Aggregate and distributional implications of fiscal consolidation in general equilibrium

Author

Roumpanis Eleftherios-Theodoros

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Declaration

I declare that this thesis does not incorporate without acknowledgement any material previously submitted for a degree or diploma in any university and that to the best of my knowledge does not contain any materials previously published or written by another person except where due reference is made in the text. The views expressed in this thesis are those of the author only, and do not necessarily reflect the institutions that the author is currently or previous affiliated with.

Eleftherios-Theodoros Roumpanis
June 2019

Abstract

This thesis is about the aggregate and distributional implications of fiscal consolidation in New Keynesian D(S)GE models. The thesis studies how these implications depend on the specific fiscal policy instrument used for debt consolidation. Chapter 2 presents a closed-economy New Keynesian D(S)GE model. Chapter 3 extends the model of Chapter 2 to set up a New Keynesian model of a small open economy within a monetary union facing sovereign interest rate premia. Finally, Chapter 4 builds a New Keynesian D(S)GE model consisting of two heterogeneous countries participating in a monetary union.

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INTRODUCTION

This thesis is about the aggregate, and especially the distributional, implications of fiscal consolidation in New Keynesian D(S)GE models. The thesis studies how these implications depend on the specific fiscal policy instrument used for debt consolidation over time. It studies both open and closed economies.

The thesis is structured as follows. Chapter 2 presents a closed-economy New Keynesian D(S)GE model. Chapter 3 extends the model of Chapter 2 to set up a New Keynesian model of a small open economy within a monetary union facing sovereign interest rate premia. Finally, Chapter 4 builds a New Keynesian D(S)GE model consisting of two heterogeneous countries participating in a monetary union. The main value added of the thesis is the study of distributional implications. Using models with ex ante agent heterogeneity enables me to evaluate not only the aggregate implications but also the distributional effects of fiscal consolidation over time. By over time, I mean both the short run phase of fiscal pain and the long run phase of fiscal gain once consolidation has been accomplished. The anticipation of the latter is crucial to the whole time path. A review of the related literature and how the thesis differs will be provided in each chapter. The same applies to the policy results. However, a general result seems to be that fiscal consolidation strategies, which use the fiscal gain to enhance the aggregate economy in the long run, can be beneficial to all types of agents over the time path.

CHAPTER 2. DEBT CONSOLIDATION: ITS AGGREGATE AND DISTRIBUTIONAL IMPLICATIONS

Debt consolidation: Its aggregate and distributional implications

Abstract

This chapter builds and solves numerically, by using Eurozone data, a closed-economy new Keynesian D(S)GE model in which the fiscal authorities are engaged in public debt reduction over time. The emphasis is on the aggregate and distributional implications of debt consolidation, where agent heterogeneity, and hence distribution, has to do with the distinction between ”capitalists” and ”workers”. The paper studies how these implications depend on the specific fiscal policy instrument used for debt consolidation. There are two key results. First, if the criterion is aggregate, or per capita, output (GDP), the best policy mix is to use the long term fiscal gain created by debt reduction to cut the capital tax rate and, during the early period of fiscal pain, to use spending cuts to bring public debt down. Second, if the criterion is equity in net incomes, the best recipe is to use the long term fiscal gain created by debt reduction to cut the labor tax rate and, during the early period of fiscal pain, to use capital taxes to bring public debt down.

1 Introduction

The 2008 world crisis has, among other things, brought into the spotlight the need for debt consolidation in several European economies. Proponents claim that debt sustainability is necessary for the revival of these economies (see e.g. European Commission, 2015, and CESifo, 2016). Opponents, on the other hand, claim that debt consolidation worsens the recession and may increase the public debt-to-GDP ratio at least in the short term; in addition, it is claimed that debt consolidation worsens inequality since fiscal austerity hurts the relatively poor. Distributional implications of debt reductions are an important issue since spending cuts and/or tax rises can affect different people/groups in different ways; even a uniform change in policy can have different effects simply because agents are heterogeneous.

This paper provides a quantitative study of the aggregate and distributional implications of debt consolidation in a new Keynesian D(S)GE model solved numerically using common parameter values and fiscal data from the Euro area. To study distributional implications, we obviously need a model with agent heterogeneity. There are many types of such heterogeneity. Here, we focus on a specific type which has always been popular in the related macro literature: the distinction between capitalists and workers. Capitalists are defined as those households who hold assets and own the firms. Workers are defined as those households with labor income only.[1] These two types are also called Ricardian and non-Ricardian or optimizing and liquidity constraint households in the DSGE literature. The study of distributional implications differentiates this chapter/paper from most of the existing literature on debt consolidation. The latter has focused on aggregate implications only (see e.g. Philippopoulos et al., 2015, 2017a and 2017b of the references therein).

The model is as follows. We use a rather standard New Keynesian D(S)GE model of a closed economy featuring imperfect competition and Rotemberg-type price rigidities. The model is solved numerically employing commonly used parameter values and fiscal data from the Euro area. Then, we assume that the debt policy target in the feedback fiscal policy rules is below the data average (from 95% to 60%) and we study the aggregate and distributional

[1]This type of agent heterogeneity (capitalists and workers) has been very common especially in the literature on fiscal policy. A well-known early paper is Judd (1985). Woodford (1989) has also used it discussing in detail the underlying assumptions about participation in financial markets. Lansing (2015) provides a recent review of macro models on capitalists and workers. Judd (1985) is probably the first paper on the implications of optimal tax policy on capitalists and workers.

implications of various policies aiming at such debt consolidation.

Results will be relative to the status quo where the status quo is defined as the case without debt consolidation. The main results are as follows. First, if the criterion is aggregate, or per capita, output (GDP), the best policy mix is to use the long term fiscal gain (namely, the fiscal space created once debt has been reduced) to cut the capital tax rate and, during the early period of fiscal pain, to use spending cuts to bring public debt down.

Second, the above policy mix is Pareto efficient (i.e. both capitalists and workers get better off with this type of debt consolidation). But, if we care about relative gains, there is a "social" cost: inequality (measured by the ratio of the capitalist's to the worker's net income) rises both in the new steady state and in the transition.

Third, if the criterion is equity in net incomes (although this comes at a lower benefit at aggregate level relative to the above policy mix), the recipe is to use the long term fiscal gain to cut the labor tax rate and, during the early period of fiscal pain, to use capital taxes to bring public debt down.

Fourth, using labor taxes or consumption taxes during the early period of fiscal pain is a bad idea both in terms of aggregate output and equity.

Section 2 presents the model. Section 3 presents the data, parameter values and the steady state solution. Section 4 explains how we model debt consolidation. The main results are in Section 5. Robustness checks are in Section 6, while, Section 7, which presents the conclusions, closes the paper. Details are in the appendix.

2 Model

The model is a New Keynesian closed-economy model featuring imperfect competition and Rotemberg-type nominal price rigidities(see e.g. Bi et al., 2013), which is extended to include a relatively rich menu of fiscal policy instruments as well as two social classes, called capitalists and workers.

2.1 Households

There are two types of households, a pool of identical capitalists and a pool of identical workers. The percentage of capitalists in the population is v_t^k, while that of workers is v_t^w. Hence, there are $\frac{v_t^w}{v_t^k}$ times more workers than capitalists, with the total number of capitalists normalized to one (see also Lansing, 2015). These population fractions of capitalists and workers at time

t are set exogeneously and are assumed to remain constant over time ruling out occupational choice and mobility across groups.

Capitalists own the firms, hold capital, money and government bonds and also receive labor income for their managerial services. Workers hold money and receive labor income for their labor services.

Households as capitalists

Each capitalist k acts competitively to maximize expected discounted lifetime utility:

$$E_o \sum_{t=0}^{\infty} \beta^t U\left(c_t^k, n_t^k, m_t^k, g_t\right) \tag{1}$$

where c_t^k is k's consumption at t, n_t^k is k's hours of work at t, m_t^k is k's end-of-period real money balances at t, g_t is total government spending at t divided by the number of capitalists implying that the per capita public spending is defined as $v^k g_t$, E_o is the rational expectations operator conditional on the current period information set and $0 < \beta < 1$ is the time preference rate.

In our numerical solutions, we will use a utility function of the form (see also e.g. Gali, 2008):

$$U\left(c_t^k, n_t^k, m_t^k, g_t\right) = \left[\frac{(c_t^k)^{1-\sigma}}{1-\sigma} - x_n \frac{(n_t^k)^{1+\eta}}{1+\eta} + x_m \frac{(m_t^k)^{1-\mu}}{1-\mu} + x_g \frac{(v^k g_t)^{1-\zeta}}{1-\zeta}\right] \tag{2}$$

where x_n, x_m, x_g, σ, η, μ, ζ are standard preference parameters.

The budget constraint of each k (written in real terms) is:

$$\begin{aligned}(1+\tau_t^c)c_t^k + x_t^k + b_t^k + m_t^k =& (1-\tau_t^k)[r_t^k k_{t-1}^k + d_t^k] + (1-\tau_t^n) w_t^k n_t^k \\ &+ R_{t-1}\frac{P_{t-1}}{P_t} b_{t-1}^k + \frac{P_{t-1}}{P_t} m_{t-1}^k - \tau_t^{l,k}\end{aligned} \tag{3}$$

where P_t is the price index at t and small letters denote real variables e.g. $b_t^k \equiv \frac{B_t^k}{P_t}$, $d_t^k \equiv \frac{D_t^k}{P_t}, w_t^k \equiv \frac{W_t^k}{P_t}$. Here x_t^k is k's real investment at t, b_t^k is k's end-of-period real government bonds at t, d_t^k is k's real dividends paid by firms at t, w_t^k is capitalists' real wage rate at t, k_t^k is k's end-of-period capital at t, $R_{t-1} \geq 1$ is the gross nominal return to government bonds between $t-1$ and t, r_{t-1}^k is the gross real return to inherited capital between $t-1$ and t, $\tau_t^{l,k}$ are the real lump-sum taxes/transfers to each household k from the government at t, τ_t^c is the tax rate on consumption at t, τ_t^k is the tax rate on capital income at t and τ_t^n is the tax rate on labor income at t.

The motion of physical capital for each k is:

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k \tag{4}$$

where $0 < \delta < 1$ is the depreciation rate of capital.

Details of the above problem and its solution are in Appendix A.

Households as workers

Workers have the same utility function as capitalists (see Eqs.(1) and (2)). Each worker w acts competitively to maximize expected discounted lifetime utility taking prices and policy as given.

The budget constraint of each w (written in real terms) is:

$$(1+\tau_t^c)c_t^w + m_t^w = (1-\tau_t^n)w_t^w n_t^w + \frac{P_{t-1}}{P_t}m_{t-1}^w - \tau_t^{l,w} \tag{5}$$

where again small letters denote real variables e.g.$w_t^w \equiv \frac{W^w}{P_t}$. Here c_t^w is w's consumption at t, n_t^w is w's hours of work at t, m_t^w is w's end-of-period real money balances at t, w_t^w is workers' real wage rate at t and $\tau_t^{l,w}$ are the real lump-sum taxes/transfers to each household w from the government at t.

Details of the above problem and its solution are in Appendix B.

2.2 Firms

The production sector consists of two sectors: the intermediate goods sector and the final goods sector. Following the literature on imperfect competition in product markets, we assume that the final goods sector is perfectly competitive, while each intermediate goods firm acts as a monopolist in its own market. The final goods production "technology" is a constant elasticity (CES) bundler of intermediate goods. Profit maximization in the final goods sector (which is competitive) yields a downward sloping demand curve for intermediate goods producers. Intermediate goods firms choose factors of production subject to this demand curve for their product facing Rotemberg-type nominal price rigidities(the latter allows for non-neutrality of money).

Final goods firms

Assume, for simplicity, that the single final good is produced by one firm. There is also a continuum (i.e. infinity) of intermediate goods firms that are indexed along the unit interval. The production function of the final good is a Dixit-Stiglitz type constant returns to scale technology:

$$y_t = \left[\int_0^1 [y_t(f)]^{\frac{\phi-1}{\phi}} df \right]^{\frac{\phi}{\phi-1}} \tag{6}$$

where y_t is the production of the final goods firm, $y_t(f)$ is the production of the variety f produced monopolistically by the intermediate goods firm f and $\phi > 0$ is the elasticity of substitution across intermediate goods produced.

Nominal profits of the final goods firm are defined as:

$$P_t y_t - \int_0^1 P_t(f) y_t(f) df \tag{7}$$

where $P_t(f)$ is the price of variety f.

The final goods firm chooses the quantity of every variety, $y_t(f)$, to maximize its profits (more generally it would want to maximize the sum of the expected discounted lifetime profits, but there is nothing that makes the problem interesting in a dynamic sense as it just buys the intermediate goods period by period. Hence, equivalently, the final goods firm could maximize profits period by period instead) subject to its production "technology". The objective function of the final goods firm in real terms is given by:

$$\max \left[y_t - \int_0^1 \frac{P_t(f)}{P_t} y_t(f) df \right] \tag{8}$$

Details of the above problem and its solution are in Appendix C.

Intermediate goods firms

There are intermediate goods firms, indexed by f, whose total mass is 1. Each firm f produces a differentiated good of variety f under monopolistic competition facing Rotemberg-type nominal price rigidities(see e.g. Bi et al., 2013). Nominal profits of firm f are defined as:

$$D_t(f) = P_t(f)y_t(f) - P_t r_t^k k_{t-1}(f) - W_t^w n_t^w(f) - W_t^k n_t^k(f) - \frac{\phi^p}{2}\left(\frac{P_t(f)}{P_{t-1}(f)\pi} - 1\right)^2 P_t y_t \tag{9}$$

where ϕ^p is a parameter which determines the degree of nominal price rigidity, π stands for the steady state value of the inflation rate, $n_t^k(f)$ is the demand of firm f for capitalists' hours of work at t, $n_t^w(f)$ is the demand of firm f for workers' hours of work at t and $k_t(f)$ is the demand of firm f for physical capital at t. Notice that the quadratic cost that the firm f faces when it changes the price of its product is proportional to aggregate output.

All firms use the same technology represented by the production function(similar to e.g. Hornstein et al., 2005):

$$y_t(f) = A_t\left[k_{t-1}(f)\right]^{\alpha}\left[\{n_t^k(f)\}^{\theta}\{n_t^w(f)^{1-\theta}\}\right]^{1-\alpha} \tag{10}$$

where A_t is an exogenous TFP, α is the share of capital and θ is the labor efficiency parameter of capitalist.

Profit maximization by firm f is also subject to the demand for its product that comes from the solution of the final goods firm's problem as specified above (see Appendix C for details):

$$P_t(f) = \left(\frac{y_t(f)}{y_t}\right)^{-\frac{1}{\phi}} P_t \tag{11}$$

Each firm f chooses its price, $P_t(f)$, and its inputs, $k_t(f)$, $n_t^k(f)$, $n_t^w(f)$, to maximize the sum of expected discounted lifetime real dividends, $\max E_o \sum\limits_{t=0}^{\infty} \Xi_{0,0+t}\frac{D_t(f)}{P_t}$, subject to the demand for its product and its production function. The objective function of firm f in real terms is given by:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t}\left[\frac{P_t(f)}{P_t}y_t(f) - r_t^k k_{t-1}(f) - w_t^w n_t^w(f) - w_t^k n_t^k(f) - \frac{\phi^p}{2}\left(\frac{P_t(f)}{P_{t-1}(f)\pi} - 1\right)^2 y_t\right] \tag{12}$$

where $\Xi_{0,0+t}$ is a stochastic discount factor taken as given by the firm f, which arises from the Euler of bonds and is defined as $\Xi_{0,0+t} = \prod\limits_{i=0}^{t-1}\left\{\frac{1}{R_i}\right\} = \beta^t \prod\limits_{i=0}^{t-1}\left[\left(\frac{P_i}{P_{i+1}}\right)\left(\frac{1+\tau_i^c}{1+\tau_{i+1}^c}\right)\left(\frac{c_{i+1}^k}{c_i^k}\right)^{-\sigma}\right]$.

Details of the above problem and its solution are in Appendix D.

2.3 Government budget constraint

The budget constraint of the "consolidated" public sector expressed in real terms is:

$$
\begin{aligned}
g_t + R_{t-1}\frac{P_{t-1}}{P_t}b^k_{t-1} + \frac{P_{t-1}}{P_t}\left[m^k_{t-1} + \frac{v^w}{v^k}m^w_{t-1}\right] = {} & b^k_t + \left[m^k_t + \frac{v^w}{v^k}m^w_t\right] + \\
& + \tau^c_t\left[c^k_t + \frac{v^w}{v^k}c^w_t\right] + \\
& + \tau^k_t[r^k_t k^k_{t-1} + d^k_t] + \\
& + \tau^n_t\left[w^k_t n^k_t + \frac{v^w}{v^k}w^w_t n^w_t\right] + \tau^l_t
\end{aligned} \tag{13}
$$

where $\tau^l_t \equiv \left[\tau^{l,k}_t + \frac{v^w}{v^k}\tau^{l,w}_t\right]$. All other variables have been defined above.[2] As above, small letters denote real variables.

In each period, one of the fiscal policy instruments has to follow residually to satisfy the government budget constraint (see below for details).

2.4 Decentralized Equilibrium (given policy instruments)

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of households maximizes utility; (ii) every type of firms maximizes profits; (iii) all constraints, including the government budget constraint, are satisfied; and (iv) all markets clear.

To proceed with the solution, we need to define the policy regime. Regarding monetary policy, we assume, as is usually the case, that the nominal

[2] Note that $\int_0^1 c^k_t dk \equiv c^k_t$, $\int_0^{\frac{v^w}{v^k}} c^w_t dw \equiv \frac{v^w}{v^k}c^w_t$, $\int_0^1 k^k_{t-1} dk \equiv k^k_{t-1}$, $\int_0^1 d^k_t dk \equiv d^k_t$, $\int_0^1 n^k_t dk \equiv n^k_t$, $\int_0^{\frac{v^w}{v^k}} n^w_t dw \equiv \frac{v^w}{v^k}n^w_t$, $\int_0^1 m^k_t dk \equiv m^k_t$, $\int_0^{\frac{v^w}{v^k}} m^w_t dw \equiv \frac{v^w}{v^k}m^w_t$, $\left[\int_0^1 g_t dk + \int_0^{\frac{v^w}{v^k}} g_t dw\right] \equiv \left[1 + \frac{v^w}{v^k}\right]g_t$, $\int_0^1 b^k_t dk \equiv b^k_t$, $\int_0^1 \tau^{l,k}_t dk \equiv \tau^{l,k}_t$, $\int_0^{\frac{v^w}{v^k}} \tau^{l,w}_t dw \equiv \frac{v^w}{v^k}\tau^{l,w}_t$.

interest rate, R_t, is used as a policy instrument, while money balances are endogenously determined. Regarding fiscal policy, we assume that, in the transition, tax rates and public spending, $\tau_t^c, \tau_t^k, \tau_t^n, \tau_t^l$ and g_t, are set exogenously, while the end-of-period public debt, b_t, follows residually from the government budget constraint (see Section 4 for a discussion of public financing cases).

Appendix E presents the dynamic DE system. It consists of 16 equations in 16 variables $[c_t^k, c_t^w, y_t, \pi_t, m_t^k, m_t^w, b_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, k_t^k, d_t]_{t=0}^{\infty}$. This is given the independently set policy instruments, $[R_t, \tau_t^c, \tau_t^k, \tau_t^n, \tau_t^l, g_t]_{t=0}^{\infty}$, technology $[A_t]_{t=0}^{\infty}$, and initial conditions for the state variables. All these variables have been defined above except for π_t and mc_t where π_t is the gross inflation rate, defined as $\pi_t \equiv \frac{P_t}{P_{t-1}}$, and mc_t is the intermediate goods firm's real marginal cost as defined in Appendix D.2.

2.5 Rules for policy instruments

Following the related literature, see e.g. Schmitt-Grohé and Uribe (2007) and Cantore et al.(2015), we focus on simple feedback rules for the exogenously set policy instruments, which means that the monetary and fiscal authorities react to a small number of endogenous macroeconomic indicators. In particular, we allow the nominal interest rate, R_t, to follow a standard Taylor rule meaning that it can react to inflation and output as deviations from a policy target, while we allow the distorting fiscal policy instruments, namely, government spending as share of output, s_t^g, the tax rate on consumption, τ_t^c, the tax rate on capital income, τ_t^k, and the tax rate on labor income, τ_t^n, to react to public debt, again as a deviation from a policy target. The target values are defined below.

In particular, we use policy rules of the following functional forms:

$$\log\left(\frac{R_t}{R}\right) = \phi_\pi \log\left(\frac{\pi_t}{\pi}\right) + \phi_y \log\left(\frac{y_t}{y}\right) \tag{14}$$

$$s_t^g = s^g - \gamma_l^g (l_{t-1} - l) \tag{15}$$

$$\tau_t^c = \tau^c + \gamma_l^c (l_{t-1} - l) \tag{16}$$

$$\tau_t^k = \tau^k + \gamma_l^k (l_{t-1} - l) \tag{17}$$

$$\tau_t^n = \tau^n + \gamma_l^n (l_{t-1} - l) \tag{18}$$

where ϕ_π, ϕ_y, γ_l^g, γ_l^c, γ_l^k and γ_l^n are feedback policy coefficients of positive value, variables without time subscripts denote target values, and where

$$l_t \equiv \frac{R_t b_t}{y_t} \tag{19}$$

denotes the end-of-period public debt burden as share of GDP.

2.6 Final Equilibrium system and solution methodology

The final equilibrium system consists of the 16 equations of the DE presented in Appendix E, the 5 feedback policy rules and the definition of l_t presented in Subsection 2.5. We thus end up with 22 equation in 22 variables $[c_t^k, c_t^w, y_t, \pi_t, m_t^k, m_t^w, b_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, k_t^k, d_t, R_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n, l_t]_{t=0}^{\infty}$. Among them, there are 16 non-predetermined or jump variables, $[c_t^k, c_t^w, y_t, \pi_t, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, d_t, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n]_{t=0}^{\infty}$, and 6 predetermined or state variables $[m_t^k, m_t^w, b_t^k, k_t^k, R_t, l_t]_{t=0}^{\infty}$. This is given the TFP, initial conditions for the state variables and the values of coefficients in the feedback policy rules.

To solve this non-linear difference equation system, we will take a first order approximation around a steady state and check saddle path stability. We first solve for the steady state of the model numerically employing common parameters values and data from the Euro area. The next section (Section 3) presents this steady state solution, or what we shall call status quo. In turn, we will compute each new reformed steady state and, then, study the transition dynamics, under various policy scenarios when we depart from the status quo and travel to a new reformed steady state with lower public debt.

3 Data, parameterization and steady state solution

This section solves numerically the above model economy by using conventional parameters and data from the Euro area. As we shall see, the model's steady state solution will resemble the main empirical characteristics of the Euro area.

3.1 Parameters and policy variables

Table 1 reports the baseline parameter values and Table 2 reports the values of exogenous policy variables used to solve the above model economy. The time unit is meant to be a quarter. Regarding parameters, we use relatively standard values often employed by the business cycle literature.

Let us briefly discuss the values summarized in Table 1. Using the Euler equation of bonds, the value of time preference rate, β, follows so as to be consistent with the average value of the real interest rate in the data, 0.0075 quarterly (see Table 2) or 0.03 annually. The share of capital in income, α, and the percentage of capitalists in population, v^k, are set at 0.33 and 0.2 respectively. The labor efficiency parameter of capitalists, θ, is set so that we obtain a reasonable value for the ratio of capitalists' wage to workers' wage, $\frac{w^k}{w^w}$, which, in our model, equals 1.68. The inverse of intertemporal substitution elasticity, σ, the inverse of Frisch labor elasticity, η, and the price elasticity of demand, ϕ, are set as in Andrès and Doménech(2006) and Gali (2008) in related studies. The inverse of public consumption elasticity in utility, ζ, is set at 1. The real money balances elasticity, μ, is taken from Pappa and Neiss (2005); this implies an interest-rate semi elasticity of money demand equal to -0.29 which is a common value in this literature. Regarding preference parameters in the utility function, χ_m, is chosen so as to obtain a value of real money balances as share of output equal to 1.97 quarterly, or 0.49 annually, which is close to the data (when we use the M1 measure, the average value in the annual data is around 0.5), χ_n, is chosen so as to abtain steady state labor hours equal to 0.28, while χ_g is arbitrarily set at 0.1 which is a common valuation of public goods in related utility functions. We set the Rotemberg's price adjustments cost parameter, ϕ^p, at 30 which, according to Keen and Wang (2007), corresponds to approximately 33 percent of the firms re-optimizing each quarter in a Calvo pricing model. Several related studies of the Euro area featuring Calvo price mechanism also set the probability of price readjustment at 1/3(see e.g. Gali et al., 2001). Concerning the exogenous TFP, A_t, it remains constant over time and equal to 1.

The effective tax rates on consumption, capital and labor are respectively $\tau^c = 0.2$, $\tau^k = 0.29$ and $\tau^n = 0.39$. These values are very close to the data averages for the Euro area over 2008-2011. The long-run nominal interest rate is 1.0075 quarterly for the Euro area in the same time period. Lump-sum taxes/transfers as share of output, s^l, and total public spending as share of output, s^g, are set -0.2 and 0.24 respectively so that their sum, $-s^l + s^g$, to be close to the data for the same time period as well. The public debt-to-output ratio follows residually from the steady state solution of the model and is equal to 3.8 quarterly (or 0.95 annually). This value is very close to the average value for the Euro area in 2015(3.6 quarterly or 0.94 annually).

The government imposes/gives a percentage, $\lambda_t^{l,k}$, of total lump-sum

Table 1: Parameter values

Parameter	Value	Description
v^k	0.2	share of capitalists in population
v^w	0.8	share of workers in population
α	0.33	share of capital
θ	0.2	labor efficiency parameter of capitalist
β	0.9926	time preference rate
μ	3.42	parameter related to money demand elasticity
δ	0.02	capital depreciation rate (quarterly)
ϕ^p	30	Rotemberg's price adjustments cost parameter
ϕ	6	price elasticity of demand
η	1	inverse of Frisch labor supply elasticity
σ	1	inverse of intertemporal substitution elasticity
ζ	1	inverse of public consumption elasticity in utility
χ_m	0.05	preference parameter related to real money balances
χ_n	6	preference parameter related to work effort
χ_g	0.1	preference parameter related to public spending
A	1	TFP level
ϕ_π	1.5	coefficient of nominal interest rate on inflation gap
ϕ_y	0.5	coefficient of nominal interest rate on output gap
γ_l^g	0.1	coefficient of government spending on debt gap
γ_l^c	0	coefficient of consumption tax rate on debt gap
γ_l^k	0	coefficient of capital tax rate on debt gap
γ_l^n	0	coefficient of labor tax rate on debt gap

Table 2: Policy variables (data average values)

Parameter	Value	Description
R	1.0075	long-run nominal interest rate
τ^c	0.20	consumption tax rate
τ^k	0.29	capital tax rate
τ^n	0.39	labor tax rate
s^g	0.24	government consumption spending as share of output
$-s^l$	0.2	government transfers as share of output
$\lambda^{l,k}$	0.2	percentage of total transfers to capitalists

taxes/transfers to the class of capitalists and a percentage, $\lambda_t^{l,w} \equiv 1 - \lambda_t^{l,k}$, to workers, where we set $\lambda_t^{l,k} = v^k$ and $\lambda_t^{l,w} \equiv 1 - v^k = v^w$ (See Appendix F for details). In other words, transfers are distributed to capitalists and workers according to their share in population.

Regarding the fiscal (tax-spending) policy instruments along the transition, these instruments can also react to the current state of public debt as a deviation from its steady state value,[3] where this reaction is quantified by the feedback policy coefficients in the policy rules (15)-(18). In our baseline experiments, we simply set the feedback policy coefficient of government spending at 0.1 (i.e. $\gamma_l^g = 0.1$) which is necessary for dynamic stability, while we switch off all other fiscal reactions to debt.[4] These baseline values of feedback policy coefficients are reported in Table 1. We report that our main results are robust to changes in these values (see Section 6 for details).

3.2 Steady state solution or the "status quo"

Table 3 reports the steady state solution of the model economy when we use the parameter values in Table 1 and the policy instruments in Table 2. As said, in this solution, the residual public financing instrument is public

[3]Since policy instruments react to deviations of endogenous macroeconomic indicators from their steady state values, feedback policy coefficients do not play any role in steady state solutions. Also, recall that "money is neutral" in the long run, so that the monetary policy regime also do not matter to the real economy at the steady state.

[4]In most experiments it is necessary for dynamic stability to allow at least one of the fiscal policy instruments to respond to debt. These values are close to those found by optimized policy rules in related studies (see e.g. Schmitt-Grohé and Uribe (2007) and Philippopoulos et al.(2015)). They are also consistent with calibrated or estimated values by previous research(see e.g. Leeper et al.(2010), Forni et al.(2010), Coenen et al.(2008), Cogan et al.(2013), Erceg and Lindé(2013)).

Table 3: Steady state solution or the "status quo"

Variables	Solution	Variables	Solution	Data
y	2.3255	l	3.8273	
c^k	0.5940	x^k	0.3379	
c^w	0.2089	d	0.3876	
n^k	0.1920	π	1	
n^w	0.3237	mc	0.8333	
k^k	16.0926	y^k	0.8657	
b^k	8.8342	y^w	0.2089	
m^k	1.5807	c/y	0.6147	0.57
m^w	1.1645	b/y	3.7988	3.76
r^k	0.0401	x/y	0.1453	0.18
w^k	1.3459	m/y	2.6830	
w^w	0.7981	k/y	6.9201	

Notes: Parameters and policy variables as in Tables 1 and 2.

debt. The solution makes sense and the resulting great ratios are close to their values in the actual data (recall that, since the time unit is meant to be a quarter, stock variables-like debt, capital and money balances- need to be divided by 4 to give the annual values). In what follows, we will depart from this solution and use it as benchmark to study the aggregate and distributional implications of various policy experiments.

4 How we model debt consolidation

In this section, following Philippopoulos et al.(2015), we explain how we model debt consolidation. We will assume that the government aims at reducing the share of public debt from approximately 95% $\left(\approx \frac{3.8}{4}100\%\right)$ of GDP, which is the steady state value and is also close to the data average, to the target value of say 60%. We choose the target value of 60% simply because it has been the reference rate of the Maastricht Treaty(we report however that our qualitative results are not sensitive to the value of the debt target assumed).

Obviously, debt reductions have to be accommodated by adjustments in the tax-spending policy instruments, which, in our model, are the output share of public spending, and the tax rates on capital income, labor income and consumption. Debt consolidation naturally implies a trade-off. In particular, it implies an inter-temporal trade-off between fiscal pain in the short term (i.e. spending has to fall and/or taxes have to rise) and fiscal gain in the medium and long term once the debt finally has been reduced (i.e. now spending can rise and/or taxes can fall).

This inter-temporal trade-off implies that the implications of debt consolidation depend heavily on the mix of public financing policy instruments used, namely, which policy instrument adjusts endogenously to accommodate the exogenous changes in fiscal policy (see also e.g. Leeper et al., 2010, for the USA). Specifically, these implications depend both on which policy instrument bears the cost of adjustment in the early period of adjustment and on which policy instrument is anticipated to reap the benefit, once consolidation has been achieved.

In the policy experiments considered below, we will experiment with fiscal policy mixes, which means that the fiscal authorities are allowed to use an instrument in the transition and perhaps a different one in the new steady state. For instance, let us assume that, once the public debt has been reduced in the new reformed steady state, it is the capital tax rate that reaps the benefits of the created fiscal space, then, in the transition to this particular reformed steady state, all fiscal instruments are available and, consequently, one of them can be employed, as in the policy rules in Subsection 2.5, to bring public debt down.

In particular, we will work as follows. First, we compute all possible reformed steady states and compare the status quo steady state solution to each of those new reformed steady state solutions (depending on which tax-spending instrument is residually determined by the new target value of debt-to-GDP ratio). Then, for each possible steady state case, we study the associated transitional dynamics. To compute the transition path, we log-linearize the model around the steady state solution for each reformed economy, then check its saddle-path stability and compute the equilibrium transition path towards each of the new reformed steady states using as initial conditions for the state variables their values in the status quo steady state. We will use the Matlab toolbox for computing steady state solutions, checking saddle-path stability, computing dynamics and making robustness checks of debt consolidation (Matlab routines are available upon request).

In all cases, we will study both aggregate and distributional implications. Regarding aggregate outcomes, we will look, for instance, at per capita output. Regarding distribution, we will compute separately the income of the representative capitalist vis-à-vis that of the representative worker. In the transition, we will work with present values of these variables. Next, all above variables values are compared to their respective values had we

Table 4: Values of the residual fiscal policy instruments in steady state

Residual Instrument	Status quo	New steady state
τ^k	0.29	0.27
τ^n	0.39	0.37
τ^c	0.20	0.18
s^g	0.24	0.25

Table 5: Output(**GDP**) in steady state

Residual Instrument	New steady state	% Change relative to the SQ
τ^k	2.3648	**+1.69 %**
τ^n	2.3496	**+1.04 %**
τ^c	2.3336	+0.35 %
s^g	2.3336	+0.35 %

Note: Steady state value of the output in the status quo(SQ) is 2.3255.

remained in the status quo economy permanently.

5 Results

5.1 Steady state results

We start with comparison of steady state solutions. Recall that in the SQ steady state, fiscal policy instruments were set as in the data and $\frac{b}{y}$ followed residually, while in the reformed steady state $\frac{b}{y}$ is ad hoc cut to 60% so that one of the fiscal policy instruments follows residually meaning that s^g is allowed to rise or one of τ^k,τ^n,τ^c is allowed to fall. Table 4 reports the value of the residual policy instrument in each case studied. They confirm that debt reduction allows for a tax cut and a spending rise.

Aggregate implications(efficiency)

Results for output in the SQ and the reformed economy under various public financing scenarios are shown in Table 5. As one would expect, in terms of the aggregate economy, our numerical results imply that it is better to allow capital taxes to take advantage of the fiscal space created by debt consolidation. The superiority of the capital tax rate is consistent with

Table 6: Net income of capitalists and net income of workers in steady state

Residual Instrument	Status quo			New steady state			% Changes		
	y^k	y^w	y^k/y^w	y^k	y^w	y^k/y^w	y^k	y^w	y^k/y^w
τ^k	0.866	0.209	4.145	0.905	0.212	4.261	+4.55%	+1.69%	+2.82%
τ^n	0.866	0.209	4.145	0.883	0.215	4.104	+1.99%	+3.00%	-0.99%
τ^c	0.866	0.209	4.145	0.881	0.213	4.144	+1.78%	+1.80%	-0.02%
s^g	0.866	0.209	4.145	0.873	0.210	4.164	+0.82%	+0.35%	+0.47%

Note: y^k and y^w stand for the net income of the capitalist and worker respectively in steady state.

the well-known result that capital taxes are particularly distorting in the medium-run and long-run (see e.g. Judd, 1985, Chamley, 1986 and Lucas, 1990). Therefore, the most efficient way of using the fiscal space generated, once debt has been brought down, is to cut the capital tax rate. This is as in Philippopoulos et al.(2015), who studied aggregate effects only.

Distributional implications (equity)

Results for net incomes are reported in Table 6. Since there are two different income groups in the society - capitalists and workers - the income gains from each particular structural reform may be distributed unequally. We first look at the net income of each agent, y^k and y^w, separately.[5] Our results show that, relative to the status quo, both social groups gain from debt consolidation independently of what the residual instrument in the new steady state is (see Table 6).

