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Master Thesis

Empirical Analysis on International Diversification

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Preface

The underlying thesis was conducted in the frame of my obligations as a post-graduate student in the Post Graduate Program: “M.Sc. In International Economics and Finance”, of Athens University of Economics and Business.

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Abstract

The thesis is focused on examining the scopes of international diversification by developing a stochastic framework for portfolio management in order to address the key subjects in international portfolio management: the mitigation of risks through diversification. The purpose of this approach is to achieve balance between portfolio's total exposure and expected return and demonstrate the benefits of international diversification. The study is under the scope of American-based investors (risk lover, risk averse, risk neutral) who choose to allocate income between United States, Japan and United Kingdom. In particular, the allocation of capital across markets is achieved by investing in large and small cap stocks and short term and long-term bonds in each market accordingly. The portfolio's final value is expressed in dollars and its value depends on the amount the investor chooses to invest in each financial instrument and on the financial instrument's current price. Variables in our analysis are considered indices returns and exchange rate appreciation rates. Portfolio composition is derived from repeated optimization problems for the successive months from 12/14 over 5/2018. Different optimization problems are solved for each type of investors. Through repeated optimizations, the optimal holdings for each month are attained for all investor types. We estimate portfolios' monthly returns using the optimal holdings we attained through our repeated optimizations and the returns we estimated for each index. The actual returns are estimated by multiplying the weights in each index, with their next-month return. Ex post analysis of portfolio's returns and risk adjusted performance measures revealed that the superior portfolio was risk averse's since he diversified more. Our model proved effective to demonstrate the importance of diversification through interpreting risk averse's performance but also to achieve balance between expected return and risk. Observing the ex post cumulative returns and statistical measures it was revealed that the portfolio of risk averse, not only exhibited higher return levels but also the most effective mitigation of risks. Our holistic approach of portfolio management proved effective to enhance diversification benefits.

Key Words: *Portfolio Theory, Risk, Return, International Diversification*

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Introduction

Subject

International investments have gained considerable attention the past years and they have become a conjoint part of the financial world. Many studies have shown that they enhance economic welfare of individuals and countries and promote efficiency. Furthermore, global investing broadens the scope for diversification, which is the cornerstone of financial theory. Diversification is raised by combining financial instruments across the world, which would react differently under the same event.
International portfolios benefit from foreign assets' performance, since correlation decreases across different markets. International diversification is without a doubt a very interesting part of the financial field which requires careful attention and treatment. Portfolio management is centered on the diversification process.

Our study focuses on the international portfolio management. Specifically, a development of a stochastic programming model was realised, which addresses key decisions of international portfolio management process. Key decisions of global investing are considered the international diversification and the reduction of risks.
The study aims to achieve an effective balance between the expected return and the total exposure through international investments.

Description of our Analysis

As noted previously, our study focuses on international portfolio management with the intention to achieve an effective balance between expected return and total exposure. This process involves many decisions and tactics that will be analyzed in the final chapters of our study. In this section, we will briefly discuss the key notions and elements of our analysis.

The first step of the study was the decision to allocate income across three major countries: *United States, United Kingdom and Japan.* Our analysis is under the scope of US investor who chooses to allocate income across the aforementioned countries. Portfolios combine different stock and bond indices of each of the countries and their final value is converted into the base currency (US Dollar). Key variables of the empirical analysis were the monthly exchange rates movements between the currencies with respect to the base currency, and the indices' returns over the period of 12/2014 to 5/2018. For each of the successive months we aim to construct optimal portfolios for three investor types (risk lover, risk defensive and risk neutral) with different constraints and treatment. Our optimization models involve the conditional value at risk metric and the expected return. To accurately address each type, a different approach was in order. In the case of risk lover, we implemented a maximization on the expected return whilst for the other two types a conditional value at risk minimization was imposed. Our goal is to find the optimal asset holdings for each holding period and estimate their final returns. We carefully address the results of these repeated applications as to if the effective balance between expected return and risk was achieved for all the categories of investors. Risk adjusted measures were estimated to show the final performance of each portfolios, and a comparison of results among all investor types. Finally, we derived the main conclusions of our empirical analysis.

Scientific Contribution

International diversification is a very interesting subject in the financial field. It shows in an accurate way, that investing beyond the domestic markets can enhance investor's profitability and decrease the exposure. However, this subject has received criticism from the academic world as to its actual benefits in the long run. Furthermore, many investors, firms and individuals are resistant to allocate income internationally. Home country bias is still strong and present in the financial world due to the notion that domestic investments are safer, and their progress is more easily predicted. Our purpose is not to argue with this fact, since it still holds for particular countries and under certain circumstances. Our goal is to show with accuracy that investing internationally can in fact improve portfolios performance and enhance diversification benefits. In addition, we aim to show that the stochastic programming model is effective to achieve balance between the level of expected return and total exposure. Furthermore, the analysis reveals that even for investors who live in countries like the United States, one of the strongest economies of the world with a relative strong currency, international investments can be more desirable than domestic. The importance of international diversification is revealed through the results of our analysis. Our findings were carefully estimated through optimization process and can contribute to the literature review to ensure the diversification benefits, by addressing the portfolio management in a unified manner.

Literature Review

International Diversification is a subject that has been introduced in the financial field for years. A lot of research has been raised for this issue, from many academics. It is important to review some of the key findings of the studies that have been conducted regarding the benefits, caveats and aspects of international diversification.

The basic notion behind international diversification is that it enables the investor to reduce portfolio risk while simultaneously improve portfolio returns. This section focuses on the related literature review findings about international diversification. A number of papers are examined in order to demonstrate the key findings of international diversification.

According to Solnik (1974) the total risk of the portfolio depends on the number of assets a portfolio consists of, the level of risk for each individual securities involved, as well as how these risks are interdependent. Solnik illustrated that diversification increases as more assets are added in the portfolio. This was done for several European countries (UK, Germany, France, Switzerland, Italy, Belgium and Netherlands). The results on a domestic level indicated that as diversification increases, the risk of the portfolio decreases. However, the risk of a portfolio can be mitigated to a certain level. An increase in the amount of assets in the portfolio, after a particular extent will have no further impact on the domestic level. However, by investing internationally in different countries, the portfolio risk can be reduced even further. Solnik presented how an international portfolio of the same size as a domestic portfolio of the European countries provide substantial risk reduction. Furthermore, the study of Solnik and Odier (1993) analyzed if adding bonds would improve the risk-adjusted performance of an internationally diversified portfolio. The results of their analysis show that the efficient frontier of an international stocks and bonds portfolio is better than a domestic portfolio of stocks and bonds. The choice to include international assets to a domestic portfolio depends on the correlation among the assets (Solnik & Mcleavey, 2009). In particular, the correlation of the international assets and domestic assets is not large. This has led various studies to investigate correlations. The study of Tang (1995), Login and Solnik (1995), revealed the increasing correlation among stocks and bonds. Study of Chatrath, Ramchander and Sanjay (1996), validated the belief that correlations between the U.S and emerging market returns have the tendency to increase if markets deteriorate. On the other, Koshers, and Vivek (1998) documented that emerging markets that have low correlations with other countries are a promising investment option since they contribute to diversification benefits. Similarly, the study of Gilmore and McManus (2001) examined the short- and long-term relationship between the US stock market and three Central European market and found low correlation between the markets. The study of Goetzmann, Li and Rouwenhorst (2005) analyzed the correlation structure of the global equity market over the last 150 years and found that the correlation structure changes substantially over time.

According to Solnik and Mcleavy (2009), the growing integration of the markets increases the correlation of assets through time. As mentioned earlier, the growing integration of market and their globalized trend, decreases deregulation thus markets are becoming more synchronized.

Focusing our interest on frontier markets, the paper of Berger, Pukthuanthong, and Yang (2011), analyzed the frontier markets and found that these markets have low levels of integration. It was revealed, that as opposed to developed and emerging markets, frontier markets are not going to be more integrated in the future. This fact can be beneficial for investors, since risk reduction is ensured.

International diversification depends also in exchange rate fluctuations. The study of Solnik, Boucrelle, and Fur (1996) revealed that exchange rate movements do not seem to affect portfolios value since the depreciation of one currency is neutralized by the appreciation of another. The extent of this study showed, that even an unprotected against exchange rate risk portfolio is described by lower risk levels than a domestic constructed one. This can be explained. When more currencies are involved in the process, a negative impact on one will be neutralized by the different or no movement of the other. Thus, currency risk can be reduced in general terms.

Moreover, other studies about international portfolio diversification applied methods to show the benefits of this process. The study of Nikolas Topaloglou, Hercules Vladimirou, Stavros A. Zenios (2002) focuses on applying scenario-based optimization problems that simultaneously determine the portfolio composition and the appropriate hedging level for each foreign asset. The purpose of the study is to examine the performance of the optimization models as risk control method, in order to select internationally diversified portfolios. The analysis is through the scope of a US investor, so the findings are relative to this base currency. The concluding remarks of the study showed that the scenario generation method, is superior that random sampling method since its consistency and effectiveness are higher. Furthermore, the composition of risk-return efficient frontiers remained relatively stable through the process, at any level of target return. Conceptually, the risk is not increased to a great scale when the return is higher. Volatility of returns is controlled through the process since flexible investment recommendations are achieved. The study of Nikolas Topaloglou, Hercules Vladimirou, Stavros A. Zenios (2002) is based on scenario optimization problems using the Conditional Value at risk metric and is effectively improving risk and return performance of globally diversified portfolios.

Another study by the same authors is the approach of optimizing portfolios through a stochastic programming model using options and forwards. (Optimizing international portfolios with options and forwards Nikolas Topaloglou, Hercules Vladimirou, Stavros A. Zenios, 2011). The analysis considers the presence of currency and market risk in its attempt to optimize global portfolios. The portfolios used in this study combine stocks and bonds denominated in multiple currencies, while the key variables are the asset returns for each period and the exchange rate movements. To protect against this risk, the study suggests holding European Options on stock indices to control of market risk and the use of quantum (cash-settled cross-currency derivative) options to hedge against foreign market and currency risk. Quanto is a derivative that is denominated in another currency other than the option is settled. Continuing the study, after addressing the risk a globally portfolio bears, and after finding ways to hedge against it, they carried out alternative strategies to optimize international portfolios. To approximate the empirical distribution of the random variables, the authors used a moment matching method for scenario generation. By using the proposed stochastic program using market data, and different decision strategies, the analysis concluded that the unhedged against risk portfolios exhibited the worst performance. In particular, their cumulative return was the lowest in high levels of volatility. When currency forwards were introduced to the process, the volatility of returns exhibited stability, but lowered the level of cumulative returns. In order to "heal" this problematic issue, options on stock prices were used which in fact improved the levels of return. A joint treatment of this issue according to the authors is more effective than dealing with one risk at a time. Specifically, they suggest hedging against market and currency risk at the same time by adopting options which empirically exhibits better results on returns and volatility. The empirical results of the study were promising since the model exhibited effectiveness to a holistic approach of risk in global portfolios. It is worth noticing that a key aspect of the study is the stochastic programming framework through which the assets' performance is examined. The effectiveness of it is proved through empirical results since it exhibited flexibility through the process, and it could provide accurate results for different decision-making strategies. Furthermore, the risk metric that is chosen is Conditional Value at risk since it exhibits coherence and is suitable for asymmetric and fat-tailed distributions which is proved appropriate for international diversification portfolios' problems. Through that metric, coherence is satisfied, and a better approximation of results is ensured. Concluding, the choice of a multi-stage stochastic program is able to reflect the evolution of random variables. In addition, it can affect the decision dynamics (interdependent decision-making procedure that is taken under a continuously changing environment due to factors that cannot be controlled or because of previous actions of the previous decision maker) as a response of the changing information as time passes.

The study of Esther Eiling, Bruno Gerard, Pierre Hillion, Frans A. de Roon (2012) is another approach for the international diversification issue. In particular, they use the mean-variance method to estimate the efficiency of portfolios. Their study focuses on examining the importance of country, industry, market and currency risk factors that affect stock's returns. To examine this effect, they use the International Capital Pricing

Model (investors have same opportunity sets, fully integrated markets) since it includes the world's return and involves several currencies. To prove its effectiveness, they conduct a two-step process in which they test if mean-variance analysis is efficient for these portfolios. Specifically, they aim to examine if the model of International Capital Pricing Model is the appropriate one to reflect the equilibrium international returns of stocks. The findings of this examination show that this test does not detect significant differences between country, industry and currency portfolios. Spanning is only marginally rejected for the ICAPM portfolios with respect to global industry returns. Moreover, the unconditional analysis provides no evidence that currency risk factors matter; currency returns are spanned by country, industry and world returns. Continuing the study, the authors implemented time varying means and covariances on managed portfolios to assess which factors dominate. It is worth noticing that seven major developed stock market indices (US, Canada, UK, Japan, Germany, Italy and France) were used since they were the seven largest world economies for period 1975-2005 which is the time horizon under examination. Results of the paper show that stock's returns are driven by industry and currency risk factors, so the volatility stems primarily from these types of risk. Additionally, one result of the study is that market portfolio is efficient with respect to country returns but not with respect to global industry returns. Currency returns improve the mean-variance efficiency of country, industry and word market portfolio returns.

These implications were used to examine diversification strategies. Passive country and industry-based equity strategies deliver indistinguishable Sharpe ratios. However, when portfolios are rebalanced every month the second strategy leads to significantly higher Sharpe ratios. The difference between these strategies depends on the ability to take short positions. In actively rebalanced portfolios, currency factor is crucial for achieving optimal portfolio performance. Conceptually, active currency investing contributes to international diversification and increases returns to cross border portfolios.

The study of Fathi Abid, Pui Lam Leung, Mourad Mroua and Wing Keung Wong (2014) also applied Markowitz's mean-variance analysis to examine the preferences for international versus domestic diversification from American investor's perspective.

The analysis implies no arbitrage opportunity. Moreover, stochastic dominance test is used for the same purposes and is considered an efficient approach since it provides a framework for examining portfolio choice without the need of asset-pricing benchmarks. The study involves daily data of 30 highest capitalization US stocks and 20 international market indices from Latin American and Asian financial markets and the G6. For the purposes of the study, a utility maximizing investor is considered, who holds two portfolios. An internationally diversified and a domestically diversified portfolio are ranked with respect to investor's wealth and utility. It is worth noticing, that investor is considered to be risk averse. The first step of this process is to obtain the efficient domestic and international portfolios and then study the preference. It is assumed that domestically diversified portfolios are more suitable for higher risk investors and internationally diversified are preferable for lower-risk investors. To continue, the classical optimization problem is implemented followed by stochastic dominance test to overcome the constraints of the classical optimization problem. The stochastic dominance set is closely related to utility theory since by SD approach investors always obtain higher utilities by choosing stochastic dominant assets.

The concluding remarks of the study are that some domestically diversified portfolios dominate some internationally diversified portfolios, supporting those who claim that domestic diversification is better. On the contrary, the same argument implies to those who claim that international diversification is better since the findings suggest that some internationally diversified portfolios dominate domestically diversified ones. Furthermore, some internationally and domestically portfolios don't exhibit any difference in stochastic dominance process.

In a more specific way, the authors imply that for a lower domestic risk level, domestic diversification is more efficient when the foreign risk is higher. On the other hand, when foreign market risk is low, in fact lower than domestic, international diversification is preferable. Additionally, internationally diversified portfolios with lower risk level are stochastic dominant compared to domestic ones. This satisfies the argument that international diversification is better. The paper satisfies all arguments about international diversification and the choice of portfolios' management lies to the type of the underlying investor (Risk averse, Risk lover, Risk-neutral)

Studying the benefits of international diversification, the study of Robert J. Hodrick and Xiaoyan Zhang was raised for further analysis. The authors used country index returns from 8 developed countries and 8 emerging market to re-explore the benefits of international diversification the past thirty years. Examining if international diversification is actually beneficial like in theory, the implemented Markowitz's approach for portfolio optimization. They used Sharpe ratios as risk-return trade-offs of the test in order to determine the effectiveness of global diversification. Furthermore, alternative strategies of international diversification were implemented. The strategies involved the naïve- investor approach (no short-selling) who gives equal weights to all country indices available in data set. Moreover, another strategy mentioned in the analysis is the period-by-period, mean-variance maximizer adopting the mean-variance frontier. Through this strategy, choices of weights depend on means, covariance and portfolio return each period, so a movement of them changes the weights. Because this strategy is characterized with a lot of limitations, it is suggested to start with equilibrium weights each period. This approach is justified through the continuation of performance of returns through successive periods in time, considering that current equilibrium weights are the latest weights of the previous period. Another approach is raised through the study. It is suggested to choose portfolio shares as functions of limited numbers of stock characteristics to maximize the expected utility of investor. The data set of this paper involves monthly returns of the pre-mentioned indices. Concluding, the authors referred that the Sharpe ratios from international investments especially those hedged against currency depreciation, appear to be quite better than investors can do from investing strictly in their local country index, and conceptually all the diversification strategies appeared to be to a certain extent effective for a globally diversified portfolio.

The study of Cheol S. Eun and Bruce G. Resnick (1991) focuses on the benefit of international diversification of investment portfolios from Japan and US objectives.The basic notion of the study is that currency risk might increase the benefits of international diversification and its importance as a parameter in the globally diversification process is high.

The first step of the study is an ex-post analysis of the gains from international diversification of bond and stock portfolios from a US and Japan investors' perspective. Then, exchange rate uncertainty is added to the process in order to examine its result and thus examine the ex-ante diversification strategies. The underlying research showed, that international diversification benefits are present in the absence of currency uncertainty and both types of investors receive gains. It is worth noticing, however, that US investors received higher levels of return and Japanese investors received lower levels of risk. When exchange rate uncertainty is introduced into the picture, US investors benefit from globally diversified portfolios through ex ante investment strategies. On the contrary, that is not true for Japanese investors. Japanese investors receive modem gains by the beginning of the procedure. Concluding, the paper suggested the importance of the currency risk presence in international diversification. Hedging against currency risk through bonds proved extremely beneficial especially for US-based investors since portfolios with risk hedging outperformed the international stock portfolio. Conceptually, this paper suggested that bonds are a very important tool in achieving international diversification.

As mentioned earlier, another aspect of international diversification is the different markets an investor chooses to invest. The study of Gholamreza Mansourfar, Shamsher Mohamad and Taufiq Hassan (2010) examined if the Middle East and North Africa actually provided diversification benefits for global investors. The first step of the study is to divide these countries' stock market into two main groups. These groups involve oil producing and non-oil producing countries. Both of these groups are examined to assess their ability for participating in the global diversification process. Short -term and long-term efficient frontiers were obtained. Following, the combination of these frontiers was examined. The authors proposed the usage of Multiobjective international portfolio models, in order to find the optimal portfolios of this region. The conclusion of the examination of Middle East and South Africa as possible investment markets was that stock markets of oil-producing regions provide diversification benefits for domestic and foreign investors. In addition, the joint treatment of all countries under examination provided better results with respect to international diversification, than sub-groups of the regions.

By examining the study of Gholamreza Mansourfar, Shamsher Mohamad and Taufiq Hassan (2010) we obtained useful information about some emerging markets of the world and concluded that international diversification is extremely beneficial for investors even if they invest in other than developed countries.

Furthermore, there are also studies that assess international diversification benefits before, during or after financial crisis periods. The study of Alexander Eptasa, Lawrence A. Leger (2010) uses mean-variance analysis with short selling constraints to diagnose the effects of global financial crisis by evaluating potential benefits of international diversification in the search for safe heavens. The authors used data of developed countries in addition of countries that could reasonably satisfy the safe heaven characteristics. (Safe heaven markets exhibit negative correlation or no correlation with downturn markets in the period of financial crisis). Safe heaven markets retain or increase assets' value in the time the market crash is into the picture. In a country-specific view, countries with GDP and as a result, its domestic stock prices remain relatively unaffected by the financial distress, are considered to be a safe heaven. In particular, countries with oil production were excluded and no other countries with available equity market indices appear to be dominated by a single product.

The results of the paper showed international diversification opportunities in the pre-crisis period but a disjunction between stock and bond markets in the crisis period. Two 'safe-haven' countries emerge for the crisis period, Colombia and (to a lesser extent) Israel. These countries have both the highest returns and almost the lowest correlations with other countries during the crisis period and are likely to be the only countries selected for the efficient portfolio of a $US investor. Concluding, it is noticed that Israel may be protected by special trade arrangements, 'diaspora support' and the willingness of non-resident citizens to repatriate income.

The study of Abidin et al. (2004) examined whether diversification gains in equity investment are present for the Malaysian investors. They compared two domestic portfolios based on market capitalization against several international portfolios. The portfolios were analyzed for several period of pre-, during- and post-crisis over a 17-year period. In general, the results show that the international portfolios performed better in most of the sub-periods. However, the findings in favor of international diversification for the Malaysian investor. They found that it is not always the case that an international portfolio has to perform better, because the domestic portfolios have shown to perform better in crisis periods and even non-crisis periods.

Another study about diversification with regard to financial crisis is Plamen Patev and Nigokhos Kanaryan (2003) paper. The purpose of the study was to evaluate market integration between U.S. stock market and CEEM from an American -based investor's point of view and to investigate the Central and Eastern European equity market co-movements before, during and after major emerging market crises before during and after major emerging market crises. The study is based on the concept of co-integration. Research aims to quantify the integration between markets in Central and Eastern European countries. The countries involved are Russia, Poland, Czech Republic and Hungary. The interest of this process is how market crisis affect international diversification benefits. Between the concluding remarks of the study was the notion that Central European and Eastern European Countries are considered segmented markets. Furthermore, the empirical results of the paper suggest that an international investor is able to receive extra profit through international diversification. In addition, through collapse periods, increase in co-movements between markets is occurred. Conceptually, when markets crash, the diversification benefits deteriorate since the correlation between them is increased through these periods. A rational investor observing this effect will decrease the amount invested in this portfolio. After the crisis, the co-movements between markets tend to decrease, thus, a part of diversification is restored. It is worth noticing that the pre-crisis level of diversification is higher than the post-crisis one. These results support the notion that a dollar-based investor can benefit from diversification in this area. Since these markets are not strongly correlated, they exhibit benefits for investors.

Another key aspect in international diversification process is the presence of transaction costs through this procedure.

The study of Rajiv Kalraa, Miroslav Stoicheva, and Srinivasan Sundaram (2004) examines the effectiveness of internationally diversified portfolios through the presence of transaction costs and periodic rebalancing. It is logical to presume that benefits of global diversification reduce when transaction costs are into the picture. The authors used monthly equity index data in local currencies and in U.S. dollars for the 13-year period 1988 to 2000 which are taken from Morgan Stanley Capital International (MSCI). Moreover, to evaluate the effects of rebalancing internationally diversified portfolios, they created a purely domestic portfolio (S&P 500 index) and six internationally diversified portfolios with varying international components. The performance of portfolios is tracked by monthly returns in home currency and US dollars. The countries involved in the process are *Korea, Thailand, Asia, Brazil, Argentina, South America, Austria* because of their relatively low correlation with each other. The results of the study showed that non rebalancing internationally diversified portfolios, resulted in country weights becoming different compared to their initial values. There is a positive relationship between the weights invested and the transaction cost movement. Another issue that is worth mentioning is that when rebalancing occurs more often, the performance of portfolio tends to decrease through time. Additionally, when transaction costs are relatively high and the rebalancing is often, an additional decrease to the expected return occurs. The joint presence of these components is observed to deteriorate the portfolio's performance. On the other hand, portfolios with no rebalancing procedure tended to exhibit greater volatility and the worst award to risk ratios. Not rebalancing resulted in lower Sharpe ratios for all internationally diversified portfolios at all transaction cost levels indicating that even in the presence of high transaction costs, rebalancing positively impacts risk-return performance.

The highest Sharpe ratios through the study occurred when transaction costs were relatively low, foreign component is small, and rebalancing is less frequent. When foreign component shifted upwards, the rebalancing became more frequent through the procedure. Concluding, the portfolios with large international components and high transaction costs also exhibit the largest average negative differences in Sharpe ratios relative to the purely domestic portfolio.

A general conclusion of the study is that international diversification benefits are exaggerated in theory and practice since the absence of transaction costs is frequent. On the other hand, further examinations about this issue need to validate this remark, since the time horizon of the test is crucial for making conclusions.

International Diversification is without a doubt a major issue in economic field that has received wide attention. Indeed, there are numerous amounts of literature around this subject with regard to different implications, methods, markets and perspectives. The attempt to globally diversify a portfolio of financial instruments is also subject to correlation, integration, exchange rate implications and constraints. Researches have produced a numerous amount of perspectives to achieve the desired result: international diversification. Through literature reviews, empirical findings are a source of useful information for academics and investors as well.

By reviewing the related literature about international diversification, we conclude that the majority of the research findings were in favor of international diversification. Studies have shown that benefits are more than caveats raised when investing internationally.

Our study aims to show similar results. We focus our interest in constructing international optimal portfolios that achieve an effective tradeoff between risk and expected return. In this study, different aspects of international investments and portfolio management methods are also going to be discussed.

We present the organization of our thesis:

The thesis is organized as follows. In **Chapter 1**, we present the definition of risk and we analyze the different types of financial risks. In **Chapter 2**, we focus our interest in different methods of portfolio management regarding risk. Following, in **Chapter 3**, the notion of International Diversification is presented. Moreover, the benefits and caveats of International Investments are mentioned. In **Chapter 4,** we analytically present our data our estimation process, and results about our empirical analysis. Finally, in **Chapter 5** the conclusions and findings of the empirical analysis are presented in an analytical manner.

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Chapter 1

In this chapter we will focus on analyzing the financial risks faced by investors and financial institutions. Financial risk determines the decisions made by investors and companies since it plays a dramatic role in the outcome of the actions they undertake. Furthermore, exposure to financial markets affects firms and individuals directly or indirectly. More specific, as markets become more and more global, the events and actions in one country can affect very rapidly other ones. Most companies and investors are internationally active, and they can be affected by events that have nothing to do with domestic markets. Due to globalization, loss or gain investors get, is prone to international affairs. Financial loss or gain results from the exposure of the firm or investor in the financial markets.

1.1.1 Financial Risks

1.1.2 Risk Definition

In order to continue our study, we should first determine what is the risk that companies' shareholders and investors cope up with. Risk refers to the probability that the expected return will defer than the actual return of the investment. Furthermore, risk includes the scenario in which the firm or investor losses a part, or his whole initial budget. Risk arises when financial transactions take place amongst organizations, countries etc. It is very common that risk is referred to as downside risk, meaning the potential loss and the uncertainty about the extent. Clearly, risk takes many different definitions. A definition that includes all the above meanings is **the probability of deviation from an anticipated outcome**. The implications of this definition are given below. We can attach probabilities to risk. Therefore, it can be measured, estimated or calculated in some way. Risk can therefore be quantified and expressed as a parameter, number or value. Risk is concerned not just with the extent or probabilities of potential losses but with deviations from the expected outcome. It is the extent to which the actual result may deviate from the expected result that makes a situation risky. Risk is a function of objectives. It is the consequences of the actual result deviating from the expected result that leads to risk. Without an objective or intended outcome, there is only uncertainty. A rider to this is that risk arises only where the deviation from the objective matters; that is, if it affects individuals or firms financially, or entails some other adverse consequence. It can also provide an opportunity.

1.2.1 How Risk Arises

As we mentioned earlier, the risk arises when a company or an investor perform financial transactions. Financial transactions include sales, purchases, investments, loans and other business activities. In addition to the transactions that take place, risk can arise due to factors out of the market conditions. Weather and natural disasters can lead to risk or change its prior extent. Specifically, there are three types of financial risks (Essentials of Financial Risk Management, 2005)

1. Financial risks arising from an organization's exposure to changes in market prices, such as interest rates, exchange rates, and commodity prices.
2. Financial risks arising from the actions of, and transactions with, other organizations such as vendors, customers, and counterparties in derivatives transactions.
3. Financial risks resulting from internal actions or failures of the organization, particularly people, processes, and systems.

Concluding, the risk is intertwined with business activity. In most cases it cannot be eliminated, or undertakers are willing to accept risk in order to take a higher return. Since it cannot be out of the picture, firms and investors are called to manage it. But for that to take place, it must first be understood and measured. The first step in risk management, is the right definition of risk and how does it appear in the business process. This process can be quite difficult in action, since it includes specific evaluation, prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability or impact of unfortunate events or to maximize the realization of opportunities.

1.3.1 Types of Risk

This section focuses on different types of risk a company or an investor undertakes. It is very important to understand, that risk in order to be managed it must be clearly evaluated and it should include all types of risks that a firm is exposed. Risk can be categorized into the following types:

1. **Business Risks:** They arise from the internal and external environment of the company and are linked with strategic decisions. Specifically, they are taken by organizations or investors in order to maximize profit or value.
2. **Non-Business Risks:** These are not under the control of the company. Political and economic imbalances, weather conditions, natural disasters can be described as non-business risks.
3. **Financial Risks:** Risks that originate from the way that the firm or investor finances their activities and involves the potential loss. Financial risk arises due to the instability of the stock market, fluctuations in interest rates and movements in the value of currencies.

In our analysis, we will focus on the financial risk a portfolio is exposed to. Financial risk is described as the probability that a certain event can negatively and unexpectedly affect the return of an asset or diminish its value. A portfolio's return can be affected by several financial risks and we will examine it further as to which these risks are.

1.4.1 Financial Risks

Along with the different types of risks we discussed, the companies and investors face a variety of financial risks. The most important financial risks are the following:

i. Market Risk
ii. Interest Rate Risk

iii. Credit Risk
iv. Operational Risk
v. Liquidity Risk
vi. Foreign Exchange Risk
vii. Commodity Price Risk
viii. Equity Price Risk
ix. Country Risk

1.4.2 Market Risk

The first category of financial risk we will analyze is market risk. Market risk is referred to as the possibility that an investor will experience losses, due to factors that affect the overall performance of the financial markets he or she is involved. Market risk is often called "systemic" risk and it cannot be eliminated with diversification. Investors and companies in order to deal with market risk, hedge against it. As we discussed, market risk covers all the possible losses that arise from the fluctuation of prices and the market index. Specifically, fluctuation of interest rates, exchange rates, stock prices and commodity prices are considered as market risk. The effect of these movements can be direct when it refers to the operation of the firm, or indirect when it refers to customers, vendors or competitors (Lhabitant and Tinguely,2001).The European Banking Authority defines risk as to the danger that a fluctuation of a certain asset is higher or lower compared to the market and the general market risk.

Financial institutions are exposed to risk when they negotiate assets, liabilities or derivatives. (Saunders&Cornett,2003). Consequently, market risk expresses the uncertainty of future return or value of the firm. In order to deal with the market risk, financial institutions are called to mathematically express it. Specifically, firms and investors used to estimate market risk using as parameter the interest rate risk. Exposure to interest risk is estimated by two methods(Smith et.al,1989) .The first method defines the impacts of interest rate fluctuations in future cash flows, net interest income and profits. This method is called *Gap analysis*. Through this method, net interest income is calculated in different time periods, under different interest rate levels and different asset and liabilities management strategies. Furthermore, the second method focuses on the impact the interest rate had on the market value of the firm. Specifically, in this method, the *duration term* is used in order to determine the sensitivity of assets and liabilities to interest rate movements. In a parallel manner, meters such as kurtosis, beta coefficient were used to calculate the sensitivity of financial instruments.(Financial Institutions Management: A Risk Management Approach 6th edition).

