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Energy Efficiency in E.U.: A Stochastic Frontier Approach

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Mariol A. Jonuzaj

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Energy Efficiency in E.U.: A Stochastic Frontier Approach

Μάριολ Α. Γιονουζάι

ΔΙΑΤΡΙΒΗ

Που υποβλήθηκε στο Τμήμα Στατιστικής

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ως μέρος των απαιτήσεων για την απόκτηση

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DEDICATION

This thesis is dedicated to my family.

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Mariol Jonuzaj

VITA

I hold a 4 year Bachelor's degree in Economics with "Excellence" from Athens University of Economics and Business (2012-2016) and a Master's Degree in Statistics from the same university (2016-2018).

Throughout my academic studies, I attended many courses in the fields of Economics, Statistics, Finance and Data Analysis. In particular, I participated in projects focused on areas of Economic Analysis, Financial Modeling and Forecasting, Advanced Data Analysis and others. Moreover, during that time, I had the opportunity to learn about issues at the frontier of current research, taught by some of the most prominent scientists in the field.

Apart from my studies, I did an internship as a Junior Business Consultant at Niriis S.A, a Management and IT consulting firm. At Niriis, my main responsibilities included financial management assistantship in projects that needed funding by the National Strategic Reference Framework (NSRF) and the Greek Development Law.

ABSTRACT

Mariol Jonuzaj

“Energy Efficiency in E.U.: A Stochastic Frontier Approach”

June 2018

During the last decades, huge improvements on energy efficiency are occurring across the European Union. Energy Efficiency policies are delivering in terms of reducing consumption, safeguarding Europe’s security of supply, reducing CO_2 emissions, creating jobs and saving money for consumers. All this brings monetary and non-monetary benefits to Europe’s industry and consumers.

Hence, measuring and evaluating the energy efficiency progress seems to be of great importance for the EU authorities. In order to assess the improvements in energy efficiency, a typical indicator used is the Energy Intensity, defined as the ratio of Gross Inland Energy Consumption to GDP (Eurostat and EEA). Energy Intensity can be considered as a proxy of the energy efficiency of a nation’s economy and shows how much energy is needed to produce a Unit of GDP.

Nevertheless, the last years, there is a growing concern about the effectiveness of this indicator and a doubt on how well the Energy Intensity can measure the actual level of energy efficiency in each county. For this reason, many new econometric approaches have been proposed during the last years. In particular, the Stochastic Frontier Approach has been in the spotlight of many researchers in order to estimate the energy efficiency.

For this reason, in this study, we employ three parametric Stochastic Frontier models which were recently introduced for the first time in energy economics literature. More specifically, we estimate and evaluate the energy efficiency of the energy consumption in the 28 EU member states from 1990 to 2016. We employ two SF models with no distributional assumptions (LSDV model & STV model) and one with distribution assumption (SF model with Half Normal inefficiency distribution).

ΠΕΡΙΛΗΨΗ

Μάριολ Γιονουζάϊ

“Energy Efficiency in E.U.: A Stochastic Frontier Approach”

Ιούνιος 2018

Κατά την διάρκεια των τελευταίων δεκαετιών, στην Ευρωπαϊκή Ένωση έχουν σημειωθεί τεράστιες βελτιώσεις στην ενεργειακή απόδοση. Οι πολιτικές Ενεργειακής Αποδοτικότητας/ Αποτελεσματικότητας συμβάλουν στην μείωση της κατανάλωσης, στην εξασφάλιση της Ευρωπαϊκής ενεργειακής τροφοδοσίας, στην μείωση των εκπομπών CO_2, στην δημιουργία νέων θέσεων εργασίας και στην εξοικονόμηση χρημάτων στους καταναλωτές. Όλα αυτά αποφέρουν νομισματικές αλλά και μη-χρηματικά οφέλη για την Ευρωπαϊκή βιομηχανία αλλά και τους Ευρωπαίους καταναλωτές.

Επομένως, η μέτρηση και η αξιολόγηση της ενεργειακής αποδοτικότητας φαίνεται να αποτελεί σημαντικό στοιχείο για τις Ευρωπαϊκές αρχές. Προκειμένου να αξιολογηθεί η βελτίωση στην ενεργειακή αποδοτικότητα, ένας τυπικός δείκτης που χρησιμοποιείται, είναι ο δείκτης Energy Intensity (δείκτης Ενεργειακής Έντασης), ο οποίος ορίζεται ως ο λόγος της Συνολικής Εγχώριας Ενεργειακής Κατανάλωσης (Gross Inland Energy Consumption) επί του Α.Ε.Π. (GDP) (Eurostat & E.E.A). Ο δείκτης Energy Intensity, μπορεί να θεωρηθεί ως ένας καλός τρόπος προσέγγισης της ενεργειακής αποδοτικότητας μια οικονομίας και δείχνει πόση ενέργεια χρειάζεται για να παραχθεί μία μονάδα του Α.Ε.Π..

Ωστόσο, τα τελευταία χρόνια, υπάρχει μια αυξανόμενη ανησυχία για την αποτελεσματικότητα αυτού του δείκτη και μια αμφιβολία σχετικά με το πόσο καλά μπορεί ο δείκτης Energy Intensity να μετρήσει το ακριβές επίπεδο της ενεργειακής αποδοτικότητας σε κάθε χώρα. Για το λόγο αυτό, τα τελευταία χρόνια έχουν προταθεί πολλές νέες οικονομετρικές προσεγγίσεις. Ειδικότερα, η Stochastic Frontier Analysis προσέγγιση βρίσκεται στο προσκήνιο πολλών ερευνητών προκειμένου να εκτιμηθεί η ενεργειακή αποτελεσματικότητα.

Για το λόγο αυτό, σε αυτήν τη μελέτη, εφαρμόζουμε τρία παραμετρικά Stochastic Frontier μοντέλα τα οποία παρουσιάστηκαν για πρώτη φορά πρόσφατα στην βιβλιογραφία των Οικονομικών της Ενέργειας. Πιο συγκεκριμένα, αξιολογούμε και εκτιμούμε την αποδοτικότητα της ενεργειακής κατανάλωσης στις 28 χώρες της Ευρωπαϊκής Ένωσης από το 1990 έως το 2016. Χρησιμοποιούμε δύο SF μοντέλα χωρίς υποθέσεις κατανομής (μοντέλο LSDV και STV μοντέλο) και ένα με παραδοχή κατανομής (Half Normal κατανομή για τον όρο της αναποτελεσματικότητας).

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1 Introduction

1.1 Energy Efficiency in European Union

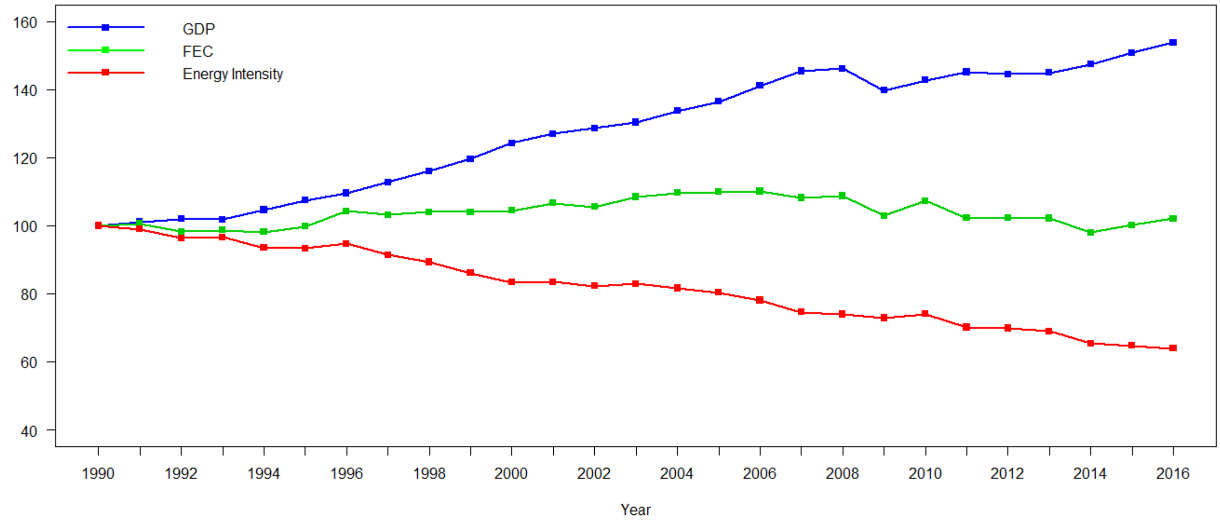
During the last decades, huge improvements on energy efficiency are occurring across the European Union. Energy Efficiency policies are delivering in terms of reducing consumption, safeguarding Europe's security of supply, reducing CO_2 emissions, creating jobs and saving money for consumers. All this brings monetary and non-monetary benefits to Europe's industry and consumers.

The EU in order to achieve these energy efficiency targets has launched several actions. Today, the 2020 Energy Target and the 2012 Energy efficiency Directives establishes a set of binding measures to help EU reach its energy efficiency targets by 2020. Under the Directives, all EU countries are required to use energy more efficiently at all stages of the energy chain, from production to final consumption. In addition, beyond the 2020 Energy Strategy, European Union has set itself an ambitious long-run energy efficiency goal, through the 2030 Energy Strategy and the 2050 Energy Roadmap which are the future energy strategies that will be launched in order to assure a compatible transition to a more energy efficient society and to meet their long-term greenhouse gas reductions targets. In particular, the 2030 Energy Strategy proposes further cuts in greenhouse emissions, an increase at the share of the renewable consumption and an additional rise in energy savings. In addition, the 2050 Energy Roadmap explores the transition of the energy system in ways that would be compatible with a radical reduction of greenhouse gas emissions while also increasing competitiveness and security of supply.

Hence, measuring and evaluating the energy efficiency progress seems to be of great importance for the EU authorities. In order to assess the improvements in energy efficiency, a typical indicator used is the Energy Intensity, defined as the ratio of Gross Inland Energy Consumption to GDP (Eurostat and EEA). Energy Intensity can be considered as a proxy of the energy efficiency of a nation's economy and shows how much energy is needed to produce a Unit of GDP.

In the next figure, we present the GDP per capita, the Final Energy Consumption and the Energy Intensity indicator from 1990 to 2016. As we can see from Figure 1.1, the GDP per capita is gradually increasing throughout these years. In particular we see a 53.77% increase from 1990 to 2016. On the other hand, the FEC seems to be constant during this period. More specifically, we observe that the highest increase from 1990 to 2016 is 10.12% with only 2.12% increase in 2016. As a result, the Energy Intensity indicator is continuously decreasing (35.31% decrease in the EU region from 1990 to 2016), indicating that energy efficiency in EU area is improving. This improvement can be considered as a result of several factors, including changes in the structure of the economy, advances in technology and the implementation of energy efficiency regulations.

Figure 1. 1: Gross Domestic Product per person (GDP per capita), Final Energy Consumption (FEC) and Energy Intensity in EU, from 1990 to 2016 (1990=100).



*Source: Eurostat and own estimation

Nevertheless, the last years, there is a growing concern about the effectiveness of this indicator and a doubt on how well the Energy Intensity can measure the actual level of energy efficiency in each county. For this reason, many new approaches have been proposed during the last years. In the next chapters, we will introduce and examine a new econometric approach, the Stochastic Frontier Approach which was introduced for the first time in energy economics literature, in 2011.

1.2 EU Energy Efficiency Directives

The EU Energy Efficiency Directives were published in 2012. The main idea is to establish a set of binding measures to help EU reach its 20% energy efficiency target by 2020. Under the directives all EU member states are required to use energy more efficiently at all stages of energy chain, from the production to final consumption.

As stated in Energy Efficiency Directives, Paragraph 1: “The Union is facing unprecedented challenges resulting from increased dependence of energy imports and scarce energy resources, and the need to limit climate change and to overcome the economic crisis. Energy efficiency is a valuable means to address these challenges”. This statement indicates the great importance of EU countries to implement the necessary measures and to achieve a better energy efficient economic environment.

Below, we present some specific measures and policies in order EU to ensure major energy savings for consumers and industry alike.

- ❖ Energy distributors or retail energy sales companies have to achieve 1.5% energy savings per year through the implementation of energy efficiency measures
- ❖ EU countries can opt to achieve the same level of savings through other means, such as improving efficiency of heating systems and others
- ❖ The public sector in EU countries should purchase energy efficient buildings, products and services
- ❖ Every year, governments in EU countries must carry out energy efficient renovations on at least 3% of the buildings they own
- ❖ Energy consumers should be empowered to better manage consumption. This includes easy and free access to data on consumption through individual metering
- ❖ National incentives for SMEs to undergo energy audits
- ❖ Monitoring efficiency levels in new energy generation capacities

1.3 Europe 2020 Target

As stated above, the Europe 2020 Energy Target constitutes a vital part for the EU energy policy in order to increase their energy efficiency level. To reach EU's 20% energy efficiency target by 2020, individual EU countries have set their own indicative national efficiency targets. Depending on country preferences, these targets can be based on primary or final energy consumption.

In the next table we present the level of Final Energy Consumption in 2016 and the 2020 Targets of each country that European Commission has defined.

Table 1. 1: National Energy Efficiency Targets

Member States	**Final Energy Consumption in Mtoe**	
	Level in 2016	**EU 2020 Target**
Austria	28.13	25.1
Belgium	36.33	32.5
Bulgaria	9.66	8.6
Croatia	6.64	7.0
Cyprus	1.76	1.8
Czech Republic	24.75	25.3
Denmark	14.45	14.4

Estonia	2.82	2.8
Finland	25.25	26.7
France	147.16	131.4
Germany	216.45	194.3
Greece	16.69	18.4
Hungary	17.86	14.4
Ireland	11.61	11.7
Italy	115.93	124.0
Latvia	3.82	4.5
Lithuania	5.11	4.3
Luxembourg	4.04	4.2
Malta	0.58	0.5
Netherlands	49.52	52.2
Poland	66.65	71.6
Portugal	16.11	17.4
Romania	22.28	30.3
Slovakia	10.42	9.0
Slovenia	4.88	5.1
Spain	82.50	80.1
Sweden	32.58	30.3
U.K.	133.69	129.2
EU 28	1107.66	1086

*Source: Eurostat Energy Database - last updated in February 2018

From Table 1.1 we see that many countries not only have already met their national Final Energy Consumption targets, but they have also reduced their consumption below their national target. For instance, the FEC in Finland, Italy and Greece in 2016 is 25.25, 115.93 and 16.69 Mtoe, respectively, almost 6%, 7% and 10% below the 2020 target. In contrast we see countries, such as Belgium, France, Germany and U.K. that are presenting a FEC above their national target (almost 11.8%, 12%, 11.4% and 3.5%, respectively), indicating that further actions should be made.

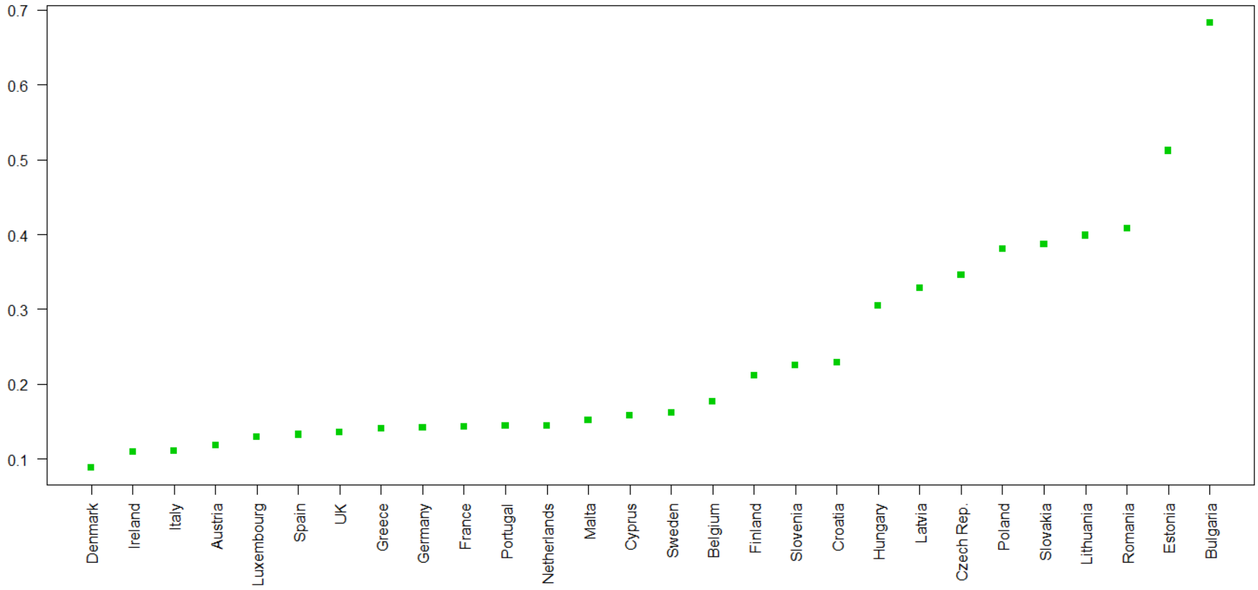
1.4 Energy Intensity

According to the European Environment Agency and the Eurostat, Energy Intensity is the ratio between the Gross Inland Consumption and the Gross Domestic Product (GDP). Energy Intensity can be considered as a good approximation of the energy efficiency of a nation's economy and shows how much energy is needed in order to produce a unit of GDP.

