Financial Firms in DSGE Models

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PART I: THE THEORY

Introduction

The failure of mainstream macroeconomics to predict the 2008-2009 crisis turned out to be a major changing point for the field. In the period that followed, macroeconomics as a discipline came under heavy criticism; the most common themes among the criticisms were the fact that conventional macro was over relying on overly mathematized models, the oversimplifying assumptions (most notably, rational expectations) and the efficient market hypothesis. Although most of these criticisms have been addressed numerous times (Wickens, 2009), it was clear to the field that changes to the conventional paradigm were necessary.

That being said, the problem was not identified to be the DSGE methodology per se or any of its fundamental assumptions, but rather the breadth and scope of the DSGE models. To be more precise, macroeconomists came to the realization that the major weakness in the models they had been using was the failure to allocate an important role to financial shocks and frictions.

Although up until the crisis a relatively big body of literature had been produced trying to combine the well established foundations of the financial frictions from the microeconomic literature with the general equilibrium models, the framework had not been part of the conventional models proposed by the scholars and employed by policy makers.

Arguably, however, up until 2008, economists had enough reasons to not explicitly allow for financial frictions in their models. The evidence until the crisis had shown that the conventional view on the sources of business fluctuations was enough to explain real activity. And the well accepted theoretical findings from the financial literature pointed to the same direction; in the competitive equilibrium growth models the Modigliani-Miller theorem applies at all times. The firm's ownership and capital structures are irrelevant and so are financial institutions and wealth distribution. In other words, there was no incentive or apparent reason for the challenging of the fundamental assumptions about the role of the financial sector. The broadly accepted hypothesis was that the financial problems had long ceased to be an integral part of the macroeconomy..

Upon the realization that the financial sector not only played a role in the economy through the already widely known to economists amplification/propagation mechanism proposed by Bernanke (1989) but it could also be the source of real fluctuations, it became apparent that a new framework was needed, one that does not stay silent about the interconnection between the real variables and the regular cyclical movements of financial variables such as balance sheet positions, liquidity ratios, and bank credit, documented by a number of economists.

One of the most popular responses to the need to model an active financial sector was the development of the field of behavioral finance. The main theme here is the effort to incorporate irrational financial behavior, ‘the animal spirits’, into the general equilibrium of the economy. The introductory book of Shiller and Akerloff (2009) provides a dense exposition of this view and also a growing body of literature has been developing. The behavioral finance framework however is more qualitative than quantitative. It lacks the precision of the mainstream approach and its failure to provide fundamental quantitative assumptions about the behavior of the individuals make it inadequate for macroeconomic forecasting.

On the other hand, far more widely accepted in the scholarly cycles have been, as mentioned before, the developments within the DSGE framework. From 2008 onwards, a large number of papers has been produced which examine the interactions of a financial sector where the Modigliani-Miller theorem assumptions are being relaxed with the real sector, within the conventional framework. This new body of work, building on already preexisting but not widely used ideas, seems to yield significant results. Notably, a model by Del Negro et al (2013) which features a combination of the Bernanke et al (1999) (from here on referred as BGG) ‘financial accelerator’ in a Smets Wouters (2007) economy showed that the 2008 crisis could have been predicted by a standard DSGE model.

Within this emerging framework, significant efforts have been dedicated in trying to model an active banking sector. While most financial frictions models assign a passive role to the banks (if they even have one) with their asset side being the focal point, models with active financial intermediaries focus on the liabilities side of their balance sheet and are capable of upgrading the financial sector from a propagation mechanism to an independent source of business fluctuations.

Following these observations, the purpose of this thesis is twofold. The first is to provide a compact survey of the main modeling modeling techniques of the banking sector. The second is to study the relation between nominal and financial frictions by simulating a modified New Keynesian version of a well known model featuring a double financial friction and liquidity problems. Our results indicate that the combination of interbank frictions, liquidity problems and nominal frictions lead to no significant differences in the responses of the variables, suggesting a disconnection between nominal and financial frictions for the type of financial sector we are considering.

Financial Frictions - The General Framework

The core problem addressed in the financial frictions literature is the identification of the mechanisms, i.e. the mechanics of the financial markets, behind the process of matching savers with investors and their macroeconomic impact. The problem is considerably complex and our understanding of it is still considered to be limited. But the problem is not merely conceptual. Many times, when a financial mechanism is revealed and understood, the practical burdens the scholars have to overcome are huge. The resulting mix between the conventional DSGE framework and the complex or even sometimes chaotic phenomena observed in the financial markets, leads to highly non linear models and to computationally cumbersome processes. Even with these difficulties however, the advances in the field have extremely significant implications for both policy makers and researchers.

Efforts to address this core problem have been made numerous times in the past, dating from Fisher's work on the debt deflation hypothesis and Keynes' 'animal spirits' in the '30's to Friedman and Schwarz's work on the causes of the Great Depression, in the '60's and Kindleberger's work on the history of financial crises in the '70's. Financial crises, being, as already mentioned, inherently non linear and chaotic phenomena however, are difficult to quantitavely model, especially in a general equilibrium context. It is not surprising therefore, that despite this for decades existing significant volume of work, only after the relatively recent advance of our knowledge of the macroeconomy and of the refinement of our mathematical and computational tools, have there been any serious attempts to formalize the interconnection between financial factors and real activity.

The first attempts to create an explicit quantitative model of financial factors were in the late '80's with the papers from Diamond (1982), Bernanke and Blinder (1988) and the seminal paper of Bernanke and Gertler (1989) which introduced a leverage constraint that the entrepreneurs are facing when borrowing from households in an OLG model. The 1989 concept was further expanded in the Bernanke, Gertler, Gilchrist paper of 1999 where they developed and incorporated the concept of the ‘financial accelerator’ in a dynamic New Keynesian model, which was later to become the dominant paradigm in this branch of literature.

Naturally, as the DSGE methodology with its emphasis on microfoundations had by that time already been enough developed, the major question the researchers had to answer was which were appropriate microfoundations, that would enable them to reach a sufficient level of abstraction without the model becoming intractable. The main pool from which the researchers drew their insights was the work on asymmetric information and moral hazard, which ultimately served as the source for the vast majority of the financial frictions microfoundations.

Next, we briefly examine the main microfoundations and provide a brief exposition of its incorporation in a general equilibrium model.

ASYMMETRIC INFORMATION AND THE FINANCIAL ACCELERATOR

Beginning with the groundbreaking work of Akerlof (1970), the body of literature on asymmetric information has provided the main toolkit in the task of formalizing credit market imperfections. By assuming imperfect information between borrowers and lenders, the relaxation of the Modigliani-Miller theorem was allowed thus paving the way for the investigation of the role that financial frictions play in the amplification of shocks to economies.

Broadly speaking, the asymmetric information problem arises when decision making is delegated by the lender to the borrower. The tensions that may arise from this interaction, such as the borrower’s behavior being influenced by the terms of the loan contract and/or the inability of the lender to monitor the risk the borrower is taking on, give rise to agency costs.

The informational asymmetry within our context is conveniently and usually expressed with credit rationing models. Here, papers from the micro literature have had considerable influence on the development of a rigorous framework. Notable and influential works on credit rationing are these from Jafee and Russell (1976), Keeton (1979) and Stiglitz and Weiss (1981).

However, the exact mechanisms of credit rationing emphasized by different authors may vary significantly. It follows therefore that the literature offers different microfoundations for different financial frictions. To illustrate, in Stiglitz and Weiss (1981) for example borrowers have more information about the payoff volatility of their project. Due to limited liability, lenders lose from lending to applicants with high volatility projects and win from the ones with low volatility. As they increase the interest rate the low volatility borrowers stop applying and the pool of applicants worsens. Likewise, Hart and Moore (1994) opened the door for models with incomplete contracts. When payments in certain states of the world are not exactly specified, debtors and financiers will try to renegotiate their obligations in the future to their favor.

The by far most widely used microfoundation for financial frictions however is the Costly State Verification (CSV) problem introduced by Townsend in 1978. This elegant and simple framework was used by Gertler and Bernanke in their 1989 paper to draw a connection between the borrower's balance sheet and the agency costs he is incurring. The concept was further developed in the 1999 paper of Bernanke, Gertler and Gilchrist which used the CSV framework in a dynamic New Keynesian model. Considering its importance for the development of the field, to motivate a better understanding of the financial frictions mechanisms, we next provide a brief exposition of the BGG (1999) model.

The Financial Accelerator

Bernanke et al consider an economy with three agents; households, entrepreneurs and retailers. Retailers, having monopoly power in price setting, are auxiliary, merely acting as the source of nominal frictions. The financial friction arises in the interaction between entrepreneurs and households. Households live forever working, consuming and saving.

The key to the model are the entrepreneurs. They are risk neutral and produce wholesale goods subject to an idiosyncratic technology shock. To finance their goods production, entrepreneurs will invest out of their own wealth and will borrow from household subject to frictions.

Each individual entrepreneur purchases at the start of each period t at price Q_t, k_{t+1} capital to use it in the period $t+1$. Subject to the shock, the entrepreneur's investment yields a return of $w R_{kt+1}$ where w is the i.i.d idiosyncratic shock with c.d.f G and $E(w)=1$. The return to capital is endogenized and in equilibrium is denoted by R_{kt+1}.

To purchase capital of value $Q_t k_{t+1}$ therefore, an entrepreneur of net worth n_t has to borrow $Q_t k_{t+1} - n_t$ from households. Under the CSV framework, the entrepreneurial return $w R_{kt+1}$ is only observable to the lenders at a verification cost of a fraction $\mu \in (0,1)$ of the amount extracted from the entrepreneurs.

We define a cutoff rate $\bar{w}$ as the threshold value the shock has to take for default. To be more specific, for the amount $Q_t k_{t+1} - n_t$ that the entrepreneur borrows from households, he promises to repay $\bar{w}_t R_{kt+1} Q_t k_{t+1}$ for realization $w \geq \bar{w}_t$ while for realizations $w < \bar{w}_t$ he will be audited and his creditors will receive his pay off $w_t R_{kt+1} Q_t k_{t+1}$ net of the auditing costs $\mu w_t R_{kt+1} Q_t k_{t+1}$ they have to incur.

Under these conditions, the entrepreneur’s maximization problem is the following:

$$max \int_{\bar{w}_t}^{\infty} (w - \bar{w}_t) d\, G(w) R_{kt+1} Q_t k_{t+1}$$

subject to the the lender’s break even condition:

$$[(1-\mu)\int_{0}^{\bar{w}_t} w\, dG(w) + (1 - G(\bar{w}_t)\bar{w}_t)] R_{kt+1} Q_t k_{t+1} = R_{t+1}(Q_t k_{t+1} - n_t)$$

where R_{t+1} is the risk free rate.

The solution to the problem yields an expression for the optimal leverage ψ :

$$Q_t k_{t+1} = \psi\left(\frac{E[R_{kt+1}]}{R_{t+1}}\right) n_t$$

which describes the link between capital expenditures and financial conditions.

The return on capital in general equilibrium is given by:

$$E[R_{kt+1}] = E\left[\frac{A_{t+1} f'(K_{t+1}) + Q_{t+1} L\left(\frac{I_{t+1}}{K_{t+1}}\right) - \frac{I_{t+1}}{K_{t+1}}}{Q_t}\right]$$

where $L()$ is the adjustment cost function capturing the technological illiquidity of capital and adding an additional amplification effect.

In this framework shocks to the entrepreneur’s aggregate net worth n_t should affect his capital holdings $Q_t k_{t+1}$ in the next period and therefore also his net worth, initializing a domino effect and thus affecting the net worth in periods n_{t+2}, n_{t+3} etc. The shock is further amplified through the endogenous reaction in the price of capital Q_t which leads to a further drop in net worth.

The model presented here is the dominant paradigm of the ‘financial accelerator’ mechanism which appears in many varieties and serves as the benchmark in a big part of the literature on financial frictions, as we shall later see.

Banking Frictions: The General Framework

Most of the work produced on frictions till at least 2010 focused on problems appearing in the non financial firm's balance sheets with approaches largely based on the financial accelerator mechanism. In other words, the reference point in the literature was a crisis in the demand of credit, while the exact mechanisms of supplying credit were treated as a veil. As it has already been mentioned, the crisis of 2008 altered this situation. It was realized that for a model to be able to better capture the financial phenomena, there need to be shocks that originate in the financial intermediation process itself, that is it must be allowed for shocks which originate in the supply of credit. Since then, and especially after 2010, a significant volume of literature has been focusing on models which are deviating from the traditional banking channel view and they are rather featuring a market based banking sector.

The traditional view of the bank lending channel, as presented below, gives a central role to the indispensability of deposits as a source of funding for the lending of commercial banks, subject to the country specific legal reserve requirements. It assumes that a monetary policy tightening raises the opportunity cost of holding deposits, which in turn leads banks to reduce lending on account of the relative fall in funding sources. In other words, it contends that after a monetary policy tightening, banks are forced to reduce their loan portfolio due to a decline in total reservable bank deposits. The relevance of this view however depends on the validity of the assumption it is making; and these assumptions were challenged however by the rapid institutional changes in the banking sector of the '80's and the '90's.

In this context, the presence of a market based financial system, that is a financial system in which intermediaries fund themselves by selling securities in competitive markets, rather than collecting deposits subject to reserve requirements, provides the foundations for more realistic and relevant models. The importance of frictions in a market based interbank market is best understood if first considered within a more general and simplified framework that for reasons of tractability abstracts from first principles. Therefore, the frame of reference in the subsequent analysis will be the textbook IS-LM model. After a brief exposition of the implications the interbank frictions have on the macro economy, it will be conceptually easier to understand the way the more involved modeling techniques of the banking system fit into the general equilibrium.

In the simplified model of the economy it is conventional to use only one interest rate. The standard narrative is that ultimate savers lend directly to ultimate borrowers such that the interest rate they are both facing are equated. To allow for frictions in general, multiple interest rates have to be introduced. The same applies for interbank frictions.

To create a framework with multiple interest rates we first allow for the existence of a financial intermediary in the economy such that we can replace direct contract between borrowers and lenders with savers who fund the intermediaries who use the funds to lend to the ultimate borrowers. Now, we can distinguish between the interest rate at which the intermediary funds himself and the interest rate he charges on the borrowers. Hence, now the supply and demand for funds are functions of two distinct interest rates. The spread between the two interest rates should be consistent with a unique volume of intermediation given the funding supply and loan demand curves.

To become more explicit, we next distinguish between the supply and the demand for intermediation. The demand for intermediation indicates the degree to which borrowers are willing to pay an interest rate higher than the one required in order to induce savers to supply funds to finance someone else’s expenditure. Symmetrically, the supply for intermediation indicates the credit spread required to induce financial institutions to intermediate a certain volume of credit between savers and ultimate borrowers and is thus the outcome of profit maximization of the intermediaries. As it was mentioned earlier, the focus of the latter models has been in trying to understand the supply side of financial intermediation and we shall thus focus on this;

The most important determinant of the supply of intermediated funds and the focal point of most of the relevant models is the capital of the intermediaries. Another important factor is the leverage ratio which imposes constraints on the intermediaries by placing a limit on the size of the losses that the intermediary would be subject to in bad states of the world, relative to its capital.

More importantly, shifts in the supply of intermediation can act as an additional source of variation in aggregate demand. An important source of problems within this context is the capital dependent nature of financial intermediation, which means that otherwise insignificant shocks can have important macroeconomic implications. This proved to be the case in the 2008-

2009 crisis when the capital losses were concentrated on highly leveraged individuals which in turn led to large amplification effects of the shock to the real economy.

Policy Implications

Before we proceed to the presentation the financial accelerator mechanism embedded in an interbank market, we first consider the importance of the framework to policy making. As it is understandable, the framework presented here has important implications not only for the understanding of fluctuations but also for the conduct of monetary policy. The notion that credit market imperfections, as sketched above, may lead to considerably more potent reactions to changes in the monetary policy was identified by Bernanke and Gertler in their 1995 paper.

Stating the complete ignorance of the field regarding the mechanisms behind the channels that transmit monetary monetary shocks in the real economy, Bernanke et al (1995) are trying to connect in a qualitative way the existing literature on imperfect information by introducing the concept of the ‘credit channel’ of monetary transmission.

With the ‘credit channel’ the main mechanism they are trying to conceptualize are the amplification effects caused by endogenous changes in the external finance premium, which is the difference in cost between funds raised externally (by issuing equity or debt) and funds generated internally (by retaining earnings). According to advocates of the credit channel, monetary policy affects not only the general level of interest rates, but also the size of the external finance premium.

The credit channel helps resolve the shortcomings of the conventional analysis of monetary policy. For example, it can explain why purchases of long lived assets such as housing, normally responsive to changes in the long term interest rates, can be seen to react so swiftly to changes in monetary policy which is assumed to have a strong effect only on short term rates. The solution seems to lie in the fact that these purchases are directly linked to consumer’s balance sheets.