The methods we analyzed to estimate market risk with respect to interest rate express some disadvantages. One weakness of these methods was to implement it in non-financial institutions. Furthermore, non-of these methods were able to create ways to calculate the whole market risk (Lhabitant and Tinguely,2001). A better method to estimate the risk is value at risk and we will analyze this and other ways in the following chapter of our study.

To end up, we will discuss the importance of the right estimation of the market risk. The specific estimation is important to the firms and investors for the following reasons (Rahl & Esseghaier,2001):

i. *The proper market risk estimation gives feedback to the firm's management. The management makes use of this information and can address the exposure at risk and compare it to the capital adequacy of the firm.*

ii. *Risk calculation and management set boundaries at purchase and sale positions of the investor or the company.*

iii. *It promotes allocation of the resources achieving a better return with respect to risk.*

iv. *Risk identification enables the parties involved to determine the capital needed according to institutional purview.*

1.4.3 Interest Rate Risk

In this section we will focus our study on interest at interest rate risk. As we discussed before, the interest rate is included in the market risk, but it should be identified separately. Market risk may include interest rate risk, but it is a more general term. One definition of interest rate risk is the following: *Interest rate risk is the probability of an adverse impact on profitability or asset value as a result of interest rate changes*.

Interest rate risk affects many organizations, both borrowers and investors, and it particularly affects capital-intensive industries and sectors. Another definition of interest rate is the following: *Interest rate exposure is generally described as the risk of a reduction in a projected or anticipated measure of the net interest income (target measure) resulting from changes in market interest rates.* Yet from a practical perspective such a definition is somewhat flawed, as the use of an anticipated (or projected) measure of net interest income is fraught with risks. Any inappropriate assumption in the projection phase will produce an inaccurate target measure and, consequently, result in an inaccurate assessment of interest rate risk.

In a more useful way, interest rate exposure could be defined as the risk that the amount of net interest income obtainable at unchanged interest rates may not be attained given an adverse change in market interest rates. Conversely, banks stand to benefit from an "interest rate opportunity" should favorable changes in market interest rates drive up net interest income. Using a "zero line" thus makes it possible to rate the risks and opportunities that arise from changes in market interest rates. Another key factor in the equation is the target measure to be used for evaluating the credit institution's performance. The Basel Committee on Banking Supervision, for instance, bases its definition on the different effects of interest rate exposure.

To sum up, interest rate exposure stems from the effect of unanticipated interest rate movements on the firm's value. Financial institutions are mostly concerned about these effects. According to banks, interest rate risk is the most important market risk for commercial banks (IFRI-CRO, 2007). Along with the banks, all firms are concerned about interest rate fluctuations since they are linked with shifts of the economic cycle and have a huge impact in the cost of capital and the investing behavior of the counterparties (Drehmann et al,2008). The future welfare of the firm is subject to the interest rates.

The interest rate changes can affect financial institutions in different ways. Regarding assessing a bank's interest rate exposure, the two most common perspectives are the earnings perspective and the economic value perspective:

- **The earnings perspective** focuses on the impact interest rate changes have on a bank's near-term earnings. After all, changes in the yield curve have a direct impact on a bank's future net interest income (including the estimated net income from asset securitization programs). Even noninterest income components, particularly fee-based income, can indirectly depend on the future development of interest rates. Hence, interest rate risk analysis from an earnings perspective will focus on assessing the earnings effects that may arise from changes in market interest rates.

- **The economic value perspective** focuses on the impact interest rate changes may have on the economic value of future cash flows and thus on the economic value of both the interest rate book and capital. The present economic value is affected in two ways by changes in interest rates: by the change in future interest cash flows included in the calculation (= primary economic value perspective) and by the change in the discount rates of all future cash flows used for this calculation (= secondary economic value perspective).

Concluding, interest rate definition and specification is very crucial for the company's welfare. With a view to capturing interest rate appropriately, the interest rate can furthermore break into four main types.

- Repricing risk, which arises from mismatches in interest rate fixation periods.
- Yield curve risk, which stems from changes in the slop and shape of the yield curve.
- Basis risk, the risks that arise from an imperfect correlation in the adjustment of the rates earned and paid on different products with otherwise similar repricing characteristics.
- Optionality risk, risks that arise primarily from options (Gamma and Vega effect) that are embedded in many banking book positions.

(Mathias Drehmann, Steffen Sorensen, Marco Stringa,2006)

1.4.4 Credit Risk

Another type of financial risk is credit risk. Credit risk can be referred to as the potential that a contractual party will fail to meet its obligations in accordance with the agreed terms (Ken Brown & Peter Moles 2016). This type of risk is also known as default, performance or counterparty risk. All the above definitions describe the same thing: the impact of a credit effect on the company's transactions. The nature, the possibility or the extent of the effect can be different, so credit risk has three characteristics:

- **Exposure** (to a party that may possibly default, or its performance might suffer from a negative effect)
- **The probability (**that the party will default its obligations)
- **The recovery rate** (if default takes place, how much can be recovered)

Logically speaking, the greater the first two elements, the greater the risk. Furthermore, the greater the last element, the lower the risk.

Formally we can express risk as:

$$Credit\ Risk\ =\ Exposure\ x\ Probability\ of\ default\ x\ (1-\ Recovery\ rate)$$

After correctly identifying risk, and which factors mainly determine it, we will analyze further why credit risk is important for investors and which issues can play an important role in it.

Credit risk has several sub-issues including **concentration** risk and **settlement** risk. **Concentration** risk arises when there are many exposures to parties that share similar characteristics. Explanatory speaking, the risk associated with any single exposure or group of exposures with the potential to produce large enough losses to threaten a bank's core operations. On the other hand, **settlement** risk arises when a clearing agent or third-party processes transactions for other parties. Indeed, financial institutions have recognized the importance of measuring credit concentration risk in addition to the credit risk of individual loans. That is, they examine not only individual risks but also the total credit risk characteristics at the portfolio level.

Given the complex environment and the globalization, credit risk is prone to other factors as well, and can be defined to further categories. There are two sub-types of credit risk, ***country*** and ***industry ri***sk.

Country risk, the risk of loss arising from a sovereign state freezing foreign currency payments (transfer/conversion risk) or when it defaults on its obligations (sovereign risk); this type of risk is prominently associated with the country's macroeconomic performance and its political stability and it is different from country to country (Different legal systems, business codes etc.). Exposure to different countries can be determined by some factors as well.

- Political risk which arises when a country is challenged externally or within its own borders. It is important to notice, that political risk affects mainly long-term contracts than short term contracts and transactions. Political instability can affect the investor's decisions as to which country they will take act in, since if a country is in the middle of political disturbances the future economic welfare is at risk.
- Economic risk is the chance that macroeconomic conditions like exchange rates, government regulation, or political stability will affect investment, usually one in a foreign country. If a nation is characterized by declining economic stability, the quality of loans or credit to that country is in question. Economic risk is often the same reason foreign investment carries more risk than domestic.
- Currency risk arises with cross-border lending. An unexpected shift to the price of foreign currency can reduce profits. To reduce currency risk, investors tend to place their money to countries with "strong" currencies. Currency risk alongside with economic risk, affect investors who take global investment opportunities.
- Enforcement risk from the debtor country. Because a creditor must go through a foreign legal system, it has been known for debtors to use their domestic legal process to stall or attempt to avoid paying, claiming that rules from their home country apply.

These are the factors that can determine the extent and the nature of country risk. In addition to country risk we will discuss the industry risk.

Industry risk is a form of concentration risk. Industry risk is referred to when international or domestic economy is in recession and the poor economic conditions affect certain industries. Firm's structure is affected by the supply chain. Credit consequences can affect relationships with vendors, customers etc. If a company experiences loss, it may neglect payments and the vendor is therefore at risk. A theoretical model framework for assessing risk is that risk is calculated by pay-offs. The ex-ante payoff of the two situations is:

$$\boldsymbol{Extend\ credit}: \boldsymbol{PV\ (Revenue - Costs)\ x\ (1 - \rho) - PV(Costs)\ x\ \rho}$$

$$\boldsymbol{Refuse\ credit}: \mathbf{0}$$

The reward is the revenue earned less the costs; the risk is the full or partial loss if a default takes place. In evaluating the desirability of taking on credit risk, different situations will have different levels of risk based on (a) the probability of default (ρ) and (b) the amount of loss that is expected or actually incurred. Thus lending, for instance, has a higher exposure (the amount at risk) for the same sized contract than does entering into an off-balance-sheet derivative agreement such as a swap. If a firm goes into liquidation, the loan might be an almost total write-off, but the risk on the swap relates to the difference between the original value and its replacement cost. Part of the evaluation process therefore needs to calculate the exposure that will arise if default takes place, and this exposure needs to be controlled by setting overall credit limits per individual counterparty and by industry type and country, if applicable. Firms gain a reward by accepting the risk that the credit may default. Alternatively, nothing is hazarded if credit is refused. Since most organizations must take risks to earn a return, the decision is slightly more complex than this simple model would suggest. In their day-to-day activities, firms seek to trade off credit risk against the potential gains and losses. These arise not just from accepting bad credits but also in rejecting good ones.

At this point it should be noted that modern finance theory suggests that rejecting credit is not necessarily the appropriate response to poor credit quality. The tenets of modern theory about risk postulate that the required return should be adjusted for the risk taken. If the risk has been correctly estimated, then, for large organizations over the medium term and where portfolio diversification effects apply, losses will be compensated by gains elsewhere. Finance theory would, in addition, suggest that only the systematic risk component need to be priced. One aim of the credit modeling process should be to provide estimates of the likely risk. The decision can then be made as to whether to provide a credit line at an appropriate risk-adjusted price to compensate for the risk or to find ways to reduce the degree of exposure but still enter into the transaction. This is a more sophisticated approach that is used in most organizations, which tend to adopt a 'yes' or 'no' view to extending credit and seek to control their exposure via limits on the amounts at risk. (Ken Brown & Peter Moles, 2016). Consequently, credit risk plays a dramatic role in business decisions and welfare. The importance given on credit risk stems from the many defaults the took place during the'80s. Since the 1980's, important changes in the institutional framework, the firm's behavior and methods of credit risk evaluation have changed significantly.

1.4.5 Operational Risk

Operational risk constitutes a wide range of risks, including losses that arise from fraud, property losses, business practices and human resources issues. Specifically, operational risk is a major concern of the firms since it includes failures that arise from the internal practices, systems or people of the firm. In order to control it, all the above should be revalued for the company to be more efficient. In the past years, more and more interest has given upon operational risk considering the losses of many financial institutions due to inadequate internal control on systems, people or technology. It is worth noticing, that there is a small possibility in severe internal problems, but they can lead to destructively results.

Due to its wide range, operational risk was initially linked to dangers that could not be categorized as market or credit risk. Nowadays, operational risk is defined as the risk that arises from inadequate or failed internal procedures, people, systems or from external events. Furthermore, for a type of risk to be considered operational, it must be specific and easy to measure. Legal risk is included in the above definition, but strategic and reputation risk are not considered operational risk.

Operational Risk is categorized in the seven following types (John C. Hull 2018):

i. *<u>Internal fraud</u>: Also known as fraud from within. Internal fraud is defined as an action that intends to deceive, embezzle property or circumvent the law or the* *policy of the firm. For fraud to be internal, a member of the firm must be involved.*

ii. *External fraud:* *The purpose of this fraud is the same as the internal (deceiving, etc.) but the fraud is caused by a third party.*

iii. *Employment practices and workplace safety:* *Unpolitical correct or unapproved actions from the syndicate.*

iv. *Clients, products, & business practices:* *Intentional (or not) failure to accomplish business obligations with respect to customers or the manufacturing procedure of the products.*

v. *Damage to physical assets:* *Losses that arise from natural disasters or other external events, for example terrorist attacks.*

vi. *Business disruption and system failures:* *Losses that result from the operation of the company or are linked to inadequate systems and inefficient changes of the work practice.*

vii. *Execution, delivery, and process management:* *transaction processing or process management, and relations with trade counterparties and vendors.*

Jarrow (2008) distinguishes operational risk into two types. The first type includes the danger of losses due to operating systems the firm follows, or the technology the company makes use of. Example of this type of risk, is the underachievement of internal processes and transactions, caused by errors or legal restrictions. The second type is focused on the losses arisen by agency costs. Ostensive facts by this type are fraud or mismanagement. Both types are described by different characteristics. In particular, the first type is based on production and transaction procedures that company is benefited from and the second is focused on management intensives.

About operational risk, many authors and researchers have categorized differently the types of operational risk, with respect to its broad range. Specifically, Saunders & Cornett, 2003 categorized operational risk into the following:

- Technology
- Workforce
- Customer Relations
- Property Assets
- External Events

On a different scope of view, Chalupka & Teply, 2008 distinguish operational risk into four major categories

- People
- Systems
- Procedures
- External Events

After careful consideration, operational risk not only is a major subject for the firm's welfare and future but is a controversial subject between researchers as to which is the right distinction between types. Each category describes a different impact on the firm since it is raised by different factors. Both internal and external events that affect production or the smooth operation of the company are described in operational risk.

In addition to types of operational risk, it should be announced that facts that cause operational risk can be further distinguished with respect to the severity and frequency of losses.

The first category with respect to frequency and severity is **high-frequency low-severity risks**. This type is also known as anticipated losses and it refers to risks than occur often, but their impact is relatively small. The other category refers to **low-frequency high-severity risks.** These risks involve fires, natural disasters or third parties' liabilities and the consequences of these facts are profound. It is worth noticing, that through management control and accurate forecasting the anticipated losses can be counterbalanced (Challepe et al, 2008). However, financial institutions face the obligation to estimate the un-anticipated risk, to the extent they can predict and accurately measure it.

In conclusion, operational risk affects the whole business environment and not only in specific areas. It is an inevitable type of risk and its abnormal shifts make the exposure to risk less predictable and more difficult to estimate it. Regarding measuring the operational risk, nowadays, financial institutions pay their attention to implement methodologies that estimate operational risk using objective factors. Jobst (2007) refers to three different ways to calculate operational risk.

Volume base approach: Exposure to risk is a function of the complexity of the business activity.

Qualitative self-assessment: It defines all types of errors made in the business activity in order to determine the severity and the frequency of internal or external events.

Quantitive techniques: Operational risk is quantified through estimating the distribution of severity and frequency of losses.

In this section we focused on analyzing the categories and how to measure operational risk. Both aspects of this study are very important since in order for a firm, investor or financial institution to decide correctly its actions, investments and other different business activities it should always pay attention to operational risk since its impacts can be enormous.

1.4.6 Liquidity Risk

Liquidity is a very important term in the economic field. It is referred to in business and in markets. Specifically, under the scope of the market, liquidity is referred with the ability of the counterparties to accomplish large volume transactions, in a small-time frame, without significant change in prices.

Given the above term, liquidity is consisted of two elements:

- The marketability of the products involved.
- The volume of and the continuity of price movements through trades.

Marketability of one product refers to the velocity it can be sold when introduced to the market. On the other hand, the continuity of price shifts stems from the diversification of successive trades in cases there is no information about a significant change in prices. Marketability and continuity can be very different between different types of products. (Christian Hugo Hoffman,2017)

More specifically, we can conclude that liquidity is how big trade-off there is between the speed of the sale and the price it can be sold for. In a liquid market, the trade-off is mild, selling quickly will not affect largely the level of prices. In a relatively liquid market, a quick sale will require a substantial change in the price. Liquidity can be measured either based on trade volume relative to shares outstanding or based on the bid-ask spread or transactions costs of trading.

Money, or cash, is the most liquid asset since there is no trade-off between speed and the price. It can be “sold” for goods or services without a loss of value. An alternative definition is that liquidity can mean the amount of cash or cash equivalents.

Liquidity can be evaluated according to three factors: ***tightness, depth and resiliency***. (Kyle 1985). Each term refers to a different scope by which liquidity is examined.

Tightness refers to the cost of turning over a position is a short period of time. This term expresses the distance between the purchase and the sale price (bid-ask spread).

Depth is the market’s ability to absorb a large volume of transactions without a significant change in the price level. Thus, the greater the depth, the smaller the effect on the price.

Resiliency is referential to the velocity prices tend to converge towards the underlying liquidation value of the commodity. Resiliency also measures the rate at which prices bounce back from an uninformative stock.

Regarding estimating liquidity, there are a series of measures used with different qualities. For the purpose of our study, we will categorize these measures into those who are depended on the transaction and those referred to the orders. On the first category of measures, the volume of transactions and profits are included that reflect liquidity after the transactions. On the second category, spreads and depth of the market are reflected, thus giving the actual liquidity in real time.

Liquidity Risk

Risk relates to the probability of having a realization of a random variable different from the realization preferred by the economic agent. In our context the economic agent would have a preference over liquidity. In that sense, the probability of not being liquid would suggest that there is liquidity risk. The higher the probability, the higher the liquidity risk. (European Central Bank, Working Paper Series No. 1008, February 2009).

In that text, we conclude that there is an inverse relationship between liquidity and liquidity risk. In a more broad sense, liquidity can be described as the financial risk that for a certain period of time a given financial asset, security or commodity cannot be traded quickly enough in the market without impacting the market price. Liquidity risk can take a lot of definitions based on the factors that affect it, the parties involved and its results on different areas. The purpose of the study is to present some forms of liquidity risk economic field has examined among years

Central Bank Liquidity Risk

Central Bank is broadly known as the monopoly supplier of liquidity. Therefore, it can never be illiquid. A central bank can only be illiquid to the extent that there is no demand for domestic currency, and therefore the supply of base money from the central bank could not materialize. This could happen in cases of hyperinflation or an exchange rate crisis.

It is worth noticing that Central Bank as a liquidity provider, is subject to some costs. These risks include counterparty credit risk related to collateral value, monetary policy related risks (e.g. risks of wrong signaling) or wider risks to financial stability (i.e. the moral hazard issue that relates to emergency liquidity assistance in turbulent periods). Nevertheless, these risks do not affect the ability of the central bank to provide liquidity. The liquidity risk the Central Bank comes against is to provide liquidity of its currency to another country's Central Bank (Central Bank Liquidity Swap).

Funding Liquidity Risk

Financial institutions are subject to funding liquidity risk. Funding liquidity risk is referred to as the danger a financial institution will fail to accomplish anticipated or unanticipated obligations. Other definitions involve the time period parameter. Specifically, the probability of becoming illiquid is typically measured for a given period ahead, and it can differ according to the length of the period. (Matz and Neu, 2006; Drehmann and Nikolaou, 2008). Generally, the definition given to this type of risk is the inability to satisfy the budget constraint over the respective period.

Measuring funding liquidity risk is not trivial. In most cases practitioners construct various funding liquidity ratios, which reveal different aspects of the availability of funds within a certain time horizon ahead and use them as proxies for funding liquidity risk. Such measures can be produced either by static balance sheet analysis or by dynamic stress testing techniques and scenario analysis. The latter is more cumbersome to calculate if only because it relies on complicated calculations and a wider set of information and hypotheses. Recently, Drehmann and Nikolaou (2008) suggest a simple and more straightforward proxy, based on the role of the central bank as a potential funding liquidity source. They argue that bidding behavior in central bank auctions can reveal the funding liquidity risk of banks over a one-week horizon and construct proxies of funding liquidity risk from bidding data. Drehmann and Nikolaou (2008) suggest that funding liquidity risk bears similarities to market liquidity risk, in the sense that it is low and stable most of the times, but subject to occasional spikes (e.g. funding liquidity risk appears elevated during the current turmoil period). This finding is supported by Matz and Neu, (2006), who view liquidity risk as a consequential risk, because it increases following one or more spikes in other financial risks (i.e. market liquidity risk). Brunnemeier and Pedersen (2007) provide further theoretical support and rationalize linkages between market and funding liquidity risk (for traders), which are validated empirically by Drehmann and Nikolaou (2008).

To conclude, the literature has shown that funding liquidity risk can for example be driven by aggregate shocks, incomplete markets and information asymmetries which lead to stochastic prices and stochastic volumes of central bank funds. Bidding in the central bank auction is one mechanism to partly hedge this risk as it allows the bank to obtain central bank money before liquidity shocks materialize.

Market Liquidity Risk

Market liquidity risk is a term broadly used in the economic field. In business, economics or investment, market liquidity is a market's feature whereby an individual or firm can quickly purchase or sell an asset without causing a drastic change in the asset's price. It is the systematic, non-diversifiable component of liquidity risk.

Academic interest has been noticed on the properties of market liquidity risk. An interesting finding is the market commonalities in market liquidity. There is a positive covariance between individual stock liquidity and the overall market liquidity. Specifically, Paul Hanouna, Jon Novak, Tim Riley, Christof Stahel show that financial objects even fewer liquid ones are strongly affected by the market’s liquidity, but stocks are more prone to convert to the same direction as the market. There is a positive covariance between individual stock liquidity and overall market liquidity. Moreover, Chordia et al. (2000, 2005) have documented that liquidity is correlated across markets, namely across stocks and across stocks and bonds. In fact, Brunnemeier and Pedersen (2005) provide a theoretical framework which rationalizes commonality of liquidity across assets and markets in general through the microstructure analysis of the behavior of traders. Moreover, Stahel (2005) points to the existence of a global (market) liquidity factor. Finally, the literature also records a negative (positive) relation between liquidity and asset prices (returns) (Acharya and Pedersen, 2005).

In addition to the properties of market liquidity risk, it is worth noticing the factors that affect the market liquidity risk. They can be divided into two categories: **exogenous factors** and **endogenous factors** (Cossin et al 2003).

Exogenous factors are linked with the market’s characteristics and present commonalities for all the third-parties involved. For instance, liquid markets are characterized by a large volume of transactions, small spreads and great depth. On the other hand, less liquid markets present fluctuations in the volume of transactions and depth. Exogenous factors are not affected by a third-party; only with a deliberate large volume action.

Endogenous factors are position-specific and are affected by every single action. Exogenous factors are defined by the magnitude of each position. Specifically, the larger the depth, the bigger the endogenous liquidity.

In conclusion, focusing on liquidity has become more and more significant in the last years. As the volume of international transactions increases and financial institutions expand their actions internationally, the dependence with international market goes up. As a result, the importance of focusing on liquidity with respect to many markets, different currencies are raised. The dependence across markets can cause a domino effect on liquidity problems thus reassurance of liquidity is nowadays very important.

1.4.8 Foreign Exchange Risk

Foreign exchange risk arises when a financial transaction is denominated in a currency other than that of the base currency of the company. Risk arises when there is a probability of appreciation of the base currency in relation to the denominated currency or depreciation of the denominated currency in relation to the base currency. The risk is that there may be an adverse movement in the exchange rate of the denomination currency in relation to the base currency before the date when the transaction is completed. Investors and businesses are prone to exchange risk since the formers accomplish transactions in different markets, and companies import, export goods or undertake international investments. By undertaking international transactions and investments, companies cope up with the probability of losses in flows, assets, obligations, profits and as a conclusion its value, as a result of the unfavorable shift of the exchange rate (Papaioanou 2006). Foreign exchange risk is developed by several sources including the following:

• where the business imports or exports

• where other costs, such as capital expenditure, are denominated in foreign currency

• where revenue from exports is received in foreign currency

• where other income, such as royalties, interest, dividends etc., is received in foreign currency

• where the business's loans are denominated (and therefore payable) in foreign currency

• where the business has offshore assets such as operations or subsidiaries that are valued in a foreign currency, or foreign currency deposits

(A guide to managing foreign exchange risk)

Foreign exchange risk can also be categorized in the following types:

- **Transaction cost**: A firm experiences transaction cost when company's flows (payable/receivable amounts) are subject to unfavorable changes in the exchange rate level. All companies, investors that act in countries with different currencies, are affected by this type of risk. Furthermore, transactions that are known to public but have not been completed are also prone to exchange rate fluctuations. Unfortunately, this lag between the execution and the settlement, may lead to losses. The risk arises when exchange rate shifts unfavorable on this time lag. The fluctuation problem can be reduced, using hedging techniques. These techniques include forward contracts and options. When a forward contact is decided as a protection technique, an exchange rate is decided in advance for a specific date in the future. On the other hand, when a company undertakes an option; the company is able to set a rate "at worst" for the transaction. Options are a very popular hedging technique for the companies, and relatively cheap. In addition to these hedging processes, holding an account of foreign currency is also a measure to reduce transaction risk. These techniques are very useful for companies that accomplish a large volume of transactions in different currencies.
- **Translation Risk*:*** Translation risk is the exchange risk danger that firms deal with foreign currencies or is raised when their balance sheet contains foreign assets. Since companies own assets internationally, they will be called to convert foreign currency into the domestic currency. If fluctuations of exchange rate are significant, there would be a substantial change in the income and value of assets held outside the home country. The risk is created by the inability to predict how relative these currencies are, and if they tend to move in the same direction. The greater the assets, liabilities, currency holdings outside the home country, the greater the translation risk. This type of risk is also known as accounting risk because it is raised when the translation of books of accounts into the home currency takes place. Unlike the transaction risk, translation risk cannot be hedged against.
- **Economic Risk:** Economic Risk is a long-term effect of transaction exposure. Specifically, economic risk is defined as the effect of the exchange rate on the firm's value and the future flows of the company. In order to calculate economic risk, changes in a firm's value and investment decisions must be devalued with respect to exchange risk fluctuations. The impact can be substantial, as

unanticipated change can affect a company's competitive position. Firms are subject to economic risk since they import or export commodities, undertake investments or hold assets in foreign countries. Companies face exchange risk even if they do not take international activities, since competition can stem from foreign countries.

Exposure to foreign exchange risk is the impact on the future flows of the firm and its assets. Foreign exchange risk reflects the sensitivity the firm has in fluctuations of the currency value. Between the factors that affect the exchange rate risk the main ones are the level of exports and the size of the firm.

Foreign exchange risk management was given great importance after the breakdown of the Bretton Woods system. Bretton Woods system was referred to as monetary management in which gold was pegged to the dollar. Factors such as globalization, market deregulation raised the level of the international functions of the firms. As the international exposure is raised, the importance of managing the exchange rate is raised as well. Foreign exchange risk management includes the specification of the risk the firm is exposed to, the definition of the hedging strategy and the selection of the right technique. Specifically, managing exchange rate risk techniques include (Michael G. Papaioannou, 2006):

1. Identification of the types of exchange rate risk that a firm is exposed to and measurement of the associated risk exposure. As mentioned before, this involves the determination of the transaction, translation and economic risks, along with specific reference to the currencies that are related to each type of currency risk. In addition, measuring these currency risks—using various models (e.g. VaR)—is another critical element in identifying hedging positions.
2. Development of an exchange rate risk management strategy. After identifying the types of currency risk and measuring the firm's risk exposure, a currency strategy needs to be established on how to deal with these risks. In particular, this strategy should specify the firm's currency hedging objectives—whether and why the firm should fully or partially hedge its currency exposures. Furthermore, a detailed currency hedging approach should be established. It is imperative that a firm details the overall currency risk management strategy on the operational level, including the execution process of currency hedging, the

hedging instruments to be used, and the monitoring procedures of currency hedges.

3. Creation of a centralized entity in the firm's treasury to deal with the practical aspects of the execution of exchange rate hedging. This entity will be responsible for exchange rate forecasting, the hedging approach mechanisms, the accounting procedures regarding currency risk, costs of currency hedging, and the establishment of benchmarks for measuring the performance of currency hedging. (These operations may be undertaken by a specialized team headed by the treasurer or, for large multinational firms, by a chief dealer).
4. Development of a set of controls to monitor a firm's exchange rate risk and ensure appropriate position taking. This includes setting position limits for each hedging instrument, position monitoring through mark-to-market valuations of all currency positions on a daily basis (or intraday), and the establishment of currency hedging benchmarks for periodic monitoring of hedging performance (usually monthly).
5. Establishment of a risk oversight committee. This committee would approve limits on position taking, examine the appropriateness of hedging instruments and associated Var positions, and review the risk management policy on a regular basis.

Exchange risk hedging has given prominence the last years as a result of a large number of currency crises. To prevent firms from experiencing a great exposure to exchange risk, there are some techniques to diminish the negative effects of an unexpected exchange rate change. These methods are:

- On balance sheet hedging. This method aims to decrease the exchange rate risk by affecting the balance sheet assets. Specifically, this technique dwells on the importance of holding assets in strong currencies and obligations in weak currencies.
- Off-balance sheet hedging. The main method for off-balance sheet hedging are the derivatives. Derivatives include futures, options in foreign currency. It is worth noticing that studies suggest that on balance sheet hedging is considered more effective in reducing exchange rate exposure.

- Foreign denomination debt. Multinational companies construct loans in foreign currency and as a result, flows of foreign currency are presented. It is a very desirable method in case of capital need and desirable interest rates.

This section was focused on defining the exchange rate risk and the ways to manage it. This type of risk is very important especially the last decade and more and more researchers are focusing on the subject.

1.4.9 Commodity Price Risk

In this section, we will focus our interest on commodity price risk. Commodity price risk refers to the uncertainty of the future market value and the future income caused by the fluctuations in prices. Commodity risk includes the price of finished goods and the price of raw materials. As a result, these prices affect the number of products produced. Commodity prices are the most volatile in comparison to equities, bonds etc. (Lamn 1996). Commodities are somewhat unusual in the risk management world. Unlike financial objects, commodities are physical objects with unique attributes. Commodities generally fall into three categories:

- **Soft commodities** include agricultural products such as coffee, sugar etc.
- **Metals** include gold, silver, aluminum.
- **Energy commodities** include gas, oil, coal.

Firms should consider managing commodity risk when fluctuations in prices infect profitability. Companies that their core operations are the production and the distribution of products, commodity risk is essential to be properly managed. An unexpected change in the price level can lead to a drastic decrease in sales. These fluctuations affect the cost of production, and as a result, the prices of the finished goods. Commodity risk can be categorized further to the following categories:

- **Price Risk:** Due to adverse movement in prices of commodities as determined by macro-economic factors.

- **Quantity Risk:** This risk arises due to changes in the availability of commodities.
- **Cost Risk:** Arises due to unfavorable movements in the prices of commodities which impact business costs.
- **Regulatory Risk:** This type of risk arises due to changes in laws and regulations which is having an impact on prices or availability of commodities.

Commodity risk uncertainty can affect a broader group of agents:

1. **Producers** (farmers, plantation companies, and mining companies) face price risk, cost risk (on the prices of their inputs) and quantify risk.
2. **Buyers** (cooperatives, commercial traders and trait ants) face price risk between the time of up-country purchase buying and sale, typically at the port, to an exporter.
3. **Exporters** face the same risk between purchase at the port and sale in the destination market; and may also face political risks with regard to export licenses or foreign exchange conversion.
4. **Governments** face price and quantity risk regarding tax revenues, particularly where tax rates rise as commodity prices rise (generally the case with metals and energy exports) or if support or other payments depend on the level of commodity prices.

When a company has exposure to commodity risk, it must decide how to manage its financial activities associated with price movements. Management of commodity price risk is essential in order to get a consolidated view of their risk, reduce volatility and improve their forecasting. Since unexpected changes can lead to reduced profit margins, investors and companies have developed several financial strategies. These financial strategies can either guarantee a commodity price in the future or lock a position in the worst-case scenario. Futures and options are two financial instruments commonly used to hedge against commodity price risk. Commodity risk hedging is often achieved by using derivatives but in some cases the risk cannot be completely eliminated. For companies that have a lot of liabilities or small profit margins the elimination of risk is of great importance.