The Energy Intensity indicator is the basic tool used by the European authorities, in order to evaluate each country's energy efficiency. As stated above, we see a gradual decrease in the

Energy Intensity throughout the last years, indicating huge advances in the EU's energy efficiency. In order to assess the energy efficiency of each country, in the following figure we present the average level of Energy Intensity Indicator for each country, from 1990 to 2016.

Figure 1. 2: Average Energy Intensity in the EU from 1990 to 2016



From Figure 1.2, we see that the most efficient country is Denmark, with an average Energy Intensity indicator of 0.089. According to the definition of the indicator, Denmark in order to produce a unit of GDP needs to consume 0.089 units of energy. Ireland, Italy, Austria, U.K. etc. are following Denmark in the group of the most energy efficient states. In contrast, the worst energy efficient country during this period seems to be Bulgaria, with an average Energy Intensity indicator of 0.683, indicating that it is necessary to consume 0.683 units of energy in order to produce a unit of GDP. Other countries, such as Estonia, Romania, Lithuania, Slovakia and Poland are also presenting relatively high levels of Energy Intensity.

However, we should highlight that the last few years, countries such as Bulgaria, Estonia, Romania and Lithuania have faced a huge development in their Energy Intensity indicator. In particular, in 1990 the energy intensity of these specific countries was 1.01, 0.86, 0.62 and 0.63, respectively. In contrast, in 2016 the energy intensity of these specific countries reduced at 0.42, 0.35, 0.22 and 0.20, respectively.

1.5 Energy Statistics

In this section, we present some vital energy statistics that are part of the Energy Balance Sheet. In particular, we present data on the Final Energy Consumption, Primary Energy Production and Energy Net Imports. Moreover, we include subsection 1.5.3, where we argue on a vital EU energy policy which is the Energy Dependence from third countries.

1.5.1 Final Energy Consumption

Measuring and regulating the Final Energy Consumption, is a fundamental policy tool for the European Commission in order to steer the EU markets in a more efficient environment. In particular, the 2020 Energy Target and the Energy Efficiency Directives are aiming to a significant reduction in the levels of FEC. In section 1.3 we illustrated the levels of FEC in each country, which EU has defined as target levels. In particular, in Table 1.1 we illustrated that the European Union FEC 2020 target is 1086 Mtoe. In the next figure, we present the levels of FEC from 1990 to 2016

Figure 1. 3: Final Energy Consumption in EU

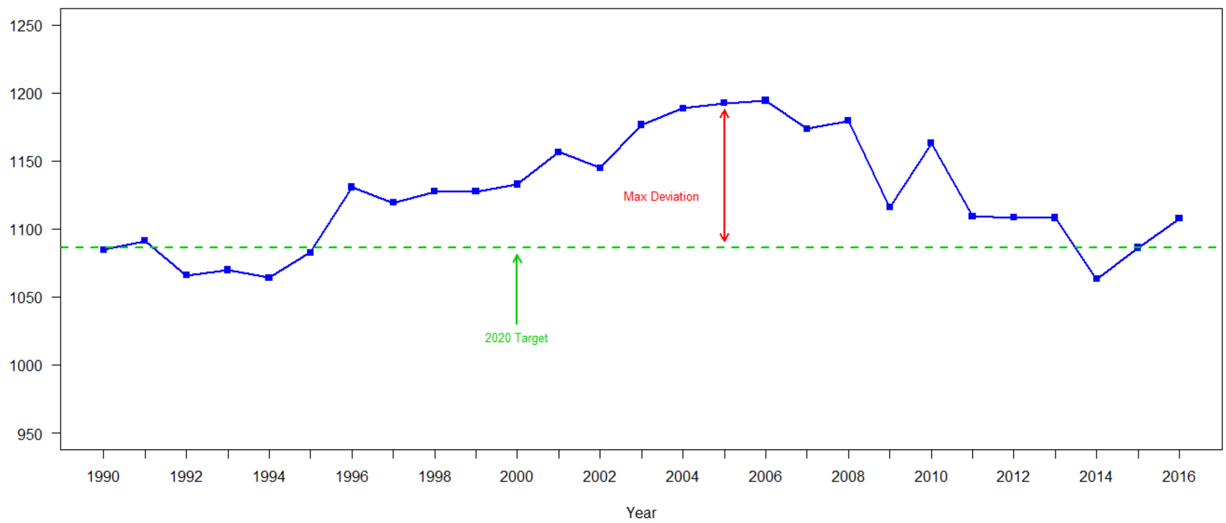


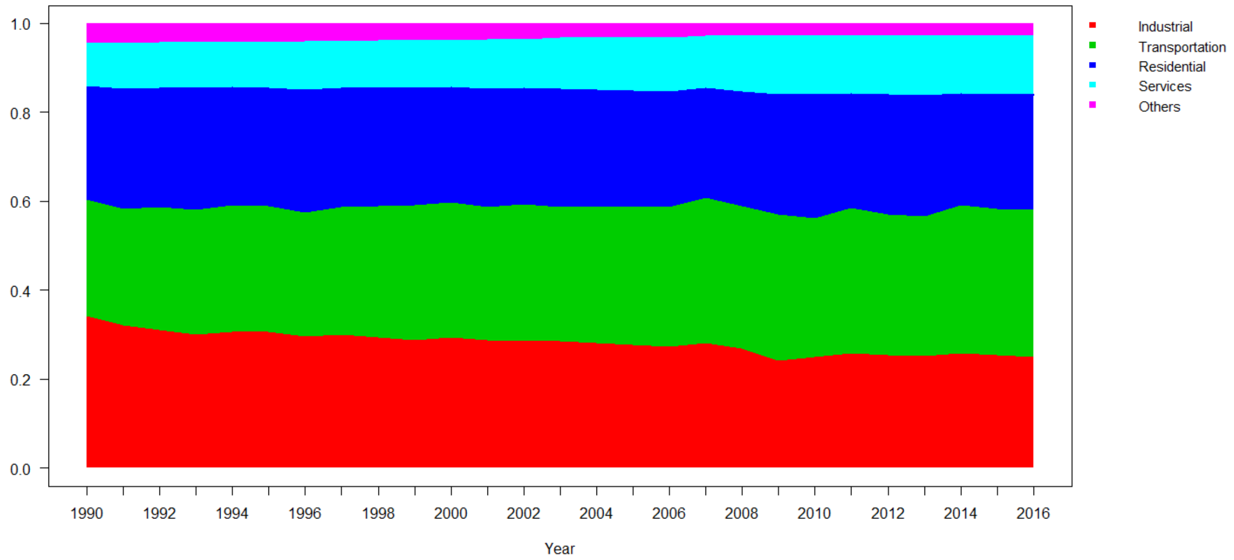
Figure 1.3 illustrates that from 1994 to 2005 we see a rapid increase in levels of energy consumption. In particular, we can see that in 2005 the FEC reached its peak and accordingly its maximum deviation from the 2020 Target. Since then, in general there is a gradual decrease in the FEC with some fluctuations among that time. Hence, we can argue that convergence to the 2020 target has been succeeded, even from the last 5 years. However, we should point out that the FEC in 2016 is 1107 Mtoe, 2% above the 2020 target.

Another point of interest is to examine the FEC for each economic sector. The official authorities (Eurostat & EEA) are analyzing the energy data and are publishing studies according to the following five sectors:

i. Industrial Sector
ii. Transportation Sector
iii. Residential Sector
iv. Services
v. Others

In the next figure, we present the EU energy consumption of each sector as percentage of the FEC, from 1990 to 2016.

Figure 1. 4: Final Energy Consumption by Sector



As we can see from Figure 1.4, the Industrial, the Transportation and the Residential sector include the largest energy consumption share. However, throughout these years we see some significant diversification in the structure of the Final Energy Consumption.

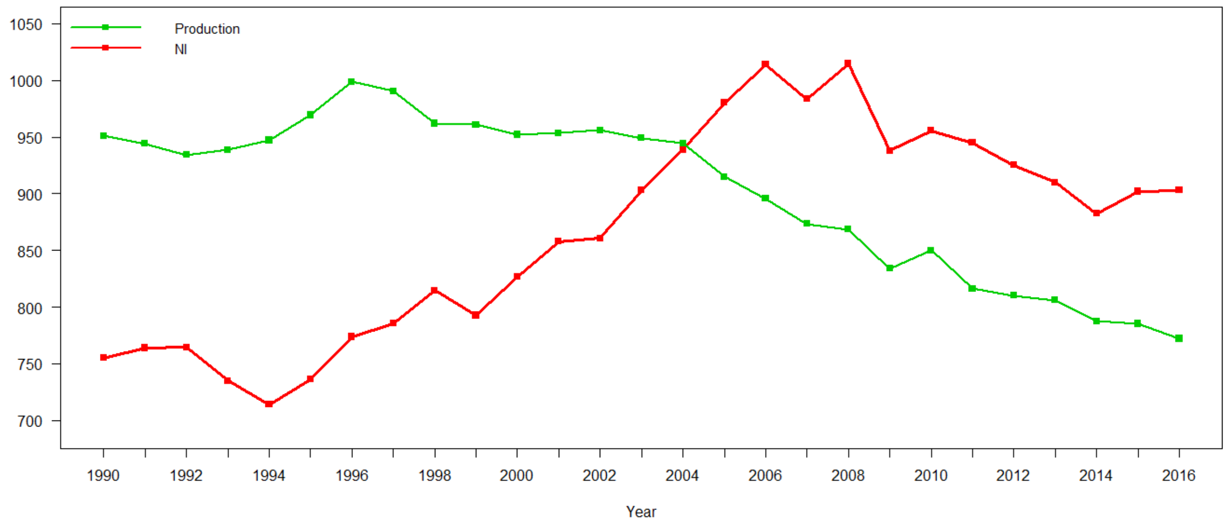
To begin with, the share of the Industrial consumption gradually decreased through these years. We see that in 1990 the Industrial share was 34.19%, in 2000 was 29.43% and eventually in 2016 was 24.99%. In contrast, the Transportation consumption is presenting a continuous increase. More specifically, the Transportation share in 1990 was 26.21% and 33.15% in 2016, indicating an almost 27% increase in this specific period. In addition, the Residential sector seems to present a constant share throughout these years, with an average share of 26%. Last, we notice a slight decrease in the share of the Services consumption throughout these last years. In particular, in 1990 the share of the Services was almost 10%, and finally in 2016 was over 13%.

1.5.2 Primary Energy Production and Net Imports

The Primary Energy Production and the Energy Net Imports are obviously the most crucial component, which EU tries to satisfy its energy demand. In the recent years there is a general downward development in energy's production level. As a result, the downturn in primary energy production has led to a situation where EU has become increasingly reliant on primary energy imports in order to satisfy its demand. As a result, the last 20 years the Net Energy Imports have skyrocketed.

In the next figure, we present the Primary Energy Production and the Energy Net Imports in EU, from 1990 to 2016.

Figure 1. 5: Primary Energy Production and Net Imports in EU



From Figure 1.5, we observe the different paths of the Primary Energy Production and the Energy Net Imports. The primary energy production reaches its highest levels in 1996, almost 1000 Mtoe, and since then it is reduced in a great scale. In particular, in 2016, the level of the primary energy production in EU was 772.109 Mote, which means that faced a reduction more than 22%. In contrast, we see that the Net Energy Imports have increased dramatically since 1994. The level of Net Imports in 1994 was 714.08 Mtoe and reached its peak in 2008, at 1014.47 Mtoe. Since then, there is a modest turndown in its levels. In 2016, the Net Imports were almost 903 Mtoe, which means that since 2008, EU Energy Net Imports faced 11% fall.

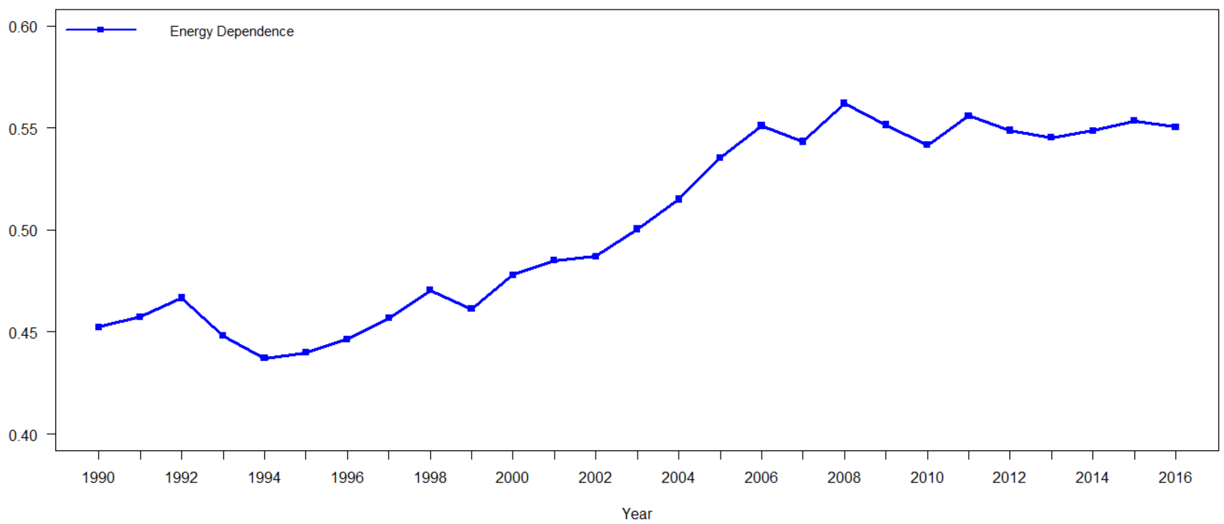
Hence, we conclude that Energy Net Imports are playing a vital role in EU energy efficiency policy. European Union as an import oriented region should secure its energy supply. In 2010, an initiative titled "Energy 2020 a strategy for competitive, sustainable and secure energy" was adopted by the European Commission. This strategy defines energy priorities for a period of 10 years and puts forward actions that may be taken in order to tackle a variety of challenges, including: achieving a market with competitive prices, secure supplies and effectively negotiating with international partners.

1.5.3 Energy Dependence in EU

European Union is mainly an energy import oriented region. As a result, European authorities should secure its energy supplies throughout the next years. As stated in the above subsection, throughout the last years, Energy Net Imports have increased dramatically. More specifically, the Net imports from 1994 to 2008 increased more than 34%. Therefore, consuming in a more efficient way not only will increase our energy efficiency level but also will reduce our energy reliance from third countries.

In order to measure the EU Energy Dependence from third countries, European Commission has defined the Energy Dependence rate, as the ratio of Energy Net Imports to Gross Inland Consumption. In the Figure 1.6, we present the Energy Dependence rate time path, from 1990 to 2016.

Figure 1. 6: Energy Dependence rate in EU



We see that the Energy Dependence rate in constantly increasing throughout the years. In particular, in 1990 the Dependence rate was 45.24% and in 2016 was increased to 55.05%. This means that more than half of EU’s energy needs were met by external imports. However, inside the EU, there are a lot of diversifications among the countries. For instance, in 2016 the Energy Dependence rate in Germany and in Greece was 64% and 78.90%, respectively. On the other hand, in Sweden and in U.K. the rate was almost 33%.

Another point of interest is to examine the regions from where the EU imports are coming from. The idea here is to see if EU's energy supply may be threatened if a high proportion of imports are concentrated among relatively few external partners. In the next table, we present a summary on the EU energy imports by product in 2016.

Table 1. 2: Energy Imports by country in 2016

Crude Oil		**Solid Fuel**		**Natural Gas**	
Country	Share (%)	Country	Share (%)	Country	Share (%)
Russia	31.9	Russia	30.2	Russia	40.2
Norway	12.4	Colombia	23.4	Norway	24.9
Iraq	8.3	Australia	14.6	Algeria	12.1
Saudi Arabia	7.8	United States	14.1	Qatar	5.5
Kazakhstan	6.8	South Africa	5.1	Others	17.3
Nigeria	5.7	Others	12.6		
Azerbaijan	4.5				
Algeria	2.8				
Mexico	2.6				
Angola	2.5				
Libya	2.3				
Others	12.4				

*Source: Eurostat

It is obvious that EU's energy imports are based on a small number of external partners. For instance, EU imports from Russia the 31.9% of crude oil, the 30.2% of solid fuel and 40.2% of natural gas, which means that only from a single country EU imports a huge amount of energy products. This indicates that energy dependence from third countries may affect not only the amount of energy imports but also the energy prices of each product.

