2015) efined as $\text{IRARL} = \text{E}\left[\frac{ARL_C(\delta)}{ARL_{op}(\delta)}\right] = \int \frac{ARL_C(\delta)}{ARL_{op}(\delta)} dF(\delta)$ where C is the signature of any compared control chart, $\text{ARL}_c(\delta)$ represents the ARL_1 of control chart C under the mean shift δ, ARL_{op} (δ) represents the ARL_1 of MCUSUM chart with $k=\delta/2$ under the mean shift δ and $F(\delta)$ is the cumulative distribution function (CDF) of shifts δ. If there is no prior information above the mean shift, the CDF of uniform distribution $U[\delta\text{min},\delta\text{max}]$ can be used as $F(\delta)$. Dai et al. (2011) presented comparison results for the AC-MCUSUM chart and three conventional C-MCUSUM chart for deferent values of the reference value($k=\delta_{\min}$ / 2 $k=(\delta_{\min}+\delta_{\max})$ / 4 $k=\delta_{\max}$ / 2). The AC-MCUSUM chart has the best performance than the other three MCUSUM charts for any of the three range shift, which means that the AMCUSUM chart has either the shortest or nearly the shortest ARL for every shift magnitude within the specified range, and performs more robustly than MCUSUM chart. Similar conclusions can be made for AMCUSUM charts designed for other moderate and small ranges of process shifts

4.3Adaptive Pignatiello's MCUSUM method (AP-MCUSUM)

P-MCUSUM chart was first introduced by Rignatiello and Runger[83].Let $C_t = \sum_{t-n_t+1}^{t}(X_t - \boldsymbol{\mu}_0)$ where n_t can be interpreted as the number of subgroups since the most recent renewal (i.e., zero values). Note that $n_t = n_{t-i} + 1\; if MC_{t-1} > 0\; or\; n_t = 1\; otherwise$. Let $\frac{C_t}{n_t} = \left(\frac{1}{n_t}\sum_{t-n_t+1}^{t} X_t\right) - \boldsymbol{\mu}_0$ *be* the difference between the accumulated sample average and the targeted mean value. Consequently, at time t, the multivariate process can be estimated to

have the mean of $\frac{C_t}{n_t} + \boldsymbol{\mu}_0$.$\|C_t\| = \sqrt{C_t{}'\Sigma^{-1}C_t}$ is the distance between the estimate of the mean vector and the target mean of the process. Finally MC_t is defined, as the cumulative sum of the process at time *t*, Pignatiello's MCUSUM scheme uses the statistic $MC_t = max\{\|C_t\| - kn_t, 0\}$ giving signal when $MC_t > H_P$. Note than, H_P depends on the desired ARL_0.and on the value of k. Wang and Huang[84] showed that the optimal value for k if it is known is δ/2 where δ is the non-centrality parameter.

4.3.1 Design of the AP-MCUSUM chart

Wang and Huang[84] added in the above chart the adaptive feature and presented the AP-MCUSUM chart. This chart is based on the idea that the reference value k is being chosen adaptively. Let $[\delta_{min,}\delta_{max}]$ be the range of the possible shifts in the process. At each step, the value of the parameter k is $k_t = \delta_t/2$, where δ_t is the shift at step t. Note that δ_t is estimated using the EWMA method .the AP-MCUSUM is defined by the statistic:

$MC_t = max\{\|C_t\| - k_t n_t, 0\}$ where $C_t = \sum_{t-n_t+1}^{t}(X_t - \boldsymbol{\mu}_0)$ *and* $n_t = n_{t-i} + 1\; if MC_{t-1} > 0\; or\; n_t = 1\; otherwise$,giving signal when $MC_t > h(k_t)$.Since $h(k_t)$ is a decreasing function of k_t and is called the operating function in the adaptive MCUSUM control charts Wang and Huang(2016) developed a non-linear model to estimate it .Also they mentioned that the use of a statistic where the control limit is a fixed value is preferable. They presented the statistic $y_t = \frac{MC_t = max\{\|C_t\| - k_t n_t, 0\}}{h(k_t)}$ where $n_t = n_{t-i} + 1\; if y_{t-1} > 0\; or\; n_t = 1\; otherwise$, giving signal when $y_t > H$,where H is a threshold to maintain the desired ARL_0. Its value is close to one but not exactly due to the estimating errors of $h(k_t)$. Wang and Huang(2016) using simulation the mentioned that setting the initial value for $\delta_0 = \frac{\delta_{min}+\delta_{max}}{2}$ the value of the smoothing parameter on the EWMA method at 0.2 (λ =0.2) since the ARL performance is the best for shifts between (0.5,2.5) .In conclusion, the following steps should be taken to get an adaptive multivariate AP-MCUSUM.

1. Set up ARL_0.

2. Determine the range of shifts desired to be detected [$\delta_{\min}$, $\delta_{\max}$].

3. Choose the smoothing parameter λ: we recommend $\lambda = 0.20$.

4. Choose the initial value $\hat{\delta}_0$: the average of $\delta_{\min}$ and $\delta_{\max}$ is a good choice.

5. Select an appropriate control limit H to achieve desired ARL_0. Generally, if the ARL_0 does not need to be very exact, the control limit can be simply set to be $H = 1$.

6. Signal the process when $y_t > H$.

4.3.2 Performance of the AP-MCUSUM chart

Wang and Huang(2016) using simulation for different values for the reference value k ,examined the performance of the AP-MCUSUM chart and compared it with the conventional P-MCUSUM, C-MCUSUM(Crosier's MCUSUM chart) and the adaptive C-MCUSUM(adaptive Crosier's MCUSUM chart).The comparisons between these charts is based on the zero states values of ARL.Comparing the two non-adaptive charts Wang and Huang(2016) mentioned that P-MCUSUM and C-MCUSUM have similar performance. Note that the value of k affects the performance of these charts. .In general for the above two charts, small values of k are preferable in detecting small shifts and large values for k for large shifts. Furthermore, the AC-MCUSUM chart has the shortest or nearly the shortest value of ARL for every shift. Finally the AP-MCUSUM chart provides a significant improvement over the previous charts and has the best performance for all the shifts.

CHAPTER 5

ADAPTIVE LINEAR PROFILES

5.1 Introduction

Statistical profile monitoring can be considered as a potential subarea of statistical process control that has attracted attention of many researchers and practitioners in recent years. In many industries, the quality of processes or products can be characterized by a profile that describes a relationship or a function between a response variable and one or more independent variables. A change in the profile relationship can indicate a change in the quality characteristic of the process or product and, therefore, needs to be monitored for control purposes depending on the relationship between the explanatory and response variable(s). There are different types of profiles and there is a growing interest in monitoring cases which are characterized by simple linear regression profiles in which a single explanatory variable X is used to describe the behavior of the response variable Y.