1.4.10 Equity Price Risk

Equity risk refers to the financial danger of holding equity. This danger refers to the probability that the stocks' price will diminish due to factors that affect the stock market. Equity markets like other financial markets always bear an important risk in terms of a market correction. Various equity crashes (1929, 1973, 1987) have had important macroeconomic consequences such as general recessions and rising unemployment. Investors and companies are subject to equity price risk and their profitability and sustainability are directly linked to the equity price. Large profit or losses can occur by the movements of the equity prices. Moreover, the return on equities can be subject to unexpected events. The impact on the return depends on the nature of the equity, the firm's function or its financial strategy. The position in a stock is prone to risk. The total risk is defined by systematic or non-systematic factors. As is well known from the Capital Asset Pricing Model (CAPM) there are two types of risk to an equity position in individual stock i:

$$Total\ Risk\ =\ Systematic\ Risk\ +\ Unsystematic\ Risk$$

Systematic Risk reflects the co-movement of that stock with the market portfolio reflected by the stock's beta and the volatility of the market portfolio while the unsystematic risk is specific to the firm itself. Beta coefficient is a measure of the volatility or systematic risk of an individual stock in comparison to the unsystematic risk of the entire market. Beta is used in the Capital Asset Pricing Model (CAPM), which calculates the expected return of an asset using beta and expected market returns. In statistical terms, beta represents the slope of the line through a regression of data points from an individual stock's returns against those of the market.

In a very well diversified portfolio, unsystematic risk can be largely diversified away, leaving behind the systematic (undiversifiable) market risk.

Haq & Heaney (2009) examined the impact on bank equity risk of the Economic Monetary Union and the deregulation and integration that accompanied EMU. Furthermore, the examined various studies related to risk factors. Specifically, by reviewing these studies the main conclusions are the following:

By the review of the paper it is revealed that systematic danger can be reduced by mergers, acquisitions and lessening of the institutional framework. On the other hand, the unsystematic danger is diminished by acquisitions and mergers as well, but increased through leverage, financial innovations and competition. Concluding, diversification is observed to decrease or increase the systematic and the unsystematic part of the risk depending on other factors.

Investors and companies holding risky assets should be compensated. The equity risk premium describes the excess return that investing in the stock market provides over the risk-free rate. (Cappiello et.al 2008). This excess return compensates the investors for undertaking higher risk investments such as stocks. The size of the premium is analog to the level of risk in a portfolio and changes over time through market risk fluctuations. As a rule, higher risk investments are compensated with higher risk premiums. Furthermore, the investor's expectations should be based on economic growth factors and inflation levels. (Kizys & Spencer, 2006)

Equity Risk Management

A very important issue we need to analyze is equity risk management. Since millions of companies and investors hold positions in stocks for their economic welfare, they are called to manage the level of risk they undertake. Equity risk management is an extremely important subject of the globalized world since equities' fluctuations affect not only companies but also countries and governments.

The first approach to equity risk management is the diversification of the portfolio. Diversification is a risk management strategy that mixes different investments in one portfolio in order to reduce the danger. Moreover, an international portfolio can reduce to a further extent the portfolio exposure since it combines assets that are less correlated, but it also other types of risk.

The second approach involves the asset allocation method (Obiyathulla Ismath Bacha,2004). This method focuses on changing the composition of the portfolio according to the fluctuations of the market. For example, if an investor anticipates the prices of equities to increase, the portfolio will include more stocks and may decrease holdings in other assets (bonds, etc.). It is worth noticing that for this method to be implemented successfully, specific stocks should be included at the appropriate time frame. Concluding, since asset allocation method is based on expectations, the level of uncertainty is considerably high.

An approach that overcomes the limitations of the above methods is using derivatives. (Beder, 1993). Derivatives are financial contracts between two or more parties and its price is determined by movements in the underlying asset. By using derivatives, investors can confine the risk by sustaining their assets or benefit from international diversification. The most common forms of derivatives are:

- ***Futures*:** Futures contract is an agreement between parties for the purchase and delivery of an asset at an agreed upon price at a future date.
- ***Forwards:*** A forward contract is a contractual agreement between a buyer and a seller at time 0 to exchange a prespecified asset for cash at later date.
- ***Swaps:*** Swaps are agreements between counterparties to exchange specified periodic cash flows in the future based on some underlying instrument of or price.
- ***Options:*** An option is a contract that gives the holder the right but not the obligation to buy or sell an underlying asset at a prespecified price for a specified time period.

These forms of derivatives provide risk hedging, and diversification. On the downside, derivatives are sensitive to demand and supply factors and are complex to understand and value.

1.4.12 Country Risk

As noted, globally active investors and companies are subject to foreign assets and liabilities and foreign interest rate risks. Country Risk is the risk that repayments from foreign borrowers may be interrupted because of the interference of foreign governments. Country or also known as Sovereign Risk is a type of credit risk that is faced by an investor that purchases assets such as loans, bonds of foreign corporations. For example, when a domestic corporation is unable or unwilling to repay a loan, a foreign investor usually has recourse to the domestic bankruptcy courts and eventually may recoup at least a portion of its original investment when the assets of the defaulted firm are liquidated or restricted. By comparison, a foreign corporation may be unable to repay the principal or interest on a loan even if it would like to. Most commonly the government of the country in which the corporation is headquartered may prohibit or limit debt payments of foreign currency shortages and adverse political reasons.

The uncertainty related to sovereign risk can be raised not only by political factors but also economics, such as exchange rate movements or technological influences. In a broader sense, country risk is the degree that economic and political factors affect financial instruments in a country.

Companies that undertake investments in foreign countries should be compensated for bearing the risk of a foreign country. The country risk premium is the additional return or premium demanded by investors to compensate them for investing in another country. Foreign investments are accompanied by higher risk since geopolitical and macroeconomic factors must be considered. The country risk premium is higher in developing rather than developed countries since the level of the exchange rate and interest rate is subject to more fluctuations. Moreover, political instability is an important factor the country risk premium is needed. There are three types of markets for international investments:

Developed markets consist of the largest, most industrialized economies. In these economies, the economic system is well developed and efficient. There is political stability and rules of the law are applied. Developed markets are usually considered the safest investment decisions. Investment analysis of developed markets usually concentrates on the current economic and market cycles. Political considerations are often less important. Examples of developed markets include the United States, Canada, France, Japan and Australia.

Emerging markets experience rapid industrialization and often demonstrate extremely high levels of economic growth. Their strong economic growth often outperforms the developed markets. However, emerging markets are considered also riskier than developed markets. There is often more political uncertainty in emerging markets, and their economies may be more prone to unfortunate events. In addition to carefully evaluating an emerging market's economic and financial fundamentals, investors should pay close attention to the country's political climate and the potential for unexpected political developments. Many of the fastest-growing economies in the world, including China, India and Brazil, are considered emerging markets.

Frontier markets represent "the next wave" of investment destinations. These markets are generally either smaller than traditional emerging markets or there are in countries with limitations on foreign investment. Although frontier markets can be exceptionally risky and often suffer from low liquidity, they also offer the potential for above-average returns over time due to higher risk levels. Frontier markets are also not well correlated with other developed markets, which means that they provide additional diversification benefits when held in global- investment portfolio. As with emerging markets, investors in frontier markets must pay careful attention to the political environment, as well as to economic and financial developments. Examples of frontier markets include Nigeria, Botswana and Kuwait.

Estimation of Sovereign Risk

Country risk is critical for the decision of investors to take investments in foreign countries.

For the right decisions to be implemented, estimating sovereign risk is important.

In evaluating country risk, a foreign investor can use alternative methods, varying from the highly quantitative to the very qualitative. Moreover, as in domestic credit analysis, an investor may rely on outside evaluation services or develop its own internal evaluation or sovereign risk models. For more specified and right assessment, many models and sources together should be used.

Outside Evaluation Models

Euromoney Country Risk Index

The Euromoney Country Risk Index rates sovereign risk of more than 180 countries based on the opinions of a global network of economists and policy analysts. The index is based on a large number of economic and political factors, including a country's economic characteristics, political characteristics structural characteristics, access to capital and credit ratings, and debt indicators. ECR scores are scaled from 0 to 100 (0=maximum risk, 100= no risk) and are put into one of five tiers that are updated quarterly.

The Economist Intelligence Unit

A sister firm to the economist, the Economist Intelligence Unit rates country risk by combined economic and political risk on a 100-point (maximum) scale. The higher the number, the worse the sovereign risk rating of the country.

The institutional Investor Index

Normally published twice a year, this index is based on surveys of the loan officers of major multinational banks. These officers give subjective scores regarding the credit quality of the given countries. Originally, the score was based on 10, but since 1980 it has been based on 100, with a score of 0 indicating certainty of default and 100 indicating no possibility of default.

Internal Evaluation Models

Statistical Models

By far, the most common approach to evaluating sovereign country risk among large foreign investments has been to develop sovereign risk scoring models based on key economic ratios for each country. An analyst begins to select a set of macro- and microeconomic variables and ratios that might be important in explaining a country's probability of rescheduling. It is very important to pick the right variables to estimate sovereign risk levels.

The most commonly used variables in calculating sovereign risk are the following (Financial Institutions Management, A Risk Management Approach 8th Edition, Anthony Saunders & Marcia Millon Cornett).

- Dept Service Ratio: The ratio of a country's interest and amortization obligations to the value of its exports.
- Import Ratio: The ratio of a country's imports to its total foreign currency reserves.
- Investment Ratio: The ratio of a country's real investment to its GNP.

Investing Overseas, is a very important step that requires careful consideration and implementation. The next step after the country analysis has been completed, is to decide which investments approach a firm must choose. This process may differ from one investor to others due to different investment profiles, initial budget, etc.

1.5.1 Chapter Epilogue

The first chapter of our study was focused completely on analyzing the financial risks an investment is exposed to. Furthermore, approaches to mitigate each type of risk were discussed shortly.

It is very important to understand that financial risks are a key subject of investments and their right representation is the first step of portfolio management process. Each type of risk affects investments in a different manner, thus careful consideration is in order. In later chapters of our study, the effects of these risks become clearer and their right treatment is more analytically approached.

Chapter 2

2.1.1 Portfolio Management Methods

This chapter focuses on portfolio management methods. Portfolio management is the science of making decisions about investments. Specifically, portfolio management aims to match investments to objectives, asset allocation for individuals and institutions, and balancing risk against performance. Portfolio management is about determining strengths, weaknesses, different investment opportunities with the aim to maximize return in a given level of risk.

Investment management was developed at the late 1950s. It is an extension of the financial study and involves a number of investment opportunities between which the right combination to maximize return is aimed.

Portfolio theory is based on the paper of H. Markowitz that deals with the selection of the best portfolio. Portfolio theory focuses on the right actions an investor should take to ensure the initial amount in the investment. In this process, different securities are combined in one portfolio which differs from one investor to another based on different investing profiles and needs. Furthermore, portfolio management includes the observation of the portfolio's advance over time and its return valuation.

There are three steps in the portfolio management process. These are the following:

- **Security analysis:** In this stage securities with the highest return or minimum risk are selected.
- **Portfolio analysis:** The portfolio's return and the possibility of risk is estimated.
- **Selection of portfolio:** At this stage, among portfolios with the lower risk at a given level of return, a specific number of portfolios that match the investor's investing profile are selected. The investing profile is determined by the amount of money the investor is willing to invest and the time frame of the investment.

2.1.2 Conventional and Modern Portfolio Management Theory

The conventional Portfolio Management Theory refers to the creation of an effective portfolio which includes a broad range of securities. Conventional investors are risk-averse and tend to invest in known firms for three reasons. Firstly, broadly known companies are linked with profitability and sustainability and are considered to be lower risk investments than other firms. Furthermore, securities held by large firms are more liquid and their volume is higher. Finally, the reputation of successful companies is a strong and conceptually, investors are more able to find clients to invest in them.

The modern portfolio theory approach uses several fundamental statistical measures to construct a portfolio. The statistical diversification is a critical factor in the selection of securities for the construction of the portfolio. Modern portfolio approach or mean-variance analysis is a mathematical framework for assembling a portfolio of assets such that the expected return is maximized for a given level of risk. Risk-averse investors construct portfolios to optimize expected returns in specific risk levels. This theory was pioneered by Harry Markowitz in his paper "Portfolio Selection" published in 1952 in the Journal of Finance. There is a rule that implies both that the investor should diversify and that he should maximize expected return. The rule states that the investor does or should diversify his funds among all those securities which give a maximum expected return. The law of large numbers will ensure that the actual yield of the portfolio will be almost the same as the expected yield. This rule is a special case of the variance of the expected return of returns rule (to be presented below). It assumes that there is a portfolio which gives both maximum expected return and minimum variance, and it commands this portfolio to the investor. This presumption, that the law of large numbers applies to a portfolio of securities, cannot be accepted. The returns from securities are too intercorrelated. Diversification cannot eliminate all variance. The portfolio with the maximum expected return is not necessarily the one with minimum variance. There is a rate at which the investor can gain expected return by taking on variance or reduce variance by giving up an expected return. (Harry Markowitz, Portfolio Selection, Journal of Finance, March 1952).

Concluding, portfolio management is an essential subject for investors and firms since it aims to maximize their long-term profitability. The modern approach is the benchmark to the portfolio management techniques and has helped a lot to optimize portfolios over time.

Next section is focused on addressing the main methods to manage a portfolio of financial assets. Methods involve ways to quantify and mathematically express risk parameters, in order to treat them properly. We shall review the most popular ways to manage financial risks:

2.2.1 Operational Risk Quantification

The first approach of portfolio management we will discuss is the operational risk quantification. Identifying and assessing operational risk, enables companies to undertake changes with assurance or risk reduction. Risk managers have become in desperate need of reliable methods for measuring and managing operational risks. Before addressing the models to mathematically express operational risk, we present their characteristics since they affect their implications on modeling methods of operational risk. (Samir Sah,2002).

2.2.2 Characteristics of Operational Risks

Characteristics of Op. Risks	Implication
1. Op risks are endogenous, i.e., specific to the facts and circumstances of each company. They are shaped by the technology, processes, organization, personnel, and culture of the company. By contrast, market, credit and	Need to gather company-specific data. However, most companies do not have a long history of relevant data. In banking, industrywide data is being gathered, but it may not be representative.

insurance risks are driven generally by exogenous factors.	
2. Op risks are dynamic, continuously changing with business strategy, processes, technology, competition, etc.	Even a company's own historical data may not be representative of current and future risks.
3. The most cost-effective strategies for mitigating op risks involve changes to business processes, technology, organization, and personnel.	Need a modeling approach that can measure the impact of operational decisions. For example, "how will op risks change if the company starts selling and servicing products over the Internet, or if a key function is outsourced?

The endogenous and dynamic nature of op risks suggests a greater reliance on expert input and professional judgment to fill data gaps—at least until companies gather enough historical data over varying business environments. Use of operational strategies to mitigate op risks suggests a causal modeling approach that managers can use to perform "what-if" analyses. After all, the goal of risk management is to reduce op risks, not just measure them.

2.3.1. Risk Modeling Methods

There is a continuum of methods to model risks. Although there are many ways to classify these modeling methods, for the purposes of our study, is useful to organize methods based on the extent to which they rely on historical data versus expert input. This list of methods is by no means exhaustive. However, it illustrates very nicely that there is a large inventory of risk modeling methods across finance, engineering, and decision science disciplines that can be drawn on to suit a particular circumstance. In particular, there are three methods that operational risk can be estimated. We describe them as follows:

Methods Based on Statistical Analysis of Historical Data

Market, credit, and insurance risks rely heavily on a statistical analysis of historical data for quantification. Actuarial approaches based on convoluting frequency and severity probability distributions

- Simulation using stochastic differential equations
- Extreme value theory to model the tail of a probability distribution

Operational risks can also be modeled using these methods, when there is an adequate amount of representative historical data. High-frequency, low-severity op risks, such as bank settlement errors for example, usually generate enough data to use methods based on statistical analysis. Although even in this example, as banks implement straight-through-processing (STP), the risk will change, and the historical data may not be a reliable indicator of prospective risks.

Methods Based on Expert Input

Decision scientists have long relied on methods listed to quantify risks when there is little or no objective data. They have had to rely almost exclusively on expert input to quantify risks, such as likelihood of success or failure of a new drug in early stages of research. These include:

- Delphi method to elicit information from a group of experts
- Decision trees, which lay out decision points and resulting discrete uncertain outcomes
- Influence diagrams, which also map out cause-effect relationships

Over time, they have refined these methods to minimize the pitfalls and biases arising from estimating subjective probabilities, thereby increasing the reliability of these approaches.

Methods Based on a Combination of Data and Expert Input

The methods rely on a combination of historical data, to the extent it is available, and expert input as needed to fill data gaps. They include, for example:

- Fuzzy logic, which uses linguistic variables and rules based on expert input
- System dynamics simulation, which uses non-linear system maps to represent the causal dynamics of a system
- Bayesian Belief Networks (BBN), which rely on a network of cause-effect relationships quantified using conditional probabilities

2.3.2 Further analysis of Risk Modeling Methods

After categorizing the methods, the next step is to define which methods are mostly used by investors and organizations and analytically describing their process and use. In this section, we will present the efficiency of these methods in estimating risk.

Operational Risk under Basel II

The management of operational risk has undergone significant changes in the last decade in the banking sector. Globalization, deregulation, usage of complex financial instruments and changes in information technology have resulted to increase in different kinds of exposure. In response to those events, the Basel Committee for banking supervision has developed a regulatory framework referred to as Basel II that introduced operational risk (OpRisk) category and corresponding capital requirements against OpRisk losses.

OpRisk is defined by Basel II as: “the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk but excludes strategic and reputational risk. A conceptual difference between OpRisk and market/credit risk is that it represents a downside risk with no upside potential. OpRisk is significant in many financial institutions. Examples of extremely large OpRisk losses are: Barings Bank in 1995 when the actions of one rogue trader caused a bankruptcy as a result of GBP 1.3 billion derivative trading loss; Enron bankruptcy in 2001 considered as a result of actions of its executives with USD 2.2 billion loss. In 2012, a capital against OpRisk in major Australian banks is about AUD 1.8-2.5 billion (8-10% of the total capital). Under the Basel II framework, three approaches can be used to quantify the OpRisk annual capital charge.

1. **The Basic Indicator Approach:** C = α 1 n P3 j=1 max(GI(j), 0), where GI(j), j = 1, . . . , 3 are the annual gross incomes over the previous three years, n is the number of years with positive gross income, and α = 0.15.
2. **The Standardized Approach:** C = 1 3 P3 j=1 max[P8 i=1 βiGIi(j), 0], where βi , i = 1, . . . , 8 are the factors for eight business lines and GIi(j), j = 1, 2, 3 are the annual gross incomes of the i-th BL in the previous three years.
3. **The Advanced Measurement Approaches (AMA**): a bank can calculate the capital charge using an internally developed model subject to regulatory approval.

A popular method under the AMA is the loss distribution approach (LDA). We will examine it as follows:

Loss Distribution Approach

For financial institutions, risk has several components: liquidity, market, and other types of risk such as operational risk. Nowadays, there is growing attention from supervision authorities on the management of operational risk. As a result, operational risk will receive the same supervision and treatment as market and credit risk. The regulatory treatment is to proceed to computations of the expected loss. One of the most typical ways to estimate the loss, in the loss distribution approach relies on a database of loss events enhanced with external data. (A. Frachot, P. Georges & T. Roncalli,2001)

In particular, under the loss distribution approach, banks quantify distributions for frequency and severity of operational risk losses for each risk cell for a 1-year horizon. The banks can use their own risk cell structure but must be able to map the losses to the Basel II risk cell.

Operational Loss Data

Consistent with the Basel II Standards on Operational Risk AMA, this methodology provides the EC estimates based on the combined use of internal and the relevant external loss data. Internal loss data is essential to represent the bank's own business activities, technological processes and risk management procedures. Hence, it is utilized both by the 'Firm-Specific Component' and the 'Industry Component'. However, due to limited historical internal loss data, the methodology also incorporates relevant external loss data to estimate tail events from a more robust dataset and to capture industry-wide operational risk trend. (Sabri Guray Uner, 2008). Some of the main features of the required data are summarized as follows:

~ Internal Data

Internal data should be collected over a minimum five-year period to be used for capital charge calculations (when the bank starts the AMA, a three-year period is acceptable). Due to a short observation period, typically the internal data for many risk cells contain few low-frequency/high-severity losses or none. A bank must be able to map its historical internal loss data into the relevant Basel II risk cells. The data must capture all material activities and exposures from all appropriate sub-systems and geographic locations. A bank can have an appropriate low reporting threshold for internal loss data collection, typically of the order of EURO 10,000. Aside from the information on gross loss amounts, a bank should collect information about the date of the event, any recoveries of gross loss amounts, as well as some descriptive information about the drivers or causes of the loss event.

~ External Data

A bank's operational risk measurement system must use relevant external data (either public data and/or pooled industry data). These external data should include data on actual loss amounts, information on the scale of business operations where the event occurred, and information on the causes and circumstances of the loss events. Industry data are available through external databases from vendors (e.g. Algo OpData provides publicly reported operational risk losses above USD 1million) and consortia of banks (e.g. ORX provides operational risk losses above EURO 20,000 reported by ORX members). External data are difficult to use directly due to different volumes and other factors. Moreover, the data have a survival bias as typically the data of all collapsed companies are not available.

~ Scenario Analysis/Expert Opinion

A bank must use scenario analysis in conjunction with external data to evaluate its exposure to high-severity events. Scenario analysis is a process undertaken by experienced business managers and risk management experts to identify risks, analyze past internal/external events, consider current and planned controls in the banks, etc. It may involve workshops to identify weaknesses, strengths and other factors; opinions on the severity and frequency of losses; opinions on sample characteristics or distribution parameters of the potential losses. As a result, some rough quantitative assessment of the risk frequency and severity distributions can be obtained. Scenario analysis is very subjective and should be combined with the actual loss data. In addition, it should be used for stress testing, for example to assess the impact of potential losses arising from multiple simultaneous loss events.

~ Business environment and internal control factors

A bank's methodology must capture the key business environment and internal control factors affecting operational risk. These factors should help to make forward-looking estimates, account for the quality of the controls and operating environments and align capital assessments with risk management objectives.

Data important for modeling but often missing in external databases are risk exposure indicators and near-misses.

Exposure indicators. The frequency and severity of operational risk events are influenced by indicators such as gross income, number of transactions, number of staff and asset values. For example, the frequency of losses typically increases with an increasing number of employees.

Near-miss losses. These are losses that could occur but were prevented. Often these losses are included in internal datasets to estimate the severity of losses but excluded in the estimation of frequency.

Methodology of Loss Distribution Approach

The first step of the approach is to draw, for each business line and each type of loss event, 2 curves of the probability distribution for loss, one which represents the frequency of loss events over a time interval (loss frequency distribution), the other the severity of these same events (loss severity distribution). To do so, we sort loss events by frequency on one hand, and by cost on the other hand, and we represent the result graphically (using histograms).

For each of the resulting distributions, we look for the mathematical model that best represents the shape of the curve. In order to validate the choice of a mathematical model, we compare the result (frequency or loss) predicted by the model to the output of the curve built from real data: if both curves overlap, the model is considered as reliable.

Then we combine both distributions, using a Monte-Carlo simulation, in order to obtain for each business line and each type of event, an aggregated curve of the loss distribution for a given time horizon. For each of those, the Value at Risk (VAR) is the maximum loss incurred with a probability of 99.9%.

The required capital in the Basel II framework is then the sum of the calculated VARs.

- Modeling Risks using Scenario Generation

The technique most commonly used to model macroeconomic risks is scenario generation. As the name suggests, it involves developing a scenario for the possible path of a variable, e.g., oil price, over the period of study, e.g., 5 years. The term scenario is often used with the assumption there is a widespread understanding of what the term means. Scenarios have been used as part of risk management for years. In particular, scenario usually includes an assessment of the likelihood of the event occurring and estimate of the severity of the adverse consequences. (Mark Laycock, 2016). Just as for forecasting the path of a hurricane, hundreds or thousands of scenarios are generated using computer simulation. Each one represents a unique path that the variable can take. The scenarios are then summarized into probability distributions for the variable. It is important to keep in mind that this is not a prediction of the movement of the price or rate. Rather, scenario generation is a method for measuring the size and frequency of the possible deviation of the movement from its expected path. Scenarios are related to stress tests. One of the key differences is that the assumed likelihood of the stress test events occurring is 100%. As a result, firms are expected to have measures in place to enable them to survive the events described in the stress test. Additionally, scenarios are required as part of AMA (Advanced Measurement Approach) models — for example, as direct inputs or to validate results from loss data. (Mark Laycock, 2016).

To model a macroeconomic variable, it is first necessary to understand its fundamental characteristics. The following are some typical characteristics of macroeconomic variables.

Long-term trend. For most variables, looking at historical time series data over a long time period reveals secular trends reflecting the fundamentals underlying the variable. For example, the long-term trend of oil prices is based on fundamental factors like the current worldwide supply, exploration activities, and worldwide demand for oil.

Jumps versus mean reversion. Although variables may have varying long-term trends, in all cases there are deviations from that trend caused by market events, like the OPEC production cut, or the Federal Reserve Board announced a change in its lending rate. In some cases, the deviation from the long-term trend is permanent and establishes a new level and trend. In others, the deviation is short-term, and the value reverts back to the long-term trend. The former type is called a jump process whereas the latter is called a mean reverting process. For example, individual stocks sometimes exhibit jumps, whereas interest rates and many commodity prices tend to exhibit mean reversion.

Relationship to other variables. There are generally two kinds of relationships with other variables: causal and correlation. A causal relationship is one where the cause-effect relationship between the variables can be explicitly stated. For example, currency exchange rates are directly affected by interest rates within the respective countries. As the interest rates in the United States move up, so must the value of the U.S. dollar, otherwise it would create arbitrage opportunities. There is a formal mathematical relationship between currencies and interest rates called the interest rate parity. In other cases, two variables may exhibit some correlation, although it is difficult to explicitly connect the two through cause-effect relationships. For example, oil and natural gas prices move independently, although they exhibit some correlation. Reflecting these relationships is vital to the generation of scenarios for all the variables that are internally consistent.

Seasonality. Many commodities exhibit seasonal cycles in their price movement. For example, natural gas cycles between summer and winter prices as the demand shift each season.

Randomness. Even after accounting for the above characteristics, most variables still exhibit a significant degree of uncertain behavior because of "noise." The noise is the aggregate effect of a multitude of small causal factors. It results in many temporary and independent movements that are uncertain in both direction and size.

Mathematics of Scenario Generation

The mathematics most often used to capture these characteristics in developing scenarios is called stochastic differential equations (SDEs). Differential equations are so named because they express the difference between the value of a variable at a time, t, and the value 1 time period later, t+1. Thus, the process of generating scenarios involves starting with the current value and using the differential equation to generate by iteration the values for each subsequent time period. It is a stochastic differential equation because the difference has a random element representing our uncertainty as to its movement at each time step.

A generic form of the SDE is:

$$dVt = f(Vmean - Vt) + g(Vt, Wt, Xt, Yt) * dt + h(Vt) * dZ$$

Let us briefly go over each term of this equation to relate it to the characteristics we are trying to model.

- The first term, dV_t, is the change in the value of the variable, V, from time t to t+1, i.e., V_{t+1} - V_t. This, of course, is what we are solving for.
- $f(V_{mean} - V_t)$ is a mean reversion function. It indicates the degree to which the variable tends to return back to its long-term fundamental level whenever it moves away from it. Whenever the current value, V_t, is above its mean level, V_{mean}, this function produces negative values, and vice versa. For variables exhibiting "jumps," this term is replaced by a function that models infrequent large jumps to a new level.
- $g(V_t, W_t, X_t, Y_t)$ is a function that indicates how the movements of the variable V are related to other potential variables W, X, and Y. This function is used to capture causal relationships, secular trend, short-term deviations, and seasonality. Often this is accomplished by breaking this term into several functions, each of which addresses one of these characteristics.
- $h(V_t) * dZ$ is the term that captures the random movement or noise. By contrast, the other terms are all deterministic. dZ is the value of a standard normal random variable. Its probability distribution has the familiar bell curve shape centered at zero with a standard deviation of 1. When the SDE is used in an iterative fashion, at each time step, a value for dZ is determined by randomly sampling this probability distribution. $h(V_t)$ is a scaling factor for the random movement. This is the term in the SDE that has different values every time we generate a new scenario. This term is also used to capture the correlation with other variables.

Each term on the right side of the SDE is a mathematical function whose structure and parameters must be determined specifically for the variable that is being studied. For example, the mean reversion function for crude oil may look like this: $-1.10 * (V_{mean} - V_t) * dt$. The mean reversion coefficient (in this case it is 1.1) describes the rate at which a short-term deviation is expected to disappear. The "half-life" of the deviation—the time in which a deviation away from the fundamental level is expected to halve—is equal to ln(2)/mean reversion coefficient. In our example, this translates to 0.63 years or about 7.5 months. Once the deviation is halved, it takes another 7.5 months for it to halve again (¼ of the original deviation). This means that the rate at which the value changes decreases as it gets closer to the fundamental level. Note also that the rate of mean reversion is not dependent on the value of V_{mean}, the fundamental level.Defining the functions and the parameter values of the SDE involves rigorous statistical analysis of historical data. SDEs are validated by mathematically analyzing how well they capture the fundamental characteristics discussed above.

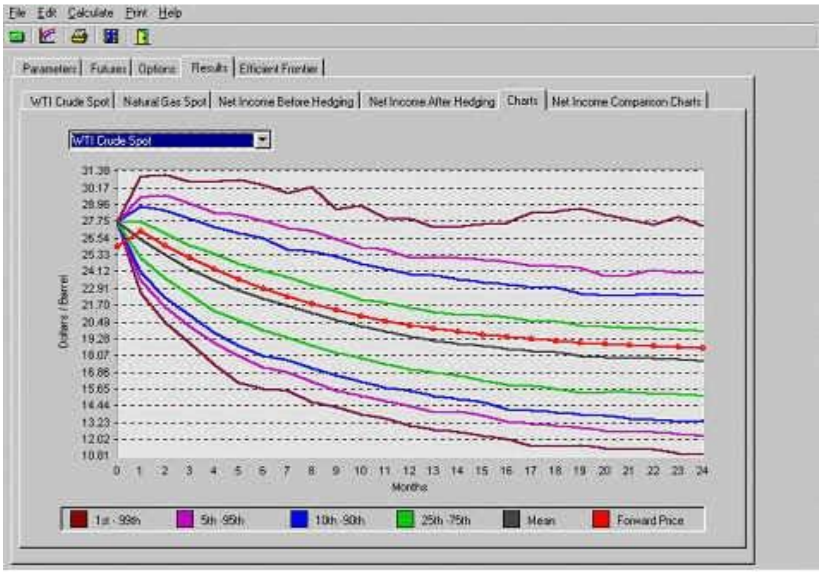

FIGURE 2.1

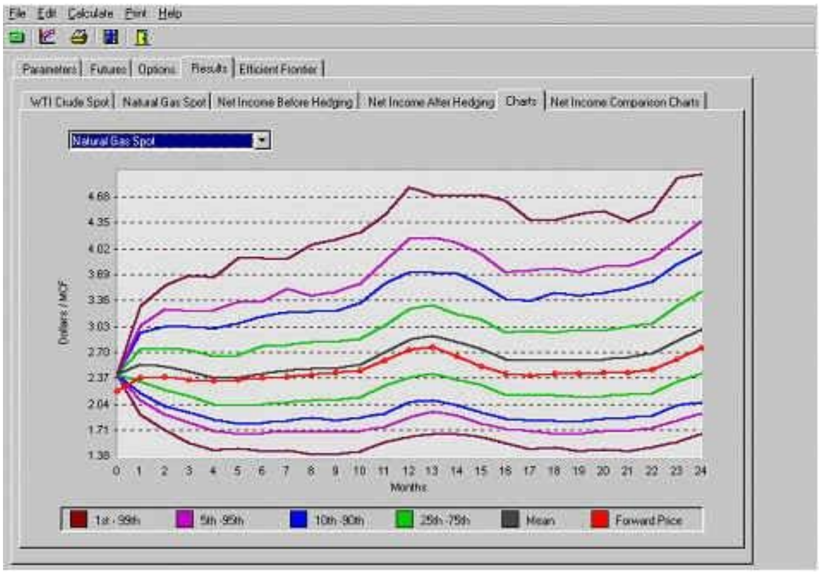

FIGURE 2.2

Figures 2.1 and 2.2 illustrate summary statistics from generating 1,000 scenarios for the movement of crude oil and natural gas, respectively, over the next 24 months (this was done as of late November 1999). The black curve indicates, for each month, the mean value of the price overall 1,000 scenarios. The other colored lines indicate various percentiles for the price each month. Although only certain percentiles are plotted, since we have all the data from the 1,000 scenario simulations, we can plot a full probability distribution of the price for each month. (Samir Shah,2000)

Uses of Scenarios

Scenarios are used in a broad range such as managing credit, liquidity, market or enterprise risk but in our study, we will focus on operational risk scenarios. Although, some of the points raised might be applicable to other types of dangers as well.