2 Literature Review

In energy economics literature, an important issue which should be dealt with is to measure the energy efficiency and to determine Energy Demand Function. Over the past few years, some published academic papers have attempted to measure the level of efficient use of energy, based on the economic theory of production and used empirical methods for measuring productive efficiency. However, these studies had not generally provided a systematic discussion of the theoretical basis of energy efficiency. In particular, neither had they clearly defined energy efficiency nor clearly shown how it should be empirically measured using parametric methods.

Recently, for the first time, a Stochastic Frontier Approach was implemented in energy economics literature. The stochastic frontier analysis (SFA) is widely used to estimate firm's individual efficiency scores and has been suggested by Aigner, Lovell and Schmidt (1977) and Meeusen and van Den Broeck (1977). The basic idea is the introduction of an additive error term consisting of a noise and an inefficiency term. For the error as well as the inefficiency term, distributional assumptions are made, most often the normal and the half-normal assumption.

The novel implementation of Stochastic Frontier Analysis in energy economics literature, to the best of our knowledge, was introduced by Filippini and Hunt (2011). In this study, they tried to model the energy efficiency through an "Energy Demand Frontier" perspective. In order to build the underlying energy demand frontier they used a parametric Stochastic Frontier Approach. The basic model that was employed was the Pooled model as it is proposed by Aigner et al. (1977). In this study, they used an unbalanced dataset for a panel of 29 OECD countries from 1978 to 2006.

Following this first attempt, Filippini and Hunt (2012) estimated a US frontier residential aggregate energy demand function. In particular, they employed they employed the Pooled model as proposed by Aigner et al. (1977), the Random Effects model as introduced by Pit and Lee (1981) and the True Random Effects model as proposed by Greene (2005a, 2005b). However, they point out, as discussed in Farsi et al. (2005), all these approaches can suffer from the 'unobserved variables bias', because the unobserved characteristics may not be distributed independently of the explanatory variables. In order to address this econometric problem, Filippini and Hunt (2012) follows the approach taken by Farsi et al (2005) by using a Mundlak version of the REM based upon Mundlak (1978).

Furthermore, Filippini et al. (2014) following the two previous novel studies, they employ the stochastic frontier analysis in order to assess the EU residential sector. In particular, they apply the a panel data model as introduced by Battese and Coelli (1995), the True Random Effects and the True Fixed Effects model as proposed by Greene (2005a, 2005b). In this study, Filippini et al. (2014) allow the level of inefficiency to be a function of several explanatory variables. More specifically, following Battese and Coelli (1995) the inefficiency term was modified in order to have a systematic component associated with a vector of policy measures and a random component.

A last attempt was made by Filippini and Zhang (2016) in order to measure the persistent and transient energy efficiency. The aim of the study was to estimate an aggregate energy demand frontier, based on data on 29 Chinese provinces over the period 2003 to 2012. According to them, there is no clear and straightforward econometric model that can be estimated in order simultaneously obtain information on persistent and transient inefficiency while controlling for unobserved heterogeneity bias. However, Kumbhakar et al (2014), Tsionas and Kumbhakar (2014) and Colombi et al. (2014) have proposed some econometric approaches that provide separate estimates of these two components. In this empirical analysis, Filippini and Zhang (2016) follow another approach based on the estimation of different independent econometric approaches. In particular, they estimate the basic version of REM based on Pit and Lee (1981), a Mundlak version of this model, the TREM proposed by Greene (2005a, 2005b) and a Mundlak version of the TREM. The first two models provide information on the level of persistent inefficiency, whereas the last two models give information on transient inefficiency.

Moreover, an additional similar effort was presented by Filippini and Hunt (2016). In this study, they used the same method as in Filippini and Zhang (2016) study, estimating an aggregate US energy demand function using panel data for 49 states over the period 1995 to 2009.

3 Stochastic Frontier Analysis

3.1 General Theory

The Stochastic Frontier Analysis (SFA) was first introduced by Aigner Lovell and Schmidt (1977) and Meeusen and Van Broeck (1977). The basic idea of this framework is that the deviations of the production from the frontier might not be entirely on the control of the firm/individual. In order to model such a production process, it is assumed that there exists a Cobb-Douglas[1] production process, a two sided error component and a positive one sided disturbance. This one sided disturbance is called the "inefficiency term" and its goal is to measure the difference of the observed productivity of a firm from its maximum feasible production. In order to measure this inefficiency, different distributional assumptions are made for both the two sided and the one sided error component.

In section 3.2, we present the theory of the Stochastic Frontier Production function. In section 3.3 we generalize the theory in the case of Cost function. Finally, in section 3.3 we present an example in order to argue on the different distributional assumptions made for the inefficiency term.

3.2 Stochastic Frontier Production Function

To begin with, we present the theory of SFA in the case of a production function. In particular, we examine what is called productivity efficiency. The "Production Frontier" examines how far the production of a firm is from its maximum feasible production. The main concept of this theory is that the firms cannot achieve their maximum production due to different obstacles and restrictions.

Firstly, we should point out that our analysis is under the assumption that firms produce only a single output because they actually do produce only a single output or because it is possible to aggregate their multiple outputs into a single output. Hence, performance is evaluated by means of output-oriented measure of technical efficiency.

In this section we assume that we have data on the quantities of inputs used to produce a single output are available for each of N firms. The general stochastic production frontier model is described in the following equation.

[1] A Cobb-Douglas production function is used, as proposed by Zellner et al. (1966). In particular, they assume a production function which is stochastic (it contains a random disturbance representing different factors).

$$y_i = f(x_i;\beta)\,TE_i \tag{3.2.1}$$

where y_i is the observed production output of firm i, $f(x_i;\beta)$ is a Cobb-Douglas production function, x_i is a vector of N inputs used by firm i, β is a vector of technological parameters to be estimated and TE_i is the Technical Efficiency of each firm. Hence, rewriting equation (3.2.1) we have the following form.

$$TE_i = \frac{y_i}{f(x_i;\beta)} \tag{3.2.2}$$

Equation (3.2.2) defines Technical Efficiency as the ratio between the observed output to maximum feasible output. From equation (3.2.2), we see that the only way for a firm to produce in its frontier, would be only if the Technical Efficiency equals to 1 ($TE = 1$). Otherwise $TE < 1$ provides of a shortfall of the observed output from the maximum feasible output, which is allowed to vary across producers/countries/individuals. Technical Efficiency can be estimated by using the stochastic production frontier model, from equation (3.2.1). Here, we should highlight that since we are in the case of a stochastic frontier function, Technical Efficiency can be written as $TE_i = \exp(-u_i)$.

Therefore, taking the natural logarithm in equation (3.2.1) we have the final form of the stochastic production frontier.

$$\ln(y_i) = \beta_0 + \sum_{i=1}^{k} \beta_i \ln(x_i) + v_i - u_i \tag{3.2.3}$$

where v_i is the classical symmetric error disturbance and u_i is the non-negative one sided error term, which represents the inefficiency of firm i. Moreover the restriction that the u_i is a non negative term, guarantees that $y_i \le f(x_i;\beta)$. The objective of equation (3.2.3) is to obtain estimates of the parameter vector β, which describes the structure of the production frontier and therefore, to obtain estimates of the u_i, which can be used to obtain estimates of the Technical Efficiency of each firm i by means $TE_i = \exp(-u_i)$.

3.3 Stochastic Frontier Cost Function

In this section we present the theory of SFA in the case of the cost function. In particular, our aim is to estimate the cost efficiency which is provided by the cost frontier. In order to achieve this, we adopt an input-oriented oriented approach to the estimation of cost efficiency.

Between the estimation of output-oriented technical efficiency and the input-oriented cost efficiency, several significant differences should be noted. The first difference concerns data requirements. The estimation of technical efficiency requires information on input use and output provision, whereas the estimation of cost efficiency requires information on input prices, output quantities and total expenditure. The data requirements for the estimation of cost efficiency seem to be onerous in some situations. Another crucial difference concerns the number of inputs. Estimation of a cost frontier can be accomplished in situations which producers produce multiple outputs, whereas the estimation of cost efficiency requires that producers produce a single output.

However, in this section we assume that cross section data on the prices of inputs employed, the quantities of outputs produced and total expenditure are available for each firm. Hence, our cost frontier can be expressed as

$$E_i = c(y_i, w_i; \beta)\ \frac{1}{CE_i} \tag{3.3.1}$$

where $E_i = w_i' x_i = \sum_{i=1}^{k} w_i\, x_i$ is the expenditure incurred producer i, y_i is a vector of outputs produced by producer i, w_i is a vector of input prices faced by producer i, $c(y_i, w_i; \beta)$ is the cost frontier common to all producers, β is a vector of parameters to be estimated and CE_i is the Cost Efficiency of each producer. As in the section 3.2, the Cost Efficiency is defined as the ration of the observed expenditure to the minimum feasible expenditure. Since $E_i \geq c(y_i, w_i; \beta)$ it follows that $CE_i \leq 1$. Hence, $CE_i = 1$ if, and only if the firm's expenditure will be at the minimum feasible cost. Otherwise, $CE_i < 1$ provides a measure of the ratio of the minimum cost to observed expenditure.

Taking the natural logarithm in equation (3.3.1), we obtain the final form of the stochastic cost frontier.

$$\ln(E_i) = \beta_0 + \sum_{i=1}^{k} \beta_i \ln(y_i) + \sum_{i=1}^{p} \gamma_\iota \ln(w_i) + v_i + u_i \tag{3.3.2}$$

where v_i is the two sided error term and u_i is the inefficiency term of each producer. Since a cost frontier must be linearly homogeneous in input prices we should impose the parameter restriction $\sum_{i=1}^{p} \gamma_\iota = 1$, prior to the equation estimation. Therefore, the equation (3.3.2) is reformulated as

$$\ln\left(\frac{E_i}{w_k}\right) = \beta_0 + \sum_{i=1}^{k} \beta_i \ln(y_i) + \sum_{i=1}^{p} \gamma_\iota \ln\left(\frac{w_i}{w_k}\right) + v_i + u_i \tag{3.3.3}$$

Using equation (3.3.3) we can obtain the inefficiency term and hence, the cost efficiency of each firm. In both formulations of the stochastic cost frontier, the error term $\varepsilon_i = v_i + u_i$ is asymmetric, being negatively skewed since $u_i \geq 0$. Apart from the homogeneity restriction and the direction of the skewness of the error term, the analysis is similar with the stochastic production frontier.

3.4 Do Distribution Assumptions Matter?

An important part of the Stochastic Frontier Analysis theory, is the distribution assumptions made in order to define the inefficiency term in either the production or the cost function. As we stated in the above sections, the form of the error term is a crucial part of the analysis. In the past years, different assumptions were made and many distributions were used in order to evaluate this term. The half normal distribution (Aigner et al., 1977), the normal truncated (Stevenson, 1980), the exponential (Meeusen and Van Broeck, 1977) and the gamma (Greene, 1990) distribution are some of the assumptions that different researchers have made during these years.

In order to evaluate if these different assumptions matter in the final interpretation, Kumbhakar and Lovell (2000) present an example based on Greene (1990), who estimated a stochastic cost frontier for a cross section of 123 U.S. electric utilities, using all four of the preceding one sided error components. In their study, they calculated the rank correlation coefficients between pairs of efficiency estimates for all sample observations. In particular, they found that the lowest rank correlation coefficient was 0.7467 (between exponential and gamma distribution) and the highest was 0.9803 (between the half normal and the truncated normal distribution). As a result, they conclude that the differences between these assumptions are relatively small, highlighting that the choice between the two one-parameter densities (half-normal and truncated normal) is completely immaterial.

4 Empirical Study

This study aims to measure the Energy Efficiency of the 28 EU member states, in order to assess their energy efficiency level and to evaluate their future energy potentials. In particular, we examine the level of efficiency of the Final Energy Consumption[2]. The study is based on a balanced dataset of the 28 member states ($i = 1,2, \dots ,28$), over the period 1990 and 2016 ($t = 1,2, \dots ,27$). The data we use are based on information taken by the general database of Eurostat and the European Environment Agency[3].

The dependent variable we use is the Final Energy Consumption (from now on FEC). The explanatory variables of our analysis are the GDP per capita, the Area of each country, the Heating Degree Days, the Cooling Degree Days, the Population of each country, the percentage of Energy Consumption in the sectors of Industry-Residence-Transport[4], the percentage of the Energy Consumption coming from the Renewables and the Net Imports of energy as percentage of the FEC[5]. More specifically, we have:

- **GDP per capita** [at 2010 market prices]
- **Area of each country** [in km^2]
- **Population** [in millions]
- **Heating Degree Days**[6]
- **Cooling Degree Days**
- **Final Energy Consumption in Industrial Sector (%)**
- **Final Energy Consumption in Residential Sector (%)**
- **Final Energy Consumption in Transportation Sector (%)**
- **Energy Consumption by Renewables products as percentage of the aggregate consumption (%)**
- **Net Energy Imports as a percentage of the FEC (%)**

[2] As we presented in Chapter one, the Final Energy Consumption, is a crucial indicator in order the European authorities to evaluate and to watchdog the EU Energy Efficiency policy.

[3] Different energy data can be found to the following address: **https://ec.europa.eu/energy/en/data-analysis/country**

[4] In the study, we decide to include the share of the Industrial, Residential and Transportation energy consumption, in order to highlight the economic features and the structural differences among the countries. For instance, Filippini and Hunt (2011) used the share of value added of the industrial and service sector of GDP.

[5] We use this variable, in order to account for the implications of Net Imports in the FEC and the consequences of energy dependency on the EU energy efficiency.

[6] According to EEA definition, a Heating Degree Day (HDD) is a proxy for the energy demand needed to heat a home or a business. Respectively, a Cooling Degree Day (CDD) is the indicator to measure the energy demand in order to cool a home or a business. Hence, the HDD and CDD are considered to be good indicators for climate in each country.

In the next table we present the original variables we use in our analysis and their descriptive statistics.

Table 4. 1: Descriptive Statistics of Original Data

Variable						
Description	Name	Mean	Median	Min	Max	Stand. Dev.
Final Energy Consumption	E	40.165	18.130	0.335	231.08	53.791
GDP per capita	GDP	21606	18310	2696	85217	14910
Area	Area	237844	88316	316	2211000	414670
Population	Pop	17.598	8.657	0.352	82.537	22.185
Heating Degree Days	HDD	2921.269	2912.71	322.12	6413.2	1155.669
Cooling Degree Days	CDD	97.702	21.865	0.01	781.42	164.67
Industrial Consumption %	IC	28.43	27.48	0.00	63.03	9.81
Residential Consumption %	RC	25.594	25.30	8.40	44.68	7.142
Transportation Consumption %	TC	31.05	29.36	7.52	71.45	11.50
Renewables %	Renew	7.91	6	0	27.15	6.52
Net Energy Imports	NI	89.61	86.33	-65.49	470.18	62.99

Nevertheless, in order to secure that our analysis is consistent and comparable, we will examine our data in a transformed form. In particular, as dependent variable will be the FEC per capita and as a result, the Population variable would be excluded from our analysis.

The main idea in our analysis will be to construct an Underlying Energy Demand Frontier. In principle, the aim here is to apply the frontier function concept in order to estimate the baseline energy demand, which is the frontier that reflects the demand for energy consumption. This approach allows the possibility to identify, if a member state is, or is not, on the frontier. Moreover, if a country is not on the frontier, the distance between the observed level of energy consumption and the frontier, measures the level of energy consumption above the baseline demand. This distance is defined as the level of energy inefficiency. The 'underlying energy efficiency' could incorporate a number of factors that will differ across states, including the different economic characteristics, different regulations as well as different social norms, lifestyles and values[7]. Hence, a low level of 'underlying energy inefficiency' implies an inefficient use of energy, so that in this situation, awareness of energy conservation could be increased in order to reach the optimal energy demand function. Nevertheless, from an empirical perspective, the aggregate energy efficiency is not observed explicitly. Therefore, this 'underlying energy inefficiency' has to be estimated.

