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Combinatorial Auctions: Implementation and Analysis of Allocation and Payment Rules

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Abstract

The last decades, auctions played a significant role in the allocation of both public and private goods. Companies like Google implement auctions to allocate advertisement slots while platforms such as eBay also use them for online sales. Auctions are applicable in various cases such as spectrum allocation, airspace system allocation and allocation of financial products. Therefore, their popularity and their extensive use by governments as well as businesses made research around auctions imperative. The research focuses on studying characteristics of auctions such as transparency, truthfulness and maximization of social welfare and revenue.

Combinatorial auctions are a type of auctions, which allows the participants to place package bids i.e. to submit bids on combinations of items and not just on individual items. Research around combinatorial auctions has been popular during the last fifteen years as there have been numerous applications such as truckload transportation, bus routes and radio spectrum allocation. Nevertheless, combinatorial auctions come with their challenges, because computing the allocation and the payments to the bidders are computationally hard problems in general.

The purpose of this thesis is to focus mainly on the class of knapsack auctions and compare VCG and the so-called "core-selecting auctions" performance in that setting. On the one hand, the celebrated VCG mechanism is an extension of the second-price sealed-bid auction or Vickrey auction. On the other hand, core-selecting auctions select an allocation and a pricing rule in a way that no coalition of participants, including the auctioneer, have incentives to deviate and achieve a better outcome. Furthermore, the concern about computational tractability remains unresolved in some scenarios. Hence, our focus is on evaluating the performance of VCG and core-selecting auctions in the knapsack setting by simulations, with respect to criteria such as generated revenue, computational complexity and social welfare.

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Περίληψη

Τις τελευταίες δεκαετίες, οι δημοπρασίες διαδραμάτισαν σημαντικό ρόλο σε αγοραπωλησίες τόσο δημόσιων όσο και ιδιωτικών αγαθών. Εταιρείες όπως η Google υλοποιούν δημοπρασίες για να διαθέσουν διαφημιστικό χώρο ενώ πλατφόρμες όπως το eBay τις χρησιμοποιούν για διαδικτυακές πωλήσεις. Οι δημοπρασίες έχουν εφαρμογή σε διάφορες περιπτώσεις, όπως στην διαχείριση εφοδιαστικής αλυσίδας και στα δρομολόγια των λεωφορείων. Η εκτεταμένη χρήση τους από κυβερνήσεις καθώς και από επιχειρήσεις κατέστησε την έρευνα γύρω από τις δημοπρασίες επιτακτική. Η έρευνα επικεντρώνεται στη μελέτη χαρακτηριστικών των δημοπρασιών όπως η διαφάνεια, η φιλαλήθεια και η μεγιστοποίηση του κοινωνικού οφέλους και των εσόδων.

Οι συνδυαστικές δημοπρασίες είναι ένας τύπος δημοπρασιών, ο οποίος επιτρέπει στους συμμετέχοντες να καταθέσουν προσφορές για συνδυασμούς προϊόντων και όχι μόνο για μεμονωμένα προϊόντα. Η έρευνα γύρω από τις συνδυαστικές δημοπρασίες είναι δημοφιλής τα τελευταία δεκαπέντε χρόνια, καθώς έχουν πολυάριθμες εφαρμογές. Ωστόσο, οι συνδυαστικές δημοπρασίες συνοδεύονται από προκλήσεις, επειδή ο υπολογισμός της ανάθεσης και των πληρωμών συνιστά δύσκολο υπολογιστικά πρόβλημα.

Σκοπός αυτής της εργασίας είναι να μελετηθεί κυρίως η κατηγορία των knapsack δημοπρασιών και να συγκριθεί η απόδοση του VCG με τις "core-selecting" δημοπρασίες. Από τη μία πλευρά, ο μηχανισμός VCG αποτελεί επέκταση της δημοπρασίας δεύτερης τιμής (Vickrey). Από την άλλη πλευρά, οι core-selecting δημοπρασίες υπολογίζουν την ανάθεση και τις πληρωμές με τέτοιο τρόπο ώστε καμία ομάδα από τους συμμετέχοντες να μην έχει κίνητρο να αποκλίνει για να επιτύχει καλύτερα αποτελέσματα. Επομένως, η παρούσα διπλωματική εργασία εστιάζει στην αξιολόγηση της απόδοσης των VCG και core-selecting δημοπρασιών στην περίπτωση των knapsack δημοπρασιών με κριτήρια όπως τα συνολικά έσοδα, η υπολογιστική πολυπλοκότητα και το κοινωνικό όφελος.

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Chapter 1

Introduction

Auctions constitute a traditional method of selling and buying goods and services since antiquity. They had been taking place in Babylon since 500 B.C. Indeed, the ancient Greek historian, Herodotus, describes the Babylonian marriage market in which women were sold for wives [DB02]. Around 30 A.D. auctions were also popular in Rome, and they were used for selling war plunder and furniture. For instance, furniture was sold by Marcus Aurelius to satisfy debts while roman soldiers sold war prizes in auctions. The strangest auction in ancient times took place in 193 A.D., during which the whole Roman Empire was looted and then was sold. More specifically, on the 28$^{\text{th}}$ of March 193 A.D. the Roman Empire was auctioned by the Praetorian guards to the wealthy senator Didius Julianus [Gib90]. The price was 6250 drachmas per soldier.

Through centuries, auctions evolved in order to satisfy people's needs and requirements. English auctions firstly appeared in 1674. They were used to sell books and paintings as well as for real estate, household goods and slaves while they usually took place in coffee houses [DB02]. After the 17$^{\text{th}}$ century auctions evolved even more and instead of being a small event, they attracted more people in larger gatherings. During that period, another type of auction appeared, i.e. the Dutch auction. The Dutch auction has its roots in the Dutch flower market of the 17$^{\text{th}}$ century. Both the English and the Dutch auction are used until today.

In the following years, auctions emerge as a significant tool for expressing the needs of the public and it was widely used in real estate and for selling personal items. Most auctioneers prefer to sell items through auctions as they can set their own rules for the auctions and possibly earn more money. On the other hand, players are those who ultimately set the prices the items will be sold.

However, auctions have not been used only for private goods. Indeed, governments have employed auctions in transactions of public goods and services. An example is the U.S. Treasury Department, which uses auctions for Treasury bills since they were introduced in 1929 [Tre]. In 1974 a uniform price or the "Dutch" auction format was employed to sell long-term securities. Every successful bidder received securities at the lowest accepted price. Additionally, in 2000 the Treasury conducted the first buyback operation. Buybacks are basically a form of reversed auction, where the Treasury bought back securities in order to reduce the amount of interest paid.

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Apart form the U.S. Treasury Department other government organizations and institutions employ auctions. The U.S. Mineral Management Service (MMS) uses auctions in order to allocate exploration and drilling rights for oil and gas on federal lands on the Outer Continental Shelf (OCS) [HHP09]. The program about offshore leasing began to operate in 1954. Offshore oil and gas consist nowadays a third of U.S. production and the leasing program has generated significant revenue for the government. During the program's operation, there have been modifications in the auction mechanism. The most common type of auction format for those mineral rights was first-price sealed-bid auctions. Indeed, due to the profitability of the sector, firms often collude in order to increase the possibility of winning and reduce the cost of investment. That way they can exploit together neighboring regions and thus reduce the auctioneer's revenue [Por95]. For this reason, measures, such as banning joint bidding for large firms, were proposed in order to face those cases.

In combinatorial auctions, players can place bids on combinations of items rather than individual items. They emerged around 1980. Indeed, the Reynolds Metals Company employed combinatorial auction settings in order to ameliorate routing and delivery times while at the same time reducing its operation cost. In those auctions, carriers bid for combinations of lanes [MWG91]. The aim is to reduce the routes they travel without carrying anything in order to reload trucks for a new delivery. Combinatorial auctions have also been employed in shipping industry. Shipping companies have been offering through auctions contracts to transportation companies. The use of combinatorial auctions can lead to significant reductions in the operating cost for the shippers, while at the same time improve the operations of the carriers [She04].