The level of acceptable uncertainty over the result may depend upon why the scenario has been created. Among the primary reasons for creating scenarios are:

Decision Making

To aid with decision making, the scenario indicates the scale of the issue concerned and helps to explore the circumstances behind that specific scenario. For example, if a risk and control self-assessment indicates a topic of potential concern, then a scenario might be completed in order to gain a better understanding of the wider issues at play. The scenario creation process and outputs are effectively the analysis and comparison against benchmark elements of the risk management cycle. The next stage is to take a decision about whether to accept, avoid, reduce, transfer or transform the risk.

Scenarios can also be used as inputs to projects. The scenario might determine the change (if any) in the risk profile after completion of the project, compared to the status quo. If the project is for risk mitigation, then having reasonably accurate results will be helpful, but if the project is proceeding for other reasons, then an indication of scale or magnitude might be sufficient.

Scenarios are also useful when exploring an emerging risk. A risk might emerge or have the potential to significantly raise its an assessment of the likelihood of the event occurring and an estimate of the severity of the adverse consequences. A strategy scenario may have similar elements, including the likelihood of occurrence, and the impact on the firm's balance sheet and revenue. In this paper, "scenario" should be taken to mean "risk scenario".

Capital Calculations

Scenarios can be used as direct inputs into the calculation or to validate the capital estimate. A firm could use scenarios as direct inputs into its capital calculation as a means to fill in knowledge gaps. For example, the firm may have peers that have experienced large losses but may not (yet) have experienced such losses itself. The firm may decide that it has an exposure to the events that have led to the losses and that these losses could materialize over the following 12 months (the AMA time horizon). The firm may therefore create a scenario to provide a data point in the form of a loss or a distribution to render the AMA estimate more complete. Toward one end of the spectrum of AMA methodologies, a firm's AMA result may be dominated by scenario results as opposed to historic loss data originating with the firm itself or its peers.

The indirect use of scenarios in relation to the AMA is to help validate the AMA estimate. In this situation a number of scenarios are needed to cover the main unexpected losses. The portfolio of scenarios is aggregated, and values are taken from the aggregate loss distribution and compared to the loss of data-dominated AMA.

Potential Surprises

The "search for potential surprises" is sometimes known as a reverse scenario. For reverse scenarios, an aggregate loss number is provided (for example, $1 billion) and the business is asked for examples of how they could potentially lose that amount over a 12-month horizon and as a result of a single event. The aim is to arrive at an extreme, but plausible scenario — chemical attacks by rogue states are out of scope.

The reverse scenario challenge requires the business to survey its activities and processes and to arrive at a combination or convergence of events that could give rise to a loss of a specific size. The loss could be expressed as a specific amount, for example $1 billion, or as a percentage of the previous year's profits/ revenues.

Scenario Generation Elements

Quantification

Quantification can be the most challenging part of completing a scenario. The degree of accuracy (not necessarily precision) and the level of uncertainty are linked to the purpose for which the scenario is being created. For some purposes the users may just want to know about operational increases or decreases, for example the current environment versus the post-project implementation environment. To help the creators and users of scenarios, it may help to clarify what is needed, as opposed to what is wanted by various users.

Data

The creators of scenarios tend to be asked for estimates of losses or the frequency and severity of losses. Sometimes the data request is couched in terms of confidence intervals, such as 99% over a particular time horizon, e.g., 12 months.

- Tail -risk stress tests

Another common practice for estimating operational risk is a tail-risk stress test. As `mentioned before, stress testing helps companies assess major changes in one or two specific variables whose effects would be major and immediate, although the exact time is not predictable. financial services firms use stress tests to assess, for example, how an event such as the tripling of oil prices, a large swing in exchange or interest rates, or the default of a major institution or sovereign country would affect trading positions and investments.

The benefits from stress-testing, however, depend critically on the assumptions—which themselves may be biased—about how much the variable in question will change. The tail-risk stress tests of many banks in 2007–2008, for example, assumed a worst-case scenario in which U.S. housing prices leveled off and remained flat for several periods. Very few companies thought to test what would happen if prices began to decline—an excellent example of the tendency to anchor estimates in recent and readily available data. Most companies extrapolated from recent U.S. housing prices, which had gone several decades without a general decline, to develop overly optimistic market assessments. (Robert S.Kaplan, Anette Mikes,2012)

Fuzzy Logic

This method is used when there is no available data to apply to statistical approaches that are commonly used for assessing market risks. There is greater reliance on expert input rather than historical data to assess the risk. Fuzzy logic has been implemented for years in the engineering sciences and is described by a broad range of applications. This method offers a promising alternative for measuring operational risk.

Many companies that are in the process of implementing ERM are assessing operational risks using qualitative methods. The approaches are typically some variation of creating a list of "Top 10" risks based on expert input. The "Top 10" lists are often developed at a low level in the organizational hierarchy (e.g., department, region, or business unit) and consolidated at various levels of the hierarchy to an ultimate corporate "Top 10" list. However, there is not enough information gathered for each risk for managers to assess the relative magnitude of the risks and their interaction across the enterprise. This makes it difficult for managers to decide how much to spend on managing each risk. There is also a possibility that major risks fall through the cracks simply because in isolation within a department, region, or business unit they are not deemed critical, when, in fact, their accumulation and interaction across the enterprise raises the potential of significant losses.

The approach described here is to apply fuzzy logic modeling to assess risk on the "Top 10" list. The assessment provides a more thorough definition of each risk and its interaction with other risks than the current methods. This provides local risk managers a decision tool for managing risks within their organizational unit. It also allows corporate risk managers clearer information to more reliably distill local Top 10 lists to a corporate Top 10 list and appropriately allocate investment to manage each risk.

The following describes the steps undertaken when adding a new risk to the Top 10 list. These steps apply fuzzy logic techniques for developing a causal model that relates the risk to its key drivers or indicators. The causal model is then used to develop a distribution of losses based on expectations for the levels of its key drivers.

Steps of Fuzzy Logic Method

Step 1-Specify Key Risk Indicators

For each of the top 10 risks, several risk indicators are specified. A KRI is an operational or financial variable that provides a reliable basis for estimating the loss corresponding to the risk. A KRI can be a specific causal variable or a proxy for the drivers of the loss attributed to risk. Ideally, KRIs should be chosen that are regularly measured on an ongoing basis so that data can be easily gathered.

Step 2—Calibrate Fuzzy Representation of KRIs and Loss Amount

The essential advantage offered by fuzzy logic techniques is the use of linguistic variables to represent KRIs and the loss amount corresponding to a risk. In this step, linguistic descriptors such as High, Low, Medium, Small, Large, for example, are assigned to a range of values for each KRI and the loss amount. Since these descriptors will form the basis for capturing expert input on the impact of KRIs on the loss amount, it is important to calibrate them to how they are commonly interpreted by the experts providing input. Referring to a variable as High, for example, should evoke the same understanding among the experts. The calibration may vary by region so that "High" employee turnover may mean different things in different regions.

Step 3—Specify Impact of KRIs on Loss Amount

Having specified the risk and its KRIs, the logical next step is to specify how the loss amount varies as a function of the KRIs. Experts provide fuzzy rules in the form of *if … then* statements that relate loss amounts to various levels of KRIs based on their knowledge and experience.

Step 4—Calculate the Expected Loss Amount

Since the fuzzy rules cover all possible combinations of KRI levels, the estimated loss amount can be calculated for the current levels of each KRI. A fuzzy calculator applies the math based on the fuzzy rules to generate the expected loss.

Step 5—Calculate Distribution of Losses

Probability distribution of expected losses next year can be derived by representing the KRIs as a probability distribution of their levels expected next year. Since the KRIs are typically operational or financial variables, an empirical distribution based on historical data can be developed for each KRI. To the extent that historical data is not representative, professional judgment is used to modify the distribution as appropriate. Applying the same fuzzy rules-based calculation produces a distribution of the expected losses that capture the uncertainty underlying the KRIs.

The fuzzy logic approach provides more information and helps managers to effectively manage operational risks rather than other qualitative approaches. A strong benefit of this method is that it uses a combination of historical data and expert input. It is worth noticing that operational risk might be more difficult to measure than market risk, the relative measurement of it provides useful information for the possible investment. (Samir Shah,2003)

In this section, we analyzed the meaning of operational risk, its importance for the managers and investors and the most typical ways in order to measure it. In the next pages of the chapter we will focus on more methods the risk is calculated when it is present in investment decisions.

2.4.1 Variance

In statistical science, the variance is the expectation of a squared root deviation from its mean or also called, the expected value. Standard deviation is the square root of variance. Variance is a powerful tool used in descriptive statistics, statistical influence hypothesis testing and Monte Carlo simulations. The variance formula is as follows:

$$SD^2 = \frac{\Sigma(X - \overline{X})^2}{N}$$

Where:
SD^2 = the variance
$\sum$ = the sum of
X = the obtained score
$\overline{X}$ = the mean score of the data
N = the number of scores
Thus, $\Sigma(X - \overline{X})^2$ = the sum of squared deviations

A small variance indicates that numbers from the set of data are close to the mean, whereas a high variance indicates that numbers from the set are spread at a wide range from the mean. A variance of zero, indicates that all values within the set are identical. Furthermore, variance shows how individual numbers relate to each other on a data set. It might be more difficult to be interpreted but it treats all deviations from the mean the same regardless of direction.

2.4.2 Variance as a risk measure

Variance is used at a large scale in the financial field. In particular, variance plays a very important role in investing activities. An investor may invest in bonds, equities, derivatives, and among these financial instruments risk is present. Variance is used in the economic study to estimate the risk. The squared deviation from the mean, or in this case returns, is the risk investors face undertaking investment activities. Conceptually, with the term variance in the financial field we illustrate the squared deviation from the expected return of the financial instrument.

2.4.3 Measuring Portfolio Risk using Variance

In this section, we will assume that investors take investment decisions by examining the level of variance with respect to the return of the financial instrument. A portfolio can contain combinations of different financial instruments such as bonds, stocks etc. Each asset has exposure to risk. The total portfolio risk depends on the percentages invested in each asset, and their risk level. To examine variance as a risk measure, some clarification is in order.

Normal Distribution

By only looking at mean and variance, we are necessarily assuming that no other statistics are necessary to describe the distribution. Investors may have different types of utility functions (preferences of economic entities with respect to risk and return), quadratic etc. In most cases, we assume that returns have a normal distribution, which can be completely described by mean and variance. This is the bell-shaped probability distribution that many other phenomena obey. The normal distribution is perfectly symmetric, and 50% of the probability lies above the mean. Because of its symmetry, the variance and semi variance (a measurement of data that can be used to estimate the potential downside risk of an investment portfolio) are equivalent measures of risk for the normal distribution. Moreover, if you know the mean and the standard deviation of the normal distribution, you know the likelihood of every point in the distribution. This would not hold if the distribution were not symmetric. The variance and semi variance would not be equivalent.

Often a normal distribution is converted into a unit normal distribution that always has a mean of zero and standard deviation of one. Most probability tables are based on a unit normal distribution. (Financial Theory and Corporate Policy ,4th edition)

Properties of Risky Assets and Portfolios *(*Bodie, Kane, Marcus,2005)

To continue our study, we must describe the properties of risky assets bear.

Rule 1. The mean or expected return of an asset is a probability-weighted average of its return in all scenarios.

Rule 2. The variance of an asset's returns is the expected value of the squared deviations from the expected return.

Rule 3. The rate of return on a portfolio is a weighted average of the rates of return of each asset comprising the portfolio, with portfolio proportions as weights. This implies that the expected rate of return on a portfolio is a weighted average of the expected rate of return on each component asset.

Rule 4. When a risky asset is combined with a risk-free asset, the portfolio standard deviation equals the risky asset's standard deviation multiplied by the portfolio proportion invested in the risky asset.

Portfolio Risk

In a portfolio that involves stocks or a combination of other financial instruments, it is difficult to predict its future value. The method that estimates the risk to a certain extent is to estimate the future range and the likelihood of the portfolio's values. The measure of it how much the future values will deviate from the predicted values. Most investors relate to the hazard that the future portfolio value will be less than anticipated. A particularly useful way to quantify the uncertainty about portfolio return is to specify the probability associated with each of the possible future returns. Assume, for example, that an investor has identified five possible outcomes for his portfolio return during the next year. Associated with each return is a subjectively determined probability, or relative chance of occurrence. The five possible outcomes are:

Possible Return	Subjective Probability
50%	0.1
30%	0.2
10%	0.4
-10%	0.2

-30%	0.1

Note that the probabilities sum to 1 so that the actual portfolio return is confined to take one of the five possible values. Given this probability distribution, we can measure the expected return and risk for the portfolio. The expected return is simply the weighted average of possible outcomes, where the weights are the relative chances of occurrence. The expected return on the portfolio is 10%, given by

$$E(Rp) = 2\ Pj\ R\ j = 1$$

$$= 0.1(50.0) + 0.2(30.0) + 0.4(10.0) + 0.2\ (-10.0) + 0.2(-30.0)$$

where the R.'s are the possible returns and the Pj's the associated probabilities. (The expected terminal market value of the portfolio is equal to M 0 (1 + .10), where M0 is the initial value.)

If the risk is defined as the chance of loss or achieving returns less than expected, it would seem to be logical to measure risk by the dispersion of the possible returns, below the expected value. However, risk measures based on below-the-mean variability are difficult to work with, and furthermore are unnecessary as long as the distribution of future return is reasonably symmetric about its expected values.

It now remains to develop a specific measure of total variability of returns. The measures which are most commonly used are the variance and standard deviation of returns. Measuring risk by standard deviation and variance is equivalent to defining risk as total variability of returns about the expected return, or simply, variability of returns. The variance of return is a weighted sum of the deviations from the expected return. The variance, designated p for the portfolio in the previous example is given by:

$$\text{Var}(r) = p_1 * (\text{E}(r) - r_1)^2 + p_2 * (\text{E}(r) - r_2)^2 + \ldots + p_S * (\text{E}(r) - r_S)^2$$

$=0.1(50.0- 10.0)^2+ 0.2(30.0- 10.0)^2 + 0.4(10.0- 10.0)^2 + 0.2(-10.0- 10.0)^2 + 0.1(-30.0- 10.0)^2 = 484$ percent squared.

The standard deviation is defined as the square root of the variance. It is equal to 22%. The larger the variance or standard deviation, the greater the possible dispersion of future realized values around the expected value, and the larger the investor's uncertainty. As a rule of thumb, it is often suggested 'that two-thirds of the possible returns on a portfolio will be within one standard deviation of the return either side of the expected value; ninety-five percent will lie with plus or minus two standard deviations of the expected return.

Thus, far our discussion of portfolio risk has been confined to a single-period investment horizon such as the next year. That is, the portfolio is held unchanged and evaluated at the end of the year. An obvious question relates to the effect of holding the portfolio for several periods, such as the next 20 years: will the 1-year risks tend to cancel out over time? Given the random walk nature of security prices, the answer to this question is no. If the risk level (standard deviation) is maintained during each year, the portfolio risk for longer horizons will increase with the horizon length. The standard deviation of possible terminal portfolio values after N years is equal to; N times the standard deviation after 1 year. Thus, the investor cannot rely on the "long run" to reduce his risk of loss. A final remark before leaving portfolio risk measures. We have implicitly assumed that investors are risk averse, i.e., they seek to minimize risk for a given level of return. This assumption appears to be valid for most investors in most situations. The entire theory of portfolio selection and capital asset pricing is based on the belief that investors on the average are risk averse. (Franco Modigliani, Gerald A. Pogue,1973)

2.4.4 Diversification

Diversification is one of the most known terms in financial theory. It describes a risk management strategy which combines a broad range of financial instruments within a portfolio. The rationale behind this approach is when different assets are combined, the long-term return will be higher, and risk will be reduced. Diversification diminishes the unsystematic danger of a portfolio, so the positive performance of one asset will out-perform negative of another. For diversification to hold, assets among portfolio need be not perfectly correlated.

In order to understand better the method of diversification, a few terms need further explanation.

2.4.5 Covariance

Covariance is a measurement of the directional relationship between variables. a positive covariance means that asset returns move together while a negative covariance means returns move inversely. Covariance is calculated by analyzing at-return surprises (standard deviations from the expected return) or by multiplying the correlation between the two variables by the standard deviation of each variable.

In particular, covariance evaluates how the means of two variables move together. The covariance is an extremely useful concept because it is the appropriate measure of the contribution of a single asset to portfolio risk. The variance of a random variable is really the same thing as its covariance with itself. Covariance can also help to select which assets to include in one portfolio. The covariance formula is as follows:

Covariance:

$$\mathrm{Cov}[X,Y] = \mathrm{E}[(X - \mathrm{E}[X])(Y - \mathrm{E}[Y])]$$

Where:

X, Y two random variables

E the expected value operator

We can conclude now that the Variance for the portfolio of two assets is:

$VAR(Rp)=a^2Var(x)+b^2Var(y)+2abCOV(x,y)$

2.4.6 Correlation Coefficient

The correlation coefficient is a statistical measure that estimates how strong the relationship is between the relative movements of these variables. The values of correlation coefficient range between -1 and 1. The value -1 implies a perfect negative correlation and the value of 1 a perfect positive correlation. A value of 0 implies no relationship between the variables. There are several types of correlation coefficients but for the purposes of our study we will use the Pearson correlation (r). This coefficient captures the strength and direction of the linear relationship between two variables. It takes the same values but when -1 or 1 it is implied a perfect linear negative or positive relationship.

This statistical coefficient is extremely useful in the financial field. It shows the direction and the strength between two financial instruments and it is also used to determine how a financial instrument is related to its benchmark index.

Correlation Coefficient Formula is as follows:

$$r = \frac{Covariance\ (x,y)}{S.D.(x)S.D.(y)}$$

$$r = \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}}$$

After analyzing these statistical terms, it is clearer how diversification is achieved in portfolio theory. When combining assets that are negatively related to each other, a decrease in one’s instrument return will not entrain the other assets and the overall performance will be higher. In addition, risk is reduced. A well-diversified portfolio will outperform portfolios that are described with assets of the same nature and positive correlations among instruments. Diversification also hedges against market volatility.

2.4.7 The minimum variance portfolio

To build a minimum variance portfolio, an investor would need to have a combination of low volatility investments or a combination of volatile investments with low correlation to each other. The latter portfolio is common in relation to building minimum variance portfolios. Investments that have low correlation could be described as those that perform differently (or at least not too similar) given the same market and economic environment. It is a prime example of diversification. When an investor diversifies a portfolio, they are essentially seeking to reduce volatility, and this is the basis of the minimum variance portfolio—a diversified portfolio of securities.

This application of variance is extremely useful since it is used to find the combination of variables for example X, Y that provides the portfolio of minimum variance. This portfolio is the one where changes in variance with respect to changes in the percentage of X, Y are zero.

Therefore, the variance can be written as: (sum weights add up to 1)

$VAR(Rp)= a^2\sigma^2_x+(1-a)^2b^2\sigma^2_y+2a(1-a)\ rxy\sigma_\chi\sigma_y$

To find the minimum variance portfolio, we set the first derivative equal to zero.

2.4.8 The minimum variance opportunity set

Another key concept in financial theory is the minimum variance opportunity set. The minimum variance opportunity set is the locus of risk and returns combinations offered by portfolios of risky assets that yield the minimum variance for a given level of return.

This concept enables investors to select portfolios that maximize their utility. The portfolio's mean returns, and variances are the measures of returns and risks respectively. Investors choose percentages of their wealth to invest in each security in order to obtain the required risk and return.

The minimum variance opportunity set is the down limit for investors, since they will never choose a portfolio below the minimum variance point.

2.4.9 Efficient Set/Frontier

In order to compare investment options, analysts have developed a system to describe each investment or each asset class with math, using unsystematic risk statistics. Risk scores are applied to the portfolios that contain the various investment options and asset classes. We then look at the expected rate-of-return and the expected volatility for each investment. The financial services industry has commonly referred to this as The Efficient Frontier. (Wealth management, 2014) The efficient set is the set of mean-variance choices from the investment opportunity set were for given variance no other investment opportunity has a higher return. The notion of efficient set narrows down the number of portfolios an investor might choose. In other words, the efficient set is the locus of highest returns with a given level of risk, so no rational investor would choose to invest elsewhere.

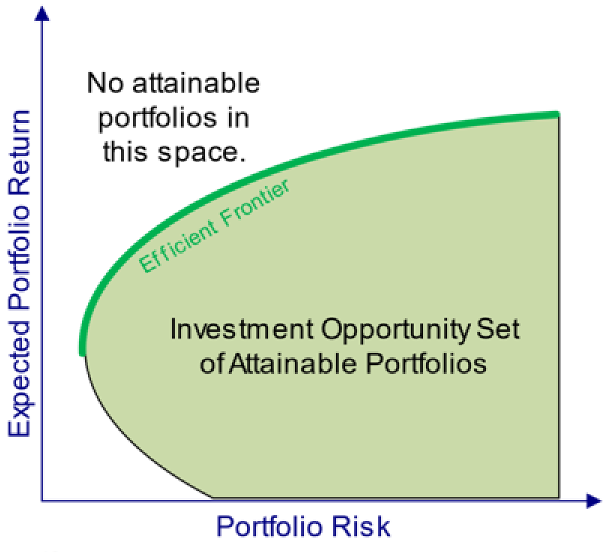

Figure 2.3: Efficient Frontier

On the efficient set, there is a portfolio with minimum risk (minimum variance portfolio) and a maximum return portfolio with the highest level of variance. Portfolios below the efficient set, are not selected by rational investors because they offer lower levels of returns with the same level of risk.

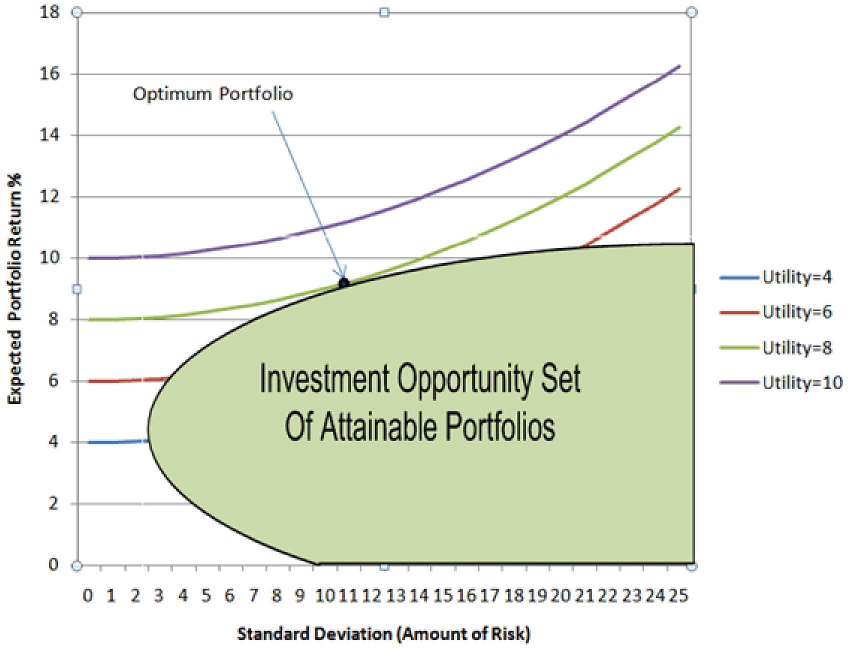

Figure 2.4: Efficient set

The efficient frontier, consists of the set of all efficient portfolios that yield the highest level of return for a given level of risk. The efficient frontier is combined with an investor's utility function to find the investor's optimal portfolio, the portfolio with the highest return for the risk the investor is willing to accept.

2.4.10 Different Types of Investors

It is assumed that higher levels of risk will accompany a higher level of return. However, investors vary as to which level they accept riskier portfolios.

For example, investors who are risk-averse will accept an investment with a lower risk for a given level of return. On the other hand, risk lovers, are willing to accept lower returns in exchange for greater levels of risk. A risk neutral on the contrary, is indifferent between two investments that have a different level of risk. His decision is based on the expected return of the investment.

In this section, we focused on analyzing the definition and the importance of variance in the financial field. It is clear that variance has a key role in investment decisions, and it is a powerful tool for selecting investment opportunities. In the modern portfolio theory, the variance has a lot of applications and should be studied with careful attention.

2.5.1 Mean Absolute Deviation (MAD)

Risk as we described earlier is the cornerstone of the financial field. The importance of accurate estimation is extremely high. Another way to assess the risk is by using mean absolute deviation as a measure of danger. Mean absolute deviation is an alternative to estimate risk after the development of Markowitz's Theory of Portfolio selection in 1952. While Markowitz's theory on mean-variance analysis has received wide acclaim, in practice it had been not been used extensively due to early computational difficulties associated with large-scale quadratic programming.

In 1992, Hiroshi Konno and Hiroaki Yamazaki provided an alternative to Markowitz's Mean-Variance model through their proposed Mean-Absolute Deviation (MAD) model. As the name implies, this model defines risk by using mean absolute deviation. In statistical science, the mean absolute deviation of the data set is the average distance between each data value and the mean. By calculating this term, we are able to observe how spread the values are on the data set. The formula of mean absolute deviation is as follows:

$$\text{Mean Absolute Deviation (MAD)} = \frac{\sum_{i=1}^{n} |x_i - \mu|}{n}$$

As we notice, by construction the MAD model is easier to solve since the portfolio problem is a linear function as opposed to the mean-variance model.

2.5.2 Mean Absolute Deviation Model

In 1992, Konno introduced an alternative to the Mean-Variance model in which risk is defined by the L1 risk (absolute deviation) function. Before introducing the model, let us show a simple relation between the MAD and Mean-Variance models. (Geo Rey Kasenbacher, Jordan Lee, and Klod Euchukanonchai,2017)

Proposition 1. *Suppose* $R(\overrightarrow{w})$ *is normally distributed with mean* $E[R(\overrightarrow{w})] = r(\overrightarrow{w})$ *and variance* $V(\overrightarrow{w}) = \sigma^2(\overrightarrow{w})$, *then:*

$$K(\overrightarrow{w}) = \sqrt{\frac{2}{\pi}}\sigma(\overrightarrow{w})$$

Proof.

$$\begin{aligned} K(\overrightarrow{w}) &= \frac{1}{\sqrt{2\pi}\sigma(\overrightarrow{w})}\int_{-\infty}^{+\infty} |z - r(\overrightarrow{w})| exp\{\frac{(z - r(\overrightarrow{w}))^2}{2\sigma^2(\overrightarrow{w})}\}dz \\ &= \frac{2}{\sqrt{2\pi}\sigma(\overrightarrow{w})}\int_{0}^{+\infty} t\ exp\{-\frac{t^2}{2\sigma^2(\overrightarrow{w})}\}dt \\ &= \sqrt{\frac{2}{\pi}}\sigma(\overrightarrow{w}) \quad \blacksquare \end{aligned}$$

For normally distributed R(w) implies that the Mean-Absolute Deviation model generates the optimal Portfolio as the corresponding Mean-Variance Model.

Here we provide the motivation for comparison under the assumption of normality. However, in practice this assumption is not often satisfied, implying the models are no longer equivalent.

Proposition 2. *The* L_1 *risk measure is less than or equal to the* L_2 *risk measure. That is,* $K(\overrightarrow{w}) \leq \sigma(\overrightarrow{w})$.

Proof. By Jensen's inequality

$$E[|R(\overline{w}) - E[R(\overline{w})]|]^2 \leq E[|R(\overline{w}) - E[R(\overline{w})]|^2]$$
$$E[|R(\overline{w}) - E[R(\overline{w})]|]^2 \leq Var[R(\overline{w})]$$

Since both sides are strictly positive and the square root is an increasing monotone function, we have

2.5.3 Mean Absolute Deviation

$$\sqrt{E[|R(\overline{w}) - E[R(\overline{w})]|]^2} \leq \sqrt{Var[R(\overline{w})]}$$
$$K(\overrightarrow{w}) \leq \sigma(\overline{w}) \quad \blacksquare \qquad (1)$$

Based on the result of (1) we hypothesize that for equal expected return the MAD model will generate lower risk. Equivalently, for the equal risk we expect the MAD model to have a higher expected return. The MAD model can be constructed as follows:

$$\begin{aligned} &\text{Minimize} && K(w) \\ &\text{Subject to} && E[R(\overrightarrow{w})] = \gamma \\ & && \overrightarrow{w}^T I = 1 \\ & && \overrightarrow{w} \geq 0 \end{aligned}$$

2.5.4 Empirical Results by using MAD versus Mean-Variance Method

Mad model in general outperforms the Mean-Variance Model and provides a larger risk-adjusted return especially in small-case portfolios. The explanation behind this result is that Mad Model is a less complicated Method to use. As for the size of portfolio increases, Mad becomes increasingly quicker than investing in a large-scale covariance matrix.

Another observation by the empirical implementation of both models is that the assumption of the normally distributed portfolio is unlikely to be satisfied. Only when the assumption of normally distributed portfolios is held, they tend to present similar results.

Furthermore, the linear model of Mad, gives portfolios which involve fewer non-zero components and hence reduces the numerous small transactions that are likely to appear in the Mean-Variance Model.

Simaan (1997) stated that although the minimization of Mean Absolute Deviation is close to the Mean-Variance Model formulation, they lead to different efficient sets. The divergence between the two models is due to the fact that each model utilizes different sample statistics and consequently relies on a different set drawn from the sample.

Both models have also been compared with out-of-sample data from shares traded in the Stockholm Stock Exchange (Papahristodoulou and Dotzauer, 2004) and the (MV) model yields higher utility levels and higher degrees of risk aversion in very similar computing times.