5.2Variable sampling sizes and variable intervals T^2 scheme (VSSI-T^2)

Based on the control chart introduced by Kang and Albin[85] for monitoring liner profiles Abdella and Kai Yang[86] presented the VSSI-T^2 chart for monitoring linear profiles using the adaptive feature in the sample size and sampling interval. Let the outgoing quality, y ,be a random variable which is a linear function of an explanatory variable x. Assume that the j^{th} random sample collected over time is (x_i, y_{ij}), $i = 1, 2, . . . , n_j$. When the process is in-control, the relationship between the response variable and the explanatory variable is assumed to be $y_{ij} = A_0 + A_1 x_i + \varepsilon_{ij}$. where i=1,2,….n

,$\varepsilon_{ij} \sim N(0, \sigma^2)$. The parameters A_0, A_1 are the parameters of intercept and slope respectively. Using the least square method the parameters are estimated as $\alpha_{0j} = \bar{y} - \alpha_{ij}\bar{x}$, $\alpha_{1j} = S_{xy(j)}S_{xx}^{-1}$,$\sigma_0^2 = \sigma^2 n^{-1} + \bar{x}^2 S_{xx}^{-1}$, ,$\sigma_1^2 = \sigma^2 S_{xx}^{-1}$,$S_{xx} = \sum_1^n (x_i - \bar{x})^2$ and $MSE_j = (n-2)^2 \sum_1^n e_{ij}^2$the estimates of A_0, A_1 *and* σ^2 where $e_{ij} = y_{ij} - \alpha_{0j} - \alpha_{1j}x_i$ is the difference between the observed and predicted values. Kang and Albin[85] using a modification of the traditional Hotellings T^2 statistic, proposed the use of the following statistic for monitoring linear profiles,

$T_j^2 = (Z_j - U)'\Sigma^{-1}(Z_j - U)$ where $\Sigma = \begin{bmatrix} \sigma_0^2 & \sigma_{01}^2 \\ \sigma_{01}^2 & \sigma_1^2 \end{bmatrix}$, $U = (A_0 A_1)'$ and $Z_j = (\alpha_{0j}\alpha_{1j})'$.It should be noted that when the parameters A_0, A_1, and σ^2 are unknown they can be estimated from Phase I sample data, but in Phase II are assumed to be known. Furthermore the non-central parameter for the above chart is defined as $\delta = (\lambda + \beta\bar{x})^2 n + \beta^2 S_{XX}$ when the intercept A_0 shifts to $A_0 + \lambda\sigma$ and the slope A_1 to $A_1 + \beta\sigma$.Finally, this chart gives signal when $T^2 \geq UCL = \chi^2_{2,\alpha}$.

Suppose that (n_1,t_1) and (n_2,t_2) are the pairs of minimum sampling size with the longest sampling interval and maximum sampling size with the shortest sampling interval respectively($n_1 < n_0 < n_2$, $t_2 < t_o < t_1$) and let w be the warning limit, which identifies when to change the sampling interval, where $0 < w < CL$. At each sampling point the value of the sampling interval and the sample size is based on the following rule (Aparisi and Haro,[34]).

$$n(i), t(i) = (n_2, t_2) \quad \text{if } w < T_{i-1}^2 < CL$$

or

$$n(i), t(i) = (n_1, t_1), \text{ if } 0 < T_{i-1}^2 \leq w$$

5.3 *EWMA₃ chart*

Kim et al [87] introduced a scheme for monitoring simple linear profiles in phased II. They presented a modification of the model proposed by Kang and Albin[85] using coded explanatory values,

$y_{ij} = B_0 + B_1 x_i^* + \varepsilon_{ij}$ i=1,2,….n where $B_0 = A_0 + A_1\bar{x}, B_1 = A_1, x_i^* = (x_i - \bar{x})$ and $\bar{x} = \frac{1}{n}\sum_1^n x_i$.Using the least square method the parameters of the above model are estimated as $MSE(j) = \sum_1^n \left(y_{ij} - b_{1j}x_i^* - b_{0j}\right)^2$,$b_{0j} = \bar{y}_j, b_{1j} = \frac{S_{xy}(j)}{S_{xx}}$,the estimations of B_0,B_1 and σ^2 respectively where $\bar{y}_j = \frac{1}{n}\sum_1^n y_{ij}$, $S_{xx} = \sum_1^n (x_i - \bar{x})^2$ and $S_{xy}(j) = \sum_1^n (x_i - \bar{x}) y_{ij}$.In order to detect shifts in the model parameters, B_0,B_1 and σ^2, the EWMA$_3$ scheme uses the beneath three EWMA charts:

$$EWMA_I(j) = \lambda b_{0j} + (1-\theta)EWMA_I(j-1)$$
$$EWMA_S(j) = \lambda b_{1j} + (1-\theta)EWMA_S(j-1)$$
$$EWMA_E(j) = \max\{ln\lambda MSE(j) + (1-\theta)EWMA_E(j-1)ln(\sigma^2)\}$$

for monitoring shifts in , B_0,B_1 and $\ln\sigma^2$ respectively. The EWMA$_3$ gives a signal as soon as one or more of the three conditions hold:

$$|EWMA_I(j) - B_0| > L_I \sigma \sqrt{\frac{\lambda}{(2-\lambda)n}}$$

$$|\, EWMA_S(j) - B_1| > L_S \sigma \sqrt{\frac{\lambda}{(2-\lambda)S_{xx}}}$$

$$EWMA_E(j) > L_E \sqrt{Var[ln(MSE_i)]}, \text{ where}$$

$$Var[ln(MSE_j)] \approx \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{2}{3(n-2)^3} - \frac{16}{15(n-2)^2}$$

L_I, L_S and L_E are chosen to give a specified in-control ARL .Note that, in practice, the practitioners are concerned at the process deterioration rather than improvement so the statistic for monitoring the variance, $EWMA_E(j)$, is designed to detect increases in variance.

5.4 VSI *EWMA*$_3$ *chart*

Li and Wang [88] based on the above scheme used the adaptive feature in the sampling interval and proposed the VSI EWMA$_3$ chart .It should be mentioned that, using a variable sampling interval in the EWMA$_3$ scheme, provides quite robust and satisfactory performance in detecting shifts under various cases, including intercept, slope and standard deviation shifts (Li and Wang [88]). Note that, using two possible values for the sampling interval the best choice in order to achieve good statistical properties in VSI control charts, (Costa [89],Wu, Zhang, and Wang [21] , Reynolds et al[24]) .Let t_1, t_2 be the two possible sampling intervals which are used in the MEWMA scheme, where $t_2 < t_o < t_1$. At (i-1)th sampling point, the value of the ith sampling interval is determined based on the decision rule used in the VSI T^2 control chart proposed by Aparisi[35]**.** It should be mentioned that unlike VSI control charts for $\bar{X}$ control charts, CUSUM control charts, or EWMA$_3$ control charts, it is quite difficult to specify the regions warning and action regions . The reason is that the monitoring statistics EWMA$_I$(j), EWMA$_S$(j) and EWMA$_E$(j) do not have an explicit in-control distribution even though the parameters of the slope and intercept are normally distributed . To overcome these problem Li and Wang [88] proposed to divide the above monitor statistics by the three corresponding control limits. As a result, the statistics used in the VSI EWMA$_3$ chart are:

$$SE_I(j) = \frac{|EWMA_I(j) - B_0|}{L_I\sigma\sqrt{\dfrac{\lambda}{(2-\lambda)n}}}$$

$$SE_S(j) = \frac{|\ EWMA_S(j) - B_1|}{L_S\sigma\sqrt{\dfrac{\lambda}{(2-\lambda)S_{xx}}}}$$

$$SE_E(j) = \frac{EWMA_E(j)}{L_E\sqrt{Var[ln(MSE_i)]}}$$

Then VSI $EWMA_3$ can use warning limit ω and control limit h =1 to divide the chart into the central region Rc = (0, ω), warning region Rw = [ω, 1) and action region [1, +∞). In this condition, the VSI $EWMA_3$ control chart signals as soon as Δ(j) > 1, where $\Delta(j) = MAX\{SE_I(j), SE_S(j), SE_E(j)\}$