Given the discussion above, it is assumed that there exists an EU aggregate energy demand function, as follows:

$$e_{it} = f(GDP_{it}, Area_i, HDD_{it}, CDD_{it}, Ind, Transp, Resid, Renew, NI, Year, EF_{it}) \tag{4.1}$$

where e_{it} is the FEC per capita, $Year$ is a time trend variable and EF_{it} is the level of 'underling energy efficiency' for each country at a particular time. From equation (4.1), taking the natural logarithm we obtain the final form of our equation. In particular, we have:

$$\begin{aligned} \ln(e_{it}) = \beta_0 + \beta_1\, gdp_{it} + \beta_2\, area_i + \beta_3\, hdd_{it} + \beta_4\, cdd_{it} + \beta_5\, Ind_{it} + \beta_6\, Transp_{it} \\ + \beta_7\, Resid_{it} + \beta_8\, Renew_{it} + \beta_8\, NI_{it} + \beta_9\, Year + v_{it} + u_{it} \end{aligned} \tag{4.2}$$

where gdp_{it} is the natural logarithm of the GDP of country i and year t, $area_i$ is the natural logarithm of Area variable, and hdd_{it} and cdd_{it} is the natural logarithm of HDD and CDD, respectively. In summary, equation (4.2) is estimated in order to estimate the 'underlying energy efficiency' for each member state. To estimate the inefficiency term, we employ the LSDV model, the STVM and the Pooled SF model.

This chapter is organized as follows. In section 4.1 we employ the Ordinary Least Squares model in order to evaluate the OLS residuals of the model. In section 4.2 we use the Least Square Dummy Variable model (Fixed Effects) as proposed by Schmidt and Sickles (1984). In section 4.3 we present the Time Varying Simple model (TVSM), which is a simple version of Battese and Coelli (1992), without distributional assumptions. Finally, in section 4.4 we use the Pooled Stochastic Frontier model (PSFM) as proposed by Aigner et al. (1977). In order to measure the energy efficiency, we use the mean and the mode of the conditional distribution as introduced by Jondrow et al. (1982).

[7] For further information, see Filippini and Hunt (2011, 2012).

4.1 Ordinary Least Squares Method

The Ordinary Least Square method is a good measure and a simple test, in order to evaluate the skewness and the existence of inefficiency in our data. As stated above, the noise component v is assumed to be iid and symmetric, distributed independently of u. On the other hand, we assume that the inefficiency term is non-negative ($u \geq 0$).

Therefore, since $\varepsilon_{it} = v_{it} + u_{it}$ and $u_{it} \geq 0$, we conclude that the OLS residuals are presenting a left skewed distribution. The left skewness is an indicator of existence of cost inefficiency. In particular, we have that $E(\varepsilon_{it}) = E(v_{it} + u_{it}) = E(v_{it}) + E(u_{it}) = E(u_{it}) > 0$. On the other hand, if $u_{it} = 0$, then $\varepsilon_{it} = v_{it}$, the error term is symmetric and the data do not support a cost inefficiency assumption. Other tests, of the stochastic frontier specification are presented in Schmidt and Lin (1984). According to them, one in order to evaluate the residual's skewness could use any of the standard error normality tests in a regression, such as the Kolmogorov-Smirnov test or the Shapiro-Wilk test. However, Schmidt and Lin (1984) highlight the fact that the most obvious difference between a normal and the sum of a normal and a half-normal (or other one-sided distributional assumption) is the skewness of the latter, and it may be preferable to base the test on the sample skewness of the OLS residuals. Schmidt and Lin (1984), in order to evaluate the skewness of the OLS residuals, propose the following test statistic:

$$\sqrt{b_1} = \frac{m_3}{{m_2}^{2/3}} \tag{4.1.1}$$

where m_2 and m_3 are the second and the third moments of the residuals. The distribution of the above statistic is widely tabulated and follows asymptotically a standard normal distribution. Since, the v_{it} are symmetrically distributed, the m_3 is simple the third sample moment of the u_{it}. Thus a positive m_3 indicates that the OLS residuals are negatively skewed and suggests the existence of the cost inefficiency (in our case, energy inefficiency).

Furthermore, Coelli (1995) proposed an alternative test statistic that is asymptotically distributed as $N(0,1)$. Under the null hypothesis of zero skewness of the errors, the form of the test statistic is:

$$\frac{m_3}{\left(\frac{6\, m_2}{N\, T}\right)^{\frac{1}{2}}} \tag{4.1.2}$$

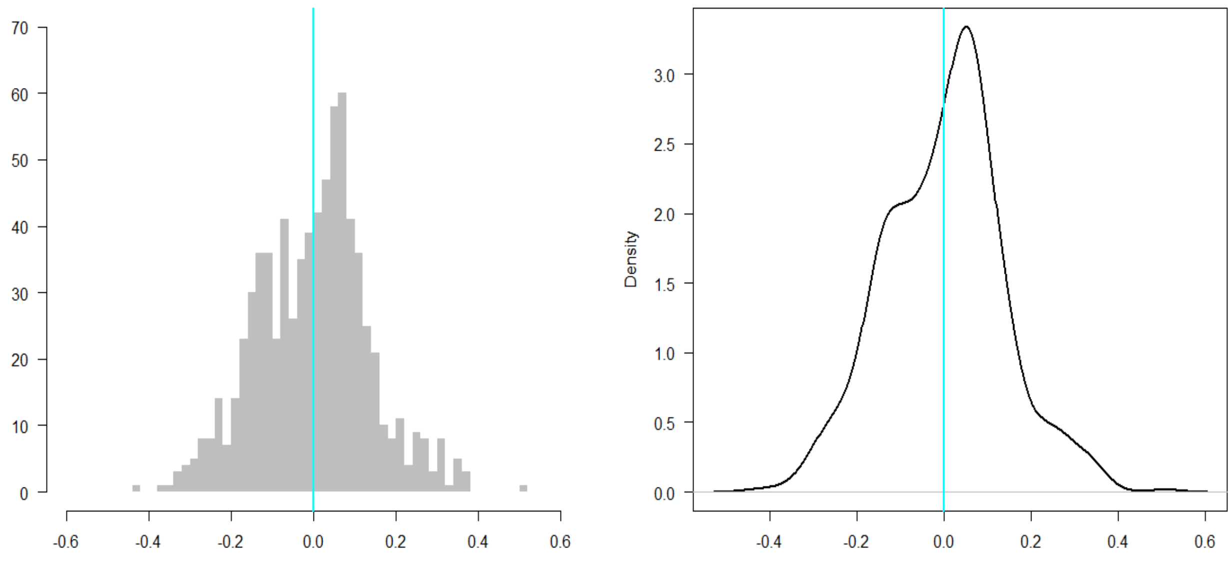
where N is the number of countries, T is the length of the time period of our study, and therefore $N\, T$ is the sample size.

The advantage of both tests is that they are based on the OLS residuals which are easy to obtain. The main disadvantage of both tests is that they rely on the asymptotic theory, and many samples are relatively small[8].

In the following figure, we present the histogram of the OLS residuals of our study. As we can see from Figure 4.1, the OLS residuals of our data are presenting a left skewness. Moreover, both the mean value and the third moment of the OLS residuals, confirm the above theory. Furthermore, the no symmetric assumption is confirmed by the standard normality tests such as the Kolmogorov-Smirnov test and Shapiro-Wilk test. However, the assumption of left skewness is not supported by the Schmidt and Lin (1984) and Coelli (1995) test statistics.

To conclude, we see that the different test statistics are very sensitive to the sample data and level of the inefficiency term. In particular, we see that the inefficiency terms are taking a very small value (very close to zero) and both asymptotic tests [Schmidt and Lin (1984) and Coelli (1995)] cannot capture the cost inefficiency assumption. Furthermore, according to Greene (2008) the skewness of the residuals is merely a sample statistic subject to sampling variability, and m_3 can be negative even if the stochastic frontier model is correct.

Figure 4. 1: OLS residuals



[8] In our study, the length of our data is 756, which is generally considered a satisfactory sample size.

4.2 Least Square Dummy Variable Model

The Least Square Dummy Variable model (LSDV) is the first model we employ with no-distributional assumptions. The LSDV model was first introduced by Schmidt and Sickles (1984) and is the simplest panel data model. According to this model, we assume that the two sided error term is following a normal distribution ($v_{it} \sim iid\, N(0, {\sigma_v}^2)$) and the inefficiency term is non-negative and constant across time ($u_i \geq 0$). In addition, we allow the inefficiency term to be correlated with either the regressors or with the v_{it}. Hence, our model will have the following form:

$$\ln{(e_{it})} \ = \beta_0 + \sum_{j=1}^{k} \beta_j \, x_{jit} + v_{it} + u_i \tag{4.2.1}$$

where e_{it} is the FEC per capita and x_{it} are the explanatory variables, as presented is equation (4.1). Rewriting the equation (4.2.1), we can obtain the following form:

$$\ln{(e_{it})} \ = (\beta_0 + u_i) + \sum_{j=1}^{k} \beta_j \, x_{jit} + v_{it} = \alpha_i + \sum_{j=1}^{k} \beta_j \, x_{jit} + v_{it} \tag{4.2.2}$$

where $\alpha_i = (\beta_0 + u_i)$ are the country's specific intercepts. From the above equation, we observe that the u_i's can be treated as fixed effects and can be estimated along with the slope parameters β_i's employing the OLS method. However, in order to evaluate these intercepts (α_i) and the inefficiency term for each country (u_i), $(N-1)$ country dummy variables can be added. Here, we should point out that from this model we exclude the Area variable. The reason why we exclude this variable is that since we use the country's dummy variable, and the Area variable is constant for each state throughout the time, our model presents the multicollinearity problem.

In the following equation we present our final model, including the $(N-1)$ country dummy variables.

$$\ln{(e_{it})} \ = \beta_0 + \sum_{j=1}^{k} \beta_j \, x_{jit} + \sum_{c=2}^{N-1} \gamma_c \, d_{c,it} + v_{it}$$
$$where \ d_{c,it} = \begin{cases} 1 & , Country = c \\ 0 & , otherwise \end{cases} \tag{4.2.3}$$

In the above equation, we assume that the control state is Austria. Hence, from model (4.2.3) we can obtain the country's specific intercepts, as follows:

$$
\begin{gathered}
\hat{\alpha}_1 = \hat{\beta}_0 \\
\hat{\alpha}_2 = \hat{\beta}_0 + \hat{\gamma}_1 \\
\hat{\alpha}_3 = \hat{\beta}_0 + \hat{\gamma}_2 \\
\vdots \\
\hat{\alpha}_N = \hat{\beta}_0 + \hat{\gamma}_{N-1}
\end{gathered} \tag{4.2.4}
$$

According to Schmidt and Sickles (1984) and Kumbhakar and Lovell (2000), the LSDV estimates of β_i's are consistent as either $N \to \infty$ or $T \to \infty$, and the consistency property does not require the u_i's to be uncorrelated with the regressors. The α_i's estimations are consistent as $T \to \infty$, although consistency of the LSDV estimates of u_i's requires both $N \to \infty$ and $T \to \infty$. After employing this model, we can obtain the inefficiencies estimations u_i from equation (4.2.4), as follows:

$$\hat{u}_i = \{\hat{\alpha}_i - \min(\hat{\alpha}_i)\} \tag{4.2.5}$$

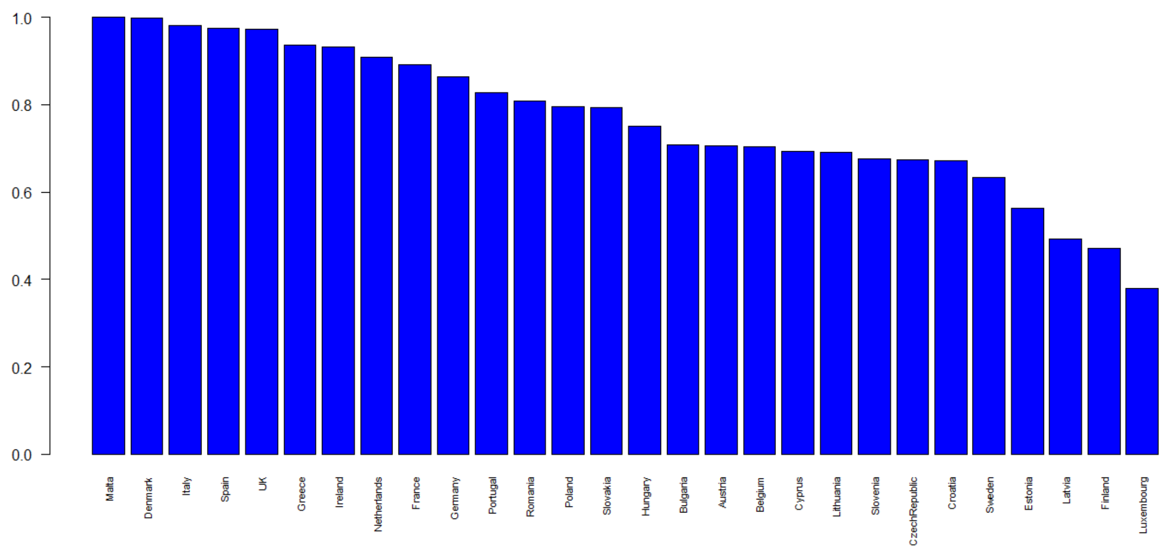
From the above equation, it turns out that at least one inefficiency term will be equal to zero. As a result, at least one country will be exactly on the cost frontier ($\hat{u}_i{}^* = 0$). Finally we obtain our Energy Efficiency estimations for each country, as:

$$\widehat{EF}_i = \exp(-\hat{u}_i) \tag{4.2.6}$$

Hence, we conclude that in the LSDV model, at least one country will be 100% efficient, since the most efficient country will have an inefficient term equal to zero ($\hat{u}_i{}^* = 0$).

From the LSDV we obtain an average energy efficiency of 76.78%. In particular, in our model, the most efficient country seems to be Malta. In addition, it seems that the group with the most efficient countries consists of Denmark, Italy, Spain and U.K with an energy efficiency of 99.89%, 98.11%, 97.59% and 97.21%, respectively. On the other hand, we see that countries such as Luxembourg, Finland, Latvia and Estonia are presenting the lowest levels of energy efficiency with an energy efficiency of 37.86%, 47.10%, 49.29% and 56.34%, respectively. Moreover, we should highlight the fact that Greece is ranked in the 6th position, higher than countries with well known energy efficiency policies such as Netherlands, Germany and Belgium which are ranked in the 8th, 10th and 18th position, respectively. Further conclusions are presented in section 4.5 where detailed information is presented. In the next figure, we sum up our energy efficiency estimates for all EU member states.

Figure 4. 2: Energy efficiencies according to the LSDV model



4.3 Simple Time Varying Model

In the previous model, we assumed that energy efficiency is constant across time. Therefore, a natural extension of the model is to allow inefficiency to vary in time. This can be achieved by introducing a function of time $h_{it} = \beta_i(t)$ where $u_{it} = u_i + h_{it}$. One specific parameterization has been suggested by Battese and Coelli (1992) which introduce only one additional parameter for the time path as $e^{h_{it}} = e^{\zeta_i (T-t)}$, which in logarithm takes the following form:

$$h_{it} = \zeta_i (T - t) \; where \; t = 1,2, \dots , T \tag{4.3.1}$$

We note that in the final period the $(T - t) = 0$, and hence h_{it} equals to zero. Therefore, this specific parameterization assumes that throughout the time, the varying inefficiency term h_{it} is gradually decreasing, and therefore the energy efficiency increases. In particular, the model assumes that at the final period, the inefficiency terms for each country would be converged to zero and hence all countries would be 100% energy efficient. Despite its simplicity, the function $\exp\big(\zeta_i (T - t)\big)$ can generate and provide a variety of time paths.

In addition to the above, Battese and Coelli (1992) introduced a variety of different parameterization. More specifically, an alternative two parameter specification method is $h_{it} = 1 + h_1 (t - T) + h_2 (t - T)^2$, where h_1 and h_2 are unknown parameters. This specification permits country effects to be convex or concave. Here, we should highlight that in the case where h_1 and h_2 equals to zero, we go back to the simple time-invariant model (LSDV model). An alternative time varying model was introduced by Kumbhakar (1990), where the time specification $\gamma(t)$ has the form $\gamma(t) = [1 + \exp(bt + ct^2)]^{-1}$. This model has values for $\gamma(t)$ between zero and one and be monotone decreasing (or increasing) or convex (or concave) depending on the values of the unknown parameter b and c.