Spectrum auctions are also used often in practice revealing interesting insights for auction theory. In spectrum auctions, governments employ auctions to sell the rights for transmitting signals over specific bands of the spectrum and to allocate scarce spectrum resources. In New Zeland, usage rights for television signal were sold for first time in 1990, using the second-price sealed-bid auction [Mil04]. The Department of Telecom of India conducted auctions for cellular services during 1991 [Jai01].

The U.S. Federal Communications Commission (FCC) designed auctions, which are considered to be successful and which are characterized as innovative because they incorporated learnings from auction theory. The FCC auctions began in 1994 and in those auctions, the combination of spectrum bands could be more valuable than an individual spectrum band, as adjacent frequencies are more desirable. The existence of complement relationships makes even more complex determining the winners of the auction and their payments. In order to cope with that problem, the U.S. Federal Communications Commission used the incentive auction. Indeed, the FCC incentive auction of the last year yielded significant revenue for the U.S. government, which was used to reduce the deficit in the government's budget [Com].

Auctions burst into cyberspace in the middle of 1990s. The Japanese company, Aucnet, was the first to implement online auctions for selling automobiles. The same year, 1995, Onsale and eBay followed with eBay becoming the leader in online auctions [Tre]. eBay employed second-price sealed bid auctions with proxy bidding. Indeed, that eBay's automatic-bid feature stands in so that a player's bid rises

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incrementally in response to other bidders' bids. The use of online auctions generates significant revenue for many companies. Many sellers implement nowadays online auctions to meet the customer's needs near and far. Technological advancement enables players to participate in the auction without even being there physically.

Apart from goods, services are also sold in online auctions. Ad-auctions are designed and implemented by large search engines such as Google, Bing and Yahoo [Mil04]. Those engines determine which advertisements should appear with a fast-auction, taking place every time someone searches a term. Ad-auctions work as follows: advertisers submit their advertisement, relevant keywords and their bids. Each time a user searches a term or a combination of terms, the search engine determines the advertisements, which match the user's query. If the user clicks on an advertisement, the advertiser pays the price determined by an auction mechanism. Google AdWords, for instance, employ the generalized second-price auction (GSP) mechanism, which is a natural extension of the Vickrey auction.

Over the years, auctioneering has evolved and altered, and even today it remains more popular than ever. It is obvious from the analysis above that auctions have applications in many different sectors. Governments as well as businesses employ auctions to sell and buy goods and services daily. The advent of the Internet and ecommerce transformed businesses and had also an impact on auctions. The extensive use of auctions in various environments lead to extensive scientific research around their properties and their challenges. Ultimately, auctions are used to determine how goods and services will be allocated and what the winners will pay for those goods and services. Despite the fact that auctions are more complex than direct selling, they have advantages such as increased revenue and flexibility, which outweigh their disadvantages and challenges. Issues like transparency, computational and communication complexity as well as strategic behavior and optimization objectives are studied from the scientific community since 1960 [Vic61].

The aim of this thesis is to examine a specific setting of auctions, i.e. the knapsack setting. For this reason, it provides information around auctions in general as well as the challenges faced. Moreover, it focuses in knapsack auctions, which can be important in real-world applications such as in the case of television ad slots. Two celebrated mechanisms for determining the payments are compared and their results are analyzed in terms of complexity and generated revenue.

The structure of the thesis is as follows: the second chapter provides an overview of auctions in general with a focus on their properties and the optimization objectives, the third chapter introduces the notion of knapsack auctions after defining the well-known Knapsack Problem and in the fourth chapter the experimental setup is described while the results and the conclusions of the experiments are discussed.

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Chapter 2

Auctions

Auctions have been used since antiquity for selling various sorts of items. Since then, however, there have been significant advances in auction theory. The origins of the economic theory of auctions lie in the seminal work of W. Vickrey [Vic61]. In this chapter, an overview of auctions will be presented. The simplest type of auctions is single-item auctions, while there are more complex types of auctions involving multiple items at the same time.

2.1 Single-item auctions

The most elementary type of auctions entails allocating a single item among various bidders. There is a single item to be allocated and a set of bidders, each of whom has a private valuation v_i, which expresses their willingness to acquire that item. When it comes to single-item auctions different mechanisms have been designed to determine the winner as well as the payment.

Definition 1. ***(Mechanism)***
A deterministic mechanism M, is defined as a tuple (X, P), where

- $X = (x_1, x_2, ..., x_n)$ *is the allocation function that determines which bidder wins the item. In the single-item case, for every non-winner* x_i *is 0,*
- $P = (p_1, p_2, ..., p_n)$ *is the payment rule that determines how much the winner of the auction should pay.*

The final utility of each winner is calculated as the difference between the valuation and price, i.e. $v_i - p_i$. When bidders submit their bids, they might not declare their true valuations. Therefore, we use b_i for their bids, as they can be different from v_i. All reasonable mechanisms satisfy the following relationship: $p_i \leq v_i$.

A common type of auctions is the first-price sealed-bid auction. In this format, all bidders submit sealed bids at the same time. Therefore, no bidder knows the bid of any other bidder. The bidder with the highest bid wins the item (allocation rule) while the winner pays the bid she submitted (payment rule). A bidder in that format aims to bid the smallest amount possible that can ensure her win, as long as this amount is smaller than her valuation for the item. For example, if there are two bidders A and B and if bidder A bids a, then bidder B would like to bid $a + \varepsilon$

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(where ε is the smallest amount, that it can be added). Nevertheless, bidder B does not know the valuations of the other bidders as well as what the other bidders are going to bid. This format of auctions offers bidders incentives to report a different valuation than their true one. This is not desired. Below truthfulness, a significant aspect of auctions, is defined.

Definition 2. ***(Truthful Auction)***
An auction, or a mechanism in general, is considered truthful or incentive-compatible [VNRT07] if declaring the true value is a dominant strategy for every player. Dominant strategy is a strategy, which is better than any other strategy for one bidder, no matter how that bidder's opponents might play. In other words, truthfulness ensures that players act according to their true preferences.

While designing auctions, one should bear in mind the importance of truthfulness. Indeed, the auctioneer does not know the true valuations of the bidders, as they are private information. Therefore, truthfulness is a way to prevent the manipulation of the market, which can create problems for both the auctioneer and the bidders. However, truthfulness is also significant from a bidders' perspective. They save time, as they do not need to consider how they will play; they just submit their true valuations.

A fundamental mechanism is the second-price sealed-bid auction, which is also known as Vickrey auction [CSS06]. In that format, each bidder i submits a bid b_i, while all the bids are submitted at the same time. The allocation rule is straightforward: the bidder with the highest bid wins the item. However, she has to pay the second-highest bid. In a second-price sealed bid auction the bid does not determine how much the winner bidder will pay. It is just used to determine who the winner is. For example, if there are two bidders in an auction and the first's bid is 15 while the second's 20, then the winner is the second bidder, who pays 15 for the item.

Theorem 1.
For single-item settings the second-price sealed-bid auction is a truthful mechanism.

The above theorem can be proven with a simple case analysis, which can be found in Vickrey's seminal paper [Vic61].

2.2 Single-parameter environments

A single-parameter auction environment consists of n bidders as well as a private valuation v_i of each bidder i. In other words, single-parameter auctions refer to the cases where every bidder's valuation function can be described by just one single parameter, which is a private information of the bidder. Single-item auctions is an example of single-parameter auctions. However, before analyzing examples of single-parameter auctions, the notion of mechanism should be redefined in that context.

Definition 3. ***(Mechanism - Generalized Definition)***
A deterministic mechanism M, is defined as a tuple (X, P), where

- $X = (x_1, x_2, ..., x_n)$ *is the allocation function that determines to which players the items are assigned,*

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- $P = (p_1, p_2, ..., p_n)$ *is the payment rule that determines how much each player should pay for what she won.*

2.2.1 Multi-unit auctions

Auctions of multiple identical items is a type of single-parameter auctions. The items in that case can be offered either in continuous or discrete quantities. In that format, bidders submit at the same time again sealed bids consisting of their demand curves. Here, the valuation v_i of each player i expresses the value per unit of an item. The auctioneer determines the allocation as well as the clearing price by combining the individual demand curves. Then, each bidder is offered the quantity she demanded at the determined clearing price paying the opportunity cost for that quantity.