Specifically, two linkages are proposed between the actions of the monetary authority and the external finance premium. The first is the balance-sheet channel which stresses the potential impact of changes in monetary policy on borrowers' balance sheets and income statements, including variables such as borrowers' net worth, cash flow and liquid assets. The second linkage, the bank lending channel, focuses more narrowly on the possible effect of monetary policy actions on the supply of loans by depository institutions. These two channels together form the credit channel.

In particular, the balance sheet channel ties the wedge to the borrower's financial condition. The better the borrower's financial condition, the lower the external finance premium that he is facing should be. The balance sheet channel of monetary policy arises because shifts in Fed policy affect not only the market interest rates per se but also the financial positions of borrowers, both directly and indirectly. For example, rising interest rates induced by a tightening of the monetary policy, should weaken the borrower's balance sheet through a reduction of their net cash flows and through falling asset prices. The essence of this mechanism is captured by the BGG model.

The 'bank lending' channel works in a similar way. The assumption here is that monetary policy may affect the external finance premium by shifting the supply of intermediated credit. Underlying this channel is the idea that certain classes of borrowers-those for whom the added costs of finance induced by incentive problems are large relative to their funding needs-may find it prohibitively expensive to obtain financing by directly issuing securities on the open market. Financial intermediaries help overcome this friction by exploiting scale economies in the evaluation and monitoring of borrowers.

The emphasis is then on the special nature of bank credit and the role of banks in the economy's financial structure. Commercial banks in particular play an indispensable role as sources of credit for certain kinds of borrowers. Deposits similarly were held to be the main source of funding for the lending of commercial banks and were subject to the legal reserve requirements of the respective country. Key then to the bank lending channel is the lack of close substitutes for deposit liabilities on the liability side of the banking sector's balance sheet and the lack of close substitutes for bank credit on the part of borrowers. In this context, the question is through which mechanisms monetary policy affects the supply of loans.

The bank lending channel was cast aside by mainstream research until recently, starting with Bernanke and Gertler. It was considered to be narrower in its focus and thus less useful than the balance sheet channel. An additional weakness of the bank lending channel was that it was prone to institutional changes. Not surprisingly therefore, with the rapid financial innovations of the '80's and '90's, with the most important being the development of securitization which led non bank financial intermediaries becoming more important sources of credit, its assumptions started to look less and less defensible. The crisis however generated a new stream of research which increasingly focused on the importance of the channel.

As we saw, the new conceptual framework of a market based interbank market presented leads to the natural conclusion that decisions about the interest rate setting should take into account the interest rate spread. Moreover, the framework allows for the introduction of unconventional monetary policy tools in extreme crisis events when the traditional interest policy is adequate to reply to a disturbance in credit supply. As we shall later see, the way the Central Bank can intervene can either be in the form of extension of credit to intermediaries on easier terms than it is available in the private sector or it can take the form of direct purchase of debt claims issued by private borrowers, such that the total credit available to the private sector exceeds the size of the intermediary balance sheets.

PART II: MODELING THE BANKING SECTOR

Having examined the broad implications of an imperfect banking sector, we can proceed to a more detailed presentation of the mechanisms causing them. Broadly speaking, the main mechanisms are very similar to the BGG paradigm. Informational asymmetries provide the main motivation, although they are taking various forms depending on the structure of the model as we shall see. The focal point here is the financial intermediary's liability side of the balance sheet.

To model the balance sheet of the intermediary, a representative agent approach suffices. However, in the financial sector intra agent interactions create their own distinct dynamics. We should therefore broaden our analytical scope to include models with heterogeneous banks. The introduction of trading between agents provides a more realistic view of the world with a trade off however the loss of tractability. Later, we shall base on this observation the main division in our presentation between two types of models; those with a representative bank and those with heterogeneous banks. Purpose here is to provide a compact overview of the main ideas for modeling a market based interbank market. Accordingly, we provide only the relationships necessary for the understanding of the financial frictions mechanisms.

Four models are presented, two with a representative bank and two with heterogeneous banks. Each model was chosen based on the feature it tries to capture. We first consider two representative bank models with the friction appearing on the liability side. The first model captures the idea of how a more complex incentive constraint structure can be integrated in a general equilibrium environment. The second model despite the fact that is using a significantly simpler friction source, is important in its contribution at introducing a framework for a comprehensive interaction between the central bank and the banking sector.

Next, we turn to two models with heterogeneous banks; the first introduces a complicated structure of the banking sector although it does not become explicit in its modeling of frictions. The second features a banking sector considerably more simplified but it provides a way of explicitly modeling asymmetric information problems between the banks.

All of the models presented below are featuring as a general setup a standard New Keynesian model (see Part III for reference), which for reasons of compactness is presented in section 3. Throughout we try to keep the notation used is the same as in the original papers.

Representative Bank

The main theme here is trying to quantitatively capture the hypothesis that a drop in bank net worth causes a rise in interest rate and a drop in investment and intermediation. In the next two models, the friction arises in the relationships between the intermediaries and the borrowing or lending end of the model.

MODELING THE BALANCE SHEET

Meh and Moran(2009) provide a framework where the bank's capital is endogenized to solve the standard asymmetric information problem between the bankers and their creditors, thus giving rise to a bank capital channel of transmission incorporated in a medium-scale version of the baseline New Keynesian model.

By transferring the source of frictions to the endogenously determined balance-sheet of the banks, the financial sector seizes to be a mere amplification mechanism to the shocks of the ultimate borrowers or lenders. Rather, the financial sector may now act as the source of the shock, as we shall see. Next, we provide a review of the model. Throughout we abstract significantly from describing the otherwise standard features of the model, focusing instead on the modeling of the banking sector.

First we consider the general environment of the model; there are three classes of economic agents : households, entrepreneurs, and bankers. Also, there are firms and a monetary authority, but their role is only auxiliary in the context of the model.

The households in the model have the typical New Keynesian features; they consume, allocate their money holdings between currency and bank deposits, supply units of specialized labor, choose a capital utilization rate, and purchase capital goods. They also have habit formation in their utility and hold money.

The wage setting is typical; households provide differentiated labor to perfectly competitive labor aggregators who make zero profits, at differentiated wage levels. They set their wages a la Calvo with a fraction of them reoptimizing and the rest indexing to last period's inflation.

Regarding the production sector, there are 3 types of goods in the economy; final goods, intermediate goods and capital goods. Final goods are assembled by competitive firms using the intermediate goods as input with a standard Dixit Stiglitz aggregation technology. Intermediate goods are produced by monopolistically competitive firms facing nominal rigidities in price setting using labor services not only from the households but also from both entrepreneurs and bankers, allowing in this way these agents to always have non-zero wealth to pledge in the financial contract. The price setting follows the standard Calvo scheme, i.e. each period, a firm receives the signal to reoptimize its price with probability $1-\theta_p$; it follows that with probability θ_p the firm simply indexes its price to last period's aggregate inflation. Their production function takes then the following form:

$$y_{it}=A_t k_{it}^{a} l_{it}^{v_k} l_{it}^{e\,v_e} l_{it}^{b\,v_b}$$

where l_{it}^{e} and l_{it}^{b} represent labor committed by entrepreneurs and bankers and $v_k+v_e+v_b=1-a$ are the labor shares.

Key in the production sector are the entrepreneurs who are producing capital goods. Their behavior and their financing decisions provide the setting for the first moral hazard problem in the model. To be more specific, entrepreneurs may borrow from the banks to undertake a project. Similarly to BGG, they have access to a technology producing capital goods subject to idiosyncratic shocks which yields zero production if it fails and $R_k i_t$ output for a volume of investment i_t if it succeeds. Additionally, entrepreneurs can gain private benefits from the production process. Moreover, there are three different projects available to entrepreneurs each of them producing the same public return R_k but with different probabilities of success; one with high probability of success a^g but with very low private benefits for the entrepreneur, one with lower probability of success $a^b<a^g$ but with private benefits proportional to the project size by a factor b and a third project with a^b probability of success again but with a larger proportionality factor $B>b$ in private benefits.

Banks can monitor the entrepreneurs but inefficiently. They have access to an imperfect monitoring technology which allows them to detect only the third project. Even when monitored therefore, there is nothing preventing the entrepreneur from choosing the second project. The optimal contract between the bank and the entrepreneur must therefore ensure that the the project with the high probability of success will be picked.

The second moral-hazard problem in the model arises within the context of the banker's function as the intermediary of funds from the investors to the entrepreneurs. In particular, investors lack the ability to monitor entrepreneurs so they deposit funds at banks and delegate the task of monitoring entrepreneurs to their bank. The problem here is not that the banker may abscond funds as is the conventional way of modeling frictions. Instead, it is that bankers may exert too little effort to make sure that assets under management are invested wisely.

To become more explicit, to prevent the entrepreneur from choosing the bad projects, the bank must pay a monitoring cost μ , proportional to the project size. Banks may therefore not properly monitor, because this activity is not costly and not publicly observable, and any resulting risk in their loan portfolio would be mostly borne by investors. Bankers must then be given an incentive to exert the efficient amount of effort. As a result, investors require banks to invest their own net worth (their capital) in entrepreneurs' projects. The moral hazard problem is mitigated when banks invest their own net worth (their capital) in entrepreneurial projects, so that they also have a lot to lose from loan default.

Next, we proceed with the formal representation of the problem. An entrepreneur of net worth n_t^e wishing to undertake a project of size i_t has to borrow from the intermediary $i_t - n_t^e$. The bank provides the funding by combining from its own funds and from the investors. The entrepreneur seeks an optimal contract such that his returns will be maximized subject to incentive, participation and feasibility constraints.

We let d_t represent the deposits i.e. the real value of funds from the investors, Q_t the price of capital produced by entrepreneurs and n_t^b the net worth of the bank. Then, under these terms, the objective function of an entrepreneur seeking to maximize the value of his contract is:

$$\max_{i_t, n_t R_{kt}^e, R_{kt}^b, R_{kt}^h} Q_t a^g R_{kt}^e i_t \quad (1)$$

subject to:

$$Q_t a^g R_{kt}^e i_t \geq Q_t a^b R_{kt}^e i_t + Q_t b i_t \ (2)$$

$$Q_t a^g R_{kt}^b i_t - \mu i_t \geq Q_t a^b R_{kt}^b i_t \ (3)$$

$$Q_t a^g R_{kt}^b i_t \geq (1 + R_t^n) n_t^b \ (4)$$

$$Q_t a^g R_{kt}^b i_t \geq (1 + R_t^d) d_t \ (5)$$

$$n_t^b + d_t - \mu i_t \geq i_t - n_t^e \ (6)$$

$$R_{kt}^e + R_{kt}^b + R_{kt}^h = R_k \ (7)$$

The interpretation of the constraints is straightforward. The first constraint (equation 2) ensures that the expected return for the entrepreneur R_{kt}^e from choosing the good project should be higher than the return from choosing the second project plus the private benefits that it would yield. Equation 3 ensures that the expected return for the bank R_{kt}^b when monitoring is at least as high as the return if not monitoring. Equations 4 and 5 are the participation constraints for the bank and the household respectively; 4 ensures that the return for the bank by investing in the project should be higher than the market determined return from investing its net worth n_t^b at the market determined rate R_t^n . Similarly equation 5 ensures the same for the household. Expression 6 ensures that the bank's loanable funds, net of monitoring, must cover the entrepreneur's financing needs and finally the last constraint indicates that the returns distributed to the economic agents should equal the total return.

In equilibrium constraints 2 and 3 bind to equality. After substituting 2 and 3 into 7, solving 7 for R_{kt}^h and plugging the resulting expression in the household's participation constraint 5, the following expression is yielded:

$$(1+R_t^d)\left(\frac{d_t}{i_t}\right)=Q_t a^g\left(R_k-\frac{b}{\Delta a}-\Delta a\frac{\mu}{Q_t}\right) \quad (8)$$

where R_t^d is the market-determined return the investing households earn, d_t are the deposits i.e. the real value of funds from the investors, i_t is the investment size, Q_t the price of capital produced by entrepreneurs, a^g the project's probability of success, $\Delta a \equiv a^g - a^b > 0$ with a^b being the probability of success on most dangerous projects and the whole term in the RHS parenthesis represents the returns on the project credibly promised to the households.

Proceeding, solving for i_t and using 6 to eliminate d_t the total project size is given by:

$$i_t=\frac{n_t^b+n_t^e}{1+\mu-\frac{Q_t a^g}{1+R_t^d}\left(R_k-\frac{b}{\Delta a}-\frac{\mu}{\Delta a\, Q_t}\right)}=\frac{a_t+n_t}{G_t} \quad (9)$$

where $1/G_t$ is the leverage achieved by the financial contract over the combined net worth of the bank and the entrepreneur, independent of idiosyncratic characteristics of the entrepreneurs.

We see therefore, that the aggregate projects entrepreneurs can undertake in the economy, depend positively on their net worth n_t^e and the net worth the bank pledges in the project n_t^b and negatively on the price of capital Q_t .

Further, we define a capital adequacy ratio as:

$$\kappa_t \equiv \frac{n_t^e}{n_t^e + d_t} \quad (10)$$

This ratio should determine in equilibrium how many funds the banks can attract based on their solvency condition. Using the FOCs and with some algebra the capital adequacy ratio can be expressed as:

$$\kappa_t = \frac{\mu}{\mu + \frac{Q_t \Delta a (1+r_t^a)\left(R - \frac{b}{\Delta a} - \frac{\mu}{\Delta a Q_t}\right)}{1+r_t^d}} \quad (11)$$

Next, to be able to get a sense of the dynamics of this economy, we need to derive expressions for the evolution of the banker's as well as the entrepreneur's net worth. To do this, the following assumptions are imposed to ensure that there is no infinite wealth accumulation; each period, a fraction $1-\tau^e$ of entrepreneurs and $1-\tau^b$ of bankers exit the economy at the end of the period's activities. The new agents replacing them have zero start up funds.

Assuming that the agents are saving all their wealth the beginning of period $t+1$ asset holdings for bankers and entrepreneurs can be expressed as:

$$k_{t+1}^e = \tau^e a^g R_{kt}^e I_t \ (12) \quad \text{and} \quad K_{t+1}^b = \tau^b a^g R_{kt}^b I_t \ (13)$$

Taking into account the fact that banks and entrepreneur can commit their labor to intermediate firms in return of a compensation and using the expressions 12 and 13 for capital, the aggregate levels of entrepreneurial N_t^e and banking net worth N_t^b may be expressed as:

$$N_{t+1}^e = [R_{t+1} + Q_{t+1}(1-\delta)]\tau^e a^g R_{kt}^e \left(\frac{N_t^b + N_t^e}{G_t}\right) + \eta^e w_{t+1}^e \quad (14)$$

$$N_{t+1}^b = [R_{t+1} + Q_{t+1}(1-\delta)]\tau^b a^g R_{kt}^b \left(\frac{N_t^b + N_t^e}{G_t}\right) + \eta^b w_{t+1}^b \quad 15$$

where η^e, η^b represent the population masses of entrepreneurs and bankers respectively, τ^e, τ^b their probability of survival in the next period and w_t is the wage received by the intermediate goods producers.

To demonstrate how the mechanism works, the authors consider the effects of a technology and a monetary shock in this economy. A negative technology shock should first lead to a decrease in the production of intermediate goods which is expected to persist for several periods. This should reduce the rental income from capital as well as the price of capital Q_t which would in turn reduce the depositor's returns as is evident from the FOC. Further, this would decrease deposits. This would in turn lead to a fall in deposits thus meaning that the banks now have to hold more capital per unit of loan they make. Investment must fall reducing earning for banks and entrepreneurs, leading to lower levels of net worth in the next period. This sets the stage for second-round effects on investment in subsequent periods, as the lower levels of bank net worth further reduce the ability of banks to attract loanable funds.

Similarly a monetary policy tightening should lead to a recession. The increase in the interest rate R_t^d increases the cost of loanable funds d_t thus leading the banks to rely more on their own capital and decreasing the ratio d_t/I_t and increasing the capital to loan ratio κ_t . Because bank capital cannot react much initially, bank lending decreases, which also leads to investment decreases. Lower investment causes declines in bank earnings and thus bank capital decreases in subsequent periods, which propagates the negative effects of the shock through time.

ALLOWING FOR AN ACTIVE CENTRAL BANK

The next model we are considering is the 2010 model by Gertler and Karadi which contains a more simplified moral hazard problem than the previous model. Similarly to Meh and Moran, the financial sector here is again modeled via the representative agent approach and the frictions arise as a result of deteriorations in the bank's balance sheet which are disrupting the flow of funds between borrowers and lenders. In modeling the other sectors of the economy, the standard New Keynesian approach is used with the usual rigidities. The model had a significant contribution to the literature by developing a comprehensive framework for unconventional monetary policy.