From the above, the Simple Time Varying (STV) model can be obtained from the simple log linear equation.

$$\ln (e_{it}) = \beta_0 + \sum_{j=1}^{k} \beta_j \, x_{jit} + v_{it} + u_{it} \tag{4.3.2}$$

Inserting the parameterization $u_{it} = u_i + h_{it}$ and $h_{it} = \zeta_i (T - t)$ into Equation (4.3.2), we obtain our final form.

$$\ln\,(e_{it}) \;= a_i + \sum_{j=1}^{k} \beta_j\, x_{jit} + \zeta_i\,(T-t) + v_{it} \qquad (4.3.3)$$

where $a_i = (\beta_0 + u_i)$ is the specific country's intercepts as in the LSDV model. The model can be estimated by the OLS method and differs only from the LSDV model in the additional inclusion of the time trend. More specifically, in order to estimate the STV model we include $(N-1)$ country dummy variables, in order to estimate the a_i (as in the case of the LSDV) and additional $(N-1)$ country dummy variables in order to estimate the ζ_i parameters. Hence, our final model can be rewritten in the following form:

$$\ln\,(e_{it}) \;= \beta_0 + \sum_{j=1}^{k} \beta_j\, x_{jit} + \sum_{c=2}^{N-1} \gamma_c\, d_{c,it} + \sum_{c=2}^{N-1} \zeta_c\, d_{c,it}\,(T-t) + v_{it} \qquad (4.3.4)$$

$$where\; d_{c,it} = \begin{cases} 1 & , Country = c \\ 0 & , otherwise \end{cases}$$

In order to estimate the inefficiencies terms, firstly we should estimate the model's country specific intercepts. In particular, we have:

$$\hat{a}_{it} = \hat{a}_i + \hat{\zeta}_i\,(T-t) \qquad (4.3.5)$$

where $\hat{a}_i$'s are the country's specific intercepts as estimated in the LSDV model and $\hat{\zeta}_i$'s are the country's time trend specific intercepts. Therefore, the inefficiency terms are obtained, according to the equation, below:

$$\hat{u}_{it} = \hat{a}_{it} - \min(\hat{a}_{it} | \mathrm{t} = \mathrm{t}') \qquad (4.3.6)$$

The most efficient country at time t is the country with the minimum value $\hat{a}_{it}$ at time period t. This country is used as a benchmark in order the inefficiency terms of the other ones, to be estimated. Therefore, according to Equation (4.3.6), at each time period t, we estimate the country with the minimum value and afterwards the inefficiency terms of the remaining EU member states. As a result, this model assumes that each time period, we have a fully efficient country with ${\hat{u}_{it}}^{*} = 0$.

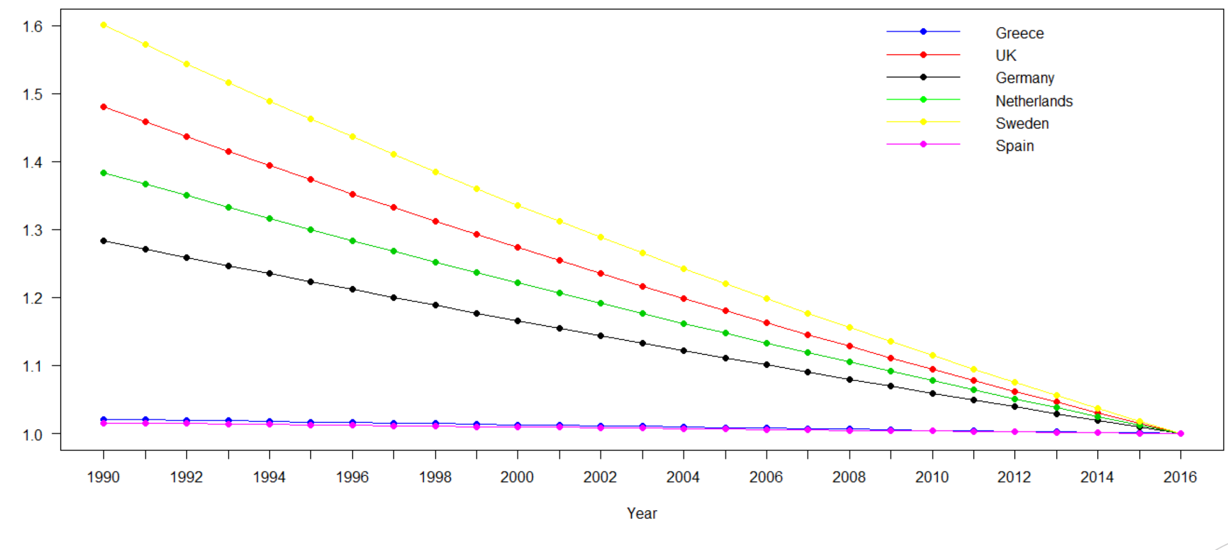
Finally, in order to estimate the Energy Efficiency of each country, we use the well known method as presented in Equation (4.2.6), but assuming time varying energy efficiency. In particular we obtain the energy efficiency levels according to the following form:

$$\widehat{EF}_{it} = \exp(-\hat{u}_{it}) \tag{4.3.7}$$

According to the STV model, we obtain a mean energy efficiency of 61.43%. The most efficient country seems to be Denmark with an average energy efficiency of 100% (fully efficient country). In the next four higher positions, we see U.K., Ireland, Italy and Spain with an average energy efficiency of 87.52%, 85.57%, 84.92% and 82.07%, respectively. In contrast, the worst energy efficient countries are Latvia, Bulgaria, Estonia, Lithuania and Romania. In this model, Greece seems to hold the 9th position with an average energy efficiency of 74.93% and Germany the 8th with an average energy efficiency of 77.66%.

Another point of interest is to examine the time paths of each country. We see that countries which present higher levels of inefficiency in the beginning of the time period, present a higher rate of zero inefficiency convergence. In the figure below, we present different inefficiency time paths. More specifically, we present the inefficiency time paths of Greece, U.K., Germany, Netherlands, Sweden and Spain. From these countries, we observe that Sweden is presenting the higher level of inefficiency in 1990. In contrast, Sweden presents the highest rate of zero inefficiency convergence, in order to reach zero energy inefficiency in 2016.

Figure 4. 3: Inefficiency Time Paths according to STV Model



4.4 Pooled Stochastic Frontier Model

In this section we employ the Pooled SF model, as introduced by Aigner et al. (1977). This is the first parametric stochastic frontier model that was employed by Filippini and Hunt (2011) in order to evaluate and to estimate the "Underlying Energy Demand Frontier". As stated in the beginning of this chapter, the concept is to formulate an Energy Demand Cost Frontier and to estimate the diversifications of each country at each time period, from the minimum feasible values on the frontier.

According to this model that was also presented in detail by Kumbhakar and Lovell (2000), we make the following assumptions:

- The two sided error term follows a Normal distribution with: $v_{it} \sim N(o, \sigma_v)$
- The one sided error term follows a Half Normal distribution: $u_{it} \sim N^{+}(0, \sigma_u)$
- v_{it} and u_{it} are distributed independently of each other, and of regressors.

From the econometric specification perspective, the literature on the estimation of stochastic frontier models using panel data needs to be considered. The first use of panel data in stochastic frontier models goes back to Pit and Lee (1981) who interpreted the panel data random effects as inefficiency rather than heterogeneity. Another panel data model is the fixed effects model (LSDV) as we employed in Section 4.2. According to Filippini and Hunt (2011), a major drawback of these models is that any unobserved, time-varying, group-specific heterogeneity is considered as inefficiency. In order to solve this problem using panel data, Greene (2005a and 2005b) proposed to extend the SFA model in its original form by adding a fixed or random individual effect in the model. However, they note that these models do not estimate the levels of persistent energy inefficiency, and as a result, the estimates of these models provide relatively high levels of energy efficiency.

Hence, for this reason, coming in line with Filippini and Hunt (2011), in this section we use the Pooled model as proposed by Aigner et al. (1977). In this model the fixed or random effects proposed by Greene (2005a, 2005b) are not included. It is obvious that, by not including the country effects in the econometric specification, we may arrive to the "unobserved variable bias" problem (e.g. a situation where correlation between observables and unobservables could bias some coefficients of the explanatory variables). Nevertheless, by introducing to our model several explanatory variables such as the Area variable for each country and some variables depending on the structure of each state, it is possible to reduce this problem.

Below we present the Pooled SF model. In the beginning, in order to estimate our model we should obtain the joint probability distribution of the v_{it} and the u_{it}. Then, from the joined PDF, we obtain the marginal density function of $\varepsilon_{it} = v_{it} + u_{it}$. In equation (4.4.1), we present the marginal density function[9].

$$f(\varepsilon_{it}) = \frac{2}{\sigma}\varphi\left(\frac{\varepsilon_{it}}{\sigma}\right)\Phi\left(\frac{\varepsilon_{it}\,\lambda}{\sigma}\right) \tag{4.4.1}$$

where $\sigma = \sqrt{{\sigma_v}^2 + {\sigma_u}^2}$, $\lambda = \frac{\sigma_u}{\sigma_v}$, φ(.) is the standard normal PDF and Φ(.) is the standard normal cumulative distribution. Hence, λ provides an indication of the relative contributions of v and u to ε. As $\lambda \to 0$ either $\sigma_v \to \infty$ or $\sigma_u \to 0$ and the symmetric error component dominates the one sided error term in the determination of ε. On the other hand, as $\lambda \to \infty$ either $\sigma_u \to \infty$ or $\sigma_v \to 0$ and the one sided error component the two sided error term in the determination of ε. If we are in the former case, then we realize that we have a cost function frontier with no energy efficiency and hence, our stochastic cost frontier model collapses.

Since $\varepsilon_{it} = v_{it} + u_{it}$, then the marginal density of ε, will be asymmetrically distributed with mean and variance

$$\begin{gathered} E(\varepsilon) = E(u) = \sigma_u\sqrt{\frac{2}{\pi}} \\ Var(\varepsilon) = \frac{\pi-2}{\pi}\,{\sigma_u}^2 + {\sigma_v}^2 \end{gathered} \tag{4.4.2}$$

Therefore, in order to estimate the model parameters, we should provide the log-likelihood function of ε and afterwards, to obtain our MLE's estimations. Below, we illustrate the final form of the log-likelihood function.

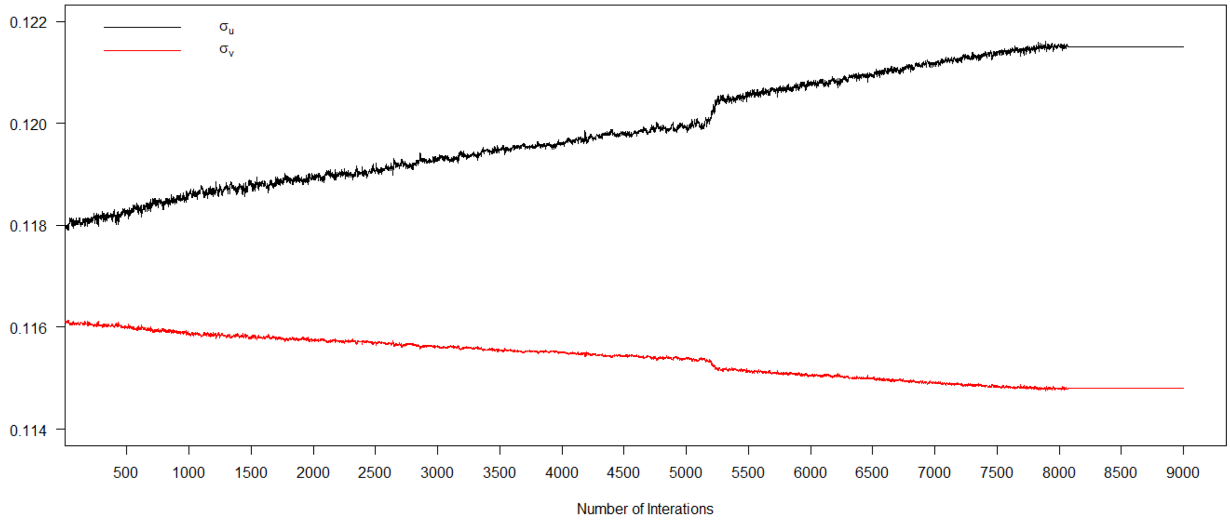
$$l(\varepsilon|\beta,\lambda,\sigma) = NT\left(\frac{\sqrt{2}}{\pi}\right) - NT\ln(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{NT}{\varepsilon_i}^2 + \sum_{i=1}^{NT}\ln\left(\Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right)\right) \tag{4.4.3}$$

In order to obtain our estimated maximum likelihood parameters, we maximize our log-likelihood function by applying the Nelder-Mead algorithm. In particular, the Nelder-Mead method was proposed by Nelder and Mead (1965) and is a commonly applied numerical method used to find the minimum or the maximum of a function in a multidimensional space. Below, in the following figure, we present the Nelder-Mead iterations and the values of σ_v and σ_u, where our

[9] For further details see Appendix.

algorithm converged. At this point, we should highlight that as initial values in the algorithm, we choose the OLS estimates.

Figure 4. 4: Nelder-Mead Iterations



From Figure 4.4, we see that after a number of iterations the algorithm totally converges (almost 8000 iterations). In particular, we see that as we increase the number of iterations the values of σ_v and σ_u diverge and hence, $\lambda = \frac{\sigma_u}{\sigma_v}$ is taking greater and greater values. In particular, at the end of the algorithm we see that $\sigma_u = 0.1215$, $\sigma_v = 0.1148$ and $\lambda = 1.058$.

The next step is to obtain estimates of the level of energy efficiency for each country. We already have estimates of ε_{it}, which obviously contains information on u_{it}. If $\varepsilon_{it} < 0$, chances are that u_{it} are nor large. In contrast, if $\varepsilon_{it} < 0$, chances are that u_{it} is large and country i at time t is energy inefficient. Hence, the problem is to extract the information that ε_{it} contains on u_{it}. In order to solve this problem, we estimate the energy efficiencies according to Jondrow et al. (1982). According to them the conditional distribution of u given ε is:

$$f(u_{it}|\varepsilon_{it}) = \frac{1}{\sqrt{2\pi\sigma_*}} \frac{\exp\left(-\frac{(u_{it}-\mu_*)^2}{2{\sigma_*}^2}\right)}{\left[1-\Phi\left(\frac{-\mu_*}{\sigma_*}\right)\right]} \tag{4.4.4}$$

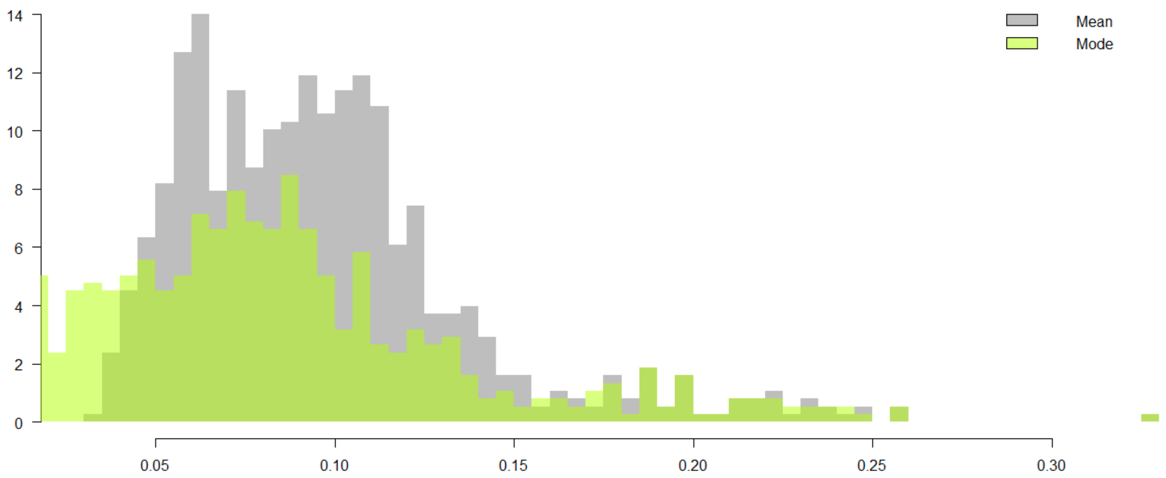
where $\sigma_* = \frac{\sigma_u \sigma_v}{\sigma}$ and $\mu_* = \frac{\varepsilon_{it} {\sigma_u}^2}{\sigma^2}$. Since $f(u|\varepsilon)$ follows is distributed as $N(\mu_*, {\sigma_*}^2)$ we can use either the mean or the mode of this distribution in order to do a point estimation of u_{it}. Filippini and Hunt (2011, 2012) used only the mean method in order to estimate their energy inefficiencies. However, in this section we will use the Mean and the Mode in order to estimate our energy inefficiencies according to these two methods and also to compare their final findings. Below we present the mean and the mode of the conditional distribution $f(u|\varepsilon)$.

$$E(u_{it}|\varepsilon_{it}) = \sigma_* \left[\frac{\varphi\left(\frac{\varepsilon_{it}\,\lambda}{\sigma}\right)}{1-\Phi\left(-\frac{\varepsilon_{it}\,\lambda}{\sigma}\right)} + \left(\frac{\varepsilon_{it}\,\lambda}{\sigma}\right) \right] \tag{4.4.5}$$

$$M(u_{it}|\varepsilon_{it}) = \begin{cases} \varepsilon_{it}\left(\frac{{\sigma_u}^2}{\sigma^2}\right) & if\ \varepsilon_{it} \geq 0 \\ 0 & otherwise \end{cases} \tag{4.4.6}$$

Once the point estimates of u_{it} are obtained, we can estimate with the well known method (Equation (4.3.7)) the energy efficiencies of each country. To conclude, in the following figure, we present the histogram of the inefficiencies as obtained from the two methods.

