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Collusion in Markets with Syndication

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Abstract

Many markets are syndicated, in the sense that firms compete when they set their prices for an identical good, but after the buyer chooses one of the firms, then the latter hires the rest of the firms in order to smooth the convex production cost. Because of this dependence, the firms have a credible threat over each other so to support a price higher than the perfect-competition price. We model syndicated markets as a repeated extensive form game, and show that standard intuitions from industrial organization can be violated. Collusion may become easier as market concentration falls, and market entry may in fact facilitate collusion. We use this subgame perfect Nash equilibrium to rationalize the findings of the IPO market. We extend previous results that hold in monopsony context to markets that are accessed by an arbitrary number of consumers.

Keywords: Syndication, Collusion, Repeated games with public information, Antitrust

Contents

1 Introduction

This handout is closely related to the literature on reputation effects in repeated games and the structure of the IPOs market. In order to establish our model, we used the results of ***Reputation in Long-Run Relationships*** (Atakan and Ekmekci, 2011) and ***Do Investment Banks Compete in IPOs? The Advent of the "7% plus contract"*** (Hansen, 2000).

Concerning the first paper, Atakan and Ekmekci model a long-run relationship as an infinitely repeated game played by two equally patient agents. Player 2 has incomplete information about Player 1's type, while Player 2's type is common knowledge. They show that if Player 1 is patient enough can use the incomplete information of Player 2 to convince him that he has a certain type, which will lead Player 1 to the highest payoff for him. To convice Player 2 of this, he has to act like the strong type, even if this will have a cost for him until the reputation's establishment.
Developing a reputation lends credibility to future threats or promises, thus Player 1 can manipulate Player 2's behaviour so to achieve the optimal payoff. For example a reputation that the incumbent firm is tough can prevent new firms from entering this market (Kreps and Wilson, 1982), or it can make the government's promises about monetary and fiscal policies more credible (Barro, 1986). However, the opponent player can be equally patient thus he can afford losses in the first periods testing Player 1's threats, aiming to make Player 1's mimic strategy not profitable. To emphasize on this conflict, Atakan and Ekmekci focus on equally patient agents.
They consider an infinitely repeated game played by two equally patient agents. They assume that Player 2 has incomplete information about Player 1's type (e.g. a distribution among all possible types), but Player 1 has perfect information about his opponent's type. With these assumptions they prove the following result concerning reputation effects: a Player 1 who is sufficiently patient can ensure for himself his optimal payoff compatible with the individual rationality of Player 2. This happens in any perfect Bayesian equilibrium of the game. For example, we examine the infinitely repeated version of the Battle of the Sexes game. In each period they have to decide whether they will choose Player 1's or Player 2's preferred place to hang out. Unanimity is required in order the players to receive any utility.

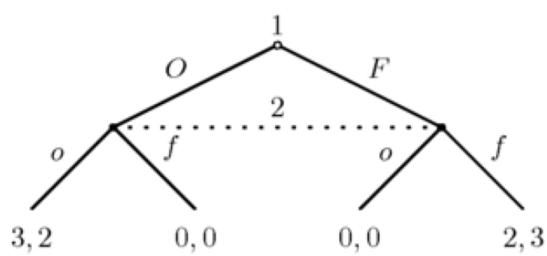


Figure 1: Battle of the Sexes

In case that Player 2 trusts that Player 1 is the strong type (i.e. he will stick to his optimal choice no matter what), then he has no choice but to choose Player 1's favorite choice, internalizing that Player 1 can afford to receive no utility by sticking to his preferred choice. This gives to a patient Player 1 a motive to convince Player 2 that his type is the strong one, regardless of his true type, in order to build reputation and maximize his payoff. However, Player 2 can internalize this motive for Player 1, so his best strategy, given his information set, is to test up to a certain period whether Player 1 is the Stackelberg type indeed, or he is just mimicking.

Taking these two opposing effects into account, Atakan and Ekmekci derive under which assumptions Player 1 can convince Player 2 that his type is the strong one and under which assumptions Player 2's screening can be succesfull, leading a rational Player 1 to reveal his type in the first period. Their main finding is that if the players are equally patient, then Player 1 can always convince Player 2 that his type is the strong one and achieve the optimal payoff, regardless of who moves first and the initial Player 2's uncertainty distribution about Player 1's type.

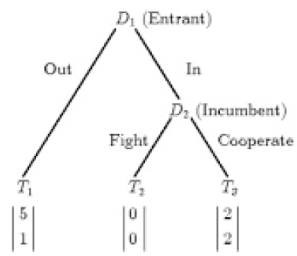
In the previous example, Player 1's strategy was simply to pretend that his type is the strong one until Player 2 stops screening. However, Player 1 may have other strategies, like trigger or tit-for-tat strategies. To be more general, Atakan and Ekmekci assume that Player 1 can be either fully rational or one of many commitment types. Commitment type means that the player sticks to a certain repeated game strategy. The commitment type that constitutes the milestone in their analysis is a dynamic pretending-to-be-strong type. The best case scenario for Player 1 would be to make his opponent believe that his future actions will be the actions that the strong type would do.

Previous literature on reputation is constrained be the assumption that Player 1 is patient and his opponent is myopic. A result by Fudenberg and Levine (1989) is that if Player 1 can commit that he will mimic the Stackelberg type with a positive probability in every period then in equilibrium he receives the highest static payoff while Player 2 is individually rational. Schmidt (1993) extends the previous result to a context where Player 2 is not myopic but he is still less patient than Player 1. However, none of these papers assume that all the players are long run players, thus Atakan's and Ekmekci's research is truly innovative. In this case Player 1 uses a stationary strategy that rewards or punishes Player 2, in order to achieve the highest possible payoff for himself. Hence, Player 2 may expect punishments or rewards either from the rational type of Player 1 after he chooses a move that would not be chosen by the strong type (Celentani, 1996) or from a commitment type other than the strong type (Schmidt, 1993). These difficulties lead all the findings of the previous literature to a precarious situation.

Atakan and Ekmekci make the following main contributions to the already existing results. First, they extend all the previous results to repeated extensive-form games of perfect information. In addition, they point the importance of perfect information for a reputation result in this context. Finally, they propose original techniques to analyse all the possible reputation results in infinitely repeated games. Their findings are on the context of repeated extensive-form games of perfect information, and according to their assumptions, they prove a reputation result for stage games with the LNCI or SCI, i.e. if a game that has the LNCI or the SCI and it is played in a complete information environment, then the Folk Theorem applies under a full dimensionality condition (Wen, 2002). Games with the LNCI have a common value part, while games with the SCI are characterized by the opposing interests of two players.

A game has LNCI if the unique payoff profile where Player 1 receives his highest stage game payoff is strictly individually rational for Player 2. For example the Battle of the Sexes game, where Player 2 moves first has the LNCI, because Player 1 receives his highest payoff.

Moreover, Player 2's payoff is strictly higher than his minimax.
A game has SCI, if Player 1 can successfully pretend that his type is the strong one and ensure to himself his highest payoff that respects Player 2's individual rationality constraint and Player 2's payoff is equal to his minimax. For example the classic Chain-Store game and the Battle of the Sexes game where Player 2 moves second have the SCI. We can verify that the Chain-Store game has the SCI because, if Player 1 can stick to the fight action regardless of Player 2's behaviour and Player 2 best responds to this fight action, then Player 1 receives the optimal payoff for him, while Player 2 receives exactly his minimax payoff.



	Entrant In	Entrant Out
Cooperate	2, 2	5, 1
Fight	0, 0	5, 1

Figure 2: Chain Store Paradox

The same applies to the battle of the Sexes game, where if Player 1 chooses the optimal option for him, then a rational Player 2 who wants to respond best will play accordingly and the outcome will be the maximum payoff for Player 1 and the minimax for his opponent. Their second important result highlights the reason why effects that have to do with reputation are especially principal in repeated games with perfect information and the LNCI, while this kind of effects are not present in some repeated simultaneous move games characterized by the LNCI property, e.g. their reputation result shows that there is a unique equilibrium payoff set in the Battle of the Sexes game (repeated sequencial move version). On the other hand, in case where a game with the LNCI and repeated sequencial move property is played every period, then the Folk Theorem applies.
Their third important result is the innovative method that they aplied so to prove their result related with reputation effects. The technique of Fudenberg and Levine (1989), which up to now was commonly used to prove reputation effect results, is not valid anymore with two equally patient agents, as Atakan and Ekmekci assume, hence a new method was important to be developed. Their technique depends on providing the information sets in which Player 1's standard type shows his rationality by singletons. Perfect information and sequencial rationality context impose strict bounds on Player 1's continuation payoff values in every subgame. In addition, for the games compatible with their assumptions, if there is a strict bound on Player 1's payoff, then it is immediate that there is a strict bound on Player 2's payoffs too. These bounds render impossible the event where Player 1 builds a reputation period by period and imposes a punishment on Player 2 when he responds optimally to his threats, as Atakan and Ekmekci note.

Their reputation result follows:

Theorem. Assume perfect information and the LNCI and SCI. For any $\delta \in [0, 1)$, any $\mu \in \Delta(\Omega)$ such that $\mu(S) > 0$, and any PBE strategy profile σ of $\Gamma^\infty(\mu, \delta)$, we have

$$U_1(\sigma, \delta) \geq \bar{g}_1 - f(\underline{z})max(1-\delta, \phi),$$

where $\underline{z} = \mu(S), \phi = \mu(\Omega_-)/\mu(S)$, and f is the decreasing, positive valued function defined by

$$f(z) = K(z)^{\bar{n}(z)}$$

where $K(z) = max(\frac{4\rho}{zl}, \frac{8M}{zl}(\rho n^p + 2), 2)$,

and $\bar{n}(z)$ is the smallest integer j such that $(1 - \frac{zl}{4\rho})^{j-1} < z$.

In conclusion, Atakan and Ekmekci note that Cripps, Dekel and Pesendorfer (2005) prove a result related with reputation concerning Bayes-Nash equilibria in the context of repeated games with simultaneous move and the SCI property. An equivalent conclusion can be reached by applying the method presented in Atakan's and Ekmekci's paper. Especially, one can difine from scratch the payoff function in the context of Bayes Nash equilibrium instead of Perfect Bayesian equilibrium. Concerning upper bound they obtained, it is still correct for Bayes Nash equilibria. The reason for this is that every argument was constructed and used on the equilibrium path without assuming sequential rationality or perfect information. Hence, the reputation result still holds. They also point out, as a field for further research, that they reached no conclusion on whether this reputation result applies to a wider class of dynamic games where a different static game takes place every period. Nevertheless, their reputation effects result holds in the following less general group of dynamic games: every finite number of perfect information static games takes place in each period. All of them have the SCI and the LNCI properties. A transition correspondence determines the static game that will be played in a certain period, the transition correspondence is characterized by stationarity and the Markovian property concerning that game that was played in the previous period, but not its outcome.