As an example, if the units are offered in discrete quantities, then each bidder submits sealed bids, which express value per item. The seller can determine the allocation by selecting the top k bids (allocation rule); the payment can be determined based on the bids of the rejected bidders, i.e. if a bidder wins m units, then she pays the highest m rejected bids (payment rule). Therefore, if a bidder wins three units of an item and the three highest rejected bids are 10, 8 and 7, then she pays the sum of them i.e. 25 for the three units. This corresponds to the opportunity cost for those three units. That is the case of the aforementioned second-price sealed-bid. In the case of first-price sealed-bid, the allocation rule remains the same. However, if the winner's bid is 12 then the winner should pay 12 for each unit of the item, i.e. 36 in total.

An example of single parameter auctions is single-minded auctions. Suppose again that there are N bidders and a set of items. For each bidder i there is a publicly known bundle of items A_i that the bidder desires and a private valuation v_i, which expresses the willingness of the bidder to pay for that bundle. In other words, a single-minded bidder is willing to acquire a unique bundle of items and bids only for that bundle. Another example of single parameter auctions is knapsack auctions, which will be present in detail in the next chapter. In that thesis, we will focus on the knapsack setting.

2.3 Combinatorial auctions

2.3.1 Overview

Combinatorial auctions are important in practice. They have been used for multiple applications such as allocating take-off and landing slots at airports, while there are also examples of government spectrum auctions. Combinatorial auctions seem to prevail the last decades [DR07] in government allocation problems as well as in business environments. However, they present significant difficulties in both theory and practice. They allow the participants to place package bids i.e. to submit bids on combinations of items and not just on individual items.

A combinatorial auction setting consists of a set of m indivisible, distinct items and a set of n bidders. Let M be the set of items and N the set of bidders. Each

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bidder i is required to submit a valuation function $v_i(S)$ for every subset $S \subseteq M$ of the items. This constitutes the highest price the bidder i is eager to pay in order to acquire the subset S. It is assumed that $v_i(\emptyset) = 0$. An allocation $X = (x_1, x_2, ..., x_n)$ is defined as a vector of subsets assigned to bidders with $x_i \cap x_j = \emptyset \ \forall \, i \neq j$.

Let P be the set of payments p_i. Bidders report their valuations to the auction b_i, which might not be truthful, as b_i can be different from v_i. Given the bids b_i, the auction mechanism will determine both the allocation X of the items to the bidders and the payment p_i for every bidder i. It is assumed [BLS18] that bidders submit bids only for items for which they have reported positive value. The aim of the mechanism is to determine an allocation, which maximizes either the social welfare or the revenue.

Definition 4. ***(Social welfare)***
Social welfare refers to the sum of the bidders' values, i.e. $\sum_{i \in N} b_i(x_i)$. *An auction is social-welfare maximizing or efficient if it [RS07] maximizes the sum of the bidders' values. Social-welfare maximization is sometimes compromised in favor of truthfulness as it is challenging to design the most efficient truthful mechanism. An allocation* X^* *is considered social welfare maximizing if and only if*

$$X^* = argmax_X \sum_{i \in N} b_i(x_i) \tag{2.1}$$

Definition 5. ***(Revenue)***
Revenue refers to the sum of the winner's payments, i.e. $\sum_{i \in N} p_i(x_i)$. *An auction is revenue maximizing if it [RS07] maximizes the sum of the winners' payments to the auctioneer. An allocation* X^* *is considered revenue maximizing if*

$$X^* = argmax_X \sum_{i \in N} p_i(x_i) \tag{2.2}$$

An auction might aim to maximize both social welfare and revenue simultaneously. However, in most cases those two goals are conflicting. There are auctions maximizing social welfare (e.g. Vickrey auctions) as well as auctions maximizing the auctioneer's revenue (e.g. Myerson optimal auctions). There is discussion around whether auctions should optimize social welfare instead of revenue. Indeed, maximizing social welfare seems rational from a government's perspective as it encourages the participation of more players in the auction and enables the governments to allocate effectively resources such as in spectrum auctions. However, even from a business perspective maximizing social welfare can be considered rational. Indeed, maintaining a high market share in a competitive international environment is the only way for the companies to achieve their long-term goals and maximize their revenue ultimately.

Although these two goals seem conflicting, there is evidence that the difference between the two optimization goals, i.e. maximizing social welfare and maximizing the revenue, is not so large if it is compared to the number of the bidders. Suppose that there is only one indivisible good in an auction and there are n bidders with a commonly known valuation distribution. The revenue of a social-welfare maximizing allocation with $n+1$ bidders is at least as high as the revenue of a revenue maximizing auction with n bidders [BK96]. Recent work [AGM09] extends that point concluding

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that the social welfare of a revenue maximizing auction with $n + \log n$ bidders is at least as high as the social welfare of a social welfare maximizing auction with n bidders.

2.3.2 The VCG mechanism

The Vickrey-Clark-Groves mechanism determines the optimal social welfare allocation by maximizing the total utility of the bidders. It is a generalization of the aforementioned second-price sealed-bid auction mechanism. That mechanism works for both homogenous and heterogeneous items and there is no requirement for the bidders to have non increasing marginal values [CSS06]. The most significant difference with the simpler second-price sealed-bid auction mechanism is that the payments cannot be expressed in most cases as the sums of bids for the items.

According to the VCG mechanism the auctioneer in order to determine the optimal allocation has to solve an optimization problem aiming to optimize the social welfare, i.e. the total value to the auction's participants $\sum_{i \in N} b_i(x_i)$, subject to constraints.

Definition 6. ***(The VCG Payment Rule)***
Let X^* be an optimal allocation and b_i the reported values of the bidders. The payment of each bidder in the VCG mechanism is the externality he imposes on the other bidders by consuming the items allocated to him [HIK+18, Cla71]. Formally:

$$P_{VCG,i} = \sum_{i \neq j} b_j(X^{-i}) - \sum_{i \neq j} b_j(X^*) \tag{2.3}$$

where X^{-i} is the allocation, which maximizes the social welfare, when all bidders apart from bidder i are present in the auction. [BLS18, Cla71] Ultimately, the VCG payment is the difference between the optimal welfare for the other bidders if bidder i was not participating in the auction and the welfare of the other bidders from the chosen outcome.

Definition 7. ***(The VCG Payoffs)***
The VCG payoff is the difference between the reported value of a bidder b_i and the VCG payment. Formally:

$$\pi_{VCG,i} = b_i(X_i^*) - p_{VCG,i} \tag{2.4}$$

Virtues of the VCG mechanism

The VCG mechanism, although it is not usually used in practice [CSS06], presents a number of theoretical properties, which make it useful for comparison. Firstly, in the Vickrey auction there is no need for a bidder to have information about the valuations of the other bidders in order to bid in an optimal way. In other words, bidding the true value constitutes the optimal strategy in the VCG mechanism.

In comparison with other welfare-maximizing auction formats [DR07] the VCG auction is a payment rule that offers incentives to the bidders to submit truthful bids as a dominant strategy. This dominant strategy property reduces the auction's

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costs as it makes it easier for the bidders to find out the optimal bidding strategy without spending resources to learn about their competitors' strategies and values.

The dominant strategy property is also associated with other positive properties of the VCG mechanism. Indeed, it ensures that the outcome of the VCG auction is not sensitive to assumptions of bidders' knowledge about the values and the strategies of their competitors. It is proven that the VCG auction is the only direct reporting auction format ensuring dominant strategies and efficient payments as well as zero payments for non-winners.

An additional virtue of the VCG mechanism is that it can be applied in various cases. The mechanism itself is flexible and allows the auctioneer to define additional constraints to the auction. For instance, in the case of a spectrum auction there might be a constraint on the concentration of the spectrum ownership. In a procurement auction for machinery the auctioneer could restrict the purchases from a specific group of bidders or require that the suppliers should have a specific capacity in their inventory. All those cases are suitable for VCG due to its flexible structure. Finally, a significant theoretical property of the Vickrey auction is that the average revenues are never less than the average revenues of any other efficient auction format.