Figure 4. 5: Inefficiencies according to the Mean and the Mode



From Figure 4.5 we see that the inefficiencies from the Mean method are greater than the Mode of the conditional distribution. As a result, the average energy efficiency of the mode method will be higher that the Mean method. In particular, we see that the average energy efficiency of the Mean and Mode method is 90.84% and 94.21%, respectively (further details of our analysis are presents in section 4.5).

Another point of interest is to present the estimated energy efficiency of each country throughout the years. For instance, we observe many similarities between countries that present similar socio-economic characteristics. In Figure 4.6 we present the estimated energy efficiency from the Mean method of Austria, Netherlands, Germany, U.K., Greece and Spain from 1990 to

2016. Furthermore, in Figure 4.7 we present the energy efficiencies of four countries, viz. Greece, Spain, Portugal and Italy that present the same socio-economic features.

Figure 4. 6: Estimated Energy Efficiencies from 1990 to 2016

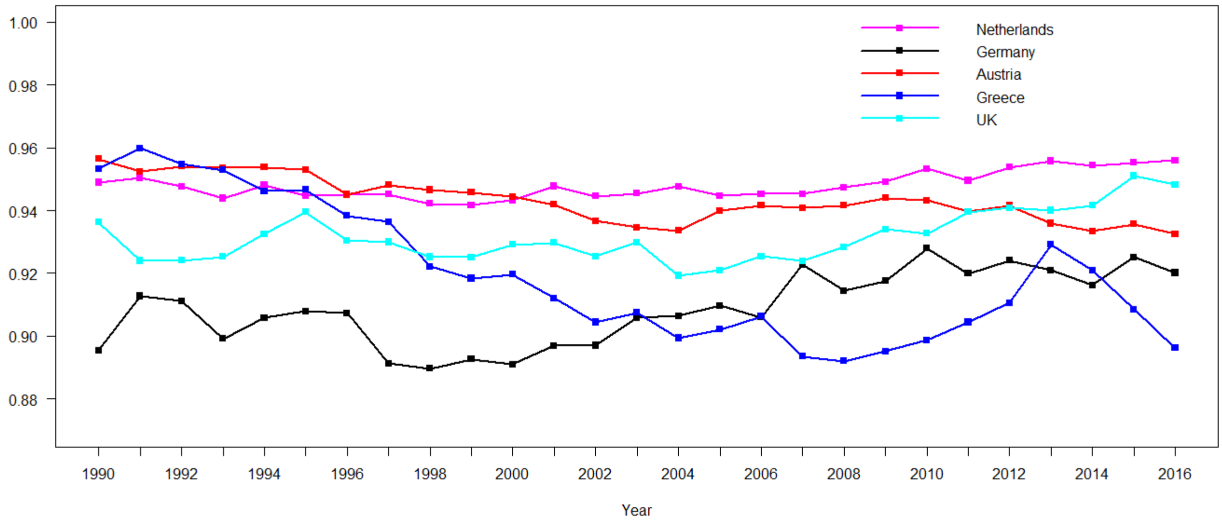
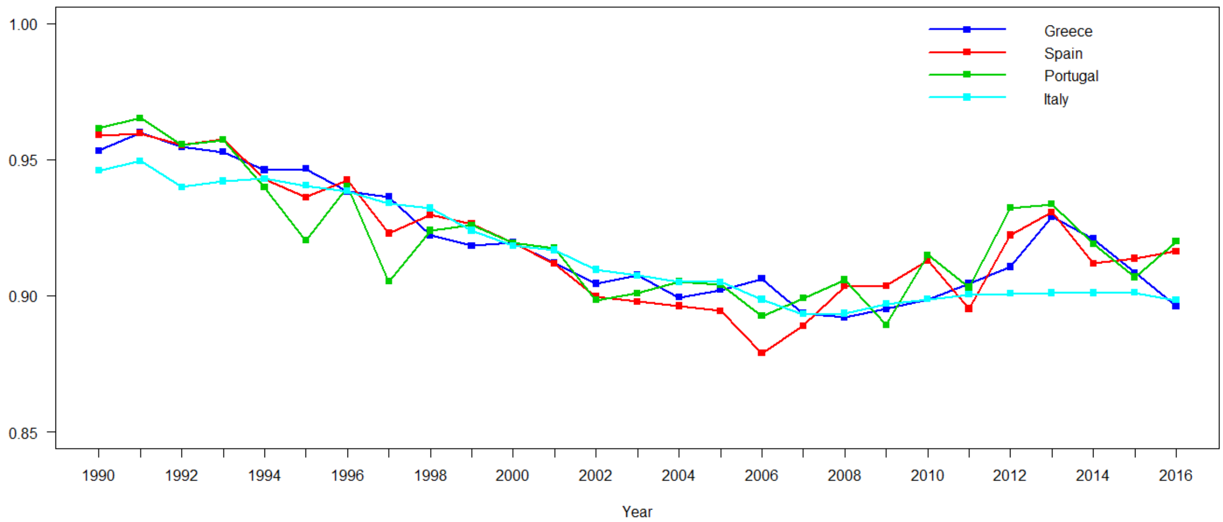


Figure 4. 7: Estimated Energy Efficiencies of Greece-Spain-Portugal-Italy



First of all, from Figure 4.6, we see that that between the different countries there are some important differences on the trend of the energy efficiency throughout the years. To begin with, from our selected countries we see that throughout the years there are a lot of fluctuations. We see that Germany, Netherlands and U.K. are presenting a continuous energy efficiency increase. In contrast, we observe that Austria is presenting a slight decrease in its efficiency. In the case of Greece, the graph illustrates great diversifications between the time periods. Until 2008 we see a significant decrease in its efficiency, and then an increase until 2013[10]. Unfortunately, since 2013 we see that Greece is presenting again a fall in its energy efficiency levels. These results, also confirm the differences between the countries due to the social and economic features.

Another point of interest is to comment Figure 4.7. As stated above, in this figure, we illustrate four countries, viz. Spain, Portugal, Greece and Italy; that are presenting some similar features in their economic and social norms. We see that these countries are presenting almost the same trend in their energy efficiency levels. We observe that from 1990 until 2006-2008, they are presenting a gradual decrease in their energy efficiency. In contrast, since 2009, they are presenting a general increase in their efficiency. As argued before, this result may occurred due to the financial crisis that these counties were dealing with and the lack of efficient energy efficiency regulations. Hence, it seems that it is necessary for them to continue the efforts in order to introduce and to legislate some crucial and sufficient regulations.

[10] In our opinion, the increase in the energy efficiency was happened because of the great decrease in the Greek's FEC, due to the financial crisis that the country faced these specific years. This argument also can be relying on the fact that after 2013, Greece is facing again a fall in its energy efficiency.

4.5 Estimation Results

In this chapter, we employed three parametric stochastic frontier models, in order to formulate and to estimate an "Underlying Energy Demand Frontier" as proposed by Filippini and Hunt (2011, 2012). As stated above, the main idea of this theory is that there exists a minimum demand frontier and each country due to different restrictions is unable to consume at its minimum level. Hence, the diversification between the observed and the minimum value, it is defined as inefficiency.

More specifically, in the above sections, we employed two models assuming no distributional assumptions for the one sided error term, viz., the LSDV and the STV model, and one model with distributional assumptions on the one sided error term, viz., the Pooled SF model, where the inefficiency term is following a half normal distribution.

To begin with, in section 4.1 we employed the simple OLS regression model in order to estimate the OLS residuals and to examine the inefficiency assumption of the model. In particular, we used different methods such as the Schmidt and Lin (1984), Coelli (1995) and different standard normality tests such as the Kolmogorov-Smirnov and Shapiro-Wilks test. In section 4.2, the Least Squares Dummy Variable model (LSDV) was employed as introduced by Schmidt and Sickles (1984), which suggests that the inefficiency term for each country is non-negative and remains constant across time. In section 4.3, the Simple Time Varying model (STV) was employed as introduced by Behr (2015). The model assumes a time trend specification as proposed by Battese and Coelli (1992). In this model, the different inefficiency time paths can be evaluated in order to examine the speed of zero inefficiency convergence.

Last, in section 4.4, the Pooled SF model was employed as suggested by Aigner et al. (1977). This was the first parametric SF model which was employed by Filippini and Hunt (2011). Here, in order to obtain the levels of the energy efficiency for each country, we assume a half normal distribution for the one sided error term, a normal distribution for the error component and in addition, we use the mean and the mode of the conditional distribution as introduced by Jondrow et al. (1982).

In the next section, we present our final findings. In Table 4.2 we present the parameter estimations for each model. In Table 4.3 we present the levels of the underlying energy efficiency of the three models (in the case of the Pooled SF model, we present only the energy efficiency levels according to the mean of the conditional distribution). In Table 4.4 we illustrate the energy efficiency levels of the Pooled SF Model according to the mean and the mode of the conditional distribution. Finally, in Figure 4.8 and in Figure 4.9 we present the Kernel densities of the inefficiencies and their pair wise inefficiency correlations.

Table 4. 2: Estimated Stochastic Frontier Models (Standard Errors in Parentheses)

	Pooled OLS	LSDV	STVM	Pooled SFM
$const$	6.8360*** (1.5376)	3.0475** (1.3454)	-7.5231*** (0.0190)	6.5104*** (0.2195)
gdp	0.4726*** (0.0104)	0.4520*** (0.0257)	0.7273*** (0.0015)	0.4726*** (0.0102)
$area$	-0.0963*** (0.0042)	n/a	n/a	-0.0918*** (0.0042)
hdd	0.4377*** (0.0218)	0.0741** (0.0310)	0.1776*** (0.0013)	0.4384*** (0.0219)
cdd	0.0069** (0.0022)	-0.0013 (0.0019)	0.0006*** (0.0001)	0.007*** (0.0021)
Ind	0.0000 (0.0014)	-0.0059*** (0.0012)	0.0004*** (0.0001)	-0.0002 (0.0013)
$Transp$	-0.0065*** (0.0014)	-0.0134*** (0.0015)	-0.0019*** (0.0001)	-0.0067*** (0.0014)
$Resid$	-0.0164*** (0.0018)	-0.0137*** (0.0015)	-0.0054*** (0.0001)	-0.0161*** (0.0018)
$Renew$	0.0000 (0.010)	-0.0179*** (0.0016)	-0.0118*** (0.0001)	-0.0001 (0.0009)
NI	-0.1024*** (0.0129)	-0.0658*** (0.0115)	-0.0659*** (0.0001)	-0.0958*** (0.0125)
$Year$	-0.0061*** (0.008)	-0.0030*** (0.007)	n/a	-0.0061*** (0.0001)
Log Likelihood	435.42	961.12	-Inf	3.4467
σ	0.137	0.0695	0.0026	0.1672
σ_v	n/a	n/a	n/a	0.1148
σ_u	n/a	n/a	n/a	0.1215
λ	n/a	n/a	n/a	1.0584
Mean Efficiency	n/a	0.7678	0.6143	[1] 0.9084 [2] 0.9421

***, **, * - significant at 1%, 5%, 10% level, respectively.

From Table 4.2, we can observe some significant differences between the three models. Firstly, as stated in the above sections, we see that the first two models (LSDV and STV model) are presenting lower average energy efficiency in comparison to the Pooled SF model. In particular, the LSDV and the STV model present a mean energy efficiency of 76.78% and 61.43%, respectively. On the other hand, according to the mean and the mode of the conditional distribution, we see that Pooled SF model presents an average energy efficiency of 90.84% and

94.21%, respectively. These results indicate the differences on the inefficiency estimation between the models. Hence, it turns out that assuming a distribution assumption for the inefficiency term (a half normal distribution) turns to underestimate the one sided error component (or without assuming distributional assumptions we overestimate the inefficiency term) and therefore, to obtain higher efficiency estimations.

Secondly, we should highlight that our estimated parameters can be interpreted as elasticities, since we have a log-log model. In the case of the variables that are not included in a logarithm form, the inference does not change at all, since they are added in the models in percentage form. For instance, in the LSDV and Pooled SF model we see that the value of gdp variable is 0.4520 and 0.4726, respectively. This means, that an increase of 1% in the GDP per capita, will increase the FEC per capita by 0.4520% and 0.4726%, respectively. On the other hand, in the STV model, we see that the value of the parameter is 0.7273, greater than the other ones. This means, that according to the STV model, an increase by 1% of GDP per capita will increase the FEC per capita by 0.7273%. In the same way, we can interpret the other variables, such as the Area, the HDD and the CDD variable.

In addition, we see that in all the above models, most of the explanatory variables seem to be statistically significant. However, in the Pooled SF model we see that the Ind and the $Renew$ variables are not statistically significant. This means that the percentage of Industrial Consumption and the percentage of Renewables do not affect the energy demand function. In contrast, we see that the Transportation and the Residential consumption are crucial factors of the energy demand frontier. In addition to these variables, the Net Imports seem to affect the FEC per capita. This indicates the great importance of a country to be energy autonomous or by using the energy more efficiently to reduce its energy reliance to other states. These results, also, come in line with the energy statistics that introduced in Chapter 1 (Introduction).

Moreover, the parameter λ is equal to 1.05. This indicates the relationship between the one sided error term and the statistical error disturbance. In a previous section, we defined the parameter lambda as $\lambda = \frac{\sigma_u}{\sigma_v}$. Hence, since $\lambda = 1.05$, we can interpret that these two errors terms are almost the same ($\sigma_u = 0.1215$ and $\sigma_v = 0.1148$). Furthermore, since the parameter lambda is different from zero, this indicates that $\sigma_u > 0$ and hence, there exists an energy demand frontier with energy efficiency.

Last but not least, as stated above, the differences on the efficiencies between the mean and the mode method seem to be immaterial. This can be confirmed from Table 4.4, where the energy efficiency for each country is presented, according to the mean and the mode of the conditional distribution.

Table 4. 3: Estimated energy efficiencies of the three models

	LSDV		**STVM**		**Pooled SF Model***	
	Level	**Ranking**	**Level**	**Ranking**	**Level**	**Ranking**
Austria	0.7051	17	0.7037	10	0.9433	2
Belgium	0.7030	18	0.6212	14	0.8991	20
Bulgaria	0.7081	16	0.3645	27	0.8913	23
Croatia	0.6726	23	0.4743	20	0.9091	16
Cyprus	0.6941	19	0.5557	15	0.8308	27
Czech Rep.	0.6434	22	0.4676	21	0.9062	18
Denmark	0.9989	2	1.0000	1	0.9134	15
Estonia	0.5634	25	0.3920	26	0.9023	19
Finland	0.4710	27	0.4656	22	0.8780	25
France	0.8921	9	0.7905	6	0.8966	21
Germany	0.8629	10	0.7766	8	0.9087	17
Greece	0.9366	6	0.7493	9	0.9196	12
Hungary	0.7514	15	0.4643	23	0.8807	24
Ireland	0.9317	7	0.8557	3	0.9297	8
Italy	0.9811	3	0.8492	4	0.9161	14
Latvia	0.4929	26	0.3278	28	0.8766	26
Lithuania	0.6920	20	0.4543	25	0.9283	9
Luxembourg	0.3786	28	0.4920	17	0.8293	28
Malta	1.0000	1	0.6852	11	0.9359	4
Netherlands	0.9084	8	0.7815	7	0.9480	1
Poland	0.7947	13	0.4744	19	0.8945	22
Portugal	0.8285	11	0.6394	13	0.9206	11
Romania	0.8081	12	0.4604	24	0.9301	7
Slovakia	0.7932	14	0.4746	18	0.9307	6
Slovenia	0.6765	21	0.5349	16	0.9385	3
Spain	0.9759	4	0.8207	5	0.9196	13
Sweden	0.6332	24	0.6487	12	0.9274	10
U.K.	0.9721	5	0.8752	2	0.9315	5

*The results of the Pooled SF model are presented according to the mean value of the conditional distribution.

Table 4.3 illustrates in detail the differences between the three models. We see that there are a lot of significant differences in the ranking positions of each model.

To begin with, as stated in section 4.2, the LSDV model illustrates that Malta seems to be the most efficient country. In contrast, according to the STV and Pooled SF model, the most efficient countries are Denmark and Netherlands. Furthermore, we see countries such as Austria, Lithuania, Romania, Slovenia and Slovakia which however they are presenting high level of energy efficiency according to the Pooled SF model, in the case of the LSDV and STV model are presenting a very low level. In addition, we observe countries such as Belgium, Bulgaria, Cyprus, Finland and Luxembourg to take a low ranking position in all models. In contrast, U.K. is all model is presenting a very high level of energy efficiency. Another point that we should mention is that Denmark in the first two models in ranked in the 1st and 2nd position, respectively. However, in the 3rd model is ranked in the 15th position. These examples can totally reflect this diversification.