5.5 VSS *$EWMA_3$ chart*

Due to the fact that, $EWMA_3$ cannot easily incorporate adaptive features, because three warning limits and three control limits must be chosen simultaneously so that all charts have the same individual in-control average sampling rate, there have been made many studies in order to improve these scheme. (Zou et al. [90]).Kazemzadehet al [91] proposed the VSS $EWMA_3$ control scheme in which a different approach is used in order to monitor the sample variances. Firstly, it should be mentioned that the VSS $EWMA_3$(Kazemzadehet al [91]) uses a standardized version of statistics introduced by Mahmoud et al. [92] for intercept and slope:

$$Z_I = \frac{b_0 - B_0}{\sigma/\sqrt{n}} \ , \quad Z_S = \frac{b_1 - B_1}{\sigma/\sqrt{nS_{xx}}}$$

In this way, control limits would be independent from sample size. Moreover, the above chart which was first developed by Castagliola [93] is a two sided EWMA chart as an extension of the Crowder and Hamilton[94] approach, but based on a three parameter logarithmic transformation . Mahmoud et al. [92] used combined the above approaches in the case of profile monitoring with some modifications. The main advantage of this chart is that it has the capability of monitoring shifts in both decreases and increases of the standard deviation. Crowder and Hamilton [94] suggested the application of the EWMA approach on the logarithmof the sample variances. Their proposed statistic is

$$z_j = (1-\lambda)z_{j-1} + \lambda T_j$$

$$T_j = \ln MSE_j$$

Moreover,Castagliola [93] improved the above statistic presenting the statistic $T_j = a + bln(MSE_j + c$ where a, b and c>0 are three constants and if selected cautiously. Note that this statistic, T_j, approximately follows normal distribution better than the approach in Crowder and Hamilton.[94] As a result, closing the distribution of the T_j to normal distribution, makes the control limits of the chart symmetric and so the approach will be able to detect decreases and increases in standard deviation in the same way. The control limits of the chart are:

$$LCL = E(T_j) - K\left(\frac{\lambda}{2-\lambda}\right)^{\frac{1}{2}} \sigma(T_j)$$

$$LCL = E(T_j) + K\left(\frac{\lambda}{2-\lambda}\right)^{\frac{1}{2}} \sigma(T_j)$$

where K>0 and $E(T_j), \sigma(T_j)$ the expectance and standard deviation of the T_j statistic. Also, since the adaptive feature in the sample size is used ,warning limits also need to be defined.

$$LWL = E(T_j) - W\left(\frac{\lambda}{2-\lambda}\right)^{\frac{1}{2}} \sigma(T_j)$$

$$LWL = E(T_j) + W\left(\frac{\lambda}{2-\lambda}\right)^{\frac{1}{2}} \sigma(T_j)$$

Moreover, in this approach the smaller sample size is used whenever the three statistics related to intercept, slope and standard deviation fall in the safe region simultaneously, otherwise the larger sample size is used.

5.6 VSI MEWMA chart

The MEWMA approach in order to monitor a general linear profile was first introduced by Zou et al[90]. Moreover, in order to improve the performance of the proposed scheme, Zou et al. [90] extended this approach by adding VSI feature of the MEWMA control chart. As a result, the time required to detect small and moderate shifts was substantially reduced. Let the outgoing quality y be a n_j-variate random vector which is a linear function of an explanatory variable **X**, where **X** is an $n_j \times p$ matrix. Taking a sample at time j $\left(x_{ij1}, x_{ij2}, \ldots\ldots\ldots x_{ijp}, y_{ij}\right)$ for $i = 1, 2, \ldots, n_j$, where i shows the i^{th} observations within each profile, and j is the j^{th} profile collected over time. When the process is in-control, the underlying model is $\boldsymbol{y}_j = \boldsymbol{X_j}\boldsymbol{\beta} + \boldsymbol{\varepsilon_j}$ where $\boldsymbol{\beta} = (\beta^{(1)}, \beta^{(2)}, \ldots, \beta^{(p)})$ be the coefficient vector and $\boldsymbol{\varepsilon_j} \sim (\mathbf{0}, I)$.Setting the vectors $\boldsymbol{z_j}(\beta) = \frac{\widehat{\boldsymbol{\beta}}_j - \boldsymbol{\beta}}{\sigma}$ and $Z_j(\sigma) = \Phi^{-1}\left\{F\left(\frac{(n-p),\hat{\sigma}_j^2}{\sigma^2}; n-p\right)\right\}$ where Φ^{-1} is the inverse of the standard normal cumulative distribution function, and $F(.; \upsilon)$ is the chi- squared distribution function with υ degrees of freedom $(\chi^2{}_{\upsilon})$.The parameters $\boldsymbol{\beta}$ and σ^2**,**are estimated by using the least square method:

$\widehat{\boldsymbol{\beta}}_j = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}_j$, $\hat{\sigma}_j^2 = \frac{1}{n-p}\left(\boldsymbol{y}_j - \boldsymbol{X}\widehat{\boldsymbol{\beta}}_j\right)'\left(\boldsymbol{y}_j - \boldsymbol{X}\widehat{\boldsymbol{\beta}}_j\right)$**.** Finally**,** this chart uses the statistic,

$$U_j = \boldsymbol{w}_j'\Sigma^{-1}\boldsymbol{w}_j$$

with $\Sigma = \begin{bmatrix} (\boldsymbol{X}'\boldsymbol{X})^{-1} & 0 \\ 0 & 1 \end{bmatrix}$ the in-control covariance matrix of $\mathbf{z}_j$, giving signal if $U_j > L\frac{\lambda}{2-\lambda}$, L>0.Furthermore, ,The $\boldsymbol{w}_j$vector is calculated at each sampling point using the following EWMA statistic:

$$\boldsymbol{w}_j = \lambda\boldsymbol{z_j} + (1-\lambda)\boldsymbol{w}_{j-1}\ {}_{,j=1,2,\ldots}\ ,\ \boldsymbol{w}_0 = \mathbf{0}, \boldsymbol{z_j} = \left(\boldsymbol{z}'_j(\beta), Z_j(\sigma)\right)'$$