As before, we first consider the general environment and we are abstracting from the standard features. The economy is populated by five agents; households, financial intermediaries, non-financial goods producers, capital producers, and monopolistically competitive retailers. The structure is similar to the previous model where the retailers are the source of price rigidities.

There is also a central bank which is endowed with the ability to perform both conventional and unconventional monetary policy, which is the distinctive feature of the model.

The households consume, deposit their savings (i.e. they are lending to financial intermediaries) and supply labor. At any moment in the household there are $1-f$ workers who supply labor and f bankers who manage a financial intermediary and transfer its earnings back to the household. Each banker has θ probability to stay a banker in the next period. The objective function of the household allows for habit formation but unlike the previous model, it contains no money balances.

Coming to the production side of the economy, we first consider the intermediate goods firms. They produce using the standard Cobb-Douglas technology in a perfectly competitive market and then sell their products to the retail firms. They acquire capital from the capital producers by obtaining funds from the financial intermediaries. An extra friction is added here by allowing the firms to choose a capital utilization rate U_t . A capital quality shock ξ_t is also allowed for. The production function is thus:

$$y_t = A_t (U_t \xi_t k_t)^a l_t^{1-a} \quad (1)$$

To obtain funds, the firms issue S_t claims equal to the capital it plans to acquire k_{t+1} at price Q_t . Assuming no frictions in the process of obtaining funds from the intermediaries, by perfect arbitrage we are led to the following relation:

$$Q_t S_t = Q_t k_{t+1} \quad (2)$$

The firm earns zero profits in equilibrium and therefore it just pays its ex post return on capital to the intermediary which can be expressed as:

$$R_{kt+1} = \frac{\left[P_{mt}\, a \frac{Y_{t+1}}{\xi_{t+1} k_{t+1}} + (Q_{t+1} - \delta(U_{t+1}))\right] \xi_{t+1}}{Q_t} \quad (3)$$

Capital producing firms operate in a competitive market, buying capital from intermediate goods producing firms, then repairing the depreciated old capital and building new. The friction here is again the neoclassical standard adjustment cost of refurbishing new capital. They then sell the new and the refurbished capital at the price Q_t , thus allowing for the endogenous determinations of the capital price.

The final economic actor in the production sector are the retail firms who simply repackage input from intermediate goods firms using a Dixit Stiglitz technology and set the prices a la Calvo as in the previous model.

Next, we turn to the financial sector of the economy. Financial intermediaries lend funds obtained from households to non financial firms. They also hold long term assets to fund those with short term liabilities. The intermediary's balance sheet is given by:

$$Q_t S_{jt} = N_{jt} + B_{jt} \quad (4)$$

where B_{jt} are the deposits the intermediary holds, or in other words his debt to households, and N_{jt} the intermediary's net worth. Accordingly, $Q_t S_{jt}$ is the total value of claims the intermediary holds against non financial firms. The intermediary earns the stochastic return R_{kt+1} on his assets $Q_t S_{jt}$ and pays interest R_{t+1} on the deposits, both of which are endogenously determined.

Given these, the evolution of the banker's net worth may be described as follows:

$$N_{jt+1} = R_{kt+1} Q_t S_{jt} - R_{t+1} B_{jt} = (R_{kt+1} - R_{t+1}) Q_t S_{jt} + R_{t+1} N_{jt} \quad (5)$$

It follows that the intermediary will choose to operate in period *i* if the discounted for the stochastically discounted values(SD) of the returns the following inequality holds: $R_{kt+i}^{SD} \geq R_{t+i}^{SD}$. If it doesn't hold, then the bank simply chooses invest its funds at the riskless rate and abstains from any investment in the real sector.

Next, we consider the intermediary's optimization problem. The intermediary, having a probability θ of surviving in the next period, maximizes the following objective function:

$$V_{jt} = E_t \sum_{i=0} (1-\theta) \theta^i \beta \Lambda_{t,t+i} N_{jt+i+1} \quad (6)$$

To motivate the existence of frictions, a simple moral hazard problem is introduced. At the beginning of each period the intermediary may choose to divert a fraction λ of the funds available for the project and transfer them to his household. In this case however, the depositors will force the intermediary to bankruptcy and will reclaim the $1-\lambda$ fraction of the assets. Thus, for the smooth operation of the financial market the following incentive constraint is necessary to hold:

$$V_{jt} \geq \lambda Q_t S_{jt} \quad (7)$$

V_{jt} may then be expressed as follows:

$$V_{jt} = v_t Q_t S_{jt} + \eta_t S_{jt} \ (8)$$

where:

$$v_t = E_t[(1-\theta)\beta \Lambda_{t,t+1}(R_{kt+1} - R_{t+1}) + \beta \Lambda_{t,t+1}\theta x_{t,t+1} v_{t+1}] \ (9)$$

$$\eta_t = E_t[(1-\theta) + \beta \Lambda_{t,t+1}\theta z_{t,t+1}\eta_{t+1}] \ (10)$$

where $x_{t,t+i} \equiv \frac{Q_{t+i} S_{jt+i}}{Q_t S_{jt}}$ and $z_{t+i} = \frac{N_{jt+i}}{N_{jt}}$ represent the gross growth rate in assets and in net worth respectively. The variable v_t has the interpretation of the expected discounted marginal gain to the banker of expanding assets $Q_t S_{jt}$ by a unit, holding net worth N_{jt} constant, and while η_t is the expected discounted value of having another unity of N_{jt} , holding S_{jt} constant.

The incentive constraint may then be expressed as:

$$v_t Q_t S_{jt} + \eta_t S_{jt} \geq \lambda Q_t S_{jt} \ (11)$$

Manipulating 11, we can derive the key expression of the model:

$$Q_t S_{jt} = \frac{\eta_t}{\lambda - v_t} N_{jt} = \phi_t N_{jt} \ (12)$$

where ϕ_t is the leverage ratio.

The above expression provides the relationship for the total demand of assets. Roughly speaking, the greater the value of v_t , the greater the opportunity cost of the intermediary being forced into bankruptcy. If v_t increases above λ then the constraint does not bind, since the value of keeping operations alive of the intermediary always exceeds the value of diverting funds. We see that the constraint limits the banker's leverage ratio to the point where the banker's incentive to cheat λ is exactly balanced by the cost v_t , motivating thus an endogenous capital constrain.

We can express the aggregate relationship as: $Q_t S_t = \phi_t N_t$. In the equilibrium variations in the net worth N_t will cause fluctuations in the overall asset demand of the intermediaries.

The distinctive feature of the model however lies, as already mentioned, not in the mechanism it uses to motivate financial frictions, which is relatively unsophisticated and exogenous, but rather in its ability to accommodate an active role for direct central bank intermediation.

Under the assumption that the Central Bank is willing to facilitate lending, we denote the amount of assets it is willing to intermediate as S_{gt} . Then, the total value of intermediated assets should be:

$$Q_t S_t = Q_t S_{pt} + Q_t S_{gt} \ (13)$$

To be more specific, the process of central bank intermediation works as follows; the central bank issues government debt to households (for which government debt and deposits are perfect substitutes) at the riskless rate R_{t+1} . Then it uses the funds to lend to non-financial firms at the market lending rate $R_{k\,t+1}$. Assuming that the central bank is not as efficient as the market in this process, an efficiency cost τ per units supplied is imposed. To guarantee that there are no agency costs arising in the interaction between thee Central Bank and the households it is assumed that the government, unlike the private sector, will always honor its debt. This also guarantees that the government faces no balance sheet constraint.

Assuming that the central bank is willing to finance a fraction ψ of intermediated assets, we can now express the total demand for intermediated assets as:

$$Q_t S_t = \phi_t N_t + \psi_t Q_t S_t = \phi_{ct} N_t \quad (14)$$

where ϕ_t is the leverage ratio for privately intermediated funds as defined above, and where ϕ_{ct} is the leverage ratio for total intermediated funds, public as well as well private which can in turn be expressed as:

$$\phi_{ct} = \frac{1}{1-\psi_t} \phi_\tau \quad (15)$$

In this economy, a negative shock in capital quality ξ should first hit via reducing the effective quantity of capital leading thus to a decrease in the asset prices. This should in turn lead to a weakening of the banker’s balance sheet. The leverage ratio here acts as the amplifier of the shock; the weakening of the balance sheet leads to a decline in asset demand by a multiplicative factor equal to the leverage constraint. This further reduces the asset price Q , further shrinking the balance sheet leading to a vicious cycle effect. Associated with the drop in intermediary capital, a sharp increase in the spread between the expected return on capital and the riskless rate should be observed. The shock hits the real economy by leading to a drop in investment and output as a result. The Central Bank can alleviate the shock by increasing the fraction of intermediated assets ψ_t it controls and thus weakening the amplification effect as it is evident form equation 14.

Heterogeneous Banks

Having seen how the risk of default may be incorporated into a full blown DSGE, we may now proceed to more complex setups by presenting two representative approaches of modeling heterogeneous financial agents. A heterogeneous banking sector allows for more sources of frictions and a more realistic interaction between the economic agents.

The groundwork for the introduction of heterogeneity was laid by Goodhart et al (2009) who in a general setting developed a framework which accounted for heterogeneous households and banks as well as for the existence of money. Other models incorporated variations of this framework into the standard New Keynesian framework. We consider two of these models; a model by Gerali et al (2010) which considers a heterogeneous banking sector enriched with the standard New Keynesian characteristics of adjustment costs, monopolistic competition and stickiness abstracting however from establishing microfounded frictions as the ones studied above. And a model by Hilberg et al (2011) which establishes a distinction between commercial and investment banks.

MONOPOLISTIC BANKS

Gerali et al (2010) develop a framework which allows for interaction between financial and credit markets, on the one hand, and the rest of the economy, on the other. The crucial element in their model lies in recognizing the existence of imperfect market structure within the financial market, which in turn can act as the source of friction. This approach allows for a more flexible interest rate behavior and provides a tractable interpretation of the different speeds at which banks interest rates adjust to changing conditions in money market interest rates. The model provides a good paradigm for further work in modeling interbank relations.

The economy described is populated by patient and impatient households, entrepreneurs, capital goods producers, commercial banks and investment banks. As usual, there is also a central bank which sets the interest rate by following the standard Taylor rule.

We first consider the two types of households in the model. Patient households maximize a standard utility with habit formation, housing services and hours worked and they deposit their funds to banks at the riskless rate. Impatient households maximize the same utility function but with the difference that they have a higher discount rate from both entrepreneurs and the patient households. Also, they are not allowed to hold deposits or own retail firms as the patient households do. Each household type supplies a differentiated variety of labor. For this, there are two unions each of which sets nominal wages for workers of its labor type and then sell it to perfectly competitive “labor packers” who transform them into an homogeneous composite labor input and sell it, in turn, to intermediate-good-producing firms.

Next we turn to the production sector of the economy. It consists, as usual by the entrepreneurs, the retailers and the capital goods producers. There is an infinity of perfectly competitive entrepreneurs of unit mass who care only for their consumption. In order to maximize lifetime consumption, entrepreneurs choose the optimal stock of physical capital, the degree of capacity utilization, the desired amount of labor input and borrowing. They use effective capital and labor as inputs to produce intermediate goods with a Cobb Douglas technology. The entrepreneurs have access to loan contracts offered by banks with which they implement their borrowing decisions. The amount of resources the entrepreneurs are able to obtain through borrowing is constrained by the value of collateral they are able to pledge which is given by their holdings of physical capital.

Retailers operate in monopolistic competition facing quadratic price adjustment costs, thus providing the source of price stickiness. They buy the intermediate good from entrepreneurs at the wholesale price and differentiate the goods at no cost. Each retailer then sells his unique variety at a mark-up over the wholesale price.

Finally, capital goods producers buy at the beginning of each period an amount of final good from the retailers and a stock of depreciated capital from the entrepreneurs. Depreciated capital can be refurbished to new capital at a one-to-one ratio while the transformation of the final good is subject to quadratic adjustment costs. The new capital is then sold back to the entrepreneur at the end of the period.

Next, we consider the demand for loans and deposits in this economy. The assumption is imposed that units of deposit and loan contracts bought by households and entrepreneurs are a composite CES basket of slightly differentiated products -each supplied by a branch of a bank j - with the markups (or markdowns) the banks are charging when setting interest rates being ε_t^d , ε_t^{bH} and ε_t^{bE} , for deposits, household loans and entrepreneurial loans.

The aggregate demand for loans by households at bank j is expressed as:

$$B_{jt}^{H}=\left(\frac{r_{jt}^{bH}}{r_{t}^{bH}}\right)^{-\varepsilon_t^{bH}} B_t^I \text{ (1)}$$

where B_t^I is the aggregate demand for real loans, r_t^{bH} is the average interest rates on loans to households which is defined as:

$$r_t^{bH}=\left[\int_0^1 r_{jt}^{bH\left(1-\varepsilon_t^{bH}\right)} dj\right]^{\frac{1}{1-\varepsilon_t^{bH}}} \text{ (2)}$$

Symmetrically, we can express the demand for loans by the entrepreneurs as well as the household's demand for deposits. Therefore, demand for deposits at bank j is expressed as:

$$d_{jt}^{P}=\left(\frac{r_{jt}^{d}}{r_{t}^{d}}\right)^{-\varepsilon_t^{d}} d_t \text{ (3)}$$

where, like before, d_t is the aggregate demand for deposits by households and r_t^d is the average deposit rate across the economy defined as:

$$r_t^{d}=\left[\int_0^1 r_{jt}^{d\,1-\varepsilon_t^{d}} dj\right]^{\frac{1}{1-\varepsilon_t^{d}}} \text{ (4)}$$

Next, we turn our attention to the financial sector. The financial sector is divided in a wholesale and a retail branch. In the retail branch a monopolistically competitive structure is introduced. In general, the entirety of the banking sector is characterized by the following balance-sheet constraint:

$$B_t=D_t+N_t$$

where B_t are the loans the banking sector makes, D_t the level of deposits and N_t the level of bank equity. Each bank has to adhere to maintaining an optimal leverage ratio ϕ , given exogenously, deviations from which come at a penalty.

The wholesale branch issues wholesale loans (asset side) by using its bank capital (accumulated through retained earnings) and the accumulated deposits (liability side). The assumption of capital accumulation through retained earnings allows for the existence of a feedback loop between the real and the financial side of the economy. A deterioration of the macroeconomic conditions should lead to a negative shock on bank profits thus weakening their ability to raise new capital.

The bank capital accumulation identity may be expressed as (expressed below in nominal terms):

$$N_{jt}^{n} = (1-\delta^{b})\, N_{jt-1}^{n} + w^{b} J_{jt-1}^{b,n} \quad (5)$$

where $1-w^{b}$ is the exogenously fixed dividend policy of the bank and $J_{jt-1}^{b,n}$ are the overall profits made by the branches of bank j .

The objective function the wholesale bank has to maximize over its deposits D_{jt} and loans subject to the balance-sheet constraint presented above is:

$$max\, E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^{p} [(1+R_t^{b}) B_{jt} - (1+R_t^{d}) D_{jt} - N_{jt} - \frac{\kappa_{\kappa b}}{2} (\frac{N_{jt}}{B_{jt}} - \phi)^{2} N_{jt}] \quad (6)$$

where the net wholesale loan rate R_t^{b} and the net loan deposit rate R_t^{d} are taken as given.

The FOC to the maximization of the wholesale bank is:

$$R_t^{b} = R_t^{d} - \kappa_{Kb} (\frac{N_{jt}}{B_{jt}} - \phi) (\frac{N_{jt}}{B_{jt}})^{2} \quad (7)$$

where it is assumed that the net deposits rate is equal to the central bank rate of the depository facility such that $R_t^{d} = r_t$. Imposing a symmetric equilibrium, the equation shows that when there is a considerable spread $S = R_t^{b} - r_t = -\kappa_{Kb} (\frac{N_{jt}}{B_{jt}} - \phi) (\frac{N_{jt}}{B_{jt}})^{2}$ between the loan and the deposits rate, the wholesale bank will choose to increase loans and thus leverage.

Equation 7 highlights the role of capital in determining loan supply condition. As long as there is a spread between the loan rate and the policy rate, the bank is willing to extend as many loans as possible by increasing leverage. The increase in leverage however moves the bank away from the optimal leverage ϕ at a cost. In equilibrium therefore, the marginal cost of reducing the the capital to loans ratio should exactly be matched with the loans to policy rate spread. This is the source of a financial accelerator effect similar to that of BGG. The increase in leverage however increases the distance from the optimal leverage ratio thus reducing profits.

The retail branch operates in a monopolistically competitive environment, behaving thus much like the retail firms in the benchmark New Keynesian framework. It obtains wholesale loans B_{jt} from the wholesale branch at the rate R_t^{b} , differentiates them costlessly and resells them to households and firms applying two distinct interest rate mark ups on the rates the implementation of which has a quadratic adjustment cost.