Last but not least, we should highlight that most of our final findings come in line with Filippini et al. (2014), where the EU residential sector is examined. Filippini et al. (2014) classify the EU member states, according to their energy efficiency level in three groups: (I) efficient states (above 93% energy efficiency), (II) moderate efficient states (from 86% to 93% energy efficiency) and (III) inefficient states (below 86% energy efficiency). The first group consists of countries such as Ireland, Netherlands and U.K., the moderate efficient states consist of Austria, France, Poland and Sweden, and the last group consists of countries such as Belgium, Greece, Germany, Denmark, Finland and Portugal. From our empirical study, we can notice many similarities in our final estimations. For instance, from the Pooled SF model, we see that Austria, Netherlands and U.K. are the most efficient countries. Moreover, we see that countries such as France, Sweden and Greece can be determined also from our study, as moderate efficient states. Last, we see that countries such as Germany and Finland to be classified in low ranking positions.

Nevertheless, we should point out that there are a few differences between the two studies. For instance, according to the Pooled SF model, countries such as Slovenia and Slovakia seem to present a high level of energy efficiency. On the other hand, Luxembourg is ranked at the last position. In contrast, according to Filippini et al. (2014), Slovenia and Slovakia are considered as moderate energy efficient countries. However, we should underline the fact that between the Residential and the Final energy consumption, the final interpretation is expected to be different since the Residential sector is affected by many other factors.

Below we present Table 4.4, where the levels of energy efficiencies and the ranking positions of the Pooled SF model according to the mean and the mode of the conditional distribution are presented.

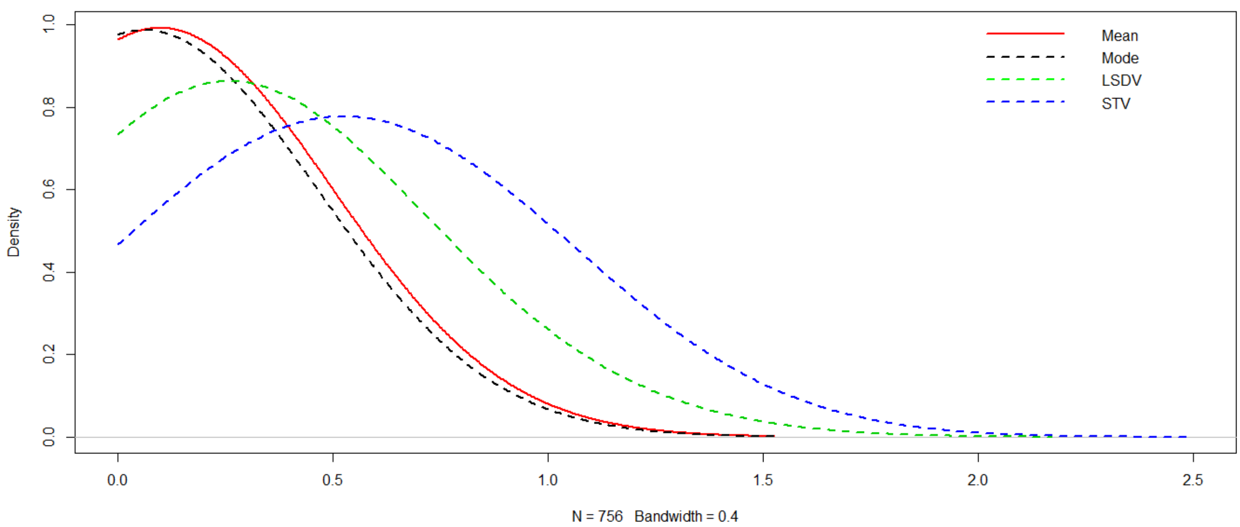
Table 4. 4: Estimated energy efficiencies of the Pooled SF model according to the Mean and the Mode of the conditional distribution

	Mean		Mode	
	Level	**Ranking**	**Level**	**Ranking**
Austria	0.9433	2	0.9991	2
Belgium	0.8991	20	0.9241	20
Bulgaria	0.8913	23	0.9121	23
Croatia	0.9091	16	0.9382	17
Cyprus	0.8308	27	0.8370	27
Czech Rep.	0.9062	18	0.9346	19
Denmark	0.9134	15	0.9489	15
Estonia	0.9023	19	0.9360	18
Finland	0.8780	25	0.8906	26
France	0.8966	21	0.9184	21
Germany	0.9087	17	0.9398	16
Greece	0.9196	12	0.9558	13
Hungary	0.8807	24	0.8947	24
Ireland	0.9297	8	0.9756	8
Italy	0.9161	14	0.9530	14
Latvia	0.8766	26	0.8933	25
Lithuania	0.9283	9	0.9775	7
Luxembourg	0.8293	28	0.8330	28
Malta	0.9359	4	0.9783	6
Netherlands	0.9480	1	1.0000	1
Poland	0.8945	22	0.9178	22
Portugal	0.9206	11	0.9574	11
Romania	0.9301	7	0.9826	5
Slovakia	0.9307	6	0.9742	10
Slovenia	0.9385	3	0.9925	3
Spain	0.9196	13	0.9561	12
Sweden	0.9274	10	0.9745	9
U.K.	0.9315	5	0.9842	4

In Table 4.4 we observe, that between these two methods there are no significant differences. More specifically, there are some diversifications between the level of energy efficiency among the countries and their final rankings. First of all, we see that in the most cases the mode seems to estimate a higher level of energy efficiency. Moreover, we see that some countries take a higher ranking position and others a lower one. More specifically, Germany, Latvia, Lithuania, Romania, Sweden and U.K. take a higher position in the case of the mode (from 17th to 16th position, 26th to 25th position, 9th to 7th position, 7th to 5th position, 10th to 9th position and 5th to 4th position, respectively). In contrast, Croatia, Czech Republic, Finland, Malta and Slovakia are presenting a lower ranking position in the case of the mode. From these, we confirm that the differences between the two methods can be defined as immaterial.

To conclude all the above, in the next figure we present the inefficiency Kernel densities[11] of all models and methods.

Figure 4. 8: Kernel densities of the inefficiency terms

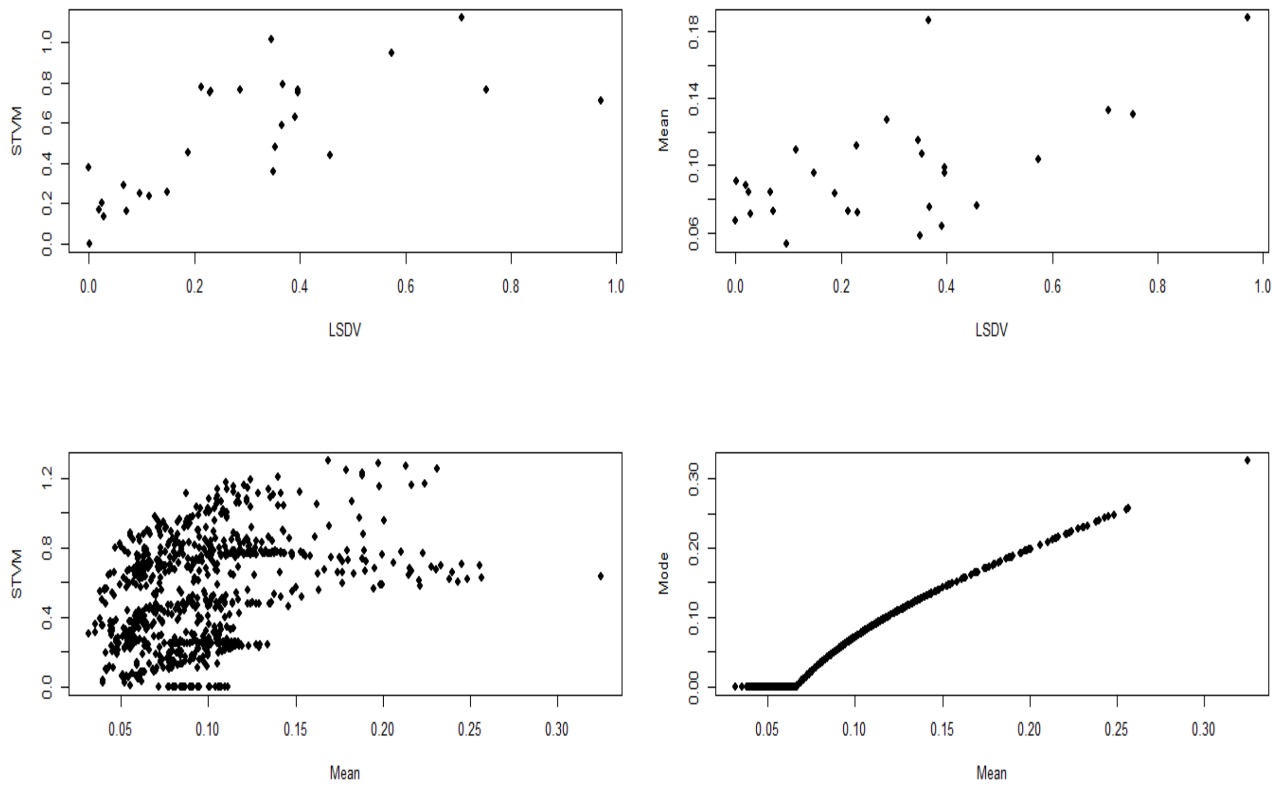


The above figure confirms the final findings presented in Tables 4.3 and 4.4. In particular, as stated above, we see that both the LSDV and the STV model are overestimating the inefficiency terms (the maximum value of the inefficiency terms of the two models are 0.9714 and 1.2959, respectively). On the other hand, we see that the Pooled SF model with its distribution assumptions, restricts the inefficiency term and therefore we receive a higher energy efficiency level (maximum value of the inefficiency terms, according to the mean and the mode is 0.3251 for both methods).

[11] The value of the bandwidth was chosen in order to visualize the differences in Kernel densities between the three models.

Another point of interest is to evaluate the correlation of the inefficiency terms between the estimated models. In Figure 4.9 we present the correlations between the models. In addition, we provide the Pearson correlation coefficients. From this specific figure, we see that there are some differences between the models. First of all, we observe that the inefficiencies between the mean and the mode method seem to be highly linear correlated. Specifically, we see that the Pearson correlation coefficient is 0.987. Moreover, we see that there exists a modest linear correlation between the LSDV and the STVM, with a Pearson correlation coefficient of 0.71. Here we should point out that, in order to estimate the correlation between the LSDV and the other time varying models, we estimate the average inefficiency for each country. Last, between the Mean-LSDV model and the STVM-Mean we observe a Pearson correlation coefficient of 0.606 and 0.394, respectively.

Figure 4. 9: Correlation of the inefficiency terms between the models



5 Conclusions and Future Research

5.1 Conclusions

In this study, we estimated an "Underlying Energy Demand Function" in order to measure and to evaluate the energy efficiency across the 28 EU member states. In particular, we employed three parametric Stochastic Frontier Approaches in order to estimate the inefficiency terms for each country. As dependent variable we choose the Final Energy Consumption per capita that is one of the most crucial tools of the EU energy efficiency policy.

Firstly, we see that the Energy Intensity is not a good indicator to estimate and to evaluate the energy efficiency among the 28 EU countries. In particular, we see a lot of diversifications in the final rankings among the empirical approach and the different econometric approaches. This indicates the great importance of different economic factors, the structure of each economy, the regulations, the social norms in each country and their environmental characteristics. For instance, we observed that countries that are presenting different socioeconomic similarities, are presenting also the same energy efficiency trend. In particular, we saw that Greece, Spain, Portugal and Italy are presenting almost the same trend in their energy efficiency level.

Secondly, we see that the GDP is a crucial factor for determining the Energy Demand Frontier. From the Pooled SF model we see that an increase by 1% in GDP per capita will increase the FEC per capita by 0.47%. As a result, we propose that the decisions on the FEC should be a function of the GDP throughout the years. For instance, since the GDP variable is statistical significant, we propose that in the case we have a growth by 1% we could let an increase in the FEC at most by 0.47%. In addition, we should also mention that a continuous decrease in the FEC may have a reverse effect on the economic activity.

Thirdly, from our study it turns out that the most significant economic sectors are the Transportation and the Residential sector. This totally makes sense, since these specific sectors are continuously increasing their share in the FEC. Therefore, the EU authorities should focus their future energy investments in these two specific areas (for further information see Policy Contribution).

Moreover, we should highlight the necessity of having energy independent economies (Energy Autonomy). From the empirical study, we see that the amounts of Energy Imports are statistical significant for determining the Energy Demand Frontier and hence, the level of energy efficiency. As stated in Chapter 1, European Union is importing huge amounts of energy products from a concentrated group of third countries. For instance, we saw that EU imports over 30% of its energy products from only a single country. Hence, from our study we conclude, that countries should use their energy amounts as efficient as possible, in order to not only to increase their energy efficiency level but also to reduce their external energy dependence. Furthermore, this would result to the minimization of the threat to different external shocks to energy prices. However, EU in the short term should take the right actions in order to secure its energy supply from the international partners.

Last but not least, from the econometric point of view, we understand the significant differences among the different models. In general, creating the "right" frontier and choosing the appropriate variables is an onerous task. For this reason, in the literature (e.g. energy efficiency, banking efficiency etc.) we see many different models that are estimated in order to obtain the efficiencies for the same frontier. For instance, in our study, we see that the models that do not make any distributional assumption for the inefficiency term, overestimate the one sided error term and therefore they estimate a lower level of energy efficiency. In contrast, assuming a distributional restriction for the inefficiency term (the half normal) we obtain lower levels of inefficiency and hence higher level of energy efficiency.

5.2 Policy Contribution

As stated until now, by using energy more efficiently, Europeans can low their energy bills, reduce their reliance on external suppliers of oil and gas, and help protect the environment. By 2020 (2020 Energy Strategy), the EU aims to reduce its greenhouse gas emissions by at least 20%, increase the share of renewable energy to at least 20% of consumption, and achieve energy savings of 20%. All EU countries must also achieve a 10% share of renewable energy in their transport sector.

In addition, beyond the 2020 strategy, European authorities have launched the 2030 and 2050 Energy Strategy in order to encourage a significant cut in greenhouse gas emissions, an increase in the share of renewable energy consumption, a further increase in energy savings, boosting private investments in early energy efficiency infrastructures as the construction of new electricity networks and investments in low carbon technology.

As a result, in order to achieve these goals European authorities have increased the amount of public funds available for energy efficiency. At the EU level, the European Structural and Investment Funds (ESIF) will allocate €18 billion to energy efficiency in period 2014-2020. In addition, the EU has developed a significant number of other support schemes and funding programmes aiming to businesses, regions and countries successfully implement energy efficiency projects. Nevertheless, it is estimated that an additional €177 billion per year will be necessary over the period 2021-2030 to reach the EU's energy and climate objectives for 2030.

Therefore, we understand the great importance of allocating the energy efficiency funds in an optimum way and giving the right directions to public authorities and private investors in order to allocate their funds in the right economic and energy sectors. Hence, the above conclusions can be a good guidance for them in order to make their energy efficiency decisions.

For instance, from our empirical analysis we arrive to the conclusion that the Residential and the Transportation sector are the most vital parts of the EU energy demand. As a result, we conclude that a high proportion of these available funds should be allocated in investments in infrastructure of the Residential sector (e.g. measures that will enhance the energy efficiency in buildings) and the Transportation sector (e.g. energy efficiency actions in shipping, railway, ports, roads etc.).

5.3 Future Research

From our empirical study, we see that many countries are facing a high level of energy efficiency. For instance, in the Pooled SF model we observed that some countries are presenting an energy efficiency level over 95%. As a result, it would have no qualitative difference if we would define these countries as fully energy efficient.

A solution to this was given by Kumbhakar et al. (2013), where they introduce a new method that is called Zero Inefficiency Stochastic Frontier model (ZISF). According to this method, we can accommodate the presence of both fully efficient and inefficient countries in our sample. In order to achieve this, they propose that some countries would be fully efficient with probability p and some others would be inefficient with probability $1 - p$. This model was based on Lambert (1992) who developed the zero-inflated Poisson model. This kind of application is able to provide us with information not only on the energy efficiency level, but also to give us a probabilistic determination on the efficient and non-efficient countries. As a result, we would be able to assess in which regime (efficient or inefficient) each country belongs.