Also, in these schemes, when the regression coefficients of a linear profile change, i.e., from β to β^* the non-centrality parameter, δ, is

$\delta = \frac{1}{\sigma}\sqrt{(\boldsymbol{\beta}^* - \boldsymbol{\beta})'(\boldsymbol{X}'\boldsymbol{X})(\boldsymbol{\beta}^* - \boldsymbol{\beta})}$.Let t_1, t_2 be the two possible sampling intervals which are used in the MEWMA scheme, where $t_2 < t_o < t_1$.Note that , using only two possible values for the sampling interval , the VSI chart has the optimal performance (Reynolds [95]). Also, let $L_1 < 0 < L$ be the additional warning limits $0 \leq L_1 \leq L$ inside the control limits. So at each sampling point if the prior sample falls inside the warning limits of $L_1\frac{\lambda}{2-\lambda}$ then at the current sample a long sampling interval, t_1, is used . On the other hand, if the prior sample falls outside of these limits but inside the control limits of $L\,\frac{\lambda}{2-\lambda}$ then, a short sampling interval, t_2, should be used .Finally an out-of-control signal is given when the statistic U_{j-1} falls outside of the control limits.

5.7 VSS MEWMA chart

Kazemzadeh et al[91] based on the previous studies proposed a MEWMA scheme with variable sample size at each sampling point, using the MEWMA approach investigated by by Zou et al[90].Assume that n_1 and n_2 represent two possible sample sizes where $n_1 < n_0 < n_2$ and n_0 is the fixed sample size in a conventional linear profile monitoring. Also, let W be the warning limit ($0 < W < CL$) which is used to determine the next sample size. Generally when the sample statistic is in the safe region, a smaller sample size (n_1) is used as the next sample size, otherwise if the sample statistic is in warning region, larger sample size (n_2) is used. When the process is just starting or after a false alarm, the first sample size is chosen at rand

5.8 Performance comparisons

5.8.1 Fixed, VSS and VSI MEWMA charts

In general, for shifts in the slope, intercept and decreases in the variance, σ, the VSS MEWMA chart has better performance than the conventional MEWMA chart. However, for large increases in the variance the VSS chart performs slightly worse. So, the use of the VSS MEWMA chart is recommended in cases where there is a small or moderate shift in the variance. It should be mentioned that the value of the large sample size (n_2) affects the performance of the charts. In fact, as the value of n_2 is increased the value of the metric ATS is improved significantly.

5.8.2 Fixed, VSS and VSI $EWMA_3$ charts

Comparing the performance between the fixed $EWMA_3$, VSS $EWMA_3$ and the VSI $EWMA_3$ charts it should be noticed that the VSI $EWMA_3$ chart has the best performance in detecting any shifts in the A_0, A_1 and σ.Furthermore, as the value of n_2 increases, improvements in the ATS would be greater, especially for large shifts. Also this chart has a good ability to detect any decreasing shifts in the standard deviation. Finally it should be noted that , although the performance of the approaches is dependent highly on the selection of design parameters in each method, generally it could be said that for detecting shifts in A_0, and for shifts in σ, the VSI chart has better performance than the VSS $EWMA_3$ chart in most cases.

5.8.3 VSS T^2, EWMA3 and MEWMA charts

In general, the VSS MEWMA chart has the best performance in detecting shifts in A_0,A_1 except in large shifts. However, it should be noted that for increasing shifts in the standard deviation and under different values of shifts the VSS $EWMA_3$ chart perform better than the others, although the VSS T^2 has the same ability for detecting large shifts. Note that the T^2 chart is not able to detect decreases in variance, so it is not comparable with the other methods. The VSS MEWMA has the best ability in this case.

CHAPTER 6

ILLUSTRATIVE EXAMPLES

6.1 Introduction

In the first chapter, the multivariate adaptive T^2 control charts was presented. As it was stated, in these charts the adaptive feature in the design parameters has been applied and using the Hotelling's T^2 statistic the have quite good performance in detecting moderate to large mean shifts. In order to examine their performance, many metrics have been proposed, defining how fast these chart can detect a shift during the process. In this section we will present the VSS,VSI and VSSI T^2 control chart and using simulation data we will demonstrate how these charts can detect shifts in the mean. Two cases will be presented where in the first, the process is in control and in the second there is a sudden mean shift during the process equal to 2 (δ=2). Finally, the chart's graph and the computation of the T^2 statistic at each sampling point have been conducted using the R programmin g language .Also, for the chart's presentation the library "ggplot" has been used.

6.2 VSS chart

Let us assume that we intend to control three quality characteristics X_1, X_2 and X_3 from a product, for instance a rectangle box, where X_1, X_2 ,X_3 represent the length, width and height respectively. The averages vector and the variance-covariance matrix, when the process is under control, are, respectively:

$$\overrightarrow{\mu_0} = \begin{pmatrix} 12 \\ 3 \\ 15 \end{pmatrix}, \Sigma_0 = \begin{bmatrix} 0.2 & 0.054 & 0.162 \\ 0,054 & 0,09 & 0.042 \\ 0.162 & 0.042 & 0.31 \end{bmatrix}$$

We are interested to monitor shifts of magnitude equals to 2 (d=2) with an in-control sample size equals to 2 (n_o = 2). Firstly, in order to construct the VSS T^2 control chart, the design parameters need to be defined. Aparisi(1996) using the Markov chain approach, proposed by Cinlar[96], presented the optimal values for the parameters of the chart for different numbers of characteristics and shift magnitudes. So we will use the vector $\overrightarrow{\mu_1} = (7.64, 3, 15)$ which has d=2 from the in-control mean. According to Aparisi(1996), the optimal values of the warning limit and the small and large sample size are: $w = 4.1089, n_1 = 1, n_2 = 5$.Moreover, the value of the control limit is $CL = X^2_{p=3,\alpha=0.005} = 12.8488$. Note that, in multivariate charts the error type I is fixed at $0.005(a = 0.005.)$.

6.2.1 Case one

In the first case, 30 subgroups will be used (m=30), where each subgroup follows the multivariate normal distribution with mean $\overrightarrow{\mu_0}$ and variance-covariance matrix Σ_0 ,$MN(\overrightarrow{\mu_0}, \Sigma_0)$

Sample	**T^2 statistic**	**Sample size**	**Sample**	**T^2 statistic**	**Sample size**	**Sample**	**T^2 statistic**	**Sample size**
1	1.6821	n_2	**11**	2.4491	n_2	**21**	2.1707	n_1
2	1.6670	n_1	**12**	0.2481	n_1	**22**	1.7366	n_1
3	0.6894	n_1	**13**	9.6905	n_1	**23**	0.2573	n_1
4	0.8161	n_1	**14**	4.7554	n_2	**24**	2.2777	n_1
5	1.9207	n_1	**15**	3.7579	n_2	**25**	2.9134	n_1
6	2.5453	n_1	**16**	5.2856	n_1	**26**	6.3518	n_1
7	2.6720	n_1	**17**	11.936	n_2	**27**	3.5031	n_2
8	1.0029	n_1	**18**	1.8272	n_2	**28**	3.4645	n_1
9	1.8492	n_1	**19**	8.8004	n_1	**29**	7.4547	n_1
10	5.1393	n_1	**20**	0.6846	n_2	**30**	1.2364	n_2