But a key question is who gains more. Even if a policy reform produces a win-win outcome (Pareto efficient) here, in the sense that both y^k and y^w rise relative to SQ, relative outcomes can also be important. Actually, the political economy literature has pointed out several reasons for this, including political ideology, envy, habits, etc. In our model, distributional implications can be measured by changes in the ratio of net incomes, y^k/y^w.

Relative to the status quo, the ratio y^k/y^w rises, or equivalently inequality rises, when the instrument that takes advantage of the fiscal space created in the new reformed steady state is the tax rate on capital. Thus, this policy is Pareto efficient, but not ”equitable”. For this reason, perhaps, we

[5] The net income of the capitalist is defined as $y_t^k = -\tau_t^c c_t^k + (1-\tau_t^k)[r_t^k k_{t-1}^k + d_t^k] + (1-\tau_t^n) w_t^k n_t^k - \tau_t^{l,k}$, while that of the worker is defined as $y_t^w = -\tau_t^c c_t^w + (1-\tau_t^n) w_t^w n_t^w - \tau_t^{l,w}$.

Table 7: Present value of output (GDP) over different time horizons **when the residual instrument in the new steady state is the tax rate on capital(τ^k).**

Adj.Instr.	$\widetilde{y}_5$	$\widetilde{y}_{10}$	$\widetilde{y}_{20}$	$\widetilde{y}_{40}$	$\widetilde{y}_{60}$	$\widetilde{y}_\infty$
τ^k	11.29	22.33	43.44	81.43	114.14	220.20
τ^n	10.70	22.05	43.40	81.40	114.10	220.16
τ^c	11.11	22.27	43.52	81.50	114.21	220.27
s^g	11.74	23.06	44.40	82.45	115.17	221.26
Status quo	**11.30**	**22.23**	**43.02**	**80.43**	**112.71**	**217.37**

Note: $\widetilde{y}_t$ stands for the sum of the discounted expected values of output (GDP) for the next t periods after the fiscal consolidation takes place.

often observe workers opposing to such a reform. In terms of equity, the best outcome takes place when we use the fiscal space created by debt consolidation in the medium- and long-run to cut the labor tax rate. Such a policy causes the ratio y^k/y^w to fall, or equivalently inequality to fall.

In sum, in the reformed steady state, a policy that increases all net incomes and, at the same time, reduces income inequality, is to cut the labor tax rate. On the other hand, if we focus on efficiency only, the best way of using the fiscal space is to cut the capital tax rate; the cost of this is an increase in income inequality.

5.2 Transition results

We next study what happens in the transition as we depart from the status quo steady state and travel towards each one of the new reformed steady states with lower public debt.

Aggregate implications(efficiency)

Results for the present value of output over different time horizons for all cases (depending on what the adjusting instrument in the transition to a new reformed steady state is) are shown in Tables 7 and 8. Every table corresponds to a different new reformed steady state depending on which fiscal policy instrument takes advantage of the fiscal space created by debt consolidation. Specifically, the fiscal space created by debt consolidation is used to cut the tax rate on capital in Table 7 and the tax rate on labor in Table 8. Every row of these tables shows the present discount value of output over different time horizons depending on which instrument adjusts

Table 8: Present value of output (**GDP**) over different time horizons **when the residual instrument in the new steady state is the tax rate on labor(τ^n).**

Adj. Instr.	$\widetilde{y}_5$	$\widetilde{y}_{10}$	$\widetilde{y}_{20}$	$\widetilde{y}_{40}$	$\widetilde{y}_{60}$	$\widetilde{y}_{\infty}$
τ^k	11.36	22.36	43.30	80.99	113.48	218.87
τ^n	10.88	22.12	43.31	81.06	113.54	218.92
τ^c	11.06	22.15	43.25	81.00	113.49	218.88
s^g	11.81	23.10	44.33	82.15	114.67	220.08
Status quo	**11.30**	**22.23**	**43.02**	**80.43**	**112.71**	**217.37**

Note: $\widetilde{y}_t$ stands for the sum of the discounted expected values of output (GDP) for the next t periods after the fiscal consolidation takes place.

Table 9: Ratio of the present value of the net income of the capitalist to that of the worker over various time horizons **when the residual instrument in the steady state is the tax rate on capital**(τ^k).

Steady state value in the status quo is **4.1446**						
Adj. Instr.	$\frac{\widetilde{y}_5^k}{\widetilde{y}_5^w}$	$\frac{\widetilde{y}_{10}^k}{\widetilde{y}_{10}^w}$	$\frac{\widetilde{y}_{20}^k}{\widetilde{y}_{20}^w}$	$\frac{\widetilde{y}_{40}^k}{\widetilde{y}_{40}^w}$	$\frac{\widetilde{y}_{60}^k}{\widetilde{y}_{60}^w}$	$\frac{\widetilde{y}_{\infty}^k}{\widetilde{y}_{\infty}^w}$
τ^k	3.40	3.52	3.72	3.94	4.03	4.13
τ^n	4.12	4.19	4.24	4.27	4.27	4.27
τ^c	3.99	4.08	4.15	4.20	4.22	4.24
s^g	4.14	4.19	4.23	4.25	4.25	4.26

Note: $\widetilde{y}_t^k$ and $\widetilde{y}_t^w$ stand for the PV of the net income of the capitalist and the worker respectively for the next t periods after the fiscal consolidation.

to bring public debt down.

Inspection of the results in Tables 7 and 8 implies that if the criterion is aggregate, or per capita, output (GDP), the best policy mix is to use the long term fiscal gain (namely, the fiscal space created once debt has been reduced) to cut the capital tax rate and, during the early period of fiscal pain, to use spending cuts to bring public debt down.

Distributional implications (equity)

Results for the ratio of the present value of the net income of capitalists to that of workers over different time horizons for all cases (depending on what is the adjusting instrument in the transition to a new reformed steady

Table 10: Ratio of the present value of the net income of the capitalist to that of the worker over various time horizons **when the residual instrument in the new steady state is the tax rate on labor(τ^n).**

Steady state value in the status quo is **4.1446**						
Adj. Instr.	$\frac{\widetilde{y}^k_5}{\widetilde{y}^w_5}$	$\frac{\widetilde{y}^k_{10}}{\widetilde{y}^w_{10}}$	$\frac{\widetilde{y}^k_{20}}{\widetilde{y}^w_{20}}$	$\frac{\widetilde{y}^k_{40}}{\widetilde{y}^w_{40}}$	$\frac{\widetilde{y}^k_{60}}{\widetilde{y}^w_{60}}$	$\frac{\widetilde{y}^k_{\infty}}{\widetilde{y}^w_{\infty}}$
τ^k	3.41	3.48	3.61	3.78	3.87	3.98
τ^n	4.08	4.11	4.14	4.15	4.14	4.13
τ^c	3.81	3.90	3.98	4.03	4.05	4.08
s^g	4.14	4.19	4.23	4.25	4.25	4.26

Note: $\widetilde{y}^k_t$ and $\widetilde{y}^w_t$ stand for the PV of the net income of the capitalist and the worker respectively for the next t periods after the fiscal consolidation.

state) are shown in Tables 9 and 10. Every table corresponds to a different new reformed steady state (depending on what is the fiscal policy instrument that takes advantage of the fiscal space created by debt consolidation). Specifically, the fiscal space created by debt consolidation is used to cut the tax rate on capital in Table 9 and the tax rate on labor in Table 10. Every row of these tables shows the ratio of the present value of the net income of capitalists to that of workers over different time horizons. Furthermore, we also check whether these values are lower or higher than the steady state value in the status quo, 4.1446 (if they are lower than the corresponding SQ steady state value for a reform, then this reform improves equality relative to the status quo).

Inspection of the results in Tables 9 and 10 reveals that the best policy mix in terms of equity arises when we use capital taxes to bring public debt down and this is combined with labor tax cuts in the steady state. Nevertheless, as said, this is not the most efficient.

6 Robustness report

We finally check the sensitivity of our results (everything reported here is available upon request). Our results are robust to changes in all key parameter values. In particular, we have extensively experimented with changes in the values of the percentage of capitalists in population, v^k, the Rotemberg adjustment pricing cost parameter in the intermediate goods firm's problem, ϕ^p, the feedback coefficient of the capital tax rate on public debt, γ^k_l, the feedback coefficient of the labor tax rate on public debt, γ^n_l, the feedback coefficient of the consumption tax rate on public debt, γ_l^c, the feedback coefficient of the government spending on public debt, γ_l^g, the feedback coefficient of the nominal interest rate on inflation, ϕ^π, whose values are relatively unknown empirically. We report that our main results do not change qualitatively within these ranges $0.15 \leq v^k \leq 0.3$, $5 \leq \phi^p \leq 105$, $0.05 \leq \gamma_l^k \leq 0.30$, $0.05 \leq \gamma_l^n \leq 0.30$, $0.05 \leq \gamma_l^c \leq 0.30$, $0.05 \leq \gamma_l^g \leq 0.30$, $0.1 \leq \phi^\pi \leq 0.30$. It is also worth mentioning that there is no stability with or without debt consolidation when γ_l^q , with $q \in (k, n, c, g)$, is zero (i.e. some feedback reaction to debt is necessary for stability, as is common in the literature).

7 Closing the chapter and possible extensions

In this chapter was built and solved numerically, by using Eurozone data, a closed-economy new Keynesian D(S)GE model in which the fiscal authorities were engaged in public debt reduction over time. The emphasis was on the aggregate and distributional implications of debt consolidation, where agent heterogeneity, and hence distribution, had to do with the distinction between ”capitalists” and ”workers”. Since the results have already been written in the introduction, here I just mention a possible extension. It would be interesting to examine what happens in an open economy context. This is studied in the next chapter.

Appendix A Households as capitalists

This appendix provides details and the solution of capitalist k's problem. The mass of this type of household is 1. Each capitalist k acts competitively to maximize expected discounted lifetime utility.

A.1 The capitalist k's problem

Each capitalist k's expected discounted lifetime utility is:

$$E_o \sum_{t=0}^{\infty} \beta^t U\left(c_t^k, n_t^k, m_t^k, g_t\right) \tag{20}$$

where c_t^k is k's consumption at t, n_t^k is k's hours of work at t, m_t^k is k's end-of-period real money balances, g_t is government spending at t divided by the number of capitalist, E_o is the rational expectations operator conditional on the current period information set and $0 < \beta < 1$ is the time preference rate.

We will use a utility function of the form(see also e.g. Gali, 2008):

$$U\left(c_t^k, n_t^k, m_t^k, g_t\right) = \left[\frac{(c_t^k)^{1-\sigma}}{1-\sigma} - x_n \frac{(n_t^k)^{1+\eta}}{1+\eta} + x_m \frac{(m_t^k)^{1-\mu}}{1-\mu} + x_g \frac{(v^k g_t)^{1-\zeta}}{1-\zeta}\right] \tag{21}$$

where x_n, x_m, x_g, σ, η, μ, ζ are standard preference parameters.

The budget constraint of each k (written in nominal terms) is:

$$\begin{aligned} (1+\tau_t^c)P_t c_t^k + P_t x_t^k + B_t^k + M_t^k =& (1-\tau_t^k)[r_t^k P_t k_{t-1}^k + D_t^k] + \\ &+ (1-\tau_t^n) W_t^k n_t^k + R_{t-1} B_{t-1}^k + \\ &+ M_{t-1}^k - T_t^{l,k} \end{aligned} \tag{22}$$

where P_t is the price index, x_t^k is k's real investment at t, B_t^k is k's end-of-period nominal government bonds at t, M_t^k is k's end-of-period nominal money holdings at t, D_t^k is k's nominal dividends paid by firms at t, W_t^k is capitalists' nominal wage rate at t, k_t^k is k's end-of-period capital at t, $R_{t-1} \geq 1$ is the gross nominal return to government bonds between $t-1$ and t, r_{t-1}^k is the gross real return to inherited capital between $t-1$ and t, $T_t^{l,k}$ are the nominal lump-sum taxes/transfers to each capitalist k from the government at t, τ_t^c is the tax rate on consumption at t, τ_t^k is the tax rate on capital income at t and τ_t^n is the tax rate on labor income at t.

Dividing by P_t the above equation, the budget constraint of each k in real terms is:

$$
\begin{aligned}
(1+\tau_t^c)c_t^k + x_t^k + b_t^k + m_t^k =& (1-\tau_t^k)[r_t^k k_{t-1}^k + d_t^k] + \\
& + (1-\tau_t^n) w_t^k n_t^k + R_{t-1}\frac{P_{t-1}}{P_t} b_{t-1}^k + \\
& + \frac{P_{t-1}}{P_t} m_{t-1}^k - \tau_t^{l,k}
\end{aligned} \tag{23}
$$

where, as above, small letters denote real variables, i.e. $b_t^k \equiv \frac{B_t^k}{P_t}, m_t^k \equiv \frac{M_t^k}{P_t}, d_t^k \equiv \frac{D_t^k}{P_t}, w_t^k \equiv \frac{W_t^k}{P_t}, \tau_t^{l,k} \equiv \frac{T_t^{l,k}}{P_t}$.

The motion of physical capital for each k is:

$$
k_t^k = (1-\delta)k_{t-1}^k + x_t^k \tag{24}
$$

where $0 < \delta < 1$ is the depreciation rate of capital.

A.2 The capitalist k's optimality conditions

Each capitalist k acts competitively taking prices and policy as given.

The first order conditions include the budget constraint, the law of motion of physical capital above and:

$$
\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}[(1-\delta) + (1-\tau_{t+1}^k) r_{t+1}^k] \tag{25}
$$

$$
\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t \frac{P_t}{P_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{26}
$$

$$
x_n (n_t^k)^{\eta} = (c_t^k)^{-\sigma} \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t^k \tag{27}
$$

$$
x_m (m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \frac{P_t}{P_{t+1}} \tag{28}
$$

Eqs.(25) and (26) are the Euler equations of capital and bonds respectively, Eq.(27) is the optimality condition for work hours and Eq.(28) is the optimality condition for money balances.

Appendix B Households as workers

This appendix provides details and the solution of worker w's problem. The mass of this type of household is $\frac{v^w}{v^k}$. Each worker w acts competitively to maximize expected discounted lifetime utility.

B.1 The worker w's problem

Workers have the same utility function as capitalists (see Eqs.(20) and (21)).

The budget constraint of each worker w in nominal terms is:

$$(1+\tau_t^c)P_t c_t^w + M_t^w = (1-\tau_t^n)W_t^w n_t^w + M_{t-1}^w - T_t^{l,w} \tag{29}$$

where c_t^w is w's consumption at t, n_t^w is w's hours of work at t, M_t^w is w's end-of-period nominal money holdings at t, W_t^w is workers' nominal wage rate at t and $T_t^{l,w}$ are the nominal lump-sum taxes/transfers to each worker w from the government at t.

Dividing by P_t the above equation, the budget constraint of each w in real terms is:

$$(1+\tau_t^c)c_t^w + m_t^w = (1-\tau_t^n)w_t^w n_t^w + \frac{P_{t-1}}{P_t} m_{t-1}^w - \tau_t^{l,w} \tag{30}$$

where small letters denote real variables e.g. $w_t^w \equiv \frac{W_t^w}{P_t}, m_t^w \equiv \frac{M_t^w}{P_t}, \tau_t^{l,w} \equiv \frac{T_t^{l,w}}{P_t}$.

B.2 The worker w's optimality conditions

Each worker w acts competitively taking prices and policy as given.

The first order conditions include the budget constraint above and:

$$\frac{(c_t^w)^{-\sigma}}{x_n (n_t^w)^{\eta}} = \frac{1+\tau_t^c}{(1-\tau_t^n)w_t^w} \tag{31}$$

$$\frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} = \beta \frac{P_t}{P_{t+1}} \left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m \frac{(m_t^w)^{1-\mu}}{1-\mu} \tag{32}$$

Eq.(31) is the optimality condition for work hours and Eq.(32) is the optimality condition for money balances.

Appendix C Final goods firms

This appendix provides details and the solution of the final goods firm's problem. There is a final goods firm that produces a single good and operates in a perfectly competitive environment.

C.1 The final goods firm's problem

Nominal profits of the final goods producer are:

$$P_t y_t - \int_0^1 P_t(f) y_t(f) df \tag{33}$$

where $P_t(f)$ is the price of variety f, $y_t(f)$ is the production of the variety f produced monopolistically by the intermediate goods firm f and y_t is the production of the final goods firm.

There is a final goods firm and a continuum (i.e. infinity) of intermediate goods firms. The latter are indexed along the unit interval. The production function of the final goods is a Dixit-Stiglitz type constant returns to scale of this form:

$$y_t = \left[\int_0^1 [y_t(f)]^{\frac{\phi-1}{\phi}} df \right]^{\frac{\phi}{\phi-1}} \tag{34}$$

where $\phi > 0$ is the elasticity of substitution across intermediate goods.

C.2 The final goods firm's optimality conditions

Under perfect competition, the final goods firm chooses the quantity of every variety, $y_t(f)$, to maximize its profits (more generally it would want to maximize the present value of expected discounted lifetime profits, but there is nothing that makes the problem interesting in a dynamic sense as it just buys the intermediate goods period by period. Hence, equivalently, the final goods firm could maximize profits period by period instead) subject to its production function taking prices as given. The solution to the profit maximization problem gives:

$$y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\phi} y_t \tag{35}$$

or equivalently:

$$P_t(f) = \left(\frac{y_t(f)}{y_t}\right)^{-\frac{1}{\phi}} P_t \tag{36}$$

Notice that, the zero profit condition, $P_t y_t = \int_0^1 P_t(f) y_t(f) df$, along with Eq.(35) imply for the price index:

$$P_t = \left\{ \int_0^1 [P_t(f)]^{1-\phi} df \right\}^{\frac{1}{1-\phi}} \tag{37}$$

Appendix D Intermediate goods firms

This appendix provides details and the solution of intermediate goods firm f's problem. The mass of these firms is normalized to 1. Each firm f produces a differentiated good of variety f under monopolistic competition facing a Rotermberg-type nominal price rigidities.

D.1 The intermediate goods firm f's problem

Due to Rotemberg pricing, to the extent that an increase of firm f's price differs from the steady state value of inflation rate, π, this firm faces a quadratic price adjustment cost, $\frac{\phi^p}{2}\left(\frac{P_t(f)}{P_{t-1}(f)\pi} - 1\right)^2 y_t$. As stressed in Rotemberg (1982), this adjustment cost accounts for the negative effects of price changes on the customer-firm relationship and, consequently, creates an inefficiency wedge between aggregate output and demand, which is reflected by the term $\left\{1 - \frac{\phi^p}{2}\left[\frac{P_t(f)}{P_{t-1}(f)\pi} - 1\right]^2\right\}^{-1}$.

Nominal profits of intermediate goods firm f are(see e.g. Bi et al., 2013):

$$D_t(f) = P_t(f) y_t(f) - P_t r_t^k k_{t-1}(f) - W_t^w n_t^w(f) - W_t^k n_t^k(f) - \frac{\phi^p}{2}\left(\frac{P_t(f)}{P_{t-1}(f)\pi} - 1\right)^2 P_t y_t \tag{38}$$

where $n_t^k(f)$ is the demand of firm f for capitalists' hours of work at t, $n_t^w(f)$ is the demand of firm f for workers' hours of work at t, $k_t(f)$ is the demand of firm f for physical capital at t and ϕ^p is a standard parameter which determines the degree of nominal price rigidity.

Dividing by P_t the above equation, the intermediate goods firm f's profits in real terms are:

$$d_t(f) \equiv \frac{D_t(f)}{P_t} = \frac{P_t(f)}{P_t} y_t(f) - r_t^k k_{t-1}(f) - w_t^w n_t^w(f) - w_t^k n_t^k(f) - \frac{\phi^p}{2}\left(\frac{P_t(f)}{P_{t-1}(f)\pi} - 1\right)^2 y_t \tag{39}$$

All firms use the same technology represented by the production function(similar to e.g. Hornstein et al., 2005):

$$y_t(f) = A_t \left[k_{t-1}(f)\right]^{\alpha} \left[\{n_t^k(f)\}^{\theta}\{n_t^w(f)^{1-\theta}\}\right]^{1-\alpha} \tag{40}$$

where A_t is an exogenous TFP, α is the share of capital and θ is the labor efficiency parameter of capitalist.

Under imperfect competition, profit maximization by f is also subject to the demand function coming from the solution to the final goods firm's problem, as specified above, namely:

$$P_t(f) = \left(\frac{y_t(f)}{y_t}\right)^{-\frac{1}{\phi}} P_t \tag{41}$$

D.2 The intermediate goods firm f's optimality conditions

Following the related literature, we follow a two-step procedure. We first solve a cost minimization problem, where each intermediate goods firm f minimizes its cost by choosing factors of production given technology and prices. The solution will give a minimum nominal cost function, which is a function of production factor prices and output produced by the firm f. In turn, given this cost function, we solve each intermediate goods firm f's maximization problem by choosing its price, $P_t(f)$.

Each f chooses its factors of production, $k_{t-1(f)}$, $n_t^k(f)$, $n_t^w(f)$, to minimize its real cost. The above cost minimization is subject to the production function of f, Eq.(40).

The solution to the cost minimization problem gives the following input demand functions:

$$r_t^k = mc_t \alpha \frac{y_t(f)}{k_{t-1}(f)} \tag{42}$$

$$w_t^k = mc_t \theta(1-\alpha) \frac{y_t(f)}{n_t^k(f)} \tag{43}$$

$$w_t^w = mc_t(1-\theta)(1-\alpha)\frac{y_t(f)}{n_t^w(f)} \tag{44}$$

From the three above equations, it arises that the associated minimum real cost function of f equals $mc_t y_t(f)$, where mc_t is its real marginal cost. It can be shown that the real marginal cost equals:

$$mc_t = \frac{1}{A_t}\left[\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\left\{\frac{w_t^k}{\theta(1-\alpha)}\right\}^{\theta} \times \left\{\frac{w_t^w}{(1-\theta)(1-\alpha)}\right\}^{1-\theta}\right]^{1-\alpha} \tag{45}$$

implying that mc_t is common for all intermediate goods firms since it only depends on production factor prices, parameters and technology which are common for all these type of firms.

Then, intermediate goods firm f chooses its price, $P_t(f)$, to maximize the sum of discounted expected lifetime real profits:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t}\left[\frac{P_t(f)}{P_t}y_t(f) - r_t^k k_{t-1}(f) - w_t^w n_t^w(f) - w_t^k n_t^k(f) - \frac{\phi^p}{2}\left(\frac{P_t(f)}{P_{t-1}(f)\pi} - 1\right)^2 y_t\right] \tag{46}$$

where $\Xi_{0,0+t}$ is a stochastic discount factor which arises from the Euler of bonds and is defined as $\Xi_{0,0+t} = \prod_{i=0}^{t-1}\left\{\frac{1}{R_i}\right\} = \beta^t \prod_{i=0}^{t-1}\left[\left(\frac{P_i}{P_{i+1}}\right)\left(\frac{1+\tau_i^c}{1+\tau_{i+1}^c}\right)\left(\frac{c_{i+1}^k}{c_i^k}\right)^{-\sigma}\right]$. The above profit maximization is also subject to the demand equation that the monopolistically competitive firm f faces, $y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\phi} y_t$.

The first order condition gives:

$$(1-\phi)\frac{P_t(f)}{P_t}y_t(f) + \phi mc_t y_t(f) - \phi^p\left[\frac{P_t(f)}{P_{t-1}(f)\pi} - 1\right]\frac{y_t P_t(f)}{P_{t-1}(f)\pi} =$$

$$\beta\phi^p\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1 - \frac{P_{t+1}(f)}{P_t(f)\pi}\right]\frac{P_{t+1}(f)}{P_t(f)\pi}y_{t+1} \tag{47}$$

Thus, the behavior of intermediate goods firm f is summarized by Eqs.(42), (43), (44) and (47).

All intermediate goods firms solve the identical problem and, consequently, set the same price, $P_t(f)$, which implies through the Eq.(37) that $P_t(f) = P_t$.

Appendix E Decentralized equilibrium (given policy instruments)

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible monetary and fiscal policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of households maximizes utility; (ii) every type of firms maximizes profit; (iii) all constraints, including the government budget constraint, are satisfied; and (iv) all markets clear.

The DE is summarized by the following conditions:[6]

$$x_n(n_t^k)^\eta(c_t^k)^\sigma = \frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^k \tag{D1}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}[(1-\delta)+(1-\tau_{t+1}^k)r_{t+1}^k] \tag{D2}$$

$$x_m(m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta\frac{P_t}{P_{t+1}}\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D3}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t\frac{P_t}{P_{t+1}}\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D4}$$

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k \tag{D5}$$

$$c_t^k + \frac{v^w}{v^k}c_t^w + x_t^k + g_t = y_t\left\{1-\frac{\phi^p}{2}\left[\frac{P_t}{P_{t-1}\pi}-1\right]^2\right\} \tag{D6}$$

$$x_n(n_t^w)^\eta(c_t^w)^\sigma = \frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^w \tag{D7}$$

$$x_m(m_t^w)^{-\mu} = \frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} - \beta\frac{P_t}{P_{t+1}}\left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c}\right] \tag{D8}$$

$$(1+\tau_t^c)c_t^w + m_t^w = \frac{P_{t-1}}{P_t}m_{t-1}^w + (1-\tau_t^n)w_t^w n_t^w - \tau_t^{l,w} \tag{D9}$$

[6] I have aggregated over all agents, divided by the total number of agents and, in turn, divided all terms by v^k.

$$r_t^k = mc_t \alpha \frac{y_t}{k_{t-1}^k} \tag{D10}$$

$$w_t^k = mc_t \theta (1-\alpha) \frac{y_t}{n_t^k} \tag{D11}$$

$$w_t^w = mc_t (1-\theta)(1-\alpha) \frac{v^k}{v^w} \frac{y_t}{n_t^w} \tag{D12}$$

$$y_t = A_t [k_{t-1}^k]^\alpha \left[\{n_t^k\}^\theta \times \left\{ \frac{v^w}{v^k} n_t^w \right\}^{1-\theta} \right]^{1-\alpha} \tag{D13}$$

$$d_t = y_t - mc_t y_t - \frac{\phi^p}{2} \left(\frac{P_t}{P_{t-1}\pi} - 1 \right)^2 y_t \tag{D14}$$

$$\begin{aligned} &(1-\phi) + \phi mc_t - \phi^p \left[\frac{P_t}{P_{t-1}\pi} - 1 \right] \frac{P_t}{P_{t-1}\pi} = \\ &\phi^p \beta \left[\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c} \right) \left(\frac{c_{t+1}^k}{c_t^k} \right)^{-\sigma} \right] \left[1 - \frac{P_{t+1}}{P_t \pi} \right] \frac{y_{t+1}}{y_t} \end{aligned} \tag{D15}$$

$$\begin{aligned} g_t + R_{t-1} \frac{P_{t-1}}{P_t} b_{t-1}^k + \frac{P_{t-1}}{P_t} \left[m_{t-1}^k + \frac{v^w}{v^k} m_{t-1}^w \right] = b_t^k &+ \left[m_t^k + \frac{v^w}{v^k} m_t^w \right] + \\ &+ \tau_t^c \left[c_t^k + \frac{v^w}{v^k} c_t^w \right] + \\ &+ \tau_t^k \left[r_t^k k_{t-1}^k + d_t \right] + \\ &+ \tau_t^n \left[w_t^k n_t^k + \frac{v^w}{v^k} w_t^w n_t^w \right] + \\ &- \left[\tau_t^{l,k} + \frac{v^w}{v^k} \tau_t^{l,w} \right] \end{aligned} \tag{D16}$$

where $n_t^k = n_t^k(f)$, $n_t^w(f) = \frac{v^w}{v^k} n_t^w$, $k_t^k = k_t^k(f)$, $b_t = b_t^k$, $d_t^k = d_t(f) \equiv d_t$, $P_t(f) = P_t$, $y_t(f) = y_t$.

Thus, we have a system of 16 equations [(D1)-(D16)] in the 16 following endogeneous variables

$$[c_t^k, c_t^w, y_t, P_t, m_t^k, m_t^w, b_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, k_t^k, d_t]_{t=0}^{\infty}$$

Conclusively, the Decentralized Equilibrium is a sequence of

$$[c_t^k, c_t^w, y_t, P_t, m_t^k, m_t^w, b_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, k_t^k, d_t]_{t=0}^{\infty}$$

satisfying the equations [(D1)-(D16)], given:

a) technology $[A_t]_{t=0}^{\infty}$,

b) initial conditions for state variables,

c) policy.

Appendix F Decentralized equilibrium

We now rewrite the above equilibrium conditions, first, by using the inflation rate rather than price level and, second, by writing total public spending and total lump-sum taxes/transfers as shares of GDP, which are more convenient forms.

F.1 Variables expressed in ratios

We define the gross inflation rate, $\pi_t \equiv \frac{P_t}{P_{t-1}}$. Defining above the exogenous total public spending divided by the number of capitalists as g_t, we also find it convenient to express it as ratio of GDP, $g_t = s_t^g y_t$. From this equation arises that the per capita public spending is defined as $v^k s_t^g y_t$. Additionally, the total lump-sum taxes/transfers divided by the number of capitalists, $\left[\tau_t^{l,k} + \frac{v^w}{v^k}\tau_t^{l,w}\right]$, equal $s_t^l y_t$ where as s_t^l are defined the total lump-sum taxes/-transfers as share of output. The government imposes/gives a percentage, $\lambda_t^{l,k}$, of the total lump-sum taxes/transfers to the class of capitalists and, at the same time, a percentage, $\lambda_t^{l,w} \equiv 1 - \lambda_t^{l,k}$, to workers, where we set $\lambda_t^{l,k} \equiv v^k$ and, consequently, $\lambda_t^{l,w} = 1 - v^k = v^w$. In other words, transfers are distributed to capitalists and workers according to their share in population. From the above, it arises that $\tau_t^{l,k} = \tau_t^{l,w} = v^k s_t^l y_t$.

F.2 Final equations

Using the above, the final non-linear stochastic system is:

$$x_n (n_t^k)^{\eta} (c_t^k)^{\sigma} = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t^k \tag{D1'}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} [(1-\delta) + (1-\tau_{t+1}^k) r_{t+1}^k] \tag{D2'}$$

$$x_m (m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta \frac{1}{\pi_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D3'}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t \frac{1}{\pi_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D4'}$$

$$k_t^k = (1-\delta) k_{t-1}^k + x_t^k \tag{D5'}$$

$$c_t^k + \frac{v^w}{v^k} c_t^w + x_t^k + s_t^g y_t = y_t \left\{ 1 - \frac{\phi^p}{2} \left[\frac{\pi_t}{\pi} - 1 \right]^2 \right\} \tag{D6'}$$

$$x_n (n_t^w)^{\eta} (c_t^w)^{\sigma} = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t^w \tag{D7'}$$

$$x_m (m_t^w)^{-\mu} = \frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} - \beta \frac{1}{\pi_{t+1}} \left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c} \right] \tag{D8'}$$

$$(1+\tau_t^c) c_t^w + m_t^w = \frac{1}{\pi_t} m_{t-1}^w + (1-\tau_t^n) w_t^w n_t^w - v^k s_t^l y_t \tag{D9'}$$

$$r_t^k = mc_t \alpha \frac{y_t}{k_{t-1}^k} \tag{D10'}$$

$$w_t^k = mc_t \theta (1-\alpha) \frac{y_t}{n_t^k} \tag{D11'}$$

$$w_t^w = mc_t (1-\theta)(1-\alpha) \frac{v^k}{v^w} \frac{y_t}{n_t^w} \tag{D12'}$$

$$y_t = A_t[k^k_{t-1}]^\alpha \left[\{n^k_t\}^\theta \times \left\{ \frac{v^w}{v^k} n^w_t \right\}^{1-\theta} \right]^{1-\alpha} \tag{D13'}$$

$$d_t = y_t - mc_t y_t - \frac{\phi^p}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 y_t \tag{D14'}$$

$$\begin{aligned}(1-\phi) + \phi mc_t - \phi^p \left[\frac{\pi_t}{\pi} - 1 \right] \frac{\pi_t}{\pi} = \\ \phi^p \beta \left[\left(\frac{1+\tau^c_t}{1+\tau^c_{t+1}} \right) \left(\frac{c^k_{t+1}}{c^k_t} \right)^{-\sigma} \right] \left[1 - \frac{\pi_{t+1}}{\pi} \right] \frac{y_{t+1}}{y_t}\end{aligned} \tag{D15'}$$

$$\begin{aligned}s^g_t y_t + R_{t-1} \frac{1}{\pi_t} b^k_{t-1} + \frac{1}{\pi_t} \left[m^k_{t-1} + \frac{v^w}{v^k} m^w_{t-1} \right] = b^k_t + \left[m^k_t + \frac{v^w}{v^k} m^w_t \right] + \tau^c_t \left[c^k_t + \frac{v^w}{v^k} c^w_t \right] + \\ + \tau^k_t [r^k_t k^k_{t-1} + d_t] + s^l_t y_t + \\ + \tau^n_t \left[w^k_t n^k_t + \frac{v^w}{v^k} w^w_t n^w_t \right]\end{aligned} \tag{D16'}$$

$$\log\left(\frac{R_t}{R} \right) = \phi_\pi \log\left(\frac{\pi_t}{\pi} \right) + \phi_y \log\left(\frac{y_t}{y} \right) \tag{D17'}$$

$$s^g_t = s^g - \gamma^g_l (l_{t-1} - l) \tag{D18'}$$

$$\tau^c_t = \tau^c + \gamma^c_l (l_{t-1} - l) \tag{D19'}$$

$$\tau^k_t = \tau^k + \gamma^k_l (l_{t-1} - l) \tag{D20'}$$

$$\tau^n_t = \tau^n + \gamma^n_l (l_{t-1} - l) \tag{D21'}$$

$$l_t \equiv \frac{R_t b_t}{y_t} \tag{D22'}$$

The final equilibrium system consists of the 17 equations in 17 endogenous variables $[c^k_t, c^w_t, y_t, \pi_t, m^k_t, m^w_t, b^k_t, x^k_t, mc_t, w^k_t, n^k_t, w^w_t, n^w_t, r^k_t, k^k_t, d_t, l_t]^\infty_{t=0}$. This is given the 5 independently set monetary and fiscal instruments, $[R_t, s^g_t, \tau^c_t, \tau^k_t, \tau^n_t]^\infty_{t=0}$, technology, $[A_t]^\infty_{t=0}$, and initial conditions for the state variables, $k^k_{-1}, b^k_{-1}, A_{-1}, m^k_{-1}, m^w_{-1}, R_{-1}, l_{-1}$. Recall that $[R_t, s^g_t, \tau^c_t, \tau^k_t, \tau^n_t]^\infty_{t=0}$ follow the feedback rules specified above, while $[s^l_t]^\infty_{t=0}$ remains constant and close to its average value in the data.

Conclusively, we have a system of 22 equations [(D1')-(D22')] in the 22 following endogeneous variables

$$[c^k_t, c^w_t, y_t, \pi_t, m^k_t, m^w_t, b^k_t, x^k_t, mc_t, w^k_t, n^k_t, w^w_t, n^w_t, r^k_t, k^k_t, d_t, R_t, s^g_t, \tau^c_t, \tau^k_t, \tau^n_t, l_t]^\infty_{t=0}$$

Conclusively, the Decentralized Equilibrium is a sequence of

$$[c^k_t, c^w_t, y_t, \pi_t, m^k_t, m^w_t, b^k_t, x^k_t, mc_t, w^k_t, n^k_t, w^w_t, n^w_t, r^k_t, k^k_t, d_t, R_t, s^g_t, \tau^c_t, \tau^k_t, \tau^n_t, l_t]^\infty_{t=0}$$

satisfying the equations [(D1')-(D22')], given:

a) technology $[A_t]^\infty_{t=0}$,

b) initial conditions for state variables $k^k_{-1}, b^k_{-1}, m^k_{-1}, m^w_{-1}, R_{-1}, l_{-1}$.