- Mean Absolute Deviation Model is very successfully implemented when dealing with investments that combine financial instruments internationally. International diversification is achieved by using the Mad Model.
- In the economic field a general assumption is that investors and customers always make their decisions rationally. Mean Absolute Deviation is consistent with this assumption.
- Mad Model is a more robust optimization technique subject to real market conditions.

  (Geo Rey Kasenbacher, Jordan Lee, and Klod Euchukanonchai,2017), (Hiroshi Konno, Hiroaki Yamazaki,1991)

2.5.5 Semi-Absolute Deviation

Another term we will analyze in this section is semi-absolute absolute deviation. Semi absolute deviation is a method of measuring the below-mean fluctuations in the returns on investment. Semi deviation is an alternative measurement of standard deviation but captures only the negative fluctuations of the values. Thus, it is an alternative measure of risk in particular, the downside risk of the investment.

This term is used in the financial field in order to capture the negative changes in the price. It captures the dispersion of an asset's value from an expected value. Semi deviation shows the worst-case scenario for the portfolio.

The semi-deviation formula:

$$\text{Semideviation} = \sqrt{\frac{1}{n} * \sum_{r_t < \text{Average}}^{n} (\text{Average} - r_t)^2}$$

2.5.5 Mean Semi-Absolute Deviation Model

In this section, we establish new portfolio optimization models in the uncertain environment by employing the semi-absolute deviation of the portfolio as the measure of risk. Let ξi (i = 1, 2, . . ., n) be the uncertain return of the ith security, and xi be the proportion of the total amount of funds invested in security i. By uncertain arithmetic, the total return of the portfolio is ξ1x1 + ξ2x2 + · · · + ξnxn which is also an uncertain variable. This means that the portfolio is risky. If an investor wants to maximize the expected return at the given risk level, then it can express in a single-objective non-linear programming model as follows:

$$\begin{cases} \text{maximize} & E[\xi_1 x_1 + \xi_2 x_2 + \cdots + \xi_n x_n] \\ \text{subject to:} & Sa[\xi_1 x_1 + \xi_2 x_2 + \cdots + \xi_n x_n] \leq d, \\ & x_1 + x_2 + \cdots + x_n = 1, \\ & x_i \geq 0, \quad i = 1, 2, \ldots, n, \end{cases}$$

where r is the minimum expected return level accepted by the investor. A risk-averse investor always wants to maximize the return and minimize the risk of the portfolio. However, these two objects are inconsistent. To determine an optimal portfolio with a given degree of risk aversion, we formalize the following optimization model,

$$\begin{cases} \text{maximize} & E[\xi_1 x_1 + \xi_2 x_2 + \cdots + \xi_n x_n] - \phi \cdot Sa[\xi_1 x_1 + \xi_2 x_2 + \cdots + \xi_n x_n] \\ \text{subject to:} & x_1 + x_2 + \cdots + x_n = 1, \\ & x_i \geq 0, \quad i = 1, 2, \ldots, n, \end{cases}$$

where ϕ ∈ [0, +∞) represents the degree of absolute risk aversion. Here, the greater the value of ϕ is, the more risk-averse the investors are. Note that ϕ = 0 means that the investor does not consider risk, and ϕ approaching infinity means that the investor will allocate all the money to risk-less securities. Note that in uncertain environment, E[ξ1x1 + ξ2x2 · · · + ξnxn] ≠ x1E[ξ1] + x2E[ξ2] + · · · + xnE[ξn] for general uncertain variables ξ1, ξ2, . . . , ξn. However, the inequality will become equality when ξ1, ξ2, . . ., ξn are independent. In particular, if security returns are all linear uncertain variables, denoting the return of security i by ξi = (ai, bi). It follows that the portfolio return ∑n i=1 ξixi = (∑n i=1 xiai , ∑n i=1 xibi) is also a linear uncertain variable. Based on this, the model may be converted into the following quadratic programming.

$$\begin{cases} \text{minimize} & \left(\sum_{i=1}^{n} x_i (b_i - a_i) \right)^2 \\ \text{subject to:} & \sum_{i=1}^{n} x_i (a_i + b_i) \geq 2r, \\ & x_1 + x_2 + \cdots + x_n = 1, \\ & x_i \geq 0, \quad i = 1, 2, \ldots, n. \end{cases}$$

Further, we assume that security returns are all zigzag uncertain variables, denoting the return of security i by ξi = (ai, bi, ci). It follows from Theorem that the portfolio return

∑n i=1 ξixi = (∑n i=1 xiai, ∑n i=1 xibi , ∑n i=1 xici) is also a zigzag uncertain variable. Then the model may be converted into the following deterministic form

$$\begin{cases} \text{minimize} & \dfrac{\left(\sum\limits_{i=1}^{n} 2x_i(c_i - a_i) + \left|\sum\limits_{i=1}^{n} x_i(2b_i - a_i - c_i)\right|\right)^2}{\sum\limits_{i=1}^{n} x_i(c_i - a_i) + \left|\sum\limits_{i=1}^{n} x_i(2b_i - a_i - c_i)\right|} \\ \text{subject to:} & \sum\limits_{i=1}^{n} x_i(a_i + 2b_i + c_i) \geq 4r, \\ & x_1 + x_2 + \cdots + x_n = 1, \\ & x_i \geq 0, \quad i = 1, 2, \ldots, n. \end{cases}$$

When the security returns are all linear or zigzag uncertain variables, both models can be converted into deterministic programming problems in a similar way.

2.5.6 Empirical Results from Semi-Absolution Deviation Model

- This model is very appropriate for uncertain portfolio optimization problems.

- If the semi-absolute deviation is extended to an interval case (fuzzy number and is a good method to deal with uncertainty) the following conclusions hold: An interval semi absolute deviation model with no short selling and no stock borrowing in a fictional market is proposed for portfolio selection. By introducing the concepts of the pessimistic satisfactory index and optimistic satisfactory index of interval inequality relation, an approach to compare interval numbers is given. By using the approach, the interval semi-absolute deviation model can be converted into two parametric linear programming problems. One can find a satisfactory solution to the original problem by solving the corresponding parametric linear programming problems. An investor may choose a satisfactory investment strategy according to an optimistic or pessimistic attitude. The model is capable of helping the investor to find an efficient portfolio that is in the closest possible accord with his goals. (Yong Fang and Shouyang Wang, 2006)

2.6.1 Value at Risk

Another method that managers use to assess the level of exposure in an investment is the Value of Risk, known as (VaR). Value-at-risk purports to indicate a quantile of the probability distribution for a portfolio's loss over a specified time period. Nowadays, Var is gaining attention as a risk measure. Part of it is explained by the fact that JP Morgan decided to create a transparent risk model, called risk metrics. (Risk metric= is the attribute of risk that is being measured, for example volatility). Another reason behind the widespread adoption of VaR was the introduction in 1985 by the Bank for International Settlements (BIS) of international bank capital requirements that allowed relatively sophisticated banks to calculate their capital requirements based on their own internal modes such as VAR. (Understanding1 Market, Credit and Operational Risk 1st edition)

In addition, Var has many applications in the financial field. It is a very important tool that is implemented in many functions in finance. In particular, Var is used in **risk management, financial reporting, financial control,** and **computing regulatory capital.** Moreover, Var is also used in stress tests, back testing, etc.

2.6.2 Definition of Var

Value at risk is a risk measure of the downside risk. In particular, it estimates how much a set of investments might lose, with a given probability for a given time period and normal market conditions. A more formal definition of Var is the following:

VaR is the maximum loss over a target horizon such that there is a low, prespecified probability that the actual loss will be larger. (Financial Risk Manager Handbook, 2nd Edition**)**

2.6.3 Parameters of Var (Financial Risk Manager Handbook, 2nd Edition)

To further specify Value at risk three things, need careful clarification:

- For the estimation of Var, a specific time horizon needs to be specified. This time horizon is called value-at-risk horizon.

- A quantile of loss should be prespecified.
- The currency in which the possible loss is denominated should be chosen.

Time horizon

The greater the time horizon, the greater the Var measure. This extrapolation depends on two factors, the behavior of the risk factors, and the portfolio positions.

To extrapolate from a one-day horizon to a longer horizon, we need to assume that returns are independently and identically distributed. This allows us to transform daily volatility to multiple-day volatility by multiplication by the square root of time. We also need to assume that the distribution of daily returns is unchanged for longer horizons, which restricts the class of distribution to the so-called "stable" family, of which the normal is a member. If so, we have:

$$\text{VAR}(T \text{ days}) = \text{VAR}(1 \text{ day}) \times \sqrt{T}$$

This requires:

- The distribution should not vary on the time horizon.
- The distribution should remain the same for various horizons.
- Innovations to be independent across days.

The choice of the appropriate time horizon depends on the characteristics of the portfolio. If the positions change quickly or if exposures change as prices change, increasing the horizon will create slippage in the Var measure. In addition, the choice of time horizon depends on the use of Var. If the purpose is to provide an accurate benchmark of the downside risk, the horizon should be relatively short. On the other hand, if the purpose of the calculation of Var is to avoid future financial problems a longer horizon is needed, in order to decide corrective actions. In practice, the time horizon should not be less than the frequency of losses or profits. Time horizon varies based on the nature of the firm or the financial institution.

Confidence Level

The higher the confidence level, the greater the Var measure. Changing the confidence level provides useful insight into the return distribution and the potential extreme losses. It is not clear, however, whether one should stop at 99%, 99.9%, 99.99% and so on. Each of these values will create an increasingly larger loss, but less likely. Another problem is that, as confidence level increases, the number of occurrences below VAR shrinks, leading to poor measures of large, but unlikely losses. With 1000 observations, for example, VAR can be taken as the 10th lowest observation for a 99% confidence level. If the confidence level increases to 99.9%, VaR is taken from the lowest observation only. Finally, there is no simple way to estimate a 99.99% VAR from this sample.

The use of Var can change the confidence level. In most cases, Var is a benchmark of the downside risk. If so, the consistency of the Var confidence level is needed.

Another use of Var is to decide how much capital is needed to avoid bankruptcy. In this case, a high confidence level of Var is advisable.

Another important point is that VaR models are only useful as far as they can be verified. This is the purpose of back testing, which systematically checks whether the frequency of losses exceeding VaR is in line with 1. For this purpose, the risk manager should not choose a value of that is too high. Picking, for instance, c = 99 99% should lead, on average, to one exceedance out of 10,000 trading days, or 40 years. In other words, it is going to be impossible to verify if the true probability associated with VAR is indeed 99.99 percent. For all these reasons, the usual recommendation is to pick a confidence level that is not too high, such as 95 to 99 percent.

2.6.4 Mathematical definition of Var

Value-at-Risk (VaR)). Let X be a random variable representing loss. Given a parameter $0 < \alpha < 1$, the α-VaR of X is

- $VaR\alpha(X) := \min \{c : P(X \leq c) \geq \alpha\}$.

- VaRα(X) is the minimum loss that will not be exceeded with probability α. VaRα(X) is the α-quantile of the distribution of X.
- VaRα(X) is the smallest loss in the $(1-\alpha) \times 100\%$ worst cases.
- VaRα(X) is the highest loss in the $\alpha \times 100\%$ best cases

2.6.5 Elements of Var Systems

In this section, we will focus our interest on the steps of a Var system.

- After collecting market data, a distribution of risk factors is chosen (empirical, normal or other).
- Collection of the portfolio positions and mapping them to risk factors.
- Choosing a Var method (Monte Carlo, etc.).

Portfolio Positions

The first step of the procedure is to choose portfolio positions. An important assumption is that portfolio positions need to be constant over the time horizon. Of course, Var in practice can be greater or lower than the anticipated. It depends on the management of investment opportunities. For example, traders and managers can take riskier decisions or enforce loss limits to reduce the risk.

Risk factors

Risk is present in securities. There are tens of thousands of securities available, but a much more restricted set of useful risk factors. Market factors affect portfolios, so the key is to choose the appropriate ones for the specified portfolio. The choice of risk factors is based on the composition of the portfolio. For example, for a fixed-income portfolio one bond market risk is inadequate, whilst from a more complex one, a greater number of factors need be included (Financial Risk Manager Handbook, 2nd Edition).

Var Methods (Financial Risk Manager Handbook, 2nd Edition)

The appropriate choice of the Var method depends on the nature of the portfolio. For fixed income portfolios, a linear method is advisable but if the portfolio consists of options, a nonlinear method should be used. The importance of risk management is clear since in order to make reasonable approximations they need to find with an appropriate measure of risk. There are a number of Var methods available:

All methods have a common base but then diverge in how they actually calculate Var. In general, there are three primary methods used for calculating Value at risk.

- Variance/Covariance Method
- Historical Simulation Method
- Monte Carlo Simulation Method

Variance/Covariance Method

Since Value at risk measures the probability that the value of an asset or portfolio will drop below a specified value in a particular time period, it should be relatively simple to compute if a probability distribution of potential prices can be derived. This method is relatively simple but is limited by the difficulty of deriving probability distributions.

The variance/covariance method assumes that daily price returns of assets follow a normal distribution. The first step of the method is to take each asset in a portfolio and map it to simpler instruments. Following, each financial asset is stated as a set of positions in the standardized market instruments. Once the standardized instruments that affect the asset or assets in a portfolio been identified, we have to estimate the variances in each of these instruments and the covariances across the instruments in the next step. In practice, these variance and covariance estimates are obtained by looking at historical data. They are key to estimating the VaR. Value at risk, is a function of variance and the desired confidence level. The volatility of returns can be calculated in two ways: a) Simple Moving Average or b) Exponentially Moving Average.

The difference between these methods is that the first put equal weights to all returns in the series while the second-place greater importance to returns with recent duration. In the final step conceptually, Value at Risk (VaR) for a specific confidence interval is then calculated by multiplying the standard deviation by the appropriate normal distribution factor.

The accuracy of this method depends on the accuracy of the assumption when starting the procedure.

Historical Simulation Method

Historical Simulation is one simple way to calculate Value at Risk for many portfolios. This approach requires fewer statistical assumptions for market factors. In this approach, the VaR for a portfolio is estimated by creating a hypothetical time series of returns on that portfolio, obtained by running the portfolio through actual historical data and computing the changes that would have occurred in each period. Historical simulation is a non-parametric approach since the returns are not subjected to any functional distribution. Var is estimated through data without deriving parameters or making assumptions about normality. The idea behind this method is that past returns are indicative of the future returns of an asset or portfolio. Furthermore, all returns in this method are treated equally.

The simplification of calculating Var by this Model is out of the question, but some downsides are raised. To begin with, historical data might not reflect the current situations of the market thus, incorrect estimations may occur. Moreover, trends in the data are not considered.

Monte Carlo Simulation Method

The approach used in this section to calculate Value at risk focuses on probabilities of losses exceeding a specified value rather than the entire distribution. The process of Monte Carlo Simulation Method begins as the Variance/Covariance Method where we identify the market's risks that affect the asset or assets in a portfolio and convert individual assets into positions in standardized instruments. After identifying and converting the assets, probability distributions for each of the market risk factors are specified and determine how these market risk factors move together. The absence of normality is an important downside of this method, the flexibility of choosing different distributions for each variable is extremely useful. In addition, subjective judgment can be brought to modify these distributions. Once the distributions are specified, the simulation process starts. In each run, the market risk variables take on different outcomes and the value of the portfolio reflects the outcomes. After repeated series of runs, the distribution of the portfolio's values is estimated that can be used to calculate Value at risk. Monte Carlo is effective, when the probability distribution is correctly estimated. Without a doubt, it is a more sophisticated and difficult method, since it is difficult to estimate distributions for a lot of variables, and a lot number of simulations are needed. The benefits might counterbalance the disadvantages since no assumptions need to be held, subjective judgment can be considered, and its forecasting ability is stronger.

Var system

To conclude, a Var system alone does not ensure effective protection against market risk. It needs to be used in combination with limits on notionals and on exposures and, in addition, should be supplemented by stress tests.

2.6.6 Conclusions

Without a doubt, Value at risk is a very useful tool to measure risk and has acquired a strong following in the risk management community. Var is easy to be understood and is often available in financial software. Furthermore, the adaptation of Var in stocks, bonds, currencies, derivatives, enables their comparison through Var. On the other hand, downsides of Var include that Var does not describe losses on the left tail, it just indicates the probability of that event occurring. Moreover, Var is measured with some error. Different methods of estimation may lead to different Value at risk results, depending on time horizon, statistical assumptions etc. In addition, Var suffers from being unstable and difficult to work with numerically when losses are not "normally" distributed. (R. Tyrrell Rockafellar, Stanislav Uryasev,2002) Despite its limits, VaR is one of the most popular methods used to measure and prevent the manifestation of market risks. In an attempt to capture this risk, we used non-parametric models, based on simulation to calculate VaR, but also parametric models, with their advantages and disadvantages, but also models for calculating the volatility and correlations. In order to implement a successful VAR estimation, the accuracy of this depends on the portfolio return distribution. Although the normal distribution is the easiest to use in practice, it may lead to an underestimation of the risk and capital allocation, because in the reality the data series have elongated tails corresponding to extreme market movements. It is appropriate to identify specific Value at Risk models for each portfolio, but we must never forget that each of these models has its own advantages and disadvantages, that relate to the probable maximum loss to a certain degree of confidence. (Advantages and Limitations of Var Models used in Managing Market Risk in Banks,2011)

2.7.1 Conditional Value at Risk

In this section, our interest focuses on another risk assessment method called Conditional Value-at risk. Conditional Value-at-risk is a new approach to optimizing or hedging a portfolio of financial instruments. This method of risk estimation is used broadly in the financial field. It was introduced after the understanding that Var does not consider the magnitude of potential losses. Conditional Value at risk, measures the amount of tail risk a portfolio has. (Tail risk= returns will move more than three standard deviations from the mean). In other words, CVaR is defined as the mean losses excess of Var. CVaR measures approximately or exactly; based on the case, the average of some percentage of the worst-case loss scenario. (Sergey Sarykalin, 2008). For continuous distributions, the risk measure is also known as Mean Excess Loss, Mean Shortfall, or Tail at Value at risk.

2.7.2 Conditional Value at risk as Coherent Risk Measure

In the financial field there are many ways risk can be defined. A way introduced by theoreticians to define risk measures is by describing some properties a risk measure may or may not have. A risk measure that has the following four properties is said to be coherent.

For the definitions of all axioms, X and Y are random variables representing loss, $c \in R$ is a scalar representing loss, and ρ is a risk function, i.e. it maps the random variable X (or Y) to R, according to the risk associated with X (or Y).

Properties

- **Monotonicity (Higher losses mean higher risk)**

A risk measure ρ is monotone, if for all X, Y: $X \leq Y \Rightarrow \rho(X) \leq \rho(Y)$.

- **Translation Equivalence (Increasing (or decreasing) the loss increases (decreases) the risk by the same amount.)**

A risk measure ρ is translation equivariant, if for all X, c: $\rho(X + c) = \rho(X) + c$

- **Subadditivity (Diversification decreases risk)**

A risk measure ρ is subadditive, if for all X, Y: $\rho(X + Y) \leq \rho(X) + \rho(Y)$.

- **Positive Homogeneity (Doubling the portfolio size doubles the risk.)**

A risk measure ρ is positively homogeneous, if for all X, $\lambda \geq 0$: $\rho(\lambda X) = \lambda\rho(X)$.

Conditional Value at risk satisfies all the above propositions. Conceptually, it is described as a coherent risk measure which is a desirable type of risk. Its wide acclaim is justified for the above reason as opposed to Value at risk which is not considered as a coherent risk measure since it does not satisfy subadditivity.

2.7.3 Mathematical Definition of Conditional Value at risk

Conditional Value-at-Risk (CVaR) in the continuous case, Let X be a continuous random variable representing a loss. Given a parameter $0 < \alpha < 1$, the α-CVaR of X is

$$CVaR\alpha(X) := E\,[X \mid X \geq VaR\alpha(X)]$$

In particular, in the continuous case of conditional value at risk, the Cvar at a given confidence level is the expected loss given will be greater than the Var at that level.

On the other hand, for distributions with possible discontinuities, it has a more subtle definition and can differ from either of those quantities, which for convenience in comparison can be designated by $CVaR^+$ and $CVaR^-$, respectively.

$CVaR^+$ has been called mean shortfall.

$CVaR^-$ is called tail Var.

$CVaR^+$ and $CVaR^-$ are simply the upper and the lower CVAR limits respectively.

Generally:

$$CVaR^- \leqslant CVaR \leqslant CVaR^+$$

It is worth noticing that while CVaR is considered a coherent risk measure $CVaR^+$ and $CVaR^-$ are not considered coherent. The lack of coherence of $CVaR^+$ and $CVaR^-$ in the presence of discreteness does not seem to be widely appreciated, although this shortcoming was already noted for $CVaR^-$ by the authors Artzner et al. (1999). They suggested, as a remedy, still another measure of risk which they called ‘‘worst conditional expectation’’ and proved to be coherent. That measure is impractical for applications, however, because it can only be calculated in very narrow circumstances. In contrast, CVaR is not only coherent but eminently practical by virtue of our minimization formula for it. That formula opens the door to computational techniques for dealing with a risk far more effectively than before. (R. Tyrrell Rockafellar, Stanislav Uryasev, 2002).

For dealing with risk effectively in distributions with the presence of discontinuity, CVaR can be viewed as a weighted average of Var and CVAR$^{+.}$ (with the weights depending, like these values themselves, on the decision x).

2.7.4 Using CVaR in portfolio optimization, Rockafellar and Uryasev Model

Based on the fact that variance in sot a coherent risk measure since it is not monotone and tail losses arising from skewed loss distributions are not considered, CVaR optimization problem has received wide acclaim because it ignores the above problems. It is preferable for a risk manager to optimize a portfolio with regard to CVaR. Rockafellar and Uryasev proposed a linear program to optimize the CVaR of a portfolio. They also proved that under certain conditions the CVaR optimization will give the same optimal portfolio as the minimum variance optimization. The rest of this section introduces its notation and presents their results.

To derive later results, Rockafellar and Uryasev labeled the cumulative distribution function of losses Ψ (x, c), so that for any given decision x ∈ S, random asset losses r ∈ R n, and loss distribution X (x, r)

$$\Psi(\mathrm{x}, c) = F_X(c) = P(X(\mathrm{x}, \mathrm{r}) \leq c) \qquad \text{in the general case, and}$$

$$\Psi(\mathrm{x}, c) = F_X(c) = \int_{\mathrm{r}:X(\mathrm{x},\mathrm{r})\leq c} p(\mathrm{r})\,d\mathrm{r} \qquad \text{in the continuous case,}$$

The function Ψ (x, c) can be interpreted as the probability that the losses do not exceed threshold c. Continuing with the notation of Ψ (x, c) as the threshold of losses, VaRα and CVaRα of an investment decision x can be then written as

$$\mathrm{VaR}_\alpha(\mathbf{x}) = \mathrm{VaR}_\alpha(X(\mathbf{x},\mathbf{r})) = \min\{c : \Psi(\mathbf{x}, c) \geq \alpha\}, \text{ and} \tag{1.1}$$

$$\mathrm{CVaR}_\alpha(\mathbf{x}) = \mathrm{CVaR}_\alpha(X(\mathbf{x},\mathbf{r})) = \mathbb{E}_\mathbf{r}[X(\mathbf{x},\mathbf{r}) \mid X(\mathbf{x},\mathbf{r}) \geq VaR_\alpha(\mathbf{x})]. \tag{1.2}$$

Rockafellar and Uryasev characterized Equation 1.1 and Equation 1.2 in terms of a function

$$\phi_\alpha(\mathbf{x}, c) := c + \frac{1}{1-\alpha}\mathbb{E}[(X(\mathbf{x},\mathbf{r}) - c)^+], \tag{1.3}$$

where E [·] is the expectation and (t) + = max {0, t}. Based on Equation 1.3, they formulated Theorem 1.

Theorem 1

As a function of c, φα (x, c) is convex and continuously differentiable. The CVaRα of the loss associated with any x ∈ S can be determined from the formula:

$$\mathrm{CVaR}_\alpha(\mathbf{x}) = \min_{c \in \mathbb{R}} \phi_\alpha(\mathbf{x}, c). \tag{1.4}$$

Furthermore, let Φ ∗ α(x) := arg minc φα (x, c), i.e. Φ ∗ α(x) is the set of minimizers of φα (x, c). Then:

$$\mathrm{VaR}_\alpha(\mathbf{x}) = \min\{c : c \in \Phi^*_\alpha(\mathbf{x})\}. \tag{1.5}$$

And following from Equation (1.4) and Equation (1.5), the following equation always holds:

$$\mathrm{CVaR}_\alpha(\mathbf{x}) = \phi_\alpha(\mathbf{x}, \mathrm{VaR}_\alpha(\mathbf{x})). \tag{1.6}$$

Based on Theorem 1, Rockafellar and Uryasev stated another theorem, which is useful for the computational calculation to find a CVaR optimal portfolio x ∗ ∈ S

Theorem 2

Let S be a convex set of feasible decisions x and assume that X (x, r) is convex in x. Then minimizing the CVaRα of the loss associated with decision x ∈ S is equivalent to minimizing φα (x, c) overall (x, c) ∈ S × R, in the sense that

$$\min_{\mathbf{x}\in S} \mathrm{CVaR}_{\alpha}(\mathbf{x}) = \min_{(\mathbf{x},c)\in S\times\mathbb{R}} \phi_{\alpha}(\mathbf{x},c)\,, \tag{1.7}$$

where, moreover, a pair (x ∗, c∗) achieves the right-hand side minimum if and only if x ∗ achieves the left-hand side minimum and c ∗ ∈ Φ ∗ α(x). Therefore, in circumstances where the interval Φ ∗ α(x) reduces to a single point (as is typical), the minimization of φα (x, c) produces a pair (x ∗, c∗) such that x ∗ minimizes the CVaRα and c ∗ gives the corresponding VaRα.

Theorem 2 not only gives a way to express the CVaR minimization problem in a tractable form, but also allows to calculate CVaRα without having to calculate VaRα. More remarkably, finding the CVaR by using Theorem 2, gives the corresponding VaR as a by-product.

Applying Theorem 2 with Equation 1.3, the investment decision x that minimizes the Conditional Value-at-Risk of a portfolio at the confidence level α can be expressed as

$$\min_{\mathbf{x}\in S} \mathrm{CVaR}_{\alpha}(\mathbf{x}) = \min_{\mathbf{x}\in S, c\in\mathbb{R}} \left(c + \frac{1}{1-\alpha}\mathbb{E}\left[(X(\mathbf{x},\mathbf{r}) - c)^{+} \right] \right). \tag{1.8}$$

This approach shows how a portfolio can be optimized with regards to minimizing the portfolio's tail risk.

2.7.5 Conclusions about Conditional Value at risk

- Conditional Value at risk has superior mathematical properties than other risk measures such as Var. Cvar as mentioned before, is coherent. Furthermore, the Cvar of a portfolio is a continuous and convex function with respect to optimization variables.
- Cvar can be optimized and constrained with convex and linear programming methods.
- Cvar accuracy depends on the accuracy of tail modeling, since it is a tail loss.
- Cvar is a strong competitor of standard deviation and variance.
- A downside of Cvar is that is more prone to estimation errors.
- Minimizing the CVaR of a portfolio is closely related to minimizing the Var of a portfolio.
- When applying minimization of CVaR in one portfolio, Var also reduces since CVaR ≥ VaR.
- More volatile investments require more careful consideration. CVaR gives a better picture of this kind of investments since it gives a whole picture of the risks.
- Conditional value at risk corresponds to high losses, but not high profits because it aims at reshaping one tail of the loss distribution.
- Cvar risk management constraints can be used in various applications.
  (Pavlo Krokhmal, Jonas Palmquist, and Stanislav Uryasev, 2005)
- Cvar is estimated by implementing the same methods as Value at risk approach.

2.8.1 Chapter Epilogue

In this chapter we focused on analyzing some of the most important management risk methods for financial instruments and portfolios. Portfolio theory first introduced by Markowitz is a core subject of the financial field and a lot of researchers have described their interest in it. As mentioned earlier, portfolio theory focuses on the estimation of risk a portfolio bears and, in this chapter, we described a number of methods to carefully estimate risk and manage risk. All methods are characterized by benefits and downsides, and the selection of the right method depends on the case and the investor's preferences and targets.

Chapter 3

3.1.1 International Diversification

In this chapter, our study goes beyond domestic markets to examine issues of international and extended diversification. International investing may be viewed as a generalization of the portfolio treatment and management with a larger variety of assets and variables. The investor faces the same issues as investing in a domestic market: security analysis, security selection, and asset allocation, but with further problems that are absent in domestic markets. These problems involve exchange rate risk presence, restrictions of capital flow across national boundaries, political risk and country-specific regulations, and different accounting practices across different countries. Thus, this chapter examines the prospects of portfolio theory with regard to international generalization and primarily focuses on the cornerstone of financial theory: the diversification. Global diversification offers many opportunities for investors to better achieve a return-risk balance. Furthermore, under the scope of our analysis will be how political and exchange risk affect the selection of international investments. Another important issue that international diversification considers, is the correlation across markets and assets between other assets. (Bodie, Kane, Markus, 2005)

3.1.2 International Diversification Definition

To continue our analysis, international diversification needs a proper definition for a better understanding of the purposes of our study. The international diversification refers to the attempt to reduce risk by structuring portfolios that combine assets from different markets. By diversifying across nations, investors aim to reduce the variability of returns selecting financial instruments and markets that they are not perfectly correlated. In particular, investors structure an international portfolio that focuses on international markets rather than domestic ones. The purpose of international diversification is the international grouping of assets which combines instruments in order to maximize investor's return by carefully selecting markets that would react differently to the same event. (Bodie, Kane, Markus, 2005, Clifford S. Asness, Roni Israelov, John M. Liew 2011).

3.1.3 Types of Markets

International diversification is achieved by structuring an international portfolio of assets, to give the investor exposure to different types of markets. In this section, different types of markets are examined.

1. <u>Developed Markets</u>

In general, developed markets are less risky than others. The developed markets involve countries with well-developed capital markets and the economy. Furthermore, they are described by high income and market efficiency. In developed markets countries with openness to foreign ownership and ease of capital movement are involved. Moreover, regulatory authorities actively monitor the market, there are fewer restrictions and free and well-developed equity and foreign exchange markets are held.

2. <u>Emerging Markets</u>

Emerging markets are riskier than developed markets. They bear the same characteristics with the developed markets, but their performance does not satisfy the obligation of characterization of a developed market. Emerging markets could be developed markets in the past and due to financial difficulties, they no longer bear the definition of developed markets or they might become developed in the future.

3. <u>Frontier Markets</u>

This type of markets is the one with the highest level of risk. In general, frontier markets involve countries that have signs of development, but their poor signs of performance do not enable them to become emerging markets. This term describes markets with riskier, smaller and more illiquid assets.

An investor can choose among these types of markets to place his funds. The investing decisions vary depending on the preferences of the contextual investor. Some investors prefer to invest in developed countries, to ensure a specific level or risk and return and others tend to invest in emerging or frontier markets to achieve higher levels of return.

3.1.4 Risk factors in International Investments

It is clear that international investments give investors a wide range of opportunities, but it is worth noticing that these opportunities do not come free of risk. The risk factors we are going to analyze, are specific to international investments.

Exchange rate risk

Exchange rate risk affects international investors. This risk is present when losses occur due to currency fluctuations. A more proper definition of exchange rate risk is: *The risk that exchange rate risk changes affect the value of foreign investment and liabilities denominated in foreign currencies*. (Anthony Saunders, Marcia Millon Cornett, 2014).