The problem of the retail bank is then to maximize the following function over the lending rates to entrepreneurs and households:

$$\max_{r^{bH}_{jt},\, r^{bE}_{jt}} = E_0 \sum_{t=0}^{\infty} \lambda^{p}_{0,t} \left[r^{bH}_{jt} B^{bH}_{jt} + r^{bE}_{jt} B^{bE}_{jt} - R^{b}_{t} B_{jt} - \frac{\kappa_{bH}}{2} \left(\frac{r^{bH}_{jt}}{r^{bH}_{jt-1}} - 1 \right)^{2} r^{bH}_{t} B^{H}_{t} - \frac{\kappa_{bE}}{2} \left(\frac{r^{bE}_{jt}}{r^{bE}_{jt-1}} - 1 \right)^{2} r^{bE}_{t} B^{E}_{t} \right] \ (8)$$

subject to the demand schedules:

$$B^{H}_{jt} = \left(\frac{r^{bH}_{jt}}{r^{bH}_{t}} \right)^{-\varepsilon^{bH}_{t}} B^{H}_{t} \ (9) \text{ and } B^{E}_{jt} = \left(\frac{r^{bE}_{jt}}{r^{bE}_{t}} \right)^{-\varepsilon^{bE}_{t}} B^{E}_{t} \ (10)$$

The FOC to its maximization problem yields (with s denoting both households H and entrepreneurs E) :

$$1 - \varepsilon^{bs}_{t} + \varepsilon^{bs}_{t} \frac{R^{b}_{t}}{r^{bs}_{t}} - \kappa_{bs} \left(\frac{r^{bs}_{t}}{r^{bs}_{t-1}} - 1 \right) \frac{r^{bs}_{t}}{r^{bs}_{t-1}} + \beta_{p} E_{t} \left(\frac{\lambda^{p}_{t+1}}{\lambda^{p}_{t}} \kappa_{bs} \left(\frac{r^{bs}_{t}}{r^{bs}_{t-1}} - 1 \right) \left(\frac{r^{bs}_{t}}{r^{bs}_{t-1}} \right)^{2} \frac{B^{E}_{t+1}}{B^{E}_{t}} \right) = 0 \ (11)$$

Apart from repackaging and selling wholesale loans, retail banks also collect funds from households in the form of deposits and then they forward these funds to the wholesale branch. Symmetrically with the loans operation, they take deposits D_t from the wholesale branch at the rate r^{d}_{t} and then at passing the funds to the wholesalers they apply a monopolistic mark down to the policy rate.

Their problem is then to choose r^{d}_{t} in order to maximize the following function:

$$\max_{r^{d}_{jt}} E_0 \sum_{t=0}^{\infty} \lambda^{p}_{0,t} \left[r_t D_{jt} - r^{d}_{jt} d_{jt} - \frac{\kappa_d}{2} \left(\frac{r^{d}_{jt}}{r^{d}_{jt-1}} - 1 \right)^{2} r^{d}_{t} D_t \right] \ (12)$$

subject to the demand for deposits:

$$d_{jt} = \left(\frac{r^{d}_{jt}}{r^{d}_{t}} \right)^{\varepsilon^{d}_{t}} D_t \ (13)$$

The FOC for the optimal deposit rate is then:

$$-1 + \varepsilon^{d}_{t} - \varepsilon^{d}_{t} \frac{r_t}{r^{d}_{t}} - \kappa_d \left(\frac{r^{d}_{t}}{r^{d}_{t-1}} - 1 \right) \frac{r^{d}_{t}}{r^{d}_{t-1}} + \beta_p E_t \left[\left(\frac{\lambda^{p}_{t+1}}{\lambda^{p}_{t}} \kappa_d \left(\frac{r^{d}_{t}}{r^{d}_{t-1}} - 1 \right) \left(\frac{r^{d}_{t}}{r^{d}_{t-1}} \right)^{2} \frac{d_{t+1}}{d_t} \right) \right] = 0 \ (14)$$

Finally, to close the financial sector an expression for the overall profits made by the bank is needed. Overall profits of a bank are the sum of earnings from the wholesale unit and the retail branches (deleting the interbank transactions) which may be expressed as:

$$J_{jt}^{b}=r_{jt}^{bH}B_{jt}^{H}+r_{jt}^{bE}B_{jt}^{E}-r_{jt}^{d}d_{jt}-\frac{\kappa_{\kappa b}}{2}\left(\frac{N_{jt}}{B_{jt}}-\phi\right)^{2}N_{jt}-Adj_{jt}^{B} \quad (15)$$

where Adj_{jt}^{B} indicates adjustment costs for changing interest rates on loans and on deposits.

In this setup, the authors consider as a shock a sudden contraction in bank capital ϕ . To compensate for their loss wholesale banks should increase their deposits D_t thus leading to an increase of the deposits rate. At the same time the bank has to increase its loan rate which in turn leads to a demand fall for loans by households and firms. Impatient households which face a borrowing constraint now have to cut up on consumption and investment which ultimately leads to an output contraction.

INVESTMENT AND COMMERCIAL BANKS

The final model we consider is by Hilberg et al (2011). The model is less involved than Gerali et al but it introduces more explicit frictions in the interaction between the commercial and the investment bank.

As usual, the economic agents in the model are the households, an entrepreneur and the retailer in the real sector and commercial and investment banks in the financial sector. The economy is also populated by a Central Bank, which sets interest rate with a Taylor rule.

Entrepreneurs in the model are perfectly competitive and using input from capital producers, they produce output using a Cobb-Douglas technology sold to the retailers. Each period the entrepreneur buys capital to use it in the next period. He covers the funds needed by borrowing from the financial sector. The amount B_t is expressed as:

$$B_t=Q_t k_{t+1}-N_t \quad (1)$$

To allow for the formation of bubble, a distinction is introduced between the fundamental price of capital Q_t charged by the capital producers and the market price of capital S_t .

The interest rate normally charged by banks on loans is R_t^B . In the model however, following Bernanke et al (1999) in the contract between the banker and the entrepreneur a finance premium may arise. The markup on the interest rate charged on the entrepreneur is given by:

$$R_{t+1}^{S}=\frac{R_t^B}{\Pi_{t+1}}\left(\frac{S_t k_{t+1}}{N_t}\right)^{\psi} \quad (2)$$

where $\frac{S_t k_{t+1}}{N_t}$ is the leverage ration and ψ is the indexation parameter (if it is 0 the interest charged is equal to the real benchmark borrowing rate).

In the fashion of BGG, the evolution of the aggregate net worth position of the entrepreneurs can be expressed as:

$$N_t=\sigma[R_t^s S_{t-1}k_t-(R_t+\frac{\mu\int w\,dF(w)R_t^s S_{t-1}k_t}{S_{t-1}k_t-N_t})(S_{t-1}k_t-N_t)]+(1-a)(1-\Omega)A_t k_t^a H_t^{(1-a)\Omega} \quad (3)$$

where σ is the survival probability of the entrepreneur, w is the default probability of the project the entrepreneur invests in, respectively and μ is the parameter controlling the supervising bank costs and as in BGG(1999) $l_t=H_t^{\Omega}H_t^{e1-\Omega}$ with H_t being the labor committed by the household and H_t^e the labor committed by the entrepreneur.

The relationship between the fundamental price of capital Q and the market price of capital S is given by:

$$R_t^S=R_t^Q(b+(1-b)(1-(1-a)\frac{(S_{t-1}-Q_{t-1})}{S_t})+e_t^{SQ}) \quad (4)$$

where the parameter a determines the speed of convergence back to the fundamental price and $b\equiv a(1-\delta)^2$.

The problem of the capital producers is the standard one as presented in the previous models. The retailers also have the function as in the previous models.

The commercial banks take deposits from households and they borrow an amount IB in the interbank market at a rate R_t^{IB} which they are taking as given from investment banks (liability side) and they make loans of volume B_t to the entrepreneurs (asset side) at a loan rate R_t^B . They can change the interest rate charged subject however to quadratic adjustment costs. Then the bank is maximizing the following objective function over the loan rate R_t^B and the deposit rate R_t^D

$$\Pi_t^{CoB}=\frac{R_{t-1}^B}{\pi_t}B_{jt-1}-\frac{R_{t-1}^D}{\pi_t}D_{jt-1}-\frac{R_{t-1}^{IB}}{\pi_t}IB_{jt-1}-\frac{\kappa_d}{2}(\frac{R_{t-1}^D}{R_{t-2}^D}-1)^2\frac{R_{t-1}^D}{\pi_t}D_{jt-1}-\frac{\kappa_b}{2}(\frac{R_{t-1}^B}{R_{t-2}^B}-1)^2\frac{R_{t-1}^B}{\pi_t}B_{jt-1} \quad (5)$$

Assuming imperfect substitution between deposits and loans of different banks, the problem is then to maximize subject to the following demand equations for deposits and entrepreneurial loans:

$$D_{jt}=\left(\frac{R_{jt}^{D}}{R_{t}^{D}}\right)^{\varepsilon^{D}} D_t \ (6) \quad \text{and} \quad B_{jt}=\left(\frac{R_{jt}^{H}}{R_{t}^{H}}\right)^{\varepsilon^{h}} B_t \ (7)$$

When the commercial bank is making a loan of volume $Q_t K_t - N_t$ to the entrepreneur, the entrepreneurs pledge as collateral the value of the capital they possess measured in the fundamental price $Q_t K_t$. The commercial bank then proceeds to securitize the collateral turning it into Asset Backed Securities (ABS) taking the market value of capital into account, not the fundamental. Moreover the securitization technology is imperfect allowing only a partial process. The value of the securitized loans is then:

$$AB_{jt}^{CoB}=\left(K_{jt}\, E_t\left(S_{t-1}\right)\right)^{\tau}-\left(N_t\right)^{\tau-1} \ (8)$$

Further, a borrowing constraint is introduced in the fashion of Gerali et al. Specifically, when the commercial bank needs to obtain funds in the interbank market from the investment bank, it faces a borrowing constraint which depends on the value of collateral it pledges, which in this case is its ABS portfolio. The constraint the commercial bank faces is the following:

$$R_t^{IB}\, IB_t \leq m_t\, AB_t^{CoB} \ (9)$$

where m_t is the loan-to-value ratio that responds to deviations of the market price of capital from the fundamental price to incorporate the reluctance of “investment banks” to provide interbank loans in the presence of asset price bubbles, following an AR(1) process.

Next, we turn to the investment branch of the banking sector. In the model investment banks are the only agents able to interact with the Central Bank. Their liabilities side consists of Central Bank loans M_{kt}^{D} while the assets side consists of excess reserves X_{kt} and interbank loans IB_{kt} .

The investment banks taking the policy rate R_t as given maximizes with respect to interbank lending and excess reserves. The objective function it maximizes is then:

$$R_t^{IB}\, IB_{kt}-R_t\left(M_{kt}^{D}-X_{kt}\right) \ (10)$$

subject to the bank’s demand for liquidity and Repo loans, as given below.

The investment bank’s demand for liquidity depends on the excess reserves and the interbank loans:

$$M_{kt}=IB_{kt}^{\zeta}X_{kt}^{1-\zeta} \quad (11)$$

The investment bank also faces a constraint when taking out a Repo loan from the central bank which can be expressed as follow:

$$M_{kt}^{D}=G_{kt}+(1-h_t)AB_{kt}^{PD} \quad (12)$$

where M^{D} is the liquidity obtainable by each individual bank and the right hand side involves the two types of assets accepted as collateral by the investment bank in its repo transactions, that is government bonds and ABS's. Government bonds are set such that the demand for liquidity by the investment banks is satisfied.

To close the model of the financial market, we need a characterization of the behavior of the Central Bank. Following the standard stylized facts, the Central Bank balance-sheet sheet consists of government bonds and ABS on the asset side and money in circulation and equity on the liabilities side. Apart from using a standard interest rate rule, the Central Bank, as it became clear in the characterization of the investment bank, also has responsibilities concerning the liquidity of the economy. To 'steer' the liquidity it regulates how much it grants to the investment banks using a haircut rule expressed as:

$$h_t=\rho_h h_{t-1}+cS_t-e_t^{h} \quad (13)$$

Thus, once the interest rate hits the zero lower bound, it is assumed that the CB will attempt to fine tune the economy by regulating the direct liquidity.

The dynamics in an economy of this kind may unfold as follow; a rise in the policy rate induced by the Central Bank would lead to a decline in liquidity demanded by the investment banks and a rise in its desired holdings of excess reserves. This would in turn lead to an increase in the interest rate for interbank loans leading to a decline in demand for interbank loans by the commercial bank. This implies that in response to an asset pricing bubble the policy rate should be risen.

A positive technology shock should have its impact on the banking sector through the CB policy rule which should decrease, meaning that the interbank lending rate decreases as well which in turn leads to higher interbank lending activity.

PART III: A FLEXIBLE FRAMEWORK

In this section we are performing comparative analysis between variations of a model, in order to get insights about the interconnection that nominal frictions have with the financial frictions. To perform this task, we are considering a model by Gertler and Kiyotaki (2011) to which we add some standard nominal frictions.

Broadly speaking, the model tries to combine the modeling techniques and insights of 3 seminal papers in financial frictions. It tries to bring together in a unified framework a financial accelerator a la BGG, a banking sector and a Central Bank a la Gertler, Karadi (2010) and it introduces liquidity problems through agents heterogeneity as Kiyotaki and Moore (1997, 2008). It unifies the work on the bank's balance sheet as an amplification mechanism and on heterogeneous agents accommodating at the same time for numerous unconventional policy measures. An important contribution of the model is that it introduces liquidity problems in a rich banking sector, providing us with significant insights about monetary policy in a much more complicated world than the standard New Keynesian environment.

As every model trying to address a complex phenomenon, the model has its shortcomings. As we'll see its financial accelerator mechanism is considerably unsophisticated and it can not allow for endogenous tightening margins as the benchmark BGG accelerator. The structure of the banking sector also gives rise to significant non linearities which make the process of finding a numerical solution a significantly more difficult task. Moreover, the structure of the real sector of the model abstracts from any nominal rigidities whatosever (without the financial sector the model collapses to a benchmark RBC), failing thus to provide significant insights about the interaction between the real and the financial sector.

The model is well suited for comparative analysis due to the flexibility it offers in parameterizing the degree of frictions in the interbank market and the degree of liquidity problems. In what follows, we shall add to the model nominal frictions as in the models examined above and by parameterizing the degree of frictions, we shall simulate different variations of both the original and our augmented models to derive insights about the interconnection between nominal and financial frictions.

The Model

PHYSICAL ENVIRONMENT

We first consider the general environment in the model. To motivate the existence of liquidity problems Gertler and Kiyotaki employ a Lucas island model setup. They consider an economy with a continuum of islands on each of which investment opportunities arrive randomly each period. Specifically, investment arrives each period iid on a fraction π^i of the islands. Symmetrically on a fraction $\pi^n = 1-\pi^i$ no investment arrives and the production sector on these islands turns instead on refinancing already existing projects.

There is continuum of firms located on a continuum of islands. We assume perfect labor mobility but capital is immobile across islands.

Capital is owned by households in the form of deposits, as it will be explained later, and it follows the following law of motion:

$$K_{t+1} = \psi_{t+1}\pi^n(1-\delta)K_t + \psi_{t+1}[I_t + \pi^i(1-\delta)K_t] = \psi_{t+1}[I_t + (1-\delta)K_t] \quad (1)$$

where ψ_{t+1} is a shock to the quality of capital, following a Markov process and it acts as an exogenous source of variation in the value of capital.

HOUSEHOLDS AND LABOR SUPPLIERS

There is a continuum of households in the economy. They lend to non financial firms via the financial intermediaries and do not own capital directly but rather in the form of deposits. Within each household there are $1-f$ workers, who supply differentiated labor at a real wage and f bankers who manage a financial intermediary and transfer dividends to the household. To avoid an infinite accumulation of funds by the bankers, we assume that a banker exits next period with i.i.d probability $1-\sigma$.

Each household, indexed by i, supplies differentiated labor to a competitive labor supplier who aggregates it. They maximize a lifetime utility function over consumption and labor supplied. They also deposit funds to banks and hold riskless government debt, which are assumed to be perfect substitutes. We allow for habit formation with the parameter h controlling the habit persistence over one period. Households then maximize:

$$E_o \sum \beta^s [(c_{it})^{-1} - (hc_{it-1})^{-1} - \varphi_t \chi \frac{l_{it}^{1+\gamma}}{1+\gamma}] \quad (2)$$

where γ is the inverse of the Frisch labor supply elasticity, χ is the labor disutility parameter and φ is a labor supply shock that follows an AR(1) process,

$$log\varphi_\tau = \rho_\varphi log\varphi_{t-1} + \sigma_\varphi \varepsilon_{\varphi t} \quad (3)$$

The budget constraint of the i-th household is then given by:

$$c_{it} + \frac{d_{it+1}}{p_t} = w_{it} l_{it} + R_{t-1} \frac{d_{it}}{p_t} + \Pi_t + TR_t - T_t \quad (4)$$

where w_{it} is the real wage earned by the household supplying differentiated labor,

$\int_0^1 d_{it} di = D_{At} = D_{gt} + D_t$ is the aggregate quantity of riskless debt held by the households (either as government bonds D_{gt} or deposits D_t which are assumed to be perfect substitutes), R_t is the nominal riskless interest rate, T is a lump sum tax, Π are the dividends that are transferred to the households by the bankers an TR_t is a lump sum government transfer.