Appendix G Tables

In this appendix we present, in form of tables, the outcomes of our experiments that are not presented in the main text. These tables have been used for comparison reasons and led to the conclusions of our study, as they are presented in the main text. In particular Tables 11 and 12 show what the aggregate implications in the transition when the residual instrument in the new reformed steady state is the tax rate on consumption and government spending respectively. Tables 13 and 14 show what the distributional implications in the transition when the residual instrument in the new reformed steady state is the tax rate on consumption and government spending respectively.

Table 11: Present value of output (**GDP**) over different time horizons **when the residual instrument in the new steady state is the tax rate on consumption(τ^c).**

Adj. Instr.	$\widetilde{y}_5$	$\widetilde{y}_{10}$	$\widetilde{y}_{20}$	$\widetilde{y}_{40}$	$\widetilde{y}_{60}$	$\widetilde{y}_\infty$
τ^k	11.30	22.22	43.02	80.43	112.70	217.37
τ^n	10.79	21.97	43.01	80.49	112.76	217.42
τ^c	11.00	22.01	42.96	80.45	112.73	217.39
s^g	11.74	22.97	44.06	81.62	113.92	218.62
Status quo	**11.30**	**22.23**	**43.02**	**80.43**	**112.71**	**217.37**

Note: $\widetilde{y}_t$ stands for the sum of the discounted expected values of output (GDP) for the next t periods after the fiscal consolidation takes place.

Table 12: Present value of output (**GDP**) over different time horizons **when the residual instrument in the new steady state is the public spending(s^g).**

Adj. Instr.	$\widetilde{y}_5$	$\widetilde{y}_{10}$	$\widetilde{y}_{20}$	$\widetilde{y}_{40}$	$\widetilde{y}_{60}$	$\widetilde{y}_\infty$
τ^k	11.30	22.23	43.02	80.43	112.71	217.37
τ^n	10.79	21.96	43.00	80.49	112.75	217.42
τ^c	10.97	21.98	42.94	80.42	112.70	217.36
s^g	11.74	22.97	44.06	81.62	113.92	218.62
Status quo	**11.30**	**22.23**	**43.02**	**80.43**	**112.71**	**217.37**

Note: $\widetilde{y}_t$ stands for the sum of the discounted expected values of output (GDP) for the next t periods after the fiscal consolidation takes place.

Table 13: Ratio of the present value of the net income of the capitalist to that of the worker over various time horizons **when the residual instrument in the new steady state is the tax rate on consumption(τ^c).**

Steady state value in the status quo is **4.1446**

Adj. Instr.	$\frac{\widetilde{y}^k_5}{\widetilde{y}^w_5}$	$\frac{\widetilde{y}^k_{10}}{\widetilde{y}^w_{10}}$	$\frac{\widetilde{y}^k_{20}}{\widetilde{y}^w_{20}}$	$\frac{\widetilde{y}^k_{40}}{\widetilde{y}^w_{40}}$	$\frac{\widetilde{y}^k_{60}}{\widetilde{y}^w_{60}}$	$\frac{\widetilde{y}^k_\infty}{\widetilde{y}^w_\infty}$
τ^k	3.46	3.52	3.64	3.81	3.90	4.02
τ^n	4.13	4.16	4.19	4.20	4.19	4.17
τ^c	3.89	3.96	4.03	4.08	4.10	4.12
s^g	4.10	4.12	4.14	4.15	4.15	4.15

Note: $\widetilde{y}^k_t$ and $\widetilde{y}^w_t$ stand for the PV of the net income of the capitalist and the worker respectively for the next t periods after the fiscal consolidation.

Table 14: Ratio of the present value of the net income of the capitalist to that of the worker over various time horizons **when the residual instrument in the new steady state is the government spending(s^g).**

Steady state value in the status quo is **4.1446**

Adj. Instr.	$\frac{\widetilde{y}_5^k}{\widetilde{y}_5^w}$	$\frac{\widetilde{y}_{10}^k}{\widetilde{y}_{10}^w}$	$\frac{\widetilde{y}_{20}^k}{\widetilde{y}_{20}^w}$	$\frac{\widetilde{y}_{40}^k}{\widetilde{y}_{40}^w}$	$\frac{\widetilde{y}_{60}^k}{\widetilde{y}_{60}^w}$	$\frac{\widetilde{y}_{\infty}^k}{\widetilde{y}_{\infty}^w}$
τ^k	3.47	3.53	3.65	3.83	3.92	4.03
τ^n	4.14	4.18	4.21	4.22	4.21	4.19
τ^c	3.85	3.94	4.03	4.09	4.11	4.13
s^g	4.11	4.13	4.15	4.17	4.17	4.17

Note: $\widetilde{y}_t^k$ and $\widetilde{y}_t^w$ stand for the PV of the net income of the capitalist and the worker respectively for the next t periods after the fiscal consolidation.

References

[1] Andrès, J., and Doménech, R. Automatic stabilizers, fiscal rules and macroeconomic stability. *European Economic Review 50*, 6 (2006), 1487–1506.

[2] Bi, H., Leeper, E. M., and Leith, C. Uncertain fiscal consolidations. *The Economic Journal 123*, 566 (2013), F31–F63.

[3] CESifo. EEAG Report on the European Economy, Munich(2016).

[4] Chamley, C. Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. *Econometrica 54*, 3 (May 1986), 607–622.

[5] Coenen, G., Mohr, M., and Straub, R. Fiscal consolidation in the euro area: Long-run benefits and short-run costs. *Economic Modelling 25*, 5 (2008), 912–932.

[6] Cogan, J. F., Taylor, J. B., Wieland, V., and Wolters, M. H. Fiscal consolidation strategy. *Journal of Economic Dynamics and Control 37*, 2 (2013), 404 – 421. Fiscal Policy in the Aftermath of the Financial Crisis.

[7] Erceg, C. J., and Lindé, J. Fiscal consolidation in a currency union: Spending cuts vs. tax hikes. *Journal of Economic Dynamics and Control 37*, 2 (2013), 422–445.

[8] EuropeanCommission. Public Finances in EMU, Brussels(2015).

[9] Forni, L., Gerali, A., and Pisani, M. The macroeconomics of fiscal consolidations in euro area countries. *Journal of Economic Dynamics and Control 34*, 9 (September 2010), 1791–1812.

[10] Gali, J., et al. Introduction to monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework [monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework]. *Introductory Chapters* (2008).

[11] Gali, J., Gertler, M., and Lopez-Salido, J. D. European inflation dynamics. *European Economic Review 45*, 7 (2001), 1237–1270.

[12] Hornstein, A., Krusell, P., and Violante, G. L. The Effects of Technical Change on Labor Market Inequalities. In *Handbook of Economic Growth*, P. Aghion and S. Durlauf, Eds., vol. 1 of *Handbook of Economic Growth*. Elsevier, July 2005, ch. 20, pp. 1275–1370.

[13] Judd, K. L. Redistributive taxation in a simple perfect foresight model. *Journal of Public Economics 28*, 1 (October 1985), 59–83.

[14] Keen, B., and Wang, Y. What is a realistic value for price adjustment costs in New Keynesian models? *Applied Economics Letters 14*, 11 (2007), 789–793.

[15] Lansing, K. J. Asset pricing with concentrated ownership of capital and distribution shocks. *American Economic Journal: Macroeconomics 7*, 4 (2015), 67–103.

[16] Leeper, E. M., Plante, M., and Traum, N. Dynamics of fiscal financing in the United States. *Journal of Econometrics 156*, 2 (June 2010), 304–321.

[17] Lucas, Robert E, J. Supply-Side Economics: An Analytical Review. *Oxford Economic Papers 42*, 2 (April 1990), 293–316.

[18] Neiss, K. S., and Pappa, E. Persistence without too much price stickiness: the role of variable factor utilization. *Review of Economic Dynamics 8*, 1 (January 2005), 231–255.

[19] Philippopoulos, A., Varthalitis, P., and Vassilatos, V. Optimal fiscal and monetary policy action in a closed economy. *Economic Modelling 48*, C (2015), 175–188.

[20] Philippopoulos, A., Varthalitis, P., and Vassilatos, V. Fiscal consolidation and its cross-country effects. *Journal of Economic Dynamics and Control 83*, C (2017a), 55–106.

[21] Philippopoulos, A., Varthalitis, P., and Vassilatos, V. Fiscal Consolidation in an Open Economy with Sovereign Premia and without Monetary Policy Independence. *International Journal of Central Banking 13*, 4 (2017b), 259–306.

[22] Rotemberg, J. J. Monopolistic price adjustment and aggregate output. *The Review of Economic Studies 49*, 4 (1982), 517–531.

[23] Schmitt-Grohé, S., and Uribe, M. Optimal simple and implementable monetary and fiscal rules. *Journal of monetary Economics 54*, 6 (2007), 1702–1725.

[24] Woodford, M. *Imperfect financial intermediation and complex dynamics, in Economic Complexity: chaos, sunspots, bubbles, and nonlinearity edited by William A. Barnett, John Geweke, Karl Shell.* Cambridge University Press Cambridge, 1989.

CHAPTER 3. DEBT CONSOLIDATION IN AN SMALL OPEN ECONOMY: AGGREGATE AND DISTRIBUTIONAL IMPLICATIONS

Debt consolidation in an small open economy: Aggregate and distributional implications

Abstract

This chapter builds and solves numerically a new Keynesian D(S)GE model of a small open economy within a monetary union facing sovereign interest rate premia due to high debt problems. In this model the fiscal authorities are engaged in public debt reduction over time. The emphasis is on the aggregate and distributional implications of debt consolidation, where income heterogeneity, and hence distribution, has to do with the distinction between ”capitalists” and ”workers”. The paper focuses on how these implications depend on the specific fiscal policy instruments used for debt consolidation. There are two key results. First, if the criterion is aggregate, or per capita, output, the best policy mix is to use the long term fiscal gain created by debt reduction to cut the tax rate on capital and, during the early period of fiscal pain, to use public spending cuts to bring public debt down. Second, if the criterion is equity in net incomes, the best recipe is to use the long term fiscal gain created by debt reduction to cut the labor tax rate and, during the early period of fiscal pain, to use capital taxes to bring public debt down.

1 Introduction

One of the consequences of the financial crisis in 2008 has been the emergence of the high public debt problem faced by most eurozone periphery countries. Thus, the need for debt consolidation has come to the center of attention emerging as a controversial issue. On one hand, proponents claim that debt consolidation is necessary to reduce borrowing costs, restore confidence and signal solvency. On the other hand, opponents claim that debt consolidation in a period of recession further dampens demand leading to a vicious cycle, at least, in the short term. Things become even worse for countries in a currency union regime because they lack monetary independence. Besides, opponents of debt consolidation claim that it worsens inequality since it is believed that fiscal austerity hurts especially the relatively poor social classes.[1]

In this paper, we study how public debt consolidation in a country with high debt, sovereign premia and loss of monetary policy independence affects aggregate macroeconomic outcomes and income distribution. The study of distributional implications differentiates this chapter/paper from most of the existing literature on debt consolidation. Most of the latter has focused on aggregate implications (see e.g. Philippopoulos et al., 2017, Coenen et al., 2008, Forni et al., 2010, Erceg and Lindé, 2013, etc.).

Considering the above, this paper provides a quantitative study of the aggregate and distributional implications of debt consolidation in a new Keynesian D(S)GE model of a small open economy within a monetary union. Obviously, to study the distributional implications of debt consolidation on incomes, we need a model with heterogeneous households. There are many types of income heterogeneity in the literature. Here, we focus on the distinction of households between ”capitalists” and ”workers” (see also the previous chapter). Capitalists are defined to be those households who hold assets, own the firms and get labor income for their managerial services, while workers are defined to be those households that have labor income only. On the production side, firms enjoy monopoly power and face Rotemberg-type nominal price rigidities.

As is well known, the standard small open economy model with incomplete asset markets faces problems of stationarity. There are several ways of inducing stationarity and convergence to a well-defined steady state in the standard small open economy model (see e.g. Schmitt-Grohé and Uribe,

[1]The latter can be a valid argument since it is believed that spending cuts and/or tax rises can affect different people/groups in different ways. Even a uniform change in policy can have different effects simply because agents are heterogeneous.

2003). In our paper we follow the device of debt-elastic sovereign interest rate premia. Namely, as the public debt-to-GDP ratio increases, the interest rate at which the country borrows from the international asset market rises.

Regarding macroeconomic policy, being in a monetary union, the economy lacks monetary policy independence. Nevertheless, this country is free to follow independent or national fiscal policy. The national fiscal authority conducts its policy through simple and implementable feedback policy rules for public spending and a number of tax rates(on consumption, capital and labor). In particular, we assume that public spending and the tax rates on consumption, capital and labor are all allowed to respond to the inherited public debt-to-GDP ratio as a deviation from a target value. Assuming that the debt policy target in the feedback policy rules is below the data average (from around 110% to 90%), we study the aggregate and distributional implications of various policies aiming at such debt consolidation. In general, debt consolidation implies an intertemporal trade-off: Fiscal pain in the short term (i.e. public spending has to fall and/or taxes have to rise) and fiscal gain in the medium and long term once debt has been reduced (i.e. now public spending can rise and/or taxes can fall).

Experimenting with various policy mixes, we study the implications of debt consolidation at steady state as well as during the transition from the status quo steady state to a new reformed steady state. As status quo steady state is defined the solution in which the fiscal policy instruments are set at their data averages for a country like Italy over the euro period, while as new reformed steady state is defined a solution in which, relative to status quo, public spending rises or one of the tax rates is cut as a result of the fiscal space created by lower debt and zero sovereign interest-rate premia.

The model is solved numerically employing commonly used parameter values and fiscal data from the Italian economy during 2001-2016. It is natural to quantify our model based on Italy because, although it belongs to eurozone periphery countries facing a debt problem, it continues to participate in the world capital market without any official foreign financial aid at least so far.

The main results are as follows. First, if the criterion is aggregate, or per capita, output, the best policy mix is to use the long term fiscal gain (namely, the fiscal space created once debt has been reduced) to cut the tax rate on capital and, during the early period of fiscal pain, to use spending cuts to bring public debt down.

Second, the above policy mix is Pareto efficient (i.e. both capitalists and workers get better off with this type of debt consolidation) both in the new steady state and in the transition. But, if we care about relative gains, there is a "social" cost: inequality (measured by the ratio of the capitalist's to worker's net income) rises both in the new steady state and in the transition.

Third, if the criterion is equity in net incomes (although this comes at a lower benefit at aggregate level relative to the above policy mix), the recipe is to use the long term fiscal gain to cut the tax rate on labor and, during the early period of fiscal pain, to use capital taxes to bring public debt down.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the data, the parameterization and the status quo solution. Section 4 discusses methodology. The main results are in Section 5. Section 6 suggests some possible extensions and closes the chapter. Technical details are in an appendix.

2 Model

Our setup is a small open economy New Keynesian model with imported and domestic goods featuring imperfect competition and Rotemberg-type nominal price rigidities. A review of small open economy models, real or new Keynesian, can be found in the book by Uribe and Schmitt-Grohé, 2017. Following Schmitt-Grohé and Uribe, 2003, we make the assumption of a debt-elastic interest-rate premium as a way to induce stationarity and close the model. Departing from homogeneous households, we assume that this country is populated by two types of households, capitalists and workers.

The number of each type of households and their percentages in the population as well as the number of firms are as follows. The economy is composed of N^k identical capitalists indexed by $k = 1, 2, ..., N^k$, of N^w identical workers indexed by $w = 1, 2, ..., N^w$, of N^h domestic firms indexed by $h = 1, 2, ..., N^h$. Similarly, there are N^f foreign firms indexed by $f = 1, 2, ..., N^f$ where each one of them produces a variety f and owned by a foreign investor. Assuming that each capitalist owns one domestic firm, the total number of capitalists equals that of domestic firms, that is $N^h = N^k$. Also, we assume that the number of domestic firms equals that of foreign firms implying that $N^f = N^k$. Hence, the number of domestic investors (capitalists) equals that of foreign investors, of domestic firms and of foreign firms. For simplicity, we assume that the population remains constant over time and of size, N. Furthermore, we assume that the number of capitalists and workers in the population remains also constant over time ruling out occupational choice and mobility across groups. Finally, the share of capitalists and workers in the population are defined as $v^k \equiv \frac{N^k}{N}$ and $v^w \equiv \frac{N^w}{N}$ respectively.

2.1 Households

There are two types of households, called capitalists and workers. Capitalists own the domestic firms, hold physical capital, money, internationally traded assets, domestic government bonds and also receive labor income for their managerial services, while workers just hold money and receive labor income for their labor services.

2.1.1 Consumption bundles

Every household of type $i \in \{k, w\}$ in the economy can be either a capitalist, indexed by k, or a worker, indexed by w. The quantity of variety h produced at home country by domestic firm h and consumed by household i is denoted as $c_t^{i,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite domestic good consumed by household i, $c_t^{i,H}$, consists of h varieties and is given by (see also e.g. Forni et al., 2010):[2]

$$c_t^{i,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{i,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{1}$$

where $\phi > 0$ is the elasticity of substitution across varieties produced in the domestic country.

Similarly, the quantity of imported variety f produced abroad by foreign firm f and consumed by household i is denoted as $c_t^{i,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite imported good consumed by household i, $c_t^{i,F}$, consists of f varieties and is given by:[3]

$$c_t^{i,F} = \left[\sum_{f=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{i,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{2}$$

In turn, having defined $c_t^{i,H}$ and $c_t^{i,F}$, household i's consumption bundle, c_t^i, is defined as:

[2] Recall that, in the introduction of Section 2, we have assumed that the number of domestic firms (and, consequently, of domestic varieties) equals that of capitalists.

[3] Recall that, in the introduction of Section 2, we have assumed that the number of foreign firms (and, consequently, of imported varieties) equals that of domestic firms and, consequently, that of capitalists.

$$c_t^i = \frac{\left(c_t^{i,H}\right)^v \left(c_t^{i,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{3}$$

where v is the degree of preference for domestic goods (if $v > 1/2$, there is a home bias).

2.1.2 Consumption expenditures, prices and terms of trade

Each household i's total consumption expenditure is:

$$P_t c_t^i = P_t^H c_t^{i,H} + P_t^F c_t^{i,F} \tag{4}$$

where P_t is the domestic consumer price index (CPI), P_t^H is the price index of home tradables and P_t^F is the price index of foreign tradables (expressed in domestic currency).

Each household i's total expenditure on home and foreign goods are respectively:[4]

$$P_t^H c_t^{i,H} = \sum_{h=1}^{N^k} P_t^H(h) c_t^{i,H}(h) \tag{5}$$

$$P_t^F c_t^{i,F} = \sum_{f=1}^{N^k} P_t^F(f) c_t^{i,F}(f) \tag{6}$$

where $P_t^H(h)$ is the price of variety h produced at home and $P_t^F(f)$ is the price of variety f produced abroad, both denominated in domestic currency.

We assume that the law of one price holds meaning that each tradable good sells at the same price at home and abroad. Thus, $P_t^F(f) = S_t P_t^{H*}(f)$, where S_t is the nominal exchange rate (where an increase in S_t implies a depreciation) and $P_t^{H*}(f)$ is the price of variety f produced abroad denominated in foreign currency. A star denotes the counterpart of a variable or a parameter in the rest-of-the world. Note that the terms of trade are denoted as $\frac{P_t^F}{P_t^H}(= \frac{S_t P_t^{H*}}{P_t^H})$, while the real exchange rate is denoted as $\frac{S_t P_t^*}{P_t}$.

2.2 Households as capitalists

This subsection presents the problem of capitalists, $k = 1, 2, ..., N^k$.

[4] Recall that, in the introduction of Section 2, we have assumed that the number of foreign firms(and, consequently, of imported varieties) equals that of domestic firms(and, consequently, that of domestic varieties) as well as that of capitalists.

2.2.1 Capitalists' optimization problem

Each capitalist k acts competitively to maximize discounted expected lifetime utility:

$$E_o \sum_{t=0}^{\infty} \beta^t U\left(c_t^k, n_t^k, m_t^k, g_t\right) \tag{7}$$

where c_t^k is k's consumption bundle at t as defined above, n_t^k is k's hours of work at t, m_t^k is k's end-of-period real money balances at t, g_t is total government spending at t divided by the number of capitalists implying that the per capita public spending is defined as $v^k g_t$, E_o is the rational expectations operator conditional on the current period information set and $0 < \beta < 1$ is the time preference rate.

In our numerical solutions, we use a utility function of the form (see also e.g. Gali, 2008):

$$U\left(c_t^k, n_t^k, m_t^k, g_t\right) = \left[\frac{(c_t^k)^{1-\sigma}}{1-\sigma} - x_n \frac{(n_t^k)^{1+\eta}}{1+\eta} + x_m \frac{(m_t^k)^{1-\mu}}{1-\mu} + x_g \frac{(v^k g_t)^{1-\zeta}}{1-\zeta}\right] \tag{8}$$

where x_n, x_m, x_g, σ, η, μ, ζ are standard preference parameters.

The budget constraint of each k (written in real terms) is:

$$\begin{aligned} &(1+\tau_t^c)c_t^k + \frac{P_t^H}{P_t}x_t^k + \frac{S_t P_t^*}{P_t} f_t^k + \frac{\phi^h}{2}\left(\frac{S_t P_t^*}{P_t} f_t^k - \frac{S P^*}{P} f^k\right)^2 + b_t^k + m_t^k = \\ &(1-\tau_t^k)\left[r_t^k \frac{P_t^H}{P_t} k_{t-1}^k + \widetilde{\omega_t}^k\right] + (1-\tau_t^n) w_t^k n_t^k + Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*} f_{t-1}^k + R_{t-1}\frac{P_{t-1}}{P_t} b_{t-1}^k + \\ &+ \frac{P_{t-1}}{P_t} m_{t-1}^k - \tau_t^{l,k} \end{aligned} \tag{9}$$

where x_t^k is k's real investment at t, f_t^k is the real value of k's end-of-period internationally traded assets at t denominated in foreign currency (if negative, it denotes foreign private debt), b_t^k is the real value of k's end-of-period domestic government bonds at t, r_{t-1}^k is the gross real return to inherited physical capital between $t-1$ and t, k_t^k is k's end-of-period physical capital at t, $\widetilde{\omega_t}^k$ is k's real dividends paid by domestic firms at t, w_t^k is capitalists' real wage rate at t, Q_{t-1} is the gross nominal return to international assets between t-1 and t, $R_{t-1} \geq 1$ is the gross nominal return to domestic government bonds between $t-1$ and t, $\tau_t^{l,k}$ are real lump-sum taxes/transfers to each

k from the government at t, $0 \leq \tau_t^c \leq 1$ is the tax rate on consumption at t, $0 \leq \tau_t^k \leq 1$ is the tax rate on capital income at t and $0 \leq \tau_t^n \leq 1$ is the tax rate on labor income at t. Small letters denote real variables e.g. $f_t^k \equiv \frac{F_t^k}{P_t^*}$, $b_t^k \equiv \frac{B_t^k}{P_t}$, $\widetilde{\omega_t}^k \equiv \frac{\widetilde{\Omega_t}^k}{P_t}$, $w_t^k \equiv \frac{W_t^k}{P_t}$, $\tau_t^{l,k} \equiv \frac{T_t^{l,k}}{P_t}$. Also, letters without time subscripts denote steady state values and, again, letter with a star as superscript denotes the counterpart of a variable in the rest-of-the world, e.g. P_t^* stands for the consumer price index (CPI) abroad. The parameter $\phi^h \geq 0$ measures adjustment costs related to private foreign assets as a deviation from their steady state value, f^k; these adjustment costs help us to avoid excess volatility and get plausible (in line with the data) short-term dynamics for private foreign assets following a policy reform.

The motion of physical capital for each k is:

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)^2 k_{t-1}^k \tag{10}$$

where $0 < \delta < 1$ is the depreciation rate of physical capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to physical capital.

Therefore, each capitalist k chooses $\{c_t^k, x_t^k, n_t^k, m_t^k, b_t^k, f_t^k, k_t^k\}_{t=0}^{\infty}$ to maximize Eqs.(7) and (8) subject to Eqs.(9) and (10), by taking as given prices $\{r_t^k, w_t^k, Q_t, R_t, P_t, P_t^H, P_t^*\}_{t=0}^{\infty}$, dividends $\{\widetilde{\omega_t}^k\}_{t=0}^{\infty}$, policy variables $\{S_t, \tau_t^c, \tau_t^n, \tau_t^k, \tau_t^{l,k}\}_{t=0}^{\infty}$, and initial conditions, $\{m_{-1}^k, b_{-1}^k, k_{-1}^k, f_{-1}^k\}$.

The first order conditions include the budget constraint of k, Eq.(9), the law of motion of physical capital, Eq.(10), and:

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}\frac{P_t^H}{P_t}\left[1+\xi\left(\frac{k_t^k}{k_{t-1}^k}-1\right)\right] = \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_{t+1}^H}{P_{t+1}}\times \tag{11}$$

$$\times\left[(1-\delta)+(1-\tau_{t+1}^k)r_{t+1}^k - \frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k}-1\right)^2 + \xi\left(\frac{k_{t+1}^k}{k_t^k}-1\right)\frac{k_{t+1}^k}{k_t^k}\right]$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}S_t\frac{P_t^*}{P_t}\left[1+\phi^h\left(S_t\frac{P_t^*}{P_t}f_t^k - S\frac{P^*}{P}f^k\right)\right] = \tag{12}$$

$$= \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}Q_t S_{t+1}\frac{P_{t+1}^*}{P_{t+1}}\frac{P_t^*}{P_{t+1}^*}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t\frac{P_t}{P_{t+1}}\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{13}$$

$$x_n(n_t^k)^\eta = (c_t^k)^{-\sigma}\frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^k \tag{14}$$

$$x_m(m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_t}{P_{t+1}} \tag{15}$$

Eqs.(11), (12) and (13) are respectively the Euler equations of physical capital, internationally traded assets and domestic government bonds, Eq.(14) is the optimality condition for work hours and Eq.(15) is the optimality condition for real money balances.

Next, each capitalist k chooses $\{c_t^{k,H}, c_t^{k,F}\}$ to minimize its total consumption expenditure, Eq.(4) for k, subject to its consumption bundle, Eq.(3) for k, by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^k.

The first order conditions include the consumption bundle of k, Eq.(3) for k, and:

$$\frac{c_t^{k,H}}{c_t^{k,F}} = \frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{16}$$

which is the optimality condition for sharing the total consumption between domestic and imported products.

Eq.(3) and Eq.(4) for k combined with Eq.(16) imply the following relation for domestic consumer price index(CPI):

$$P_t = (P_t^H)^v(P_t^F)^{1-v} \tag{17}$$

Finally, each capitalist k chooses $\{c_t^{k,H}(h), c_t^{k,F}(f)\}$ to minimize the sum of its consumption expenditure on home and foreign goods, sum of Eqs.(5) and (6) for k, subject to composite domestic and foreign good consisting of varieties, Eqs.(1) and (2) for k, by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{k,H}$ and $c_t^{k,F}$.

The first order conditions include Eqs.(1) and (2) for k, and:

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^\phi \tag{18}$$

$$c_t^{k,F}(f) = \frac{c_t^{k,F}}{N^k}\left(\frac{P_t^F}{P_t^F(f)}\right)^\phi \tag{19}$$

Plugging Eqs.(18) and (19) into Eqs.(1) and (2) for k respectively, we get the following relations for price indexes:

$$P_t^H = \left\{ \sum_{h=1}^{N^k} \frac{1}{N^k} [P_t^H(h)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{20}$$

$$P_t^F = \left\{ \sum_{f=1}^{N^k} \frac{1}{N^k} [P_t^F(f)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{21}$$

Details of the above problem and its solution are in Appendix A.

2.3 Households as workers

This subsection presents the problem of workers, $w = 1, 2, ..., N^w$.

2.3.1 Workers' optimization problem

Workers have the same utility function as capitalists (see Eqs.(7) and (8)). Each worker w acts competitively to maximize discounted expected lifetime utility taking prices and policy as given.

The budget constraint of each w (written in real terms) is:

$$(1+\tau_t^c)c_t^w + m_t^w = (1-\tau_t^n)w_t^w n_t^w + \frac{P_{t-1}}{P_t} m_{t-1}^w - \tau_t^{l,w} \tag{22}$$

where c_t^w is w's consumption bundle at t as defined above, n_t^w is w's hours of work at t, m_t^w is w's end-of-period real money balances at t, w_t^w is workers' real wage rate at t and $\tau_t^{l,w}$ are real lump-sum taxes/transfers to each w from the government at t. Again small letters denote real variables, e.g. $m_t^w \equiv \frac{M_t^w}{P_t}$, $w_t^w \equiv \frac{W_t^w}{P_t}$.

Therefore, each worker chooses $\{c_t^w, n_t^w, m_t^w\}_{t=0}^{\infty}$ to maximize Eqs.(7) and (8) indexed by w, subject to Eq.(22), by taking as given prices $\{w_t^w, P_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,w}\}_{t=0}^{\infty}$, and initial condition, m_{-1}^w.

The first order conditions include the budget constraint above, Eq.(22), and:

$$\frac{(c_t^w)^{-\sigma}}{x_n (n_t^w)^{\eta}} = \frac{1+\tau_t^c}{(1-\tau_t^n)w_t^w} \tag{23}$$

$$\frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} = \beta \frac{P_t}{P_{t+1}} \left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m (m_t^w)^{-\mu} \tag{24}$$

Eq.(23) is the optimality condition for work hours and Eq.(24) is the optimality condition for real money balances.

Next, each worker w chooses $\{c_t^{w,H}, c_t^{w,F}\}$ to minimize its total consumption expenditure, Eq.(4) for w, subject to its consumption bundle, Eq.(3) for w, by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^w.

The first order conditions include the consumption bundle of w, Eq.(3) for w, and:

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{25}$$

which is the optimality condition for sharing the total consumption between domestic and imported products.

Eq.(3) and Eq.(4) for w combined with Eq.(25) imply the following relation for domestic consumer price index(CPI):

$$P_t = (P_t^H)^v (P_t^F)^{1-v} \tag{26}$$

which, as expected, coincides with the equation of CPI derived from the capitalist k's problem, Eq.(17).

Finally, each worker w chooses $\{c_t^{w,H}(h), c_t^{w,F}(f)\}$ to minimize the sum of its consumption expenditure on home and foreign goods, sum of Eqs.(5) and (6) for w, subject to composite domestic and foreign good consisting of varieties, Eqs.(1) and (2) for w, by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{w,H}$ and $c_t^{w,F}$.

The first order conditions include Eqs.(1) and (2) for w, and:

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{27}$$

$$c_t^{w,F}(f) = \frac{c_t^{w,F}}{N^k}\left(\frac{P_t^F}{P_t^F(f)}\right)^{\phi} \tag{28}$$

Plugging Eqs.(27) and (28) into Eqs.(1) and (2) for w respectively, we get the following relations for price indexes:

$$P_t^H = \left\{\sum_{h=1}^{N^k} \frac{1}{N^k}[P_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{29}$$

$$P_t^F = \left\{ \sum_{f=1}^{N^k} \frac{1}{N^k} [P_t^F(f)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{30}$$

which, as expected, coincide with the equations of price indexes derived from the capitalist k's problem, Eqs.(20) and (21).

Details of the above problem and its solution are in Appendix B.

2.4 Firms

This subsection presents the problem of domestic firms. There are N^k domestic firms indexed by $h = 1, 2, ..., N^k$. Each firm h produces a differentiated tradable good of variety h under monopolistic competition facing Rotemberg-type nominal price rigidities (see e.g. Walsh, 2010, Wickens, Chapter 9, 2008, and Bi et al., 2013).

2.4.1 Demand for the firm *h*'s product

Each firm h faces demand for its product, $y_t^{H,d}(h)$. The latter comes from domestic households' consumption and investment, $c_t^H(h)$ and $x_t(h)$ respectively, where $c_t^H(h) \equiv \sum_{k=1}^{N^k} c_t^{k,H}(h) + \sum_{w=1}^{N^w} c_t^{w,H}(h)$ and $x_t(h) \equiv \sum_{k=1}^{N^k} x_t^k(h)$, from the government, $g_t(h)$, and from foreign households' consumption of the domestic good, $c_t^{F^*}(h)$. Thus, aggregate demand for each good h is:

$$y_t^{H,d}(h) = \left[c_t^H(h) + x_t(h) + g_t(h) + c_t^{F^*}(h) \right] \tag{31}$$

Since we have:

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{32}$$

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{33}$$

$$x_t^k(h) = \frac{x_t^k}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{34}$$

$$g_t(h) = \frac{N^k g_t}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{35}$$

$$c_t^{F^*}(h) = \frac{c_t^{F^*}}{N^k}\left(\frac{P_t^{F^*}}{P_t^{F^*}(h)}\right)^{\phi} \tag{36}$$

we can rewrite the relation (31) as:

$$y_t^{H,d}(h) = \frac{1}{N^k}\left[c_t^H + x_t + N^k g_t + c_t^{F^*}\right] \times \left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{37}$$

where $c_t^H \equiv \sum\limits_{k=1}^{N^k} c_t^{k,H} + \sum\limits_{w=1}^{N^w} c_t^{w,H}$ is domestic households' total consumption of home goods, $x_t \equiv \sum\limits_{k=1}^{N^k} x_t^k$ is capitalists' total investment, $N^k g_t$ denotes total government purchases of domestic output and $c_t^{F^*} \equiv N^k \overline{c}_t^{F*}$ is total consumption of home goods by households in the rest of the world (i.e. domestic country's exports) implying that $\overline{c}_t^{F*}$ stands for this total consumption divided by the number of domestic capitalists. Also notice that the law of one price implies that in Eq.(36):

$$\frac{P_t^{F^*}}{P_t^{F^*}(h)} = \frac{\frac{P_t^H}{S_t}}{\frac{P_t^H(h)}{S_t}} = \frac{P_t^H}{P_t^H(h)} \tag{38}$$

Since aggregate demand of the economy, $N^k y_t^{H,d}$, is:

$$N^k y_t^{H,d} = \left[c_t^H + x_t + N^k g_t + N^k \overline{c}_t^{F*}\right] \tag{39}$$

then aggregate demand for each good h is rewritten as:

$$y_t^{H,d}(h) = y_t^{H,d}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{40}$$

where $y_t^{H,d}$, as implied by above, denotes the aggregate demand of the economy divided by the number of capitalists.

2.4.2 Firms' optimization problem

Nominal profits of each firm h are defined as:

$$P_t\widetilde{\omega}_t(h) = P_t^H(h)y_t^H(h) - P_t^H r_t^k k_{t-1}(h) - W_t^w n_t^w(h) - W_t^k n_t^k(h) - \frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right)^2 P_t^H(h)y_t^H(h) \tag{41}$$

where $y_t^H(h)$ stands for the production of domestic firm h, π^H stands for the steady state value of the gross domestic goods inflation rate, $k_{t-1}(h)$ denotes the physical capital input chosen by firm h, $n_t^w(h)$ denotes workers' labor input chosen by firm h, $n_t^k(h)$ denotes the capitalists' labor input chosen by firm h and $\phi^P \geq 0$ is a parameter which determines the degree of nominal price rigidity.