Weaknesses of the VCG mechanism

Despite the fact that the dominant strategy property is associated with various virtues of the VCG mechanism, there are a couple of weaknesses of the mechanism, which may arise and which impose certain difficulties.

Those weaknesses are mainly caused by the presence of complementarities. If there are only substitute preferences, then the weaknesses of the VCG mechanism will not present. It may be the case that a bidder considers two goods as identical i.e. he or she regards any one unit of one good as exactly the same as any one unit of the other good. In other words, the bidder cannot tell any difference between the two goods. For instance, a bidder may not be able to distinguish two different types of beers. This is the case of perfect substitute preferences, where a bidder considers the goods as perfectly interchangeable.

The other extreme is perfect complement preferences. A bidder now in that case considers that two goods should be consumed together. In other words, one item should be consumed every time the other one is consumed. Consuming just this item alone seems pointless. For instance, having just one shoe and not a pair of shoes is pointless.

One major disadvantage of the VCG auction is its complexity. For large-scale application with heterogeneous and indivisible items, it is computationally intensive to determine the optimal allocation and the payments. For many types of problems, this means that computing the bidders' payments and the output of the mechanism requires exponential time. Therefore, in such cases using the VCG mechanism is clearly impractical, especially when the number of bidders is quite large. Even in cases where the payments and allocation can be computed in polynomial time, the total computation time for the payments of a truthful mechanism is a major obstacle. However, this is not the only reason that the VCG mechanism is not widely used

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in practice. There are cases of small-scale combinatorial auctions, where the VCG mechanism is not employed and other mechanisms are used.

Although most analyses focus on the complexity of the VCG mechanism, its major weaknesses are associated with the revenues of the auction. Especially, in the private sector, revenues play a crucial role. However, even in auctions implemented by governments, revenues are significant as they constitute a measure of the governments' performance. Sometimes the VCG mechanism can lead to extremely low or zero revenues, despite the fact that there might be many competitors and the items in auction might be valuable.

Consider an example where there are three bidders and two items, A and B. The first bidder is willing to pay 1 dollar for both items while the second bidder is willing to pay 1 dollar for item A and the third bidder 1 dollar for item B. The VCG mechanism would select the second and the third bidder to assign the items A and B respectively. The price the second bidder has to pay is equal to the difference of the value of the first bidder and the third bidder; she has to pay zero. Similarly, for the third bidder the price is zero. Therefore, the auction revenues are zero. If the items have been offered as indivisible then each bidder would be eager to pay 1 dollar for the indivisible item. Thus, an ascending auction would lead to revenues of one dollar. This deficiency of the VCG auction is the reason, why it is not applied even in small-scale auctions.

Another weakness of Vickrey auction is the fact that the revenues of the auctioneer are not monotonous in the set of bidders and in the bids. For instance, if in the aforementioned example the third bidder did not participate in the auction or if his value were 0 dollars instead of 1, then the revenues of the auctioneer would increase to 1 dollar. Thus, this deficiency reveals another flaw of the VCG mechanism, which could be possibly exploited by the bidders.

Another drawback of the VCG auction is that it is vulnerable to collusion by a coalition of losing bidders. Consider again the above example. If the value of the first bidder remains unchanged, while the values of the second and the third bidder are reduced to 0.2 dollars, then the winner would be the first bidder, changing the initial optimal allocation. Nevertheless, if the example remained unchanged then the outcome would be again zero revenues for the auctioneer. A related deficiency is shill bidding, where a bidder uses different identities in an auction. For example, in the above case if the third and the second bidder were merged to a single bidder offering 0.2 dollars for one the item A or B and 0.5 dollars for the items A and B, then the winner would be again the first bidder. However, the merged bidder could take part in the auction as two bidders, with each one bidding 1 dollar for a different item. The outcome would be zero revenues for the auctioneer ant the combined bidder would have won both items for zero dollars.

If we changed the above example, by making the items substitutes for the first bidder, then the first bidder would be willing to pay 0.5 dollars for item A as well as for item B. That alteration would radically affect the outcome of the auction. The second bidder would be assigned to item A and he would have to pay 0.5 dollars while the third bidder would be assigned item B and he would have to pay 0.5 dollars. Thus, the revenues of the auctioneer will now be 1 dollar instead of 0 in the unmodified example.

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In general, if each one of the bidders has substitute preferences then none of the problems mentioned above would arise [CSS06]. With that modification, the revenues of the auctioneer will be monotonous, bidders will have no incentive to use multiple identities and there will be no coalitions of losing bidders. In case substitute preferences do not exist and bidders have also additive values then there are value profiles, which can lead to such problems.

Additionally, in the Vickrey auction it is assumed that the payoffs are quasilinear. Because of this assumption, the payoffs should be expressed as the difference between the value of the goods and the payment of the bidder. More precisely, it means that bidders should have no budget limit in order to have a dominant strategy and the buyer in a case of a procurement auction should have no limit on the cost of the procurement.

Another problem that could arise in Vickrey auctions is that payments for the same item might be different. For this reason, the Vickrey auctions are sometimes considered unfair. For instance, let A and B be two identical items and let X and Y be two bidders participating in the auctions for those items. Suppose that the Bidder X bids 5 for either of the two identical items and 8 for both items and Bidder Y bids also 5 for either of the two identical items and 9 for their combination. In that case, both of the bidders will win one item. However, Bidder X will pay 4 while Bidder Y will pay 3 for the same item. Obviously, although the items are identical and the values of the bidders for a single item are equal, the payments are different, showing that the outcome is unfair.

The VCG auction can also have problems with privacy. [RTK90] In other words, bidders might not be eager to report their true values of the items due to their fear that their estimations could be revealed to the other bidders, who could use that information against them. Indeed, when it comes to government auctions there have been cases that public was outraged, because bidders payed ultimately significantly less than their value [McM94].

The impact of public reactions to an auction's outcome is significant and can lead even to the cancelation of the auction in order to renegotiate. However, this restriction of the Vickrey auction has been resolved in a great extent due to modern technological advancement and the advent of encryption techniques. Moreover, proxy bidders, which are used widely in online auctions, can also ensure the privacy of the bidders. Hence, this restriction is not as critical as the aforementioned ones.

2.3.3 Core selecting auctions

In the previous section, the Vickrey-Clarke-Groves (VCG) auction was extensively discussed. Both its virtues as well as its weaknesses have been analyzed. The analysis showed that the VCG auction suffers from a variety of significant weaknesses, although its theoretical implications can be proven useful. One of the main weaknesses is that it can lead to extremely small or even zero revenues for the auctioneer, as it was illustrated in the examples. Consequently, there have been efforts to mitigate those problems associated with VCG by designing and implementing new techniques.

A way to overcome this challenge, [DM02] is to deviate from the property of

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truthfulness and employ the concept of the core to compute the payments. In core auctions, there is no coalition of bidders, who would pay more than the auctioneer's revenue. Auctions that select core allocations with respect to reported values generate competitive levels of sales revenues at equilibrium and limit bidder incentives to use shills [DM08]. Among core-selecting auctions, the ones that minimize seller revenues also maximize incentives for truthful reporting, produce the Vickrey outcome when that lies in the core and, in contrast to the Vickrey auction, create no incentive for an auctioneer to exclude qualified bidders.

Non-core payments may be considered unfair since there are bidders willing to pay more than the winners' payments. Moreover, non-core payments make the auction vulnerable to defections, as the seller can attract better offers afterward. Given bids, a core-selecting mechanism finds an efficient allocation, and determines core-payments.

Ultimately, the concept of the core entails that there is no group of bidders that can deviate together in order to ameliorate their outcomes as well as the auctioneer's revenue [DR07]. The payments must be sufficiently high that no subset of players can together provide an alternative allocation and a vector of payments, which makes the auctioneer and all players in the subset weakly better off and makes at least one of them strictly better off. Payment rules that avoid outcomes leading to coalitions of bidders are considered core-selecting payments.