Note that the households are not optimizing over capital. That is because they hold capital indirectly in the form of deposits, and the accumulation is possible via the accumulation of funds that the bankers are doing.

The FOCs in the household's consumption savings problem are:

$$\lambda_{it} = (c_{it} - hc_{it-1})^{-1} - h\beta E_t (c_{it+1} - hc_{it})^{-1} \quad (5)$$

$$\lambda_{it} = \beta E_t \lambda_{it+1} \frac{R_t}{\Pi_{t+1}} \quad (6)$$

where λ_{it} denotes the marginal utility of consumption for each household *i* and Π_{t+1} is the inflation rate.

The FOCs with respect to labor are more involved due to wage rigidity. To derive them we first consider the structure of the labor market. The households supply differentiated labor to a competitive labor supplier who aggregates using the standard Dixit – Stiglitz technology:

$$l_t^d = \left(\int_0^1 l_{it}^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{1-\eta}} \quad (7)$$

where η is the elasticity of substitution among different types of labor and l_t^d is the aggregate labor demand.

The labor supplier taking as given differentiated labor wages w_{it} and wage w_t maximizes his profits subject to the Dixit Stiglitz technology. The labor supplier then maximizes the following objective function subject to 7:

$$\max_{l_{it}} w_t l_t^d - \int_0^1 w_{it} l_{it} di \text{ (8)}$$

The solution to his maximization problem yields an expression for input demand and for aggregate wage. Specifically, demand for type i of differentiated labor is given by:

$$l_{it} = \left(\frac{w_{it}}{w_t}\right)^{-\eta} l_t^d \text{ (9)}$$

where w_{it} denotes the differentiated labor wages and w_t is the aggregate wage level given by :

$$w_t = \left(\int_0^1 w_{it}^{1-\eta} di\right)^{\frac{1}{1-\eta}} \text{ (10)}$$

Next, we need an expression for wages. Households set their wages following a la Calvo. In particular, $1-\theta_w$ of households can optimally readjust their wage in each period while the rest of them index their wages to inflation with an indexation parameter $x_w \in [0,1]$. Hence, the relevant labor supply optimization problem of the household is given by:

$$\max_{w_{it}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_w)^{\tau} \left(-\varphi_{\tau} \psi \frac{l_{it+\tau}^{1+\gamma}}{1+\gamma} + \lambda_{it+\tau} \prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{x_w}}{\Pi_{t+s}} w_{it} l_{it+\tau}\right) \text{ (11)}$$

subject to the labor demand for each individual household indexed to inflation:

$$l_{it+\tau} = \left(\prod_{s=1}^{\tau} \frac{\Pi_{t+s-1}^{x_w}}{\Pi_{t+s}} \frac{w_{it}}{w_{t+\tau}}\right)^{-\eta} l_{t+\tau}^d \text{ (12)}$$

Manipulating the FOC of the problem we can express the law of motion of wages it in recursive form (as in Fernandez-Villaverde(2006)):

$$f_t^1 = \left(\frac{\eta-1}{\eta}\right)\left(\check{w}_t^{(1-\eta)} \lambda_t w_t^n l_t^d + \beta\theta_w E_t \left(\frac{\Pi_t^{x_w}}{\Pi_{t+1}}\right)^{1-\eta} \left(\frac{\check{w}_{t+1}}{\check{w}_t}\right)^{\eta-1} f_{t+1}^1\right) \text{ (13)}$$

$$f_t^2 = \psi\varphi_t\left(\left(\frac{\check{w}_t}{w_t}\right)^{\eta(1+\gamma)} (l_t^d)^{1+\gamma} + \beta\theta_w E_t \left(\frac{\Pi_t^{x_w}}{\Pi_{t+1}}\right)^{-\eta(1+\gamma)} \left(\frac{\check{w}_{t+1}}{\check{w}_t}\right)^{\eta(1+\gamma)} f_{t+1}^2\right) \text{ (14)}$$

where $f_1 = f_2 = f$ and $\check{w}$ is the optimally readjusted wage.

To close the problem of the household (having imposed symmetric equilibrium in wages and consumption), we need an expression for the evolution of wages, which is given by:

$$w_t^{1-\eta}=\theta_w\left(\frac{\Pi_{t-1}^{x_w}}{\Pi_t}\right)^{1-\eta} w_{t-1}^{1-\eta}+(1-\theta_w)\check{w}_t^{1-\eta} \quad (15)$$

where, as usual, a fraction θ_w of households index their wages to past inflation while the rest optimally re adjust it.

Finally, we need to derive an expression for the aggregate labor supply l_t . To do this, we integrate over all households i:

$$\int_0^1 l_{it}\, di=\int_0^1 \left(\frac{w_{it}}{w_t}\right)^{-\eta} di\, l_t^d=l_t \quad (16)$$

Defining the wage dispersion term as

$$v_w \equiv \int_0^1 \left(\frac{w_{it}}{w_t}\right)^{-\eta} di \quad (17)$$

we get the following expression for aggregate labor demand:

$$l_t^d=\frac{l_t}{v_w} \quad (18)$$

NON FINANCIAL FIRMS

Next, we consider the production sector of the economy. We assume a goods market with the following structure: four economic agents operate in it, intermediate goods producers, retailers, who are the source of price stickiness, final goods producers and capital producers. Intermediate producers and final goods producers operate in a perfectly competitive environment while the retailers operate in monopolistic competition, buying intermediate goods at marginal cost, differentiating them and setting prices in a Calvo scheme similar to the way that households set their wages. Then, final goods producers buy the differentiated goods from the retailers, they assemble them and they sell them in a perfectly competitive market.

Intermediate Goods Producers

Intermediate goods firms produce goods in a perfectly competitive market. Throughout we assume that they face no frictions in obtaining funds. They take wages w_t as given. At the end of period t they acquire k_{t+1} capital for production. We assume no adjustment costs in their production process, thus the problem they are facing is static.

The firms that have an opportunity to invest, finance their capital acquisition by issuing state contingent securities S_t^i at price Q_t^i . They then use the funds to buy capital from the capital goods producer. Each security is a claim of future returns r_t from one unit of investment. By imposing the assumption of perfect competition in the labor supply market, labor is perfectly mobile across islands.

Firms sell their intermediate output to retailers at the perfectly competitive price p_{mt} . The problem the competitive firm is facing is thus:

$$max\ p_{mt}\, y_{it} - r_t Q_t k_{it} - w_t l_{it}^d \quad (19)$$

subject to a Cobb Douglas production technology

$$y_{it} = A_t k_{it-1}^a (l_{it}^d)^{1-a} \quad (20)$$

where A_t is a transitory technology shock following an AR(1) process and l_{it}^d is the amount of packed labor input rented by the firm.

The FOCs to this problem are standard. The firms choose labor demand such that:

$$w_t = p_{mt}(1-a)\frac{y_{it}}{l_{it}^d} \quad (21)$$

The expected profits per units of capital may be expressed as follows:

$$r_t = p_{mt}\, a A_t \left(\frac{l_{it}}{k_{it-1}}\right)^{1-a} \quad (22)$$

The CRS assumption together with the perfect labor mobility assumption mean that there is no need to keep track of the distribution of capital across islands. In addition, perfect competition implies zero profits earned state by state.

Retailers

There is a continuum of retailers indexed by *i* in the economy. They buy intermediate goods at marginal cost, they differentiate them and then they sell them in a monopolistically competitive market, setting their prices a la Calvo. Each retailer resets his price p_{it} with probability $1-\theta_p$. They are maximizing the following objective function with respect to p_{it} :

$$\max_{p_{it}} E_t \sum_{\tau=0}^{\infty} (\beta\theta_p)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_{\tau}} \left[\left(\prod_{s=1}^{\tau} \Pi_{t+s-1}^{\chi} \frac{p_{it}}{p_{t+\tau}} - p_{mt+\tau}\right) y_{it+\tau}\right] \quad (23)$$

subject to the demand for each intermediate good:

$$y_{it+\tau}=\left(\prod_{s=1}^{\tau}\Pi_{t+s-1}^{\chi}\frac{p_{it}}{p_{t+\tau}}\right)^{-\varepsilon}y_{t+\tau}^{d} \quad (24)$$

where ε is the elasticity of substitution and y_t^d is the aggregate demand in the economy.

As with the wage problem of the household, we can express the FOCs of the retailer in recursive form where:

$$g_t^1=\lambda_\tau p_{mt}\, y_t^d+\beta\theta_p E_t\sum_{\tau=0}^{\infty}\beta\theta_p\left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{-\varepsilon}g_{t+1}^1 \quad (25)$$

$$g_t^2=\lambda_t\check{\Pi}_t\, y_t^d+\beta\,\theta_p E_t\left(\frac{\Pi_t^{\chi}}{\Pi_{t+1}}\right)^{1-\varepsilon}\left(\frac{\check{\Pi}_t}{\check{\Pi}_{t+1}}\right)g_{t+1}^2 \quad (26)$$

$$\varepsilon\, g_t^1=(\varepsilon-1)\, g_t^2 \quad (27)$$

where with $\check{p}_t$ being the optimally readjusted price, we define $\check{\Pi}_t=\frac{\check{p}_t}{p_t}$.

Given Calvo pricing, the price index evolves as:

$$1=\theta_p\left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{1-\varepsilon}+(1-\theta_p)\check{\Pi}_t^{1-\varepsilon} \quad (28)$$

Final Goods Producers

The final goods producers operate in a perfectly competitive market. They buy the differentiated intermediate goods from the retailers and they assemble them into a final bundle using the standard Dixit Stiglitz technology:

$$y_t^d=\left(\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (29)$$

Taking as given the intermediate goods prices and the final good price they maximize profits subject to 26. The FOCs to their problem are standard:

$$y_{it}=\left(\frac{p_{it}}{p_t}\right)^{-\varepsilon}y_t^d \quad (30) \text{ and } \quad p_t=\left(\int_0^1 p_{it}^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}} \quad (31)$$

where 30 is the relationship for aggregate demand y_t^d and 31 expresses the aggregate price level.

Aggregation

Next, to obtain a characterization for the production sector, we need to aggregate over the intermediate producers. We know that:

$$\int_0^1 A_t k_{it}^a (l_{it}^d)^{1-a} di = A_t k_t^a (l_t^d)^{1-a} \quad (32)$$

Furthermore from the final goods producer’s FOC and using the equilibrium condition:

$$y_{it} = (c_t + i_t) \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} \quad (33)$$

Integrating out:

$$A_t k_t^a (l_t^d)^{1-a} = (c_t + i_t) \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di \quad (34)$$

We can define the price dispersion term as

$$v_t^p = \int_0^1 \left(\frac{p_{it}}{p_t}\right)^{-\varepsilon} di \quad (35)$$

where by the properties of Calvo pricing:

$$v_t^p = \theta_p \left(\frac{\Pi_{t-1}^{\chi}}{\Pi_t}\right)^{-\varepsilon} + (1-\theta_p)(\check{\Pi}_t)^{1-\varepsilon} \quad (36)$$

Therefore, we get:

$$y_t^d = \frac{A_t k_t^a (l_t^d)^{1-a}}{v_t^p} \quad (37)$$

Capital Goods Producers

To close the production sector, we need a characterization of the capital goods producers. They operate in a national market making new capital using inputs of final output and subject to adjustment costs. As it has already been noted, they sell new capital to firms on investing islands at the the price of Q_t^i .

The problem they have to solve can be formulated as follows:

$$\max_{i_t} E_t \sum_{\tau=t}^{\infty} \Lambda_{\tau,t} \left[Q_\tau^i i_\tau - \left[1 + f\left(\frac{i_\tau}{i_{\tau-1}}\right)\right] i_t\right] \quad (38)$$

From profit maximization the price of new capital goods should be equal to the marginal cost of investment goods production,

$$Q_t^i = 1 + f\left(\frac{i_t}{i_{t-1}}\right) + \frac{i_t}{i_{t-1}} f'\left(\frac{i_t}{i_{t-1}}\right) - E_t \Lambda_{t,t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 f'\left(\frac{i_{t+1}}{i_t}\right) \quad (39)$$

FINANCIAL INTERMEDIARIES

In the model there exists a heterogeneous banking sector. Banks finance their lendings each period by raising funds from households from a national financial market. The national market is split between the deposits market (retail), where banks raise funds from households and the interbank market (wholesale) where banks borrow and lend to one another.

The frictions in the model are motivated via a simple moral hazard problem that the financial intermediaries are facing when they are obtaining funds from households and when they are trading with each other.

At the beginning of period t , before the investment arrival (or non arrival) is realized, each bank raises deposits from households at the risk free rate R_{t+1} . After the investment shock is realized, they start operating in the interbank market borrowing at cost R_{bt} .

This kind of active interbank market allows for the introduction of a liquidity problem in the model. Normally, as we'll see, when there are no frictions in the market, funds flow from non investing islands to the investing ones, leading equalization of asset prices and thus allowing to treat the banking sector as homogeneous.

After obtaining funds the banks can make loans to non financial firms located on the same island. After learning about its lending opportunities, each bank decides the amount of lending it wants to make to non financial firms as well as the amount of interbank borrowing. Each bank cannot borrow more than its current net worth and its borrowed funds. Thus we can express the flow of funds constraint each bank is facing as:

$$Q_t^h s_t^h = n_t^h + b_t^h + d_t \quad (40)$$

where Q_h^t is the price of a loan and s_t^h the volume of equity bought from nonfinancial firms, n_t^h is the net worth of the bank, b_t^h is the amount of interbank borrowings and d_t are the deposits. The heterogeneity is introduced by indexing each variable by $h=i,n$ which indicates whether the intermediary is operating on an investing (i) or a non investing island (n).

The evolution of each individual's bank net worth can be expressed as follows:

$$n_t^h = [r_t + (1-\delta) Q_t^h] \psi_t s_{t-1} - R_{bt} b_{t-1} - R_t d_{t-1} \quad (41)$$

where r_t are the dividends paid to the intermediary for each unit of loan they made to the firms in the previous period (since non financial firms operate in perfect competition they pay all of their ex post returns to the bank), $(1-\delta)Q_t^h$ represents the value of a depreciated one unit of capital, $R_{bt}b_{t-1}$ is the amount repaid for the bank loans borrowed in the previous period and $R_t d_{t-1}$ the repayment to the depositors.

Next, we turn to the banker's optimization problem. Remembering that each banker exits with i.i.d probability $1-\sigma$, we can express the objective of the bank at the end of period t as follows:

$$V_t = E_t \sum_{i=1}^{\infty} (1-\sigma)\sigma^{i-1} \Lambda_{t,t+i} n_{t+i}^h \quad (42)$$

where $\Lambda_{t,t+i} = \beta^i \frac{\lambda_{t+i}}{\lambda_t}$ is the stochastic discount factor.

We now need to introduce the moral hazard problem. The constraint either in the wholesale or the retail interbank market is motivated, as usual, by an agency problem. After a bank obtains funds, the banker managing the bank can divert a fraction θ of these funds to his family after the realization of the idiosyncratic shock. The creditors may then reclaim the remaining $1-\theta$ of their assets. In this way, a borrowing constraint arises.

Mathematically, the problem may be expressed as follows. Let $V_t(s_t^h, b_t^h, d_t)$ be the maximized value of the bank's objective. Then to ensure that the bank does not divert funds, the following inequality must hold:

$$V_t(s_t^h, b_t^h, d_t) \geq \theta (Q_t^h s_t^h - w b_t^h) \quad (43)$$

This expression indicates that the bank's maximized profits must be larger than the funds the bank would obtain if it were to divert a fraction θ of its assets.

The parameter w controls for the degree of frictions in the interbank market. It takes values $w \in (0,1)$. The intuition here is the following; the interbank rate lies between the return on loans and the return on deposits and the higher the value w takes, the lower the degree of interbank frictions, that is the banks can divert less funds from the interbank market. It follows that with $w=0$ we have symmetric frictions in the interbank and the retail financial markets, that is, the interbank and the rate are the same, while with $w=1$ the interbank market is frictionless, i.e. banks are able to reclaim 100% of their assets from the banker who tried to divert them.