Concerning the second paper, Hansen (2000) claims that the fact that the majority of initial public offerings (IPOs) are characterized by a 7% spread, suggests either that the investment bankers collude and syndicate to ensure a 7% profit or that a 7% spread is the efficient level of spreads that occurs by the clearance of the competitive market. Hansen's results seem to support that the latter is more likely to be the case. Hansen points out that 7% is not an excessive level and also that public institutions have not noticed any discrepancy. In addition, the competitions among banks based on reputation and placement service is still strong, a fact that supports the efficient allocation claim. Hansen also provides an structural experiment to test whether the IPO market is efficient or collusive. One can notice that since the early 1990's the percentage of the IPOs with 7% spread has stabilized in a very high level. Hansen compares two contradicting theories, the cartel theory which claims that there is collusion in the IPO market and the efficient market theory which supports that the 7% IPO is the sole winner of competition that determines the optimal IPO contract.
Collusion in the IPO market takes two forms, explicit or implicit, in either case the profit yielded by the investment banks in 7%, which supposedly exceeds the normal profit level. Chen and Ritter (2000) suggest an explicit form of cartel in the IPO market and their findings

have led many of the IPO clients to initiate legal procedures concerning the investment banks plus a thorough investigation by the U.S. Department of Justice. Hansen claims that it is empirically difficult to distinguish between the explicit and the implicit form of collusion, as their impact on the observable data is similar. Hence, Hansen tests whether there is a collusion scheme (can be either explicit or implicit) or the competition hypothesis can stand. Explaining the survivorship principal, a type of contract can survive in the competitive market if and only if it is efficient. As Hansen explains, the investment banks compete each other on the reputation field an many other. Thus, the IPO contract market is surprising convoluted, taking into account price changes, insurance and advertising services, hence when the spread price is stabilised there is not strong evidence of collusive strategies because the contract's price will be determined by the competitive structure of the market. Assuming that the level of insurance and advertising would lead the spread above 7%, then the underpricing effect (due to competition) may reduce the price of the contract to a 7% equivalent. One can ask how might is the competition mechanism that drives the spread to 7% levels. Considering that the spread original value plus underpricing effect exceeds 15% in general(Carter, 1998), there can be spreads other than 7% that can survive in the competitive environment. The convergence on the 7% level may be a psychological effect or a common agreement, driven by a fortuitous strange use, or by its mighty allure, during the 1990's when IPO market skyrocketed. Taking into account that a constant spread could benefit the function of the whole IPO market, then one can claim that the 7% spread is the most agreeable number. A most agreeable number is the mean spread in IPOs with price other than 7%, although it is near 7%, which after rounding makes 7% a strong convergence point.

Finally, Hansen concludes that the IPO market is not concentrated, entry is sufficiently high and the 7% spread does not lead the firms to excessive profit. In addition, U.S. Department of Justice claims that there is absolutely no evidence supporting collusion, after a thorough investigation. In addition the empirical results are consistent with the efficient market hypothesis. However Hatfield, Kominers and Lowery (2016) have published a model in which collusion constitutes a subgame perfect Nash equilibrium, given that the investment banks can syndicate. Hence, we can not rule out yet any of the two hypotheses as there is evidence supporting both of them.

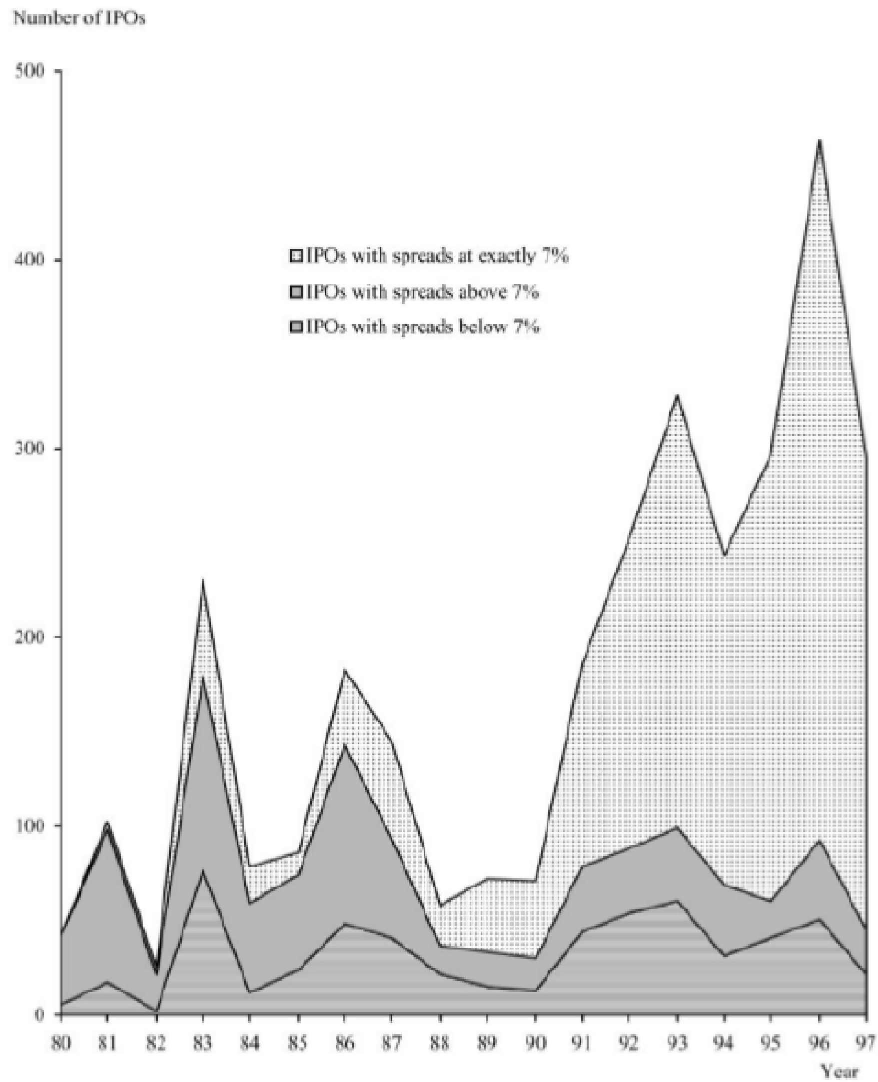


Figure 3: IPO spreads

At this point, a reference to three strongly relevant papers could not be ommited. The first is ***The Contractor's Game*** (Lang and Rosenthal, 1991). The framework is similar, as we describe, but the result differs in the crucial point of collusion existence. In more detail, the paper assumes:

- Symmetric information auction model
- Fixed cost for each job, as there is no way to split it
- No collusion

The two last assumptions are fundamentally different to this handout's ones, as we assume that the cost function is continuous and differentiable and so collusion is possible and profitable due to the convexity of the cost function.
Due to continuity and differentiability we can get an equilibrium in pure strategies, while Lang and Rosenthal provide an equilibrium in mixed strategies. In their equilibrium, each

firm submits a bid for a certain job with a probability p which depends on the model parameters. In addition, the value of the bid is random as well, it is drawn by a CDF which depends on the model parameters too. In our handout the strategy of each firm is fully deterministic.
The main difference of our handout's result and Lang and Rosenthal is the driving force behind their common output. Both results imply that as the number of firms increases the price for consumers increases as well, which is counterintuitive as it opposes to the main market competition idea. However, Lang and Rosenthal show this in a competitive market environment with no collusion, while we get our results from a anticompetitive market, where collusion between firms is common and deviation from it is punishable. Having those two results, one can conclude that when this price behavior is present, it is not clear whether there is or not a collusive environment in the market. In other words, this price movement does not constitute decisive evidence for or against an anticompetitive market.

The second relevant paper is ***Split Awards, Procurement, and Innovation*** (Anton and Yao, 1989). They develop a format of split award auctions, where the split choice is endogenous. They assume perfect information and investigate the existence of collusive equilibria. Similarly to our handout, they prove that collusion of the bidders can be profitable in equilibrium. But at this point the similarities end, as there are some important characteristics that are fundamentally different to our work.
First, in our handout, a job can be undertaken only by one firm. This rule is to ensure that the buyer will not suffer from the collusive equilibrium described above. However, it turns out that the firms will split the job by themselves in equilibrium.
Second, collusion in Anton and Yao is easily achievable, as they can receive an order for a fraction of the total job. However, in our work the firms can only split the award after the auction, which makes deviation tempting. This is the reason why our equilibrium needs an infinite amount of periods and a convoluted punishment scheme in order to ensure a viable collusion.
Third, Anton and Yao claim that the increase of the bidders leads to a cheaper price for the buyer. In our handout, the opposite happens, so one can see that our results are different to Anton and Yao in their core.
In conclusion, one can see that *Collusion in Markets with Syndication* is different to *Split Awards, Procurement, and Innovation*, as they discuss non relevant parts of game theory. Anton and Yao investigate whether the split award auction can hurt the buyer, while Hatfield, Cominers and Lowery prove that the buyer can be hurt even when the award cannot be split. However, we can draw a common generic result by those two and this is that collusion hurts the buyer in any case, so it should be prevented whenever possible.

The third paper is ***Dynamic Price Competition with Capacity Constraints and a Strategic Buyer*** (Anton, Biglaiser and Vettas, 2014), in which the authors analyze a dynamic durable good oligopoly model where sellers are capacity constrained. There is one buyer who places orders to two incumbent firms. These firms have to choose their capacities before the buyer places the order. Then they announce their prices, and the buyer chooses the best for him. The interesting part is that the game has two periods, which means that if the buyer uses the whole capacity of one seller in period 1, then the remaining seller will exercise a monopoly in period 2. Therefore, we have the following opposing forces in the buyer's and sellers' decisions:

- The buyer wants to have a cheap price in the first period, but he gets hurt by a monopoly in the second period. This leads him to buy from the expensive seller too, if the price does not exceed a certain threshold.

- The sellers know that the buyer can tolerate higher prices in the first period because of his preference for competition in the second period, but at the same time they can undercut their competitor and sell out their capacity in the first period.

Capacity constraints imply that a pure strategy equilibrium fails to exist. Instead, the sellers play a mixed strategy regarding their pricing. A common result with our handout is that the buyer is hurt by the dynamic nature of the game, as the sellers can enjoy higher profits than the competitive case because the buyer is willing to pay more in order to maintain the competition in the future. The important differences are that in our handout, collusion is the main reason that hurts the buyer and that the more firms exist the worse off the buyer is. One more technical difference is that the marginal cost in *Dynamic Price Competition with Capacity Constraints and a Strategic Buyer* is constant, while in our work is increasing, which forms the incentive to collude. Thus, there is collusion because of convex costs and a pure strategy equilibrium is sustainable, as there is the threat of turning the whole syndicate against the deviator.

2 Model

In this handout we consider a market consisted of two (2) consumers, in contrast with the monopsony case of Hatfield, Kominers and Lowery. The main question at this point is whether the best strategy for the firms allows both consumers to have access in the IPO's market or not. To answer this question we need to build a model, similar to HKL, enriched with some extra assumptions about the demand function.
There is a finite set of infinitely - lived identical firms F and an infinite sequence of one period - lived identical tuples of consumers c_{1t}, c_{2t}. Time is discrete and firms discount the future at the rate $\delta \in (0,1)$. Let us also define the market concentration by $\phi = \frac{1}{F}$.
Each firm is characterized by a cost function $e(s,m)$, where s is the fraction of the production made by a firm and m denotes the productive capacity owed by this firm. We impose symmetry by giving each firm the same productive capacity. We assume, intuitively, that the cost function is homogeneous of degree one. In addition, we assume that e is strictly increasing and strictly convex in s and strictly decreasing in m and that $e(0,m) = 0, \forall m$.

The stages of the game are described below:

1. Each buyer i announces her reservation price υ_i to the firms, which is equal with the utility she is about to enjoy after the contract is fulfilled.
2. Each firm f simultaneously announces to the buyers its price $p_t^f, \in [0,\infty)$ to fulfill the IPO contract.
3. All the buyers observe simultaneously all the firms's offers.
4. Each buyer accepts at most one offer; the buyer's option is immediately and publicly observed by the firms. In case that a buyer does not accept any offer, the game ends for her.
5. If an offer has been accepted by at least one buyer, then the firm f that placed that offer becomes the symdicate leader, who is responsible to fulfill the contract in order to receive the price amount that it offered. The syndicate leader can offer to any other firm $g \in F - f$ a fee equal to w_t^g so to undertake to follow through a fraction of the contract. These offers take place simultaneously and publicly.
6. Each firm $g \in F - f$ that received an offer by the leader either accepts of rejects it. From now on, we will call the set of the firms that accepted the syndication offer, the syndicate G_t. G_t is observable by any other firm.