Definition 8. ***(The Core)***
Let W be the set of winners, X^* be the allocation, which maximizes the welfare, $S \subseteq N$ refers to a coalition of bidders and X^S is the allocation which would be chosen if only the bidders of the coalition S participated in the auction. Given those definitions, a payment vector P is considered to lie in the core, if apart from the condition that payoffs should be greater than or equal to zero, it also satisfies the following relationship:

$$\sum_{i \in W \setminus S} p_i \geq \sum_{i \in S} u_i(X^S) - \sum_{i \in S} u_i(X^*) \quad \forall\ S \subseteq N \tag{2.5}$$

In combinatorial auctions with complements, the payments of the Vickrey-Clarke-Groves (VCG) auction are often outside the core and in the most extreme situation are zero. Therefore, the core is essentially the set of payments that are feasible for the coalition of the whole and at the same time unblocked by any other coalition. [CSS06, BLS18]

A minimum-revenue core-selecting (MRCS) auction computes an allocation and a payment, which minimizes the auctioneer's revenue while at the same time ensuring that payoffs are in the core with respect to bids b_i. The auction mechanism computes the vector of payoffs $\pi^b = (\pi_0^b, \pi_1^b; ..., \pi_n^b)$, so that $\pi_i^b = b_{i,y} - p_i$ if bundle y is allocated to bidder i, who pays p_i. The seller's revenue is $\pi_0^b = \pi_0 = \sum_{i \in N} p_i$, while $W(\cdot)$ represents the total utility derived for bidders. Therefore, any MRCS auction chooses payoffs [OB13] that solve:

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$$\begin{aligned} &\text{minimize } \pi_0 \\ &\text{subject to } \pi_0 + \sum_{i \in N} \pi_i^b \leqslant W(N) \\ &\qquad \pi_i^b \geq 0 \; \forall \; S \subseteq N \\ &\qquad \pi_0 + \sum_{i \in S} \pi_i^b \leqslant W(S) \; \forall \; S \subseteq N \end{aligned}$$

The problem can be rewritten in terms of payments instead of payoffs. The losing bidders pay nothing, as they have not won anything from the auction.

$$\begin{aligned} &\underset{(p_1,\ldots,p_n)}{\text{minimize}} \sum_{i \in N} p_i \\ &\text{subject to } b_i(x_i^*) \geq p_i \; \forall \; i \in N \\ &\qquad p_i \geq 0 \; \forall \; i \in N \\ &\qquad \sum_{j \notin S} p_j \geq w(S) - \sum_{i \in S} b_i(x_i^*) \; \forall \; S \subseteq N \end{aligned}$$

Apart from MRCS, there is an infinite family of core-selecting payment rules. Designing the optimal core-selecting payment rule is a challenging task, as there exists no strategy-proof core-selecting payment rule [GL16]. One of the most prominent directions in finding the optimal payments is the minimization of a distance metric to the VCG payment [Par02]. A rule that is widely used in practice nowadays is the Quadratic Rule, proposed by Day and Cramton [DP12]. That rule seems to be successful in practice aand it has been uses by the UK telecommunications regulator and for landing rights auctions in New York City airports [AB17]. According to that rule, the payments should have the following characteristics:

- they satisfy the core constraints,
- they minimize the total revenue for the auctioneer,
- they minimize the Euclidean distance to the VCG payments.

Despite the fact that until now there is no complete understanding of the properties of the Quadratic rule, it has been used extensively in auctions both in business and government environments. Indeed, it has been used by governments to allocate resources with value more than $20 billion [AB17]. However, recently there has been research about the characteristics and the incentive properties of the Quadratic rule, showing that the equilibrium strategies are far from truthful [AB10]. Therefore, there is a need to define and explore better core-selecting payment rules.

It is suggested from some researchers that we should depart from the use of the VCG payment as a reference point [EK10]. There also cases where the objective is not to minimize the Euclidean distance to the VCG payment but to the zero reference point [DP12]. It is shown with the use of computational experiments, in which truthful bidding is assumed, that on average the use of the zero reference point can affect the distribution of payoffs in favor of the winners having higher valuations. However, it is not examined how the use of the zero reference point affects social welfare and revenue. Other research work with a focus on comparing different payment rules [AB10], concluded that core-selecting payment have a better

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performance in comparison with the VCG payment rule when bidders' value are more correlated.

Overall, in order to design a new payment rule or improve an already existing one an experienced auction designer is required. In some cases, there is evidence in favor of or against a payment rule, but in general, there is no comprehensive analysis and comparison of the various payment rules. Therefore, it is not clear yet which core-selecting payment rule has the best performance.

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Chapter 3

Knapsack Auctions

3.1 The Knapsack Problem

The knapsack or rucksack problem is a significant problem in combinatorial optimization. Suppose there is a set of items, each of which are assigned a value and a weight. The aim is to determine the items that should be included in the knapsack in order to maximize the total value without exceeding the knapsack's capacity. The name of the problem refers to the case that a person has a knapsack with a fixed size and he has to select the most valuable items to carry. The problem is often faced in cases of resource allocation where there are financial constraints and it can be encountered in various scientific fields such as combinatorics, computer science, complexity theory, cryptography and applied mathematics.

More specifically, there are two major variants of the problem; the 0-1 knapsack problem and the fractional knapsack problem.

- **0-1 Knapsack Problem**
  Suppose a thief finds n items in a store. However, he has to choose to choose which items he will steal as he cannot carry all of them because his knapsack has a weight limit W. Each item i is worth v_i euros and weighs w_i. The weight limit of the knapsack as well as the weight and the value of each item are integers. The problem is known as 0-1 knapsack problem because the thief must decide whether he will take an item or leave it in the store. It is a binary decision, as the thief cannot take a fractional amount of an item or multiple units of an item. The problem can be expressed as following:

$$\begin{aligned} \text{maximize} \sum_{i=1}^{n} x_i v_i \\ \text{subject to} \sum_{i=1}^{n} x_i w_i \leqslant W \\ x_i \in \{0,1\} \forall i \in \{1,...,n\} \end{aligned}$$

- **Fractional Knapsack Problem**
  The setting is the same as in the case of the 0-1 knapsack problem. In the Fractional Knapsack Problem, there is no constraint according to which the

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thief has to take a binary decision for each item. Nevertheless, the thief can take fractions of each item. Of course, since those problems present such a major difference, there are also differences in the approach used to solve them. The problem can be expressed as follows, with q_i referring to the fraction of item i:

$$\begin{aligned} \text{maximize} \quad & \sum_{i=1}^{n} q_i v_i \\ \text{subject to} \quad & \sum_{i=1}^{n} q_i w_i \leqslant W \\ & 0 \leq q_i \leq \ \forall\ i \end{aligned}$$

Both variants of the knapsack problem mentioned above have the optimal substructure property.

Definition 9. ***(Optimal Substructure)***
A problem has the optimal substructure if an optimal solution can be computed by optimal solutions of its subproblems. That property is of paramount importance to evaluate the applicability of greedy algorithms as well as dynamic programming algorithms.

In the 0-1 knapsack problem [CLRS09], suppose the most valuable combination of items that the thief can steal weighs at most W. If an item i weighing w_i is taken away from that combination, then the remaining combination should be the most valuable combination weighing at most $W - w_i$, that the thief can choose among the $n-1$ items of the store apart from item i. Similarly, for the fractional knapsack problem, it can be shown that it has the optimal substructure property.

Despite the fact that both problems exhibit the property of optimal substructure and share some common elements, we cannot use the same technique to solve them. Indeed, a greedy approach is suitable for solving the fractional knapsack problem, but we cannot apply a greedy approach also for the 0-1 knapsack problem. More precisely, to solve the fractional problem, we should compute the ratio $\frac{v_i}{w_i}$, i.e. the value of each item i per weight unit. Under a greedy approach, the thief is going to select all those items with the highest ratio of value per weight until he reaches, the limit W. In order to optimize the process the items can be sorted by that ratio.