Next, we need to derive an analytical solution for the bank's problem. To solve the problem we first set up the Bellman equation as follows

$$V_{t-1}(s_{t-1}, b_{t-1}, d_{t-1}) = E_{t-1} \Lambda_{t-1,t} \sum_{h=i,n} \pi^h [(1-\sigma) n_t^h + \sigma \underset{d_t}{Max} (\underset{s_t^h, b_t^h}{Max} V_t(s_t^h, b_t^h, d_t))] \quad (44)$$

The intuition behind 44 is that if the banker exits he leaves with his net worth n_t and if he stays in the market he needs to first maximize over the loans he makes to the households and afterwards over his interactions with the real and the interbank sector. Since the probability of investment arriving on his island is i.i.d and constant he is taking into account into his problem the weighted average of his objective in the two stages of the world.

The decision problem is then solved via the method of undetermined coefficients. For this, it is first necessary to guess a form that the value function takes and then to verify that guess. The value function is hypothesized to have the following linear form:

$$V_t(s_t^h, b_t^h, d_t) \geq v_{st} s_t^h - v_{bt} b_t^h - v_t d_t \quad (45)$$

where v_{st} is the marginal value of assets, v_{bt} is the marginal cost of the interbank debt and v_t is the marginal cost of deposits.

The problem may then be expressed as:

$$max\, V_t(s_t^h, b_t^h, d_t) + \bar{\lambda}_t (V_t(s_t^h, b_t^h, d_t) - \theta(Q_t^h s_t^h - w b_t^h)) \quad (46)$$

where λ_t^h is the Lagrangian for the incentive constraint, dependent on the type of island that the intermediary operates and $\bar{\lambda}_t$ is the average of the Lagrange multipliers across the states of the islands $\bar{\lambda}_t \equiv \pi^i \lambda_t^i + \pi^n \lambda_t^n$.

The FOCs to the problem (using the fact that $b_t^h = q_t^h s_t^h - n_t^h - d_t$) are then the following:

$$d_t: \ (v_{bt} - v_t)(1 + \bar{\lambda}_t) = \theta w \bar{\lambda}_t \quad (47)$$

$$s_t^h: \ \left(\frac{v_{st}}{Q_t^h} - v_{bt}\right)(1 + \lambda_t^h) = \lambda_t^h \theta (1 - w) \quad (48)$$

$$\lambda_t^h: \ \left[\theta - \left(\frac{v_{st}}{Q_t^h} - v_t\right)\right] Q_t^h s_t^h - [\theta w - (v_{bt} - v_t)] b_t^h \leq v_t n_t^h \quad (49)$$

We first examine the case with an imperfect interbank market, that is with $0 < w < 1$ which yields a highly non linear system. Afterwards we examine the simplified cases with $w = 0$ and $w = 1$.

Imperfect Interbank Market

The incentive constraint 49 can be then rewritten as:

$$[(\theta(1-w) + v_{bt}) Q_t^h - v_{st}] s_t^h \leq (v_{bt} - \theta w) n_t^h - (\theta w + v_t - v_{bt}) d_t \quad (50)$$

Then 48 may be re expressed as:

$$\lambda_t^h = \frac{\frac{v_{st}}{Q_t^h} - v_{bt}}{\theta(1-w) - (\frac{v_{st}}{Q_t^h} - v_{bt})} \quad (51)$$

We assume that the abundance of supply on investing islands leads to lower security prices, ie $Q_t^i < Q_t^n$ and thus we learn from 41 that $\lambda_t^i > \lambda_t^n > 0$.

Manipulating 47 we get:

$$v_{bt} - v_t = \frac{(\theta w \bar{\lambda}_t)}{(1+\bar{\lambda}_t)} > 0 \quad (52)$$

Thus the marginal cost of interbank borrowing exceeds the marginal cost of deposits.

Using the FOCs the incentive constraint can be represented as:

$$Q_t^h s_t^h \leq \frac{1}{\theta(1-w) - (\frac{v_{st}}{Q_t^h} - v_{bt})} [(v_{bt} - \theta w) n_t^h - \frac{\theta w}{1+\bar{\lambda}_t} d_t] \quad (53)$$

Equation 53 gives us a relationship for asset demand. We see that the demand for assets is positive in the excess of the marginal value of holding assets v_{st} relative to the marginal cost of interbank borrowing v_{bt} as well as with bank net worth n_t^h and negative w.r.t. deposits d_t , i.e. with the amount that the bank borrows from households.

Plugging the FOCs and the incentive constraint into 46 and with some algebra and using the fact that $b_t^h = q_t^h s_t^h - n_t^h - d_t$ we get:

$$V_t(s_t^h, b_t^h, d_t) = [v_{bt} + \lambda_\tau^h (v_{bt} - \theta w)] n_t^h + \theta w \frac{\bar{\lambda}_t - \lambda_t^h}{1+\bar{\lambda}_t} d_t \quad (54)$$

Plugging 54 into the Bellman equation for time $t+1$ we get:

$$V_t(s_t^h, b_t^h, d_t) = \underset{h'}{E_t} \Lambda_{t,t+1} \Omega_{t+1}^{h'} n_{t+1}^{h'} \quad (55)$$

where $\Omega_t^h = 1 - \sigma + \sigma [v_{bt} + \lambda_t^h (v_{bt} - \theta w)]$ (56)

Next, we employ the method of matching coefficients to see which values correspond to the guessed form:

$$V_t(s_t^h, b_t^h, d_t) = \underset{h'}{E_t} \Lambda_{t,t+1} \Omega_{t+1}^{h'} n_{t+1}^{h'} = \underset{h'}{E_t} \Lambda_{t,t+1} \Omega_{t+1}^{h'} [[r_{t+1} + (1-\delta) Q_{t+1}^{h'}] \psi_{t+1} s_t^h - R_{bt+1} b_t^h - R_{t+1} d_t] \quad (57)$$

where in the second equality the expression for the evolution of bank net worth was used.

Consequently, given the guessed function 45, the method of matching coefficients yields the following values:

$$v_{bt} = R_{bt+1} \underset{h'}{E_t} \Lambda_{t,t+1} \Omega_{t+1}^{h'} \quad (58)$$

$$v_t = R_{t+1} \underset{h'}{E_t} \Lambda_{t,t+1} \Omega_{t+1}^{h'} = \frac{R_{t+1}}{R_{bt+1}} v_{bt} \quad (59)$$

$$v_{st} = \underset{h'}{E_t} \Lambda_{t,t+1} \Omega_{t+1}^{h'} [r_{t+1} + (1-\delta) Q_{t+1}^{h'}] \psi_{t+1} \quad (60)$$

where we condition on h' , that is the type the island is going to be in the next period.

Taking the derivative of 53 w.r.t bank net worth n_t^h yields the marginal propensity of the banks to buy assets, which can be interpreted as the optimal leverage ratio of the banks. Thus the expression for the bank's leverage is given by:

$$\phi_t^h = \frac{[v_{bt} - \theta w]}{\theta(1-w) - (\frac{v_{st}}{Q_t^h} - v_{bt})} \quad (61)$$

Next, we can aggregate the investing constraint for all the banks in each type of island and letting D_t be the aggregate value of bank deposits, obtain the following relationships:

$$Q_t^i s_t^i = \frac{1}{\theta(1-w) - (\frac{v_{st}}{Q_t^h} - v_{bt})} [(v_{bt} - \theta w) n_t^h - \frac{\theta w}{1+\bar{\lambda}_t} \pi^i D_t] \quad (62)$$

$$Q_t^n s_t^n \leq \frac{1}{\theta(1-w) - (\frac{v_{st}}{Q_t^h} - v_{bt})} [(v_{bt} - \theta w) n_t^h - \frac{\theta w}{1+\bar{\lambda}_t} \pi^n D_t] \quad (63)$$

As Gertler and Kiyotaki point out, if the constrain in 63 binds tightly, then banks in non-investing islands will be more inclined to use their funds to re-finance existing investments rather than lend them to banks on investing islands. This raises the likelihood that banks on non-investing islands will earn zero excess returns on their assets.

Symmetric wholesale and retail frictions

The case where $w=0$ yields a more simplified system. From the FOCs we learn that $v_{bt} = v_t$. Proceeding with the methods of undetermined coefficients as before we learn the following:

$$v_t = \underset{h'}{E_t} \Lambda_{t,t+1} \Omega_{t+1}^{h'} R_{t+1} \quad (68)$$

$$\frac{v_{st}}{Q_t^h} - v_t \equiv \mu_t^h = \underset{h'}{E_t} \Lambda_{t,t+1} \Omega_{t+1}^{h'} \left(R_{kt+1}^{hh'} - R_{t+1} \right) \quad (69)$$

$$\Omega_t^h = 1 - \sigma + \sigma \left(v_t + \phi_t^{h'} \mu_t^{h'} \right) \quad (70)$$

$$R_{kt+1}^{hh'} = \psi_{t+1} \frac{r_{t+1} + (1-\delta) Q_{t+1}^{h'}}{Q_t^h} \quad (71)$$

where Ω_t^h is the marginal value of net worth, $R_{kt+1}^{hh'}$ is the return on the bank's assets conditioned on the state of the island next year h' , μ_t^h is the excess value of a unit of assets relative to deposits and v_t is the marginal cost of deposits.

By imposing the assumption $Q_t^i < Q_t^n$, we have that $\mu_t^i > \mu_t^n$ and we can then express the leverage ratios of the banks as follows:

$$\frac{Q_t^i s_i^t}{n_i^t} = \phi_t^i = \frac{v_t}{\theta - \mu_i^t} \quad (72)$$

$$\frac{Q_t^n s_t^n}{n_t^n} \leq \phi_t^n = \frac{v_t}{\theta - \mu_{tt}^n} \text{ and } \left(\frac{Q_t^n s_t^n}{n_t^n} - \phi_t^n \right) \mu_t^n = 0 \quad (73)$$

As in the case with $0 < w < 1$, the expression for the leverage constraint on the non investing islands indicates that if there is positive excess value of assets relative to deposits then it will bind tightly. On the other hand, if the excess return on assets is zero then the leverage constraint will be an inequality.

Frictionless Interbank Market

Finally, for the case where $w = 1$ things are considerably more simplified. Banks are now unable to divert funds from other intermediaries. This implies that funds flow freely from island to island leading through perfect arbitrage to the equalization of the asset prices $Q_t^i = Q_t^n = Q_t$. From the FOCs we now learn that $\frac{v_{st}}{Q_t} = v_{bt}$, that is, the marginal value of assets is equal to the marginal cost of borrowing. It follows that the excess value of assets relative to deposits has to be positive;

$$\mu_t \equiv \frac{v_{st}}{Q_t} - v_t > 0 \quad (74)$$

Undetermined coefficients in this case yield:

$$v_t = E_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} \text{ (75)}$$

$$\mu_t = E_t \Lambda_{t,t+1} \Omega_{t+1} \left(R_{kt+1} - R_{t+1} \right) \text{ (76)}$$

$$\Omega_t = 1 - \sigma + \sigma \left(v_t + \phi_t \mu_t \right) \text{ (77)}$$

$$R_{kt+1} = \psi_{t+1} \frac{r_{t+1} + (1-\delta) Q_{t+1}}{Q_t} \text{ (78)}$$

and finally the incentive constraint is:

$$\frac{Q_t s_t}{n_t} - b_t = \phi_t$$

or aggregating:

$$Q_t S_t = \phi_t N_t \text{ (79)}$$

with the leverage ratio being:

$$\phi_t = \frac{v_t}{\theta - \mu_t} \text{ (80)}$$

Bank Net Worth

To close the imperfect interbank market, we need to derive an expression for the evolution of aggregate bank net worth.

Total bank worth is given by:

$$N_t^h = N_{ot}^h + N_{yt}^h \text{ (81)}$$

where N_{ot}^h is the net worth of old bankers and N_{yt}^h is the net worth of new bankers.

Specifically, the net worth of existing bankers should evolve according to:

$$N_{ot}^h = \sigma \pi^h \left[\left(r_t + (1-\delta) Q_t^h \right) \psi_t s_{t-1} - R_t D_{t-1} \right] \text{ (82)}$$

Similarly, the net worth of the new bankers should be equal to:

$$N_{yt}^h = \xi \left(r_t + (1-\delta) Q_t^h \right) \psi_t s_{t-1} \text{ (83)}$$

where ξ is the fraction of the total net worth of existing entrepreneurs that the household is willing to transfer to the new bankers as start up funds.

Finally, the aggregate bank balance sheet is given by:

$$D_t = \sum_{h=i,n} \left(Q_t^h S_t^h - N_t^h \right) \text{ (84)}$$

that is deposits equal the difference between total assets and bank worth.

POLICY INTERVENTIONS

Next, we consider the implications that a direct intervention of the Central Bank would have in this kind of economy. We consider the case where the central bank directly channels funds to intermediaries in times of distress.

To obtain the funds, the central bank issues government debt at the riskless rate R_{t+1} . It then proceeds by lending the funds at rate $R_{kt+1}^{hh'}$ conditioned on the state of the next period.

Letting S_{pt}^{h} be the privately intermediated securities and S_{tg}^{h} the publicly intermediated, the total intermediation of assets can be expressed as:

$$Q_t^h S_t^h = Q_t^h \left(S_{pt}^h + S_{tg}^h \right) \quad (85)$$

The central bank intermediate only a fraction ϕ_{ct} of total securities such that:

$$S_{tg}^h = \phi_{ct}^h S_t^h \ (86)$$

Then the leverage constraint is expressed as:

$$Q_t^i S_t^i = \frac{1}{1-\phi_{ct}^i} \phi_t^i N_t^i \ (87) \quad \text{and} \quad Q_t^n S_t^n = \frac{1}{1-\phi_{ct}^n} \phi_t^n N_t^n \ (88)$$

iff banks on non investing islands are also leverage constrained, that is if $\mu_t^n > 0$.

By manipulating ϕ_{ct} therefore, we see that the central bank is able to manipulate asset demand and use it as an extra tool in times of distress.

GOVERNMENT AND CENTRAL BANK

We now need a characterization of the government budget constraint. Given direct Central Bank lending, total government expenditures g_t are given by:

$$g_t = \bar{g} + \tau \sum_{h=i,n} S_{gt}^h \ (89)$$

where $\bar{g}$ are the normal government expenditures and $\tau \sum_{h=i,n} S_{gt}^h$ the expenditures for intermediating assets given the government's efficiency cost τ .

It then follows that the government's budget constraint is given by:

$$g_t + \sum_{h=i,n} Q_t^h \left(S_{gt}^h - (1-\delta) \psi_t S_{gt-1} \right) = T_t + D_{gt} - R_t D_{gt-1} + r_t \psi_t S_{gt-1} \ (90)$$

which tells us that government spending and asset purchase are financed by bonds, lump sum taxes and the returns on intermediated assets.

Next, we need a Taylor rule that characterizes the behavior of the Central Bank:

$$\frac{R_t}{R}=\left(\frac{R_{t-1}}{R}\right)^{\gamma_R}\left(\left(\frac{\Pi_t}{\Pi}\right)^{\gamma_\Pi}\left(\frac{y_t^D}{y_{t-1}^D}\right)^{\gamma_Y}\right)^{1-\gamma_R} m_t \quad (91)$$

where R and Π are the steady state interest and inflation rates and m_t is a monetary policy shock which is characterized as follows:

$$m_t=\sigma_m \varepsilon_{mt} \quad (92) \quad \text{with} \quad \varepsilon_{mt} \sim N(0,1)$$

MARKET CLEARING

Let government consumption is denoted by g_t . Then the aggregate resource constraint becomes:

$$y_t^d=c_t+\left(1+f\left(\frac{i_t}{i_{t-1}}\right)\right)i_t+g_t \quad (93)$$

For the model to close, we need equilibrium in the labor market which is obtained if

$$(1-a)\frac{y_t^d}{l_t^d}\lambda_t p_{mt}=\chi\left(l_t^d\right)^{\varepsilon} \quad (94)$$

Finally, closing the model, the securities market clear if the total volume of securities issued on investing and non investing islands corresponds to aggregate capital acquired by each type:

$$S_t^i=i_t+(1-\delta)\pi^i K_t \quad (95)$$

$$S_t^n=(1-\delta)\pi^n K_t \quad (96)$$

Simulations

In this section we will present the effects arising from a productivity and a capital quality shock in the setup we presented above. We will compare the responses of the New Keynesian economy to the baseline RBC economy and we will draw conclusions about the interaction of financial and nominal frictions.

To this end, we will compare three variations of each model; a baseline case with a perfect interbank market and no unconventional central bank intervention, a perfect interbank market case with unconventional monetary policy and a case with symmetric frictions in the wholesale and retail financial markets with no central bank intervention.

CALIBRATION

To make a meaningful comparison between the original model (throughout we refer to it as the benchmark) and our modified version of it, we replicated the original model and regarding the New Keynesian version, we try to be consistent with the calibrated values from the original paper. In the table below, we present the calibrated values.