Therefore, the payoffs for each party after period t will be:

- A consumer i that accepted some firm's offer will receive $\upsilon_i - p_t^f$.

- A consumet that did not accept any offer will receive 0.
- The syndicate leader f will receive $p_t^f - e(s_f, m_f) - \sum_{g \in G_t} w_t^g$.
- A syndicate member g will receive $w_t^g - e(s_g, m_g)$.
- The firms that do not belong to the syndicate receive 0.

Because of the convex cost function, each firm in the syndicate receives ex post an equal share of the whole contract, for efficiency reasons. We also normalize the whole production volume and total productive capacity to unity. Thus, the production costs of each firm participating in the syndicate, leader included, is $e(\frac{1}{|G_t|}, \phi)$. The payoffs can be represented as follows:

$p_t^f - e(\frac{1}{|G_t|}, \phi) - \sum_{g \in G_t} w_t^g$, for the leader and $w_t^g - e(\frac{1}{|G_t|}, \phi)$, for the syndicate members.

3 Optimal Collusion

In this section we calculate the highest sustainable price via collusion. We are going to prove that there is a strategy that yields more profit than the strategy of HKL model, given that there are two buyers. In HKL model, the price that characterizes the subgame perfect Nash equilibrium is a convex function of ϕ.

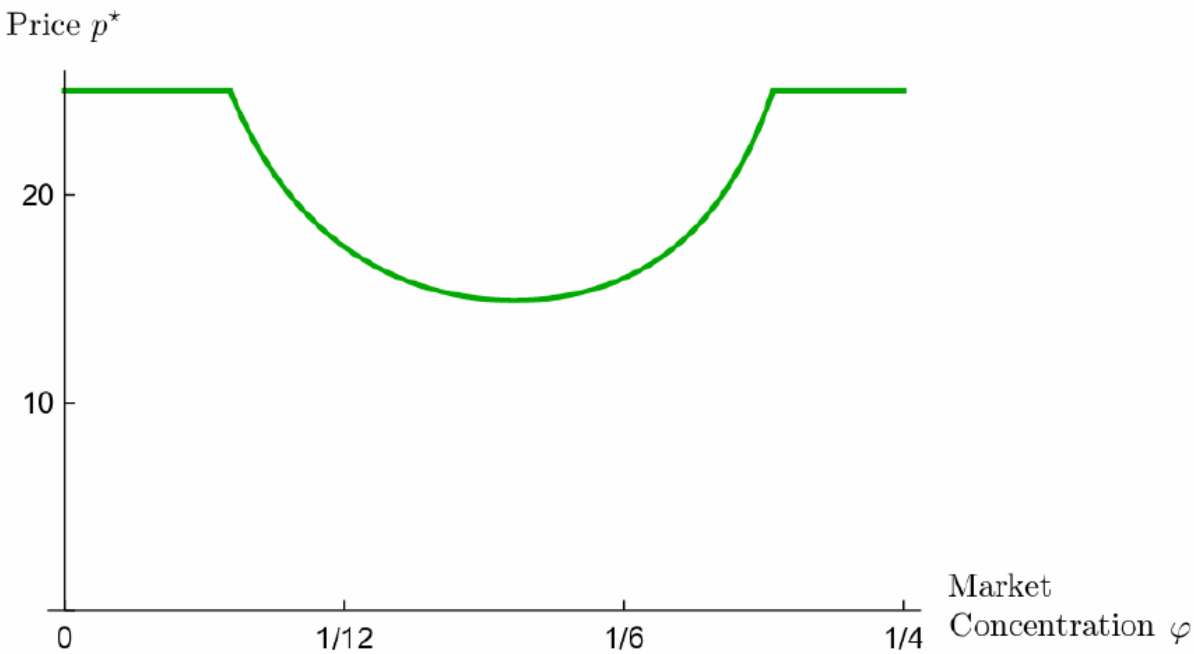


Figure 4: Highest sustainable Price in Collusion

We need to distinguish two seperate cases, concerning the reservation price of each buyer. In each of them, we are going to proof that the strategy shown below is a subgame perfect Nash equilibrium that yields more profits to the firms compared with the HKL strategy, in fact this strategy indicates the highest sustainable price.
First we need to define some parameters:

- We define as υ the highest reservation price among the two and with υ' the smallest one.
- We define as λ the fraction of the total production that the buyer with reservation price υ needs. By definition, the buyer with reservation price υ needs $1 - \lambda$.
- It is also realistic to assume that $\lambda > \frac{1}{2}$, because it yields greater utility to the same type of buyer.
- We also define as ϕ_1, ϕ_2 as the concentration of two groups of the syndicate, when the syndicate decides to form two groups, where each one follows through the contract of a different buyer.

- We define the constant c, given some ϕ_1, ϕ_2 as: $c = e(\lambda, \frac{\phi}{\phi_1}) + e(1-\lambda, \frac{\phi}{\phi_2}) - e(\lambda, 1)$
  We prove later that c is always a positive quantity, due to the convexity of the cost function e.

At this point it is necessary to make the following assumptions, in order a subgame perfect Nash equilibrium to be feasible:

1. $\delta \geq \frac{1}{2}$
   This assumption is reasonable, because the large majority of firms in IPO market operate for many years, hence we can assume that they do not discount the future heavily.

2. $\upsilon \geq e(\lambda, 1)$
   This ensures that a firm has not negative profits if it follows through the contract of the highest reservation price buyer, even by itself. This assumption is intuitively correct because in any other case, every firm would ask for a price higher than υ and no buyer would accepted it, leading to the end of the game.

3. $\upsilon' \geq e(\lambda, \frac{\phi}{\phi_1}) + e(1-\lambda, \frac{\phi}{\phi_2})$
   for some ϕ_1. Note that if ϕ_1 is defined, then $\frac{1}{\phi_2} = \frac{1}{\phi} - \frac{1}{\phi_1}$.

4. $\upsilon' \geq \frac{\upsilon + c}{2}$
   This is the most crucial assumption and it ensures that the reservation prices are close enough in order that both buyers accept an offer. We prove that for the cost functions that are not characterized by an extreme level of concavity, the constant c is not significant compared to the reservation prices.

5. $e(1-\lambda, \phi) \geq c$

6. $(2\delta-1)\cdot[e(\lambda,\phi)+e(1-\lambda,\phi)]+\delta\cdot[e(\lambda,\phi)-e(1-\lambda,\phi)] \geq (2\delta-1)\cdot[e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2})]$

The last two assumptions have also to do with the convexity level of the cost function.

Theorem 1. *Given that the assumptions* $(1)-(6)$ *hold, the highest price sustainable in a subgame perfect Nash equilibrium,* p'*, is given by*

$$p' = \begin{cases} \upsilon' & \phi \in [1-\delta, 1] \\ min[\upsilon', \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)}] & \phi \in [0, 1-\delta) \end{cases}$$

Moreover, p' *is quasiconvex in* ϕ *and* $\lim_{\phi\to 0} p' = \upsilon'$

Figure 5 plots the highest sustainable price p' as a function of ϕ.

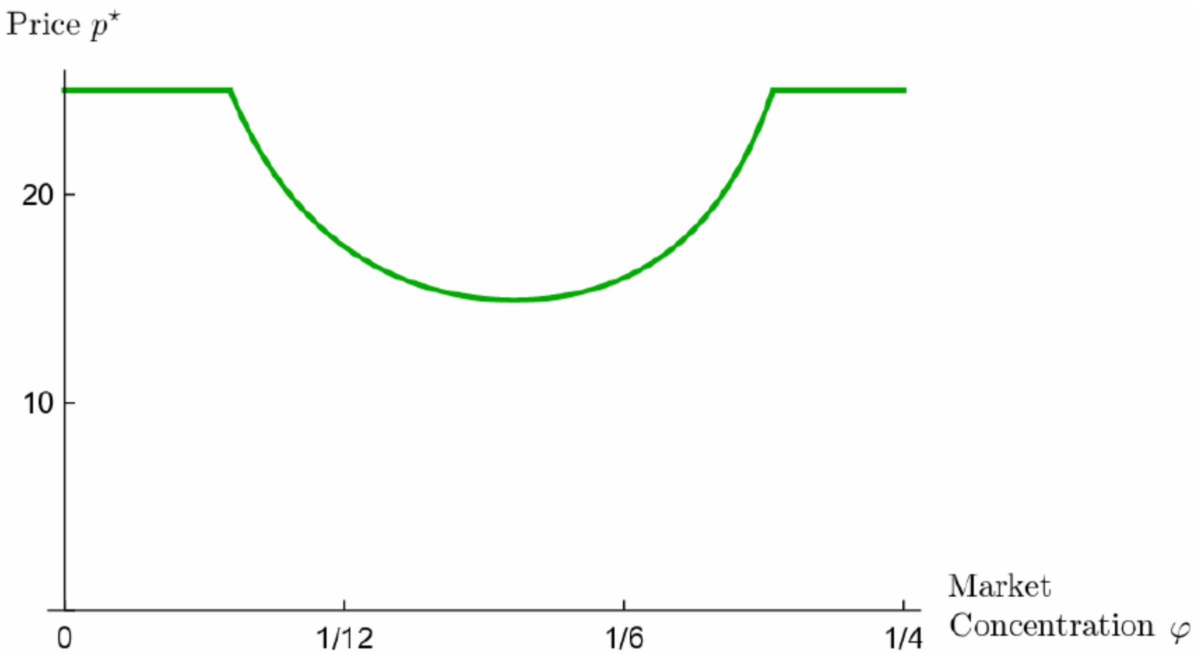


Figure 5: Highest sustainable Price curve as a function of ϕ

We prove Theorem 1 by considering two cases concerning the reservation prices of the buyers. In the first case the buyer with the lowest reservation price υ' is characterized by $\upsilon' > p^*, \forall \phi \in [0, 1]$, where p^* is the price in the subgame perfect Nash equilibrium in HKL model, denoted by:

$$p^* = \begin{cases} \upsilon & \phi \in [1-\delta, 1] \\ min[\upsilon, \frac{(1-\delta)\cdot(e(1,\phi))-\phi\cdot(e(1,1))}{(1-\delta-\phi)}] & \phi \in [0, 1-\delta) \end{cases}$$

Where υ is the reservation price of the only buyer. In the second case, $\exists \phi \in [0, 1]$ such that $\upsilon' \leq p^*$.

3.1 Case 1

- $\upsilon' > p^*, \forall \phi \in [0, 1]$

First, we need to compare the intertemporal profit that every pricing method yields.

3.1.1 HKL pricing $p = p^*$

Given this pricing method, only the buyer with the high reservation price will buy, because $\upsilon' > p^*, \forall \phi \in [0, 1]$. Hence, given that $\delta > \frac{1}{2}$ and $e(\lambda, 1) \leq \upsilon$) (Assumptions 1, 2):

$$profit_{HKL} = \frac{1}{1-\delta} \cdot \phi \cdot (p - e(\lambda\phi, \phi) - (|F| - 1) \cdot e(\lambda\phi, \phi) = \frac{\phi}{1-\delta}(p - |F| \cdot e(\lambda\phi, \phi)) = \frac{\phi}{1-\delta} \cdot (p - e(\lambda, 1))$$

in a subgame perfect Nash equilibrium context, as it is proved in HKL model.