Exchange rate risk is further analyzed in the previous section, but it should be noticed that exchange rate affects differently investors, since they tend to invest in different countries and exchange rates are not perfectly correlated across countries.

Country- Specific Risk

Countries are not always described by the same types or levels of risk. Every country bears different types of risks at a different level; thus, further analysis of dangers is needed. It is difficult to receive information about assets held in a foreign country and expensive at the same time. Moreover, the probability of a false estimation is relatively high. A way to assess a country's risk is examining the political risk the country has. Political risk is the danger that an investment will perform losses because of the country's political instability. Other approaches to estimate the country-specific risk is examining at the same time the financial and economic risk. Financial risk is a more general term that refers to the loss of money. Economic risk refers to the danger that an investment will lose its value because of changes in the macroeconomic conditions of the country. To assess the country-specific risk, all the above terms are used and given values in order to help the investor choose among countries. Credit rating agencies provide investors useful information regarding markets and therefore enables them to choose wisely when faced with a variety of options as to which country they will invest.

Transaction Cost

Transaction cost can be raised when international transactions occur. Transaction costs involve management fees, taxes, commissions, etc. Fortunately, the trend of the markets to become more and more synchronized and open to foreign investors, has reduced transaction costs significantly.

3.1.5 Benefits of International Investments

After analyzing the dangers an international investment bears, we focus on the benefits of international investment. The benefits of international investments though, differ depending on the size or nature of the company, and among investors as well. In this section, we will provide a generalization of benefits that are raised by international investments.

Diversification of funds.

The most important benefit of the realization of global investments is the diversification of funds. This term has its own benefits also and we will further analyze it in a later section. When undertaking a global investment, an investor will have alternative sources of stability.

Variety of options

The choices under the notion of a global market are almost infinite. An investor is not limited by a given amount of choices and can make decisions among a wide variety of financial instruments.

Investment Growth on an International level

Investing internationally has the potential for more growth investment which translates to higher return potential in overseas investments.

Currency Diversification Strengthens Portfolios

Much like international investing gives portfolio safety in numbers as opposed to having all assets invested in one country's economy, so do currency differences from country to country.

A reduction in taxes.

A reduction in taxes takes place when countries across the world offer attractive tax incentives to foreign investors. (Otherwise known as tax heavens). These incentives are given to strengthen the investing environment of the country and attract foreign wealth.

New technologies are developed.

Innovative firms tend to invest abroad. New technologies are brought to countries by investing overseas.

3.1.6 Benefits of International Diversification

We proceed our analysis by addressing the benefits of international diversification.

Overcoming home country bias.

Most investors are reluctant to invest beyond the domestic market since they tend to be ensured by the known environment of their country. This tendency is known as home country bias and leaves investors with little exposure to a world with a variety of potential investments. By investing internationally, diversification is achieved as the exposure increases, thus, home country bias problem is solved by diversifying internationally.

Income portfolios receive a better return.

The correlation coefficients between a stock index of one country and bond portfolios of another are very low, suggesting that income portfolios that are balanced between stocks and bonds would greatly benefit from international diversification. (Bodie, Kane, Marcus,2005)

Capitalize on emerging markets.

Sophisticated investors who seek big potential upsides often seek out international investments—especially in emerging markets—which can be purchased at a discounted cost with measured risk.

Counterbalance domestic negative effects

When holding a portfolio with international positions of assets that are not correlated (or negatively correlated) with each other, a domestic factor that causes losses on domestic assets will not reduce portfolio's value since other assets will remain unaffected.

Benefit from exchange rate fluctuations

Investing globally is considered also beneficial when the exchange rate changes in favor of the portfolio. Exchange rate fluctuation can increase a portfolio's value. Moreover, currency diversification is ensured, thus lower currency volatility and therefore lower risk levels are achieved.

Higher profit

The prospect of higher profit is achieved when a favorable event occurs in a foreign market under the assumption that the investor holds a global portfolio. (Nikolas Topaloglou, Hercules Vladimirou, Stavros A. Zenios, 2002)

3.1.7 Caveats of International Diversification

Diversification is one of the most important terms in financial theory. In the context of global investing under specific assumptions, diversification implies that international diversification should produce an adjusted return superior than one of a domestic-constructed portfolio. Yet, there are some caveats when an investor invests internationally.

- Given the recent market downturn, it has been observed an increasing correlation between international markets. Furthermore, it has been noticed that correlation is even higher when market downturns occur. Conceptually, diversification is weakest when the need of it is even greater.
- Markets exhibit skewness (measure of asymmetry of the probability distribution of a real random variable) and tendency to crash together. Thus, the ability of portfolio to protect investors towards risk is impaired, especially in the short-term horizon.
- Questions have been raised in investing overseas, especially in developed countries. Through globalization, the world is getting smaller by far and thus there might be no diversification benefit from investing in developed markets.

The correlation between developed countries is relatively high, so the diversification is not easily achieved. (Ken Slow, 2017)

- During pre-crisis periods, investors have no overexposure to relatively uncorrelated assets. (Robert Vermeulen,2011)

Without a doubt, the performance of markets during the recent crisis has been disturbing for investors, companies, governments and researchers. On the other hand, it should be noticed that an investment decision usually involves a longer-term horizon. Long term results tend to estimate economic performance better than the short run. In the long run, international investments enhance investor's welfare and diversification through investing globally is achieved efficiently.

3.2.1 Chapter Epilogue

The analysis conducted in this chapter involved the accurate definition of international diversification, and the key elements that determine it. All types of markets were discussed, alongside with benefits and caveats of global investing. Furthermore, the benefits of international diversification were analyzed as well in an analytical manner.

Without a doubt, international investing and the process of diversifying internationally are complex issues that need careful treatment. A correct definition of the risks and benefits in the process is extremely important to ensure their effectiveness.

Chapter 4

4.1.1 Empirical Analysis

This chapter is focused on our empirical analysis of international diversification. Since global investments increase the variety of investment decisions and broaden the scope of diversification, it is considered a very interesting issue. Our analysis focuses on constructing optimal portfolios, that are characterized by a good balance between risk and return.

In the process of international portfolio management several decisions and consideration are in order. The first step is the decision to allocate funds in specific markets and in particular securities. An initial allocation of funds is decided in markets. The next step of portfolio management is the appropriate combination of securities in each market. This decision is made through specialists who manage the asset mix in the portfolio. The performance of the assets and the portfolio is compared to a certain benchmark. It is worth noticing that the major risk under consideration is the market risk all assets are exposed to, and currency risk is considered a subordinate type of risk.

The aim of our study is the development of a stochastic framework that reflects the key decisions in portfolio management: the optimal allocation of funds under the presence of market and currency risk. Furthermore, by investing internationally we aim to achieve an effective balance between total exposure and expected return.

4.2.1 Introduction

Portfolios

For the purposes of our study, we consider portfolios that consist indices that are denominated in different currencies. The key risks are the market risk of positions in stocks and bonds indices, and the currency risk of foreign investments. Both types of risk can be mitigated through diversification.

Countries

Portfolios were constructed by choosing to invest in three major countries. As mentioned earlier, the countries involved in our analysis are the *United States, United Kingdom and Japan.* The aspect of study is under the scope of an American-based investor. Conceptually, foreign investments are considered taking positions in United Kingdom and Japan indices.

The choice of the above countries is based on the following:

- *Availability of data.*
- *No strong correlation among countries in order to achieve diversification in portfolios.*
- *Representative sample of the world's indices.*
- *Economic development and relative stability of the countries.*
- *Choice of countries with different currency progress and relative value.*

Indices

In each market certain indices were chosen. In particular, large and small capitalization stock indices were used in addition to short term and long-term bonds indices. The indices considered are the following:

United States

i. S&P 500 Composite – Price Index (US1)
ii. S&P 600 Small Cap-Price Index (US2)
iii. US Citigroup WGBI US 1-3Y- Price Index (US3)
iv. US Citigroup WGBI US 7-10Y- Price Index (US4)

Japan

i. Topix Large 70- Price Index (JP1)
ii. Topix Small -Price Index (JP2)
iii. Citi Japanese Government Bond 1-3Y- Price Index (JP3)
iv. Citi Japanese Government Bond 7-10Y Price Index (JP4)

United Kingdom

i. FTSE 100-Price Index (UK1)
ii. FTSE Small Cap -Price Index (UK2)
iii. UK Citigroup WGBI UK 1-3Y-Price Index (UK3)
iv. UK Citigroup WGBI UK 7-10Y-Price Index (UK4)

For each country, indices with similar characteristics were chosen. The indices under consideration are a representative part of the market for each country. Portfolios are constructed by choosing to invest in United States, United Kingdom and Japan among the indices. The investor chooses to allocate income between stock and bond indices for each country based on the performance of each asset and the investor's tolerable level of exposure.

Currencies

The approach in our study is by a US investor who chooses to allocate income in the prementioned countries. A major issue of the study is the currencies involved. Our reference currency is US dollar. Since investment decisions involve United Kingdom and Japan, currencies under consideration, are the Japanese Yen and the British Pound. Foreign investments involve currency risk, which refers to the probability that the domestic currency will deteriorate in relation to other currencies. The choice of foreign investments involves currency risk parameter, since the probability of losses or gains can be affected by the movement of exchange rates. Another key fact of the study is the correlation between currencies. If the correlation is low, a negative effect on one currency will not affect the other. In next section, we will provide a more holistic view of the exchange rates involved in our study.

Types of investors

Portfolio management can be viewed differently depending the type of investor. In our analysis, we consider three types of investors who choose to allocate funds in a different way. Specifically, empirical analysis is under the scope of the following types of investors:

1. *Risk averse* is considered a type of investor who has high-risk aversion. These investors avoid investment decisions that involve stocks with high volatility and prefer to invest in safer securities. In finance, risk averse investors are considered investors who, when faced with two investment options with the same level of expected return, they choose the investment with the lower risk. Risk averse tend to combine assets in order to achieve diversification and achieve lower levels of exposure.

2. *Risk neutral.* This category involves investors that their decisions are not affected by the level of uncertainty. A risk neutral party is indifferent between investments that yield the same return, even if the level of risk is different. In portfolio theory, a risk neutral chooses to invest in securities that yield higher return even if their risk is higher. Risk neutral tend to construct portfolios with higher level of return, but their exposure is higher. Contrary to risk averse, they

take investment decisions with regard to expected return, not levels of exposure. Risk neutrals are completely insensitive to risk.

3. *Risk lovers.* In economics and finance risk lovers are considered investors who focus their interest in gaining higher levels of risk. Investors who are considered risk lovers are willing to be exposed to higher levels of uncertainty. They tend to invest in extremely risky investments (such as stocks with high volatility) in order to receive higher level of return. In addition, risk lovers do not diversify their portfolio, their main focus is to achieve higher levels of return, even if exposure increases significantly.

Every type of investor has a different approach when constructing a portfolio. The choice between assets in order to construct an optimal portfolio will be viewed in a different manner and therefore the results will not be the same. Portfolio management may differ based on the preferences of each investor, and in our analysis, we construct portfolios for each type of investor in order to view the problem in a more integrated manner.

4.3.1 Data analysis

As mentioned previously, the problem of portfolio management in our analysis involves indices denominated in United States, United Kingdom and Japan. In this section, we focus our interest in analyzing the statistical characteristics of each index and explaining their importance. Furthermore, it is worth mentioning that exchange rates are considered a very important aspect of our study, since the allocation of funds is affected not only by the performance of assets but also by the movement of exchange rates under question. Thus, our description will involve exchange rates.

4.3.2 Descriptive Statistics

Descriptive Statistics can help describe data collected in order to address the international optimization portfolio issue. Statistical measures help to quantify key parameters s in portfolio theory such as the expected return and risk. The first step of our study is to calculate the monthly asset returns and exchange rate variations. Based on the asset returns and variations, we computed several statistical measures. We address data by estimating the most important statistical measures and their significance. In particular, for each index and exchange rate we estimated:

- *Mean.* Reflects the expected level of return. Conceptually, higher mean, higher return level.

- *Standard Deviation.* This measure reflects the risk of investment decisions. A high level of standard deviation implies a high volatility in assets and portfolios, thus investments with high standard deviation are considered risky.

- *Kurtosis* is also a measure of risk. Specifically, it is a statistical measure to describe the distribution. Kurtosis is used to value extreme values in both tails of the distribution. For investors, high kurtosis of the return distribution implies that the investor will experience occasional extreme returns (either positive or negative), more extreme than the usual + or - three standard deviations from the mean that is predicted by the normal distribution of returns. This phenomenon is known as *kurtosis risk*. In normal distribution, kurtosis takes the value of 3. When assets exhibit kurtosis exceeding the level of three, they are considered riskier. Securities with lower levels of kurtosis, tend to be less risky.

- *Skewness.* Skewness is a statistical measure that estimates the asymmetry of the probability distribution of a real random variable. It can take both negative, and positive values. A zero value implies a symmetry in the distribution. In finance, it is considered a measure of risk. A negative skewness implies that returns will be most frequently less that the average, while a positive skewness implies that returns will be more than anticipated.

- *Correlation Coefficient*. In our study we used the Pearson correlation coefficient to study the co-movement of assets and exchange rates. Pearson correlation measures the degree and direction of correlation between assets. A value of 1 implies a perfect linear relationship, a value of -1 implies a perfect linear negative relationship while zero implies no relationship between variables. This measure is the cornerstone of financial field since through low levels of correlation, diversification can be achieved.

We present the statistics of asset returns and the exchange rate appreciations.

Table 4.1

Sample statistics of monthly local returns of assets and monthly appreciation of exchange rates over period 1/1994-5/2018.

Statistics of monthly returns of assets

	Mean(%)	Standard Deviation(%)	Kurtosis	Skewness	Jarque Bera Statistic
US1	0,6887618	4,14759877	1,35759253	-0,6863825	2,585826337
US2	0,927209	5,1798155	1,67023554	-0,5365863	1,035937934
US3	0,2819214	0,43944386	0,9170503	0,565598515	2,824094433
US4	0,4632642	1,80071921	1,32256077	0,085965543	0,04231044
JP1	0,2929033	5,21875549	0,63291908	-0,29901372	1,01932707
JP2	0,2660243	5,43745357	0,14223928	-0,01210735	0,002435877
JP3	0,0680237	0,25583358	9,67659232	1,442645377	188,7704548
JP4	0,2689475	1,16765853	6,20029041	-0,65109899	8,834418271
UK1	0,3528463	3,88652499	0,59948395	-0,57109737	3,824147939
UK2	0,5073941	4,7254481	4,93750689	-0,31538954	0,759777123
UK3	0,3505315	0,45857639	0,88534556	0,481126332	2,106212559
UK4	0,5102463	1,5738661	0,39021544	-0,13610012	0,256703289
Yen to Dollar	0.051554691	3.153110931	3.244975516	0.640035888	0.07503267
Pound to Dollar	-0.009327834	2.420954574	1.282898656	-0.342844833	1.057751041

* The Jarque–Bera statistic has a χ^2 distribution with two degrees of freedom. Its critical values at the 5% and 1% confidence levels are 5.99 and 9.21, respectively. The normality hypothesis is rejected when the Jarque–Bera statistic has a higher value than the corresponding critical value at the respective confidence level.

**Normality hypothesis at 5% confidence level holds for all assets and exchange rates except JP3.

***Normality hypothesis at 1% confidence level holds for all assets and exchange rates except JP3 and JP4.

Table 4.1 lists the corresponding statistics of asset returns and exchange rate variations. A key observation of the estimation of statistics is that most variables exhibit high levels of standard deviation in relation to their means. Conceptually, the volatility of them is high. Stock indices tend to exhibit the highest levels of standard deviation, as one can anticipate. An interesting finding is that although bonds are considered less volatile, their standard deviation is considerable. Long term bonds indices are characterized by higher levels of risk (standard deviation) than short term bonds indices. Furthermore, the stock index with the highest level of volatility is the Topix Small Price Index in Japan, while the stock index with the highest level of expected return is S&P 600. Comparing stock indices, we can argue that the stock index with the lowest volatility is FTSE 100 in United Kingdom. With regard to bond indices, the less volatile option is the short-term bond index in Japan.

Another key observation is that some indices listed in Table 4.1 tend to exhibit excess kurtosis. Excess kurtosis implies heavier tails than normal distribution. Specifically, among indices with high levels of excess kurtosis were the bond indices in Japan and the small capitalization stock index in United Kingdom. Normality holds for all indices except Citi Japanese Government Index (1-3Y) in 5% and 1% significance level, and Citi Japanese Government Index (7-10Y) at 1% significance level.

Focusing our interest in exchange rates, one of the key findings is that both of them tend to hold the normality assumption since they do not exhibit high levels of excess kurtosis. From Table 4.1, it is observed that British Pounds' appreciation rates exhibit a negative mean. Negative Mean implies that most appreciation rates under the period we examined, are negative. Thus, the value of pound in relation to the US dollar deteriorates under the examination period. However, comparing the exchange appreciation rates (British Pound to US dollar, Yen to US dollar) it is clear that the appreciation rates of Pound to Dollar exhibited lower levels of standard deviation and kurtosis. Since both statistical measures are used in finance as measures of risk, it is clear than the exchange rate with lower levels of each, is less risky. We conclude that British Pound is considered a less volatile currency but based on the mean and the measure of skewness, its value is becoming lower as the study continues.

4.3.3 Correlations

One of the key aspects of portfolio theory is correlation. In previous chapters we mentioned the concept of correlation and its significance in finance. In this section, we will present correlations between indices and exchange rates we used for the purposes of our study. Since our study focuses on international diversification, correlation is a major subject as it can affect the performance between assets and the total portfolios' structure and return. The aim is to examine the correlation between indices returns, exchange rates appreciations and correlation between indices' movements with exchange rate variations.

Table 4.2

Correlation between monthly exchange rate movements over period 1/1994-5/2018.

	MONTHLY VARIATION OF YIEN TO DOLLAR	MONTHLY VARIATION OF POUND TO DOLLAR
MONTHLY VARIATION OF YIEN TO DOLLAR	1	,100
MONTHLY VARIATION OF POUND TO DOLLAR	,100	1

Table 4.2 illustrates the correlation between the exchange rates appreciation over the prementioned period. Correlation is observed to be positive, which implies that exchange rates tend to change in the same direction. A positive or a negative effect on one of the exchange rates, will affect the other, in the same direction. The degree of correlation, however, is not considered high. Furthermore, the statistical significance of their correlation rejected.

Table 4.3

Correlation between indices returns and exchange rate movements over period 1/1994-5/2018.

Correlations

	Returns YEN TO US	Returns UK TO US
Returns US1	-0.041	,212**
Returns US2	-0.098	,195**
Returns US3	,236**	0.016
Returns US4	,250**	-0.084
Returns UK1	-0.043	-0.033
Returns UK2	-0.102	0.057
Returns UK3	,185**	-,236**
Returns UK4	,156**	-,247**
Returns JP1	-,276**	,185**
Returns JP2	-,229**	0.103
Returns JP3	0.016	-0.022
Returns JP4	0.038	-0.003

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

Table 4.3 presents the correlations between the return of each index with the movement of exchange rate. However, not all correlations are statistically significant, thus we will focus on the correlations that are important for our study. Observing Table 4.3 it is clear that Returns of the Japanese large capitalization index (JP1) are negatively related to the exchange rate of Yen to Dollar and positively related to Pound to Dollar. This result can be explained. When yen to dollar appreciates, less yen are needed to buy one US dollar. This fact enables Japanese investors to take investment activities in the US, since the currency movement is in their favor. Conceptually, they will reduce investment opportunities in Japan. Another approach to this issue, is when a country's currency depreciates (in this case the US dollar), investments become more attractive to foreign investors, so their expected return is increased. A negative correlation can be explained for the above reasons. Furthermore, in Table 4.1 it is also observed that UK3 index is also negatively correlated to the domestic exchange rate. The same explanation can hold as the previous case. The correlation between stocks and exchange rates is a very interesting case in the financial field and can be very complex to understand. The increasing globalization and the large volume of transactions has raised the issue and it is very important to understand investment decisions.

Table 4.4

Correlation between indices over 1/1994-5/2018.

		S&P 500 PRICE INDEX RETURN	S& P 600 SMALL CAP RETURN	US CITIGROUP 1-3Y RETURN	US CITIGROUP 7-10Y RETURN	TOPIX LARGE 70 RETURN	TOPIX SMALL RETURH	CITI JAPANESE GOV BOND 1-3Y RETURN	CITI JAPANESE GOV BOND 7-10Y RETURN	FTSE 100 RETURN	FTSE SMALL CAP RETURN	UK CITIGROOUP 1-3Y RETURN	UK CITIGROUP 7-10Y RETURN
S&P 500 PRICE INDEX RETURN	Pearson Correlation	1											
S&P 600 SMALL CAP RETURN	Pearson Correlation	,804**	1										
US CITIGROUP 1-3Y RETURN	Pearson Correlation	-,164**	-,227**	1									
US CITIGROUP 7-10Y RETURN	Pearson Correlation	-,179**	-,251**	,755**	1								
TOPIX LARGE 70 RETURN	Pearson Correlation	,552**	,493**	-,315**	-,290**	1							
TOPIX SMALL RETURH	Pearson Correlation	,320**	,309**	-,247**	-,172**	,827**	1						

CITI JAPANESE GOV BOND 1-3Y RETURN	Pearson Correlation	-,051	-,088	,216**	,113	-,205**	-,246**	1					
CITI JAPANESE GOV BOND 7-10Y RETURN	Pearson Correlation	-,098	-,143*	,281**	,309**	-,282**	-,283**	,725**	1				
FTSE 100 RETURN	Pearson Correlation	,794**	,642**	-,174**	-,162**	,503**	,273**	-,050	-,095	1			
FTSE SMALL CAP RETURN	Pearson Correlation	,663**	,665**	-,312**	-,270**	,558**	,407**	-,073	-,106	,733**	1		
UK CITIGROOUP 1-3Y RETURN	Pearson Correlation	-,217**	-,233**	,687**	,554**	-,325**	-,262**	,229**	,214**	-,107	-,240**	1	
UK CITIGROUP 7-10Y RETURN	Pearson Correlation	-,122*	-,166**	,520**	,745**	-,260**	-,190**	,108	,218**	-,026	-,152**	,703**	1

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

Table 4.4 illustrates the correlations between indices. The findings are particularly interesting and for the purposes of our study will help us understand and evaluate the results of our analysis. The key findings of the correlations can be described as follows:

- By observing Table 4.4, stock and bond indices are negatively related to each other. Conceptually, when the return of a stock index deteriorates, the return of a bond index will increase. An investor who aims to diversify the portfolio, will be wise to invest in stock and bond indices to mitigate the negative results of a negative effect on one index.
- Negative correlation between stock and bond indices can increase by investing globally. To illustrate this effect, we compared different combinations of indices through the markets in question. For example, combining a US stock index with UK index (in particular short -term bond with large capitalization stock) enables diversification since their negative correlation is considerable. The same result is observed when the small capitalization index of US and short-term bond of United Kingdom are combined. Another finding that leads to interesting results, is the great increase in negative correlation combining Japanese stock indices with US bonds. They exhibit high levels of negative correlation. Similar results are observed when we combined stock indices of United Kingdom with United States bond indices. The negative correlation is increased by allocating income in two markets. The degree of negative correlation is higher when we combined indices between these two markets than choose to invest in one market at the same time.

- Stock indices on the same market exhibit high levels of correlation. Specifically, the average level of correlation between stocks in the same market is close to 0.80. This implies a strong positive correlation which can impose difficulty in the process of diversification. International investment can resolve this issue. Even in the case an investor chooses to allocate income only in stocks, combining indices across countries can reduce risk. The correlation between stocks of different markets is considerably lower than indices in the same market. Furthermore, investors can benefit from the exchange rate movements by investing globally.

- Like stock indices in the same market, bond indices in the same market exhibit high levels of correlation. The average level of correlation between bond indices in the same market is close to 0.70. When foreign investments are introduced in the portfolio management process, correlation between indices decreases since different markets are involved.

4.4.1. Model Description

In the previous section we mentioned that our study is based on market data collected by three major countries United States, United Kingdom and Japan. Market data collected involve four indices in each country, with the same characteristics. The study's approach is by the scope of a US investor who chooses to allocate income among these three countries. Furthermore, we analyzed the portfolio management problem under the scope of three types of investors: *risk averse, risk lover and risk neutral.* Each type has different investing preferences and profile therefore the portfolio composition and return will be somewhat different for each. The initial cash endowment is 1 million with no holdings in each asset. The initial budget in US dollars (the investor's base currency). In this section, we will analyze the model description of our analysis.

4.4.2 Introductory Elements of the Model

Conditional Value at Risk

For the purposes of our empirical study we used the Conditional Value at Risk (Cvar) metric Rockafellar and Uryasev (2002) that measures the amount of tail risk in a portfolio. Conditional Value at Risk, also known as expected shortfall, measures the conditional expectation of portfolio losses above a prespecified quantile of the distribution (beyond Var). In Chapter 2, the mathematical interpretation of CVarR was analyzed in an analytical manner. It is again worth noticing that CvaR is considered a coherent risk measure which is used broadly in the financial field. In our analysis, CVaR minimization was implemented for two types of investors, risk averse and risk neutral. It is worth noticing, however that in the risk neutral case, a certain target was chosen for each holding period for portfolio rebalancing. The choice of the risk CVaR risk metric, according to Nikolas Topaloglou et.al (2011) is appropriate for our analysis, since returns of financial assets and variations of exchange rates exhibit asymmetric distributions and fat tails. CVaR is considered an appropriate risk measure since it achieves to control the tail of portfolio's distribution.

Expected Return

Our analysis involves different types of investors, thus different treatment is in order. One of these types is an investor who aims to maximize the expected return of the portfolio. It is clear, that in this case the appropriate approach is to implement an expected return maximization, not minimization of risk, since this type of investor is not sensitive to higher exposure.

Variables

Our empirical test involves indices denominated in multiple currencies. Portfolios are composed with indices denominated in three countries (United States, United Kingdom, Japan). Market prices of stock and bond indices were collected in addition to exchange rates for the corresponding currencies. Our analysis involves monthly data of asset returns and exchange rate variations over period of 31/12/1014 to 5/2018.

Using market data, indices' monthly returns were calculated as well as the exchange rate variations for each month. Corresponding prices were computed based on the monthly returns and variations of exchange rates. Specifically:

We compute the corresponding outcomes of indices prices and exchange rates at the end of holding period under scenario N:

$\pi_i^n = (1+ r_i^n)\pi_i \qquad \forall c \in C_0, \quad \forall i \in I_c, \quad \forall n \in N$

$e_i^n = (1+ r_i^n)\, e_i \qquad \forall c \in C, \quad \forall n \in N$

Our variables involve the corresponding outcomes of indices and exchange rate variations.

We generate scenarios of the corresponding prices and exchange rate movements for each index and exchange rate in a monthly basis.

Cash Endowment

The aspect of study is from a US investor with initial cash endowment (1 million) in his base currency. In our analysis, investor allocates income totally in the markets. In the progress of the analysis, no self-financing, borrowing or lending is permitted. Funds are absorbed by the purchases in the market no additional cash endowment is allowed.

4.4.3 Model Formulations

The problem of international portfolio optimization is addressed by structuring portfolios from the perspective of a US investor who holds positions in indices in multiple currencies. These portfolios are subject to market and currency risk and our purpose is to assess them in an integrated manner. Variables determine the recommended purchase of assets in each currency to construct a portfolio with a control on the total risk.

Portfolio rebalancing is taken at the begging of the planning period. In this process, scenarios about corresponding outcomes of indices and exchange rates are considered. In our analysis, time period involves one month, thus portfolio rebalancing is implemented in a monthly basis.

The objective of portfolio optimization is somewhat different for each type of investor, so different aspects will be analyzed. Our analysis continues to describe analytically the model we used to construct optimal portfolios.

Sets and Indices

C_o set of currencies (markets, countries)

$I_c \in C_o$ set of indices in currency $\in C_o$ (stock and bond indices in each currency)

N set of scenarios

Deterministic Parameters

T length of planning horizon

μ target return of the revised portfolio

α the prespecified confidence level (quantile) for the CvaR measure

s_{ic} initial price of the stock index $\in I_c$ in currency $\in C_o$

b_{ic} initial price of bond index $\in I_c$, in currency $\in C_o$

e_c exchange rate for currency c $\in C_o$

V_o initial value of portfolio (1000000)

Scenario dependent parameters

---

p_n probability of scenario $\in N$

e^n_c exchange rate at the end of horizon under scenario $\in N$

S_{nc} price of stock index at the end of planning horizon in currency c $\in C_o$

b_{nc} price of bond index at the end of planning horizon in currency $\in C_o$

All exchange rates are expressed in units of the base currency per one unit of foreign currency.

Computed Parameter

V_t portfolio value

$V_t = \Sigma x_i s_t + \Sigma x_i b_i + \Sigma\, e^n_c\, (\Sigma x_i s_t + \Sigma x_i b_i)$

Decision Variables

Asset purchase under portfolio revision

x_{ic} units of asset $i \in I_c$, in currency $c \in C_o$

Auxiliary Variables

V_l^n the total value of the revised portfolio at the end of the holding period under scenario $n \in N$ (in units of the base currency)

R_n holding period return of the revised portfolio under scenario $n \in N$

L_n portfolio loss during the planning horizon under scenario $n \in N$

R expected in sample holding period return of the revised international portfolio

y_n auxiliary variables used in the definition of Cvar; they measure the excess loss beyond Var in scenario $n \in N$

z the Var value of portfolio losses during holding period at a prespecified confidence level $(1-\alpha)$

In our analysis, portfolio management involves stock and bond indices. Thus, portfolio's terminal value will be affected by the units and prices of bond and stock indices purchased after converting the prices into base currency.

We present the optimization problem of portfolio management:

(1a) $\min z + 1/(1-\alpha)\, \Sigma_n p_n y_n$

(1b) s.t $V_o = \Sigma x_i s_t + \Sigma x_i b_i + \Sigma x_i$

(1c) $x_i/e_o = \Sigma x_i s_t + \Sigma x_i b_i \quad i \in I_c$, in currency c $\in C_o$

(1d) $R_n = V^n_l / V_l^o - 1$

(1e) $L_n = - R_n$

(1f) $R = \Sigma p_n R_n \geq \mu$

(1g) $y_n \geq 0$

(1h) $y_n \geq L_n - z$

(1i) $x_i \geq 0$

This model minimizes Cvar measure of portfolio losses under the planning horizon. The objective function (1a) the expected excess loss beyond the 1-a quantile over the holding period.

Constraint 1b is the cash balance condition for the base and foreign currencies. We assume that the total initial cash is absorbed through market purchases, a condition that applies for each holding period. No borrowing or self-financing is allowed. Conceptually, purchases are bounded by the value of the revised portfolio.

All currency exchanges are made through the base currency.

Portfolio value

At the end of the holding period portfolio value is computed:

$V_t = \Sigma x_i s_t + \Sigma x_i b_i + \Sigma e^n_c (\Sigma x_i s_t + \Sigma x_i b_i)$

Portfolio value at the end of period reflects the value of all asset holdings at the end of the holding period. It is expressed in terms of the base currency. Foreign asset holdings are valued in terms of the base currency, by applying the respective exchange rates at the end of horizon.

Portfolio value end period is the initial value of portfolio for the next period of study.