		Households
β	0.99	Depreciation Rate
γ	0.33	Inverse Frisch Elasticity
h	0.5	Habits Parameter
ψ	5.584	Labor Disutility Parameter
θ_w	0.68	Probability of keeping wages fixed
χ_w	0.62	Wage inflation indexation parameter
		Labor Supplier
η	10	Elasticity of substitution between labor varieties
		Financial Intermediaries
π^i π^n	0.25 0.75	Fraction of islands with investment arrivals Fraction of islands with no investment arrivals
θ	0.383 0.129	Fraction of divertable assets: perfect interbank market Fraction of divertable assets: imperfect interbank market
σ	0.972	Survival probability of the bankers
ξ	0.003 0.002	Transfer to entering bankers: perfect interbank market Transfer to entering bankers: imperfect interbank market
		Intermediate Good Firms
α	0.33	Effective Capital Share
δ	0.025	Depreciation Rate
		Retail Firms

ε	10	Elasticity of substitution between goods
θ_p	0.82	Probability of keeping prices fixed
χ	0.63	Inflation Indexation Parameter
Capital Producing Firms		
κ	1.72	Adjustment Costs Parameter
Central Bank		
γ_π	2.1	Inflation Coefficient of the Taylor Rule
γ_R	0.77	Interest Rate Coefficient of the Taylor Rule
γ_y	0.19	Output Coefficient of the Taylor Rule
ν_g	100	Policy Parameter for Direct Lending Facilities

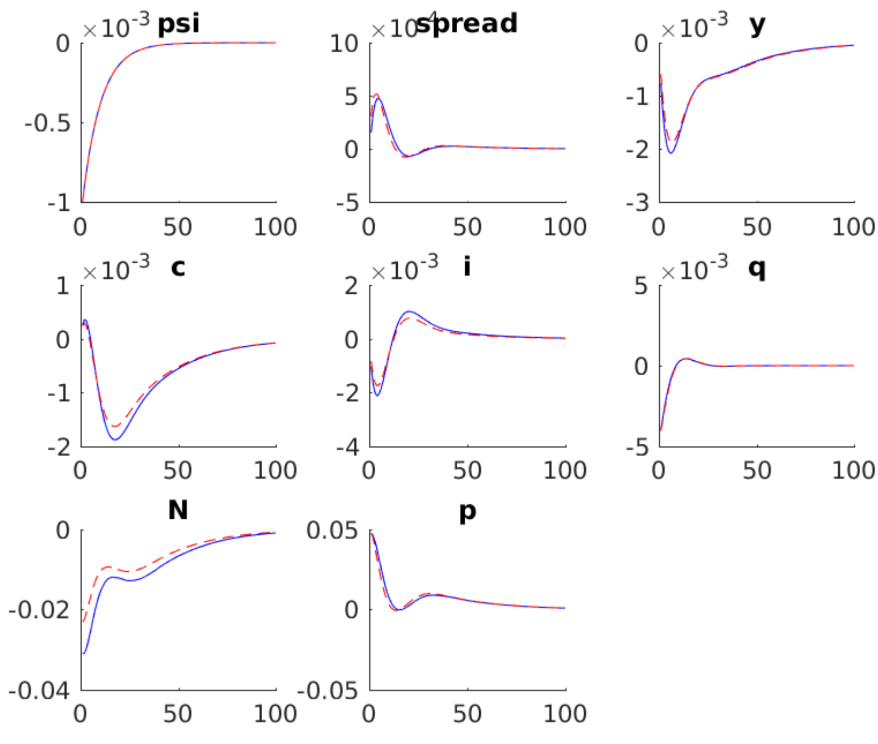
To obtain impulse response functions to the shocks, we had to log linearized the imperfect interbank market models around a region of the steady state, since its highly non linear nature made guessing initial steady state values a tedious task. The log linearized models turned out to have multiple steady states resulting to ambiguous results which were highly dependent on parameter values we gave to the steady state net worth variables that emerged in the log linear model. In the final model used, we assumed that in the steady state, the net worth and the asset demand in the investing islands was higher than in the non investing ones by a factor of approximately 1.5-2, depending on the variation of the model that we are simulating.

CRISIS EXPERIMENTS

What we are considering here is a shock in the quality of capital ξ_t . The mechanics behind the shock work as follows; a crisis in the quality of capital should first affect investment and then through the equilibrium conditions it should reduce the loans the non financial firms are issuing. This should in turn lead to a fall in the asset prices which should tighten the leverage constraint of the intermediaries forcing them to proceed to fire sales to meet the new constraint. The effect is further magnified by the leverage ratio which acts as a multiplier thus reducing the asset price even more. Note here that there is both an exogenous (through the deterioration of the capital quality) and an endogenous component (through the leverage ratio) to the fall in asset prices. This should in turn lead to a further reduction in investment leading to a vicious cycle effect and an enhanced drop in output. This process should be reflected onto a rise in the spread between the bank's return on its holdings and the risk free rate. The crucial element for the return to steady state is the behavior of the bank's net worth. As the effects of the shock attenuate, bank net worth should start increasing leading to a process of deleveraging, since the banks are now building up equity. This process of deleveraging should lead the economy back to its steady state.

Next, we consider a capital quality shock in a frictionless interbank market model with no intervention. In figure 1, the blue traces the responsiveness of the benchmark model with perfect interbank market and with no central bank intervention to the shock and the red dashed line depicts the New Keynesian variation over a time span of 100 periods.

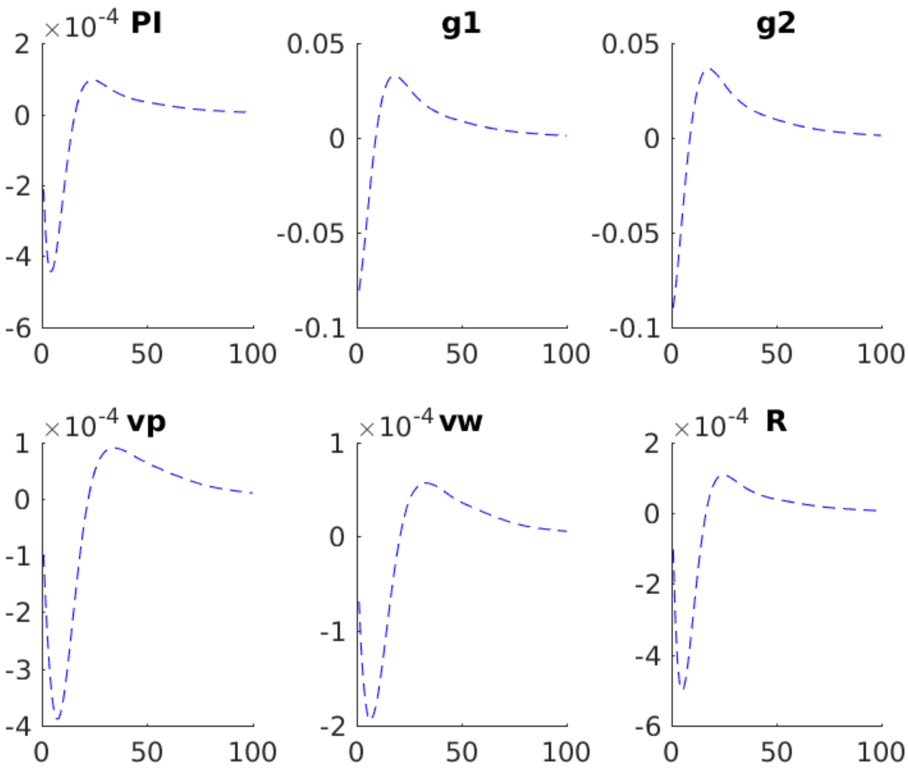
Figure 1 - Perfect Interbank Market



The model responds exactly as we expected it to; on impact we observe a rise in the spread, which indicates lighter discounting of future profits. The quantity variables of output, consumption and investment fall as does the price of capital. The deterioration in the value of the asset holdings implies a net worth fall which translates to an increase of the leverage ratio.

The process that leads the model back to its steady state is also clearly visible; the banks start building up on their net worth which leads to deleveraging (p in the figure) which ultimately leads the economy back to its steady state. What is interesting here, is that the New Keynesian model behaves almost identically both quantitatively and qualitatively with the benchmark model; adding nominal frictions in this case does not seem to affect the behavior of the model in any significant way apart from the fact that the New Keynesian model is responding slightly better to the shock. The channel explaining this is the falling price dispersion of the New Keynesian model. In the absence of liquidity problems, the crisis in the quality of capital leads to lower intermediate prices which in turn lead to a fall in inflation (PI), thus resulting to a lower price dispersion (vp). As a result of this, the output of the intermediate firms absorbs the main shock and the nominal frictions act as a filter, leading to a milder fall in the quantity variables.

Figure 2 - Inflation Variables with Perfect Interbank Market



Next, we consider the models with an intervening central bank and a perfect interbank market. Figure 3 depicts the impulse response functions to the capital quality shock. The responses of the both models in this case are significantly damped down compared to the previous ones (see

Figure 9). The intervention helps the transition to the steady state become smoother. The additional element here is the fraction of assets intermediated by the intervention, denoted by ph in the graph. In both cases (in the benchmark case barely noticeable), the fraction of government intermediated assets increases on impact, just as it was described above.

Again, we see that the two models behave almost identically, with their impulse responses reacting symmetrically to the shock. One difference here is however the reactions of output and investment to the shock. In contrary with the no intervention models, in this case the New Keynesian variables (dashed red line) are hit marginally more than the benchmark's variable. The intuition here is simple. In the absence of nominal frictions, the Central Bank intervention is much more effective, such that in the comparison the 'advantage' of the New Keynesian model in the no intervention case, that is its milder reaction to the negative shock due to the absorption effects of the prices, is offset. Nominal frictions act therefore again as a protective layer to shocks but this time in the opposite direction. Regarding the inflation variables, we see that they are identical to the ones of the no intervention case. In both cases however, the central bank intervention significantly attenuates the shock as it is evident in figure 9 and 10.

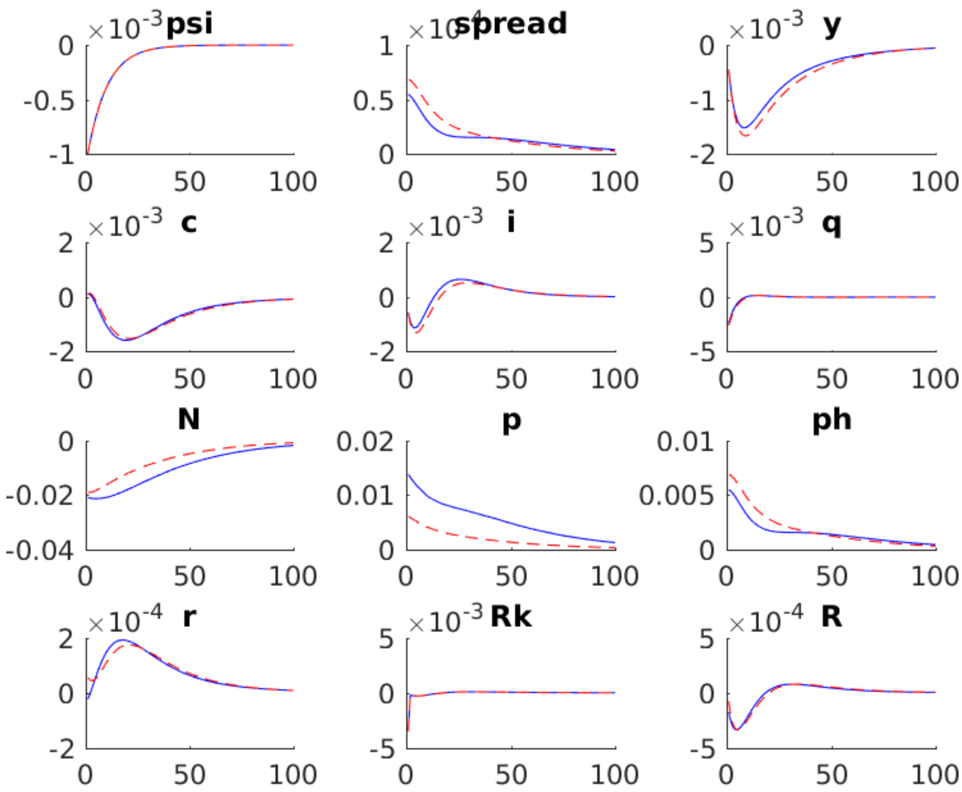
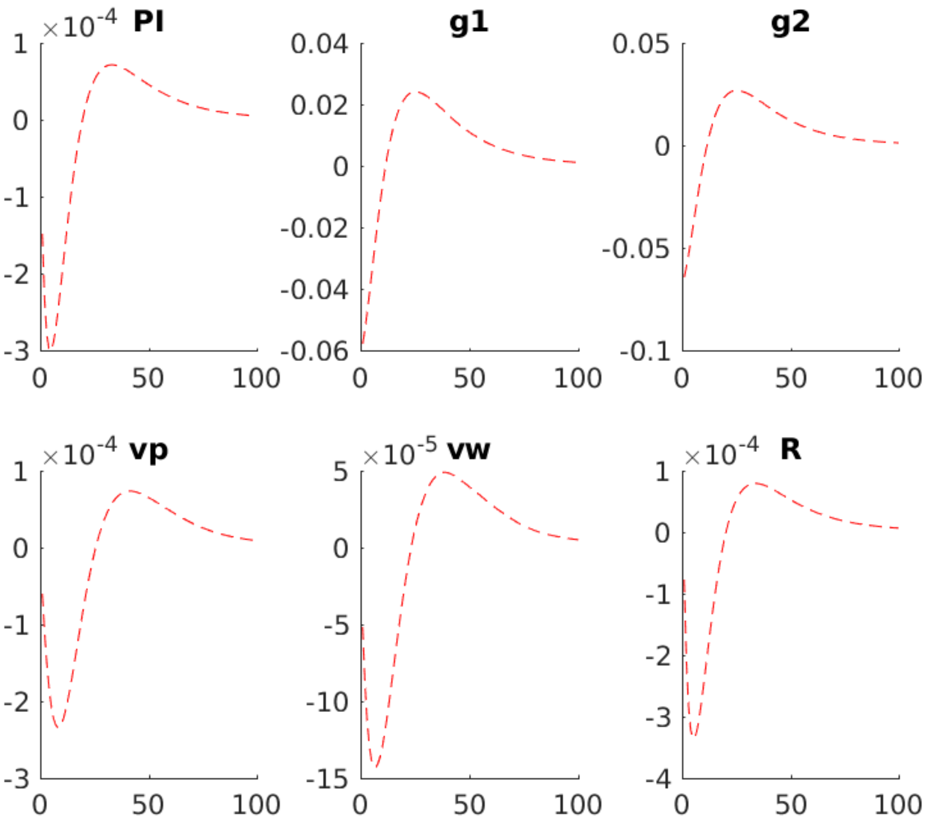


Figure 3 - Perfect Interbank with Intervention

Figure 4 - Inflation Variables with CB Intervention



The next case we consider is that of symmetric frictions in the interbank and the wholesale financial markets. As mentioned before, for this case we loglinearized the model and there is a multiplicity of steady states, depending on the values we give to the steady state parameters. Here, we are considering the case where the leverage constraint binds in both types of islands, that is $\mu_t^h > 0 \; \forall h = i, n$ (see discussion in the model section), which should lead to less volatile responses. Figure 5 illustrates the impulse responses of the two models. Again, we see that the models behave almost identically, with their differences being marginal.