3.1.2 HKL pricing $p = p'$

This pricing allows both buyers to buy, as we are going to show. Hence, first we calculate the syndicate leader's profit:

$$\begin{aligned}
&profit_{leader} = \\
&= \phi \cdot (p' - e(\lambda\phi_1, \phi) - (\tfrac{1}{\phi_1} - 1) \cdot e(\lambda\phi_1, \phi)) + \phi \cdot (p' - e((1-\lambda)\phi_2, \phi) - (\tfrac{1}{\phi_2} - 1) \cdot e(\lambda\phi_2, \phi)) = \\
&= \phi \cdot (2p' - e(\lambda, \tfrac{\phi}{\phi_1}) - e((1-\lambda), \tfrac{\phi}{\phi_2}))
\end{aligned}$$

Therefore, the intertemporal profits of each firm equal:

$$profit_{p'} = \frac{\phi}{1-\delta} \cdot (2p' - e(\lambda, \tfrac{\phi}{\phi_1}) - e((1-\lambda), \tfrac{\phi}{\phi_2}))$$

We need to compare the profit of each strategy, so to verify the dominant one. It holds that $profit_{HKL} \leq profit_{p'}$.

<u>Proof:</u>

$$\begin{aligned}
&profit_{HKL} \leq profit_{p'} \Leftrightarrow \\
&p^* - e(\lambda, 1) \leq 2p' - e(\lambda, \tfrac{\phi}{\phi_1}) - e((1-\lambda), \tfrac{\phi}{\phi_2}) \Leftrightarrow \\
&p' \geq \tfrac{1}{2} \cdot [p^* - e(\lambda, 1) + e(\lambda, \tfrac{\phi}{\phi_1}) + e((1-\lambda), \tfrac{\phi}{\phi_2})]
\end{aligned}$$

We need to verify in which cases does the condition above hold.

- $\phi \in [1 - delta, 1]$ Then, by definition, $p^* = \upsilon$ and $p' = \upsilon'$, hence we need to prove that:

  $$\begin{aligned}
  &\upsilon' \geq \tfrac{1}{2} \cdot [\upsilon - e(\lambda, 1) + e(\lambda, \tfrac{\phi}{\phi_1}) + e((1-\lambda), \tfrac{\phi}{\phi_2})] \Leftrightarrow \\
  &\upsilon' \geq \tfrac{\upsilon + c}{2}
  \end{aligned}$$

  which holds by assumption 4. We also need to examine whether c takes values such that the inequality above does not lead to a contradiction.

- $\phi \in [0, 1-\delta)$ We need to show that:

  $$p' \geq \tfrac{1}{2} \cdot [p^* - e(\lambda, 1) + e(\lambda, \tfrac{\phi}{\phi_1}) + e((1-\lambda), \tfrac{\phi}{\phi_2})]$$

  By definition, when $\phi \in [0, 1-\delta) : p^* = min[\upsilon, \frac{(1-\delta)\cdot(e(1,\phi)) - \phi \cdot (e(1,1))}{(1-\delta-\phi)}]$,
  $p' = min[\upsilon', \frac{(1-\delta)\cdot(e(\lambda,\phi) + e(1-\lambda,\phi)) - \phi \cdot (e(\lambda, \frac{\phi}{\phi_1}) + e(1-\lambda, \frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)}]$

If $p^* = \upsilon$ then $p' = \upsilon'$ and the proof is the same with the previous case where $\phi \in [1 - delta, 1]$. If $p^* = \frac{(1-\delta)\cdot(e(1,\phi))-\phi\cdot(e(1,1))}{(1-\delta-\phi)}$, then $p' = \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)}$.

Thus, we have to prove:

$p' \geq \frac{1}{2} \cdot [p^* - e(\lambda, 1) + e(\lambda, \frac{\phi}{\phi_1}) + e((1-\lambda), \frac{\phi}{\phi_2})] \Leftrightarrow$

$\frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)} \leq \frac{(1-\delta)\cdot(e(1,\phi))-\phi\cdot(e(1,1))}{2\cdot(1-\delta-\phi)} + \frac{c}{2} \Leftrightarrow$

$(1-\delta) \cdot (e(\lambda, \phi) + e(1-\lambda, \phi)) - \phi \cdot (e(\lambda, \frac{\phi}{phi_1} + e(1-\lambda, \frac{\phi}{phi_2}) \geq$
$(1-\delta) \cdot e(\lambda, \phi) - \phi \cdot e(\lambda, 1) + c \cdot (1 - \delta - \phi) \Leftrightarrow$

$(1-\delta) \cdot e(1-\lambda, \phi) \geq \phi \cdot (e(\lambda, \frac{\phi}{\phi_1}) + e(1-\lambda, \frac{\phi}{\phi_2}) - e(\lambda, 1)) + c \cdot (1 - \delta - \phi)$

But $e(\lambda, \frac{\phi}{\phi_1}) + e(1-\lambda, \frac{\phi}{\phi_2}) - e(\lambda, 1) = c$, hence:

$(1-\delta) \cdot e(1-\lambda, \phi) \geq (1-\delta) \cdot c \Leftrightarrow$

$e(1-\lambda, \phi) \geq c$, which holds by Assumption 5.

We conclude:

$$p' = \begin{cases} \upsilon' & \phi \in [1-\delta, 1] \\ min[\upsilon', \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)}] & \phi \in [0, 1-\delta) \end{cases}$$

yields $\forall \phi \in [0, 1]$ more profit than

$$p^* = \begin{cases} \upsilon & \phi \in [1-\delta, 1] \\ min[\upsilon, \frac{(1-\delta)\cdot(e(1,\phi))-\phi\cdot(e(1,1))}{(1-\delta-\phi)}] & \phi \in [0, 1-\delta) \end{cases}$$

of the HKL model, given that assumptions (1-6) hold.

Corollary: If $\upsilon' \leq \frac{\upsilon}{2}$, then the buyer with low reservation price cannot buy eventually, as HKL pricing yields more profit, as we have shown above.

3.1.3 Bertrand Reversion Nash Equilibrium

In order to delineate the strategy of the firms in which our proposed pricing constitutes a subgame perfect Nash equilibrium, we need to show that certain strategy sets are Nash equilibria in certain subgames. We first describe the Bertrand Reversion Nash equilibrium, i.e. the equilibrium in which all firms make zero profits and the buyers obtain the good at the lowest possible cost of production. In this equilibrium every firm offer a price $p^f = e(\lambda, \frac{\phi}{\phi_1}) + e(1-\lambda, \frac{\phi}{\phi_2})$, where ϕ_1, ϕ_2 are chosen that way so to minimize the expression. The buyers choose one firm each, randomly. Then the syndicate leader will offer to each

firm g that belongs to ϕ_1 a fee $w_1^g = e(\phi_1 \cdot \lambda, \phi)$ and to each firm that belongs to ϕ_2 a fee $w_2^g = e(\phi_2 \cdot (1-\lambda), \phi)$. These fees are equal to the cost of production of each firm, hence every firm will accept the offer and have zero profit. The syndicate leader will also be a member in one of the two groups, thus its total cost will be equal to $\frac{1}{\phi_1} \cdot w_1^g + \frac{1}{\phi_2} \cdot w_2^g =$
$= \frac{1}{\phi_1} \cdot e(\phi_1 \cdot \lambda, \phi) + \frac{1}{\phi_2} \cdot e(\phi_2 \cdot (1-\lambda), \phi) = e(\lambda, \frac{\phi}{\phi_1}) + e(1-\lambda, \frac{\phi}{\phi_2}) = p^f$.
Therefore the syndicate leader breaks even too.

This strategy set is a subgame perfect Nash equilibrium, because a firm will not offer a higher price, the buyers will not choose it. Also a firm will not offer a lower price because even if it distributes its offers at the most efficient way, which is the abovementioned, it will have negative profit, while its current profit is zero. In addition a firm will not offer a lower fee to the other firms, because they will not accept to have negative profit. A higher fee would lead the leader to have damage.

Our first result shows that the Bertrand reversion Nash equilibrium just described a subgame perfect Nash equilibrium, in which each firm obtains its lowest individually rational payoff.

Proposition 1. There exists a subgame perfect Nash equilibrium of the stage game, in which each firm obtains a payoff of 0, its lowest individually rational payoff.

3.1.4 Maintaining Collusion when the market is concentrated

We show that when the number of firms is small enough i.e. $\phi \geq 1-\delta$, the collusion can be sustainable under p' pricing.

Proposition 2. If $\phi \geq 1-\delta$, then the following strategy constitutes a subgame perfect Nash equilibrium:

- $p_t^f = p'(\phi) = \upsilon' \quad \forall t, \forall f \in F$
- The leader calculates ϕ_1, ϕ_2 to minimize the total cost.
- The syndicate leader offers to each firm g that belongs to ϕ_1 a fee $w_1^g = e(\phi_1 \cdot \lambda, \phi)$ and to each firm that belongs to ϕ_2 a fee $w_2^g = e(\phi_2 \cdot (1-\lambda), \phi)$
- Every firm accepts to cooperate with the leader.
- Reversion to Bertrand pricing if any firm at any t deviates from the pricing strategy above.

To prove Proposition 2 we need to show that no firm has a motive to deviate i.e. to compare the profit in case of deviation with the profit yielded by the strategy. When all the firms cooperate, one firm can be the syndicate leader at a given period t and earn profit equal to $\pi_{coop}^t = 2 \cdot p' - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})$ and the rest of the firms will have zero profit. Each firm will be the syndicate leader with an equal probability ϕ. In case of cooperation this will happen for all the periods of the game $t = 0, 1, ..., \infty$. Therefore, the total profit of each firm will be:

$\pi_{coop} = \sum_{t=0}^{\infty} \delta^t \phi \cdot (2 \cdot \upsilon' - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})) = \frac{\phi}{1-\delta} \cdot (2 \cdot \upsilon' - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2}))$

A firm has two ways to deviate. First, it can deny to join the syndicate. If this happens, the game will reverse to Bertrand phase, where the deviating firm will have zero profits. In the deviation stage, the deviator will also have zero profit as it denied to join the syndicate. Thus, a firm will not deviate in this way, as it can have positive profits in case of cooperation. Second, a firm can offer a slightly lower price so both buyers will choose it. This means that the game will reverse to Bertrand phase after this stage and everybody will have zero profit. However at the deviation stage, the deviator will offer to each firm g that belongs to ϕ_1 a fee $w_1^g = e(\phi_1 \cdot \lambda, \phi)$ and to each firm that belongs to ϕ_2 a fee $w_2^g = e(\phi_2 \cdot (1-\lambda), \phi)$ and they will accept. The profit of the deviator in this period will be $\pi_{bert} = 2 \cdot \upsilon' - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})$, as it will be the leader for certain by offering the lowest price, just below $p' = \upsilon'$.
We need to compare π_{coop} with π_{bert} so to find out whether deviating can be rational. We claim that:

$\pi_{coop} \geq \pi_{bert} \Leftrightarrow$
$\frac{\phi}{1-\delta} \cdot (2 \cdot \upsilon' - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})) \geq 2 \cdot \upsilon' - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2}) \Leftrightarrow$
$\frac{\phi}{1-\delta} \geq 1 \Leftrightarrow$
$\phi \geq 1 - \delta$, which holds.

Therefore, Proposition 2. is valid.