All firms use the same technology represented by the production function (similar to e.g. Hornstein et al., 2005, and Baxter and King, 1993):

$$y_t^H(h) = A_t\,[k_{t-1}(h)]^{\alpha}\left[\{n_t^k(h)\}^{\theta}\{n_t^w(h)^{1-\theta}\}\right]^{1-\alpha} \tag{42}$$

where A_t is an exogenous TFP, $0 < \alpha < 1$ is the share of physical capital and $0 < \theta < 1$ is the labor efficiency parameter of the capitalist.

Profit maximization by firm h is also subject to the demand for its product, Eq.(40) as derived above. But, instead of using Eq.(40), we can equivalently use the following equation, Eq.(43), which expresses the demand for good h in terms of production:

$$y_t^H(h) = y_t^H\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{43}$$

where $y_t^{H,d} \equiv y_t^H \times \left\{1 - \frac{\phi^P}{2}\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right]^2\right\}$ and with y_t^H to denote the aggregate output of the economy divided by the number of capitalists.

This equation can be derived by considering the equations of Subsection 2.4.1 and the following relation that associates aggregate demand of each good h with its production by domestic firm h:

$$y_t^{H,d}(h) = y_t^H(h) \times \left\{1 - \frac{\phi^P}{2}\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right]^2\right\} \tag{44}$$

The term in the brackets captures the Rotemberg-type pricing cost and reflects the discrepancy between production and demand, as one expected in a Rotemberg-type fashion(see e.g. Bi et a., 2013, and Lombardo et al., 2008).

Each firm h chooses its price, $P_t^H(h)$, and its inputs, $k_t(h)$, $n_t^k(h)$, $n_t^w(h)$, to maximize the sum of discounted expected real dividends, $\max E_o \sum\limits_{t=0}^{\infty} \Xi_{0,0+t}\widetilde{\omega_t}(h)$, subject to the equation which is equivalent to the demand for its product, that is Eq.(43), and its production function, Eq.(42). The objective function of firm h in real terms is given by:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \left[\frac{P_t^H(h)}{P_t} y_t^H(h) \left\{ 1 - \frac{\phi^P}{2} \left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1 \right)^2 \right\} - \frac{P_t^H}{P_t} r_t^k k_{t-1}(h) - w_t^w n_t^w(h) - w_t^k n_t^k(h) \right] \tag{45}$$

where $\Xi_{0,0+t}$ is a stochastic discount factor taken as given by the firm h. This is defined as $\Xi_{0,0+t} = \prod_{i=0}^{t-1} \left\{ \frac{1}{R_i} \right\} = \beta^t \prod_{i=0}^{t-1} \left[\left(\frac{P_i}{P_{i+1}} \right) \left(\frac{1+\tau_i^c}{1+\tau_{i+1}^c} \right) \left(\frac{c_{i+1}^k}{c_i^k} \right)^{-\sigma} \right]$ and arises from the Euler of government bonds.

2.4.3 Firms' optimality conditions

Following the related literature, instead of solving the above problem, we follow a two-step procedure. We first solve a cost minimization problem, where each firm h minimizes its cost by choosing factors of production given technology and prices. The solution will give a minimum real cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, we solve the dynamic profit maximization problem of firm h by choosing its price.

Cost minimization problem: In the first stage, we solve a static cost minimization problem, where each firm h minimizes its cost by choosing its factors of production, $k_t(h)$, $n_t^k(h)$, $n_t^w(h)$, subject to its production function, Eq.(42), given technology and prices. The cost function is defined in real terms as follows:

$$\min \widetilde{\psi} = \left[\frac{P_t^H}{P_t} r_t^k k_{t-1}(h) + w_t^w n_t^w(h) + w_t^k n_t^k(h) \right] \tag{46}$$

The solution to the cost minimization problem gives the following input demand functions:

$$\frac{P_t^H}{P_t} r_t^k k_{t-1}(h) = mc_t \alpha y_t^H(h) \tag{47}$$

$$w_t^k n_t^k(h) = mc_t \theta (1-\alpha) y_t^H(h) \tag{48}$$

$$w_t^w n_t^w(h) = mc_t (1-\theta)(1-\alpha) y_t^H(h) \tag{49}$$

where $mc_t \equiv \widetilde{\psi}'(y_t^H(h))$ (as we will show just below, by summing up these three factor demand functions, the real cost is a function of production) stands for the real marginal cost, which, by definition, is the derivative of the associated minimum real cost function, $\widetilde{\psi}(y_t^H(h))$, with respect to the production, $y_t^H(h)$.

Summing up the three above equations it arises the following relation for the associated minimum real cost function of h:

$$\widetilde{\psi}(y_t^H(h)) = mc_t y_t^H(h) \tag{50}$$

Where the real marginal cost, mc_t, it can be shown that equals:

$$mc_t = \frac{1}{A_t}\left[\frac{P_t^H}{P_t}\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\left\{\frac{w_t^k}{\theta(1-\alpha)}\right\}^{\theta} \times \left\{\frac{w_t^w}{(1-\theta)(1-\alpha)}\right\}^{1-\theta}\right]^{1-\alpha} \tag{51}$$

implying that mc_t is common for all firms since it only depends on prices, parameters and technology which are common for all firms.

Profit maximization: The solution to the cost minimization problem above gave a minimum real cost function, Eq.(50), which is a function of prices and output produced by the firm h. In turn, given this cost function, we solve a dynamic profit maximization problem where firm h maximizes discounted expected lifetime real profits by choosing its price, $P_t^H(h)$.

These profits are defined as:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t}\left[\frac{P_t^H(h)}{P_t}y_t^H(h) - \widetilde{\psi}(y_t^H(h)) - \frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right)^2 \frac{P_t^H(h)y_t^H(h)}{P_t}\right] \tag{52}$$

The above profit maximization is subject to the Eq.(43), which is equivalent to the demand equation, Eq.(40), that the monopolistically competitive firm h faces.

The first order condition gives:

$$(1-\phi)\frac{P_t^H(h)}{P_t}y_t^H(h)+\phi mc_t y_t^H(h)-\frac{\phi^P}{2}(1-\phi)\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right]^2\frac{P_t^H(h)y_t^H(h)}{P_t}- \tag{53}$$
$$-\phi^P\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right]\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}\frac{P_t^H(h)}{P_t}y_t^H(h)=$$
$$\beta\phi^P\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1-\frac{P_{t+1}^H(h)}{P_t^H(h)\pi^H}\right]\frac{P_{t+1}^H(h)}{P_t^H(h)\pi^H}\frac{P_{t+1}^H(h)}{P_{t+1}}y_{t+1}^H(h)$$

Thus, the behavior of h is summarized by Eqs.(47), (48), (49) and (53).

Since, all firms solve the identical problem, they will set the same price, $P_t^H(h)$, which, through the Eq.(20) (which coincides with Eq.(29)), implies that $P_t^H(h)=P_t^H$.

Details of the above problem and its solution are in Appendix C.

2.5 Government budget constraint

The period budget constraint of the "consolidated" public sector expressed in real terms [5] is (see Appendix D for details):

$$Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\frac{P_{t-1}}{S_{t-1}P_{t-1}^*}(1-\lambda_{t-1})d_{t-1}+\frac{\phi^g}{2}\left[(1-\lambda_t)d_t-(1-\lambda)d\right]^2+R_{t-1}\frac{P_{t-1}}{P_t}\lambda_{t-1}d_{t-1}+\frac{P_t^H}{P_t}g_t \tag{54}$$
$$+\frac{P_{t-1}}{P_t}m_{t-1}=m_t+\tau_t^c\left[\frac{P_t^H}{P_t}\left(c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}\right)+\frac{P_t^F}{P_t}\left(c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}\right)\right]+\tau_t^k\left[r_t^k\frac{P_t^H}{P_t}k_{t-1}^k+\widetilde{\omega}_t^k\right]+$$
$$+\tau_t^n\left[w_t^kn_t^k+\frac{v^w}{v^k}w_t^wn_t^w\right]+\tau_t^l+d_t$$

where $\tau_t^l\equiv\left[\tau_t^{l,k}+\frac{v^w}{v^k}\tau_t^{l,w}\right]$ are total lump-sum taxes/transfers at t divided by the number of capitalists, m_t is the end-of-period stock of real money balances at t divided by the number of capitalists, $d_t\equiv\frac{D_t}{P_t}$ is the end-of-period total domestic real public debt (held by domestic and foreign agents) at t divided by the number of domestic capitalists and $0\leq\lambda_t\leq1$ is the fraction of total real public debt held by domestic agents (capitalists) implying that

[5] I have aggregated over all agents, divided by the total number of agents and, in turn, divided all terms by v^k.

$0 \leq 1 - \lambda_t \leq 1$ is the fraction of total real public debt held by foreign agents.[6] All other variables have been defined above. The parameter $\phi^g \geq 0$ measures adjustment costs related to domestic public debt held by foreign agents and are similar to those of the capitalist in Eq.(9) above. Again letters without time subscripts denote steady state values of the corresponding variables.

In each period, one of $\{\tau_t^c, \tau_t^k, \tau_t^n, g_t, \tau_t^l, \lambda_t, d_t\}$ needs to follow residually to satisfy the government budget constraint. We assume that this role is played by total public debt divided by the number of capitalists, d_t.

2.6 Closing the model: Debt elastic interest rate

Here we assume that the interest rate premium that the country faces when it borrows from world capital market, $Q_t - Q_t^*$, is an increasing function of the end-of-period total public debt as share of GDP, $\frac{D_t}{P_t^H y_t^H}$, when the latter exceeds a certain threshold, $\overline{d}$(defined below).[7] In particular, following e.g. Schmitt-Grohé and Uribe, 2003 and García-Cicco et al., 2010, we use:

$$Q_t = Q_t^* + \psi \left(e^{\left(\frac{D_t}{P_t^H y_t^H} - \overline{d} \right)} - 1 \right) \tag{55}$$

where the world interest rate, Q_t^*, is exogenously given, $\overline{d}$ is an exogenous threshold value above which the interest rate on government debt starts rising above Q_t^* and the parameter ψ measures the elasticity of the interest rate premium with respect to deviations of total public debt-to-GDP ratio from its threshold value.[8]

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[6] Total domestic public debt differs from country's foreign debt. The end-of-period total public debt in nominal terms divided by the number of domestic capitalists, $D_t = B_t + S_t F_t^g$, can be held either by a domestic agent (capitalist), $B_t = \lambda_t D_t$, or by a foreign investor, $S_t F_t^g = (1 - \lambda_t) D_t$ (Recall that the number of domestic capitalists equals that of foreign investors.). On the other hand, the country's end-of-period net foreign debt in nominal terms divided by the number of domestic capitalists denominated in domestic currency(if negative, it denotes liabilities), $S_t(F_t^g - F_t^k) = (1 - \lambda_t) D_t - S_t F_t^k$, is the real value of domestic public debt held by each foreign investor denominated in domestic currency, $S_t F_t^g$(if negative, it denotes liabilities), plus the real value of domestic private debt owed by each domestic capitalist denominated in domestic currency, $-S_t F_t^k$ (if positive, it denotes assets). Notice that we treat $0 \leq \lambda_t \leq 1$ as exogenous, because, in our small open economy setup, we do not model the behavior of foreign investors (but only that of the domestic investors).

[7] As we have mentioned above, this assumption is also compatible with several empirical studies.

[8] The value of $\overline{d}$ can be thought of as any value of debt-to-GDP ratio above which sustainability concerns start arising.

2.7 Exchange rate and fiscal policy

To proceed with the solution of the model, we need to specify the exchange rate and fiscal policy regimes. Regarding the exchange rate regime, we assume fixed exchange rate along with loss of monetary independence so that we mimic a monetary union regime (Recall that in our model we quantify Italy over euro years). This means that we treat nominal exchange rate, S_t, as a fixed exogenous variable and the nominal interest rate of domestic government bonds, R_t, as an endogenous variable.[9] It is worth to mention that the presence of nominal price rigidities in the transition breaks money neutrality implying that monetary policy (exchange rate regime) matters to real economy. Regarding fiscal policy, as mentioned in Subsection 2.5, we assume that one of the fiscal policy instruments is endogenous in the transition as residually determined by the government budget constraint. In our experiments, this role is played by the end-of-period total real public debt divided by the number of capitalists, d_t(see below for other public financing cases at the steady state).

2.8 Decentralized Equilibrium (given policy instruments)

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of households maximizes utility; (ii) every firm maximizes profit; (iii) all constraints, including the government budget constraint and the balance of payments, are satisfied; and (iv) all markets clear.

Appendix F presents the dynamic DE system. It consists of 26 equations in 26 variables $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, y_t^H, m_t^k, m_t^w, \widetilde{\omega}_t^k, f_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, k_t^k, Q_t, d_t, R_t, P_t, P_t^H, P_t^F, P_t^*]_{t=0}^{\infty}$. This is given the independently set monetary and fiscal policy instruments,$[S_t, \tau_t^c, \tau_t^k, \tau_t^n, \tau_t^l, g_t, \lambda_t]_{t=0}^{\infty}$, technology $[A_t]_{t=0}^{\infty}$, rest-of-the-world variables, $[\bar{c}_t^{F*}, Q_t^*, P_t^{H*}]_{t=0}^{\infty}$, and initial conditions for the state variables, $[k_{-1}^k, f_{-1}^k, d_{-1}, Q_{-1}, R_{-1}, m_{-1}^k, m_{-1}^w]$.

2.9 Rules for fiscal policy instruments

Following a rule-like approach, see e.g.Schmitt-Grohé and Uribe (2007), fiscal policy is conducted through simple implementable feedback rules. Namely, the fiscal authorities adjust fiscal policy instruments according

[9] See e.g. Erceg and Lindé, 2013, for a similar modelling.

to some rules reacting to an easily observable endogenous macroeconomic indicator capturing the current liabilities state of the economy.[10] More specifically, we allow all the main spending-tax policy instruments, namely, the ratio of real government spending to real GDP, defined as s_t^g, and the tax rates on consumption, capital income and labor income, τ_t^c, τ_t^k and τ_t^n respectively, to react to the beginning-of-period public liabilities to output ratio, l_{t-1}, as a deviation from a target value, l, according to the following simple linear rules:[11]

$$s_t^g = s^g - \gamma_l^g (l_{t-1} - l) \tag{56}$$

$$\tau_t^c = \tau^c + \gamma_l^c (l_{t-1} - l) \tag{57}$$

$$\tau_t^k = \tau^k + \gamma_l^k (l_{t-1} - l) \tag{58}$$

$$\tau_t^n = \tau^n + \gamma_l^n (l_{t-1} - l) \tag{59}$$

where l_{t-1} is defined as:

$$l_{t-1} \equiv \frac{R_{t-1}\lambda_{t-1}d_{t-1} + Q_{t-1}\frac{S_t}{S_{t-1}}(1-\lambda_{t-1})d_{t-1}}{\frac{P_{t-1}^H}{P_{t-1}}y_{t-1}^H} \tag{60}$$

and where, in the above rules, Eqs.(56)-(59), variables without time subscripts denote policy target values and $\gamma_l^q \geq 0$ for $q = g, c, k, n$ are feedback policy coefficients on the public debt target. The rest of fiscal policy instruments (that is, lump-sum transfers as share of GDP, denoted as s_t^l, and the share of total public debt held by domestic capitalists, λ_t) are assumed to remain constant over time and equal to their data averages (see the next subsection).

In the above rules, a policy target value (like $s^g, \tau^c, \tau^k, \tau^n$) will be the value of the corresponding variable in the new reformed steady state (see Section 4), while the debt policy target is set to a value less than in the data (this will be the case of debt consolidation where fiscal policy systematically brings public debt down over time).

[10]Here the magnitude of these reaction coefficients is set arbitrarily in a value close to those of Philippopoulos et al., 2017, who work with optimized rules.

[11]For similar rules, see e.g Schmitt-Grohé and Uribe, 2007. See also EMU-Public Finances, 2011, by the European Commission for similar fiscal reaction functions used in practice.

2.10 Exogenous variables

Let us now define the exogenous variables of the model. We assume that foreign imports or equivalently domestic exports, are a function of the terms of trade, $\tau\tau_t \equiv \frac{P_t^F}{P_t^H}$, where both variables are expressed as deviations from their steady state values, namely:

$$\frac{\bar{c}_t^{F*}}{\bar{c}^{F*}} = \left(\frac{\tau\tau_t}{\tau\tau}\right)^{\gamma} \tag{61}$$

where $0 < \gamma < 1$ is a parameter that measures the terms of trade elasticity of foreign imports. This functional form captures the idea that as the economy becomes more competitive, due to an increase in relative prices of foreign goods, we expect an increase in its exports. Note that the steady state value of $\bar{c}_t^{F*}$, that is $\bar{c}^{F*}$, is exogenously specified in Section 3.

As for the other rest-of-the-world variables, namely, the foreign interest rate, Q_t^*, and the gross rate of domestic inflation in the foreign country, $\pi_t^{H*} = \frac{P_t^{H*}}{P_{t-1}^{H*}}$, we assume that they remain constant over time and exogenously set at $Q_t^* = 1.0115$ (which is the data average value - see below) and $\pi_t^{H*} = 1$ at all t.

As for the exogenously set policy instruments, we set the nominal exchange rate S_t at 1 (fixed exchange rate) at all t, while, the total lump-sum transfers as share of GDP, $-s_t^l = -\left(\tau_t^{l,k} + \frac{v^w}{v^k}\tau_t^{l,w}\right)/\left(\frac{P_t^H}{P_t}y_t^H\right) = -\tau_t^l/\left(\frac{P_t^H}{P_t}y_t^H\right)$, and the fraction of total public debt held by domestic capitalists, λ_t, are set at their data averages values at all t.

Finally, the TFP, A_t, remains constant over time and equal to 1.

2.11 Final Equilibrium system and solution methodology

The final equilibrium system consists of the 26 equations of the DE presented in Appendix F, the 4 feedback policy rules in Subsection 2.9, the definition of l_t presented in Subsection 2.9 and the Eq.(61) for domestic exports in Subsection 2.10. Transforming some variables into ratios as presented in Appendix G.1 and using 2 auxiliary variables to transform the system into a first order one, we thus end up with 34 equations in 34 variables $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, \bar{c}_t^{F*}, y_t^H, m_t^k, m_t^w, \widetilde{\omega}_t^k, f_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, k_t^k, klead_t, Q_t, d_t, R_t, l_t, \tau\tau_t, \tau\tau lag_t, \pi_t, \pi_t^H, \pi_t^*, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n]_{t=0}^{\infty}$. Among them, there are 25 non-predetermined or jump variables, $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, \bar{c}_t^{F*}, y_t^H, \widetilde{\omega}_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, klead_t, \tau\tau_t, \pi_t, \pi_t^H, \pi_t^*, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n]_{t=0}^{\infty}$, and 9 predetermined or state variables $[m_t^k, m_t^w, f_t^k, k_t^k, Q_t, d_t, R_t, \tau\tau lag_t, l_t]_{t=0}^{\infty}$. This is given TFP, lump-sum transfers as share of GDP, rest-of-the-world variables, initial conditions for the state variables and the values of coefficients in the feedback policy rules.

To solve this non-linear difference equation system, we will take a first order approximation around the steady state. We will work as follows. We first solve for the steady state of the model numerically employing common parameters values and data in accordance with the Italian economy over 2001-2016. The next Section (Section 3) presents this steady state solution, or what we call the status quo. In turn, we will study transition dynamics, under various policy scenarios, when we depart from the status quo and travel to a new reformed steady state with lower public debt than in the status quo solution.

3 Data, parameterization and steady state solution

This section presents the parameterization and fiscal data averages from Italy over 2001-2016 (the exact end period for each variable may vary analogous to data availability) which are used to solve the status quo steady state of our model economy. Then we present this solution which it serves as a point of departure to study policy reforms.

3.1 Parameters and policy variables

We use annual data for Italy over 2001-2016, that are taken from Eurostat. The parameterization of the model is based on the assumption that the economy is in the deterministic steady state of the decentralized equilibrium presented above with fiscal policy instruments set at their data averages and zero inflation rate. Since policy instruments react to deviations of endogenous macroeconomic indicators from their steady state values, feedback policy coefficients do not play any role at the steady state.[12] In Tables 1 and 2 are reported the baseline parameters and policy variables respectively. We report that our main results are robust to changes in these parameter values

[12]In this stage of our analysis, there is no intention of policy reforms, which means that we set as target values of these macroeconomic indicators in the policy rules their status quo steady state values. This along with the assumption that the economy is in the status quo steady state imply that the feedback policy instruments do not play any role since the debt gap in policy rules is zero.

(these results are available upon request). Thus, although our numerical experiments below are not meant to provide a rigorous quantitative study, they illustrate the qualitative dynamic features of the model in a realistic way.

The value of households' discount factor, β, follows from the Euler of government bonds in steady state (which coincides with the Euler of internationally traded assets in steady state) by setting the gross interest rate at $R = Q = 1.0225$ and the gross inflation rate at 1. Note that this value of interest rate is consistent with an interest-rate premium of 1.1% over the German 10-year bond rate, which is the average value in the data.

The value of α implies a labor share, $(1-\alpha)$, equal to 0.62, which is the average value in the Italian data for the period under consideration. The parameter θ, which stands for the capitalist's labor efficiency parameter, is set so that we obtain a reasonable value for the ratio of capitalists' wage to workers' wage, $\frac{w^k}{w^w}$, which, in our model, equals 1.69. Following the related literature, we use rather standard parameter values for the inverse of intertemporal substitution elasticity, σ, the inverse of Frisch labour elasticity, η, and the price elasticity of demand, ϕ, which are as in Andrès and Doménech, 2006, and Galí, 2008. The inverse of elasticity of public consumption in utility, ζ, is set at 1. The real money balances elasticity, μ, is taken from Pappa and Neiss, 2005; this implies an interest-rate semi elasticity of money demand equal to -0.29 which is a common value in this literature. Regarding preference parameters in the utility function, χ_n follows from the households' labour supply condition, χ_m is set at 0.001 and χ_g is arbitrarily set at 0.1, which is a common valuation of public goods in related utility functions. We set the Rotemberg's price adjustments cost parameter, ϕ^P, at 91.91 which corresponds to an average frequency of price reoptimization at 15 months (see Keen and Wang, 2007). The value of γ, in Eq.(61) for foreign imports, is set at 0.9.

As for the threshold parameter value of $\overline{d}$ (see Eq.(55)), which determines the public debt-to-GDP ratio above which sovereign interest-rate premia emerge, is set at 0.9. This value is consistent with several studies which found that in most advanced economies the adverse effects of public debt arise when it is around $90-100\%$ of GDP (see e.g. Philippopoulos et al., 2017, Reinhart and Rogoff, 2010, and Checherita-Westphal and Rother, 2012). Also, this parameter value belongs to the range of thresholds for sustainable public debt estimated by the European Commission (2011). In turn, using again Eq.(55), we derive the value of the associated interest-rate premium parameter, ψ. Specifically, the value of ψ follows from Eq.(55) by setting the value for the parameter $\overline{d}$ as said just above and using data averages over the period under consideration for the interest-rate premium as well as the public debt-to-GDP ratio. The resulting value of ψ is 0.0505, which means that a percentage point increase in the debt-to-GDP ratio leads to an increase in the interest rate premium by 5.05 basis points. Such values are in line with empirical findings for OECD countries (see e.g. Ardagna et al., 2008).

The adjustments cost parameters related to changes in private and public foreign assets (see Eqs.(9) and (54) respectively) are both set at 0.3. As said, this value gives plausible short-run dynamics for private foreign assets and, in turn, for the country's net foreign debt following a policy reform. Similarly, the value of ξ, measuring the capital adjustments cost, is set at 0.3.

Concerning the exogenous TFP, A_t, it remains at 1 for every t. Regarding the rest-of-the world variables, π_t^{H*}, Q_t^* and $\overline{c}_t^{F*}$, we set their steady state values equal to $\pi^{H*} \equiv 1$, $Q^* = 1.0115$ (which is the data average in Germany) and $\left[\overline{c}^{F*}\right] / \left[c^{k,F} + \frac{v^w}{v^k} c^{w,F}\right] = 1.01$ (which is the ratio of exports to imports in the Italian data).

The steady state values of fiscal and public finance policy instruments, $\tau_t^c, \tau_t^k, \tau_t^n, s_t^g, s_t^l$ and λ_t are set at their data averages in Italy over 2001-2016. In particular, τ^c, τ^k, τ^n are the effective tax rates on consumption, capital and labor respectively in the Italian data over 2001-2016. Moreover, s^g and $-s^l$, namely, government spending on goods/services and on transfer payments as shares of aggregate output respectively, are set at their average values in the data, 0.22 and 0.23 respectively. Note that transfer payments are distributed to capitalists and workers according to their percentage in population, which are $v^k = 0.2$ and $v^w = 0.8$ respectively. Finally, the fraction of total public debt held by domestic private agents, λ, is set at 0.64 which is, again, its data average value.

We report that our main results are robust to changes in these values. Thus, although our numerical simulations below are not meant to provide a rigorous quantitative study, they illustrate the qualitative dynamic features of the model in a realistic way.

3.2 Status quo steady state

Table 3 reports the steady state solution of the model economy when we use the parameter values in Table 1 and the policy instruments in Table 2. As said, in this steady state, which is called "status quo", all fiscal policy instruments are as in the data and total real public debt divided by the number of capitalists, d, follows residually from the government budget constraint. In the fourth column of Table 3 we also present some key ratios in the Italian data. Most of the key ratios produced by our model endogenously are meaningful and close to their actual values. For example, the solution for the country's net foreign debt as share of aggregate output, $\widetilde{f}$,[13] is 0.2100, while its average value in the data is 0.2109. Also, the solution for total public debt as share of output[14] is 1.0971, while its average value in the data is 1.098. The status quo steady state will serve as a point of departure to study various policy experiments.

4 Description of policy experiments

In this section, we define the reformed economy, then we discuss about debt consolidation and, finally, we provide our solution strategy.

4.1 Definition of the reformed economy

Our main thought experiment in this paper is the case in which the economy departs from the status quo steady state (see Subsection 3.2 above for details), where fiscal policy instruments are as in the data, and travels to a new reformed steady state with lower debt and no sovereign interest-rate premia. As new reformed steady state is defined the case in which the public debt-to-GDP ratio is permanently reduced so that there are no sovereign interest-rate premia in the new steady state. In other words, in the new reformed steady state, we set premia equal to zero, that is $Q = Q^*$, which implies that the output share of public debt reduces from around 110% (which is the status quo solution) to the threshold value, $\overline{d}$, corresponding to zero premia, that is $\frac{\tau\tau^{v-1}d}{y^H} = \overline{d} = 0.9$. To put it differently, since, in our model, sovereign premia arise whenever the public debt-to-output ratio happens to be above the 0.9 threshold value, premia are eliminated ($Q = Q^*$) once debt-to-GDP ratio reduction has reached to the value of this threshold.

In addition, we assume that, in the new reformed steady state, the country's net foreign debt position becomes zero or, equivalently, that the country

[13]Thus, $\widetilde{f} \equiv \frac{S_t(F_t^g - F_t^k)}{P_t^H y_t^H} = \frac{(1-\lambda_t)D_t - S_t F_t^k}{P_t^H y_t^H} = \frac{(1-\lambda_t)\tau\tau_t^{1-v} d_t - \tau\tau_t^{v*} f_t^k}{y_t^H}$, where $\tau\tau_t \equiv \frac{P_t^F}{P_t^H}$ is the terms of trade (Recall that the number of domestic capitalists equals that of foreign investors.). Details are in Appendix.

[14]This is $\frac{D}{P^H y^H} \equiv \frac{\tau\tau^{1-v} d}{y^H}$, where $\tau\tau_t \equiv \frac{P_t^F}{P_t^H}$ is the terms of trade. Details in Appendix.

ends up with a balanced trade.[15] In other words, in the new reformed steady state, we set the country's net foreign debt as share of output to zero, $\widetilde{f} = 0$. This means that the country's net foreign debt as share of output is permanently reduced from 0.21 (which is the status quo solution) to zero. The fiscal space created by this reduction allows government to rise public spending or to cut one of the tax rates.[16]

4.2 How we model public debt consolidation

The way we model public debt consolidation is similar to that of previous chapter. Nevertheless, it is repeated here for the reader's convenience. It is widely recognized that debt consolidation implies a tradeoff between short-term fiscal pain and medium-term fiscal gain once the debt finally has been reduced. In our model, during the early phase of the transition, debt consolidation comes at the cost of increasing one of the tax rates or reducing public spending, while in the medium- and long-run, alleviation in the debt burden allows, other things equal, a cut in one of the tax rates or a rise in public spending. Thus, one has to value the early costs of stabilization vis-à-vis the medium- and long-term benefits from the fiscal space created by debt consolidation. This intertemporal tradeoff also implies that the implications of debt consolidation depend heavily on the public financing policy instruments used, namely, which policy instrument adjusts endogenously to accommodate the exogenous changes in fiscal policy (see also e.g. Leeper et al., 2010, and Davig and Leeper, 2011). Specifically, these implications depend both on which fiscal policy instrument bears the cost of adjustment in the early period of adjustment and on which fiscal policy instrument is anticipated to reap the benefit, once debt consolidation has been achieved. In the policy experiments considered below, we will experiment with fiscal policy mixes, which means that the fiscal authority is allowed to use a fiscal policy instrument in the transition and perhaps a different one in the new reformed steady state. Notice that we use one policy instrument at a time both in the transition and in steady state to understand the logic of our results.

We examine several cases of debt consolidation where the role of policy is to improve either resource allocation or "equality" by gradually reducing the debt (public and foreign) as share of output over time as said in Subsection

[15]For a similar practice (namely, to assume a zero net foreign debt position in the steady state), see e.g. Mendoza and Tesar, 2005.

[16]Notice that here we allow only one of the fiscal policy instruments at a time to take advantage of the fiscal space created by debt reduction.

4.1. Once debt has been reduced, in a new reformed steady state there is fiscal space to rise public spending or to cut one of the tax rates. Hence, we study four possible new reformed steady state solutions analogous to which one of the four fiscal policy instruments takes advantage of the fiscal space created by debt consolidation. Then, for each one of these steady state solutions, we study four transition paths analogous to which fiscal policy instrument will adjust to bring debt down during the transition to the particularly studied new reformed steady state.[17] To compute the path towards a new reformed steady state for a case of adjusting instrument in the transition, we should determine policy targets (that is policy variables without time subscripts) and coefficients in the feedback policy rules, Eqs.(56)-(59). As for the policy targets, we set as values the new reformed steady state values of the corresponding variables. As for the coefficients of policy instruments on debt gap, we set the coefficient of the adjusting instrument in the transition at the arbitrary value 0.1,[18] switching off the corresponding coefficient of the other instruments.

Having described how we model debt consolidation, let us proceed with the solution strategy we follow. First, we take a first-order approximation of the equilibrium conditions around a new reformed steady state. Next, we set the initial values of the (endogenous and exogenous) predetermined variables equal to their status quo steady state values. Finally, we compute the equilibrium transition path travelling towards a new reformed steady state with debt consolidation. Notice that, here, it is natural to use the case without debt consolidation (status quo steady state) as a reference regime through which we compare the several policy reforms.

5 Results

5.1 Steady state results

We start with comparison of steady state solutions. Recall that in the status quo (SQ) steady state, fiscal policy instruments were set as in their data averages and the public debt-to-GDP ratio followed residually, while, in the new reformed steady state, the public debt-to-GDP ratio is cut to 90%,

[17] As said in the above subsection we experiment with policy mixes.

[18] Notice that saddle path stability is achieved under all cases studied when one of the fiscal policy instrument adjusts in the transition by setting the coefficient of the chosen instrument at 0.1 (switching off the corresponding coefficient of the other instruments). This value is close to those found by optimized policy rules in related studies (see e.g. Schmitt-Grohé and Uribe, 2007, and Philippopoulos et al., 2017).

eliminating the sovereign interest premia, and the country's net foreign debt-to-GDP ratio is cut to zero, so that one of the fiscal policy instruments can follow residually meaning that s^g is allowed to rise or one of τ^k, τ^n, τ^c is allowed to be cut. Table 4 reports the value of the associated residual fiscal policy instrument in each case of new reformed steady state studied as well as in the status quo. In the two following subsections we will investigate how the implications of debt consolidation in steady state depend on the public financing policy instrument used, examining each public financing case separately. Namely, we will examine which fiscal policy instrument should be used switching either to a more efficient economy with higher output or to a more "equitable" economy.

Aggregate implications(efficiency)

Results for output in the SQ and the reformed economy under various public financing scenarios are shown in Table 5. As one would expect, in terms of aggregate economy, our numerical results imply that it is better to allow capital taxes to take advantage of the fiscal space created by debt consolidation. The superiority of the capital tax rate is consistent with the well-known result that capital taxes are particularly distorting in the medium-run and long-run (see e.g. Chamley, 1986, and Lucas, 1990). Therefore, the most efficient way of using the fiscal space generated, once the debt has finally been reduced, is to cut the capital tax rate.

Distributional implications (equity)

Results for net incomes and their ratio in the SQ and the reformed economy under various public financing scenarios are reported in Table 6. Since there are two different income groups in the society - capitalists and workers - the income gains from each particular structural reform may be distributed unequally.

Our results for each agent's net income in steady state, y^k and y^w,[19] show that, relative to status quo, both social groups gain from debt consolidation independently of which the residual instrument in the new reformed steady state is (see Table 6).

[19]The net income of the capitalist is defined as $y_t^k = -\tau_t^c c_t^k + (1-\tau_t^k)[r_t^k \tau\tau_t^{v-1} k_{t-1}^k + \widetilde{\omega_t}^k] + (1-\tau_t^n) w_t^k n_t^k + (Q_{t-1}-1)\tau\tau_t^{v+v^*-1} \frac{1}{\pi_t^*} f_{t-1}^k + (R_{t-1}-1)\frac{1}{\pi_t}\lambda_{t-1} d_{t-1} - v^k s_t^l y_t^H \tau\tau_t^{v-1}$ and the net income of the worker is defined as $y_t^w = -\tau_t^c c_t^w + (1-\tau_t^n) w_t^w n_t^w - v^k s_t^l y_t^H \tau\tau_t^{v-1}$.

But a key question is who gains more. Even if a policy reform produces a win-win outcome(Pareto efficient), here in the sense that both y^k and y^w rise, relative outcomes can also be important. Actually, the political economy literature has pointed out several reasons for this, including political ideology, envy, habits, etc. In our model, distributional implications can be measured by changes in the ratio of net incomes, y^k/y^w.

Relative to the status quo, the ratio y^k/y^w rises, or equivalently inequality rises, when the instrument that takes advantage of the fiscal space created by debt consolidation in the new reformed steady state is the tax rate on capital. Thus, this policy is Pareto efficient, but not equitable. For this reason, perhaps, we often observe workers opposing to such a reform. In terms of equity, the best outcome takes place when the fiscal space created by debt consolidation in the medium- and long-run is used to cut the labor tax rate. Such a policy causes the ratio y^k/y^w to fall, or equivalently inequality to fall.

In sum, in the new reformed steady state, a policy that both increases all net incomes and reduces income inequality is to cut the labor tax rate. On the other hand, if we focus on efficiency only, the best way of using the fiscal space is to cut the capital tax rate. This policy, although it is Pareto efficient, it rises inequality relative to status quo.

5.2 Transition results

We next study what happens in the transition as we depart from the status quo steady state and travel towards each one of the new reformed steady states with lower (public and country's) debt and no sovereign interest rate premia.

Aggregate implications(efficiency)

Results for the present value of output over different time horizons after the fiscal consolidation takes place are shown in Tables 7 and 8. Every table corresponds to a different new reformed steady state depending on what the residually determined tax-spending instrument is.