Table 1:T^2 statistic and sample size at each sampling point for the VSS chart, case 1

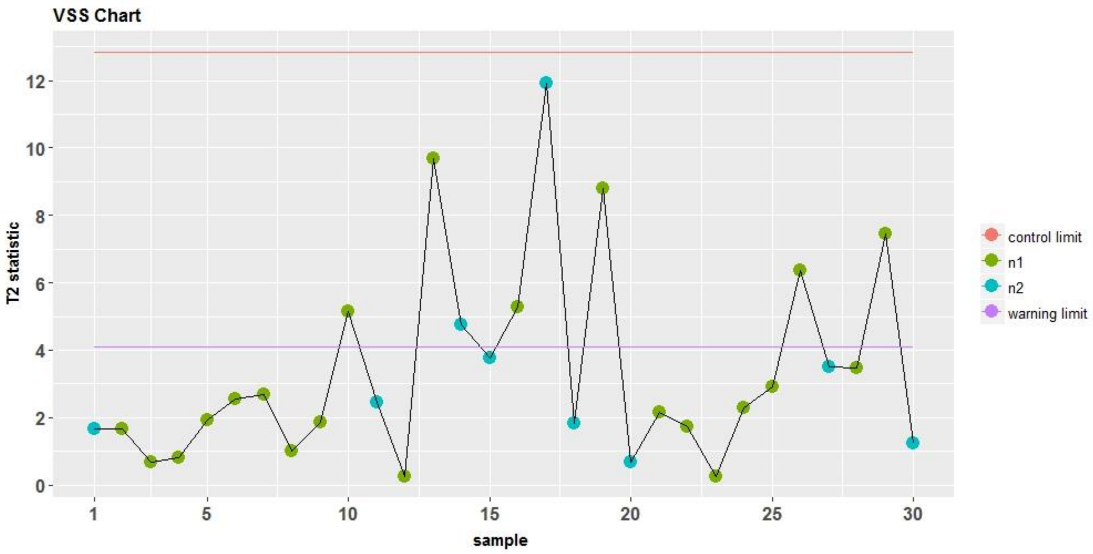

Figure 5: The VSS control chart when the process is in control

The two different colors (blue, green) in this chart represent the two different values for the sampling size (n_1=small sample size ,n_2=large sample size. As we can see the value of the sample size at sampling point i, depends on the value of the statistic at the (i-1)th sampling point. If the prior sample falls into the safe region ($0 < T_{i-1}^2 < w$) then at the next sample the small sample size will be used .On the other hand if prior sample falls into the warning region ($w < T_{i-1}^2 < CL$) then at the next sample the large sample size is used.

6.2.2 Case two

In this case a shift will occur during the process. As it was mentioned before we are interested to monitor shifts of magnitude equals to 2 (d=2).Let the first 20 subgroups be generated from a multivariate normal distribution with mean $\overrightarrow{\mu_0}$ and variance-covariance matrix Σ_0,$MN(\overrightarrow{\mu_0}, \Sigma_0)$. The other 10 will be generated from a multivariate normal distribution with mean $\overrightarrow{\mu_1}$ and variance-covariance matrix Σ_0

Sample	**T^2 statistic**	**Time units**	**Sample**	**T^2 statistic**	**Time units**	**Sample**	**T^2 statistic**	**Time units**
1	0.5737	n_2	**11**	3.6971	n_1	**21**	22.9326	n_2
2	2.9185	n_1	**12**	1.1843	n_1	**22**	17.8874	n_2
3	2.6455	n_1	**13**	0.7597	n_1	**23**	17.8059	n_2
4	10.8481	n_1	**14**	1.1829	n_1	**24**	29.4493	n_2
5	6.3906	n_2	**15**	1.7003	n_1	**25**	17.1786	n_2
6	1.9899	n_2	**16**	4.1278	n_1	**26**	22.6185	n_2
7	0.2108	n_1	**17**	2.9350	n_2	**27**	24.6621	n_2
8	3.7423	n_1	**18**	3.3068	n_1	**28**	21.1043	n_2
9	2.7510	n_1	**19**	7.4909	n_1	**29**	8.7917	n_2
10	1.6341	n_1	**20**	9.2024	n_2	**30**	23.5726	n_2

Table 2:T^2 statistic and sample size at each sampling point for the VSS chart,case 2

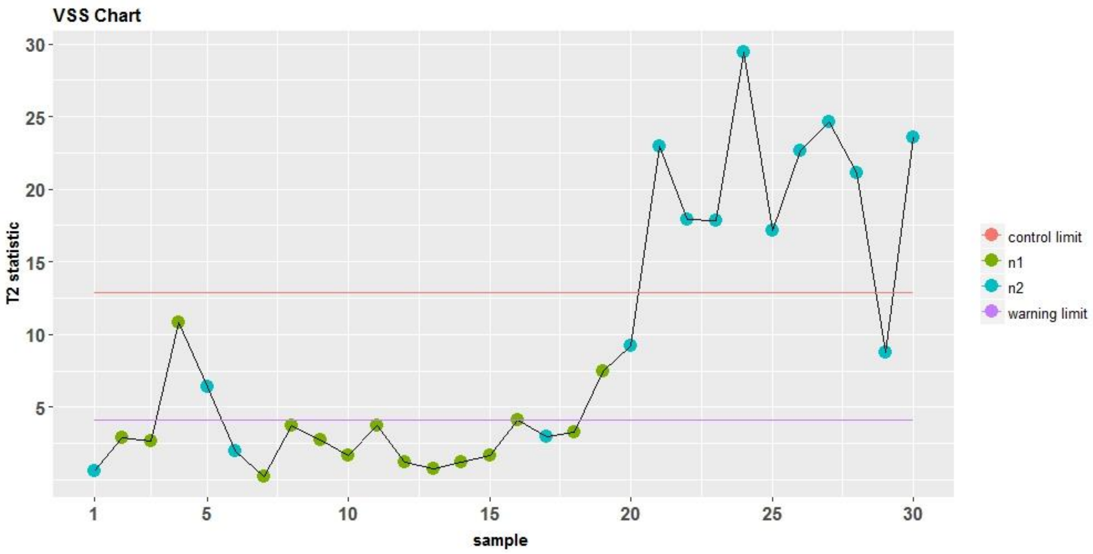

Figure 6: The VSS control chart when a shift occurs

It can be clearly seen that at the 20^{th} sampling point where the process mean changes, the chart can detect the shift giving signal for all the observations.