3.1.5 Maintaining Collusion when the market is not concentrated

We need construct a strategy that leads to a subgame perfect Nash equilibrium for the rest of values that ϕ can take i.e. $\phi \in [0, 1-\delta)$. The key idea is to construct strategies that exploit syndicate boycotting to enforce higher prices. Play begins in the cooperation phase, in which each firm offers the same price p' and a firm, upon having its offer accepted, engages in efficient syndication. The game continues in the cooperation phase as long as no one deviates. Intuitevely, other firms can punish a price deviator within period, by refusing to join its syndicate, therefore increasing the cost of production of the deviator. However, in order to incentivize firms not to join the deviator's syndicate we need to promise them rewards in future periods. For this reason, Bertrand reversion might not be the most efficient strategy to punish a deviator in this case.
After a period in which firm f was the price deviator, we enter to the collusive punishment phase, if and only if no firm accepted to join its syndicate in the deviation period. During this phase, the firms that refused to join the deviator's syndicate enjoy the rewards for their refusal to assist the deviator. This phase has also to be subgame perfect. If the price is too low, then the promised rewards may not be sufficient in order to prevent the firms to join the deviator's syndicate. If the price is too high, then the firms will have a motive to deviate from it and complete the IPO's by themselves, or by assembling a syndicate. Also, "the reward should fit the temptation" (Mailath et al., 2016), meaning that the higher the fees offered by the deviator, the higher should be the promised rewards for not joining its syndicate.
In conclusion, in this section we discuss a subgame perfect Nash equilibrium, that prevents any firm from deviating, even if the concentration is low. The first phase is the cooperation

phase, in which every firm offers a price p', as discussed previously and the buyers choose a firm randomly. Then the leader organizes efficiently the firms into syndicates and pay them fees equal to their production costs. When a firm deviates in the cooperation phase, we enter in the collusive punishment phase. In this phase, if every firm denied to join the deviator's syndicate then a new price q is offered by every firm. The syndicate leader in this case has to pay the fee equal to the production cost of each firm, plus the compensation fee ψ to everyone but the deviator. In case that some firm accepted to join the deviator's syndicate during the cooperation phase, or some firm deviates during the collusive punishment phase, then the game enters to its final phase, Bertrand Reversion, that we have proved that is subgame perfect.
Therefore, we have to prove that the collusive punishment phase is subgame perfect and that the whole strategy described above is also subgame perfect, so to conclude that pricing at p' is sustainable and no firm is likely to deviate in the cooperation phase.

We now give a formal construction of the strategy profile that supports p'. We first construct an equilibrium that support this price for $p' < v'$. We then extend to the case that $p' = v'$. The equilibrium is constructed as follows:

- There are three phases of equilibrium play:
  1. In the cooperation phase,
     - every firm submits tha same bid $p = p'$,
     - the short lived buyers then accept one offer at $p = p'$, choosing each with equal probability,
     - every firm, if it becomes the syndicate leader, offers to each firm g that belongs to ϕ_1 a fee $w_1^g = e(\phi_1 \cdot \lambda, \phi)$ and to each firm that belongs to ϕ_2 a fee $w_2^g = = e(\phi_2 \cdot (1-\lambda), \phi)$ to join the syndicate, and
     - every other firm accepts this offer.
  2. In the collusive punishment phase with continuation values ψ,
     - every firm submits the same bid $q = min(e(1-\lambda, \phi), v')$,
     - the short lived buyers then accept one offer at $p = p'$, choosing each with equal probability,
     - every firm $g \in F$, if it becomes the syndicate leader, offers to each firm $h \in F - [g]$ that belongs to ϕ_1 a fee $w_1^h = e(\phi_1 \cdot \lambda, \phi) + \psi^h$ and to each firm that belongs to ϕ_2 a fee $w_2^h = e(\phi_2 \cdot (1-\lambda), \phi) + \psi^h$ to join the syndicate, and
     - every other firm accepts the offer by the syndicate leader g to join the syndicate.
  3. In the Bertrand reversion phase, agents play the Bertrand reversion Nash equilibrium.
- Under equilibrium play, the game starts and continues in the cooperation phase until some firm deviates. When this happens, the next step is determined by the level of fees that the deviator will offer to each firm to convince it to join its syndicate. Intuitively, the offers have to be large enough, to persuade other firms to deviate by joining the syndicate and accepting the Bertrand reversion by the next period. We calculate the positive differences between the syndication offers and the production cost of completing a ϕ fraction of the IPO as $\sum_{g \in F-[f]} (w^g - e(\phi, \phi))^+$. We categorize the set of offers made by a deviating firm f into three categories: uniformly low offers, insufficient offers, and

sufficient offers.

Uniformly Low Offers: $\sum_{g\in F-[f]}(w^g - e(\phi,\phi))^+ = 0$. In this case, all syndication offers are insufficient to induce any other firm to accept the syndication because they are weakly less than their cost of production. Therefore no firm accepts the offer and the game enters its Bertrand reversion phase in the next period.

Insufficient offers:$0 < \sum_{g\in F-[f]}(w^g - e(\phi,\phi))^+ \leq \frac{\delta}{1-\delta} \cdot (2 \cdot q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})^+$. In this case, absent dynamic rewards and punishments, some firms would be tempted to accept the syndication offer. All the firms do reject the syndication offers and the game proceeds to the collusive punishment phase with

$$\psi^h = \begin{cases} \frac{w^h - e(\phi,\phi)}{\sum_{g\in F-[f]}(w^g - e(\phi,\phi))^+} \cdot (2 \cdot q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2}))^+, & h \neq f \\ 0, & h = f \end{cases}$$

Sufficient offers: $\sum_{g\in F-[f]}(w^g - e(\phi,\phi))^+ > \frac{\delta}{1-\delta} \cdot (2 \cdot q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2}))$. In this case, the game enters the Bertrand reversion phase in the next period. During the current period each firm h accepts the offer if and only if its share in the total offer exceeds its production cost given the actions of the other firms, i.e. firm accepts iff $w^h \geq \bar{w}$, where $\bar{w} = e(\sum_{g\in F-[f]} \mathbb{I}_{w^g \geq \bar{w}}, \phi)$.

Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

Figure 6 provides an automaton representation of the subgame perfect Nash equilibrium described previously.

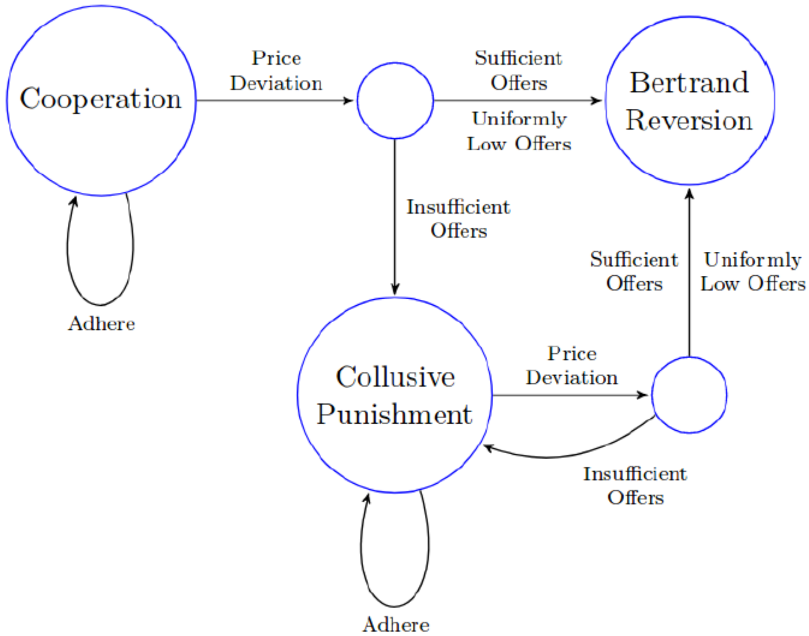


Figure 6: Nash Equilibrium Strategy Diagram

It is immediate that the conjectured equilibrium delivers a price of p' in each period. We now verify that the prescribed strategies constitute a subgame-perfect Nash equilibrium. We first show that the prescribed actions regarding accepting or rejecting syndication offers are the best responses.
In the cooperation phase, we have calculated that the intertemporal profit of the firm equals:

$$profit_{p'} = \frac{\phi}{1-\delta} \cdot (2p' - e(\lambda, \frac{\phi}{\phi_1}) - e((1-\lambda), \frac{\phi}{\phi_2}))$$

which is strictly greater than zero. Therefore, every firm should accept the prescribed syndication offer in every period, or else the game is about to transit to its Bertrand phase, where the profit of each firm is zero. Trivially, the same result holds for the uniformly low offers, where $\sum_{g \in F-[f]} (w^g - e(\phi, \phi))^+ = 0$. A firm should never accept any offer of this kind, because it does not yield to it any additional profit, perhaps even less, compared to rejection. In any case the game will transit to its Bertrand phase in the next period.
In case of insufficient offers, where
$0 < \sum_{g \in F-[f]} (w^g - e(\phi, \phi))^+ \leq \frac{\delta}{1-\delta} \cdot (2 \cdot q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})$, the best response for every firm is also to reject this offer. To show this, we calculate and compare the profits of accepting and rejecting this insufficient offer.

In case of acceptance, the firm will split the workload with the deviator, thus its profit will be:

$$w^h - e(\frac{\lambda}{2}, \phi) - e(\frac{1-\lambda}{2}, \phi) < w^h - e(\phi, \phi)$$

by the convexity of e.

In case of rejection, its profit will be derived by adding up the collusive punishment phase's rewards.

$\frac{\delta}{1-\delta} \cdot \psi^h = \frac{\delta}{1-\delta} \cdot [\frac{w^h - e(\phi,\phi)}{\sum_{g\in F-[f]}(w^g - e(\phi,\phi))^+} \cdot (2 \cdot q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2}))^+]$ (I)

The offer is insufficient, thus by definition:

$\sum_{g\in F-[f]}(w^g - e(\phi,\phi))^+ \leq \frac{\delta}{1-\delta} \cdot (2 \cdot q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})^+ \Rightarrow$
$\frac{\delta}{1-\delta} \cdot \frac{(2\cdot q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})^+}{\sum_{g\in F-[f]}(w^g - e(\phi,\phi))^+} \geq 1 \Rightarrow$

By (I):

$\frac{\delta}{1-\delta} \cdot \frac{\psi^h}{w^h - e(\phi,\phi)} \geq 1 \Rightarrow$
$\frac{\delta}{1-\delta} \cdot \psi^h \geq w^h - e(\phi,\phi) \geq w^h - e(\frac{\lambda}{2}, \phi) - e(\frac{1-\lambda}{2}, \phi)$

Therefore, no firm accepts an insufficient offer.

In case of sufficient offers, given that the game will enter to its Bertrand phase next period, the firms are about to accept the offers, as the collusive punishment compensation is not sufficient to make them reject the syndication offer of the deviator.

To sum up, we have shown:

- Cooperation Phase: All the firms accept the syndication offers.
- Deviation-Uniformly Low Offers: All the firms reject the syndication offer, as it yields zero or negative profits and the game enters the Bertrand phase.
- Deviation-Insufficient Offers: All the firms reject the syndication offer so to receive the collusive punishment compensation, which yields more profit than the offer.
- Deviation-Sufficient Offers: The firms will accept the offer and the game enters the Bertrand phase.

We need also to examine whether a firm has the incentive to deviate from the Cooperation Phase and the Collusive Punishment. The firm knows that in case of deviating, if it offers uniformly low, or insufficient offers then it will have to complete the job by itself. It can only persuate other firms to help it if and only if its offers are sufficient.