Equations and Constraints

Equation (1d) computes the portfolio return under each scenario. On the other hand, (1e) estimates the portfolio loss under scenario. Constraint (1f) poses a minimum bound for expected return. For the purposes of our study, our minimum bound was 1%.

Constraints (1g) and (1h) determine the excess losses, y_n under each scenario in relation to the variable z, which takes the value of the (1 a)-quantile of portfolio losses (i.e., is the VaR at the corresponding confidence level).

Finally, constraint (1i) ensures that asset purchases and therefore portfolio holdings are not negative.

Model Extensions

In our analysis, it is mentioned that the approach of portfolio management is under the aspect of three types of investors. In this section, we will analyze the different approaches we followed to properly reflect the investing profile and preferences of each type.

Risk Lover

Portfolio management problem is viewed differently by an investor who aims to maximize the expected return. Specifically, since he ignores risk exposure, we did not impose a constraint for the losses beyond Var. Furthermore, instead of minimizing the objective function of conditional value at risk we imposed a maximization in the expected return of the portfolio. In addition, constraint of expected return was used, with a minimum bound of 1%.

Risk averse

A risk averse investor avoids high levels of risk. In this case, portfolio management problem involved constraint of excess losses and conditional value at risk minimization. Contrary to risk lover, a constraint on expected return was not implemented.

Risk neutral

Risk neutral investors are insensitive to risk, but they are not viewed as the risk lovers investors. The seek higher levels of return than risk averse but lower than risk lovers. In this case, cvar minimization was implemented along with the constraint for losses. In addition, a constraint was imposed for the expected return. After estimating expected return for risk lover and risk averse for each holding period we continued our analysis on risk neutral. The bound for expected return in holding period is the sum of returns of risk lover and risk averse divided by two.

$R_{neutral} = (R_{lover} + R_{averse}) / 2$

It should be mentioned that returns involve the same time period of our analysis.

4.5.1 Methodology

Our analysis is based on international portfolio management problem using historical data. In order to show the actual performance of international portfolios, dynamic tests are in order. Since variables are not constant and tend to change over time, repeated applications help estimate the performance in a more integrated way. Our focus is to examine the actual performance using the data we collected for each index and exchange rates.

4.5.1 Period of analysis

Our back-testing simulations involve repeated applications of the model described earlier for successive months from 12/2014 to 5/2018 (180 months).

4.5.2 Dynamic Test

Back testing starts at 12/2014 with an initial cash endowment of one million in the investor's base currency. Initial portfolio does not have holdings in each asset. In a monthly basis, we estimate indices' returns and exchange rate variations based on market data in the preceding 180 months. Based on the indices returns, we compute the projected asset prices and exchange rate variations under each scenario. The corresponding prices at the end of one-month planning horizon are as follows:

$$\pi_i^n = (1+ r_i^n)\pi_i \qquad \forall c \in C_0, \quad \forall i \in I_c, \quad \forall n \in N$$

$$e_i^n = (1+ r_i^n)\, e_i \qquad \forall c \in C, \quad \forall n \in N$$

For each of successive months over 12/2014-5/2018 an optimization problem is solved. The equations and constraints described in the previous section are used to estimate the optimal portfolio composition and value in a monthly basis. The results of the model record the monthly purchases of the investor, the expected return, the portfolio loss, the conditional value at risk as well as the revised portfolio value. The portfolio's value at the end of the holding period, is the initial value of the portfolio in the next period of study.

It should be again mentioned that no exogenous cash flow is allowed. Besides the initial amount on the first period in the successive months' purchases are self-financed. In addition, now cash withdrawal is allowed. Furthermore, back testing was implemented for each type of investor, using different constraints and in the case of risk-seeking a different optimization problem.

Dynamic test for each investor revealed the optimal monthly positions in indices. Calculations were carried out to show the relative amount each investor spends on indices (values are expressed in the base currency). This amount is based on the investor's risk preferences and the performance of each index in a monthly basis.

The relative amount or also called *wage* or *weight* in each index, is a very important parameter for back testing simulations. It reveals the importance investors give in the corresponding positions of stock and bond indices over the period of our study.

The next step of the procedure, after estimating the relative amounts spent on positions, is to show the actual monthly performance of the portfolio. As mentioned earlier, monthly returns of indices were estimated. The actual performance of portfolio in dynamic testing is estimated by multiplying the relative values of each position in indices with the next month return on each of the corresponding indices. The total performance of portfolio is the weighted average of returns of indices that are involved in the portfolio.

These calculations reflect the actual performance of portfolio over the holding period. Ex post returns reflect the portfolio returns that would have been realised, if the recommendations of the model had been implemented during the simulations of back testing.

Back testing experiments were carried out for each type of investor. In next section, the results of the back-testing simulations will be presented

4.6.1 Results of the analysis

4.6.2 Monthly Purchases and Returns of Investors

In this section we will provide the results of the repeated optimization problem for each type of investors.

Risk Lover

We start the results analysis with the risk lover. Risk lover is a type of investor who seeks high levels of risk in order to receive higher expected return. In our analysis, the model we used to find the optimal positions in indices over the holding period is the maximization of expected return with no constraints in risk exposure. We present the results as follows:

Table 4.5

Monthly purchases of risk lover.

UK1: FTSE 100 Price Index

Dates	UK1
Dec2014	113
Jan2015	71
Feb2015	80

March2015	154
April2015	355
May2015	1072
June2015	409
July2015	170
August2015	870
Sept2015	486
Oct2015	270
Nov2015	178
Dec2015	128
Jan2016	1042
Febr2016	858
March2016	709
April2016	419
May2016	408
June2016	448
July2016	463
August2016	523
Sept2016	611
Oct2016	794
Nov2016	1005
Dec2016	127
Jan2017	170
Febr2017	238
March2017	341
April2017	478
May2017	726
June2017	117
July2017	190
August2017	689
Sept2017	116
Oct2017	205
Nov2017	373
Dec2017	678
Jan2018	124
Febr2018	262
March2018	523
April2018	1038
May2018	220

Table 4.5 illustrates the monthly purchases of risk-seeking. In our repeated simulations, we observed that each month under the period of our study risk lover, invests only in the United Kingdom's large cap stock index. This type of investor aims to earn the highest level of

expected return under each scenario. Investing in more than one index, will reduce exposure, thus the expected return, due to diversification. This fact explains the investment strategy of this investor. In addition, investing in a non-domestic market, can yield higher levels of return. Currency risk exposure of international investments increases the total risk of portfolio. As risk increases, return will also relatively increase.

Risk averse

Risk averse investors seek investment opportunities that are characterized by low risk levels. To properly reflect their portfolio performance, we minimized the metric of tail losses (Cvar) and posed a bound on the conditional value at risk under each scenario. Results of the repeated simulations are the following:

Table 4.6

Monthly purchases of risk averse

UK1: FTSE 100 Price Index

JP2: Topix Small Price Index

Dates	UK1	JP2
Dec2014	70	13994
Jan2015	51	5478
Feb2015	30	15865
March2015	104	13534
April2015	209	40924
May2015	441	175435
June2015	222	43500
July2015	65	25853
August2015	442	86618
Sept2015	242	47422
Oct2015	131	25727
Nov2015	84	16448
Dec2015	583	114293
Jan2016	461	90438

Febr2016	370	72531
March2016	298	58374
April2016	257	50448
May2016	245	47993
June2016	256	50140
July2016	257	50354
August2016	285	55943
Sept2016	321	63025
Oct2016	347	68014
Nov2016	143	181057
Dec2016	543	106500
Jan2017	70	13837
Febr2017	146	28714
March2017	207	40531
April2017	298	58370
May2017	440	86295
June2017	69	13558
July2017	148	9894
August2017	653	
Sept2017	663	129886
Oct2017	113	22250
Nov2017	200	39241
Dec2017	357	70025
Jan2018	643	126077
Febr2018	129	25289
March2018	253	49722
April2018	497	97461
May2018	102	2073

The results of back-testing simulations in the risk-averse case are shown in Table 4.6. It is observed that almost every holding period, investments in the United Kingdom’s large cap and Japan’s small cap stock indices were realised. It should be noticed, that the relative amount invested in the UK was larger. Risk defensive diversifies the portfolio combining these two indices. In previous section, the correlation matrix among indices was illustrated. Table 4.6 presented that between UK’s large cap stock index and Japan’s small cap stock index a relatively low correlation is present. The low levels of correlation can enable portfolio’s diversification. In addition, investing in different markets with different exchange rates, can mitigate currency risk under each scenario.

Risk Neutral

Risk neutral investors are a very interesting category of investors. In finance, risk neutrals are considered investors who are indifferent to risk when taking an investment decision. For the prementioned reasons, under the scope of our study this category was treated differently than the other two. In back-testing simulations, an optimization problem was solved in the similar way as risk averse. Specifically, a cvar minimization problem was solved but with a minimum bound of the expected return. The constraint on expected return was the following:

$E\ (r) \geq Risk_{lover}+Risk_{defensive}/2$

(Returns are from the same period of study)

To proper identify risk neutral's investment behavior we treated our simulations in this manner. We shall present the findings of the repeated simulations:

Table 4.7

Monthly purchases of risk neutral

UK1: FTSE 100 Price Index

JP2: Topix Small Price Index

	UK1	JP2
Dec2014	80	10970
Jan2015	59	3664
Feb2015	31	16194
March2015	107	13814
April2015	154	61153
May2015	448	178178
June2015	225	44180
July2015	66	26800
August2015	449	87972
Sept2015	244	48163
Oct2015	133	26129
Nov2015	85	16705
Dec2015	592	116080
Jan2016	1551	
Febr2016	78	15364
March2016	631	123652

April2016	545	106863
May2016	519	101663
June2016	903	
July2016	112	112038
August2016	635	124474
Sept2016	71	140223
Oct2016	88	17293
Nov2016	36	46035
Dec2016	138	27078
Jan2017	180	35183
Febr2017		124295
March2017	331	64820
April2017	476	93349
May2017	70	13800
June2017	110	21683
July2017	233	15823
August2017	293	57554
Sept2017	488	95552
Oct2017	83	16371
Nov2017	147	28873
Dec2017	263	51522
Jan2018	473	92764
Febr2018	95	18607
March2018	186	36584
April2018	366	71709
May2018	75	14769

In Table 4.7 the results of the repeated applications of the optimization problem of risk neutral are clearly illustrated. Like risk averse, risk neutral invests in FTSE 100 and Topix Small price indices. However, the relative amounts spent in the UK market are a little higher under this case. In addition, in more holding periods (3 in risk neutral, 1 in risk defensive) portfolio consists only one index, which implies higher levels of exposure under these periods. Due to the difference of portfolio composition, different returns and exposure are presented. Risk neutral exhibits lower return levels as we will show in the following page. Differences in portfolio structure can affect the overall performance of portfolio. This fact was raised due to the constraint that was imposed over the expected return.

4.6.3 Cumulative Returns and Compositions of Portfolio

In this section we will present the cumulative returns and the composition of all investor types' portfolio.

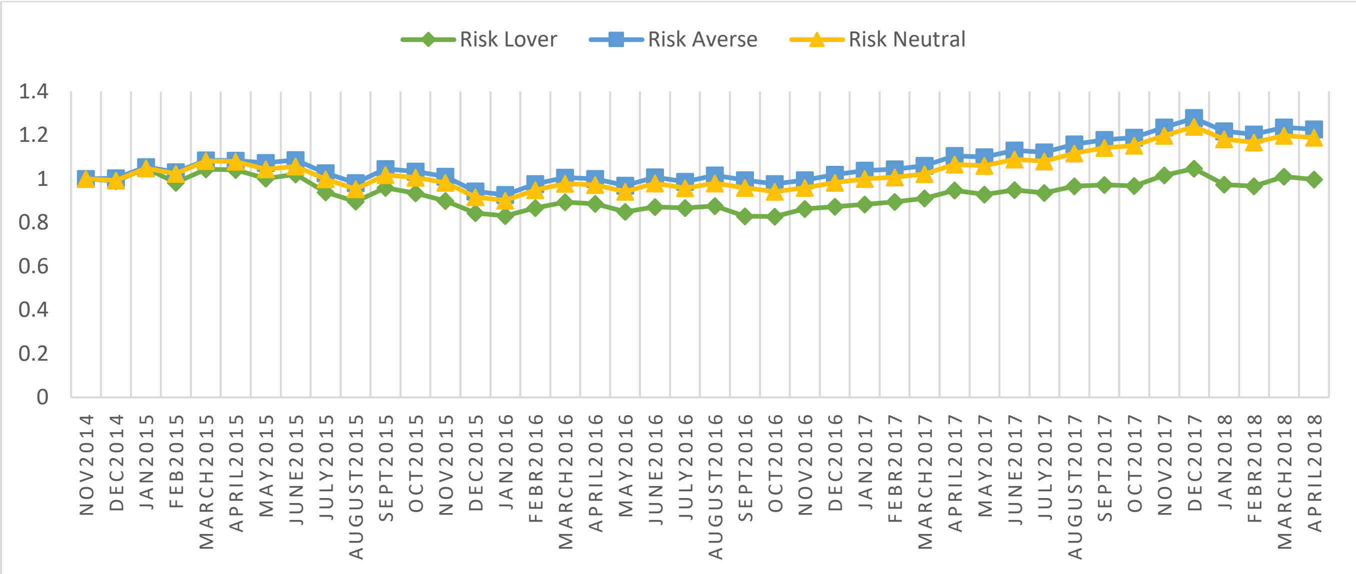

FIGURE 4.1: Ex post cumulative returns of CvaR and Expected Return- optimized international portfolios of stock and bond indices.

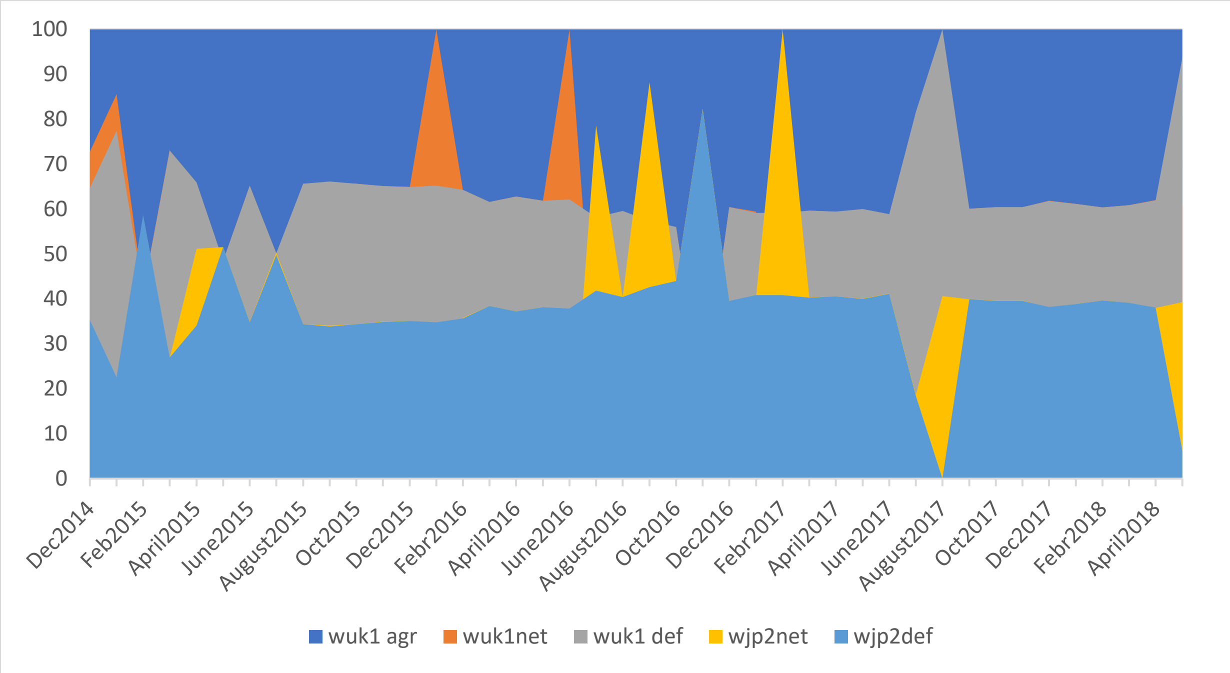

FIGURE 4.2: Compositions of optimal portfolios of international stock and bond indices. A maximization over the expected return was solved for risk lover, a minimization of CvaR was implemented for risk defensive and neutral and a minimum constraint on expected return for the second, μ ≥ (RriskLover+Rriskaverse)/2.

Figure 4.1 illustrates the cumulative returns of each investor over the period of our study. Cumulative returns help us understand and compare results derived from back-testing simulations.

Observing Figure 4.1, the investor who exhibited the worst performance, is risk lover. In each holding period he invested totally in one stock index (UK1). This investment behavior has increased exposure to risk. Firstly, by investing in one index he is not able to diversify, thus a negative effect cannot be counterbalanced. In addition, currency risk can't be mitigated since only one exchange rate is involved in the outcome. Conceptually, even if risk lover aims to maximize the expected return, he does not present the highest level of returns, due to increased volatility which stems from the absence of diversification.

Continuing our analysis on Figure 4.1, it is shown that risk averse and risk neutral exhibit a similar progress over the period of our study. However, risk averse's performance is better as shown in Figure 4.1.

In previous section, we described the monthly purchases of investors. Figure 4.2 illustrates the portfolio composition for each type of investors (in terms of percentages). As mentioned previously, risk defensive and risk neutral, construct portfolios combining FTSE 100 and Topix Small, price indices. The structure of their portfolio though, is somewhat different as shown in Figure 4.2. Specifically, risk neutral gives a greater proportion of income in the UK stock index than risk defensive and does not diversify for more holding periods. By giving a relative greater proportion in one index even for little time periods, a negative effect is more difficult to be counterbalanced. The losses of a negative movement on the UK stock index will be greater, as the relative proportion increases. Furthermore, one can note that the same holds for the Japan stock index since risk averse tends to invest more in this price index. This fact is true, however, in most holding periods, he exhibits a better balance between the amounts he invests in two indices. As noted before, investing mainly in one index can increase exposure, and return is less likely to remain high for long periods of time. Similarly, giving greater importance in one index, increases risk and imposes difficulty on the diversification process. Allocating income between financial assets in a balanced way, is proved to enhance portfolio's performance.

Figures 4.1 and 4.2 illustrated the key findings of our repeated optimization problems. By comparing cumulative returns, we can evaluate which portfolio exhibited the best performance. Figure 4.2 helps understanding the progress of each portfolio over time.

Based on cumulative returns, we can conclude that the portfolio which presented the highest return is the risk averse's. This fact helps us show the diversification benefits and the effectiveness of our optimization problem. This type of investor constructed portfolios using two indices and two different currencies, that were well balanced as to the amounts spent in each of them. This investment strategy raised diversification for the prementioned reasons. Through diversification, he was able to improve portfolio performance and lower the overall volatility. Diversification not only lowered the level of exposure, but also ensured relative expected -return stability and increase in the whole period.

4.6.4 Performance Measures

For a more accurate analysis of the results of the back-testing simulations, some performance measures are needed. This section focuses on examining the performance of portfolios under the scope of performance measures and address the key findings of this progress. Specifically, ex post returns of the simulation period will be evaluated in terms of the appropriate measures used in finance. The measures used in this section are the following:

- *Standard Deviation*

Standard Deviation is used to estimate the dispersion of a variable relative to its mean and is calculated the square root of variance. Thus, in finance, it is a measure of risk. Higher standard deviations imply higher risk levels. The performance can be evaluated through standard deviation since higher or lower levels imply higher/lower risk exposure on the investment.

- *Geometric Mean*

In finance, geometric mean is an important tool for calculating portfolio performance. Geometric mean indicates the central tendency of a set of numbers using the product of their values. It is defined as the nth root of the product of n numbers. Formula is as follows:

$\mu\text{geometric}=[(1+R1)(1+R2)\dots(1+Rn)]^{1/n}-1$

where: ·

$R1\dots Rn$ are the returns of an asset.

Geometric mean can be also referred as to compounded annual growth rate, since it considers the effects of compounding. It is also more effective for longer time periods. In our section, by estimating the average rate of returns, we will evaluate all portfolios.

- *Sharpe Ratio*

Sharpe Ratio was introduced by William Sharpe and it is one of the most popular risk measures. The Sharpe Ratio estimates the excess return of the investment in comparison to the risk-free rate, per unit of volatility. If the ratio of the fund is higher, then investors are compensated for undertaking the specified level of risk. The ratio focuses on maximizing returns and minimizing volatility. It can be used to calculate the returns investors anticipate in relation to the risk they undertake. Investors prefer higher Sharpe Ratios (Sharpe 1964) The ratio is estimated as follows:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

R_p in the equation stands for the average return of the investment, r_f is the risk-free rate of interest and σ_p is the standard deviation of the portfolio.

The Sharpe ratio is used to evaluate the portfolio's past performance, by using historical data of the investment. It can also be useful for determining portfolio's expected performance, using anticipated return of the investment and the riskless rate. Furthermore, Sharpe ratio can be insightful for determining whether the excess returns result from smart investment decisions or too much risk. High Sharpe Ratios show a well-adjusted performance of a portfolio.

On the contrary, there are some caveats in using Sharpe ratio to evaluate portfolio's performance. Specifically, it is assumed that normal distribution holds and that price movements in each direction, are equally risky.

For the purposes of our study, in order to calculate Sharpe Ratio, we use the US-Treasury Bills as benchmark.

- *Sortino Ratio*

Sortino Ratio is a variation of the Sharpe ratio. This type of risk -adjusted performance measure is estimated with the same way as Sharpe ratio, with the only difference that instead of the total standard deviation of the portfolio's returns, we use the standard deviation of the negative return's portfolios exhibit. Standard deviation of negative returns is called downside deviation.

The formula of Sortino Ratio is the following:

$$Sortino\ Ratio = \frac{R_p - R_t}{\sigma_{np}}$$

R_p = Expected portfolio/asset return

R_{rf} = Target rate of return defined by investor

σ_p = Downside of portfolio/asset standard deviation

The key difference between Sharpe and Sortino ratio is that only the downside volatility is present in the second. This is very insightful for managers and investors, since it gives a better view of the risk adjusted performance due to the fact only negative volatility is present, as opposed to Sharpe Ratio. Upside volatility is beneficial for investors; thus, this type of measure is more efficient in estimating portfolio's performance. In our study, we use US Treasury bills as a benchmark to estimate Sortino Ratio.

Table 4.8 illustrates the statistics of the realised returns of portfolios of all types of investors. The results are assessed in terms of standard deviation(volatility), geometric mean (cumulative returns), Sharpe and Sortino Ratio (risk-adjusted performance measures).

Table 4.8

Statistics of ex-post performance of portfolio returns for all types of investors.

	Standard Deviation	Geometric Mean	Sharpe Ratio	Sortino Ratio
Portfolios of risk lover	0.037298842	0.932991442	-0.003977529	-0.006198371
Portfolios of risk averse	0.030887385	1.069123692	0.146619799	0.225426765
Portfolios of risk neutral	0.030827992	1.067997853	0.137768104	0.206756146

The comparison of statistic results proves the conclusions derived in the previous section. The portfolios of risk lover exhibited the worst performance. The standard deviation was higher than the other two investors which implies higher levels of volatility. In addition, geometric mean is considerably lower, thus, average returns tend to exhibit worst performance. Sharpe and Sortino Ratios are negative which shows, that under most scenarios, returns are expected to be lower than the riskless rate, or negative. Risk seeking investing profile presented absence of diversification, since only one stock index was involved. This fact provided no protection over the exposure, and therefore the increased volatility worsened the portfolio's performance.

Continuing our study, we compare the results between risk averse and risk neutral. Geometric Mean for risk averse is higher than risk neutral's, thus, higher levels of returns are expected. On the contrary, standard deviation is relatively higher than risk neutral. Looking only in one performance measure, however, may not be insightful. In Table 4.8 it is clear that risk averse exhibits the highest Sharpe and Sortino Ratios. Sharpe and Sortino Ratios show with more accuracy the performance of portfolio per unit of volatility. Furthermore, adding diversification to portfolio, increases sharpe ratio which is important for our study. By looking these performance measures conceptually, it is clear that the portfolio with the best risk adjusted performance, is averse's.

Ex-post analysis of portfolio's returns proved that diversification can improve portfolios performance. This is illustrated by interpreting the results of risk averse. Risk averse investor repeatedly invests in two indices with two different currencies involved. Furthermore, he chooses to allocate income in a more balanced way, to offset possible negative changes in the indices' prices. The nature of optimization problem proved efficient to show the benefits of international diversification, by imposing a minimization on conditional value at risk alongside with a constraint on expected shortfall.

4.7.1 Chapter Epilogue

This chapter was focused completely on our empirical analysis of the international portfolio optimization problem. Introductory remarks about the model were analyzed in early stages of the chapter as well as data analysis of variables involved in the progress of back-testing.

It is important to review some key remarks of the chapter.

Our empirical analysis involved repeated applications of optimization models for three types of investors over 12/2014 to 5/2018. For each type, different constraints were implemented. The initial cash endowment was the same for all investment strategies (1million) and no cash inflow or outflow was allowed. Furthermore, analysis involved three markets, United States, United Kingdom and Japan and two exchange rates (Yen to Dollar, Pound to Dollar).

Different constraints due to different investing profiles lead to different results between investors. In this chapter, it was revealed that risk averse's investing preferences (to mitigate risk as possible) lead to higher returns and less exposure in the long run. Risk averse diversifies portfolio (more than risk neutral since risk seeking invests in one index under each scenario), therefore he exhibits the best performance. This fact was ensured by examining statistical and risk adjusted measures and comparing the results derived for all investor types.

Diversification benefits were clear after comparing the key measures of all investing profiles and finding that the best performance involves the investor who diversifies more. This finding is consistent with literature review of international diversification that was analyzed in previous chapters. In nowadays globalized world, international investments are becoming more popular than past years and are proved to positively affect each country's welfare and development. Thus, an empirical analysis of them is considered interesting and insightful for finance and the whole economic field.

Chapter 5

5.1.1 Conclusions

In the previous chapter, we analytically described the methodology and nature of our study. In addition, the results of back-testing simulations were illustrated and analyzed. Our repeated optimization problems for different types of investors lead to interesting results, and in this section, an analytically analysis of the results will be realised. It is very important to properly address the results, since this progress will reveal the effectiveness of our research and its contribution to other literature review about international portfolio management.

5.1.2 Brief Review of the model

We review the portfolio management issue, by the prospective of a US investor (we examine three types of a US investor*: risk lover, risk averse and risk neutral*) who allocates income between United States, United Kingdom and Japan, with initial cash endowment of one million and no additional cash inflow or outflow through the process. We aim to construct optimal portfolios that achieve balance between expected return and risk.

Our analysis involves repeated optimizations problems for the successive months over 12/2014-5/2018. For each investor, the optimal holdings for each months are estimated, alongside with the portfolio value, expected shortfall, etc. We then estimate the weights in each index and multiply this value with the next month's return of the asset. This reflects the returns that would have been realised if the recommendations of the model had been adopted during simulations.

5.1.3 Review of the Results of the analysis

Results of our back-testing analysis revealed that the investor with the best portfolio ex post-return was risk averse. As mentioned previously, under the analysis he constructed optimal portfolios combining United Kingdom's large cap stock index and Japan's small cap stock index. Risk averse's return was superior due to the fact that he allocated income among two indices in a more balanced way. Risk lover invested only in FTSE 100 under all holding periods, while risk neutral gave a relative higher importance on the aforementioned index. The returns of these investors were lower because diversification benefits deteriorated as portfolio's structure was less balanced. The appropriate balance between indices involved in a portfolio, can enhance the reduction of risk. A negative movement of one index has less impact in the final value of the portfolio if the relative importance is not comparatively high. Thus, returns in the long run exhibit a more stable and positive performance.

Our analysis continued by estimating performance measures of the portfolio's ex post returns. The estimation of the performance measures contributed to the conclusion derived from the comparison of cumulative returns. In particular, after estimating the Standard Deviation, Geometric mean, Sharpe and Sortino ratio for all portfolio's ex post returns, it became clear that the portfolio with the best risk-adjusted performance was risk averse's. Sharpe and Sortino ratios were higher than risk neutral and risk lover, alongside with the Geometric Mean. It is worth noticing, that standard deviation of the ex post returns for risk averse was relatively higher than risk neutral but looking at Sharpe and Sortino ratios we concluded that he exhibits a better performance per unit of volatility, thus, his portfolio is superior than the other two. The worst portfolio performance was risk lover's since he did not diversify for all holding periods.

5.1.4 Further observations and findings

We demonstrated that the best ex post portfolio performance was risk averse's by observing cumulative returns and performance measures. This type of investor constructed optimal portfolios combining FTSE 100 price index (United Kingdom) and Topix Small (Japan). The allocation was considered more balanced compared to risk neutral.

We address our findings by focusing our interest on indices returns through the period of study. The performance of each index can be insightful for the interpretation of results. It should be noticed that returns are expressed into units of domestic currency. Currency risk is involved since returns shift based on the movement of the exchange rate.

We estimate performance measures to address the indices' performance under our analysis. In this section, we will use different measures to estimate the performance than previously. We present the results as follows:

TABLE 5.1

Statistics of Performance Measures of stock and bond indices

Statistics of Performance Measures of Indices	Sharpe Ratio	Sortino Ratio	Geometric Mean
S&P500	0.221099694	0.355586919	1.088229149
S&P600	0.218707572	0.398750748	1.149583288
US CITIGROUP1-3Y	-0.066416137	-0.158564471	1.010875388
US CITRIGROUP 7-10Y	0.030925207	0.046861232	1.043559379
FTSE 100	-0.002046261	-0.003226702	0.907648283
FTSE SMALL CAP	0.100420232	0.123574297	1.022512635
UK CITIGROUP1-3Y	-0.125386455	-0.184128664	0.900657314
UK CITIGROUP 7-10Y	-0.035991759	-0.035108191	0.956331035
TOPIX LARGE	0.270344155	0.350496382	1.163460655
TOPIX SMALL	0.349000681	0.533755806	1.232656752
CITI GOV. BOND 1-3Y	0.017496275	0.050132495	1.049062276
CITI GOV. BOND 7-10Y	0.051286411	0.076567712	1.082607556

Table 5.1 illustrates the statistics of performance measures of indices. Observing the results, many interesting findings are derived. Our first observation is that in Japan indices, no negative Sharpe and Sortino ratios are noticed. Each index exhibits positive results of these measures and a high geometric mean, which implies that returns will be relatively high. In addition, Topix Small presents the highest Sharpe and Sortino ratio, alongside with the highest geometric mean. Conceptually, the index with the best risk-adjusted performance is the one of Japanese small cap stock index.

Focusing our interest on the United Kingdom indices, we observe that the best performance in this market involves the small cap stock index (FTSE Small Cap). All other indices exhibit negative Sharpe and Sortino Ratios and lower geometric means. This finding is particularly interesting. FTSE Small Cap outperforms not only the large cap index, but also the bond indices with regard to risk adjusted performance. Thus, this index exhibits better performance per unit of volatility. Stock indices once again outperform a bond index by observing Table 5.1.

Regarding United States indices, the best noticed performance involves long term bond index. Short term bond index does not exhibit an effective risk-adjusted performance.

Without a doubt, the best performance noticed among indices, is the performance of Topix Small index. All measures are higher than the other indices and we conclude that it rewards investors a better return per unit of volatility.