Specifically, in Tables 7 and 8, the residually determined fiscal policy instrument in steady state are respectively the tax rate on capital, τ^k, and the tax rate on labor, τ^n. Every row of a table, that corresponds to a different case analogous to what fiscal policy instrument is used for bringing public debt down during the transition, shows present values of output over different time horizons.

Inspection of the results in Tables 7 and 8 implies that if the criterion is aggregate, or per capita, output, the best policy mix is to use the long term fiscal gain (namely, the fiscal space created once debt has been reduced) to cut the capital tax rate and, during the early period of fiscal pain, to use public spending cuts to bring public debt down.

Distributional implications (equity)

Results for the ratio of the present value of the net income of capitalists to that of workers over different time horizons after the fiscal consolidation takes place are shown in Tables 9 and 10. Every table corresponds to a different new reformed steady state depending on what the residually determined fiscal policy instrument is. Specifically, in Tables 9 and 10, the residually determined fiscal policy instruments are respectively the tax rate on capital, τ^k, and the tax rate on labor, τ^n. Every row of a table, that corresponds to a different case analogous to what fiscal policy instrument is used for bringing public debt down during the transition, shows the ratio of the present value of the net income of capitalists to that of workers over different time horizons. Notice that we will check whether the value of each case is lower than the status quo steady state value of the same time period over different time horizons (if they are lower, then this case of policy reform improves equality relative to status quo). Our results show that, although the most efficient policy mix (that is, to use spending cuts during the transition and to cut the capital tax rate in the new reformed steady state) is Pareto efficient during the transition (see Table 11), it rises inequality in the transition relative to status quo (see Table 9). Alternatively, if one cares about the equity, focusing on the case where the fiscal space created by debt consolidation is used to cut the labor tax rate (see Table 10), the best recipe is to use capital taxes to bring public debt down, during the early period of fiscal pain (this holds independently of what the adjusting instrument in the new reformed steady state is - see also Table 9). It is worth mentioning that this policy mix is also Pareto efficient during the transition, as it arises from Table 12.

In sum, the policy mix that found to be the most efficient, although it is Pareto efficient, it comes at the cost of rising inequality. And all this relative to status quo. If the criterion is equity in net incomes, the best recipe, which is also Pareto efficient, is to use the long term fiscal space created by debt reduction to cut the labor tax rate, and, during the early period of fiscal pain, to use capital taxes to bring public debt down.

6 Closing the chapter and possible extensions

In this chapter was built and solved numerically a new Keynesian D(S)GE model of a small open economy within a monetary union facing sovereign interest rate premia due to debt management problem. In this model the fiscal authorities were engaged in public debt reduction over time; The emphasis was on the aggregate and distributional implications of debt consolidation, where income heterogeneity, and hence distribution, had to do with the distinction between "capitalists" and "workers". Since the results have already been written in the introduction, here I just mention a possible extension. It would be interesting to examine what happens in a two-country world economy context. This is studied in the next chapter.

Appendix A Households as capitalists

This appendix presents and solves the capitalist k's problem in some detail. There are $k = 1, 2, ..., N^k$ identical capitalists that act competitively. Each capitalist k faces a problem that can be solved following a two-step procedure. Specifically, we first solve an inter-temporal problem, in which the capitalist acts competitively to maximize discounted expected lifetime utility and, then, an intra-temporal problem, in which he minimizes his consumption expenditures.

A.1 Capitalists' optimization problem

Inter-temporal problem: Each capitalist $k = 1, 2, ..., N^k$ acts competitively to maximize discounted expected lifetime utility:

$$E_o \sum_{t=0}^{\infty} \beta^t U\left(c_t^k, n_t^k, m_t^k, g_t\right) \tag{62}$$

where c_t^k is k's consumption bundle at t as defined below in the intra-temporal problem, Eq.(69), n_t^k is k's hours of work at t, m_t^k is k's end-of-period real money balances at t, g_t is total government spending at t divided by the number of capitalists implying that the per capita public spending is defined as $v^k g_t$, E_o is the rational expectations operator conditional on the current period information set and $0 < \beta < 1$ is the time preference rate.

In our numerical solutions, we use a utility function of the form (see also e.g. Gali, 2008):

$$U\left(c_t^k, n_t^k, m_t^k, g_t\right) = \left[\frac{(c_t^k)^{1-\sigma}}{1-\sigma} - x_n \frac{(n_t^k)^{1+\eta}}{1+\eta} + x_m \frac{(m_t^k)^{1-\mu}}{1-\mu} + x_g \frac{(v^k g_t)^{1-\zeta}}{1-\zeta}\right] \tag{63}$$

where x_n, x_m, x_g, σ, η, μ, ζ are standard preference parameters.

The budget constraint of each k (written in real terms) is:

$$\begin{aligned}
&(1+\tau_t^c)c_t^k + \frac{P_t^H}{P_t}x_t^k + \frac{S_t P_t^*}{P_t} f_t^k + \frac{\phi^h}{2}\left(\frac{S_t P_t^*}{P_t} f_t^k - \frac{S P^*}{P} f^k\right)^2 + b_t^k + m_t^k = \\
&(1-\tau_t^k)\left[r_t^k \frac{P_t^H}{P_t} k_{t-1}^k + \widetilde{\omega_t}^k\right] + (1-\tau_t^n) w_t^k n_t^k + Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*} f_{t-1}^k + R_{t-1}\frac{P_{t-1}}{P_t} b_{t-1}^k + \\
&+ \frac{P_{t-1}}{P_t} m_{t-1}^k - \tau_t^{l,k}
\end{aligned} \tag{64}$$

where x_t^k is k's real investment at t, f_t^k is the real value of k's end-of-period internationally traded assets at t denominated in foreign currency (if negative, it denotes foreign private debt), b_t^k is the real value of k's end-of-period domestic government bonds at t, r_{t-1}^k is the gross real return to inherited physical capital between $t-1$ and t, k_t^k is k's end-of-period physical capital, $\widetilde{\omega_t}^k$ is k's real dividends paid by domestic firms at t, w_t^k is capitalists' real wage rate at t, Q_{t-1} is the gross nominal return to international assets between t-1 and t, $R_{t-1} \geq 1$ is the gross nominal return to domestic government bonds between $t-1$ and t, $\tau_t^{l,k}$ are real lump-sum taxes/transfers to each k from the government at t, $0 \leq \tau_t^c \leq 1$ is the tax rate on consumption at t, $0 \leq \tau_t^k \leq 1$ is the tax rate on capital income at t, $0 \leq \tau_t^n \leq 1$ is the tax rate on labor income at t, P_t is the domestic consumer price index (CPI) at t, P_t^H is the price index of home tradables at t and S_t is the nominal exchange rate (where an increase in S_t implies a depreciation) at t. Small letters denote real variables e.g. $f_t^k \equiv \frac{F_t^k}{P_t^*}$, $b_t^k \equiv \frac{B_t^k}{P_t}$, $\widetilde{\omega_t}^k \equiv \frac{\widetilde{\Omega_t}^k}{P_t}$, $w_t^k \equiv \frac{W_t^k}{P_t}$, $\tau_t^{l,k} \equiv \frac{T_t^{l,k}}{P_t}$. Also, letters with a star as superscript denote the counterpart of a variable in the rest-of-the world, e.g. P_t^* stands for the consumer price index (CPI) abroad at t, while letters without time subscripts denote steady state values, e.g. P^* stands for the steady state value of consumer price index (CPI) abroad. The parameter $\phi^h \geq 0$ measures adjustment costs related to private foreign assets as a deviation from their steady state value.

The motion of physical capital for each k is:

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)^2 k_{t-1}^k \tag{65}$$

where $0 < \delta < 1$ is the depreciation rate of physical capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to physical capital.

Therefore, in the inter-temporal problem, each capitalist k chooses $\{c_t^k, x_t^k, n_t^k, m_t^k, b_t^k, f_t^k, k_t^k\}_{t=0}^{\infty}$ to maximize Eqs.(62) and (63) subject to Eqs.(64) and (65), by taking as given prices $\{r_t^k, w_t^k, Q_t, R_t, P_t, P_t^H, P_t^*\}_{t=0}^{\infty}$, dividends $\{\widetilde{\omega_t}^k\}_{t=0}^{\infty}$, policy variables $\{S_t, \tau_t^c, \tau_t^n, \tau_t^k, \tau_t^{l,k}\}_{t=0}^{\infty}$, and initial conditions, $\{m_{-1}^k, b_{-1}^k, k_{-1}^k, f_{-1}^k\}$.

Intra-temporal problem: Each capitalist k minimizes the following total consumption expenditure:

$$P_t c_t^k = P_t^H c_t^{k,H} + P_t^F c_t^{k,F} \tag{66}$$

where $c_t^{k,H}$ is the composite domestic good consisting of h varieties consumed by capitalist k as defined below, Eq.(70), $c_t^{k,F}$ is the composite imported good consisting of f varieties consumed by capitalist k as defined below, Eq.(71), and P_t^F is the price index of foreign tradables (expressed in domestic currency).

Each capitalist k's total consumption expenditure is split into total expenditure on home and foreign goods respectively as follows:[20]

$$P_t^H c_t^{k,H} = \sum_{h=1}^{N^k} P_t^H(h) c_t^{k,H}(h) \tag{67}$$

$$P_t^F c_t^{k,F} = \sum_{f=1}^{N^k} P_t^F(f) c_t^{k,F}(f) \tag{68}$$

where the quantity of variety h produced at home country by domestic firm h and consumed by capitalist k is denoted as $c_t^{k,H}(h)$, the quantity of imported variety f produced abroad by foreign firm f and consumed by capitalist k is denoted as $c_t^{k,F}(f)$, the price of variety h produced at home is denoted as $P_t^H(h)$ and the price of variety f produced abroad is denoted as $P_t^F(f)$(denominated in domestic currency).

The consumption bundle of k is defined as:

[20] Recall that, in the introduction of Section 2 in the main text, we have assumed that the number of foreign firms(and, consequently, of imported varieties) equals that of domestic firms(and, consequently, that of domestic varieties) and, in turn, that of capitalists.

$$c_t^k = \frac{\left(c_t^{k,H}\right)^{v}\left(c_t^{k,F}\right)^{1-v}}{v^{v}(1-v)^{1-v}} \tag{69}$$

where v is the degree of preference for domestic goods (if $v > 1/2$, there is a home bias).

Using a Dixit-Stiglitz aggregator, the composite domestic good consumed by k, $c_t^{k,H}$, consists of h varieties and is given by:[21]

$$c_t^{k,H} = \left[\sum_{h=1}^{N^k}\left(\frac{1}{N^k}\right)^{\frac{1}{\phi}}\left[c_t^{k,H}(h)\right]^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{70}$$

where $\phi > 0$ is the elasticity of substitution across varieties produced in the domestic country.[22]

Similarly, using a Dixit-Stiglitz aggregator, the composite imported good produced abroad and consumed by each k, $c_t^{k,F}$, consists of f varieties and is given by:[23]

$$c_t^{k,F} = \left[\sum_{f=1}^{N^k}\left(\frac{1}{N^k}\right)^{\frac{1}{\phi}}\left[c_t^{k,F}(f)\right]^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{71}$$

Therefore, in the intra-temporal problem, each capitalist k chooses $\{c_t^{k,H}, c_t^{k,F}\}$ to minimize its total consumption expenditure, Eq.(66), subject to its consumption bundle, Eq.(69), by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^k. Next, each capitalist k chooses $\{c_t^{k,H}(h), c_t^{k,F}(f)\}$ to minimize the sum of its consumption expenditure on home and foreign goods, sum of RHS of Eqs.(67) and (68), subject to composite domestic and foreign goods consisting of varieties, Eqs.(70) and (71), by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{k,H}$ and $c_t^{k,F}$.

[21] Recall that, in the introduction of Section 2 in the main text, we have assumed that the number of domestic firms (and, consequently, of domestic varieties) equals that of capitalists.

[22] Note that, in our model, elasticity of substitution for varieties produced in the domestic country is common with that for varieties produced in the foreign country.

[23] Recall that, in the introduction of Section 2 in the main text, we have assumed that the number of foreign firms (and, consequently, of imported varieties) equals that of domestic firms and, in turn, that of capitalists.

A.2 Capitalists' optimality conditions

Each capitalist k acts competitively taking prices and policy as given.

Inter-temporal problem: The first order conditions include the budget constraint of k, Eq.(64), the law of motion of physical capital, Eq.(65), and:

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}\frac{P_t^H}{P_t}\left[1+\xi\left(\frac{k_t^k}{k_{t-1}^k}-1\right)\right]=\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_{t+1}^H}{P_{t+1}}\times \tag{72}$$
$$\times\left[(1-\delta)+(1-\tau_{t+1}^k)r_{t+1}^k-\frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k}-1\right)^2+\xi\left(\frac{k_{t+1}^k}{k_t^k}-1\right)\frac{k_{t+1}^k}{k_t^k}\right]$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}S_t\frac{P_t^*}{P_t}\left[1+\phi^h\left(S_t\frac{P_t^*}{P_t}f_t^k-S\frac{P^*}{P}f^k\right)\right]= \tag{73}$$
$$=\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}Q_tS_{t+1}\frac{P_{t+1}^*}{P_{t+1}}\frac{P_t^*}{P_{t+1}^*}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}=\beta R_t\frac{P_t}{P_{t+1}}\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{74}$$

$$x_n(n_t^k)^{\eta}=(c_t^k)^{-\sigma}\frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^k \tag{75}$$

$$x_m(m_t^k)^{-\mu}=\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}-\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_t}{P_{t+1}} \tag{76}$$

Eqs.(72), (73) and (74) are respectively the Euler equations of physical capital, internationally traded assets and domestic government bonds, Eq.(75) is the optimality condition for work hours and Eq.(76) is the optimality condition for real money balances.

Intra-temporal problem: The first order conditions include the consumption bundle of k, Eq.(69), the composite domestic and imported good consumed by k, Eqs.(70) and (71) respectively, and:

$$\frac{c_t^{k,H}}{c_t^{k,F}}=\frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{77}$$

$$c_t^{k,H}(h)=\frac{c_t^{k,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{78}$$

$$c_t^{k,F}(f) = \frac{c_t^{k,F}}{N^k}\left(\frac{P_t^F}{P_t^F(f)}\right)^{\phi} \tag{79}$$

Eq.(77) is the optimality condition for sharing the total consumption between domestic and imported products, Eqs.(78) and (79) are demand equations of capitalist for varieties produced at home and abroad respectively.

Plugging Eqs.(78) and (79) into Eqs.(70) and (71) respectively, we get the following relations for price indexes:

$$P_t^H = \left\{\sum_{h=1}^{N^k} \frac{1}{N^k}[P_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{80}$$

$$P_t^F = \left\{\sum_{f=1}^{N^k} \frac{1}{N^k}[P_t^F(f)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{81}$$

Yet, Eqs.(66), (69) and (77) imply the following relation for domestic consumer price index(CPI):

$$P_t = (P_t^H)^{v}(P_t^F)^{1-v} \tag{82}$$

Appendix B Households as workers

This appendix presents and solves the worker w's problem in some detail. There are $w = 1, 2, ..., N^w$ identical workers that act competitively. Similarly to capitalist k's problem, each worker w faces a problem that can be solved following a two-step procedure. Specifically, we first solve an inter-temporal problem, in which the worker acts competitively to maximize discounted expected lifetime utility and, then, an intra-temporal problem, in which he minimizes consumption expenditures.

B.1 Workers' optimization problem

Inter-temporal problem: Workers have the same utility function as domestic capitalists (see Eqs.(62) and (63)).

The budget constraint of each worker w is in real terms:

$$(1+\tau_t^c)c_t^w + m_t^w = (1-\tau_t^n)w_t^w n_t^w + \frac{P_{t-1}}{P_t}m_{t-1}^w - \tau_t^{l,w} \tag{83}$$

where again small letters denote real variables, e.g. $w_t^w \equiv \frac{W_t^w}{P_t}$, $\tau_t^{l,w} \equiv \frac{T_t^{l,w}}{P_t}$. Here c_t^w is w's consumption bundle at t as defined below in the intra-temporal problem, $Eq.$(87), m_t^w is w's end-of-period real money balances at t, n_t^w is w's hours of work at t, w_t^w is workers' real wage rate at t and $\tau_t^{l,w}$ are real lump-sum taxes/transfers to w from the government at t.

Therefore, in the inter-temporal problem, each worker w chooses $\{c_t^w, n_t^w, m_t^w\}_{t=0}^{\infty}$ to maximize Eqs.(62) and (63) for w, subject to Eq.(83), by taking as given prices $\{w_t^w, P_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,w}\}_{t=0}^{\infty}$, and initial conditions, m_{-1}^w.

Intra-temporal problem: Each worker w minimizes the following total consumption expenditure:

$$P_t c_t^w = P_t^H c_t^{w,H} + P_t^F c_t^{w,F} \tag{84}$$

where $c_t^{w,H}$ is the composite domestic good consisting of h varieties consumed by worker w (see also Eq.(88) below) and $c_t^{w,F}$ is the composite imported good consisting of f varieties consumed by worker w (see also Eq.(89) below).

Each worker w's total consumption expenditure is split into total expenditure on home and foreign goods respectively as follows:[24]

$$P_t^H c_t^{w,H} = \sum_{h=1}^{N^k} P_t^H(h) c_t^{w,H}(h) \tag{85}$$

$$P_t^F c_t^{w,F} = \sum_{f=1}^{N^k} P_t^F(f) c_t^{w,F}(f) \tag{86}$$

where the quantity of variety h produced by domestic firm h and consumed by worker w is denoted as $c_t^{w,H}(h)$ and the quantity of imported variety f produced by foreign firm f and consumed by worker w is denoted as $c_t^{w,F}(f)$.

The consumption bundle of w is defined as:

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v (1-v)^{1-v}} \tag{87}$$

[24]Recall that, in the introduction of Section 2 in the main text, we have assumed that the number of foreign firms(and, consequently, of imported varieties) equals that of domestic firms(and, consequently, that of domestic varieties) and, in turn, that of capitalists.

Using a Dixit-Stiglitz aggregator, the composite domestic good consumed by w, $c_t^{w,H}$, consists of h varieties and is given by:[25]

$$c_t^{w,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{w,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{88}$$

Similarly, using a Dixit-Stiglitz aggregator, the composite imported good consumed by w, $c_t^{w,F}$, consists of f varieties and is given by:[26]

$$c_t^{w,F} = \left[\sum_{f=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{w,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{89}$$

Therefore, in the intra-temporal problem, each worker w chooses $\{c_t^{w,H}, c_t^{w,F}\}$ to minimize its total consumption expenditure, Eq.(84), subject to its consumption bundle, Eq.(87), by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^w. Next, each worker w chooses $\{c_t^{w,H}(h), c_t^{w,F}(f)\}$ to minimize the sum of its consumption expenditure on home and foreign goods, sum of RHS of Eqs.(85) and (86), subject to composite domestic and foreign goods consisting of varieties h and f respectively, Eqs.(88) and (89) respectively, by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{w,H}$ and $c_t^{w,F}$.

B.2 Workers' optimality conditions

Each worker w acts competitively taking as given prices and policy.

Inter-temporal problem: The first order conditions include the budget constraint, Eq.(83), and:

$$\frac{(c_t^w)^{-\sigma}}{x_n (n_t^w)^{\eta}} = \frac{1+\tau_t^c}{(1-\tau_t^n) w_t^w} \tag{90}$$

$$\frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} = \beta \frac{P_t}{P_{t+1}} \left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m (m_t^w)^{-\mu} \tag{91}$$

[25] Recall that, in the introduction of Section 2 in the main text, we have assumed that the number of domestic firms (and, consequently, of domestic varieties) equals that of capitalists.

[26] Recall that, in the introduction of Section 2 in the main text, we have assumed that the number of foreign firms (and, consequently, of imported varieties) equals that of domestic firms and, in turn, that of capitalists.

Eqs.(90) and (91) are the optimality conditions for work hours and real money balances respectively.

Intra-temporal problem: The first order conditions include the consumption bundle of w, Eq.(87), the composite domestic and imported goods consumed by w, Eqs.(88) and (89) respectively, and:

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{92}$$

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{93}$$

$$c_t^{w,F}(f) = \frac{c_t^{w,F}}{N^k}\left(\frac{P_t^F}{P_t^F(f)}\right)^{\phi} \tag{94}$$

Eq.(92) is the optimality condition for sharing the total consumption between domestic and imported products, Eqs.(93) and (94) are demand equations of domestic worker w for varieties produced at home and abroad respectively.

Plugging Eqs.(93) and (94) into Eqs.(88) and (89) respectively, we get the following relations for price indexes:

$$P_t^H = \left\{\sum_{h=1}^{N^k}\frac{1}{N^k}[P_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{95}$$

$$P_t^F = \left\{\sum_{f=1}^{N^k}\frac{1}{N^k}[P_t^F(f)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{96}$$

which, as expected, coincide with the equations of price indexes derived from the capitalist k's problem, Eqs.(80) and (81).

Yet, Eqs.(84), (87) and (92) imply the following relation for domestic consumer price index(CPI):

$$P_t = (P_t^H)^v (P_t^F)^{1-v} \tag{97}$$

which, as expected, coincides with the equation of CPI derived from the capitalist k's problem, Eq.(82).

Appendix C Firms

This appendix presents and solves the firm h's problem. There are $h = 1, 2, ..., N^k$ identical domestic firms that each one of them produces a differentiated tradable good of variety h under monopolistic competition and Rotemberg-type nominal price rigidities (see Bi et al., 2013).

C.1 Demand for the firm h's product

Each firm h faces demand for its product, $y_t^{H,d}(h)$. The latter comes from domestic households' consumption and investment, $c_t^H(h)$ and $x_t(h)$ respectively, where $c_t^H(h) \equiv \sum_{k=1}^{N^k} c_t^{k,H}(h) + \sum_{w=1}^{N^w} c_t^{w,H}(h)$ and $x_t(h) \equiv \sum_{k=1}^{N^k} x_t^k(h)$, from the government, $g_t(h)$, and from foreign households' consumption of the domestic good, $c_t^{F^*}(h)$. Thus, aggregate demand for each good h is:

$$y_t^{H,d}(h) = \left[c_t^H(h) + x_t(h) + g_t(h) + c_t^{F^*}(h)\right] \tag{98}$$

Aggregate demand for each good h is associated with production of domestic firm h according to the following relation:

$$y_t^{H,d}(h) = y_t^H(h) \times \left\{1 - \frac{\phi^P}{2}\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right]^2\right\} \tag{99}$$

where $y_t^H(h)$ stands for the production of domestic firm h, π^H stands for the steady state value of the gross domestic goods inflation rate and $\phi^P \geq 0$ is a parameter which determines the degree of nominal price rigidity. The term in the brackets captures the Rotemberg-type pricing cost and reflects the discrepancy between production and demand as one expected in a Rotemberg-type fashion.

Since we have:

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{100}$$

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{101}$$

$$x_t^k(h) = \frac{x_t^k}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{102}$$

$$g_t(h) = \frac{N^k g_t}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{103}$$

$$c_t^{F^*}(h) = \frac{c_t^{F^*}}{N^k}\left(\frac{P_t^{F^*}}{P_t^{F^*}(h)}\right)^{\phi} \tag{104}$$

we can rewrite the Eq.(98) as:

$$y_t^{H,d}(h) = \frac{1}{N^k}\left[c_t^H + x_t + N^k g_t + N^k \bar{c}_t^{F^*}\right] \times \left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{105}$$

where $c_t^H \equiv \sum_{k=1}^{N^k} c_t^{k,H} + \sum_{w=1}^{N^w} c_t^{w,H}$ is total consumption of home goods by domestic households, $x_t \equiv \sum_{k=1}^{N^k} x_t^k$ is total investment, $N^k g_t$ denotes total government purchases of domestic output, and $c_t^{F^*} \equiv N^k \bar{c}_t^{F^*}$ is total consumption of home goods by households in the rest of the world (i.e. domestic country's exports). Also notice that the law of one price implies that in Eq.(104):

$$\frac{P_t^{F^*}}{P_t^{F^*}(h)} = \frac{\frac{P_t^H}{S_t}}{\frac{P_t^H(h)}{S_t}} = \frac{P_t^H}{P_t^H(h)} \tag{106}$$

Since aggregate demand, $N^k y_t^{H,d}$, is:

$$N^k y_t^{H,d} = \left[c_t^H + x_t + N^k g_t + N^k \bar{c}_t^{F^*}\right] \tag{107}$$

then aggregate demand for each good h is rewritten as:

$$y_t^{H,d}(h) = y_t^{H,d}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{108}$$

where $y_t^{H,d}$, as implied by above, denotes the aggregate demand of the economy divided by the number of capitalists.

Using the Eq.(99), another expression equivalent to demand for good h in terms of production can be derived:

$$y_t^H(h) = y_t^H\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{109}$$

where $y_t^{H,d} \equiv y_t^H \times \left\{1 - \frac{\phi^P}{2}\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right]^2\right\}$ and with y_t^H to denote the aggregate output of the economy divided by the number of capitalists.

Notice that solving the firm h's problem below, we should use Eq.(108) as an expression for demand of good h. However, it is more convenient for someone to work with Eq.(109), replacing demand for good h with Eq.(108).

C.2 Firms' problem

Nominal profits of each firm h are defined as:

$$P_t\widetilde{\omega_t}(h) = P_t^H(h)y_t^H(h) - P_t^H r_t^k k_{t-1}(h) - W_t^w n_t^w(h) - W_t^k n_t^k(h) - \frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right)^2 P_t^H(h)y_t^H(h) \tag{110}$$

where $k_{t-1}(h)$ denotes the physical capital input chosen by firm h, $n_t^w(h)$ denotes workers' labor input chosen by firm h and $n_t^k(h)$ denotes the capitalists' labor input chosen by firm h.

All firms use the same technology represented by the production function (similar to e.g. Hornstein et al., 2005, and Baxter and King, 1993):

$$y_t^H(h) = A_t\left[k_{t-1}(h)\right]^{\alpha}\left[\{n_t^k(h)\}^{\theta}\{n_t^w(h)^{1-\theta}\}\right]^{1-\alpha} \tag{111}$$

where A_t is an exogenous TFP, $0 < \alpha < 1$ is the share of physical capital and $0 < \theta < 1$ the labor efficiency parameter of capitalist.

Profit maximization by firm h is also subject to the demand for its product, Eq.(108), as derived above. But as we have mentioned above, instead of using Eq.(108), we can equivalently use Eq.(109).

Each firm h chooses its price, $P_t^H(h)$, and its inputs, $k_t(h)$, $n_t^k(h)$, $n_t^w(h)$, to maximize the sum of discounted expected real dividends, $\max E_o \sum\limits_{t=0}^{\infty} \Xi_{0,0+t}\widetilde{\omega_t}(h)$, subject to Eq.(109) and its production function, Eq.(111). The objective function of firm h in real terms is given by:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t}\left[\frac{P_t^H(h)}{P_t}y_t^H(h)\left\{1 - \frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right)^2\right\} - \frac{P_t^H(h)}{P_t}r_t^k k_{t-1}(h) - w_t^w n_t^w(h) - w_t^k n_t^k(h)\right] \tag{112}$$

where $\Xi_{0,0+t}$ is a stochastic discount factor taken as given by the firm h. This is defined as $\Xi_{0,0+t} = \prod_{i=0}^{t-1}\left\{\frac{1}{R_i}\right\} = \beta^t \prod_{i=0}^{t-1}\left[\left(\frac{P_i}{P_{i+1}}\right)\left(\frac{1+\tau_i^c}{1+\tau_{i+1}^c}\right)\left(\frac{c_{i+1}^k}{c_i^k}\right)^{-\sigma}\right]$ and arises from the Euler of domestic government bonds.

C.3 Firms' optimality conditions

Following the related literature, instead of solving the above problem, we follow a two-step procedure. We first solve a cost minimization problem, where each firm h minimizes its cost by choosing factors of production given technology and prices. The solution will give a minimum real cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, we solve the dynamic profit maximization problem of firm h by choosing its price.

Cost Minimization problem: In the first stage, we solve a static cost minimization problem, where each h minimizes its cost by choosing its factors of production, $k_t(h)$, $n_t^k(h)$, $n_t^w(h)$, subject to its production function, Eq.(111), given technology and prices. The cost function is defined in real terms as follows:

$$\min \widetilde{\psi} = \left[\frac{P_t^H}{P_t} r_t^k k_{t-1}(h) + w_t^w n_t^w(h) + w_t^k n_t^k(h)\right] \tag{113}$$

The solution to the cost minimization problem gives the following input demand functions:

$$\frac{P_t^H}{P_t} r_t^k k_{t-1}(h) = mc_t \alpha y_t^H(h) \tag{114}$$

$$w_t^k n_t^k(h) = mc_t \theta(1-\alpha) y_t^H(h) \tag{115}$$

$$w_t^w n_t^w(h) = mc_t(1-\theta)(1-\alpha) y_t^H(h) \tag{116}$$

where $mc_t \equiv \widetilde{\psi}'(y_t^H(h))$[27] stands for the real marginal cost which, by definition, is the derivative of the associated minimum real cost function, $\widetilde{\psi}(y_t^H(h))$, with respect to the production, $y_t^H(h)$.

[27] As we show just below, by summing up these factor demand functions, the total cost is a function of firm h's output.

Summing up the three above equations it arises the following relation for the associated minimum real cost function of h:

$$\widetilde{\psi}(y_t^H(h)) = mc_t y_t^H(h) \tag{117}$$

Where the real marginal cost, mc_t, it can be shown that equals:

$$mc_t = \frac{1}{A_t}\left[\frac{P_t^H}{P_t}\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\left\{\frac{w_t^k}{\theta(1-\alpha)}\right\}^{\theta} \times \left\{\frac{w_t^w}{(1-\theta)(1-\alpha)}\right\}^{1-\theta}\right]^{1-\alpha} \tag{118}$$

implying that mc_t is common for all firms since it only depends on prices, parameters and technology which are common for all firms.

Profit maximization: The solution to the cost minimization problem above gave a minimum real cost function, Eq.(117), which is a function of prices and output produced by the firm. In turn, given this cost function, we solve a dynamic profit maximization problem where each firm h maximizes discounted expected lifetime real profits by choosing its price, $P_t^H(h)$. The profits are defined as:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t}\left[\frac{P_t^H(h)}{P_t} y_t^H(h) - \widetilde{\psi}(y_t^H(h)) - \frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right)^2 \frac{P_t^H(h) y_t^H(h)}{P_t}\right] \tag{119}$$

The above profit maximization is subject to the Eq.(109), which is equivalent to the demand equation, Eq.(108), that the monopolistically competitive firm h faces.

The first order condition gives:

$$\begin{aligned}
(1-\phi)\frac{P_t^H(h)}{P_t} y_t^H(h) + \phi\, mc_t y_t^H(h) - \frac{\phi^P}{2}(1-\phi)\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right]^2 \frac{P_t^H(h) y_t^H(h)}{P_t} - \qquad (120) \\
-\phi^P\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right]\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}\frac{P_t^H(h)}{P_t} y_t^H(h) = \\
\beta\phi^P\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1 - \frac{P_{t+1}^H(h)}{P_t^H(h)\pi^H}\right]\frac{P_{t+1}^H(h)}{P_t^H(h)\pi^H}\frac{P_{t+1}^H(h)}{P_{t+1}} y_{t+1}^H(h)
\end{aligned}$$

Thus, the behavior of h is summarized by Eqs.(114), (115), (116) and (120).

Since all firms solve the identical problem, they will set the same price, $P_t^H(h)$, which, through the Eq.(80) (which coincides with the Eq.(95)), implies that $P_t^H(h) = P_t^H$.

Appendix D Government budget constraint

This Appendix presents the government budget constraint in some detail.

We start by presenting the government's budget constraint in nominal terms:

$$
\begin{aligned}
& N^k Q_{t-1} S_t F^g_{t-1} + P_t \frac{\phi^g}{2} N^k \left(\frac{S_t F^g_t}{P_t} - \frac{S F^g}{P} \right)^2 + N^k R_{t-1} B_{t-1} + N^k P^H_t g_t + N^k M_{t-1} = \\
& = N^k M_t + \tau^c_t \left[P^H_t c^H_t + P^F_t c^F_t \right] + \tau^k_t \left[N^k r^k_t P^H_t k^k_{t-1} + P_t N^k \widetilde{\omega}^k_t \right] + \tau^n_t \left[W^k_t N^k n^k_t + W^w_t N^w n^w_t \right] + \\
& + \left[N^k T^{l,k}_t + N^w T^{l,w}_t \right] + N^k B_t + S_t N^k F^g_t
\end{aligned}
\tag{121}
$$

where F^g_t is the end-of-period nominal public debt held by each foreign agent divided by their number[28] at t and expressed in foreign currency, B_t is the end-of-period nominal public debt held by each domestic agent (capitalist) at t, M_t is the end-of-period stock of nominal money balances divided by the number of capitalists at t and $c^F_t \equiv \sum_{k=1}^{N^k} c^{k,F}_t + \sum_{w=1}^{N^w} c^{w,F}_t$ is total consumption of imported goods by domestic households. The parameter $\phi^g \geq 0$ captures adjustment costs related to public foreign debt. Again letters without time subscripts denote steady state values, e.g. P stands for the steady state value of domestic consumer price index (CPI). The rest of the notation is as above.

Then, dividing by the domestic current CPI, P_t, and the total number of domestic capitalists, N^k, we get the government budget constraint in real terms:

$$
\begin{aligned}
& Q_{t-1} \frac{S_t P^*_t}{P_t} \frac{P^*_{t-1}}{P^*_t} f^g_{t-1} + \frac{\phi^g}{2} \left(\frac{S_t P^*_t}{P_t} f^g_t - \frac{S P^*}{P} f^g \right)^2 + R_{t-1} \frac{P_{t-1}}{P_t} b_{t-1} + \frac{P^H_t}{P_t} g_t + \frac{P_{t-1}}{P_t} m_{t-1} = \\
& = m_t + \tau^c_t \left[\frac{P^H_t}{P_t} \left(c^{k,H}_t + \frac{v^w}{v^k} c^{w,H}_t \right) + \frac{P^F_t}{P_t} \left(c^{k,F}_t + \frac{v^w}{v^k} c^{w,F}_t \right) \right] + \tau^k_t \left[r^k_t \frac{P^H_t}{P_t} k^k_{t-1} + \widetilde{\omega}^k_t \right] + \\
& + \tau^n_t \left[w^k_t n^k_t + \frac{v^w}{v^k} w^w_t n^w_t \right] + \left[\tau^{l,k}_t + \frac{v^w}{v^k} \tau^{l,w}_t \right] + b_t + S_t \frac{P^*_t}{P_t} f^g_t
\end{aligned}
\tag{122}
$$

where $f^g_t \equiv \frac{F^g_t}{P^*_t}$, $b_t \equiv \frac{B_t}{P_t}$, $m_t \equiv \frac{M_t}{P_t}$.

[28] Recall that the number of foreign investors equals that of domestic investors (capitalists).