1. **Cooperation Phase:**
   First, we prove that a firm will never deviate to make uniformly low or insufficient offers. In this case it will have to complete the job by itself:

   $\pi_{deviation} \leq \pi_{cooperation} \Leftrightarrow$

   $2p' - e(\lambda, \phi) - e(1-\lambda, \phi) \leq \frac{\phi}{1-\delta} \cdot (2p' - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})) \Leftrightarrow$

$$2(1-\delta)\cdot p' - (1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi)) \leq \phi\cdot 2p' - \phi\cdot(e(\lambda,\tfrac{\phi}{\phi_1})+e(1-\lambda,\tfrac{\phi}{\phi_2})) \Leftrightarrow$$

$$2p'\cdot(1-\delta-\phi) \leq (1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi)) - \phi\cdot(e(\lambda,\tfrac{\phi}{\phi_1})+e(1-\lambda,\tfrac{\phi}{\phi_2}))$$

If $\phi \geq 1-\delta$ then we need to show:

$$p' \leq \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2(1-\delta-\phi)}$$

which holds, because if $\phi < 1-\delta$ then $p' = min[\upsilon', \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)}]$

If $\phi \geq 1-\delta$ then by its definition, $p' = \upsilon' > 0$, therefore we need to show:

$$1-\delta-\phi \leq \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot\upsilon'}$$

which holds trivially, as the left hand side of the inequality is non-positive and the right hand side is non-negative.

In case of sufficient offers, we also prove that deviation is not profitable. When a firm is making sufficient offers, it has to compensate the syndication members not only for their production cost, but for the collusive punishment compensation they would receive in case of rejection.

$$\pi_{cooperation} \geq \pi_{deviation} \Leftrightarrow$$

$$\frac{\phi}{1-\delta}\cdot(2p' - e(\lambda,\tfrac{\phi}{\phi_1}) - e(1-\lambda,\tfrac{\phi}{\phi_2})) \geq$$
$$2p' - e(\lambda,\tfrac{\phi}{\phi_1}) - e(1-\lambda,\tfrac{\phi}{\phi_2}) - \frac{\delta}{1-\delta}\cdot(2q - e(\lambda,\tfrac{\phi}{\phi_1}) - e(1-\lambda,\tfrac{\phi}{\phi_2})) \Leftrightarrow$$

$$\phi\cdot(2p' - e(\lambda,\tfrac{\phi}{\phi_1}) - e(1-\lambda,\tfrac{\phi}{\phi_2})) \geq$$
$$(1-\delta)\cdot(2p' - e(\lambda,\tfrac{\phi}{\phi_1}) - e(1-\lambda,\tfrac{\phi}{\phi_2})) - \delta\cdot(2q - e(\lambda,\tfrac{\phi}{\phi_1}) - e(1-\lambda,\tfrac{\phi}{\phi_2})) \Leftrightarrow$$

$$2(1-\delta-\phi)\cdot p' \leq (1-\delta-\phi)\cdot(e(\lambda,\tfrac{\phi}{\phi_1})+e(1-\lambda,\tfrac{\phi}{\phi_2})) + \delta\cdot(2q - e(\lambda,\tfrac{\phi}{\phi_1}) - e(1-\lambda,\tfrac{\phi}{\phi_2})) \Leftrightarrow$$

$$2(1-\delta-\phi)\cdot p' \leq (1-2\delta-\phi)\cdot(e(\lambda,\tfrac{\phi}{\phi_1})+e(1-\lambda,\tfrac{\phi}{\phi_2})) + 2\delta q \Leftrightarrow$$

If $\phi < 1-\delta \Rightarrow 1-\delta-\phi > 0$, we need to show:

$$p' \leq \frac{(1-2\delta-\phi)}{2(1-\delta-\phi)}\cdot(e(\lambda,\tfrac{\phi}{\phi_1})+e(1-\lambda,\tfrac{\phi}{\phi_2})) + \frac{2\delta}{2(1-\delta-\phi)}\cdot q$$

But, by its definition, $q = min[\upsilon', e(1-\lambda,\phi)]$

If $e(1-\lambda,\phi) < \upsilon'$, then we need to show:

$$p' \leq \frac{(1-2\delta-\phi)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))+2\delta\cdot e(1-\lambda,\phi)}{2(1-\delta-\phi)}$$

But $p' = min[\upsilon', \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)}]$, as $\phi < 1-\delta$, thus it is sufficient to show:

$\frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)} \leq \frac{(1-2\delta-\phi)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))+2\delta\cdot e(1-\lambda,\phi)}{2(1-\delta-\phi)} \Leftrightarrow$

$(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2})) \leq$
$(1-2\delta-\phi)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))+2\delta\cdot e(1-\lambda,\phi) \Leftrightarrow$

$(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi)) \leq (1-2\delta)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))+2\delta\cdot e(1-\lambda,\phi) \Leftrightarrow$

$(1-\delta)\cdot e(\lambda,\phi)+(1-3\delta)\cdot e(1-\lambda,\phi) \leq (1-2\delta)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2})) \Leftrightarrow$

$(1-2\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))+\delta\cdot(e(\lambda,\phi)-e(1-\lambda,\phi)) \leq (1-2\delta)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2})) \Leftrightarrow$

$(2\delta-1)\cdot[e(\lambda,\phi)+e(1-\lambda,\phi)]+\delta\cdot[e(\lambda,\phi)-e(1-\lambda,\phi)] \geq (2\delta-1)\cdot[e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2})]$

Which holds, by assumption (6). We should notice that for λ sufficiently close to $\frac{1}{2}$, which is in agreement with assumption (4), the result holds anyway and we do not need to assume (6).

Towards the other case about q, if $e(1-\lambda,\phi) \geq \upsilon'$, then we conclude that $p' = \upsilon'$ and it is sufficient to show:

$\upsilon' \leq \frac{(1-2\delta-\phi)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))+2\delta\cdot\upsilon'}{2(1-\delta-\phi)} \Leftrightarrow$

$2(1-\delta-\phi)\cdot\upsilon' \leq (1-2\delta-\phi)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))+2\delta\cdot\upsilon') \Leftrightarrow$

$2(1-2\delta-\phi)\cdot\upsilon' \leq (1-2\delta-\phi)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2})) \Leftrightarrow$

$2\upsilon' \geq (e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))$

which holds by assumption (3).

If $\phi \geq 1-\delta \Rightarrow 1-\delta-\phi \leq 0$, then $q = e(1-\lambda,\phi)$, because e is a decreasing function with respect to ϕ. In other case, q should always be equal to υ'. In addition $p' = \upsilon'$ because $\phi \geq 1-\delta$. Therefore it is sufficient to show:

$\upsilon' \geq \frac{(1-2\delta-\phi)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))+2\delta\cdot e(1-\lambda,\phi)}{2(1-\delta-\phi)} \Leftrightarrow$

$\upsilon' \geq (e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))+\frac{2\delta\cdot e(1-\lambda,\phi)-(1-\phi)\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2(1-\delta-\phi)} \Leftrightarrow$

$\upsilon' \geq (e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))+\frac{(\delta-1+\phi)(e(1-\lambda,\phi)-e(1-\lambda,\frac{\phi}{\phi_2})+\delta\cdot e(1-\lambda,\phi)-(1-\phi)\cdot e(\lambda,\frac{\phi}{\phi_1})}{2(1-\delta-\phi)}$,

where the first term of the numerator is positive, because $\phi < \frac{\phi}{\phi_2}$ and function e is decreasing with respect to ϕ. The second term is also positive because $\delta > 1-\phi$ and $\phi < \frac{\phi}{\phi_1}$. The denominator is negative as $\delta > 1-\phi$, thus the whole fraction is negative. Hence it suffices to show that the inequality holds, even if we ignore the negative fraction, i.e.:

$\upsilon' \geq (e(\lambda, \frac{\phi}{\phi_1}) + e(1-\lambda, \frac{\phi}{\phi_2}))$

which holds by assumption (3).

In conclusion, we have proved that given that no firm will deviate during Collusive Punishment Phase, no firm will deviate during Cooperation Phase. To complete the proof that our strategy is a subgame perfect Nash equilibrium, we need to show that no firm has incentive to deviate during Collusive Punishment Phase.

2. **Collusive Punishment Phase**
During Collusive Punishment Phase one can deviate by offering a price just smaller than q. The deviator has to offer a fee to the rest, so to convince them to syndicate. The deviator can offer either uniformly low offers, either insufficient offers or sufficient offers. In the first 2 cases, we have proved that no firm will accept syndication, so the deviator has to complete the job by itself. The profits will be:

$$\pi_{deviation} = 2q - e(\lambda, \phi) - e(1-\lambda, \phi) \leq 2 \cdot e(1-\lambda, \phi) - e(\lambda, \phi) - e(1-\lambda, \phi) \leq 0$$

as $\lambda \geq \frac{1}{2}$

Therefore, deviation accompanied by uniformly low of insufficient offers is not profitable, as in collusive punishment the deviator would enjoy positive profit.

If the deviator wants to make a sufficient offer, then it has to compansate every firm for the future profit that they would make in Collusive Punishment Phase, because by the next period the game will enter to its Bertrand Phase, which means zero profit for every firm. Then, deviator's profit would be:

$$\pi_{deviation} = 2q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2}) - \frac{\delta}{1-\delta} \cdot (2q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})) =$$
$$= (1 - \frac{\delta}{1-\delta}) \cdot (2q - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})) \leq 0, \text{ as } \delta \geq \frac{1}{2}$$

Hence, there is no profitable deviation in the Collusive Punishment Phase. That completes the proof, that if $\upsilon' > p^*, \forall \phi \in [0,1]$ the our strategy is a subgame perfect Nash Equilibrium. It now remains to show that no price higher than p' can be sustained. But, this result is by construction. If the equilibrium price exceeded p', a firm could profitably deviate by undercutting infinitesimally on price and completing the project without recruiting any syndicate members; such a deviation would be profitable even though the firm would receive its lowest individually rational payoff in all subsequent periods. That is:

If $p > p'$ then

$\pi_{deviation} > \pi_{cooperation} \Leftrightarrow$

$2p - e(\lambda, \phi) - e(1-\lambda, \phi) > \frac{\phi}{1-\delta} \cdot (2p - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})) \Leftrightarrow$

$p > \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)} \Leftrightarrow$

$p > p'$

which holds.

Thus, we have established the highest sustainable price stated in the theorem for cases where $\upsilon' > p^*, \forall\phi \in [0,1]$.

3.2 Case 2

- $\exists\phi \in [0,1] : \upsilon' < p^*$

In this case, the HKL pricing strategy can attract both consumers for some instances of ϕ as shown in Figure 7. We show that HKL pricing strategy with price p^* is not a subgame perfect Nash equilibrium $\forall\phi$.

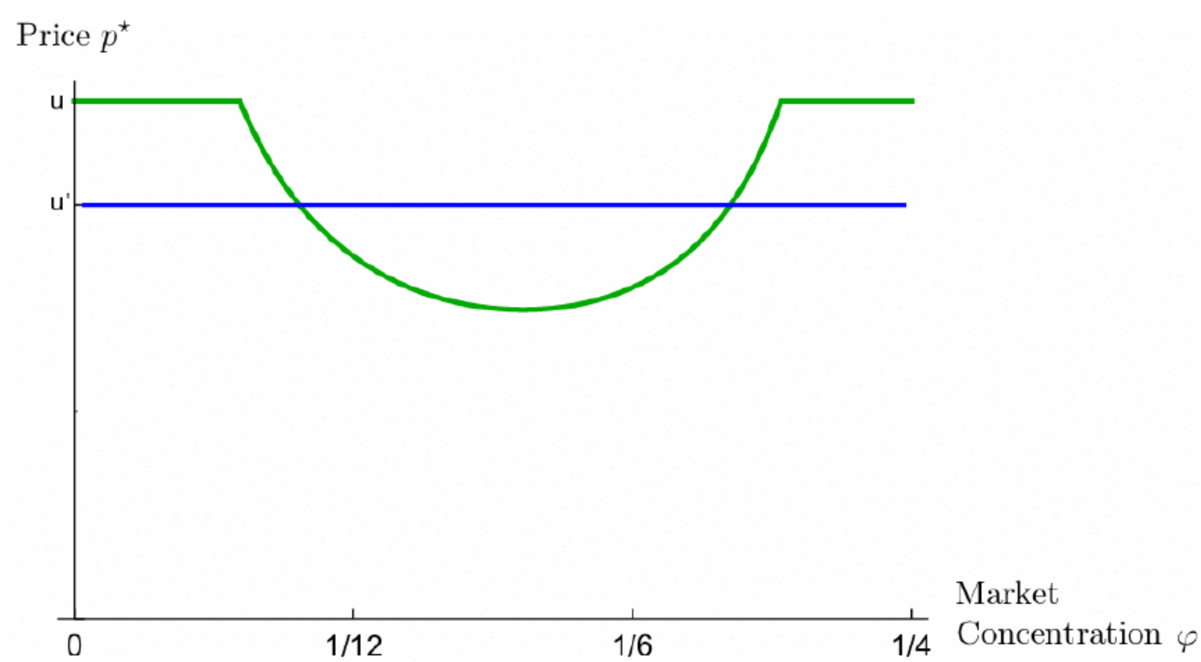


Figure 7: Case where both consumers buy the good for certain ϕ's

i. $\phi \in [1-\delta, 1]$

Then $p^* = \upsilon$, so only one consumer will buy. We have already proved that pricing at $p' = \upsilon'$ yields more profit, as long as $\upsilon' > \frac{\upsilon+c}{2}$.

ii. $\phi \in [0, 1-\delta) - [\phi_2, \phi_3]$, as they appear in the diagram.