6.3 VSI chart

Let us assume that we intend to control four quality characteristics X_1, X_2 X_3 and X_4 from a product,. The averages vector and the variance-covariance matrix, when the process is under control, are, respectively:

$$\overrightarrow{\mu_0} = \begin{pmatrix} 12 \\ 4 \\ 1.5 \\ 6 \end{pmatrix} \qquad \Sigma_0 = \begin{pmatrix} 0.2065 & 0.1269 & 0.0698 & -0.1289 \\ 0.1269 & 0.1370 & 0.0743 & 0.1418 \\ 0.0698 & 0.0743 & 0.0501 & 0.0614 \\ -0.1289 & 0.1418 & 0.0614 & 0.1790 \end{pmatrix}$$

We are interested to monitor shifts of magnitude equals to 2 (d=2).We will use the vector $\overrightarrow{\mu_1} = (12{,}2419, 4, 1.5, 6)$ which has δ=2 from the in-control mean. A sample of n =3 items is taken from the process every t_0 =30 minutes.Also,bt_0 $=0.2t_0$ is used as the shortest time between samples. According to Aparisi and Haro[34], the optimal values of the warning limit and the small and large sample size are: $w = 1.701, a = 4$,.So, the short sampling interval will be t_2=6 minutes and the long sampling interval t_1=120. Moreover, the value of the control limit is $CL = X^2_{p=3,\alpha=0.005} = 12.8488$. Note that, in multivariate processes the error type I is fixed 0.005($a = 0.005$.).

6.3.1 Case one

In the first case, 30 subgroups will be used (m=30), where each subgroup follows the multivariate normal distribution with mean $\overrightarrow{\mu_0}$ and covariance matrix Σ_0 ,

Sample	T^2 statistic	Time units	Sample	T^2 statistic	Time units	Sample	T^2 statistic	Time units
1	1.947	6 min	**11**	1.672	6 min	**21**	3.669	6 min
2	4.731	6 min	**12**	3.216	120 min	**22**	6.026	6 min
3	2.294	6 min	**13**	8.198	6 min	**23**	8.047	6 min
4	5.874	6 min	**14**	0.458	6 min	**24**	0.984	6 min
5	1.772	6 min	**15**	1.664	120 min	**25**	6.236	120 min
6	4.440	6 min	**16**	2.923	120 min	**26**	3.747	6 min
7	3.834	6 min	**17**	0.911	6 min	**27**	0.608	6 min
8	3.680	6 min	**18**	4.898	120 min	**28**	3.982	120 min
9	1.370	6 min	**19**	2.986	6 min	**29**	4.550	6 min
10	5.927	120 min	**20**	3.137	6 min	**30**	6.894	6 min

Table 3:T^2 statistic and sampling interval at each sampling point for the VSI chart, case 1

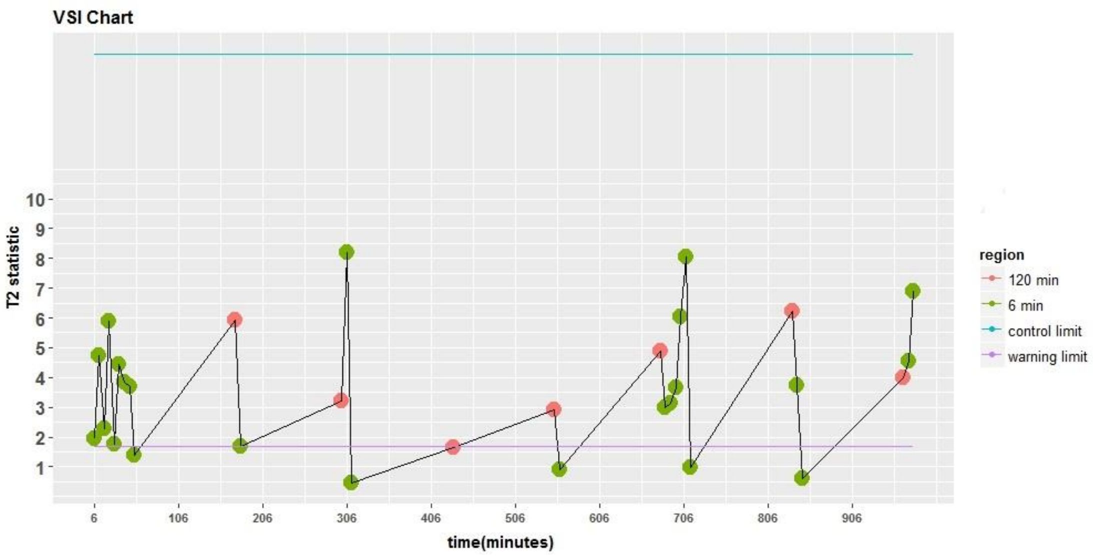

Figure 7: The VSI control chart when the process is in control

The two different colors in this chart represent the two different values for the sampling interval (t_1=long sampling interval,t_2=short sampling interval. As we can see the value of the sampling interval at sampling point i, depends on the value of the statistic at the (i-1)th sampling point. If the prior sample falls into the safe region $(0 < T_{i-1}^2 < w)$ then at the next sample the long sampling interval will be used .On the other hand if prior sample falls into the warning region ($w < T^2_{i-1} < CL$) then at the next sample the short sampling interval is used.

6.3.2 Case two

As it was mentioned before we are interested to monitor shifts of magnitude equals to 2 (d=2).Let the first 20 subgroups be generated from a multivariate normal distribution with mean $\overrightarrow{\mu_0}$ and variance-covariance matrix Σ_0, $MN(\overrightarrow{\mu_0}, \Sigma_0)$. The other 10 will be generated from a multivariate normal distribution with mean $\overrightarrow{\mu_1}$ and variance-covariance matrix Σ_0,

Sample	**T^2 statistic**	**Time units**	**Sample**	**T^2 statistic**	**Time units**	**Sample**	**T^2 statistic**	**Time units**
1	1.923	6 min	**11**	4.673	6 min	**21**	12.106	6 min
2	3.348	6 min	**12**	0.874	6 min	**22**	12.677	6 min
3	3.634	6 min	**13**	1.539	120 min	**23**	24.377	6 min
4	6.308	120 min	**14**	0.886	120 min	**24**	6.231	6 min
5	1.142	120 min	**15**	8.428	120 min	**25**	30.805	6 min
6	1.226	120 min	**16**	14.096	6 min	**26**	27.604	6 min
7	2.864	120 min	**17**	7.747	6 min	**27**	10.396	6 min
8	5.599	6 min	**18**	3.323	6 min	**28**	23.297	6 min
9	2.002	6 min	**19**	6.479	6 min	**29**	28.303	6 min
10	9.347	6 min	**20**	6.288	6 min	**30**	15.266	6 min

Table 4: T^2 statistic and sampling interval at each sampling point for the VSI chart, case 2