For convenience, let $D_t \equiv B_t + S_t F_t^g$ denote the total nominal public debt issued by the domestic government divided by the number of domestic agents (capitalists). This debt can be held either by a domestic agent (capitalist), $\lambda_t D_t$, or by a foreign agent, $(1-\lambda_t)D_t$,[29] where $0 \leq \lambda_t \leq 1$.[30] Then, the above government budget constraint is rewritten as:

$$
Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\frac{P_{t-1}}{S_{t-1}P_{t-1}^*}(1-\lambda_{t-1})d_{t-1} + \frac{\phi^g}{2}\left[(1-\lambda_t)d_t - (1-\lambda)d\right]^2 + R_{t-1}\frac{P_{t-1}}{P_t}\lambda_{t-1}d_{t-1} + \tag{123}
$$

$$
+\frac{P_t^H}{P_t}g_t + \frac{P_{t-1}}{P_t}m_{t-1} = m_t + \tau_t^c\left[\frac{P_t^H}{P_t}\left(c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H}\right) + \frac{P_t^F}{P_t}\left(c_t^{k,F} + \frac{v^w}{v^k}c_t^{w,F}\right)\right] + \tau_t^k\left[r_t^k\frac{P_t^H}{P_t}k_{t-1}^k + \widetilde{\omega}_t^k\right] +
$$

$$
+\tau_t^n\left[w_t^k n_t^k + \frac{v^w}{v^k}w_t^w n_t^w\right] + \tau_t^l + d_t
$$

where $d_t \equiv \frac{D_t}{P_t}$ and $\tau_t^l \equiv \tau_t^{l,k} + \frac{v^w}{v^k}\tau_t^{l,w}$.[31]

In each period, one of $\{\tau_t^c, \tau_t^k, \tau_t^n, g_t, \tau_t^l, \lambda_t, d_t\}$ needs to follow residually to satisfy the government budget constraint.

Appendix E Equilibrium in the status quo economy

This Appendix presents in some detail the status quo equilibrium system, given feedback policy coefficients. We will work in steps.

[29] Recall that the number of foreign investors equals that of domestic investors(capitalists).

[30] Public debt differs from foreign debt. The end-of-period total public debt, written in nominal terms, is $N^k D_t = N^k B_t + S_t N^k F_t^g$, where $B_t = \lambda_t D_t$ is domestic government bonds held by each domestic capitalist and $S_t F_t^g = (1-\lambda_t)D_t$ denotes domestic government bonds held by each foreign investor. On the other hand, the country's end-of-period net foreign debt, written in nominal terms, is $S_t(N^{k*}F_t^g - N^k F_t^k) = N^{k*}(1-\lambda_t)D_t - S_t N^k F_t^k$, where F_t^k is foreign assets held by each domestic capitalist (if negative, it denotes liabilities). Again, recall that the number of domestic capitalists equals that of foreign investors.

[31] Assuming that the lump-sum transfers are distributed to each class of households according to their percentage in the population, this implies that the lump-sum transfers of a capitalist equals those of a worker, that is $\tau_t^{l,k} = \tau_t^{l,w}$. Hence, the total lump-sum transfers divided by the number of capitalists, $\tau_t^l \equiv \tau_t^{l,k} + \frac{v^w}{v^k}\tau_t^{l,w}$, along with the above relation, $\tau_t^{l,k} = \tau_t^{l,w}$, imply the equation $\tau_t^{l,k} = \tau_t^{l,w} \equiv v^k \tau_t^l$.

E.1 Market clearing conditions and the balance of payments

The market-clearing conditions in the domestic product market, the capital market, the labor markets, the money market, the domestic government bond market and the dividend market are respectively:

$$\sum_{k=1}^{N^k} c_t^{k,H} + \sum_{w=1}^{N^w} c_t^{w,H} + \sum_{k=1}^{N^k} x_t^k + N^k g_t + N^k \bar{c}_t^{F*} \equiv N^k y_t^H \times \left\{ 1 - \frac{\phi^P}{2} \left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1 \right]^2 \right\} \tag{124}$$

$$\sum_{k=1}^{N^k} k_{t-1}^k = \sum_{h=1}^{N^k} k_{t-1}(h)$$

$$\sum_{k=1}^{N^k} n_t^k = \sum_{h=1}^{N^k} n_t^k(h)$$

$$\sum_{w=1}^{N^w} n_t^w = \sum_{h=1}^{N^k} n_t^w(h)$$

$$\sum_{k=1}^{N^k} m_t^k + \sum_{w=1}^{N^w} m_t^w = N^k m_t$$

$$\sum_{k=1}^{N^k} b_t^k = N^k b_t \equiv N^k \lambda_t d_t$$

$$\sum_{k=1}^{N^k} \widetilde{\omega}_t^k = \sum_{h=1}^{N^k} \widetilde{\omega_t}(h)$$

The balance of payments is obtained by adding the profit function of firms, the budget constraint of households and the government budget constraint. Then, the balance of payments in real terms [32] is:

$$
\begin{aligned}
&\left[\frac{P_t^H}{P_t}\left(c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}\right)+\frac{P_t^F}{P_t}\left(c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}\right)+\frac{P_t^H}{P_t}x_t^k+\frac{P_t^H}{P_t}g_t\right]-\\
&-\frac{1}{N^kP_t}\sum_{h=1}^{N^k}P_t^H(h)y_t^H(h)\left\{1-\frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right)^2\right\}+\frac{\phi^h}{2}\left(\frac{S_tP_t^*}{P_t}f_t^k-\frac{SP^*}{P}f^k\right)^2+ \qquad (125)\\
&+\frac{\phi^g}{2}[(1-\lambda_t)d_t-(1-\lambda)d]^2=\\
&Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}f_{t-1}^k-Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\frac{P_{t-1}}{S_{t-1}P_{t-1}^*}(1-\lambda_{t-1})d_{t-1}+\left[(1-\lambda_t)d_t-\frac{S_tP_t^*}{P_t}f_t^k\right]
\end{aligned}
$$

where are variables that have been defined above.

Multiplying by parts the Eq.(108), $y_t^{H,d}(h)=y_t^{H,d}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi}$, by $P_t^H(h)$ and, then, aggregating with respect to the number of firms, it arises, through the domestic price index, $P_t^H=\left\{\sum_{h=1}^{N^k}\frac{1}{N^k}[P_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}}$, that $N^kP_t^Hy_t^{H,d}=\sum_{h=1}^{N^k}P_t^H(h)y_t^{H,d}(h)$. This equation, in turn, yields:

$$
\frac{1}{N^kP_t}\sum_{h=1}^{N^k}P_t^H(h)y_t^H(h)\left\{1-\frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right)^2\right\}=\frac{P_t^H}{P_t}y_t^H\left\{1-\frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right)^2\right\} \qquad (126)
$$

since we have defined $y_t^{H,d}(h)=y_t^H(h)\times\left\{1-\frac{\phi^P}{2}\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right]^2\right\}$ and $y_t^{H,d}\equiv y_t^H\times\left\{1-\frac{\phi^P}{2}\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right]^2\right\}$.

Yet, multiplying the market clearing condition in the domestic product market, Eq.(125), by $\frac{1}{N^k}\frac{P_t^H}{P_t}$, we get:

$$
\left[\frac{P_t^H}{P_t}\left(c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}\right)+\frac{P_t^H}{P_t}x_t^k+\frac{P_t^H}{P_t}g_t+\frac{P_t^H}{P_t}\bar{c}_t^{F*}\right]=\frac{P_t^H}{P_t}y_t^H\times\left\{1-\frac{\phi^P}{2}\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right]^2\right\} \qquad (127)
$$

[32] I have divided by the total number of agents and, in turn, divided all terms by v^k.

where the LHS of this equation is equal to $\frac{1}{N^k P_t} \sum_{h=1}^{N^k} P_t^H(h) y_t^H(h) \left\{ 1 - \frac{\phi^P}{2} \left(\frac{P_t^H(h)}{P_{t-1}^H(h) \pi^H} - 1 \right)^2 \right\}$, as we have shown above, Eq.(127).

Therefore, using the Eqs.(127) and (128), the balance of payments can be written :

$$\frac{P_t^F}{P_t} \left[c_t^{k,F} + \frac{v^w}{v^k} c_t^{w,F} \right] - \frac{P_t^H}{P_t} \left[\bar{c}_t^{F*} \right] + Q_{t-1} \frac{S_t P_t^*}{P_t} \frac{P_{t-1}^*}{P_t^*} \left[\frac{P_{t-1}}{S_{t-1} P_{t-1}^*} (1 - \lambda_{t-1}) d_{t-1} - f_{t-1}^k \right] \tag{128}$$

$$= \left[(1 - \lambda_t) d_t - \frac{S_t P_t^*}{P_t} f_t^k \right] - \frac{\phi^g}{2} \left[(1 - \lambda_t) d_t - (1 - \lambda) d \right]^2 - \frac{\phi^h}{2} \left(\frac{S_t P_t^*}{P_t} f_t^k - \frac{S P^*}{P} f^k \right)^2$$

Appendix F Decentralized equilibrium (given policy)

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible fiscal policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of household maximizes utility; (ii) every firm maximizes profit; (iii) all constraints, including the government budget constraint and the balance of payments, are satisfied; and (iv) all markets clear.

The DE is summarized by the following conditions:

$$x_n (n_t^k)^\eta (c_t^k)^\sigma = \frac{(1 - \tau_t^n)}{(1 + \tau_t^c)} w_t^k \tag{D1}$$

$$\frac{(c_t^k)^{-\sigma}}{(1 + \tau_t^c)} S_t \frac{P_t^*}{P_t} \left[1 + \phi^h \left(S_t \frac{P_t^*}{P_t} f_t^k - S \frac{P^*}{P} f^k \right) \right] =$$
$$= \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1 + \tau_{t+1}^c)} Q_t S_{t+1} \frac{P_{t+1}^*}{P_{t+1}} \frac{P_t^*}{P_{t+1}^*} \tag{D2}$$

$$\frac{(c_t^k)^{-\sigma}}{(1 + \tau_t^c)} \frac{P_t^H}{P_t} \left[1 + \xi \left(\frac{k_t^k}{k_{t-1}^k} - 1 \right) \right] =$$
$$= \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1 + \tau_{t+1}^c)} \frac{P_{t+1}^H}{P_{t+1}} \left[(1 - \delta) + (1 - \tau_{t+1}^k) r_{t+1}^k - \frac{\xi}{2} \left(\frac{k_{t+1}^k}{k_t^k} - 1 \right)^2 + \xi \left(\frac{k_{t+1}^k}{k_t^k} - 1 \right) \frac{k_{t+1}^k}{k_t^k} \right] \tag{D3}$$

$$x_m(m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta \frac{P_t}{P_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D4}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t \frac{P_t}{P_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D5}$$

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)^2 k_{t-1}^k \tag{D6}$$

$$\frac{c_t^{k,H}}{c_t^{k,F}} = \frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{D7}$$

$$c_t^k = \frac{\left(c_t^{k,H}\right)^v \left(c_t^{k,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D8}$$

$$c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H} + x_t^k + g_t + \overline{c}_t^{F*} = y_t^H \left\{1 - \frac{\phi^P}{2}\left[\frac{P_t^H}{P_{t-1}^H \pi^H} - 1\right]^2\right\} \tag{D9}$$

$$x_n(n_t^w)^{\eta}(c_t^w)^{\sigma} = \frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^w \tag{D10}$$

$$x_m(m_t^w)^{-\mu} = \frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} - \beta\frac{P_t}{P_{t+1}}\left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c}\right] \tag{D11}$$

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{D12}$$

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D13}$$

$$(1+\tau_t^c)c_t^w + m_t^w = \frac{P_{t-1}}{P_t}m_{t-1}^w + (1-\tau_t^n)w_t^w n_t^w - v^k \tau_t^l \tag{D14}$$

$$\frac{P_t^H}{P_t} r_t^k k_{t-1}^k = mc_t \alpha y_t^H \tag{D15}$$

$$w_t^k n_t^k = mc_t \theta (1-\alpha) y_t^H \tag{D16}$$

$$\frac{v^w}{v^k}w_t^w n_t^w = mc_t(1-\theta)(1-\alpha)y_t^H \tag{D17}$$

$$y_t^H = A_t[k_{t-1}^k]^\alpha \left[\{n_t^k\}^\theta \times \left\{\frac{v^w}{v^k}n_t^w\right\}^{1-\theta}\right]^{1-\alpha} \tag{D18}$$

$$\widetilde{\omega}_t^k = \frac{P_t^H}{P_t}y_t^H - mc_t y_t^H - \frac{\phi^P}{2}\left(\frac{P_t^H}{P_{t-1}^H \pi^H} - 1\right)^2 \frac{P_t^H}{P_t}y_t^H \tag{D19}$$

$$\begin{aligned} & Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\frac{P_{t-1}}{S_{t-1}P_{t-1}^*}(1-\lambda_{t-1})d_{t-1} + R_{t-1}\frac{P_{t-1}}{P_t}\lambda_{t-1}d_{t-1} + \\ & + \frac{P_t^H}{P_t}g_t + \frac{\phi^g}{2}\left[(1-\lambda_t)d_t - (1-\lambda)d\right]^2 + \frac{P_{t-1}}{P_t}\left[m_{t-1}^k + \frac{v^w}{v^k}m_{t-1}^w\right] = \\ & = \left[m_t^k + \frac{v^w}{v^k}m_t^w\right] + \tau_t^c\left[c_t^k + \frac{v^w}{v^k}c_t^w\right] + \tau_t^k\left[r_t^k\frac{P_t^H}{P_t}k_{t-1}^k + \widetilde{\omega}_t^k\right] + \\ & + \tau_t^n\left[w_t^k n_t^k + \frac{v^w}{v^k}w_t^w n_t^w\right] + \tau_t^l + d_t \end{aligned} \tag{D20}$$

$$\begin{aligned} & \frac{P_t^F}{P_t}\left[c_t^{k,F} + \frac{v^w}{v^k}c_t^{w,F}\right] - \frac{P_t^H}{P_t}\left[\overline{c}_t^{F*}\right] + Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\left[\frac{P_{t-1}}{S_{t-1}P_{t-1}^*}(1-\lambda_{t-1})d_{t-1} - f_{t-1}^k\right] \\ & = \left[(1-\lambda_t)d_t - \frac{S_t P_t^*}{P_t}f_t^k\right] - \frac{\phi^g}{2}\left[(1-\lambda_t)d_t - (1-\lambda)d\right]^2 - \frac{\phi^h}{2}\left(\frac{S_t P_t^*}{P_t}f_t^k - \frac{SP^*}{P}f^k\right)^2 \end{aligned} \tag{D21}$$

$$P_t = (P_t^H)^v (P_t^F)^{1-v} \tag{D22}$$

$$P_t^F = S_t P_t^{H*} \tag{D23}$$

$$P_t^* = (P_t^{H*})^{v^*}\left(\frac{P_t^H}{S_t}\right)^{1-v^*} \tag{D24}$$

$$(1-\phi)\frac{P_t^H}{P_t}y_t^H + \phi mc_t y_t^H - \frac{\phi^P}{2}(1-\phi)\left[\frac{P_t^H}{P_{t-1}^H\pi^H}-1\right]^2\frac{P_t^H y_t^H}{P_t} - \tag{D25}$$

$$-\phi^P\left[\frac{P_t^H}{P_{t-1}^H\pi^H}-1\right]\frac{P_t^H}{P_{t-1}^H\pi^H}\frac{P_t^H}{P_t}y_t^H =$$

$$\beta\phi^P\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1-\frac{P_{t+1}^H}{P_t^H\pi^H}\right]\frac{P_{t+1}^H}{P_t^H\pi^H}\frac{P_{t+1}^H}{P_{t+1}}y_{t+1}^H$$

$$Q_t = Q_t^* + \psi\left(e^{\left(\frac{P_t}{P_t^H}\frac{d_t}{y_t^H}-\bar{d}\right)}-1\right) \tag{D26}$$

where $n_t^k = n_t^k(h)$, $n_t^w(h) = \frac{v^w}{v^k}n_t^w$, $k_t^k = k_t(h)$, $\sum_{k=1}^{N^k} b_t^k \equiv N^k\lambda_t d_t$, $\tau_t^{l,k} = \tau_t^{l,w} = v^k\tau_t^l$, $m_t = m_t^k + \frac{v^w}{v^k}m_t^w$, $\widetilde{\omega}_t^k = \widetilde{\omega_t}(h)$, $P_t^H(h) = P_t^H$, $y_t^H(h) = y_t^H$.
Thus, we have a system of 26 equations [(D1)-(D26)] in the 26 following variables

$$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, y_t^H, m_t^k, m_t^w, \widetilde{\omega}_t^k, f_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w,$$
$$r_t^k, k_t^k, R_t, Q_t, d_t, P_t, P_t^H, P_t^F, P_t^*]_{t=0}^{\infty}$$

Conclusively, the Decentralized Equilibrium is a sequence of

$$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, y_t^H, m_t^k, m_t^w, \widetilde{\omega}_t^k, f_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w,$$
$$r_t^k, k_t^k, R_t, Q_t, d_t, P_t, P_t^H, P_t^F, P_t^*]_{t=0}^{\infty}$$

satisfying the equations [(D1)-(D26)], given:
a) technology $[A_t]_{t=0}^{\infty}$,
b) rest-of-the-world variables $[\bar{c}_t^{F*}, Q_t^*, P_t^{H*}]_{t=0}^{\infty}$,
c) initial conditions for state variables,
d) policy.

Appendix G Decentralized equilibrium (given feedback policy coefficients)

We now rewrite the above equilibrium conditions, first, by using the inflation rates rather than price levels and, second, by writing total public spending and total lump-sum taxes/transfers as shares of GDP, which are more convenient forms.

G.1 Transformed variables

We first express prices in rate form. We define 5 new variables, which are the gross domestic CPI inflation rate, $\pi_t \equiv \frac{P_t}{P_{t-1}}$, the gross foreign CPI inflation rate, $\pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$, the gross domestic goods inflation rate, $\pi_t^H \equiv \frac{P_t^H}{P_{t-1}^H}$, the gross rate of exchange rate depreciation, $\epsilon_t \equiv \frac{S_t}{S_{t-1}}$, and the terms of trade, $\tau\tau_t \equiv \frac{P_t^F}{P_t^H} = \frac{S_t P_t^{H*}}{P_t^H}$.[33] In what follows, we use π_t, π_t^*, π_t^H, ϵ_t, $\tau\tau_t$ instead of P_t, P_t^*, P_t^H, S_t, P_t^F respectively.

Also, for convenience and comparison with the data, we express fiscal policy variables as shares of real GDP, $\frac{P_t^H}{P_t} N^k y_t^H$. In particular, using the definitions above, the total public spending in real terms, $\frac{P_t^H}{P_t} N^k g_t$, can be written as ratio of real GDP, as $\frac{P_t^H}{P_t} N^k g_t = s_t^g \frac{P_t^H}{P_t} N^k y_t^H$, where s_t^g denotes the output share of government spending. The total lump-sum taxes/transfers in real terms, $N^k \tau_t^l \equiv \left[N^k \tau_t^{l,k} + N^w \tau_t^{l,w} \right]$, can be written as ratio of real GDP, as $N^k \tau_t^l = s_t^l \frac{P_t^H}{P_t} N^k y_t^H$, where as s_t^l are defined the lump-sum taxes/transfers as share of output. Also, as said, assuming that the lump-sum taxes/transfers to each capitalist and worker are equal, it implies $\tau_t^{l,k} = \tau_t^{l,w} = v^k s_t^l y_t^H \frac{P_t^H}{P_t}$.

Finally, using the Eqs.(D22),(D23) and (D24), we derive the following equations that we will use them below to make some transformations:

$$\tau\tau_t = \frac{P_t^F}{P_t^H} = S_t \frac{P_t^{H*}}{P_t^H} = \frac{P_t^{H*}}{P_t^{F*}}$$

$$\frac{P_t^H}{P_t} = \tau\tau_t^{v-1}$$

$$\frac{P_t^{H*}}{P_t^*} = \tau\tau_t^{1-v^*}$$

$$\frac{P_t^F}{P_t} = \tau\tau_t^{v}$$

$$\frac{P_t^{F*}}{P_t^*} = \tau\tau_t^{-v^*}$$

[33] Thus, $\frac{\tau\tau_t}{\tau\tau_{t-1}} = \frac{\frac{S_t}{S_{t-1}} \frac{P_t^{H*}}{P_{t-1}^{H*}}}{\frac{P_t^H}{P_{t-1}^H}} = \frac{\epsilon_t \pi_t^{H*}}{\pi_t^H}$

$$S_t \frac{P_t^*}{P_t} = \tau\tau_t^{v+v^*-1}$$

$$g_t = s_t^g y_t^H$$

$$\tau_t^l = s_t^l \tau\tau_t^{v-1} y_t^H$$

G.2 Final equations

Using the above, the final non-linear stochastic system is:

$$x_n (n_t^k)^\eta (c_t^k)^\sigma = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t^k \tag{D1'}$$

$$\begin{aligned}
&\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} \tau\tau_t^{v^*+v-1} \left[1 + \phi^h \left(\tau\tau_t^{v^*+v-1} f_t^k - \tau\tau^{v^*+v-1} f^k\right)\right] = \\
&= \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} Q_t \tau\tau_{t+1}^{v^*+v-1} \frac{1}{\pi_{t+1}^*}
\end{aligned} \tag{D2'}$$

$$\begin{aligned}
&\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} \tau\tau_t^{v-1} \left[1 + \xi \left(\frac{k_t^k}{k_{t-1}^k} - 1\right)\right] = \\
&= \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tau\tau_{t+1}^{v-1} \left[(1-\delta) + (1-\tau_{t+1}^k) r_{t+1}^k - \frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k} - 1\right)^2 + \xi \left(\frac{k_{t+1}^k}{k_t^k} - 1\right)\frac{k_{t+1}^k}{k_t^k}\right]
\end{aligned} \tag{D3'}$$

$$x_m (m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta \frac{1}{\pi_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D4'}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t \frac{1}{\pi_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D5'}$$

$$k_t^k = (1-\delta) k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)^2 k_{t-1}^k \tag{D6'}$$

$$\frac{c_t^{k,H}}{c_t^{k,F}} = \frac{v}{1-v} \tau\tau_t \tag{D7'}$$

$$c_t^k = \frac{\left(c_t^{k,H}\right)^v \left(c_t^{k,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D8'}$$

$$c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H} + x_t^k + s_t^g y_t^H + \bar{c}_t^{F*} = y_t^H\left\{1 - \frac{\phi^P}{2}\left[\frac{\pi_t^H}{\pi^H} - 1\right]^2\right\} \tag{D9'}$$

$$x_n(n_t^w)^\eta (c_t^w)^\sigma = \frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^w \tag{D10'}$$

$$x_m(m_t^w)^{-\mu} = \frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} - \beta\frac{1}{\pi_{t+1}}\left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c}\right] \tag{D11'}$$

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v}\tau\tau_t \tag{D12'}$$

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D13'}$$

$$(1+\tau_t^c)c_t^w + m_t^w = \frac{1}{\pi_t}m_{t-1}^w + (1-\tau_t^n)w_t^w n_t^w - v^k s_t^l y_t^H \tau\tau_t^{v-1} \tag{D14'}$$

$$\tau\tau_t^{v-1} r_t^k k_{t-1}^k = mc_t \alpha y_t^H \tag{D15'}$$

$$w_t^k n_t^k = mc_t\theta(1-\alpha)y_t^H \tag{D16'}$$

$$\frac{v^w}{v^k}w_t^w n_t^w = mc_t(1-\theta)(1-\alpha)y_t^H \tag{D17'}$$

$$y_t^H = A_t[k_{t-1}^k]^\alpha \left[\{n_t^k\}^\theta \times \left\{\frac{v^w}{v^k}n_t^w\right\}^{1-\theta}\right]^{1-\alpha} \tag{D18'}$$

$$\widetilde{\omega}_t^k = \tau\tau_t^{v-1}y_t^H - mc_t y_t^H - \frac{\phi^P}{2}\left(\frac{\pi_t^H}{\pi^H} - 1\right)^2 \tau\tau_t^{v-1}y_t^H \tag{D19'}$$

$$
\begin{aligned}
&Q_{t-1}\tau\tau_t^{v^*+v-1}\frac{1}{\pi_t^*}\tau\tau_{t-1}^{1-v-v^*}(1-\lambda_{t-1})d_{t-1}+R_{t-1}\frac{1}{\pi_t}\lambda_{t-1}d_{t-1}+\\
&\qquad +s_t^g y_t^H\tau\tau_t^{v-1}+\frac{1}{\pi_t}\left[m_{t-1}^k+\frac{v^w}{v^k}m_{t-1}^w\right]=\\
&=\left[m_t^k+\frac{v^w}{v^k}m_t^w\right]+\tau_t^c\left[c_t^k+\frac{v^w}{v^k}c_t^w\right]+\tau_t^k\left[r_t^k\tau\tau_t^{v-1}k_{t-1}^k+\widetilde{\omega}_t^k\right]+\\
&\qquad +\tau_t^n\left[w_t^k n_t^k+\frac{v^w}{v^k}w_t^w n_t^w\right]+\left[s_t^l y_t^H\tau\tau_t^{v-1}\right]+\\
&\qquad +d_t-\frac{\phi^g}{2}\left[(1-\lambda_t)d_t-(1-\lambda)d\right]^2
\end{aligned}
\tag{D20'}
$$

$$
\begin{aligned}
&\tau\tau_t^v\left[c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}\right]-\tau\tau_t^{v-1}\left[\bar{c}_t^{F*}\right]+Q_{t-1}\tau\tau_t^{v^*+v-1}\frac{1}{\pi_t^*}\left[\tau\tau_{t-1}^{1-v-v^*}(1-\lambda_{t-1})d_{t-1}-f_{t-1}^k\right]\\
&=\left[(1-\lambda_t)d_t-\tau\tau_t^{v^*+v-1}f_t^k\right]-\frac{\phi^g}{2}\left[(1-\lambda_t)d_t-(1-\lambda)d\right]^2-\frac{\phi^h}{2}\left(\tau\tau_t^{v^*+v-1}f_t^k-\tau\tau^{v^*+v-1}f^k\right)^2
\end{aligned}
\tag{D21'}
$$

$$
\frac{\pi_t}{\pi_t^H}=\left(\frac{\tau\tau_t}{\tau\tau_{t-1}}\right)^{1-v}
\tag{D22'}
$$

$$
\frac{\tau\tau_t}{\tau\tau_{t-1}}=\epsilon_t\frac{\pi_t^{H*}}{\pi_t^H}
\tag{D23'}
$$

$$
\frac{\pi_t^*}{\pi_t^{H*}}=\left(\frac{\tau\tau_t}{\tau\tau_{t-1}}\right)^{v^*-1}
\tag{D24'}
$$

$$
\begin{aligned}
&(1-\phi)\tau\tau_t^{v-1}y_t^H+\phi mc_t y_t^H-\frac{\phi^P}{2}(1-\phi)\left[\frac{\pi_t^H}{\pi^H}-1\right]^2\tau\tau_t^{v-1}y_t^H-\\
&\qquad -\phi^P\left[\frac{\pi_t^H}{\pi^H}-1\right]\frac{\pi_t^H}{\pi^H}\tau\tau_t^{v-1}y_t^H=\\
&\beta\phi^P\left[\left(\frac{1}{\pi_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1-\frac{\pi_{t+1}^H}{\pi^H}\right]\frac{\pi_{t+1}^H}{\pi^H}\tau\tau_{t+1}^{v-1}y_{t+1}^H
\end{aligned}
\tag{D25'}
$$

$$Q_t = Q_t^* + \psi\left(e^{\left(\frac{d_t \tau\tau_t^{1-v}}{y_t^H} - \overline{d}\right)} - 1\right) \tag{D26'}$$

$$\frac{\overline{c}_t^{F*}}{\overline{c}^{F*}} = \left(\frac{\tau\tau_t}{\tau\tau}\right)^{\gamma} \tag{D27'}$$

$$l_{t-1} \equiv \frac{R_{t-1}\lambda_{t-1}d_{t-1}\tau\tau_{t-1}^{1-v} + Q_{t-1}\epsilon_t(1-\lambda_{t-1})d_{t-1}\tau\tau_{t-1}^{1-v}}{y_{t-1}^H} \tag{D28'}$$

$$s_t^g = s^g - \gamma_l^g(l_{t-1} - l) \tag{D29'}$$

$$\tau_t^k = \tau^k + \gamma_l^k(l_{t-1} - l) \tag{D30'}$$

$$\tau_t^n = \tau^n + \gamma_l^n(l_{t-1} - l) \tag{D31'}$$

$$\tau_t^c = \tau^c + \gamma_l^c(l_{t-1} - l) \tag{D32'}$$

$$\tau\tau lag_{t+1} = \tau\tau_t \tag{D33'}$$

$$klead_{t-1} = k_t^k \tag{D34'}$$

The final equilibrium system consists of the 26 equations of the DE presented in Appendix F, the 4 feedback policy rules in Subsection 2.9 in the main text, the definition of l_t presented in Subsection 2.9 in the main text, and the Eq.(61) for domestic exports in Subsection 2.10 in the main text. Transforming some variables into ratios as presented in Appendix G.1 and using 2 auxiliary variables to transform the system into a first order one, we, thus, end up with 34 equations in 34 variables $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, \overline{c}_t^{F*}, y_t^H, m_t^k, m_t^w, \widetilde{\omega}_t^k, f_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, k_t^k, klead_t, Q_t, d_t, R_t, l_t, \tau\tau_t, \tau\tau lag_t, \pi_t, \pi_t^H, \pi_t^*, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n]_{t=0}^{\infty}$. Among them, there are 25 non-predetermined or jump variables, $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, \overline{c}_t^{F*}, y_t^H, \widetilde{\omega}_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w, n_t^w, r_t^k, klead_t, \tau\tau_t, \pi_t, \pi_t^H, \pi_t^*, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n]_{t=0}^{\infty}$, and 9 predetermined or state variables $[m_t^k, m_t^w, f_t^k, k_t^k, Q_t, d_t, R_t, l_t, \tau\tau lag_t]_{t=0}^{\infty}$. This is given TFP, total lump-sum transfers as share of GDP, rest-of-the-world variables, initial conditions for the state variables and the values of coefficients in the feedback policy rules.

Conclusively, we have a system of 34 equations [(D1')-(D34')] in the 34 following variables

$$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, \bar{c}_t^{F*}, y_t^H, m_t^k, m_t^w, \widetilde{\omega}_t^k, f_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w,$$

$$n_t^w, r_t^k, k_t^k, klead_t, Q_t, d_t, R_t, l_t, \tau\tau_t, \tau\tau lag_t, \pi_t, \pi_t^H, \pi_t^*, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n]_{t=0}^{\infty}$$

Conclusively, the Decentralized Equilibrium is a sequence of

$$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, \bar{c}_t^{F*}, y_t^H, m_t^k, m_t^w, \widetilde{\omega}_t^k, f_t^k, x_t^k, mc_t, w_t^k, n_t^k, w_t^w,$$

$$n_t^w, r_t^k, k_t^k, klead_t, Q_t, d_t, R_t, l_t, \tau\tau_t, \tau\tau lag_t, \pi_t, \pi_t^H, \pi_t^*, s_t^g, \tau_t^c, \tau_t^k, \tau_t^n]_{t=0}^{\infty}$$

satisfying the equations [(D1')-(D34')], given:

a) some exogenous variables which remain constant over time such as $[A_t, Q_t^*, \pi_t^{H*}, s_t^l]_{t=0}^{\infty}$,

b) initial conditions for state variables $[k_{-1}^k, f_{-1}^k, d_{-1}, m_{-1}^k, m_{-1}^w, R_{-1}, Q_{-1}, l_{-1}, \tau\tau lag_{-1}]$.

References

[1] Andrès, J., and Doménech, R. Automatic stabilizers, fiscal rules and macroeconomic stability. *European Economic Review 50*, 6 (2006), 1487–1506.

[2] Ardagna, S., Caselli, F., and Lane, T. Fiscal Discipline and the Cost of Public Debt Service: Some Estimates for OECD Countries. *The B.E. Journal of Macroeconomics 7*, 1 (August 2007), 1–35.

[3] Baxter, M., and King, R. G. Fiscal Policy in General Equilibrium. *American Economic Review 83*, 3 (June 1993), 315–334.

[4] Bi, H., Leeper, E. M., and Leith, C. Uncertain fiscal consolidations. *The Economic Journal 123*, 566 (2013), F31–F63.

[5] Chamley, C. Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives. *Econometrica 54*, 3 (May 1986), 607–622.

[6] Checherita-Westphal, C., and Rother, P. The impact of high government debt on economic growth and its channels: An empirical investigation for the euro area. *European Economic Review 56*, 7 (2012), 1392–1405.

[7] Coenen, G., Mohr, M., and Straub, R. Fiscal consolidation in the euro area: Long-run benefits and short-run costs. *Economic Modelling 25*, 5 (2008), 912–932.

[8] Davig, T., and Leeper, E. M. Monetary-fiscal policy interactions and fiscal stimulus. *European Economic Review 55*, 2 (February 2011), 211–227.

[9] Erceg, C., and Lindé, J. Fiscal consolidation in an open economy. *American Economic Review 102*, 3 (2012), 186–91.

[10] Erceg, C. J., and Lindé, J. Fiscal consolidation in a currency union: Spending cuts vs. tax hikes. *Journal of Economic Dynamics and Control 37*, 2 (2013), 422–445.

[11] EuropeanCommission. Public Finances in EMU, Brussels(2011).

[12] Forni, L., Gerali, A., and Pisani, M. The macroeconomics of fiscal consolidations in euro area countries. *Journal of Economic Dynamics and Control 34*, 9 (2010), 1791–1812.

[13] Gali, J., et al. Introduction to monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework [monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework]. *Introductory Chapters* (2008).

[14] Garcia-Cicco, J., Pancrazi, R., and Uribe, M. Real Business Cycles in Emerging Countries? *American Economic Review 100*, 5 (December 2010), 2510–2531.

[15] Hornstein, A., Krusell, P., and Violante, G. L. The Effects of Technical Change on Labor Market Inequalities. In *Handbook of Economic Growth*, P. Aghion and S. Durlauf, Eds., vol. 1 of *Handbook of Economic Growth*. Elsevier, July 2005, ch. 20, pp. 1275–1370.

[16] Keen, B., and Wang, Y. What is a realistic value for price adjustment costs in New Keynesian models? *Applied Economics Letters 14*, 11 (2007), 789–793.

[17] Leeper, E. M., Plante, M., and Traum, N. Dynamics of fiscal financing in the united states. *Journal of Econometrics 156*, 2 (2010), 304–321.

[18] Lombardo, G., and Vestin, D. Welfare implications of Calvo vs. Rotemberg-pricing assumptions. *Economics Letters 100*, 2 (August 2008), 275–279.

[19] Lucas, Robert E, J. Supply-Side Economics: An Analytical Review. *Oxford Economic Papers 42*, 2 (April 1990), 293–316.

[20] Mendoza, E. G., and Tesar, L. L. Why hasn't tax competition triggered a race to the bottom? Some quantitative lessons from the EU. *Journal of Monetary Economics 52*, 1 (January 2005), 163–204.

[21] Neiss, K. S., and Pappa, E. Persistence without too much price stickiness: the role of variable factor utilization. *Review of Economic Dynamics 8*, 1 (January 2005), 231–255.

[22] Philippopoulos, A., Varthalitis, P., and Vassilatos, V. Fiscal Consolidation in an Open Economy with Sovereign Premia and without Monetary Policy Independence. *International Journal of Central Banking 13*, 4 (December 2017), 259–306.

[23] Reinhart, C., and Rogoff, K. Growth in a time of debt. *American Economic Review 100*, 2 (2010), 573–78.

[24] Schmitt-Grohé, S., and Uribe, M. Closing small open economy models. *Journal of International Economics 61*, 1 (October 2003), 163–185.

[25] Uribe, M., and Schmitt-Grohé, S. *Open Economy Macroeconomics*. Princeton University Press, 2017.

[26] Walsh, C. *Monetary Theory and Policy, Third Edition*, 3 ed., vol. 1. The MIT Press, 2010.

[27] Wickens, M. Macroeconomic theory. *A Dynamic General Equilibrium Approach. Princeton, NJ* (2008).