In this case, p^* still attracts only one buyer, while p' attracts both. In this case, the proof that p' yields more profit than p^* is the same as in Case 1, as long as the 6 Hypotheses hold.

iii. $\phi \in [\phi_2, \phi_3]$

In this case, both pricing strategies attract both consumers. Obviously, HKL pricing yields more profit, as $p^* > p'$ and consumption is the same. Also, the proof that p' is a subgame perfect Nash equilibrium is the same as Case 1. But one can show that HKL strategy is not a Nash equilibrium in this case.

$\phi \in [\phi_2, \phi_3] \Leftarrow p^* = \frac{(1-\delta)\cdot e(\lambda,\phi) - \phi \cdot e(\lambda,1)}{1-\delta-\phi}$, thus in case of cooperation the profit will be:

$$\pi_{cooperation} = \frac{\phi}{1-\delta} \cdot (2p^* - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2}))$$

But we show that there is a profitable deviation i.e. to offer a price just smaller that p^* and complete the job alone. Then the game enters to its Bertrand phase.

$$\pi_{deviation} = 2p^* - e(\lambda, \phi) - e(1-\lambda, \phi)$$

$$\pi_{deviation} \geq \pi_{cooperation} \Leftrightarrow$$

$$2p^* - e(\lambda, \phi) - e(1-\lambda, \phi) \geq \frac{\phi}{1-\delta} \cdot (2p^* - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})) \Leftrightarrow$$

$$(1-\delta) \cdot (2p^* - e(\lambda, \phi) - e(1-\lambda, \phi)) \geq \phi \cdot (2p^* - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})) \Leftrightarrow$$

$$2 \cdot (1-\delta-\phi) \cdot p^* \geq (1-\delta) \cdot (e(\lambda, \phi) + e(1-\lambda, \phi)) - \phi \cdot (e(\lambda, \frac{\phi}{\phi_1}) + e(1-\lambda, \frac{\phi}{\phi_2})) \Leftrightarrow$$

But $\phi < 1-\delta$, so it is sufficient:

$$p^* \geq \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi)) - \phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)} = p'$$

which holds, hence there is a profitable deviation to HKL strategy i.e. HKL pricing strategy is not a subgame perfect Nash equilibrium.

By the same proof as in case 1, we get that p^* is the highest price it can be achieved in a subgame perfect Nash equilibrium enviroment and this completes our proof.

Since p' is quasiconvex, the only regions where p' may equal υ' are low and high values of ϕ. That is, there will be a single threshold value of ϕ above which $p' = \upsilon'$ and a single threshold of ϕ below which $p' = \upsilon'$. For *phi* above the higher threshold, it is immediate that no firm will deviate from $p = \upsilon'$ in expectation of completing the job alone for $q = e(1-\lambda, \phi)$. The payoff for such deviation is strictly lower than the case where the price would be $p = \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)}$ as all payoffs in the future periods are 0, while the current payoff from the deviation is:

$$\upsilon' - e(1-\lambda, \phi) < \frac{(1-\delta)\cdot(e(\lambda,\phi)+e(1-\lambda,\phi))-\phi\cdot(e(\lambda,\frac{\phi}{\phi_1})+e(1-\lambda,\frac{\phi}{\phi_2}))}{2\cdot(1-\delta-\phi)} - e(1-\lambda, \phi)$$

The analysis from preventing firms from joining a deviator's syndicate is identical to the case where $p' \leq \upsilon'$.

In the region where ϕ is small, the analysis is slightly different. When $p' = \upsilon'$ but $e(\lambda, \phi) + e(1-\lambda, \phi)) < \upsilon'$, the analysis does not change. When $e(\lambda, \phi) + e(1-\lambda, \phi)) > \upsilon'$ it is trivial to enforce υ' by refusing the join a syndicate after a price deviation from υ', even when the price in future periods remains υ'. Preventing a deviation to join a price deviator's syndicate is identical to the lower ϕ case, except the threshold for sufficient offers and the continuation payoffs for rejecting a syndicate offer are determined by $\upsilon' - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})$ rather than $e(\lambda, \phi) + e(1-\lambda, \phi)) - e(\lambda, \frac{\phi}{\phi_1}) - e(1-\lambda, \frac{\phi}{\phi_2})$.

4 Market with N consumers

In the previous section we have prove that under certain realistic hypotheses, the subgame perfect Nash equilibrium pricing allows both consumers in a 2-consumer market to purchase the good. A question rises about under which assumptions the same result occurs to a N-consumer market i.e. under which hypotheses all N consumers can buy the offered good. We prove that as N increases, the more demanding the assumptions become so to have an equilibrium that every consumer buys the service. We show that as $N \to \infty$, it is impossible to have such an equilibrium.

Suppose the are n consumers. The only case that everyone buys is to set a price as described before adjusted to the lowest reservation price (without loss of generality, the n^{th} reservation price). The profit of each firm by this strategy must be the highest possible, therefore must be higher than the profit yielded by adjusting the price to the second lowest reservation price, i.e. the $(n-1)^{th}$.

$\pi_n \geq \pi_{n-1} \Rightarrow$

$\frac{\phi}{1-\delta} \cdot [(n-1) \cdot p_{n-1} - \sum_{i=1}^{n-1} e(\lambda_i, \frac{\phi}{\phi_i})] \leq \frac{\phi}{1-\delta} \cdot [n \cdot p_n - \sum_{i=1}^{n} e(\lambda_i, \frac{\phi}{\phi_i'})] \Rightarrow$

$p_n \geq \frac{n-1}{n} \cdot p_{n-1} - \frac{\sum_{i=1}^{n-1} e(\lambda_i, \frac{\phi}{\phi_i}) - \sum_{i=1}^{n} e(\lambda_i, \frac{\phi}{\phi_i'})}{n}$

But $0 < \frac{\sum_{i=1}^{n-1} e(\lambda_i, \frac{\phi}{\phi_i}) - \sum_{i=1}^{n} e(\lambda_i, \frac{\phi}{\phi_i'})}{n} = \frac{c}{n} < \frac{1}{n} \cdot \sum_{i=1}^{n} \upsilon_i = \bar{\upsilon}$

When $\phi \in [1-\delta, 1]$ then $p_{n-1} = \upsilon_{n-1}$, thus it has to be:

$p_n \geq \frac{n-1}{n} \cdot \upsilon_{n-1} + \frac{c}{n}$

In order to have all consumers buying it must be: $p_n \leq \upsilon_n \Rightarrow$

$\upsilon_n \geq \frac{n-1}{n} \cdot \upsilon_{n-1} + \frac{c}{n}$

which is a contradiction as $n \to \infty$, as $\upsilon_n < \upsilon_{n-1}$ by definition. We do not consider the case where $\exists i, j : \upsilon_i = \upsilon_j$ because these two consumers can be perceived as one with demand the sum of the part demands.

Therefore, we conclude that even in an enviroment with infinitely many firms, even if every reservation price is larger than the cost of the firms, some consumers will not have access to the offered good because of the syndication between the firms. The syndication transfer a part of the perfect competition consumer surplus to the firms and it also creates a deadweight loss, because of the inefficiency described above.

5 Evaluation

In order to test how the equilibrium price behaves on average as the number of consumers grows, we have evaluated the game on Matlab®. After setting up the game, we calculate the expected profit for every firm in the coordination case when the good is affordable to i out of n consumers. Then the firms choose to serve the number of consumers that yields them maximum profit. Finally, we check whether a firm has an incentive to deviate, so to accept the pricing strategy as a subgame perfect Nash equilibrium. The iteration calculates the equilibrium price for various values of ϕ and draws the corresponding diagram. The matlab code can be found in the Appendix.

First, we run the simulation with 2 consumers. When the hypotheses (1)-(6) hold, we get the following equilibrium prices:

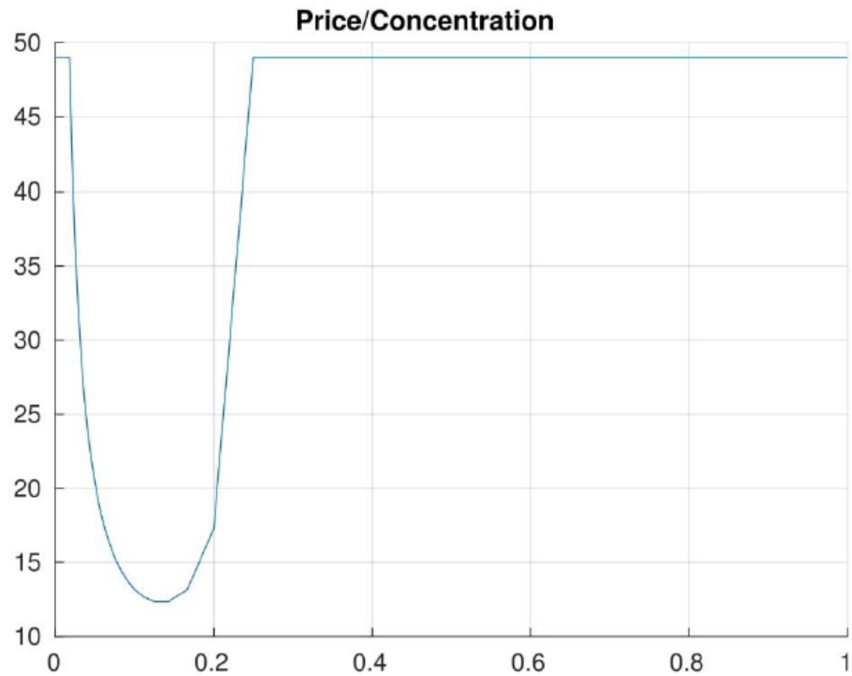


Figure 8: Case with 2 consumers, where υ is sufficiently close to υ'

In Figure 8, assumption (4) holds with strict inequality, i.e. υ' is close to υ, while in Figure 9 the assumption holds but it is unstable as $\upsilon' \approx \frac{\upsilon+c}{2}$

Next, we increase the number of consumers, to show that the pricing strategy changes as