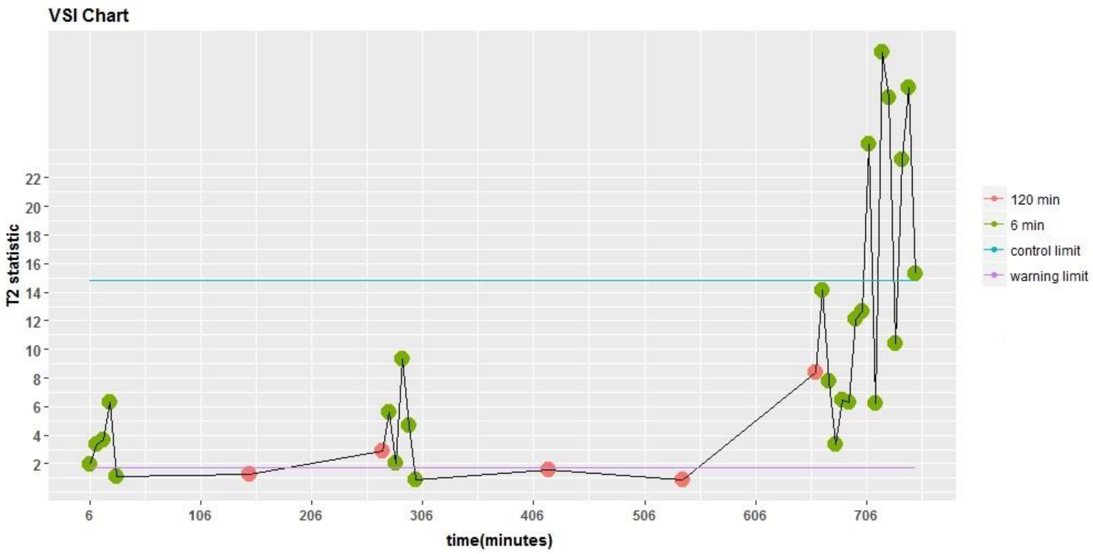

Figure 8: The VSI control chart when a shift occurs

It can be seen that when the process mean changes the VSI chart can detect the shift giving signal.

6.4 VSSI chart

Let us assume that we intend to control three quality characteristics X_1, X_2 , X_3 and X_4 from a product,. The averages vector and the variance-covariance matrix, when the process is under control, are, respectively:

$$\overrightarrow{\mu_0} = \begin{pmatrix} 12 \\ 4 \\ 1.5 \\ 6 \end{pmatrix} \qquad \Sigma_0 = \begin{pmatrix} 0.2065 & 0.1269 & 0.0698 & -0.1289 \\ 0.1269 & 0.1370 & 0.0743 & 0.1418 \\ 0.0698 & 0.0743 & 0.0501 & 0.0614 \\ -0.1289 & 0.1418 & 0.0614 & 0.1790 \end{pmatrix}$$

We are interested to monitor shifts of magnitude equal to 2 (d=2). We will us e the vector $\overrightarrow{\mu_1} = (12{,}2419, 4, 1.5, 6)$ which has d=2 from the in-control mean with an in-control sample size equals to 2 (n_o = 3). Firstly, in order to construct the VSSI T^2 control chart, the design parameters need to be defined. Aparisi and Haro using the Markov chain approach , presented the optimal values for the parameters of the chart for different numbers of characteristics and shift magnitudes Each sample is taken from the process every t_0 =25 minutes. Note that, $bt_0 = 0{:}2t_0$ is used as the shortest time between samples According to Aparisi and Haro[35], the optimal values of the warning limit the small and large sample size are: $w = 2.741, a = 4, n_1 = 2, n_2 = 4$,.So, the short sampling interval will be t_2=5 minutes and the long sampling interval t_1 =54.97 minutes. Moreover, the value of the control limit is $CL = X^2_{p=3,\alpha=0.005} = 14{,}86$. Note that, in multivariate processes the error type I is fixed 0.005($a = 0.005$.).Finally The first sample size and frequency of sampling has to be selected randomly .In this example ,at the first sampling point the short sam- pling interval and large sample size are used.

6.4.1 Case one

In the first case, 30 subgroups will be used (m=30), where each subgroup follows the multivariate normal distribution with mean $\overrightarrow{\mu_0}$ and covariance matrix Σ_0 ,

Sample	**Sample**	**T^2 statistic**	**Time units**	**Sample**	**T^2 statistic**	**Time units**	**Sample**	**T^2 statistic**
1	3.454	n_2,t_2	**11**	0.463	n_2,t_2	**21**	2.210	n_2,t_2
2	11.966	n_2,t_2	**12**	3.100	n_1,t_1	**22**	4.205	n_1,t_1
3	1.701	n_2,t_2	**13**	6.867	n_2,t_2	**23**	1.497	n_2,t_2
4	11.159	n_1,t_1	**14**	3.993	n_2,t_2	**24**	4.841	n_1,t_1
5	7.113	n_2,t_2	**15**	2.073	n_2,t_2	**25**	5.420	n_2,t_2
6	5.134	n_2,t_2	**16**	1.763	n_1,t_1	**26**	8.382	n_2,t_2
7	6.638	n_2,t_2	**17**	1.909	n_1,t_1	**27**	3.672	n_2,t_2
8	2.884	n_2,t_2	**18**	0.611	n_1,t_1	**28**	2.527	n_2,t_2
9	3.038	n_2,t_2	**19**	2.542	n_1,t_1	**29**	4.306	n_1,t_1
10	7.398	n_2,t_2	**20**	4.138	n_1,t_1	**30**	1.274	n_2,t_2

Table 5:T^2 statistic, sample size and sampling interval at each sampling point for the VSSI chart,case 1

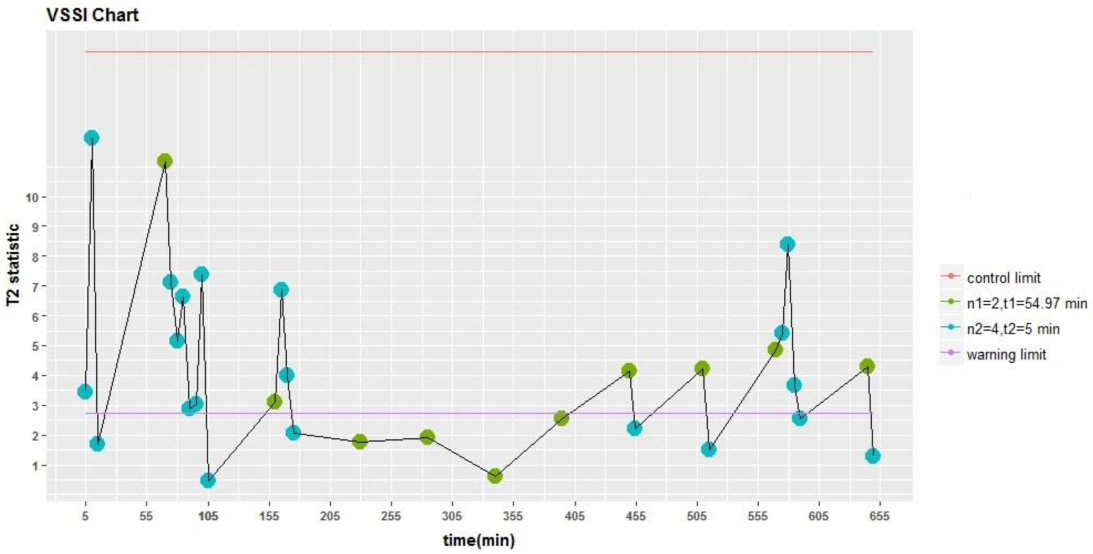

Figure 9: The VSSI control chart when the process is in control

The two different colors in this chart represent the two different values for the sampling interval and the sample size (t_1=long sampling interval,t_2=short sampling interval and,$n_1 = 2, n_2 = 4$,.for the small and large sample size respectively . As we can see the value of the sample size at sampling point i, depends on the value of the statistic at the (i-1)th samplig point. If the prior sample falls into the safe region $(0 < T_{i-1}^2 < w)$ then at the next sample the long sampling interval and the small sample size will be used .On the other hand if prior sample falls into the warning region $(w < T_{i-1}^2 < CL)$ then at the next sample the short sampling interval and the large sample size is used.