However, the greedy approach cannot work in the case of the 0-1 knapsack problem [CLRS09]. In order to illustrate that the greedy approach is not appropriate for the 0-1 knapsack problem, take for example the setting depicted in Figure 3.1. In that example, there are three items and the knapsack can have items of weight up to 25 in total. The first item weighs 5 and is worth 20 euros, the second item weighs 10 and is worth 30 euros and the third item weighs 15 and is worth 45 euros. Therefore, the first item has the highest ratio of value to weight, which equals to $20/5 = 4$, followed by the second and the third, both of which have ratio of value to weight equal to 3. Thus, following the greedy approach the thief would select the first item firstly. However, the optimal solution includes the combination of the second and the third item and not the first item. Both solutions, which include the first item, are suboptimal.

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In the fractional knapsack problem, the greedy approach, according to which the thief should select firstly the first item, is the optimal approach. Selecting the first item does not lead to an optimal solution in the 0-1 knapsack problem because the thief is not able to fill the knapsack to its full capacity.

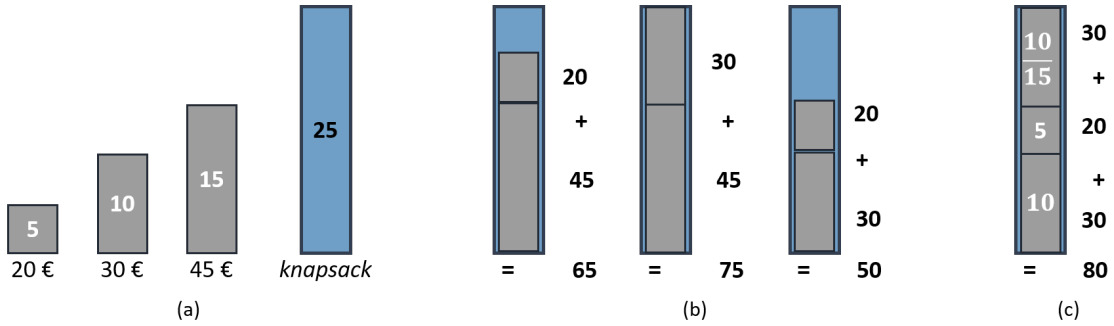


Figure 3.1: The thief should choose a combination of items in order to maximize his utility without exceeding the capacity W (a). The first item has the highest ratio of value to weight, equal to $20/5 = 4$. However, no solution that includes the first item is optimal. Indeed, the optimal solution is the combination of the second and the third item, yielding utility of 75 (b). If the problem is the fractional knapsack problem then ranking the items based on the ratio of their value to their weight and selecting the items with the highest ratio without exceeding the capacity W is the optimal solution (c).

In the case of the 0-1 problem when deciding whether an item should be included in the knapsack, the thief should compare the solution to the subproblem that contains that specific item with the solution to the subproblem that does not contain that specific item. Therefore, the 0-1 knapsack problem can be solved using dynamic programming. Using the following recurrence, a dynamic programming algorithm can be designed. [KT05]

$$OPT(i,w) = \begin{cases} OPT(i-1,w), & \text{if } w_i > w. \\ max(OPT(i-1,w), v_i + OPT(i-1, w-w_i)), & \text{otherwise.} \end{cases} \quad (3.1)$$

The dynamic programming algorithm for the 0-1 knapsack problem has a time complexity of $\mathrm{O}(nW)$, with n being the number of items and W being the capacity of the knapsack. However, we usually consider the running time of an algorithm as a function of the size of the input. The input of the 0-1 knapsack problem is:

- The number of items n, which is using $\mathrm{O}(\log n)$ bits.
- n weights. The weights of the items should be in the range $\{0, ..., W\}$ as we can ignore items with weight greater than the capacity of the knapsack. Therefore, each weight is represented using $\mathrm{O}(\log W)$ bits. In order to represent all weights we need $\mathrm{O}(n \log W)$ bits.
- n values. If V is the maximum possible value, then each value can be represented using $\mathrm{O}(\log V)$ bits. In order to represent all values we need $\mathrm{O}(n \log V)$ bits.

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As a result the total size of the input of the algorithm is $O(\log n + n \log W + n \log V) = O(n(\log W + \log V))$. If we set $k = \log W$ and $v = \log V$, the input size is $O(n(k + v))$. The running time of the dynamic programming algorithm for the 0-1 knapsack problem is $O(nW) = O(n2^k)$. Therefore, the dynamic programming algorithm for the 0-1 knapsack problem is not polynomial, but pseudopolynomial.

3.2 Knapsack Auctions

In knapsack auctions, we have again a setting similar to the auctions mentioned above. There is a set of agents $N = 1, 2, ..., n$ and each of them desires a number of items [AH06]. Suppose that w_i represents the publicly-known number of items bidder i desires. However, there are W available items. Moreover, suppose that v_i refers to the bidder's valuation for having that number of items; this is the benefit of each bidder if she wins.

In a knapsack auction associated with advertising time, for instance [Rou16], the auctioneer has a supply W which can represent the total duration of the commercial time. Additionally, each bidder i has the following attributes:

- a publicly known demand w_i, which can represent the duration of a television advertisement,
- a private valuation v_i, which can represent the company's willingness to pay in order for the advertisement to be shown during a popular television program.

An allocation $X = (x_1, x_2, ..., x_n)$ is defined as a vector of subsets assigned to bidders with $x_i \cap x_j = \emptyset \; \forall \, i \neq j$. The feasible set is defined a s the binary vector $X = (x_1, x_2, ..., x_n)$ such that $\sum_{i=1}^{n} w_i x_i \leqslant W$. Indeed, $x_i = 1$ indicates that the bidder i is a winner, while $x_i = 0$ indicates that the bidder i is a loser. Apart from the advertisement case, this auction setting can be used in various others situations. For instance, it can be used to model auctions including bidders, who want files stored on a shared server, data streams sent through a shared communication channel, or processes executable on a shared computer. Of course, in all the aforementioned cases different bidders can have different demands.

If we aim to maximize the social welfare then in order to find the optimal allocation, we have to solve the following integer problem:

$$\begin{aligned} X^* = \text{argmax} \sum_{i=1}^{n} b_i x_i \\ \text{subject to} \sum_{i=1}^{n} x_i w_i \leqslant W \\ x_i \in \{0, 1\} \forall i \in \{1, ..., n\} \end{aligned}$$

The allocation rule solves an instance of the Knapsack problem in which the bidders' (i.e., bidder) values are the given bids $(b_1, ..., b_n)$, and the item sizes are $(w_1, ..., w_n)$. When bidders bid truthfully, then this allocation rule maximizes the social welfare. As far as the payments are concerned, either the VCG payment rule or a core payment rule can be used, as described above.

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In order to determine the allocation of a knapsack auction apart from solving the aforementioned integer program there are also approximation algorithms. The major objective in approximation algorithms is to design algorithms for NP-hard problems [Rou16], which yield solutions that are as close as possible to the optimal solution, given that they have polynomial complexity. Indeed, also in the case of knapsack auctions there is research around designing approximation algorithms with the best possible approximation guarantee, subject to $P \neq NP$.

There are various approximation algorithms for the knapsack auctions, which exhibit acceptable worst-case guarantees. For example, [MN08] given the bids b_i and the sizes w_i an allocation can be computed using the following simple greedy approach:

1. Sort and re-arrange the bidders according to the ratio of their bid b_i to the sizes w_i so that:
$$\frac{b_1}{w_1} \leq \frac{b_2}{w_2} \leq \frac{b_3}{w_3} \leq ... \frac{b_n}{w_n}.$$

2. Determine the bidders with the highest ratio and add them to the solution, until no other bidder fits and then halt or continue to follow the sorted order, picking any further bidders that happen to fit.

3. Return either the solution of step-2, or the highest bidder, depending on which choice leads to higher social welfare.

It can be proven that assuming truthful bids, the social welfare of the greedy allocation rule is at least 50% of the maximum-possible social welfare. However, that approximation deviates significantly from the optimal solution. The aforementioned $\frac{1}{2}$ greedy approximation algorithm for the knapsack auctions can perform even better if specific assumptions hold. More specifically, if $w_i \leq aW$ for every bidder i and if $a \in (0, \frac{1}{2})$ then the approximation guarantee can improve to $1 - a$.