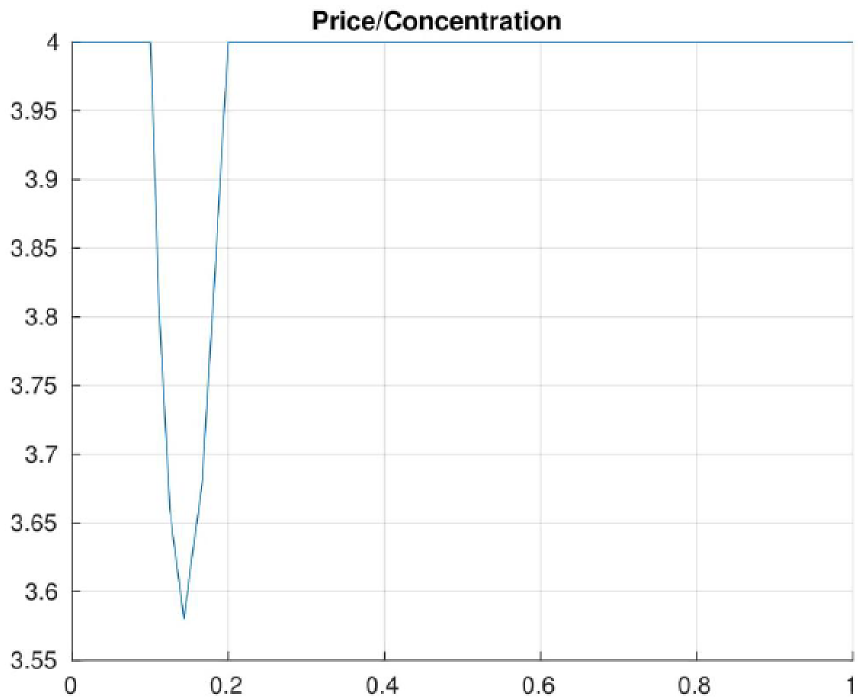


Figure 9: Case with 2 consumers, where $\upsilon' = \frac{\upsilon + c}{2}$

the number of consumers increases. An n grows, if we consider the reservation price of each consumer as a random variable, the probability of having some consumers out of the market converges to 1, as we proved in the previous part. In the simulation we can see that as n increases to 10, every simulation ends up with less than 10 consumers being in the market. We can also see the equilibrium price in the following diagram:

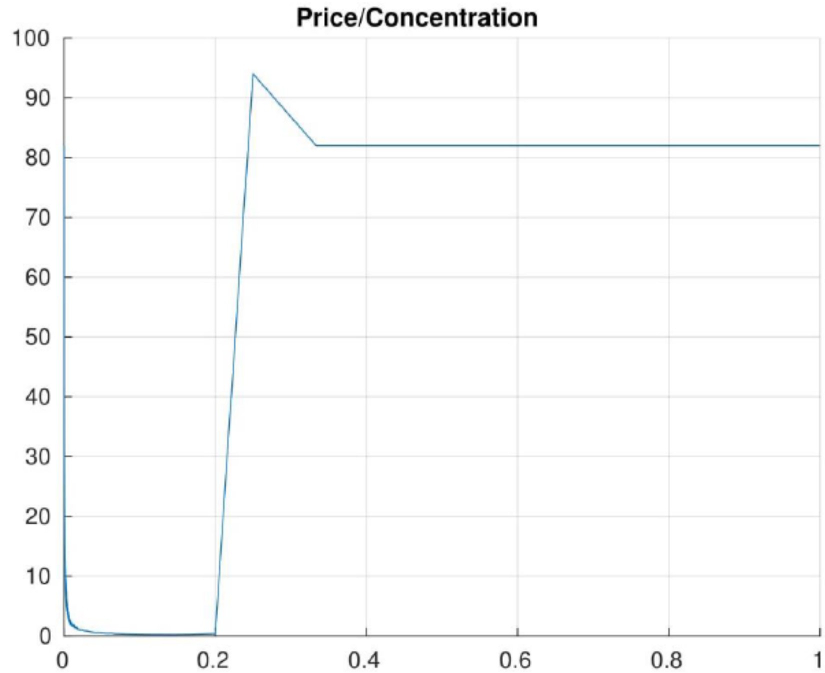


Figure 10: Case with 10 consumers

Also, for $n = 100$:

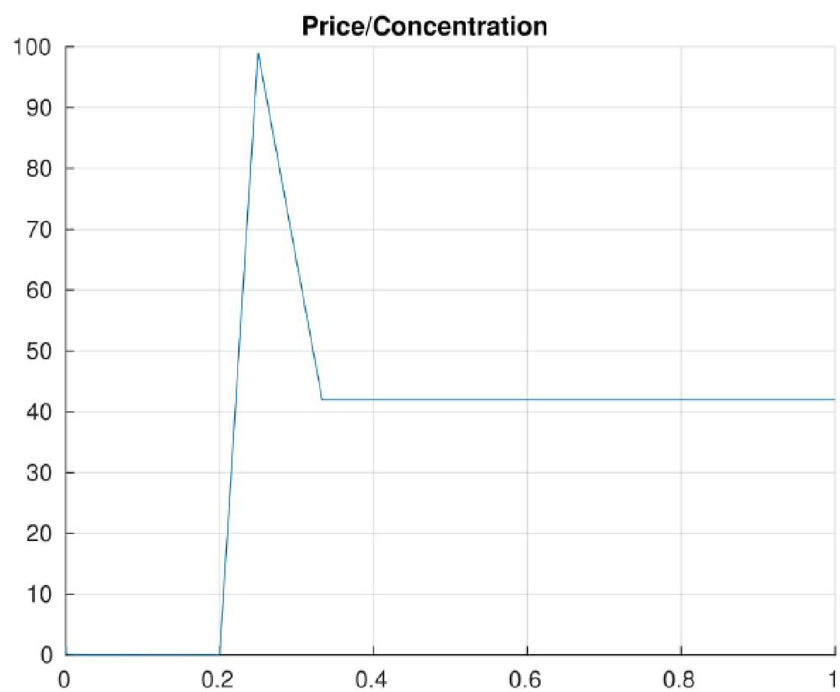


Figure 11: Case with 100 consumers

Obviously, we can see that the equilibrium price is not quasiconcave and it has a peak for $\phi = 0.25$. The intuition behind this, is that the firms are not that many so to be profitable to deviate alone, plus there are not that few so they can yield sufficient profits by sharing the production costs. They have the option to lead some consumers out of the market, hence they choose to offer a higher price so to maximize the total profit.

6 Conclusion

Our results strengthen HKL results in the sense that we confirm the price behavior and also we expand those results to consumer's surplus. In accordance with HKL paper, we show that the classic intuition in Industrial Organization does not hold in the case of syndication i.e. with infinitely many firms the market price equals the monopoly price. In other words an entry in the market can raise the price. One can say that in this type of collusion the firms compete each other in price offers, while they threaten each other not to price to low. Simultaneously the incumbent firms welcone new entries in the market, as they will allow them to impose credible threats to sustain a higher price, up to the monopolistic one.

Another important result is that in such environments, even when the firms are infinitely many, some consumers that are willing to pay more than the produced good's marginal cost have no access in the market. The latter confirms that the syndication strategy reduces consumer's surplus by raising the prices and by kicking out of the market consumers who could afford the good in a perfect competition context. In this type of market a infinite number of firms acts, as a whole, like a monopoly firm that sets prices that lower consumer's surplus.

Our analysis also adds to the ongoing scholarly debate on whether the IPO underwriting market is collusive and, if so, how collusion persists despite low market concentration in the industry. Our results offer potential insight into other features of the financial industry as well: For example, regulatory barriers routinely restrict participation in certain types of investments to investors that meet net worth or financial sophistication requirements. One might predict that the industry would oppose such restrictions, on the grounds that higher capacity (i.e., more investors) reduces the total cost of production. However, our work shows that increased capacity may reduce industry profits by making collusion more difficult. Our analysis thus suggests that the financial sector may actively support such restrictions, as they can facilitate collusion.

In addition, our work also highlights the importance of considering the full extensive form of firm interactions in industrial organization settings. Many industries are characterized by repeated, complex interactions that are best modeled as repeated extensive form games, such as IPO underwriting, debt origination, municipal auctions followed by horizontal subcontracting between bidders, and real estate transactions with agent selection. Further exploring repeated extensive form games is thus crucial to understanding subtle but important strategic interactions in these, and many other, markets.

Finally, our work can be expanded in various directions. An interesting question is whether we have the same subgame perfect Nash equilibrium when the offers are not public information but private and known only to the consumers. Another expansion could provide a closed form of a N-consumers subgame perfect Nash equilibrium, while we just show that as $n \to \infty$ at least one consumer will be driven out of the market. Furthermore, HKL paper discusses the case that firms are heterogeneous. They conclude that the intuition of the results derived by the homogeneous firms case still hold. We do not discuss this case and it would be an interesting expansion. In conclusion, one could test our theoretical results on real data in order to show that our hypotheses are realistic and correspond to the real world conditions in the IPO market.

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List of Figures

Appendix

<u>Simulation Code:</u>

```
clear all;
clc

------ Parameters's initialization ------

n = 2;
delta = 0.75;
u = round(100*rand(n,1));
u = sort(u,'descend');
q = zeros(n,1);
sum = 0;
for i=1:n
     sum = sum + u(i);
end
lambda = u/sum;
range = 5000;
fi = zeros(range,1);
solution-price= zeros(range,1);
for k=drange(1:range)
     F = k;
     phi = 1/F;
     fi(k) = phi;
     FF = round(F);

------ Function 1 (profit calculation) ------

     profit = zeros(n,1);
     price = zeros(n,1);
     sum1matrix = zeros(n,1);
     for i=1:n

          sum = 0;
          for j=1:i
               sum = sum+lambda(j);
          end

          team = zeros(j,1);
          for j=1:i
               team(j) = F*lambda(j)/sum;
          end
          cap = phi*team;
```

```
        sum = 0;
        for j=1:i
            sum = sum + e(lambda(j),cap(j));
        end
        sum1 = 0;
        for j=1:i
            sum1 = sum1 + e(lambda(j),phi);
        end

        profit(i) = (phi/(1-delta))*(i*p(sum,sum1,phi,delta,u(i),i) - sum);
        price(i) = p(sum,sum1,phi,delta,u(i),i);
        summatrix(i) = sum;
        sum1matrix(i) = sum1;
        q(i)=min(e(lambda(i),phi),u(i));
    end

------ Function 2 (sgpne with max profit) ------

    max = 0;
    index = 1;

    for i=1:n
        profitBertrand = i*price(i)-sum1matrix(i);
        profitSuff = i*price(i)-summatrix(i)-(delta/(1-delta))*(i*q(i)-summatrix(i));
        if (profit(i) >= profitBertrand) and (profit(i) >= profitSuff)
            if profit(i) >= max
                max = profit(i);
                index = i;
            end
        end
    end

    solution-price(k) = price(index);


------ Output ------

    Output = sprintf('d firms serve d out of d customers, with price f, that
yields to each of them profits of f ',FF,index,n,price(index),profit(index));
    disp(Output);
end

figure('Name','Price/Concentration','NumberTitle','off')
hold on
plot(fi,solution-price);
xlim([0,1]);
title('Price/Concentration');
```

```
grid on
```

We have used the following functions:

- ```
  function cost = e(s,m)
  if (s>1)||(m>1)
       error(’invalid inputs’);
  end
  cost = s*s/m;
  end
  ```

- ```
  function price = p(sum,sum1,phi,delta,u,i)
  if (phi>1)||(delta>1)||(delta<0)||(phi<0)||(u<0)
       error(’invalid inputs’);
  end
  if (phi<1-delta)
       price = min(u,((1-delta)*sum1-phi*sum)/(i*(1-delta-phi)));
       if price<0
            price=0;
       end
       else
       price = u;
  end
  end
  ```