6.4.2 Case two

In the first case, 30 subgroups will be used (m=30), where each subgroup foll ows the multivariate normal distribution with mean $\overrightarrow{\mu_0}$ and covariance matrix Σ_0.

Sample	T^2 statistic	Time units	Sample	T^2 statistic	Time units	Sample	T^2 statistic	Time units
1	4.700	n_2,t_2	**11**	4.520	n_2,t_2	**21**	30.845	n_1,t_1
2	7.682	n_2,t_2	**12**	12.679	n_2,t_2	**22**	19.667	n_2,t_2
3	2.358	n_2,t_2	**13**	0.627	n_2,t_2	**23**	40.874	n_2,t_2
4	7.220	n_1,t_1	**14**	3.062	n_1,t_1	**24**	8.682	n_2,t_2
5	1.311	n_2,t_2	**15**	5.352	n_2,t_2	**25**	9.105	n_2,t_2
6	5.934	n_1,t_1	**16**	5.145	n_2,t_2	**26**	11.931	n_2,t_2
7	1.186	n_2,t_2	**17**	6.177	n_2,t_2	**27**	19.318	n_2,t_2
8	3.441	n_1,t_1	**18**	3.124	n_2,t_2	**28**	55.257	n_2,t_2
9	2.943	n_2,t_2	**19**	4.953	n_2,t_2	**29**	16.640	n_2,t_2
10	11.818	n_2,t_2	**20**	0.451	n_2,t_2	**30**	26.595	n_2,t_2

Table 6:T^2 statistic, sample size and sampling interval at each sampling point for the VSSI chart,case 2

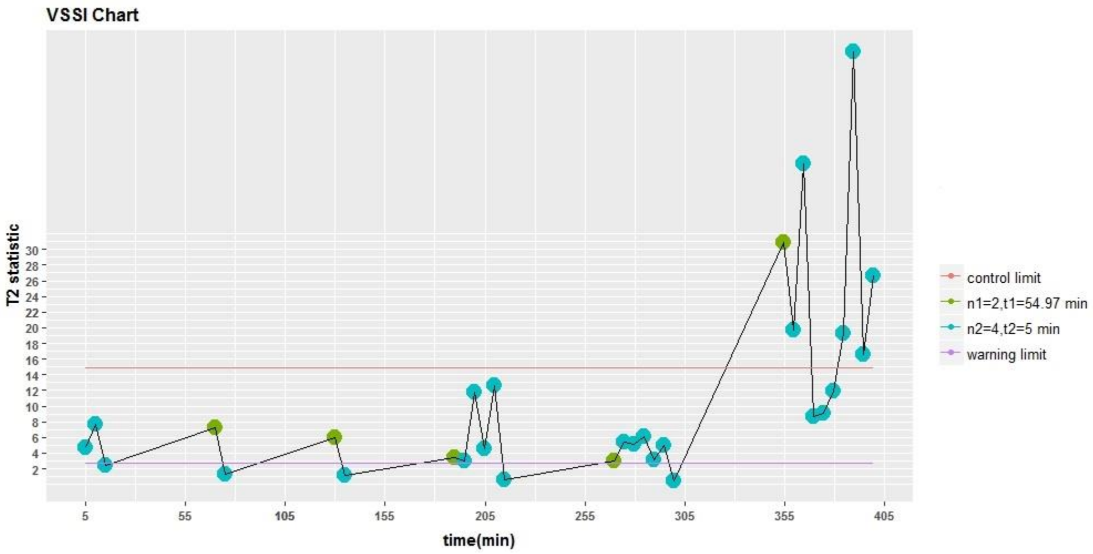

Figure 10: The VSSI control chart when a shifts occurs

It can be seen that at the 20th sampling point where the process mean changes (707 minutes) the VSI chart can detect the shift giving signal when the shift occurs.

CHAPTER 7

GENERAL CONCLUSIONS AND DISCUSSION

In this thesis, a review of the adaptive multivariate control charts was presented. In general , adaptive control charts have better performance than the conventional control charts since they can quickly detect shifts in the process mean. We presented an extension of the conventional Hotelling's, MEWMA and MCUSUM control charts when the adaptive feature is added in the design parameters. Moreover, some adaptive linear profiles were presented. Lastly the performance of the VSS, VSI and VSSI control charts for large shifts, was demonstrated. It was stated that, adaptive Hotelling's T^2 control charts are simple in design but are not appropriate in detecting small and moderate mean shifts. However, they have a good performance for large shifts (generally for $\delta>3$). It should be noted that, even though many adaptive Hotelling's T^2 control charts have been proposed, the choice of the chart with the best performance depends on the nature of the process. For instance, if the sampling interval is not practical to be used, then only charts with fixed sampling interval can be used (control charts with variable sample size, warning limits, action limits, e.t.c). On the other hand, due to the fact that MEWMA and MCUSUM control charts use information not only from current but also from past observations, they can perform better in detecting small to moderate shifts than the Hotelling's T^2 control charts. Moreover, we presented profile monitoring schemes which are used in cases where the quality of processes or products can be characterized by a profile that describes a relationship or a function between a response variable and one or more independent variables applying the adaptive feature. Finally, on the last section we demonstrated the performance of the VSS, VSI and VSSI control charts in detecting large shifts. Using simulation data, we presented two cases where in the first the process is in control and in the second a shift occurs during the process. Note that it was assumed that the process was on phase II since the design parameters and the in-control mean and covariance matrix was known. We saw that these charts can detect large shifts giving a signal when the mean is changed. As a future research, many interesting issues can be pursued. For instance, since profile monitoring is a relatively new research area in statistical process control, one can examine the performance of Hotelling's T^2 and MEWMA control charts using variable sample size and variable control limits (VSSC-T^2, VSSC-MEWMA). We also recommend extending the idea named VSSI to be used for monitoring the polynomial and non-linear quality profiles, as well. Furthermore, the idea of variable warning limits can be evaluated for attribute control charts as well as for EWMA and CUSUM charts. It should be noted that, most charts are designed and examined for their performance supposing that that the control mean and variance-covariance matrix are known. However in practice these parameters are unknown and they need to be estimated. As a result, if the process parameters (mean vector and covariance matrix) are estimated from a number of preliminary training samples, the charts' performance will be affected. So, a sensitivity analysis should be applied in these charts in order to examine the computational cost of the parameters' estimation.

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