Table 1: Baseline parameter values

Parameter	Value	Description
v^k	0.2	share of capitalists in population
v^w	0.8	share of workers in population
α	0.38	share of capital
θ	0.19	labor efficiency parameter of capitalist
β	0.9780	time preference rate
v	0.5	home goods bias parameter at home
v^*	0.5	home goods bias parameter abroad
μ	3.42	parameter related to money demand elasticity
δ	0.04	capital depreciation rate
ϕ^P	91.91	Rotemberg's price adjustments cost parameter
ϕ	6	price elasticity of demand
η	1	inverse of Frisch labor supply elasticity
σ	1	inverse of intertemporal substitution elasticity
ζ	1	inverse of elasticity of public consumption in utility
ψ	0.0505	interest-rate premium parameter
χ_m	0.001	preference parameter related to real money balances
χ_n	5	preference parameter related to work effort
χ_g	0.1	preference parameter related to public spending
$\bar{d}$	0.9	threshold parameter of public debt as share of output
γ	0.9	terms of trade elasticity of foreign imports
ξ	0.3	adjustment cost parameter on physical capital
ϕ^g	0.3	adjustment cost parameter on foreign public debt
ϕ^h	0.3	adjustment cost parameter on private foreign assets/debt
λ	0.64	fraction of total public debt held by domestic agents
$\left[\bar{c}_t^{F*}\right]/\left[c^{k,F}+\frac{v^w}{v^k}c^{w,F}\right]$	1.01	exports to imports ratio
γ_l^g	0.1	coefficient of government spending on debt gap
γ_l^c	0	coefficient of consumption tax rate on debt gap
γ_l^k	0	coefficient of capital tax rate on debt gap
γ_l^n	0	coefficient of labor tax rate on debt gap

Table 2: Policy variables (data average values)

Parameter	Value	Description
R	1.0225	long-run nominal interest rate
τ^c	0.18	consumption tax rate
τ^k	0.31	capital tax rate
τ^n	0.42	labor tax rate
s^g	0.22	government spending as share of output
s^l	-0.23	lump-sum taxes/transfers as share of output

Table 3: "Status quo" steady state solution

Variables	Description	Steady state solution	Data
y^H	output	1.9829	
c^k	consumption of capitalist	0.5366	
c^w	consumption of private worker	0.1807	
n^k	labor of capitalist	0.1908	
n^w	labor of worker	0.3361	
$v^k n^k + v^w n^w$	weight average of labor	0.3070	0.2183
k^k	physical capital	6.9148	
w^k	real wage rate of capitalist	1.0391	
w^w	real wage rate of worker	0.6165	
$\frac{w^k}{w^w}$	real wage rates ratio	1.6854	
r^k	real return to capital	0.0908	
$\tau\tau$	terms of trade	0.9952	
$Q - Q^*$	interest rate premium	0.0110	0.0110
$\frac{c^k+\frac{v^w}{v^k}c^w}{y^H\tau\tau^{v-1}}$	total consumption as share of GDP	0.6336	0.5961
$\frac{k}{y^H}$	physical capital as share of GDP	3.4872	
$\frac{d}{y^H\tau\tau^{v-1}}$	total public debt as share of GDP	1.0971	1.098
$\frac{f^k\tau\tau^{v^*}}{y_t^H}$	private foreign assets as share of GDP	0.1849	0.1039
$\widetilde{f} \equiv \frac{\frac{(1-\lambda)d}{\tau\tau^{v-1}} - f^k\tau\tau^{v^*}}{y_t^H}$	country's net foreign debt as share of GDP	0.2100	0.2109
y^k	income of capitalist	0.8139	
y^w	income of private worker	0.1807	

Note: Parameters and policy variables as in Tables 1 and 2.

Table 4: Values of the residual fiscal policy instruments in steady state

Residual Instrument	Status quo	New steady state
τ^k	0.3118	0.2929
τ^n	0.4210	0.4018
τ^c	0.1756	0.1593
s^g	0.2222	0.2306

Table 5: Output(**GDP**) in steady state

Residual Instrument	New steady state	% Change relative to the SQ
τ^k	2.2957	**+15.7754 %**
τ^n	2.2800	**+14.9842 %**
τ^c	2.2615	+14.0485 %
s^g	2.2615	+14.0485 %

Note: Steady state value of the output in the status quo(SQ) is 1.9829.

Table 6: Net income of capitalists and net income of workers in steady state

Residual Instrument	New steady state			% Changes from staus quo steady state		
	y^k	y^w	y^k/y^w	y^k	y^w	y^k/y^w
τ^k	0.9461	0.2077	4.5557	+16.2477%	+14.9218%	+1.1537%
τ^n	0.9242	0.2101	4.3983	+13.5530%	+16.2759%	-2.3418%
τ^c	0.9205	0.2075	4.4370	+13.0984%	+14.8012%	-1.4832%
s^g	0.9130	0.2046	4.4628	+12.1782%	+13.2076%	-0.9093%

Note: y^k stands for the net income of the capitalist in steady state and y^w stands for the net income of the worker in steady state. The values of y^k, y^w and y^k/y^w in status quo steady state are 0.8139, 0.1807 and 4.5038 respectively.

Table 7: Present value of output (GDP) over different time horizons **when the residual instrument in steady state is the tax rate on capital(τ^k).**

Adj.Instr.	$\widetilde{y}_5$	$\widetilde{y}_{10}$	$\widetilde{y}_{20}$	$\widetilde{y}_{40}$	$\widetilde{y}_{60}$	$\widetilde{y}_{80}$	$\widetilde{y}_{\infty}$
τ^k	10.3397	20.1413	38.4035	70.0101	95.6404	116.2145	176.2069
τ^n	10.2226	20.0098	38.3062	69.9519	95.6025	116.2000	176.2506
τ^c	10.2471	20.0707	38.3624	70.0205	95.7142	116.3489	176.4509
s^g	10.3394	20.2719	38.7149	70.4200	96.1282	116.7796	176.8966
Status quo	9.4879	17.9770	32.3684	53.1127	66.4074	74.9276	89.0556

Note: $\widetilde{y}_t$ stands for the discounted expected value of output (GDP) for the next t periods after the fiscal consolidation takes place.

Table 8: Present value of output (GDP) over different time horizons **when the residual instrument in steady state is the tax rate on labor(τ^n).**

Adj. Instr.	$\widetilde{y}_5$	$\widetilde{y}_{10}$	$\widetilde{y}_{20}$	$\widetilde{y}_{40}$	$\widetilde{y}_{60}$	$\widetilde{y}_{80}$	$\widetilde{y}_{\infty}$
τ^k	10.3462	20.1461	38.3785	69.8676	95.3634	115.8150	175.4147
τ^n	10.2320	20.0166	38.2810	69.8120	95.3320	115.8087	175.4659
τ^c	9.9797	19.9474	38.2166	69.7534	95.3146	115.8255	175.5222
s^g	10.3467	20.2844	38.7126	70.3178	95.8921	116.4164	176.1293
Status quo	9.4879	17.9770	32.3684	53.1127	66.4074	74.9276	89.0556

Note: $\widetilde{y}_t$ stands for the discounted expected value of output (GDP) for the next t periods after the fiscal consolidation takes place.

Table 9: Ratio of the present value of the net income of the capitalist to that of the worker over various time horizons **when the residual instrument in steady state is the tax rate on capital(τ^k).**

Adj. Instr.	$\frac{\widetilde{y}^k_5}{\widetilde{y}^w_5}$	$\frac{\widetilde{y}^k_{10}}{\widetilde{y}^w_{10}}$	$\frac{\widetilde{y}^k_{20}}{\widetilde{y}^w_{20}}$	$\frac{\widetilde{y}^k_{40}}{\widetilde{y}^w_{40}}$	$\frac{\widetilde{y}^k_{60}}{\widetilde{y}^w_{60}}$	$\frac{\widetilde{y}^k_{80}}{\widetilde{y}^w_{80}}$	$\frac{\widetilde{y}^k_{\infty}}{\widetilde{y}^w_{\infty}}$
τ^k	4.2116	4.3348	4.4476	4.5057	4.5192	4.5248	4.5341
τ^n	4.4115	4.4617	4.5105	4.5439	4.5555	4.5600	4.5618
τ^c	4.4907	4.4828	4.5205	4.5492	4.5573	4.5598	4.5600
s^g	4.4622	4.4707	4.5048	4.5372	4.5465	4.5494	4.5519
Status quo	4.5038	4.5038	4.5038	4.5038	4.5038	4.5038	4.5038

Note: $\widetilde{y}^k_t$ and $\widetilde{y}^w_t$ stand for the PV of the net income of the capitalist and that of the worker respectively for the next t periods after the fiscal consolidation.

Table 10: Ratio of the present value of the net income of the capitalist to that of the worker over various time horizons **when the residual instrument in steady state is the tax rate on labor(τ^n).**

Adj. Instr.	$\frac{\widetilde{y}_5^k}{\widetilde{y}_5^w}$	$\frac{\widetilde{y}_{10}^k}{\widetilde{y}_{10}^w}$	$\frac{\widetilde{y}_{20}^k}{\widetilde{y}_{20}^w}$	$\frac{\widetilde{y}_{40}^k}{\widetilde{y}_{40}^w}$	$\frac{\widetilde{y}_{60}^k}{\widetilde{y}_{60}^w}$	$\frac{\widetilde{y}_{80}^k}{\widetilde{y}_{80}^w}$	$\frac{\widetilde{y}_{\infty}^k}{\widetilde{y}_{\infty}^w}$
τ^k	4.0780	4.1911	4.2955	4.3498	4.3628	4.3683	4.3775
τ^n	4.2823	4.3235	4.3645	4.3918	4.4010	4.4044	4.4051
τ^c	4.6377	4.3754	4.3941	4.4096	4.4109	4.4100	4.4066
s^g	4.3325	4.3310	4.3565	4.3831	4.3909	4.3933	4.3953
Status quo	4.5038	4.5038	4.5038	4.5038	4.5038	4.5038	4.5038

Note: $\widetilde{y}_t^k$ and $\widetilde{y}_t^w$ stand for the PV of the net income of the capitalist and that of the worker respectively for the next t periods after the fiscal consolidation.

Table 11: Present values of the net income of the capitalist ($\widetilde{y}_t^k$) and of the worker ($\widetilde{y}_t^w$) over various time horizons (t) **when the adjusting instrument in the transition is public spending (s^g) and the residual instrument in steady state is the tax rate on capital(τ^k).**

	$t = 10$	$t = 20$	$t = 40$	$t = 80$	$t \to \infty$
$\widetilde{y}_t^k$	8.842 (7.378)	16.820 (13.285)	30.288 (21.799)	49.586 (30.753)	74.383 (36.551)
$\widetilde{y}_t^w$	1.978 (1.638)	3.734 (2.950)	6.675 (4.840)	10.899 (6.828)	16.341 (8.116)

Note: The values of the corresponding variables in the status quo are in parentheses.

Table 12: Present values of the net income of the capitalist ($\widetilde{y}_t^k$) and of the worker ($\widetilde{y}_t^w$) over various time horizons (t) **when the adjusting instrument in the transition is the tax rate on capital (τ^k) and the residual instrument in steady state is the tax rate on labor(τ^n).**

	$t = 10$	$t = 20$	$t = 40$	$t = 80$	$t \to \infty$
$\widetilde{y}_t^k$	8.437 (7.378)	16.219 (13.285)	29.360 (21.799)	48.142 (30.753)	72.323 (36.551)
$\widetilde{y}_t^w$	2.013 (1.638)	3.776 (2.950)	6.750 (4.840)	11.021 (6.828)	16.522 (8.116)

Note: The values of the corresponding variables in the status quo are in parentheses.

CHAPTER 4. DEBT CONSOLIDATION AND ITS CROSS-COUNTRY EFFECTS: AGGREGATE AND DISTRIBUTIONAL IMPLICATIONS

Debt consolidation and its cross-country effects: Aggregate and distributional implications

Abstract

This chapter builds and solves numerically a New Keynesian D(S)GE model consisting of two heterogeneous countries participating in a monetary union. We study how public debt consolidation in a country with high debt and in a country with solid public finances affects each other's aggregate macroeconomic outcomes as well as income distribution. The emphasis is on the aggregate and distributional implications of debt consolidation, where income heterogeneity in both countries, and hence distribution, has to do with the distinction among ”capitalists”, ”private workers” and ”public employees”. The paper focus on how these implications depend on the specific fiscal policy instrument used for debt consolidation. There are two key results. First, if the criterion is aggregate, or per capita, output, the best policy mix for both countries is to use the long term fiscal gain created by debt reduction to finance an increase in public investment spending and, during the early period of fiscal pain, to use public consumption spending cuts to bring public debt down. Second, if the criterion is equity in net incomes, the best recipe for both countries is to use the long term fiscal gain created by debt reduction to cut the labor tax rate and, during the early period of fiscal pain, to use capital taxes to bring public debt down.

1 Introduction

The 2008 world crisis has, among other things, brought into the spotlight the need for debt consolidation in several eurozone periphery countries.[1] For reasons related to sustainability and loss of confidence, these countries[2] have been forced to take restrictive fiscal policy measures which have further dampened demand in the short term and hurt especially relatively poor income groups. On the other hand, fiscal policy in eurozone center countries, like Germany, has been neutral.[3]

In this paper, we study how public debt consolidation in a country with high debt and sovereign premia and in a country with solid public finances (which can go for mild consolidation) affects each other's aggregate macroeconomic outcomes as well as income distribution. The study of distributional implications differentiates this chapter/paper from most of the existing literature on debt consolidation. Most of the latter has focused on aggregate implications only (see e.g. Philippopoulos et al., 2017, Coenen et al., 2008, Forni et al., 2010, Erceg and Lindé, 2013 etc.).

In light of the above, this paper provides a quantitative study of the aggregate and distributional implications of debt consolidation in a New Keynesian D(S)GE model consisting of two heterogeneous countries forming a currency union. Country heterogeneity takes the form of weak public finances and external debt in one country and sound public finances and external assets in the other country and this is reflected in sovereign interest rate premia. Obviously, to study the distributional implications of debt consolidation within each country, we need a model with heterogeneous households. There are many types of income heterogeneity in the literature. Here, we focus on the distinction among ”capitalists”, ”private workers” and ”public employees”. Capitalists are defined to be those households who hold assets, own the private firms and get labor income for their managerial services. Private workers and public employees are defined to be those households that are employed in private and public sector respectively and have labor income only. The labor of public employees, together with goods purchased from the private sector, are used by the state-owned firm as inputs in the production of public goods and services. On the private production side, firms enjoy monopoly power and face Rotemberg-type nominal price rigidities, while their productivity is enhanced by the public investment in infrastructure.

[1] See e.g. EMU-Public Finances (2015) by the European Commission.

[2] Specifically, these countries are Greece, Italy, Portugal, Spain and Cyprus.

[3] See e.g.EMU-Public Finances (2015) by the European Commission.

As for macroeconomic policy, monetary policy is common for the monetary union. On the other hand, countries in the monetary union can follow their fiscal policy independently (national fiscal policies). Following a rule-like approach to policy, we assume that fiscal policy is conducted via simple implementable feedback policy rules. In particular, we assume that, in each country, each category of public spending and tax rates is allowed to respond to the inherited public debt-to-GDP ratio as a deviation from a policy target. Debt consolidation means that the target is lower than the average in the data.

The model is solved numerically employing commonly used parameter values and fiscal data from Germany (called the home country) and Italy (called the foreign country) over the euro years. As we will see later, the steady state solution of this model gives well defined values of the great ratios for these economies over the euro years. This solution is used as a point of departure to study the dynamics driven by debt consolidation in both countries (Germany also goes for mild consolidation).

The main results are as follows. First, as expected, if fiscal policy in both countries remained unchanged as in their data averages over the examined period, then the model would be dynamically unstable. In other words, in both countries, at least one of the fiscal policy instruments (spending cuts and/or tax rises) should react to public debt imbalances for restoring dynamic stability.

Second, if the criterion is aggregate, or per capita, output, the best policy mix for both countries is to use public consumption spending cuts to bring public debt down during the early period of fiscal pain and, once debt has been reduced, to use the long term fiscal space created by debt consolidation to finance an increase in public investment spending. This policy mix, when followed by both countries, is productive for both countries, relative to status quo, along the transition to the new reformed steady state.

Third, the above policy mix, when followed by both countries, also improves equality (as measured by relative net incomes) vis-à-vis the status quo in both countries. However, a policy mix that could improve equity even further in both countries would be to use the long term fiscal space created by debt consolidation to cut the tax rate on labor and, during the early period of fiscal pain, to use the tax rate on capital to bring public debt down.

The rest of the paper is organized as follows. Section 2 presents the model. The status quo steady state solution as well as the parameterization and data used for its solution are in Section 3. Section 4 discusses the solution methodology. The main results are in Section 5. Section 6 closes the chapter and gives some possible extensions. Technical details are in an appendix.

2 A two-country model of a currency union

This section sets up a New Keynesian D(S)GE model consisting of two heterogeneous countries populated by heterogeneous households. The two countries form a monetary union. The model is as in Philippopoulos et al.(2017). However, here we differ in that we distinguish between different types of agents, including public employees, allowing for the study of the distributional implications of debt consolidation. We start with an informal description of the model and discussion of its key assumptions.

2.1 Informal description of the model and discussion of key assumptions

In this model there are two countries that form a closed system in a New Keynesian setup. In a regime of a currency union, there is a single monetary authority or central bank and a 'world' financial intermediary. In each country, there are heterogeneous households, private and state-owned firms and a national fiscal authority or government.

There are three types of households in each country, called 'capitalists', 'private workers' and 'public employees'. Capitalists own the private firms, hold private physical capital, money, internationally traded assets, domestic government bonds and also receive labor income for their managerial services. Both private workers and public employees just hold money and receive labor income for their labor services.

On the production side, as we said above, there is a state-owned firm and a number of private firms. The state-owned firm uses public employees' labor and goods purchased from the private sector as inputs to produce public goods and services. Private firms combine capitalists' and private workers' labor with private and public physical capital (public infrastructure) for the production of private goods. Each private firm produces a differentiated tradable private good and, consequently, acts monopolistically facing Rotemberg-type nominal price rigidities. Nominal price rigidities give a real role to monetary and exchange rate policy, at least in the transition path.

Monetary policy is common for both countries, while fiscal policy is conducted independently (national fiscal policies). Both monetary and fiscal policy are conducted by simple implementable state-contingent policy rules. Regarding monetary policy, the single monetary authority follows a Taylor-type rule for the nominal interest rate. Regarding fiscal policy, in each country, the national fiscal authority can use a menu of fiscal policy instruments that are allowed to respond to the inherited public debt-to-GDP ratio as deviation from a target value.

The market for internationally traded assets allows national governments to borrow from foreign capitalists (selling their bonds abroad) as well as one country's capitalists to borrow (lend) from (to) other country's capitalists.[4] All this international borrowing/lending takes place through a financial intermediary which faces a transaction cost that is proportional to the country's debt.[5] In turn, this cost creates a wedge between the interest rate that faces the agents (capitalists) of debtor's country and those of creditor's country. Consequently, capitalists in the debtor country face a higher interest rate in the international asset market than the capitalists in the creditor country.[6] Any profit of the financial intermediary is rebated lump-sum to capitalists in the creditor country.

To model a monetary union consisting of a country that is a systematic debtor (Italy) in the international asset market and another that is a systematic creditor (Germany), we need to introduce some type of heterogeneity. There are several ways to produce systematic borrowers and lenders, but, here, following e.g.Philippopoulos et al.(2017), we assume that agents (across countries) differ in their patience about the future or, equivalently, in their discount factors. Specifically, we assume that households in Germany, which is a systematic creditor, have higher discount factors than households in Italy, which is a systematic debtor.

The number of each type of households and their percentages in the population as well as the number of private firms are as follows. The home economy is composed of N^k identical capitalists indexed by $k = 1, 2, ..., N^k$, of N^w identical private workers indexed by $w = 1, 2, ..., N^w$ and of N^b identical public employees or bureaucrats indexed by $b = 1, 2, ..., N^b$. We also have N^h domestic private firms indexed by $h = 1, 2, ..., N^h$, where we assume that each domestic capitalist owns one domestic private firm, so that $N^k = N^h$. Similarly, in the foreign economy. For simplicity, we assume that the number of agents in the domestic country, N, equals that in the foreign country,

[4]See also Forni et al., 2010, and Cogan et al., 2013, and many others.

[5]There are many other ways to model financial intermediation (see e.g.Forni et al., 2010, Cogan et al., 2013, and many others who assume a transaction cost incurred when agents participate in the international asset market), but we prefer to focus on this because we find it more intuitive (see also e.g.Cúrdia and Woodford (2010, 2011).

[6]Hence, the sovereign interest rate spread between these two countries is created by transaction costs incurred by the financial intermediary.

N^*, that is $N = N^*$. We also assume that the same holds for the number of capitalists in both countries, that is $N^k = N^{k*}$. In addition, we assume that there are N^f firms in the foreign country indexed by $f = 1, 2, ..., N^f$, whose total number equals that of the foreign capitalists, since it is assumed that each foreign capitalist owns one foreign private firm. Furthermore, we assume that, in each country, the number of capitalists, private workers and public employees in the population remains constant over time, ruling out occupational choice as well as mobility across groups. Finally, the share of capitalists, private workers and public employees in the population of domestic country are defined as $v^k \equiv \frac{N^k}{N}$, $v^w \equiv \frac{N^w}{N}$ and $v^b \equiv \frac{N^b}{N}$ respectively, while the share of capitalists, private workers and public employees in the population of foreign country are defined as $v^{k*} \equiv \frac{N^{k*}}{N^*}$, $v^{w*} \equiv \frac{N^{w*}}{N^*}$ and $v^{b*} \equiv \frac{N^{b*}}{N^*}$.

Below, we present the domestic country. The foreign country will be symmetric except explicitly said. A star will denote the counterpart of a variable or a parameter in the foreign country.

2.2 Households as capitalists

This subsection presents the problem of domestic capitalists, $k = 1, 2, ..., N^k$.

2.2.1 Consumption bundles and expenditures of domestic capitalists

The quantity of variety h produced at home country by domestic private firm h and consumed by domestic capitalist k is denoted as $c_t^{k,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite domestic private good consumed by k, $c_t^{k,H}$, consists of h varieties and is given by(see also e.g. Forni et al., 2010):[7]

$$c_t^{k,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{k,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{1}$$

where $\phi > 0$ is the elasticity of substitution across private good varieties produced in the domestic country.

Similarly, the quantity of imported variety f produced abroad by foreign private firm f and consumed by domestic capitalist k is denoted as

[7] Recall that, in Subsection 2.1, we have assumed that the number of domestic firms (and, consequently, of domestic varieties) equals that of domestic capitalists.

$c_t^{k,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite imported private good consumed by k, $c_t^{k,F}$, consists of f varieties and is given by:[8]

$$c_t^{k,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{k,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{2}$$

In turn, having defined $c_t^{k,H}$ and $c_t^{k,F}$, capitalist k's consumption bundle, c_t^k, is defined as:

$$c_t^k = \frac{\left(c_t^{k,H}\right)^v \left(c_t^{k,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{3}$$

where v is the degree of preference for domestic private goods (if $v > 1/2$, there is a home bias).

Each domestic capitalist k's total consumption expenditure is:

$$P_t c_t^k = P_t^H c_t^{k,H} + P_t^F c_t^{k,F} \tag{4}$$

where P_t is the consumer price index (CPI), P_t^H is the price index of home private tradables and P_t^F is the price index of foreign private tradables (expressed in domestic currency).

Each domestic capitalist k' total expenditure on home and foreign private goods are respectively:[9]

$$P_t^H c_t^{k,H} = \sum_{h=1}^{N^k} P_t^H(h) c_t^{k,H}(h) \tag{5}$$

$$P_t^F c_t^{k,F} = \sum_{f=1}^{N^{k^*}} P_t^F(f) c_t^{k,F}(f) \tag{6}$$

where $P_t^H(h)$ is the price of variety h produced at home and $P_t^F(f)$ is the price of variety f produced abroad expressed in domestic currency.

[8] Recall that, in Subsection 2.1, we have assumed that the number of foreign firms (and, consequently, of imported varieties) equals that of foreign capitalists.

[9] Recall that, in Subsection 2.1, we have assumed that the number of domestic firms(and, consequently, that of domestic varieties) equals that of domestic capitalists as well as that the number of foreign firms(and, consequently, of imported varieties) equals that of foreign capitalists.

2.2.2 Prices and terms of trade

We assume that the law of one price holds meaning that each tradable private good sells at the same price at home country and abroad. Thus, $P_t^F(f) = S_t P_t^{H^*}(f)$, where S_t is the nominal exchange rate (where an increase in S_t implies a depreciation) and $P_t^{H^*}(f)$ is the price of variety f produced abroad denominated in foreign currency. Note that the terms of trade are defined as $\frac{P_t^F}{P_t^H}(= \frac{S_t P_t^{H^*}}{P_t^H})$, while the real exchange rate is defined as $\frac{S_t P_t^*}{P_t}$, where P_t^* stands for the consumer price index (CPI) abroad (see below). Being in a currency union, we will exogenously set $S_t \equiv 1$ at all t.

2.2.3 Domestic capitalists' optimization problem

Each domestic capitalist k acts competitively to maximize discounted expected lifetime utility:

$$E_o \sum_{t=0}^{\infty} \beta^t U\left(c_t^k, n_t^k, m_t^k, y_t^g\right) \tag{7}$$

where c_t^k is k's consumption bundle at t as defined above, n_t^k is k's hours of work at t, m_t^k is k's end-of-period real money balances at t, y_t^g are public goods and services at t divided by the number of domestic capitalists, E_o is the rational expectations operator conditional on the current period information set and $0 < \beta < 1$ is the time preference rate.

In our numerical solutions, we use a utility function of the form (see also e.g. Gali, 2008):

$$U\left(c_t^k, n_t^k, m_t^k, y_t^g\right) = \left[\frac{(c_t^k)^{1-\sigma}}{1-\sigma} - x_n \frac{(n_t^k)^{1+\eta}}{1+\eta} + x_m \frac{(m_t^k)^{1-\mu}}{1-\mu} + x_g \frac{(v^k y_t^g)^{1-\zeta}}{1-\zeta}\right] \tag{8}$$

where x_n, x_m, x_g, σ, η, μ, ζ are standard preference parameters.

The budget constraint of each domestic capitalist k (written in real terms) is:

$$\begin{aligned}(1+\tau_t^c)c_t^k + \frac{P_t^H}{P_t}x_t^k + \frac{S_t P_t^*}{P_t}f_t^k + b_t^k + m_t^k =& (1-\tau_t^k)\left[r_t^k \frac{P_t^H}{P_t}k_{t-1}^k + \widetilde{\omega}_t^k\right] + (1-\tau_t^n)w_t^k n_t^k + \\ &+ Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}f_{t-1}^k + R_{t-1}\frac{P_{t-1}}{P_t}b_{t-1}^k + \\ &+ \frac{P_{t-1}}{P_t}m_{t-1}^k - \tau_t^{l,k} + \widetilde{\pi}_t^k \end{aligned} \tag{9}$$

where x_t^k is k's real private investment at t, f_t^k is the real value of k's end-of-period internationally traded assets at t denominated in foreign currency (if negative, it denotes foreign private debt), b_t^k is the real value of k's end-of-period domestic government bonds at t, r_{t-1}^k is the gross real return to inherited private physical capital between $t-1$ and t, k_t^k is k's end-of-period private physical capital at t, $\widetilde{\omega_t}^k$ is k's real dividends paid by domestic private firms at t, w_t^k is domestic capitalists' real wage rate at t, Q_{t-1} is the gross nominal return to international assets between t-1 and t, $R_{t-1} \geq 1$ is the gross nominal return to domestic government bonds between $t-1$ and t, $\tau_t^{l,k}$ are real lump-sum taxes/transfers to each k from the government at t, $\widetilde{\pi_t}^k$ is the profits distributed in a lump-sum fashion to each k by the financial intermediary (see below) at t, $0 \leq \tau_t^c \leq 1$ is the tax rate on consumption at t, $0 \leq \tau_t^k \leq 1$ is the tax rate on capital income at t and $0 \leq \tau_t^n \leq 1$ is the tax rate on labor income at t. Small letters denote real variables e.g. $f_t^k \equiv \frac{F_t^k}{P_t^*}$, $b_t^k \equiv \frac{B_t^k}{P_t}$, $\widetilde{\omega_t}^k \equiv \frac{\widetilde{\Omega_t}^k}{P_t}$, $\widetilde{\pi_t}^k \equiv \frac{\widetilde{\Pi_t}^k}{P_t}$, $w_t^k \equiv \frac{W_t^k}{P_t}$. Also, letters with a star as superscript denote the counterpart of a variable in the rest-of-the world, e.g. P_t^* stands for the consumer price index (CPI) abroad as said above.

The motion of private physical capital for each k is:

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)^2 k_{t-1}^k \tag{10}$$

where $0 < \delta < 1$ is the depreciation rate of domestic private physical capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to domestic private physical capital.

Therefore, each domestic capitalist k chooses $\{c_t^k, x_t^k, n_t^k, m_t^k, b_t^k, f_t^k, k_t^k\}_{t=0}^{\infty}$ to maximize Eqs (7) and (8) subject to Eqs.(9) and (10), by taking as given prices $\{r_t^k, w_t^k, Q_t, R_t, P_t, P_t^H, P_t^*\}_{t=0}^{\infty}$, dividends $\{\widetilde{\omega_t}^k\}_{t=0}^{\infty}$, profits $\{\widetilde{\pi_t}^k\}_{t=0}^{\infty}$, policy variables $\{S_t, \tau_t^c, \tau_t^n, \tau_t^k, \tau_t^{l,k}\}_{t=0}^{\infty}$, and initial conditions, $\{m_{-1}^k, b_{-1}^k, k_{-1}^k, f_{-1}^k\}$.

The first order conditions include the constraints Eqs.(9), (10), and:

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}\frac{P_t^H}{P_t}\left[1 + \xi\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)\right] = \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_{t+1}^H}{P_{t+1}} \times \tag{11}$$

$$\times\left[(1-\delta) + (1-\tau_{t+1}^k)r_{t+1}^k - \frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k} - 1\right)^2 + \xi\left(\frac{k_{t+1}^k}{k_t^k} - 1\right)\frac{k_{t+1}^k}{k_t^k}\right]$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}S_t\frac{P_t^*}{P_t} = \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}Q_t S_{t+1}\frac{P_{t+1}^*}{P_{t+1}}\frac{P_t^*}{P_{t+1}^*} \tag{12}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t \frac{P_t}{P_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{13}$$

$$x_n (n_t^k)^{\eta} = (c_t^k)^{-\sigma} \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t^k \tag{14}$$

$$x_m (m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \frac{P_t}{P_{t+1}} \tag{15}$$

Eqs.(11), (12) and (13) are respectively the Euler equations of domestic private physical capital, internationally traded assets and domestic government bonds, Eq.(14) is the optimality condition for work hours and Eq.(15) is the optimality condition for real money balances.

Next, each domestic capitalist k chooses $\{c_t^{k,H}, c_t^{k,F}\}$ to minimize its total consumption expenditure, Eq.(4), subject to its consumption bundle, Eq.(3), by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^k.

The first order conditions include the consumption bundle of k, Eq.(3), and:

$$\frac{c_t^{k,H}}{c_t^{k,F}} = \frac{v}{1-v} \frac{P_t^F}{P_t^H} \tag{16}$$

which is the optimality condition for sharing the total consumption between domestic and imported private products.

Eqs.(3), (4) and (16) imply the following relation for consumer price index (CPI):

$$P_t = (P_t^H)^v (P_t^F)^{1-v} \tag{17}$$

Finally, each domestic capitalist k chooses $\{c_t^{k,H}(h), c_t^{k,F}(f)\}$ to minimize the sum of its consumption expenditure on home and foreign private goods, sum of Eqs.(5) and (6), subject to the composite domestic private good and the composite foreign private good consisting of varieties, Eqs.(1) and (2), by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{k,H}$ and $c_t^{k,F}$.

The first order conditions include Eqs.(1), (2) and:

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{18}$$

$$c_t^{k,F}(f) = \frac{c_t^{k,F}}{N^{k^*}} \left(\frac{P_t^F}{P_t^F(f)} \right)^{\phi} \tag{19}$$

Plugging Eqs.(18) and (19) into Eqs.(1) and (2) respectively, we get the following relations for price indexes:

$$P_t^H = \left\{ \sum_{h=1}^{N^k} \frac{1}{N^k} [P_t^H(h)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{20}$$

$$P_t^F = \left\{ \sum_{f=1}^{N^{k^*}} \frac{1}{N^{k^*}} [P_t^F(f)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{21}$$

Details of the above problem and its solution are in Appendix A.

2.3 Households as private workers

This subsection presents the problem of domestic private workers, w=1,2,...,N^w.

2.3.1 Consumption bundles and expenditures of domestic private workers

The quantity of variety h produced at home country by domestic private firm h and consumed by domestic private worker w is denoted as $c_t^{w,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite domestic private good consumed by w, $c_t^{w,H}$, consists of h varieties and is given by:[10]

$$c_t^{w,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{w,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{22}$$

Similarly, the quantity of imported variety f produced abroad by foreign private firm f and consumed by domestic private worker w is denoted as $c_t^{w,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite imported private good consumed by each w, $c_t^{w,F}$, consists of f varieties and is given by:[11]

$$c_t^{w,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{w,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{23}$$

[10]Recall that, in Subsection 2.1, we have assumed that the number of domestic firms (and, consequently, of domestic varieties) equals that of domestic capitalists.

[11]Recall that, in Subsection 2.1, we have assumed that the number of foreign firms (and, consequently, of imported varieties) equals that of foreign capitalists.

In turn, having defined $c_t^{w,H}$ and $c_t^{w,F}$, domestic private worker w's consumption bundle, c_t^w, is defined as:

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{24}$$

Each domestic private worker w's total consumption expenditure is:

$$P_t c_t^w = P_t^H c_t^{w,H} + P_t^F c_t^{w,F} \tag{25}$$

Each domestic private worker w's total expenditure on home and foreign private goods are respectively:[12]

$$P_t^H c_t^{w,H} = \sum_{h=1}^{N^k} P_t^H(h) c_t^{w,H}(h) \tag{26}$$

$$P_t^F c_t^{w,F} = \sum_{f=1}^{N^{k^*}} P_t^F(f) c_t^{w,F}(f) \tag{27}$$

2.3.2 Domestic private workers' optimization problem

Domestic private workers have the same utility function as domestic capitalists(see Eqs.(7) and (8)). Each domestic private worker w acts competitively to maximize discounted expected lifetime utility taking prices and policy as given.

The budget constraint of each domestic private worker w (written in real terms) is:

$$(1+\tau_t^c)c_t^w + m_t^w = (1-\tau_t^n)w_t^w n_t^w + \frac{P_{t-1}}{P_t} m_{t-1}^w - \tau_t^{l,w} \tag{28}$$

where c_t^w is w's consumption bundle at t as defined above in Subsection 2.3.1, m_t^w is w's end-of-period real money balances at t, n_t^w is w's hours of work at t, w_t^w is domestic private workers' real wage rate at t and $\tau_t^{l,w}$ are real lump-sum taxes/transfers to each w from the government at t. Again small letters denote real variables, e.g.$w_t^w \equiv \frac{W_t^w}{P_t}$.

[12] Recall that, in Subsection 2.1, we have assumed that the number of domestic firms(and, consequently, that of domestic varieties) equals that of domestic capitalists as well as that the number of foreign firms(and, consequently, of imported varieties) equals that of foreign capitalists.