Additionally, it can also be proven that for every $\epsilon > 0$ there is a $1-\epsilon$ approximation algorithm for the knapsack problem, running in time polynomial to the input ("fully polynomial-time approximation scheme (FPTAS)")[BKV05]. Nevertheless, this approach can be slow; therefore, using it instead of the integer program would not make a notable difference.

In this thesis, the aim is to examine knapsack auctions and compare different allocation and payment rules. In the next chapter, VCG and MRCS will be applied in the same datasets in order to evaluate their performance and compare the revenue generated.

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Chapter 4

Experiments and Results

4.1 Experimental Setup

The experiments presented in this chapter refer to knapsack auctions. As mentioned above knapsack auctions consist of the following elements:

- a set of agents $N = 1, 2, ..., n$,
- a publicly known demand w_i, which represents the number of items bidder i desires,
- a private valuation v_i, which represents the bidder's willingness to pay in order to acquire that number of items.

4.1.1 Data Description

In order to evaluate the VCG and the MRCS payment rules for the knapsack auctions we should decide what datasets we will use. For the evaluation process, we concluded, based on the relevant literature, that the concept of *triangles* is useful. Bidder triangles are defined as follows:

Definition 10. ***(Bidder Triangles)***
Suppose we have three bidders i, j, k in an auction and w_i, w_j, w_k represents the demand of each bidder. Also, W is the number of available items.. Then, the three bidders create a triangle if:

$$\begin{aligned} w_i + w_j &\leq W \\ w_i + w_k &> W \end{aligned} \tag{4.1}$$

Through the experiments the aim is to detect whether the existence of triangles affects the difference between the revenue of the VCG and the core payments in the knapsack setting. There has been shown that there is a correlation of triangles with VCG revenue in the case of auctions with single-minded bidders [San11].

Therefore, three different types of datasets have been created:

- datasets with bidder triangles,
- datasets without bidder triangles,

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- datasets with small demands.

If the supply is 90 an example of a bidder triangle could be the following:

bidder	demand
1	60
2	70
3	30

In that example it is obvious that:

$$\begin{aligned} w_3 + w_1 &\leq 90 \\ w_3 + w_2 &> 90 \end{aligned} \tag{4.2}$$

In order to create the datasets the following parameters should be defined:

- N: the number of bidders,
- l_bound: the lower limit of the valuations' range,
- u_bound: the upper limit of the valuations' range,
- bins: the number of bins to split the valuations' range,
- the number of available items W,
- p_lower: the percentage of bidders with low demands,
- p_medium: the percentage of bidders with medium demands.

In all datasets, the values v_i and the demands w_i are integers. Additionally, datasets with different supply are generated (i.e. $W \in [30, 45, 90, 120]$). Finally, the number of bidders can also take different values (i.e. $N \in [6, 8, 10, 12, 15]$).

Datasets with bidder triangles

In this type of datasets, there is at least one instance of a triangle, as described above. In order to ensure that, the following steps are followed in the generation of the data:

1. Split the valuation range into high, low and medium categories. For instance, if the minimum of the valuation range is 1, the maximum of the valuation range is 30 and the bins are 3, as in our case, the output is $((1, 10), (11, 20), (21, 30))$.
2. Determine the cutting points for the intervals of bidders with low, medium and high valuations. 40% of the bidders will have low valuations, 40% medium valuations and 20% high valuations in order to mimic real data.
3. Calculate how many bidders will have large demands, i.e. $\frac{W}{2} < w_i < W$. It is supposed that 20% of bidders has large demands. The majority of bidders with large demands, i.e. half of them, are also bidders with high valuation. The remaining bidders with large demands are bidders with medium and small valuation.

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4. Select randomly a bidder who has not been assigned a demand and assign her a demand equal to the difference of the supply W and the minimum assigned demand until that point. That way we ensure the existence of at least a triangle. Assign to the rest of the bidders, who have not yet been assigned a demand, a demand w_i, such that $1 < w_i < \frac{W}{3}$.

Datasets without bidder triangles

In this type of datasets there are no bidder triangles. In order to ensure that, the following steps are followed in the generation of the data:

1. Split the valuation range into high, low and medium categories. For instance, if the minimum of the valuation range is 1, the maximum of the valuation range is 30 and the bins are 3, as in our case, the output is $((1, 10), (11, 20), (21, 30))$.

2. Determine the cutting points for the intervals of bidders with low, medium and high valuations. 40% of the bidders will have low valuations, 40% medium valuations and 20% high valuations in order to mimic real data.

3. Calculate how many bidders will have large demands, i.e. $\frac{W}{2} < w_i < cW$. It is supposed that 20% of bidders has large demands. The majority of bidders with large demands, i.e. half of them, are also bidders with high valuation. The remaining bidders with large demands are bidders with medium and small valuation. Since there are no triangles, the maximum demand for the bidders with large demands is set to cW, where $\frac{1}{2} < c < 1$. In our case $c = 0.8$.

4. Since triangles should not exist, determine the maximum demand for the rest of the bidders so that there are no triangles. The maximum is calculated as the difference between the supply W and the maximum of the already assigned large demands.

Datasets with small demands

In this type of datasets all bidders have relatively small demands. In order to ensure that, the following steps are followed in the generation of the data:

1. Split the valuation range into high, low and medium categories. For instance, if the minimum of the valuation range is 1, the maximum of the valuation range is 30 and the bins are 3, as in our case, the output is $((1, 10), (11, 20), (21, 30))$.

2. Determine the cutting points for the intervals of bidders with low, medium and high valuations. 40% of the bidders will have low valuations, 40% medium valuations and 20% high valuations in order to mimic real data.

3. Each bidder will have a demand w_i, such that $1 < w_i < \frac{W}{3}$.

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4.2 Results

The output of the experiments is a table presenting the number of bidders, the supply, the VCG revenue, the core revenue, the maximum difference between the payments of the bidders as well as bidders who had similar payments in both approaches, i.e. bidders that $\frac{p_{i,core}-p_{i,vcg}}{p_{i,vcg}} < 0.01$. An example of the output is the following:

N	suppy	vcg	core	vcg_core_diff	max_diff	similar_payment
6	30	40	44	4	2	(2, 3, 4)
8	30	53	59	6	6	(1, 6, 7)
10	30	26	31	5	3	(4, 5)
12	30	27	45.33333	18.33333	5.333333	(1, 6, 7)
15	30	56	64.5	8.5	2.5	(9,)

In Figure 4.1 it is obvious that the VCG payments are smaller than the payments of the minimum-revenue core-selecting auction. It holds that $p_{i,core} \geq p_{i,vcg} \forall\, i \in N$. However, VCG seems to be a good approximation of core payments.

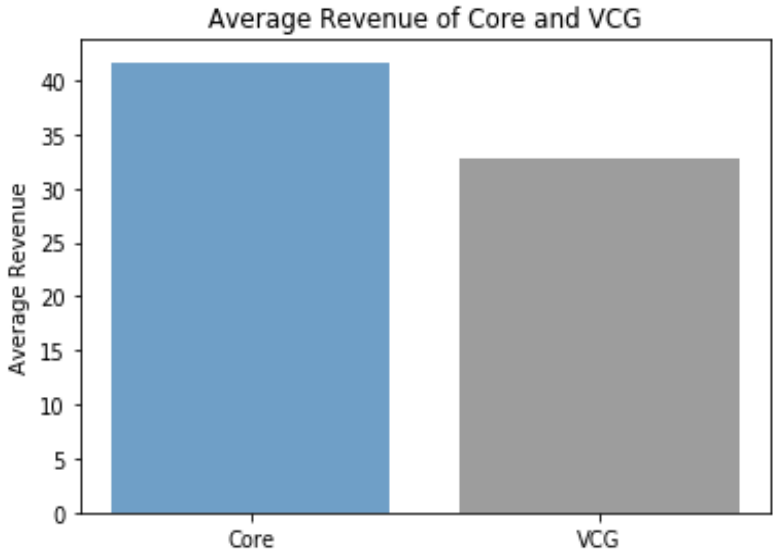


Figure 4.1: Average VCG and Core payments

The average revenue of both VCG and MRCS increase as the number of bidders increases (Figure 4.2). This is happening because, as the number of bidders gets larger, there are more bidders with small and medium demands, who have large valuations, as described above in the data generation process. Nevertheless, the difference between the two mechanisms does not seem to be affected by the number of bidders.