Performance Measures from Table 5.1 are very insightful for the interpretation of results from our back-testing simulations. Our repeated applications revealed that risk neutral and risk averse repeatedly invest in small cap stock index in Japan for most of the holding periods. Table 5.1 showed that this index outperformed all others. Thus, our model was effective to capture this risk adjusted performance, since it involved holdings of this index for these types of investors. Specifically, risk averse aims to minimize exposure when investing, and we achieved to demonstrate this fact through our research. Topix Small presented the best risk adjusted performance, and in our final results it is involved in risk averse's portfolio. Exposure was mitigated by involving holdings of this index. In addition, risk neutral also exhibited holdings on this index. However, the amount given to this index was lower conceptually, exposure was raised, and the long-term return was relatively lower.

In previous chapter, the purchases of each type of investor were demonstrated. Results revealed that United Kingdom's large cap stock index was involved in the optimal portfolio holdings.

Particularly, risk lover's portfolio involves under each scenario, this stock index. Risk lover is an investor that is willing to be exposed to higher levels of exposure. From Table 5.1, it figures that this index exhibits negative Sharpe and Sortino ratios and a comparatively lower geometric mean. FTSE 100 is the only stock index that presents negative risk adjusted performance. By nature, risk lover tends to invest in stock indices since their levels of volatility are higher. Furthermore, he seeks investments that exhibit higher risk levels as mentioned previously. Our model was effective to capture the investing preferences of risk lover. Firstly, we were able through the maximization of the expected return to show that he in fact invests in stocks. Secondly, exposure is raised, so does the expected return when all income is absorbed in one stock index. Diversification benefits are absent, and he can only benefit from an exchange rate movement in his favor. In addition, portfolio structure involved the most-risky stock index for all repeated applications. From these results, it is clear that this type of investor seeks risky investments as literature suggested. Thus, our research proved effective not only to accurately addressing the investment preferences of this investor, but also to demonstrate that in the absence of diversification, exposure increases, which affects the long term expected return in a negative manner.

Our analysis involved risk averse and risk neutral who also exhibited holdings in this index. This is a very interesting finding from our analysis. One could argue, that in the risk averse case he would be expect holdings in bonds, combination of stock and bond indices or indices with lower levels of volatility, thus a better risk adjusted performance. However, by observing in a more integrated manner statistical measures this fact can be explained.

It was revealed previously, that optimal holdings reasonably involve the small cap stock Japanese price index. Since it outperforms all others, we will consider that for the case of risk averse and risk neutral, it should always be included in the portfolio.

Another statistical measure that plays a dramatic role when constructing portfolios, is the correlation coefficient. A lot of literature review in finance is regarding the role of correlation coefficient in portfolio theory. Negative or low levels of correlation between assets and indices are considered desirable when deciding to manage a portfolio. After all, diversification is raised when assets that would react different to the same event are combined.

We estimated the correlations between Japanese small cap stock index and the other indices. The results are the following:

Table 5.2

Correlations among Topix Small and Stock and Bond Indices

Correlations between Topix Small and Indices	
S&P 500 AND Topix Small	0.621845685
S&P 600 and Topix Small	0.229776659
US Citigroup1-3Y and Topix Small	-0.196986445
US Citigroup 7-10Y and Topix Small	-0.162544984
FTSE 100 and Topix Small	0.553386639
FTSE Small and Topix Small	0.582150411
UK Citigroup 1-3Y and Topix Small	0.348249419
UK Citigroup 7-10Y and Topix Small	0.179581751
Topix Large and Topix Small	0.830999571
Japanese Gov. Bond 1-3Y and Topix Small	0.121613445
Japanese Gov. Bond 7-10Y and Topix Small	0.083567652

Table 5.2 presents the correlations between Topix Small and all indices involved in our analysis.

We observe that the lowest correlation between Topix Small and any other stock index, is between S&P600 and Topix Small. In addition, the second lowest is between Topix Small and FTSE 100.

Lower levels of correlations are exhibited between Topix Small and bond indices. In particular, the lowest correlation in Table 5.2 is between US Citigroup 7-10Y price index and Topix Small.

These results can be described relatively controversial with regard to our findings. We would anticipate a combination of Topix Small and a bond index or a price index with lower correlation levels. However, we should notice that even if the lowest correlations are presented between US bond indices and Topix Small, the portfolio we would structure combining the proceeding indices, would involve domestic investments. Thus, international diversification benefits would deteriorate under this scenario. Involving a domestic index instead of a foreign, implies that no second exchange rate would be involved in the portfolio. Thus, movements in favor of the investor regarding currencies, would not affect portfolio value.

In Chapter 4, Table 4.1 we presented the statistics of monthly returns of indices and exchange rate appreciation rates. By interpreting the results of Table 4.1, we concluded that under the period of our examination, the value of British Pound deteriorated, since its appreciation rates exhibited a negative mean. British indices are therefore cheaper from our US prospective. However, their final value decreases as British pound becomes a less expensive currency. Even if its value deteriorates, we concluded in Chapter 4, that British Pound is considered a less risky currency than Yen since it exhibits lower standard deviation and kurtosis. It is worth noticing however, that as British Pound deteriorates, Dollar strengthens. Changes in the exchange rates can work both ways for an international investor.

Table 4.1 also revealed that the exchange rate Yen to Dollar does not present many negative appreciation rates. Value of Yen does not decrease in the same manner as British Pound. Therefore, investments in Japan will not exhibit a decrease in their value as in United Kingdom.

An international portfolio of these two exchange rates can be protected from drastic changes in their value, since one movement will be mitigated by another. Furthermore, Table 4.3 showed that the correlation between United Kingdom’s large cap stock index and the British Pound appreciation rate was statistically insignificant. This implies, that returns of FTSE 100 could increase, even if Pound to Dollar deteriorates. Moreover, stock indices in the US, exhibit a positive correlation with the exchange rate of British Pound to US dollar. Since our study revealed that Pound deteriorates, this correlation implies that returns of the US stock indices will also decrease. The same argument holds for the Japan's large cap stock index. Its returns are correlated positively with Pound to Dollar movements. Thus, in our analysis, an investment in Japan would exhibit decrease in its value since as we mentioned Pound to Dollar deteriorated.

Bond indices in the United States, exhibit correlations that may be considered favorable for our study. In particular, they exhibit positive correlation with Yen to Dollar movements. We mentioned that Yen does not seem to deteriorate with regard to dollar, but by looking on its initial and its final value(Appendix C) we conclude that even if in the time analysis it exhibited high levels of volatility, its long term trend did not impose significant difference between the initial and its final value. Thus, it remains unclear if returns of United States' stock indices will increase under the period we examine.

Table 4.3 reveals favorable correlations for the investors, that involve United Kingdom's bond indices. However, Table 5.1 illustrated that they exhibit the worst risk adjusted performance.

Regarding Japan stock and bond market, the following findings were revealed. Firstly, bond indices do not exhibit an important correlation with any exchange rate, thus exchange rate movements will not affect their returns and value. Japan's large cap stock index exhibited statistically significant correlation with both exchange rates. It is positively correlated with British to Dollar movements and negatively correlated with Yen to Dollar movements. This fact would impair portfolios returns. Since Pound to Dollar depreciates and Yen to Dollar in considered not to lose its value, including this index to the portfolio would be not wise. Both correlations would drive the value of Topix Large to lower levels, thus portfolio would exhibit more negative returns.

Topix Small was previously referred as to the index with the best risk adjusted return. Even if correlation between this index and Yen to Dollar is not considered favorable, its performance ensures an increase in returns if it is involved in the portfolio.

5.1.4 Final conclusions

Observing the results of our repeated applications, the estimation of statistical measures, we will derive the main conclusions regarding our study.

Different optimization problems were solved for different investor types. The study aimed to show the differences in portfolio composition and value between different investing profiles. Furthermore, for each type of investor we aimed to attain an effective balance between expected return and risk.

It was previously mentioned that risk lover aims to be exposed to higher levels of risk. Higher risk levels are often linked to higher levels of expected return. Thus, our optimization involved an expected return maximization. The nature of the optimization problem proved effective when an interpretation of ex post returns, risk adjusted measures and understanding of statistical measures was realised. Risk lover repeatedly invested in one stock index (FTSE 100), therefore no diversification benefits were exhibited. Absence of diversification lead to the lowest ex post cumulative returns and the worst risk adjusted performance. Through these results, the importance of diversification became even more clear. However, we should notice, that even in the case of risk lover no allocation of funds in the domestic markets was realised. Investing in foreign markets was the optimal solution under this case as well. Through these results, the importance of diversification became even more clear.

Risk averse and risk neutral exhibited similar progress through the study, but it was revealed in Chapter 4 that risk averse exhibited superior portfolio returns and risk adjusted performance. Portfolio was relatively more balanced than risk neutral. This fact increased the returns and the performance in the long run.

Optimal portfolio holdings involved FTSE 100 and Topix Small price indices. Our optimization problem proved effective, since these holdings enabled a balance between expected return and risk. The following results ensure our conclusions.

1. Portfolios included Topix Small for almost all holding periods, especially in the risk averse case. The estimation of risk adjusted performance measures of each index revealed that the most effective index was Topix Small in Japan. Sharpe and Sortino ratios, alongside with Geometric Mean were higher when compared to others.

Conceptually, a better performance per unit of volatility is achieved and exposure decreases. Since the nature of risk averse is to invest in a more conservative manner to avoid risk, our optimization problem proved effective to capture his risk preferences. The repeated minimizations of CVaR showed optimal holdings in Topix Small, and by observing these measures it is clear why these results were raised.

2. Risk averse's and risk neutral's holdings involved FTSE 100 price index. In the beginning of our analysis, this result seemed controversial, but it was revealed that this also was optimal solution for these investors. In Table 5.2 the lowest correlation was exhibited between Topix Small and US Citigroup 7-10Y price indices. However, this would imply fewer foreign positions in foreign markets, thus a decrease in international diversification benefits alongside with a long-term increase in exposure.
3. Including more units of foreign currency in a portfolio usually ensures the mitigation of risk since a negative movement in one currency can be counterbalanced by a positive movement in the other currency.
4. Observing the statistics of exchange rates and the correlations between exchange rate movements and indices returns a lot of interesting results were raised. Our first finding was, that under our analysis the exchange rate of Pound to Dollar deteriorated. This fact played an important role in interpreting the results. Firstly, since United States' stock indices and Japan's large cap stock index were positively correlated to this exchange rate, based on the progress of British Pound to Dollar, it would seem logical that their returns would also decrease. Thus, holding these indices in the portfolio, would not be optimal.
5. Bond indices in Japan do not exhibit significantly important correlation with exchange rates. Conceptually, any movement of them will not affect their value. It is known from literature that international diversification is enhanced with the exchange rate presence. If exchange rate is considered unimportant, any benefits from movements will be absent, thus the investor will not gain the actual international diversification benefits. Currency risk raises the exposure but in the long run any movements may be cancelled out. In the short-term approach, benefits can be gained from exposure to currency risk. In our research bond indices in Japan were not considered optimal holdings. For the above reasons, the results seem more logical. Furthermore, regarding these indices, normality is rejected at 1% level for both, and at 5% for Citi Japanese Government Bond 1-3Y price index.

6. Topix Large presented statistically important correlation with both exchange rates. It is positively correlated with British to Dollar movements and negatively correlated with Yen to Dollar movements. Under this case portfolio returns would deteriorate. Since Pound to Dollar depreciates and Yen to Dollar in considered not to lose its value, including this index to the portfolio would be not wise. Both correlations would drive the value of Topix Large to lower levels, thus a portfolio that would involve this index would exhibit more negative returns.
7. The unfavorable correlation of Topix Small with Yen to Dollar exchange rate, is not considered important since its performance is superior of all.

The results show the effectiveness of our modeling method. The best optimal holdings are selected under each holding period, in order to lower the level of expected return. Our holistic approach to review the results showed that the best solution to achieve an effective balance between risk and expected return was combining FTSE 100 and Topix Small price indices. Key notions of our estimation technique were statistics of exchange rates and the correlations between indices returns with the appreciation rates of exchange rates. Portfolio composition return and value strongly depend on the co-movement of indices and exchange rates. After all, modern portfolio theory is based on correlations between assets involved in a portfolio. Diversification is raised through low levels of correlation since negative movements can be mitigated. Low or negative correlation is desirable not only between exchange rates, prices of assets in a portfolio, but also between assets' returns and exchange rate movements. The appropriate choice of holdings based on correlation, is extremely important for the portfolios' performance. Undesirable variations can be mitigated through this way as well.

We should notice that exchange rate is a very important parameter in international portfolio management. It can be viewed as a different asset in a portfolio, since its importance is high. Movements of indices depend on exchange rates; therefore, portfolio value is prone to exchange rate variations.

Differences in currencies among markets lowers correlation. Exchange rates are a very important parameter in the economic field which have a huge impact on the prices. However, exchange rates across world tend to exhibit differences based on the welfare of country's economy, production, politics etc. Conceptually, involving different currencies in a portfolio decreases exposure, since currencies rarely exhibit similar progress.

It is clear that the effectiveness of our optimization problem was ensured by interpreting the results of all investors. Furthermore, diversification benefits were present through our study. Specifically, risk averse is the investor who allocated funds in two markets in a balanced way (not giving greater importance in one index), thus more units were included in both markets as opposed to other investors. Diversification of funds enhanced his long-term return and risk adjusted performance.

Our analysis showed that international allocation of funds in international investment opportunities can be extremely beneficial for investors. Risk averse, not only exhibited the lowest volatility, but also gained superior levels of return in the long run.

5.2.1 Chapter Epilogue

This Chapter was focused on analytically analyzing the results from our repeated optimization problems, followed by discussing the key concluding remarks of our analysis. Conclusions were derived by viewing the results under a holistic approach which included several statistical measures for all variables involved in our empirical analysis.

Our modeling method proved effective to demonstrate risk profiles for each investor and the diversification importance and benefits. In particular, risk lover was rightfully treated since we aimed to show the repeated investments he undertakes in high volatility stock index. Absence of diversification, thus increased exposure deteriorated his ex post returns and worsened the portfolio risk adjusted performance. On the other hand, risk averse and risk neutral constructed superior portfolios. From our estimations of statistical measures and ex post cumulative returns, it was revealed that risk averse was the investor who constructed the portfolio with the best ex post performance. His investing strategy involved a better balance between amounts invested in United Kingdom and Japan since no relatively greater importance in one index was given. Conceptually, he diversified more. International Diversification benefits were clear under this case. Our analysis proved effective to demonstrate that by allocating income internationally, expected returns increase and portfolio's performance enhances. Thus, an effective balance between risk and expected return is achieved.

Furthermore, it is worth noticing that under each month of our analysis no allocation of income in the domestic market was noticed. Investors repeatedly invest in foreign markets. Since our analysis is based on historical data and the nature of our optimization problem is accurate and carefully address, we should conclude that the decision to perform foreign investments is always optimal and linked to increased benefits.

International portfolio management is a complex and very interesting issue for investors, academics, countries companies etc. In addition, the endeavor to benefit from international diversification requires special attention and organization. The approach followed in this study proved effective to address portfolio management in a unified manner and achieve balance between expected return and risk. Repeated optimizations that capture the historical evolution of random variables is a promising method to construct optimal portfolios and demonstrate international diversification benefits.

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https://www.irmi.com/articles/expert-commentary/sailing-through-rough-winds-of-macroeconomic-risks

Appendix A

Figure 2.1

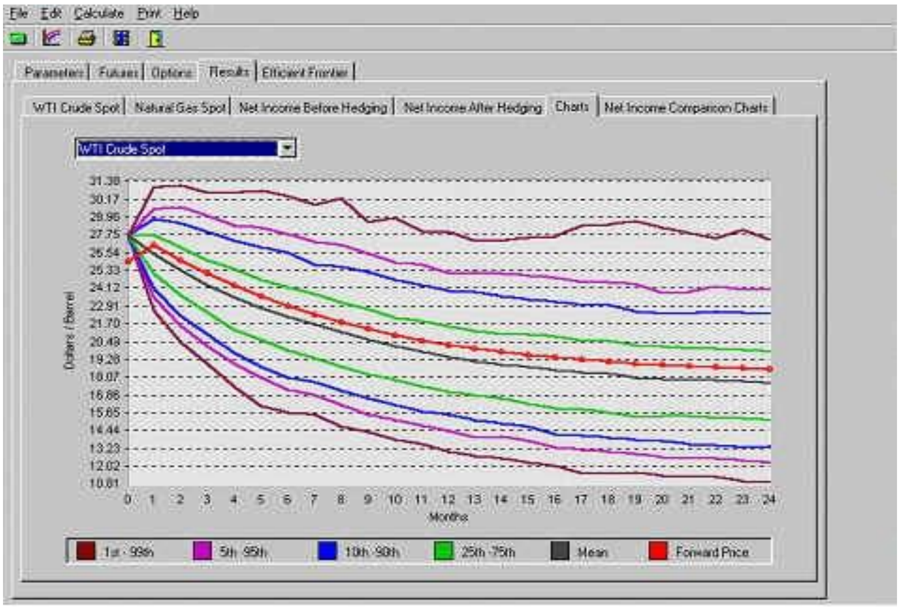
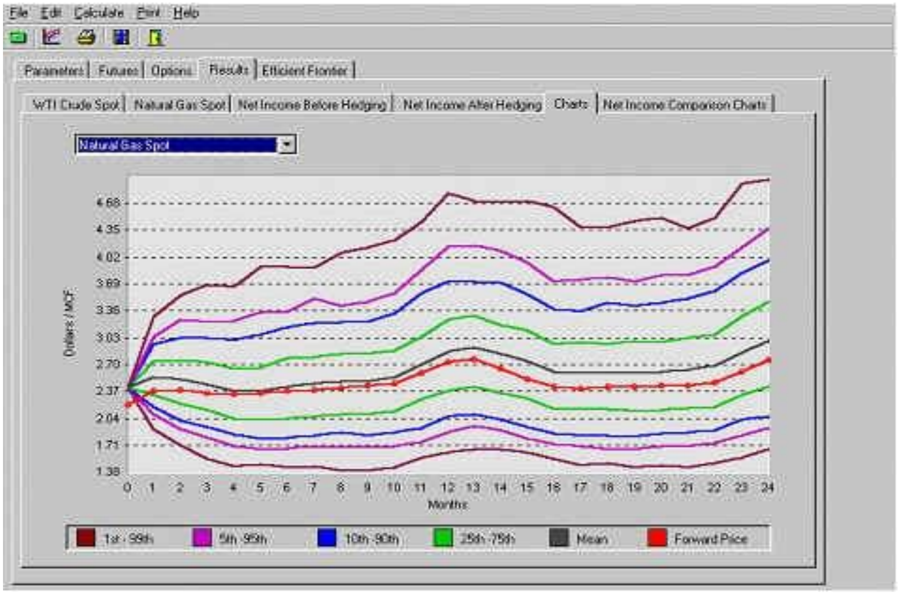


Figure 2.2



Figures 2.1 and 2.2 illustrate summary statistics from generating 1,000 scenarios for the movement of crude oil and natural gas, respectively, over the next 24 months (this was done as of late November 1999). The black curve indicates, for each month, the mean value of the price over all 1,000 scenarios. The other colored lines indicate various percentiles for the price at each month. Although only certain percentiles are plotted, since we have all the data from the 1,000 scenario simulations, we can plot a full probability distribution of the price for each month. These figures were used for illustration purposes on Chapter 2.

Appendix B

Figure 2.3 Efficient Set

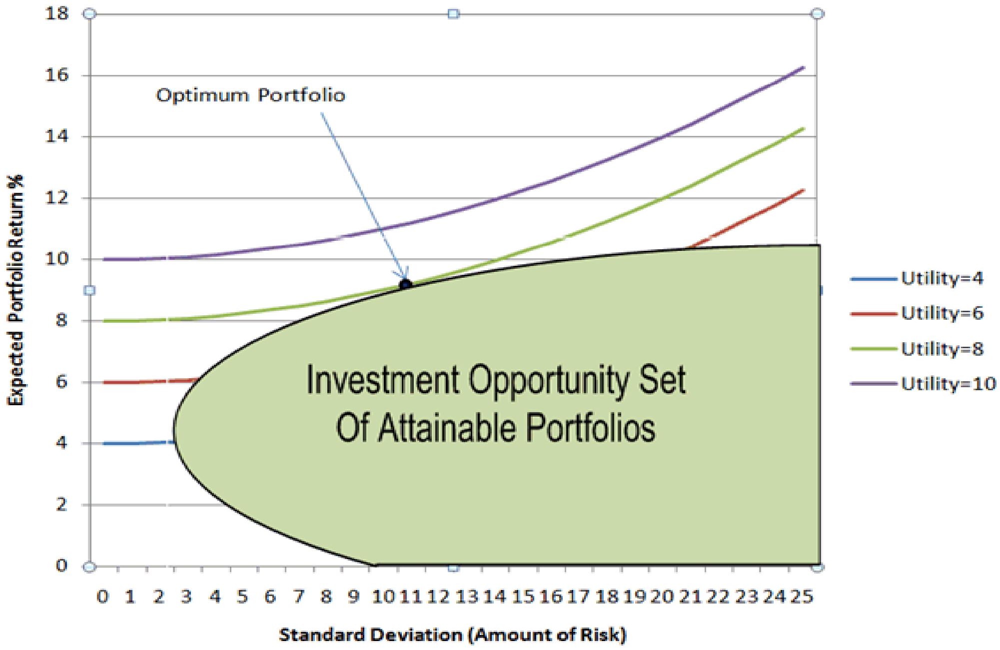
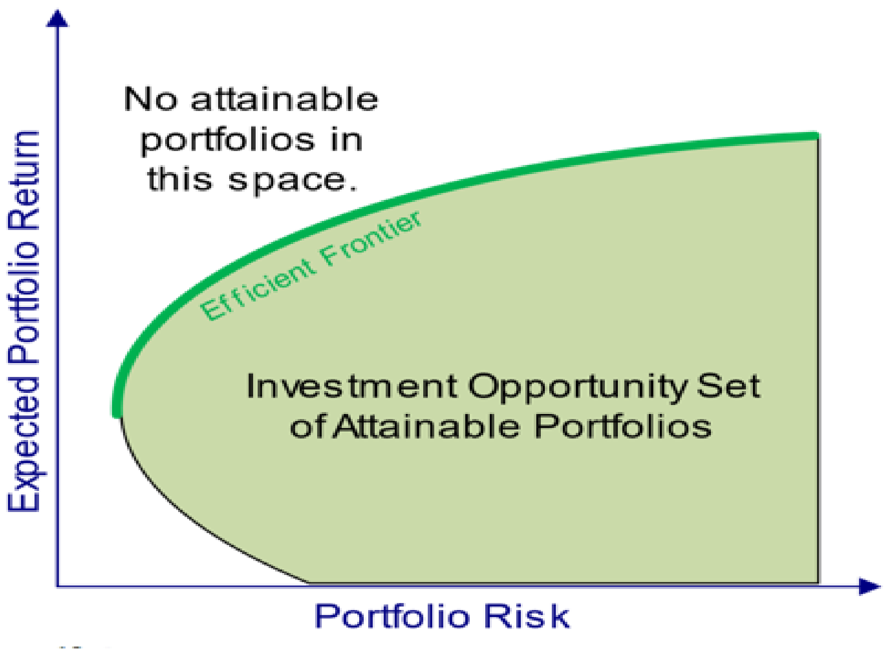


Figure 2.4 Efficient Frontier



Figures 2.3 and 2.4 demonstrate the efficient set and efficient frontier. They were presented for illustration purposes to better understand their importance in the financial study.

Appendix C

Exchange Rates (Yen to Dollar, Pound to Dollar) over period 1/1994-6/2018.

	Japanese Yen to US Dollar	British Pound to US Dollar
31/1/1994	0.009601536	1.486104919
1/2/1994	0.00973236	1.484494455
2/2/1994	0.009853673	1.516300227
3/2/1994	0.009555205	1.511692945
4/2/1994	0.010137362	1.54340042
30/6/1994	0.009982531	1.536499547
29/7/1994	0.009982531	1.536499547
31/8/1994	0.010104073	1.576988583
30/9/1994	0.010314595	1.630496812
31/10/1994	0.010106114	1.565092184
30/11/1994	0.010023054	1.564504521
30/12/1994	0.010106114	1.587503175
31/1/1995	0.010341262	1.580303102
28/2/1995	0.011574077	1.628797133
31/3/1995	0.01190193	1.60929529
28/4/1995	0.011814048	1.588587587
31/5/1995	0.011778566	1.590887397
30/6/1995	0.011358476	1.600102407
31/7/1995	0.01020929	1.549090684
31/8/1995	0.010136849	1.582704208
29/9/1995	0.009780429	1.57790927
31/10/1995	0.009849306	1.529496337
30/11/1995	0.00969415	1.552602161
29/12/1995	0.009353225	1.511007691
31/1/1996	0.009508415	1.530901242
29/2/1996	0.009363287	1.526507808
29/3/1996	0.009546084	1.50060024
30/4/1996	0.009250694	1.549402705
31/5/1996	0.009117017	1.553808384
28/6/1996	0.009369876	1.557098813
31/7/1996	0.009208103	1.562988434
30/8/1996	0.008978676	1.563403843
30/9/1996	0.008791982	1.628399284
31/10/1996	0.008787346	1.680305143
29/11/1996	0.008615491	1.711303157
31/12/1996	0.008237232	1.60251274
31/1/1997	0.008285691	1.631507676
28/2/1997	0.008076892	1.642009162
31/3/1997	0.0078796	1.622612731
30/4/1997	0.008581481	1.63599182

30/5/1997	0.008738967	1.664308896
30/6/1997	0.008445946	1.637492017
31/7/1997	0.008313243	1.621796951
29/8/1997	0.008282945	1.615404497
30/9/1997	0.008314625	1.676895731
31/10/1997	0.007835455	1.684806416
28/11/1997	0.007690829	1.645386337
31/12/1997	0.007886435	1.635002126
30/1/1998	0.007915776	1.646497077
27/2/1998	0.007498781	1.674593074
31/3/1998	0.007568304	1.671989166
30/4/1998	0.00721683	1.630709522
29/5/1998	0.00720539	1.668502019
30/6/1998	0.006926167	1.63599182
31/7/1998	0.007082655	1.67450895
31/8/1998	0.007346459	1.699408606
30/9/1998	0.008581481	1.674705252
30/10/1998	0.008135373	1.650600819
30/11/1998	0.008865248	1.663810459
31/12/1998	0.008599931	1.642791431
29/1/1999	0.00842815	1.601999295
26/2/1999	0.008443806	1.614309237
31/3/1999	0.008377665	1.610098538
30/4/1999	0.008210518	1.60330923
31/5/1999	0.008261732	1.57629256
30/6/1999	0.008713458	1.620010368
30/7/1999	0.009120343	1.608001415
31/8/1999	0.009392317	1.646903821
30/9/1999	0.009582215	1.640904467
29/10/1999	0.009786172	1.592686384
30/11/1999	0.009769918	1.611707443
31/12/1999	0.009341866	1.620903167
31/1/2000	0.009102908	1.578706408
29/2/2000	0.009749915	1.595303427
31/3/2000	0.009252834	1.564798297
28/4/2000	0.009287207	1.494701284
31/5/2000	0.009451349	1.51388994
30/6/2000	0.009125753	1.497701029
31/7/2000	0.009377344	1.454989888
31/8/2000	0.009254118	1.478502573
29/9/2000	0.009158348	1.452095374
31/10/2000	0.009027308	1.417595192
30/11/2000	0.008756567	1.493808165
29/12/2000	0.008599561	1.461091143
31/1/2001	0.008525513	1.442293824
28/2/2001	0.007979572	1.421706617
30/3/2001	0.008094872	1.430799388
30/4/2001	0.008416092	1.420595798

31/5/2001	0.00801796	1.406390639
29/6/2001	0.008007367	1.425191688
31/7/2001	0.008403008	1.450494619
31/8/2001	0.008394191	1.469702091
28/9/2001	0.008169601	1.454291615
31/10/2001	0.008122157	1.426106302
30/11/2001	0.007630093	1.455392228
31/12/2001	0.007475238	1.413307705
31/1/2002	0.007472445	1.414407151
28/2/2002	0.007545177	1.423994304
29/3/2002	0.007788162	1.457301078
30/4/2002	0.008057693	1.463207644
31/5/2002	0.008343067	1.524297299
28/6/2002	0.008350033	1.562207086
31/7/2002	0.008434192	1.546910047
30/8/2002	0.008214227	1.572599034
30/9/2002	0.008162599	1.564504521
31/10/2002	0.008162266	1.556008527
29/11/2002	0.00842673	1.609891171
31/12/2002	0.008337502	1.643709524
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28/2/2003	0.008433125	1.58065281
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29/8/2003	0.008951349	1.661405549
30/9/2003	0.009096284	1.696957355
31/10/2003	0.009130752	1.719838335
28/11/2003	0.009330969	1.790157713
31/12/2003	0.009447331	1.820233718
30/1/2004	0.00915248	1.855804027
27/2/2004	0.00961215	1.837863667
31/3/2004	0.009061665	1.773364072
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28/2/2005	0.009349289	1.88960904
31/3/2005	0.009531526	1.909891327
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31/8/2005	0.00882301	1.769097407
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30/11/2006	0.00839243	1.957138663
29/12/2006	0.008267196	1.957406827
31/1/2007	0.008439887	1.959746801
28/2/2007	0.008469193	1.961361185
30/3/2007	0.008367851	1.999840013
30/4/2007	0.00820917	1.978082842
31/5/2007	0.008097494	2.006340035
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31/8/2007	0.008694518	2.037365279
28/9/2007	0.008674155	2.077360921
31/10/2007	0.0090151	2.056132415
30/11/2007	0.008951349	1.990604347
31/12/2007	0.009405126	1.987992525
31/1/2008	0.009605687	1.989218436
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29/8/2014	0.009116186	1.621139661
30/9/2014	0.008920209	1.599846415
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28/11/2014	0.008340631	1.559259664
31/12/2014	0.008513174	1.501907422
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30/10/2015	0.008111616	1.505298651
30/11/2015	0.008312897	1.47390452
31/12/2015	0.008260026	1.418500078
29/1/2016	0.008859358	1.393592263
29/2/2016	0.008897193	1.43729788
31/3/2016	0.009346231	1.464900973
29/4/2016	0.009016726	1.455498144
31/5/2016	0.009747539	1.336791166
30/6/2016	0.009757525	1.327703869
29/7/2016	0.009667907	1.309706233
31/8/2016	0.00987508	1.299004962
30/9/2016	0.009516106	1.220896871
31/10/2016	0.008776164	1.249406532
30/11/2016	0.008573756	1.235651003
30/12/2016	0.008882967	1.258099012
31/1/2017	0.008937749	1.244400199
28/2/2017	0.008974244	1.250453289
31/3/2017	0.008971024	1.293744744
28/4/2017	0.009042818	1.290955565

31/5/2017	0.008899964	1.298954342
30/6/2017	0.009050183	1.3183568
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31/8/2017	0.008883756	1.341651573
29/9/2017	0.008800493	1.327950706
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30/3/2018	0.009138679	1.377353553
30/4/2018	0.009204289	1.330654283
31/5/2018	0.009028123	1.320254017
29/6/2018	0.009028123	1.320254017

Observing Yen to Dollar it is clear that the initial and final value of the exchange rate does not exhibit substantial difference. On the other hand, Pound to Dollar depreciated importantly.