Another option of our future research is to handle the endogeneity problem in Stochastic Frontier models. Endogeneity problems can arise in stochastic frontier models due to a couple of major reasons: First, the determinants of the cost frontier and the two-sided error term can be correlated. Secondly, the inefficiency term and two-sided error term can be correlated, or in particular, the determinants of the inefficiency can cause this correlation. Endogeneity in a stochastic frontier model would lead to inconsistent parameter estimates, and hence, it would need to be addressed properly. Kutlu (2010) seems to be the first study, which deals with the endogeneity problem in Stochastic Frontier models. In particular, is providing a framework in order to deal with the endogeneity problem in a Battese & Coelli (1992) estimator. Further studies have been introduced by Tran and Tsionas (2013, 2015), where in the first one they propose a GMM variation and in the second they use a copula approach to allow dependence between the inputs and the component error (inefficiency term and the noise term). Moreover, Amsler et al. (2016), allow endogeneity of the inputs with respect to the statistical noise and inefficiency separately.

Last but not least, a part of the current research is to obtain the levels of the persistent and the transient energy efficiency. The persistent part is related to the presence of structural problems in the economy and the transient part is related to different drawbacks that can be changed in the short term. As a result, obtaining information on these two components, it is crucial in order to plan the appropriate energy efficiency policy. As stated in Chapter 2, the last few years several approaches have been proposed. However, in the most cases are proposed some relatively complex econometric approaches that provide separate estimates of these two components. Hence, it turns out that further work should be made in order to provide more straightforward methods. For instance, a solution to this problem was introduced by Filippini and Greene (2016), where they propose a practical full information maximum simulated approach estimator.

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7 Appendix

A. The joint PDF of the Normal and the Half Normal Distribution

According to the distributional assumptions of the Pooled SFM we have the following two equations:

$$f(v_{it}) = \frac{1}{\sqrt{2\pi}\sigma_v} \exp\,(-\frac{{v_{it}}^2}{2\sigma_v}) \tag{A.1}$$

and

$$f(u_{it}) = \frac{\sqrt{2}}{\sqrt{\pi}\sigma_u} \exp\,(-\frac{{u_{it}}^2}{2\sigma_u}) \tag{A.2}$$

where (A.1) and (A.2) are the PDF of v_{it} and u_{it} , respectively. Hence, in line with the assumption that the two error components are distributed independently, the joint PDF is:

$$f(v_{it}, u_{it}) = f(v_{it})\, f(u_{it}) = \frac{1}{\sqrt{2\pi}\sigma_v} \exp\left(-\frac{{v_{it}}^2}{2\sigma_v}\right) \frac{\sqrt{2}}{\sqrt{\pi}\sigma_u} \exp\,(-\frac{{u_{it}}^2}{2\sigma_u}) \tag{A.3}$$

Below, I present analytically, the final form of the equation (A.3).

$$f(v_{it}, u_{it}) = \frac{1}{\sqrt{2\pi}\sigma_v} \frac{\sqrt{2}}{\sqrt{\pi}\sigma_u} \exp\left(-\frac{{v_{it}}^2}{2\sigma_v} - \frac{{u_{it}}^2}{2\sigma_u}\right) \Rightarrow \tag{A.4}$$

$$f(v_{it}, u_{it}) = \frac{1}{\pi\sigma_v\sigma_u} \exp\left(-\frac{{v_{it}}^2}{2\sigma_v} - \frac{{u_{it}}^2}{2\sigma_u}\right) \tag{A.5}$$

Since $\varepsilon_{it} = v_{it} + u_{it}$, we have:

$$f(\varepsilon_{it}, u_{it}) = \frac{1}{\pi\sigma_v\sigma_u} \exp\left(-\frac{(\varepsilon_{it} - u_{it})^2}{2\sigma_v} - \frac{{u_{it}}^2}{2\sigma_u}\right) \tag{A.6}$$

The marginal density function of **ε** is obtained by integrating **u** out of the equation (A.6). In particular, Weinstein (1964) for the first time derived the distribution of the sum of a symmetric normal distribution and a truncated normal distribution. In line with this proof, we have:

$$f(\varepsilon_{it}) = \int_0^{+\infty} f(\varepsilon_{it}, u_{it})\, du = \int_0^{+\infty} \left\{ \frac{1}{\pi \sigma_v \sigma_u} \exp\left(-\frac{(\varepsilon_{it} - u_{it})^2}{2\sigma_v} - \frac{{u_{it}}^2}{2\sigma_u} \right) \right\} du \Rightarrow \tag{A.7}$$

$$f(\varepsilon_{it}) = \frac{2}{\sqrt{2\pi({\sigma_v}^2 + {\sigma_u}^2)}} \exp\left(-\frac{{\varepsilon_{it}}^2}{2({\sigma_v}^2 + {\sigma_u}^2)} \right) \left(1 - \Phi\left(\frac{-\varepsilon_{it} \left(\frac{\sigma_u}{\sigma_v}\right)}{\sqrt{({\sigma_v}^2 + {\sigma_u}^2)}} \right) \right) \Rightarrow \tag{A.8}$$

$$f(\varepsilon_{it}) = \frac{2}{\sigma\sqrt{2\pi}} \exp\left(-\frac{{\varepsilon_{it}}^2}{2\sigma^2} \right) \left(\Phi\left(\frac{\varepsilon_{it}\,(\lambda)}{\sigma} \right) \right) = \frac{2}{\sigma} \varphi\left(\frac{\varepsilon_{it}}{\sigma}\right) \Phi\left(\frac{\varepsilon_{it}\,\lambda}{\sigma}\right) \tag{A.9}$$

where $\sigma = \sqrt{({\sigma_v}^2 + {\sigma_u}^2)}$, $\lambda = \frac{\sigma_u}{\sigma_v}$, φ(.) is the standard normal PDF and Φ(.) is the standard normal cumulative distribution. Hence, the final form is presented in the equation (A.10).

$$f(\varepsilon_{it}) = \frac{2}{\sigma} \varphi\left(\frac{\varepsilon_{it}}{\sigma}\right) \Phi\left(\frac{\varepsilon_{it}\,\lambda}{\sigma}\right) \tag{A.10}$$

B. The Log Likelihood of the Marginal Density Function

In equations (B.1) and (B.2), I present the likelihood function of the marginal density function of residuals **ε**.

$$L(\varepsilon|\beta, \lambda, \sigma) = \prod_{i=1}^{NT} f(\varepsilon_i) = \prod_{i=1}^{NT} \left\{ \left(\frac{2}{\sigma}\right) \varphi\left(\frac{\varepsilon_i}{\sigma}\right) \Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right) \right\} \tag{B.1}$$

$$L(\varepsilon|\beta, \lambda, \sigma) = \prod_{i=1}^{NT} \left\{ \left(\frac{2}{\sigma}\right) \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{{\varepsilon_i}^2}{2\sigma^2} \right) \Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right) \right\} \tag{B.2}$$

Hence, taking the natural logarithm of the equation (A.2), we obtain the log-likelihood function. Below, I present in detail the proof on how we obtain the final form.

$$l(\varepsilon|\beta,\lambda,\sigma) = \ln\left(\prod_{i=1}^{NT} f(\varepsilon_i)\right) = \ln\left(\prod_{i=1}^{NT}\left\{\left(\frac{2}{\sigma}\right)\ \varphi\left(\frac{\varepsilon_i}{\sigma}\right)\ \Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right)\right\}\right) \tag{B.3}$$

$$l(\varepsilon|\beta,\lambda,\sigma) = \ln\left(\prod_{i=1}^{NT}\left\{\left(\frac{2}{\sigma}\right)\ \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{{\varepsilon_i}^2}{2\sigma^2}\right)\ \Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right)\right\}\right) \tag{B.4}$$

$$l(\varepsilon|\beta,\lambda,\sigma) = \sum_{i=1}^{NT}\left\{\ln\left(\frac{2}{\sigma}\right) + \ln\left(\frac{1}{\sqrt{2\pi}}\right) + \left(-\frac{{\varepsilon_i}^2}{2\sigma^2}\right) + \ln\left(\Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right)\right)\right\} \Rightarrow \tag{B.5}$$

$$l(\varepsilon|\beta,\lambda,\sigma) = NT\ln\left(\frac{2}{\sigma}\right) + NT\ln\left(\frac{1}{\sqrt{2\pi}}\right) - \frac{1}{2\sigma^2}\sum_{i=1}^{NT}{\varepsilon_i}^2 + \sum_{i=1}^{NT}\ln\left(\Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right)\right) \Rightarrow \tag{B.6}$$

$$\begin{aligned} l(\varepsilon|\beta,\lambda,\sigma) = {} & NT\ln(2) - NT\ln(\sigma) + NT\ln(1) - NT\ln\left(\sqrt{2\pi}\right) - \frac{1}{2\sigma^2}\sum_{i=1}^{NT}{\varepsilon_i}^2 \\ & + \sum_{i=1}^{NT}\ln\left(\Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right)\right) \Rightarrow \end{aligned} \tag{B.7}$$

$$l(\varepsilon|\beta,\lambda,\sigma) = NT(\ln(2) - \ln(\sqrt{2\pi})) - NT\ln(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{NT}{\varepsilon_i}^2 + \sum_{i=1}^{NT}\ln\left(\Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right)\right) \Rightarrow \tag{B.8}$$

$$l(\varepsilon|\beta,\lambda,\sigma) = NT\left(\frac{\sqrt{2}}{\pi}\right) - NT\ln(\sigma) - \frac{1}{2\sigma^2}\sum_{i=1}^{NT}{\varepsilon_i}^2 + \sum_{i=1}^{NT}\ln\left(\Phi\left(\frac{\lambda\,\varepsilon_i}{\sigma}\right)\right) \tag{B.9}$$

We conclude that equation (B.9), represents the final form of the log-likelihood function of the marginal density function of residuals **ε**. The final form as represented in the above equation, will be used in order to obtain our Maximum Likelihood parameter estimations.

C. First Order Conditions of Log-Likelihood function

The First Order Conditions of the log-likelihood function, consist of the derivates of the function with respect to the parameters **β**, the standard error σ and the parameter λ. Below, we present in detail the FOC of the log-likelihood function.

$$(1)\colon \frac{\partial l}{\partial \sigma} = -NT\ \frac{1}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{NT} {\varepsilon_i}^2 - \frac{\lambda}{\sigma^2} \sum_{i=1}^{NT} \frac{\varphi\left(\frac{\lambda\, \varepsilon_i}{\sigma}\right)}{\Phi\left(\frac{\lambda\, \varepsilon_i}{\sigma}\right)}\ \varepsilon_i \tag{C.1}$$

$$(2)\colon \frac{\partial l}{\partial \lambda} = \frac{1}{\sigma}\ \sum_{i=1}^{NT} \frac{\varphi\left(\frac{\lambda\, \varepsilon_i}{\sigma}\right)}{\Phi\left(\frac{\lambda\, \varepsilon_i}{\sigma}\right)}\ \varepsilon_i \tag{C.2}$$

$$(3)\colon \frac{\partial l}{\partial \boldsymbol{\beta}} = \frac{1}{\sigma^2} \sum_{i=1}^{NT} \varepsilon_i x_i - \frac{\lambda}{\sigma} \sum_{i=1}^{NT} \frac{\varphi\left(\frac{\lambda\, \varepsilon_i}{\sigma}\right)}{\Phi\left(\frac{\lambda\, \varepsilon_i}{\sigma}\right)}\ x_i \tag{C.3}$$

where x_i is a (k x 1) vector consisting of elements of the ith row of data matrix X, $\varphi(.)$ and $\Phi(.)$, are respectively, the standard normal PDF and the standard normal cumulative distribution. Given the equation (C.2) we have that:

$$\frac{\partial l}{\partial \lambda} = \frac{1}{\sigma}\ \sum_{i=1}^{NT} \frac{\varphi\left(\frac{\lambda\, \varepsilon_i}{\sigma}\right)}{\Phi\left(\frac{\lambda\, \varepsilon_i}{\sigma}\right)}\ \varepsilon_i = 0 \tag{C.4}$$

at the optimum. Inserting equation (C.4) in equation (C.1), the Maximum Likelihood estimator for σ is determined through the following equation.

$$\frac{\partial l}{\partial \sigma} = -NT\ \frac{1}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{NT} {\varepsilon_i}^2 = 0 \Rightarrow \tag{C.5}$$

$$\frac{1}{\sigma^3} \sum_{i=1}^{NT} {\varepsilon_i}^2 = NT\ \frac{1}{\sigma} \Rightarrow \tag{C.6}$$

$$\hat{\sigma}^2 = \frac{1}{NT} \sum_{i=1}^{NT} {\varepsilon_i}^2 \tag{C.7}$$

Equation (C.7) determines the final estimator of the variance of the model, which is the usual ML estimator of the residuals variance in a regression model. However, we see that the determination of $\widehat{\boldsymbol{\beta}}$ is not independent of $\hat{\sigma}^2$ of the other equations. Hence, our final equations in order to solve this linear system would be:

$$\hat{\sigma}^2 = \frac{1}{NT} \sum_{i=1}^{NT} {\varepsilon_i}^2 \tag{C.7}$$

$$\frac{1}{\hat{\sigma}^2} \sum_{i=1}^{NT} \varepsilon_i x_i - \frac{\lambda}{\hat{\sigma}} \sum_{i=1}^{NT} \frac{\varphi\left(\frac{\lambda\, \varepsilon_i}{\hat{\sigma}}\right)}{\Phi\left(\frac{\lambda\, \varepsilon_i}{\hat{\sigma}}\right)} x_i = 0 \tag{C.8}$$

According to Aigner et al. (1977), from the above equations, we reach to a system of (k+1) equations that corresponds very closely to the system of first-order equations encountered to the so-called 'Tobit' model[12]. Furthermore, they argue that there are available various algorithms for finding the optimal values of the parameters $\boldsymbol{\beta}, \sigma$ and λ. Most of these, require analytical the first or the second order conditions in addition to the likelihood function itself for their best performance at reasonable cost in terms of computer time.

D. A General Discussion on Stationarity

In statistics, a stationary process is a stochastic process whose unconditional joint probability distribution does not change when is shifted in time. Since stationarity is an assumption underlying many statistical procedures used in time series analysis, non-stationary data is often transformed to become stationary. The most common cause of violation of stationarity is a trend in the mean, which can be due either to the presence of a unit root or of a deterministic trend. As a result, an absence of stationarity can cause unexpected behaviors, like t-ratios not following a t distribution.

In particular, in our study we use as dependent variable the natural logarithm of the Final Energy Consumption per capita. This variable can be considered as a time series data since it is observed in a time sequence, from 1990 to 2016. However, we should highlight the fact that in Stochastic Frontier Literature there is no comment on these types of statistical issues. Moreover, we should point out that in this specific case, our dependent variable is examined as an expenditure/cost of a production function and not as a time series data.

In this section, we present the unit root tests for the dependent variable and the model's residuals. In order to test the stationarity assumption, we use the Augmented Dickey Fuller test. In the next table, we present our final findings.

[12] See Amemiya (1973).

Table: Unit Root Test

Unit Root test on Dependent Variable		
	t	$t < t_{c,a=5\%}$
$ln(e_{it}) = \rho ln(e_{it-1}) + \varepsilon_{it}$ $H_0: \rho = 1$ $H_a: \rho < 1$	-2.131	**TRUE**
Unit Root test on Residuals		
$ln(v_{it}) = \rho ln(v_{it-1}) + \varepsilon_{it}$ $H_0: \rho = 1$ $H_a: \rho < 1$	t	$t < t_{c,a=5\%}$
OLS Model	-7.158	**TRUE**
LSDV Model	-12.113	**TRUE**
STV Model	-14.459	**TRUE**

Augmented Dickey-Fuller Table

	No constant, no trend				Constant, no trend				Constant, trend			
N	**0.01**	**0.025**	**0.05**	**0.10**	**0.01**	**0.025**	**0.05**	**0.10**	**0.01**	**0.025**	**0.05**	**0.10**
25	**-2.66**	**-2.26**	**-1.95**	**-1.60**	**-3.75**	**-3.33**	**-3.00**	**-2.62**	**-4.38**	**-3.95**	**-3.60**	**-3.24**
50	**-2.62**	**-2.25**	**-1.95**	**-1.61**	**-3.58**	**-3.22**	**-2.93**	**-2.60**	**-4.15**	**-3.80**	**-3.50**	**-3.18**
100	**-2.60**	**-2.24**	**-1.95**	**-1.61**	**-3.51**	**-3.17**	**-2.89**	**-2.58**	**-4.04**	**-3.73**	**-3.45**	**-3.15**
250	**-2.58**	**-2.23**	**-1.95**	**-1.61**	**-3.46**	**-3.14**	**-2.88**	**-2.57**	**-3.99**	**-3.69**	**-3.43**	**-3.13**
500	**-2.58**	**-2.23**	**-1.95**	**-1.61**	**-3.44**	**-3.13**	**-2.87**	**-2.57**	**-3.98**	**-3.68**	**-3.42**	**-3.13**
>500	**-2.58**	**-2.23**	**-1.95**	**-1.61**	**-3.43**	**-3.12**	**-2.86**	**-2.57**	**-3.96**	**-3.66**	**-3.41**	**-3.12**