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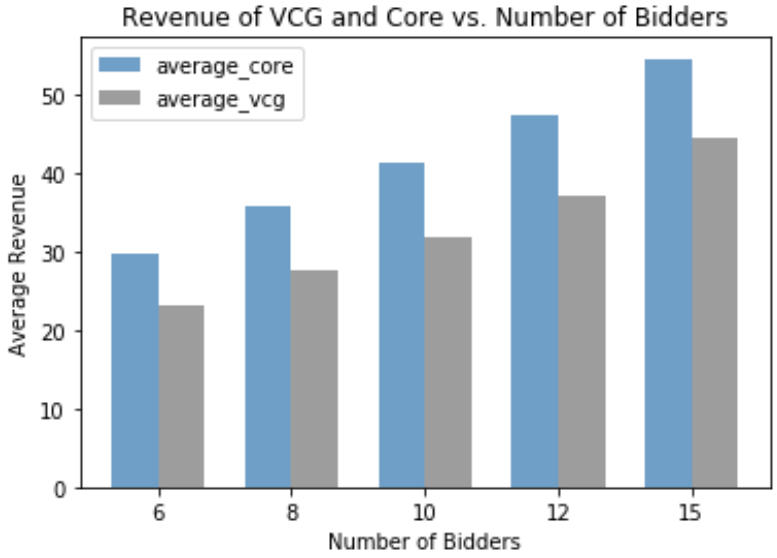


Figure 4.2: Average VCG and Core payments vs. Number of Bidders

Additionally, the supply does not seem to affect the average revenue of both VCG and MRCS (Figure 4.3), because the valuations are distributed the same way and the bidders' demands are determined based on the auction's supply. We observe that the revenues of both mechanisms remain almost the same and as a result, their difference seems to be constant as the supply changes.

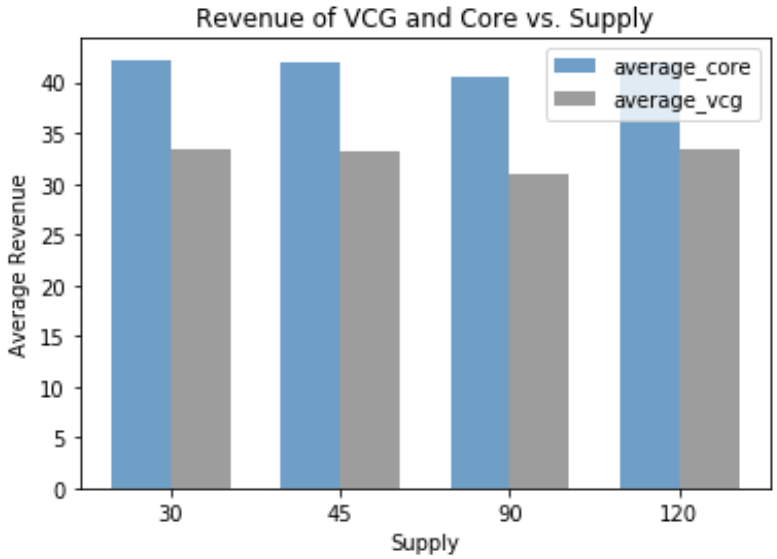


Figure 4.3: Average VCG and Core payments vs. Supply

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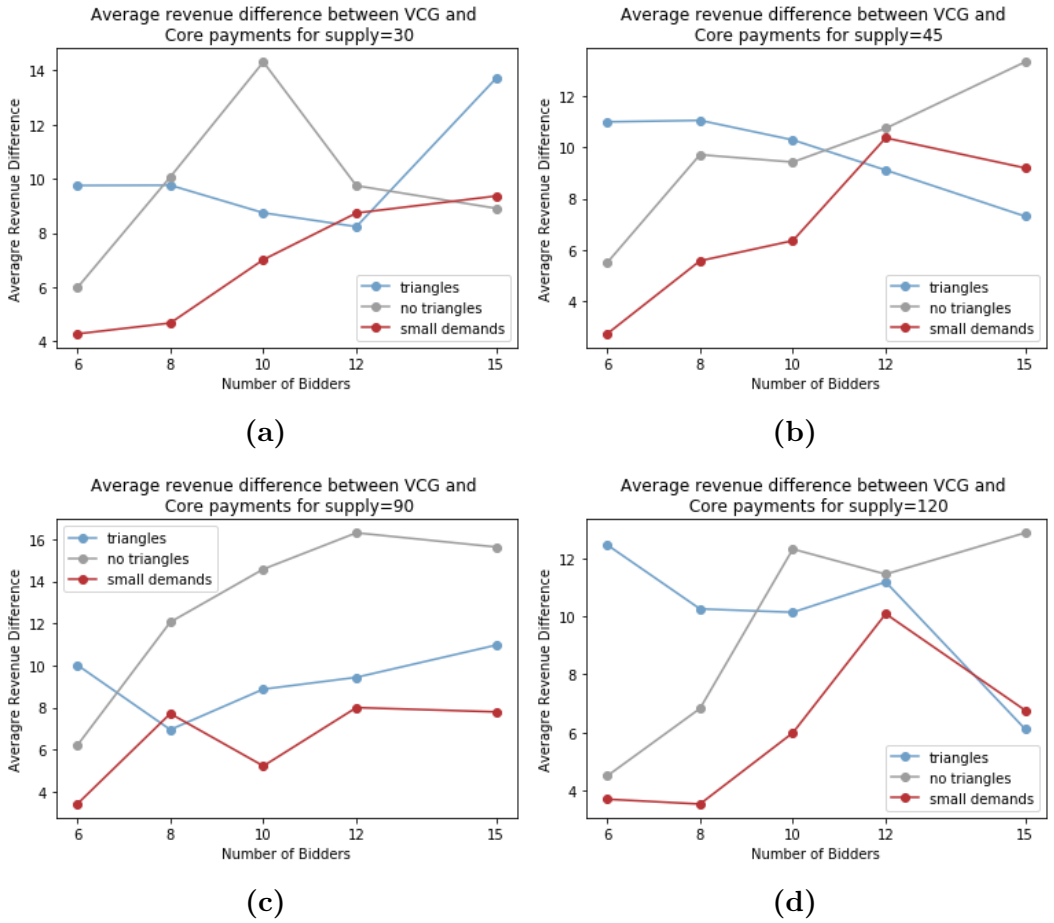


Figure 4.4: Average revenue difference between VCG and core payments vs. the number of bidders as supply increases

The existence of triangles does not seem to affect significantly the average difference between the revenue generated by the VCG mechanism and the core auctions (Figure 4.4). When bidders have small demands, then we observe that the average revenue difference between the two mechanisms is small in comparison with the case of triangles. Therefore, the existence of triangles could affect the generated revenue under certain assumptions such as that there are no triangles and bidders have small demands.

4.3 Conclusions

The implementation of the aforementioned experiments as well as the analysis of the results leads to the following conclusions:

- One of the main weaknesses of the VCG mechanism is that in the worst case, it may have zero revenue. However, judging by the results presented above it behaves well on average in the knapsack setting.
- Given that the core-selecting auctions are computationally intensive we could possibly employ the VCG mechanism in the knapsack setting as it is faster.

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- The VCG mechanism seems to be closer to the core payments when every player has small demand.
- Although there is evidence that there is correlation of triangles with VCG revenue in auctions with single-minded bidders, the existence or not of triangles is not conclusive for the comparison of the revenue of the VCG mechanism and core-selecting auctions in the knapsack setting.

In the future, more cases where the VCG mechanism can perform well in comparison with core-selecting auctions could be identified.

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