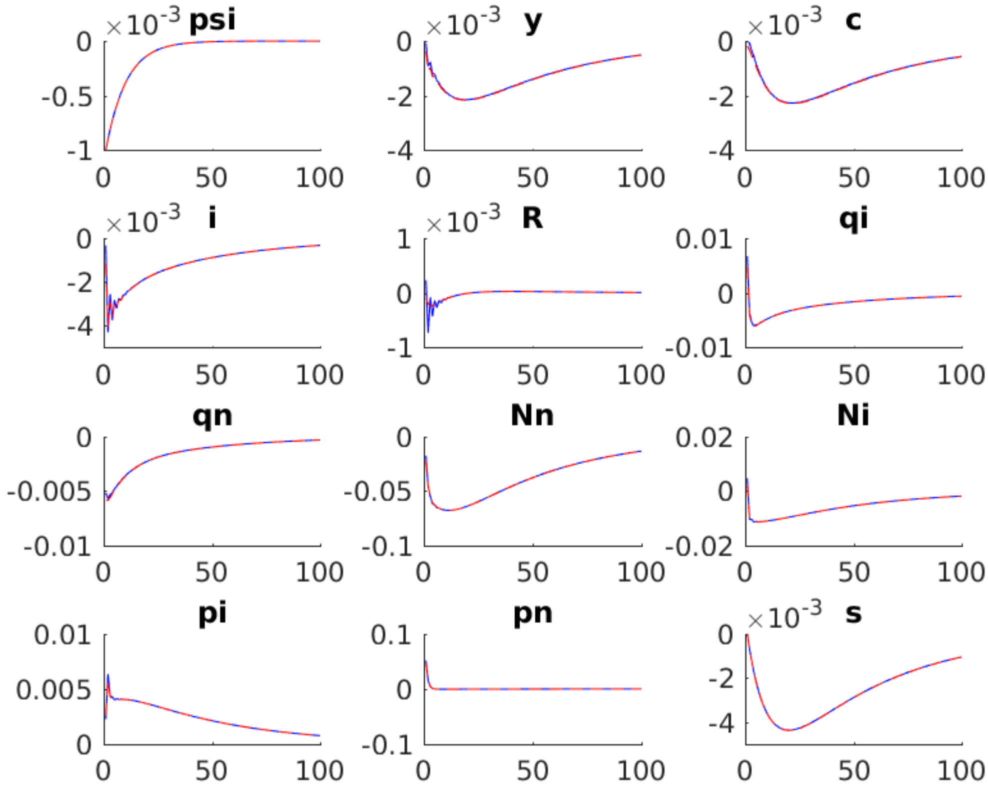


Figure 5 - Symmetric Financial Frictions

The response is exactly the same as in the previous cases; we again see a fall in asset prices in both islands, a fall in investment and the same process of building up net worth and deleveraging which leads back to the steady state path. The only noticeable difference is the behavior of the interest rate; in the New Keynesian case it finds a much smoother path to the steady state when compared with the benchmark case, where it converges to the steady state with damped oscillations. The obvious reason is the Central Bank in the New Keynesian case is stabilizing the interest rate with the Taylor rule. What is probably surprising is that here the inflation variables (Figure 6) react in a slightly less pronounced fashion than in the perfect interbank market case.

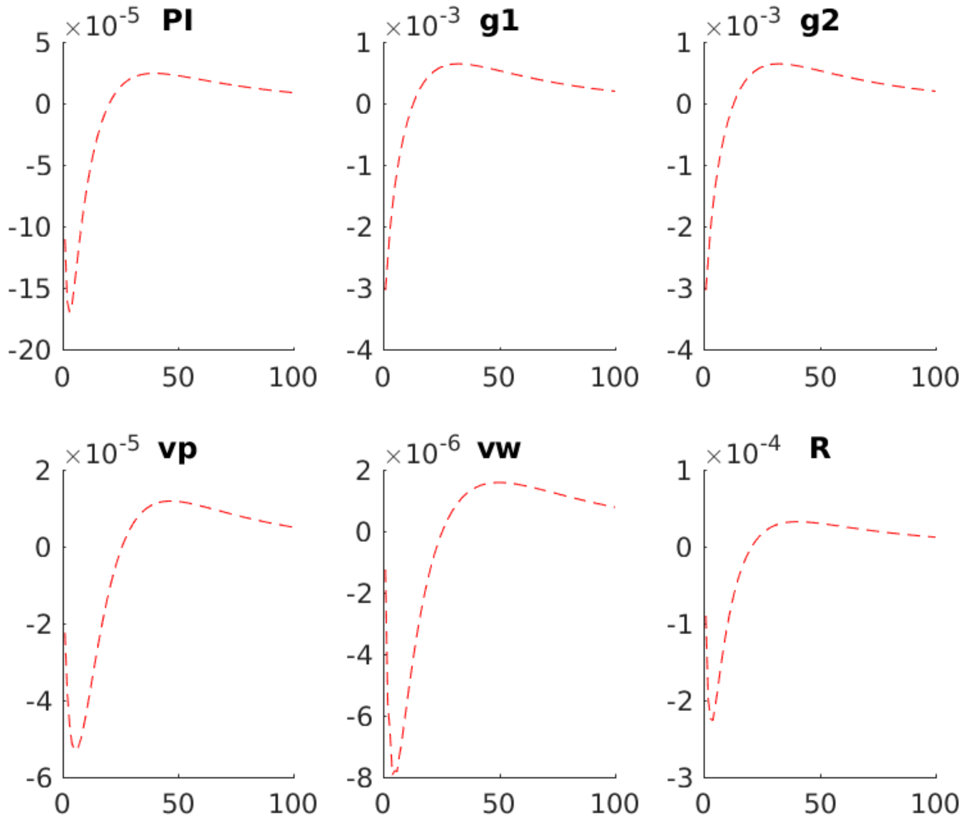
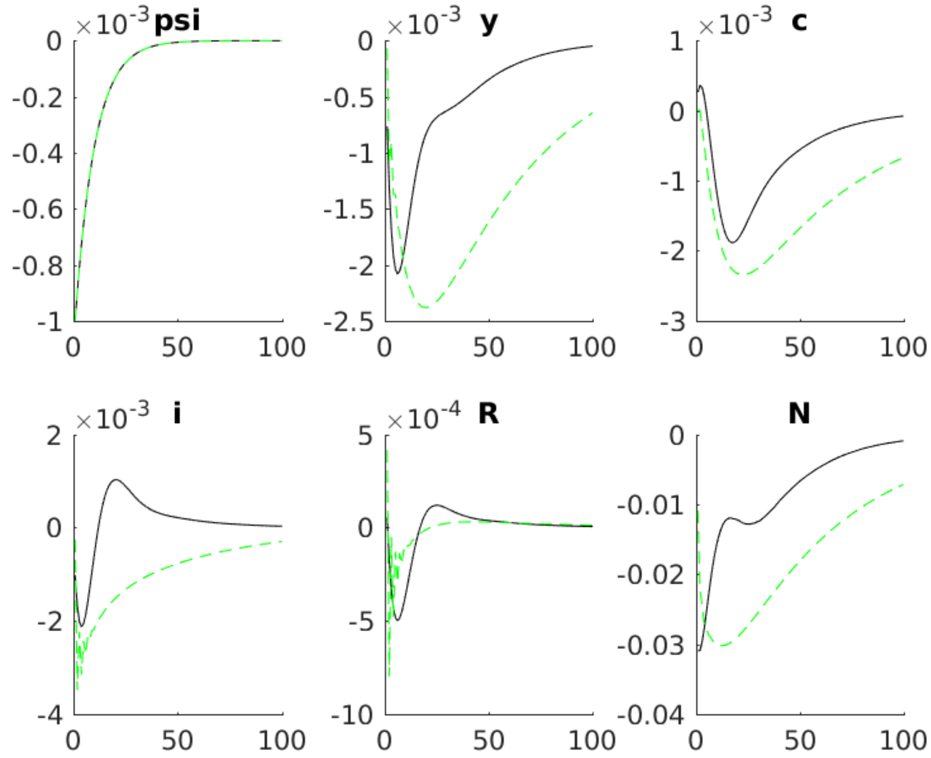


Figure 6 - Inflation Variables with Symmetric Frictions

Finally, to get a better sense of the impact of the introduction of interbank frictions, we compare the perfect interbank market models with their corresponding interbank frictions models in Figures 7 and 8. As expected, the differences between the two models are almost non existent. The black line depicts the perfect interbank market while the green dotted line depicts the economy with the imperfect interbank market. We first consider the benchmark case; In figure 7 we see that the difference in the impulse responses is not significant. As expected, the limited flow of funds from the non investing to the investing islands leads to a more exacerbated and volatile response. With the asset prices not being equalized through the islands, they fall more than in the perfect interbank case, inducing a steeper fall in bank net worth and thus, in the absence of loans, investment, leading to the larger and more protracted fall in output.

Figure 7 - Benchmark Perfect vs Imperfect Interbank



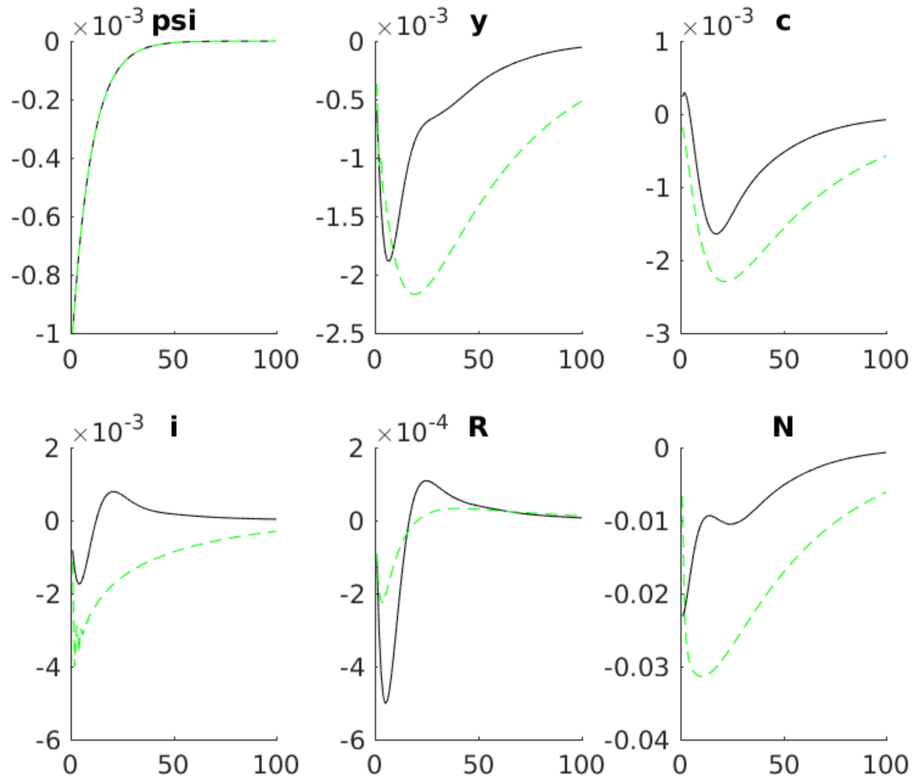


Figure 8 - NK Perfect vs Imperfect Interbank Market

Figure 9 - Benchmark Models Intervention vs No Intervention

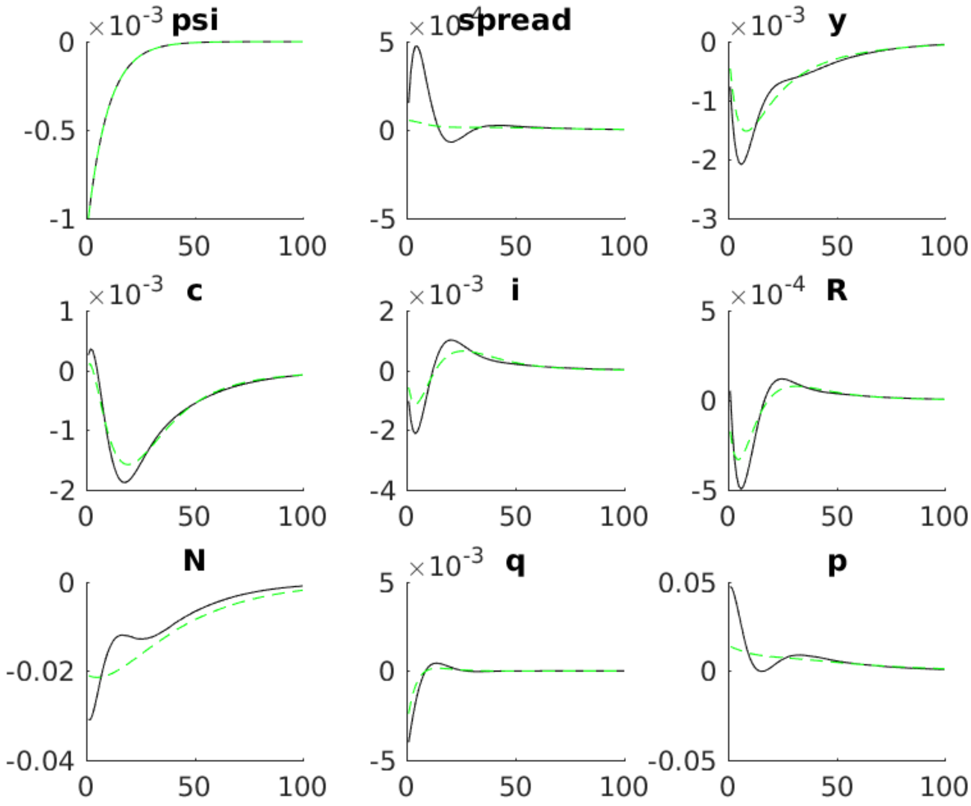
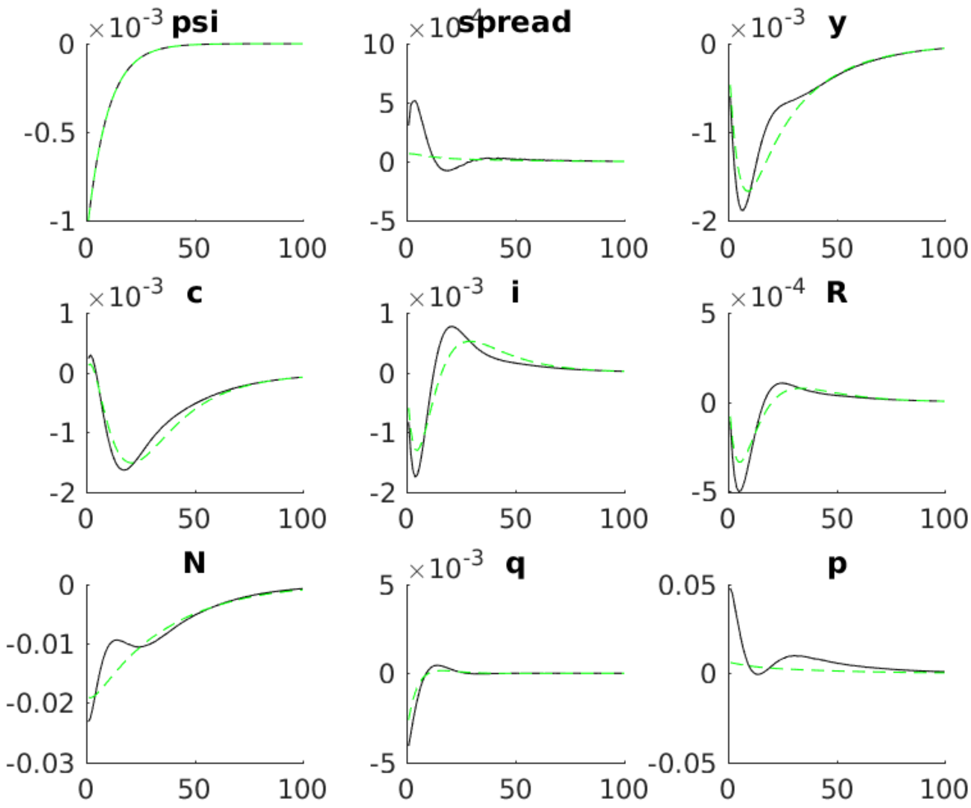


Figure 10 - NK Models Intervention vs No Intervention



PART IV: CONCLUSIONS

In this thesis, we first provided a compact review of the theory and of some modeling techniques of the banking frictions. The aim was to provide a sense of how DSGE's in the last decade have been trying to capture banking imperfections and incorporate them in the general equilibrium. In the models we examined we saw that the modeling techniques revolved around the same core; there is a considerable continuity in the literature regarding the mechanisms the scholars have been using to model the frictions. As it is understandable, the review was not completely comprehensive. During the least years many models, considerably more complex, have been trying to capture more complicated phenomena. Notable in this respect are the recent models that are broadening the scope of the financial market by trying to capture the phenomenon of bank runs (Gertler et al (2015, 2016)).

In the last part of the thesis, we simulated variations of a notable financial frictions models with the aim of drawing conclusions about the interconnections between the nominal and the financial frictions. The results were clear. The economy, whether it is flexible or nominally rigid, reacts qualitatively and quantitatively in the same way when a crisis, propagated through the financial sector, hits. As expected, the existence of frictions both in the retail and in the wholesale financial market make the negative reaction worse in both cases. In every case, with the existence of nominal frictions, the economy enters into a brief deflation. More interestingly for the policy maker, unconventional monetary policy measures significantly alleviate the shock, with their effect being slightly more pronounced in the case of a flexible economy.

In a broader scope, there is no doubt that when it comes to understanding the interconnection between the financial sector and the real economy, there is still a lot to be done. As it became clear throughout the text, even relatively simple ideas turn out to be mathematically and computationally hazardous when it comes to incorporating in the general equilibrium. And in systems with such complex networks as in the financial markets, any attempt to abstract leads inevitably to partial and many times unrealistic results. Moreover, such models give rise to significant non linearities which leads to computational and algorithmic complexities. The existence, in many cases, of multiple equilibria and steady states creates further ambiguity for the practitioner.

Despite all the difficulties however, the incorporation of financial frictions in DSGE models is a big step forward for the field of macroeconomics. And results have already showed up, notably with the model by Del Negro et al (2013) which proved to be able to have predicted the 2008 crisis. Ultimately, there is no doubt that the integration of macroeconomics and finance poses one of the greatest future challenges to the field.

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