Therefore, each domestic private worker chooses $\{c_t^w, n_t^w, m_t^w\}_{t=0}^{\infty}$ to maximize Eqs.(7) and (8) for w, subject to Eq.(28), by taking as given prices $\{w_t^w, P_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,w}\}_{t=0}^{\infty}$, and initial condition, m_{-1}^w.

The first order conditions include the budget constraint above, Eq.(28), and:

$$\frac{(c_t^w)^{-\sigma}}{x_n (n_t^w)^{\eta}} = \frac{1+\tau_t^c}{(1-\tau_t^n) w_t^w} \tag{29}$$

$$\frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} = \beta \frac{P_t}{P_{t+1}} \left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m (m_t^w)^{-\mu} \tag{30}$$

Eq.(29) is the optimality condition for work hours and Eq.(30) is the optimality condition for real money balances.

Next, each domestic private worker w chooses $\{c_t^{w,H}, c_t^{w,F}\}$ to minimize its total consumption expenditure, Eq.(25), subject to its consumption bundle, Eq.(24), by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^w.

The first order conditions include the consumption bundle of w, Eq.(24), and:

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v} \frac{P_t^F}{P_t^H} \tag{31}$$

which is the optimality condition for sharing the total consumption between domestic and imported private products.

Eqs.(24), (25) and (31) imply the following relation for consumer price index (CPI):

$$P_t = (P_t^H)^v (P_t^F)^{1-v} \tag{32}$$

which, as expected, coincides with the equation of CPI derived from the domestic capitalist k's problem, Eq.(17).

Finally, each domestic private worker w chooses $\{c_t^{w,H}(h), c_t^{w,F}(f)\}$ to minimize the sum of its consumption expenditure on home and foreign private goods, sum of Eqs.(26) and (27), subject to the composite domestic private good and the composite foreign private good consisting of varieties, Eqs.(22) and (23), by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{w,H}$ and $c_t^{w,F}$.

The first order conditions include Eqs.(22), (23) and:

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{33}$$

$$c_t^{w,F}(f) = \frac{c_t^{w,F}}{N^{k^*}} \left(\frac{P_t^F}{P_t^F(f)} \right)^{\phi} \tag{34}$$

Plugging Eqs.(33) and (34) into Eqs.(22) and (23) respectively, we get the following relations for price indexes:

$$P_t^H = \left\{ \sum_{h=1}^{N^k} \frac{1}{N^k} [P_t^H(h)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{35}$$

$$P_t^F = \left\{ \sum_{f=1}^{N^{k^*}} \frac{1}{N^{k^*}} [P_t^F(f)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{36}$$

which, as expected, coincide with the equations of price indexes derived from the domestic capitalist k's problem, Eqs.(20) and (21).

Details of the above problem and its solution are in Appendix B.

2.4 Households as public employees

This subsection presents the problem of domestic public employees, b=1,2,...,N^b.

2.4.1 Consumption bundles and expenditures of domestic public employees

The quantity of variety h produced at home country by domestic private firm h and consumed by domestic public employee b is denoted as $c_t^{b,H}(h)$. Using a Dixit-Stiglitz aggregator, the composite domestic private good consumed by b, $c_t^{b,H}$, consists of h varieties and is given by:[13]

$$c_t^{b,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{b,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{37}$$

Similarly, the quantity of imported variety f produced abroad by foreign private firm f and consumed by domestic public employee b is denoted as $c_t^{b,F}(f)$. Using a Dixit-Stiglitz aggregator, the composite imported private

[13] Recall that, in Subsection 2.1, we have assumed that the number of domestic firms (and, consequently, of domestic varieties) equals that of domestic capitalists.

good consumed by domestic public employee b, $c_t^{b,F}$, consists of f varieties and is given by:[14]

$$c_t^{b,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{b,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{38}$$

In turn, having defined $c_t^{b,H}$ and $c_t^{b,F}$, domestic public employee b's consumption bundle, c_t^b, is defined as:

$$c_t^b = \frac{\left(c_t^{b,H}\right)^v \left(c_t^{b,F}\right)^{1-v}}{v^v (1-v)^{1-v}} \tag{39}$$

Each domestic public employee b's total consumption expenditure is:

$$P_t c_t^b = P_t^H c_t^{b,H} + P_t^F c_t^{b,F} \tag{40}$$

Each domestic public employee b's total expenditure on home and foreign private goods are respectively:[15]

$$P_t^H c_t^{b,H} = \sum_{h=1}^{N^k} P_t^H(h) c_t^{b,H}(h) \tag{41}$$

$$P_t^F c_t^{b,F} = \sum_{f=1}^{N^{k^*}} P_t^F(f) c_t^{b,F}(f) \tag{42}$$

2.4.2 Domestic public employees' optimization problem

Domestic public employees have the same utility function as domestic capitalists (see e.g. Eqs.(7) and (8)). Each domestic public employee b acts competitively to maximize discounted expected lifetime utility taking prices and policy as given.

The budget constraint of each domestic public employee b (written in real terms) is:

[14] Recall that, in Subsection 2.1, we have assumed that the number of foreign firms (and, consequently, of imported varieties) equals that of foreign capitalists.

[15] Recall that, in Subsection 2.1, we have assumed that the number of domestic firms(and, consequently, that of domestic varieties) equals that of domestic capitalists as well as that the number of foreign firms(and, consequently, of imported varieties) equals that of foreign capitalists.

$$(1+\tau_t^c)c_t^b + m_t^b = (1-\tau_t^n)w_t^b n_t^b + \frac{P_{t-1}}{P_t} m_{t-1}^b - \tau_t^{l,b} \tag{43}$$

where c_t^b is b's consumption bundle at t as defined above in Subsection 2.4.1, m_t^b is b's end-of-period real money balances at t, n_t^b is b's hours of work at t, w_t^b is domestic public employees' real wage rate at t and $\tau_t^{l,b}$ are real lump-sum taxes/transfers to each b from the government at t. Again small letters denote real variables, e.g. $w_t^b \equiv \frac{W_t^b}{P_t}$.

Assuming that the domestic government exogeneously determines the total domestic public wage bill in real terms divided by the number of domestic capitalists, defined as $\bar{g}_t^w = \frac{v^b}{v^k} w_t^b n_t^b$, we can rewrite the budget constraint of b as follows:

$$(1+\tau_t^c)c_t^b + m_t^b = (1-\tau_t^n)\frac{v^k}{v^b}\bar{g}_t^w + \frac{P_{t-1}}{P_t} m_{t-1}^b - \tau_t^{l,b} \tag{44}$$

Therefore, each domestic public employee chooses $\{c_t^b, m_t^b\}_{t=0}^{\infty}$ to maximize Eqs.(7) and (8) for b, subject to Eq.(43), by taking as given prices $\{P_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,b}, \bar{g}_t^w\}_{t=0}^{\infty}$, and initial condition, m_{-1}^b.

The first order conditions include the budget constraint above, Eq.(43), and:

$$\frac{(c_t^b)^{-\sigma}}{1+\tau_t^c} = \beta \frac{P_t}{P_{t+1}} \left[\frac{(c_{t+1}^b)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m (m_t^b)^{-\mu} \tag{45}$$

Eq.(45) is the optimality condition for real money balances.

Next, each domestic public employee b chooses $\{c_t^{b,H}, c_t^{b,F}\}$ to minimize its total consumption expenditure, Eq.(40), subject to its consumption bundle, Eq.(39), by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^b.

The first order conditions include the consumption bundle of b, Eq.(39), and:

$$\frac{c_t^{b,H}}{c_t^{b,F}} = \frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{46}$$

which is the optimality condition for sharing the total consumption between domestic and imported private products.

Eqs.(39), (40) and (46) imply the following relation for consumer price index(CPI):

$$P_t = (P_t^H)^v (P_t^F)^{1-v} \tag{47}$$

which, as expected, coincides with the equation of CPI derived from the domestic capitalist's problem (and also coincides with equation of CPI derived from the domestic private worker's problem, Eq.(32))

Finally, each domestic public employee b chooses $\{c_t^{b,H}(h), c_t^{b,F}(f)\}$ to minimize the sum of its consumption expenditure on home and foreign private goods, sum of Eqs.(41) and (42), subject to the composite domestic private good and the composite foreign private good consisting of varieties, Eqs.(37) and (38), by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{b,H}$ and $c_t^{b,F}$.

The first order conditions include Eqs.(37), (38) and:

$$c_t^{b,H}(h) = \frac{c_t^{b,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{48}$$

$$c_t^{b,F}(f) = \frac{c_t^{b,F}}{N^{k^*}}\left(\frac{P_t^F}{P_t^F(f)}\right)^{\phi} \tag{49}$$

Plugging Eqs.(48) and (49) into Eqs.(37) and (38) respectively, we get the following relations for price indexes:

$$P_t^H = \left\{\sum_{h=1}^{N^k} \frac{1}{N^k}[P_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{50}$$

$$P_t^F = \left\{\sum_{f=1}^{N^{k^*}} \frac{1}{N^{k^*}}[P_t^F(f)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{51}$$

which, as expected, coincide with Eqs.(20) and (21), that is the equations of price indexes derived from the domestic capitalist's problem (which also concide with Eqs.(35) and (36), that is the equations of price indexes derived from the domestic private worker's problem).

Details of the above problem and its solution are in Appendix C.

2.5 Domestic private firms

This subsection presents the problem of private firms in the domestic economy. There are N^k [16] domestic private firms indexed by $h = 1, 2, ..., N^k$. Each

[16] Recall the assumption we have made that, in both countries, the number of capitalists equals that of private firms.

domestic private firm h produces a differentiated tradable good of variety h under monopolistic competition facing Rotemberg-type nominal price rigidities (see e.g. Walsh, 2010, Wickens, Chapter 9, 2008, and Bi et al., 2013).

2.5.1 Demand for the domestic private firm h's product

Each domestic private firm h faces demand for its product, $y_t^{H,d}(h)$. The latter comes from domestic households' private consumption and investment, $c_t^H(h)$ and $x_t(h)$, where $c_t^H(h) \equiv \sum_{k=1}^{N^k} c_t^{k,H}(h) + \sum_{w=1}^{N^w} c_t^{w,H}(h) + \sum_{b=1}^{N^b} c_t^{b,H}(h)$ and $x_t(h) \equiv \sum_{k=1}^{N^k} x_t^k(h)$, from the domestic state-owned enterprise's use of private goods as inputs in its production function, denoted as $g_t^c(h)$, from the domestic government's investment, $g_t^i(h)$, from the financial intermediary which is located in the domestic country, denoted as $\widetilde{\upsilon}_t(h)$,[17] and from foreign households' consumption of the domestic private goods, $c_t^{F^*}(h)$, where $c_t^{F^*}(h) \equiv \sum_{k^*=1}^{N^{k^*}} c_t^{k,F^*}(h) + \sum_{w^*=1}^{N^{w^*}} c_t^{w,F^*}(h) + \sum_{b^*=1}^{N^{b^*}} c_t^{b,F^*}(h)$ with a star, again, we denote the counterpart of a variable in the foreign country. Thus, aggregate demand for each variety h is:

$$y_t^{H,d}(h) = \left[c_t^H(h) + x_t(h) + g_t^c(h) + g_t^i(h) + \widetilde{\upsilon}_t(h) + c_t^{F^*}(h)\right] \tag{52}$$

Since we have:

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{53}$$

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{54}$$

$$c_t^{b,H}(h) = \frac{c_t^{b,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{55}$$

$$x_t^k(h) = \frac{x_t^k}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{56}$$

[17] See also Cúrdia and Woodford, 2010 and 2011, for a similar modelling of resources consumed by banks.

$$g_t^c(h) = \frac{N^k \overline{g}_t^c}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{57}$$

$$g_t^i(h) = \frac{N^k \overline{g}_t^i}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{58}$$

$$\widetilde{v}_t(h) = \frac{\widetilde{v}_t}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{59}$$

$$c_t^{k,F^*}(h) = \frac{c_t^{k,F^*}}{N^{k^*}} \left(\frac{P_t^{F^*}}{P_t^{F^*}(h)} \right)^{\phi} \tag{60}$$

$$c_t^{w,F^*}(h) = \frac{c_t^{w,F^*}}{N^{k^*}} \left(\frac{P_t^{F^*}}{P_t^{F^*}(h)} \right)^{\phi} \tag{61}$$

$$c_t^{b,F^*}(h) = \frac{c_t^{b,F^*}}{N^{k^*}} \left(\frac{P_t^{F^*}}{P_t^{F^*}(h)} \right)^{\phi} \tag{62}$$

we can rewrite the relation (52) as:

$$y_t^{H,d}(h) = \frac{1}{N^k} \left[c_t^H + x_t + N^k \overline{g}_t^c + N^k \overline{g}_t^i + \widetilde{v}_t + c_t^{F^*} \right] \times \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{63}$$

where $c_t^H \equiv \sum_{k=1}^{N^k} c_t^{k,H} + \sum_{w=1}^{N^w} c_t^{w,H} + \sum_{b=1}^{N^b} c_t^{b,H}$ is domestic households' total consumption of private home goods, $x_t \equiv \sum_{k=1}^{N^k} x_t^k$ is domestic capitalists' total private investment, $N^k \overline{g}_t^c$ denotes domestic private sector's total domestic goods and services that are used by the domestic state-owned enterprise for the production of total domestic public goods and services, $N^k \overline{g}_t^i$ denotes domestic public infrastructure investment, $\widetilde{v}_t$ denotes total resources consumed by the financial intermediary and $c_t^{F^*} \equiv \sum_{k^*=1}^{N^{k^*}} c_t^{k,F^*} + \sum_{w^*=1}^{N^{w^*}} c_t^{w,F^*} + \sum_{b^*=1}^{N^{b^*}} c_t^{b,F^*}$ is foreign households' total consumption of private home goods (i.e. domestic country's exports). Also notice that the law of one price implies that in Eqs.(60), (61) and (62):

$$\frac{P_t^{F^*}}{P_t^{F^*}(h)} = \frac{\frac{P_t^H}{S_t}}{\frac{P_t^H(h)}{S_t}} = \frac{P_t^H}{P_t^H(h)} \tag{64}$$

and recall that the number of domestic capitalists equals that of foreign capitalists(see Subsection 2.1).

Since domestic aggregate demand, $N^k y_t^{H,d}$, is:

$$N^k y_t^{H,d} = \left[c_t^H + x_t + N^k \overline{g}_t^c + N^k \overline{g}_t^i + \widetilde{\upsilon}_t + c_t^{F^*}\right] \tag{65}$$

then domestic aggregate demand for each variety h is rewritten as:

$$y_t^{H,d}(h) = y_t^{H,d}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{66}$$

2.5.2 Domestic private firms' optimization problem

Nominal profits of each domestic private firm h are defined as:

$$P_t\widetilde{\omega}_t(h) = P_t^H(h)y_t^H(h) - P_t^H r_t^k k_{t-1}(h) - W_t^w n_t^w(h) - W_t^k n_t^k(h) - \frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right)^2 P_t^H y_t^H \tag{67}$$

where $y_t^H(h)$ stands for the production of domestic private firm h, y_t^H stands for the total private output in the domestic economy divided by the number of domestic capitalists, π^H stands for the steady state value of the gross domestic goods inflation rate, $k_{t-1}(h)$ denotes the domestic private physical capital input chosen by domestic private firm h, $n_t^w(h)$ denotes domestic private workers' labor input chosen by domestic private firm h, $n_t^k(h)$ denotes the domestic capitalists' labor input chosen by domestic private firm h and $\phi^P \geq 0$ is a parameter which determines the degree of nominal price rigidity. The quadratic cost that the domestic private firm h faces once it changes the price of its product is proportional to aggregate domestic private output divided by the number of domestic private firms (which is equal to the number of domestic capitalists as we have said in Subsection 2.1).[18]

[18]This specification of Rotemberg-type cost is similar to that of Bi et al., 2013. Here, working with summations, instead of integrals, we should have a pricing cost which is proportional to the aggregate domestic private output divided by the number of private firms. With this modification, we can derive the same NK Philips curve as Bi at al, 2013.

All domestic private firms use the same technology represented by the production function(similar to e.g. Hornstein et al., 2005, and Baxter and King, 1993):

$$y_t^H(h) = A_t \left\{ [k_{t-1}(h)]^{\alpha} \left[\{n_t^k(h)\}^{\theta} \{n_t^w(h)^{1-\theta}\} \right]^{1-\alpha} \right\} \left(k_{t-1}^g \right)^{\theta_k} \tag{68}$$

where A_t is an exogenous TFP, k_{t-1}^g denotes the stock of domestic public infrastructure divided by the number of domestic capitalists,[19] $0 < \alpha < 1$ is the share of domestic private physical capital, $0 < \theta_k < 1$ is the output elasticity of domestic public infrastructure for domestic private firm h and $0 < \theta < 1$ is the labor efficiency parameter of domestic capitalist. We assume a positive θ_k, which implies that the production function has increasing returns to scale with respect to all inputs, as in Baxter and King, 1993. Notice that we keep CRS over private inputs.

Profit maximization by domestic private firm h is also subject to the demand for its product, Eq.(66) as derived above. But, instead of using Eq.(66), we can equivalently use the following equation, Eq.(69), which expresses the demand for domestic good h in terms of production:

$$y_t^H(h) = y_t^H \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{69}$$

where

$$y_t^{H,d} \equiv y_t^H \times \left\{ 1 - \frac{\phi^P}{2} \left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1 \right]^2 \frac{y_t^H}{y_t^H(h)} \right\}$$

This equation can be derived by considering the equations of Subsection 2.5.1 and the following relation, that associates aggregate demand for each variety h with its production by domestic private firm h:

$$y_t^{H,d}(h) = y_t^H(h) \times \left\{ 1 - \frac{\phi^P}{2} \left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1 \right]^2 \frac{y_t^H}{y_t^H(h)} \right\} \tag{70}$$

The term in the brackets captures the Rotemberg-type pricing cost and reflects the discrepancy between production and demand as one expected in a Rotemberg-type fashion(see e.g. Bi et al., 2013, and Lombardo et al., 2008).

Each domestic private firm h chooses its price, $P_t^H(h)$, and its inputs, $k_t(h)$, $n_t^k(h)$, $n_t^w(h)$, to maximize discounted expected lifetime real dividends,

[19] The stock of domestic public infrastructure is common for all domestic private firms.

$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t}\widetilde{\omega_t}(h)$, subject to Eq.(69) and its production function, Eq.(68). The objective function of domestic private firm h in real terms is given by:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t}\left[\frac{P_t^H(h)}{P_t}y_t^H(h) - \frac{P_t^H}{P_t}r_t^k k_{t-1}(h) - w_t^w n_t^w(h) - w_t^k n_t^k(h) - \frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right)^2 \frac{P_t^H y_t^H}{P_t}\right] \tag{71}$$

where $\Xi_{0,0+t}$ is a stochastic discount factor taken as given by the domestic private firm h. This is defined as $\Xi_{0,0+t} = \prod_{i=0}^{t-1}\left\{\frac{1}{R_i}\right\} = \beta^t \prod_{i=0}^{t-1}\left[\left(\frac{P_i}{P_{i+1}}\right)\left(\frac{1+\tau_i^c}{1+\tau_{i+1}^c}\right)\left(\frac{c_{i+1}^k}{c_i^k}\right)^{-\sigma}\right]$ and arises from the Euler of domestic government bonds.

2.5.3 Domestic private firms' optimality conditions

Following the related literature, instead of solving the above problem, we follow a two step procedure. We first solve a cost minimization problem, where each domestic private firm h minimizes its cost by choosing factors of production given technology and prices. The solution will give a minimum real cost function, which is a function of factor prices and output produced by the domestic private firm. In turn, given this cost function, we solve the dynamic profit maximization problem of domestic private firm h by choosing its price.

Cost minimization problem: In the first stage, we solve a static cost minimization problem, where each h minimizes its cost by choosing its factors of production, $k_t(h)$, $n_t^k(h)$, $n_t^w(h)$, subject to its production function, Eq.(68), given technology and prices. The cost function is defined in real terms as follows:

$$\min \widetilde{\psi} = \left[\frac{P_t^H}{P_t}r_t^k k_{t-1}(h) + w_t^w n_t^w(h) + w_t^k n_t^k(h)\right] \tag{72}$$

The solution to the cost minimization problem gives the following input demand functions:

$$\frac{P_t^H}{P_t}r_t^k k_{t-1}(h) = mc_t \alpha y_t^H(h) \tag{73}$$

$$w_t^k n_t^k(h) = mc_t \theta(1-\alpha) y_t^H(h) \tag{74}$$

$$w_t^w n_t^w(h) = mc_t (1-\theta)(1-\alpha) y_t^H(h) \tag{75}$$

where $mc_t \equiv \widetilde{\psi}'(y_t^H(h))$ (as we will show just below, by summing up these three factor demand functions, the real cost is a function of production) stands for the real marginal cost, which, by definition, is the derivative of the associated minimum real cost function, $\widetilde{\psi}(y_t^H(h))$, with respect to the production, $y_t^H(h)$.

Summing up the three above equations it arises the following relation for the associated minimum cost function of h in real terms:

$$\widetilde{\psi}(y_t^H(h)) = mc_t y_t^H(h) \tag{76}$$

Where the real marginal cost, mc_t, it can be shown that equals:

$$mc_t = \frac{1}{A_t (k_{t-1}^g)^{\theta_k}} \left[\frac{P_t^H}{P_t} \frac{r_t^k}{\alpha} \right]^{\alpha} \left[\left\{ \frac{w_t^k}{\theta(1-\alpha)} \right\}^{\theta} \times \left\{ \frac{w_t^w}{(1-\theta)(1-\alpha)} \right\}^{1-\theta} \right]^{1-\alpha} \tag{77}$$

implying that mc_t is common for all domestic private firms since it only depends on prices, parameters, stock of domestic public infrastructure divided by the number of domestic capitalists and technology which are common for all domestic private firms.

Profit maximization: The solution to the cost minimization problem above gave a minimum real cost function, Eq.(76), which is a function of prices and output produced by the domestic private firm. In turn, given this cost function, we solve the dynamic profit maximization problem of h by choosing its price. Specifically, in the second stage, domestic private firm h chooses its price, $P_t^H(h)$, to maximize the discounted expected lifetime real profits:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \left[\frac{P_t^H(h)}{P_t} y_t^H(h) - \widetilde{\psi}(y_t^H(h)) - \frac{\phi^P}{2} \left(\frac{P_t^H(h)}{P_{t-1}^H(h) \pi^H} - 1 \right)^2 \frac{P_t^H y_t^H}{P_t} \right] \tag{78}$$

The above profit maximization is subject to the Eq.(69), which is equivalent to the demand equation that the monopolistically competitive domestic private firm h faces, Eq.(66).

The first order condition gives:

$$
\begin{aligned}
&(1-\phi)\frac{P_t^H(h)}{P_t}y_t^H(h)+\phi mc_t y_t^H(h)-\phi^P\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right]\frac{P_t^H}{P_t}\frac{y_t^H P_t^H(h)}{P_{t-1}^H(h)\pi^H}= \\
&\quad \beta\phi^P\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1-\frac{P_{t+1}^H(h)}{P_t^H(h)\pi^H}\right]\frac{P_{t+1}^H(h)}{P_t^H(h)\pi^H}\frac{P_{t+1}^H}{P_{t+1}}y_{t+1}^H
\end{aligned} \tag{79}
$$

Thus, the behavior of h is summarized by Eqs.(73), (74), (75) and (79).

All domestic private firms solve the identical problem and they will set the same price, $P_t^H(h)$, which implies, through the Eq.(20) (or the identical Eqs.(35) and (50)), that $P_t^H(h) = P_t^H$.

More details of the above problem and its solution are in Appendix D.

2.6 Public sector

We now present the public sector. We specify the production function of public goods and services and, then, the government budget constraint.

2.6.1 The state-owned enterprise

Following most of the related literature,[20] we assume that total domestic public goods and services, $N^k y_t^g$, are produced using goods and services purchased from the domestic private sector, $N^k \overline{g}_t^c$, and total domestic public employment, l_t^g. In particular, following e.g. Linnemann (2009) and Economides et al.(2013, 2014), we use the following Cobb-Douglas production function in aggregate terms:

$$
N^k y_t^g = A_t\left(N^k \overline{g}_t^c\right)^{\theta_g}\left(l_t^g\right)^{1-\theta_g} \tag{80}
$$

where $0 \leq \theta_g \leq 1$ is a technology parameter. Notice that we assume that both domestic private and domestic public good production face the same TFP; this is because we do not want our results to be driven by exogenous factors. The total cost of domestic public production, $N^k \overline{g}_t^c + w_t^b l_t^g$, is financed by the domestic government through taxes and bonds (see the budget constraint of the domestic government, Eq.(82), below).

Similarly, we assume that total public goods and services in the foreign country, $N^{k*} y_t^{g*}$, are produced using goods and services purchased from

[20]See Economides et al., 2014, for details and a review of the literature on the production function of public goods.

the private sector, $N^{k*}\overline{g}_t^{c*}$, and total public employment, l_t^{g*}. In particular, following e.g. Linnemann (2009) and Economides et al.(2013, 2014), we use the following Cobb-Douglas production function in aggregate terms:

$$N^{k*}y_t^{g*} = A_t^*\left(N^{k*}\overline{g}_t^{c*}\right)^{\theta_g^*}\left(l_t^{g*}\right)^{1-\theta_g^*} \tag{81}$$

where $0 \leq \theta_g^* \leq 1$ is a technology parameter. Notice that we assume that both private and public good production in the foreign country face the same TFP; this is because we do not want our results to be driven by exogenous factors. The total cost of foreign public production, $N^{k*}\overline{g}_t^{c*} + w_t^{b*}l_t^{g*}$, is financed by the foreign government through taxes and bonds (see the budget constraint of the foreign government, Eq.(85), below).

2.6.2 Government budget constraint

In the domestic country, the period budget constraint of the "consolidated" public sector expressed in real terms[21] is(See Appendix E for details):

$$\begin{aligned} Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}f_{t-1}^g + R_{t-1}\frac{P_{t-1}}{P_t}b_{t-1} + \frac{P_t^H}{P_t}\overline{g}_t^c + \frac{P_t^H}{P_t}\overline{g}_t^i + \overline{g}_t^w + \frac{P_{t-1}}{P_t}m_{t-1} = & \quad (82) \\ = m_t + \tau_t^c\left[\frac{P_t^H}{P_t}\left(c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H} + \frac{v^b}{v^k}c_t^{b,H}\right) + \frac{P_t^F}{P_t}\left(c_t^{k,F} + \frac{v^w}{v^k}c_t^{w,F} + \frac{v^b}{v^k}c_t^{b,F}\right)\right] + \\ +\tau_t^k\left[r_t^k\frac{P_t^H}{P_t}k_{t-1}^k + \widetilde{\omega}_t^k\right] + \tau_t^n\left[w_t^kn_t^k + \frac{v^w}{v^k}w_t^wn_t^w + \overline{g}_t^w\right] + \tau_t^l + b_t + S_t\frac{P_t^*}{P_t}f_t^g \end{aligned}$$

where f_t^g is the real value of end-of-period domestic public debt held by each foreign capitalist at t (expressed in foreign prices),[22] b_t is the end-of-period domestic real public debt held by each domestic capitalist at t, $\frac{P_t^H}{P_t}\overline{g}_t^c$, as implied by above definitions, is total public spending on goods and services purchased from the private sector in real terms at t divided by the number of domestic capitalists, $\frac{P_t^H}{P_t}\overline{g}_t^i$ is, as implied by above definitions, total public investment in infrastructure in real terms at t divided by the number of domestic capitalists, $\tau_t^l \equiv \left[\tau_t^{l,k} + \frac{v^w}{v^k}\tau_t^{l,w} + \frac{v^b}{v^k}\tau_t^{l,b}\right]$ are total real lump-sum taxes/transfers (if positive, it denotes total lump-sum taxes paid to the

[21] I have aggregated over all agents, divided by the total number of agents and, in turn, divided all terms by v^k.

[22] That is, since the returns to bonds held by domestic capitalists and foreign capitalists can differ, our modelling implies that the government bond market can be segmented.

government; if negative, it denotes transfers received by the government) at t divided by the number of domestic capitalists and m_t is the end-of-period stock of real money balances at t divided by the number of domestic capitalists. All other variables have been defined above.

Therefore, as in e.g.Alesina et al., 2002, we include four main types of government spending (purchases of goods and services from the private sector, public investment in infrastructure, public wages and transfers to individuals). We also include three main types of taxes (taxes on consumption, labor and capital income).

Equivalently, if we define total nominal public debt in the domestic country as $N^k D_t \equiv N^k B_t + N^{k*} S_t F_t^g$, then, dividing by the number of domestic capitalists, it becomes in real terms $d_t \equiv b_t + \frac{S_t P_t^*}{P_t} f_t^g$,[23] with $b_t \equiv \lambda_t d_t$ and $\frac{S_t P_t^*}{P_t} f_t^g \equiv (1-\lambda_t) d_t$, where $0 \leq \lambda_t \leq 1$ denotes the fraction of domestic public debt held by each domestic private agent(domestic capitalist) and $0 \leq 1 - \lambda_t \leq 1$ is the fraction of domestic public debt held by each foreign private agent(foreign capitalist). Then, the above government budget constraint is rewritten in real terms as:

$$Q_{t-1} \frac{S_t P_t^*}{P_t} \frac{P_{t-1}^*}{P_t^*} \frac{P_{t-1}}{S_{t-1} P_{t-1}^*} (1-\lambda_{t-1}) d_{t-1} + R_{t-1} \frac{P_{t-1}}{P_t} \lambda_{t-1} d_{t-1} + \frac{P_t^H}{P_t} \overline{g}_t^c + \frac{P_t^H}{P_t} \overline{g}_t^i + \overline{g}_t^w + \frac{P_{t-1}}{P_t} m_{t-1} = \tag{83}$$

$$= m_t + \tau_t^c \left[\frac{P_t^H}{P_t} \left(c_t^{k,H} + \frac{v^w}{v^k} c_t^{w,H} + \frac{v^b}{v^k} c_t^{b,H} \right) + \frac{P_t^F}{P_t} \left(c_t^{k,F} + \frac{v^w}{v^k} c_t^{w,F} + \frac{v^b}{v^k} c_t^{b,F} \right) \right] + \tau_t^k \left[r_t^k \frac{P_t^H}{P_t} k_{t-1}^k + \widetilde{\omega_t}^k \right] +$$

$$+ \tau_t^n \left[w_t^k n_t^k + \frac{v^w}{v^k} w_t^w n_t^w + \overline{g}_t^w \right] + \tau_t^l + d_t$$

where $d_t \equiv \frac{D_t}{P_t}$.

In each period, one of $\{\tau_t^c, \tau_t^k, \tau_t^n, \overline{g}_t^c, \overline{g}_t^i, \overline{g}_t^w, \tau_t^l, \lambda_t, d_t\}$ needs to follow residually to satisfy the government budget constraint in the domestic country. We assume, except otherwise said, that this role is played by the end-of period total public debt divided by the number of domestic capitalists, d_t.[24]

Here, we model public infrastructure as a stock variable assuming that it accumulates like private physical capital (see also e.g. Fischer and Turnovsky,

[23] Recall that the number of domestic capitalists equals that of foreign capitalists.

[24] Here, we treat the share of domestic public debt held by foreign private agents, $0 \leq 1 - \lambda_t \leq 1$, as exogenous variable setting as value its data average. In a similar way, we treat this variable for the foreign country.

1998). Hence, the stock of domestic public infrastructure divided by the number of domestic private firms (which is equal to the number of domestic capitalists as we have assumed in Subsection 2.1), k^g_t, evolves according to:

$$\overline{g}^i_t = k^g_t - (1-\delta^g)k^g_{t-1} + \frac{\xi^g}{2}\left(\frac{k^g_t}{k^g_{t-1}} - 1\right)^2 k^g_{t-1} \tag{84}$$

where $0 \leq \delta^g \leq 1$ is the depreciation rate of domestic public infrastructure stock and $\xi^g \geq 0$ is a parameter capturing adjustment costs related to domestic public infrastructure stock.

Similarly, the period budget constraint of the "consolidated" public sector in the foreign country expressed in real terms[25] is (see Appendix E for details):

$$Q^*_{t-1}\frac{P_t}{S_tP^*_t}\frac{P_{t-1}}{P_t}f^{g*}_{t-1} + R^*_{t-1}\frac{P^*_{t-1}}{P^*_t}b^*_{t-1} + \frac{P^{H*}_t}{P^*_t}\overline{g}^{c*}_t + \frac{P^{H*}_t}{P^*_t}\overline{g}^{i*}_t + \overline{g}^{w*}_t + \frac{P^*_{t-1}}{P^*_t}m^*_{t-1} = \tag{85}$$

$$= m^*_t + \tau^{c*}_t\left[\frac{P^{H*}_t}{P^*_t}\left(c^{k,H*}_t + \frac{v^{w*}}{v^{k*}}c^{w,H*}_t + \frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right) + \frac{P^{F*}_t}{P^*_t}\left(c^{k,F*}_t + \frac{v^{w*}}{v^{k*}}c^{w,F*}_t + \frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right)\right] +$$

$$+\tau^{k*}_t\left[r^{k*}_t\frac{P^{H*}_t}{P^*_t}k^{k*}_{t-1} + \widetilde{\omega}^{k*}_t\right] + \tau^{n*}_t\left[w^{k*}_t n^{k*}_t + \frac{v^{w*}}{v^{k*}}w^{w*}_t n^{w*}_t + \overline{g}^{w*}_t\right] + \tau^{l*}_t + b^*_t + \frac{P_t}{S_tP^*_t}f^{g*}_t$$

where, as we have mentioned, a star denotes the counterpart of a variable or a parameter in the foreign country.

Let D^*_t denote the total nominal foreign public debt in foreign currency divided by the number of domestic capitalists. This can be held either by a foreign private agent(foreign capitalist), $B^*_t = \lambda^*_t D^*_t$, or by a domestic private agent(domestic capitalist), $\frac{F^{g*}_t}{S_t} = (1-\lambda^*_t)D^*_t$.[26] Then, we have:

[25] I have aggregated over all agents, divided by the total number of agents and, in turn, divided all terms by v^k.

[26] Recall that the number of domestic capitalists equals that of foreign capitalists.

$$Q^*_{t-1}\frac{P_t}{S_t P^*_t}\frac{P_{t-1}}{P_t}\frac{S_{t-1}P^*_{t-1}}{P_{t-1}}(1-\lambda^*_{t-1})d^*_{t-1} + R^*_{t-1}\frac{P^*_{t-1}}{P^*_t}\lambda^*_{t-1}d^*_{t-1} + \frac{P^{H*}_t}{P^*_t}\overline{g}^{c*}_t + \frac{P^{H*}_t}{P^*_t}\overline{g}^{i*}_t + \overline{g}^{w*}_t + \frac{P^*_{t-1}}{P^*_t}m^*_{t-1} = \tag{86}$$

$$= m^*_t + \tau^{c*}_t\left[\frac{P^{H*}_t}{P^*_t}\left(c^{k,H*}_t + \frac{v^{w*}}{v^{k*}}c^{w,H*}_t + \frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right) + \frac{P^{F*}_t}{P^*_t}\left(c^{k,F*}_t + \frac{v^{w*}}{v^{k*}}c^{w,F*}_t + \frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right)\right] +$$

$$+\tau^{k*}_t\left[r^{k*}_t\frac{P^{H*}_t}{P^*_t}k^{k*}_{t-1} + \widetilde{\omega}^{k*}_t\right] + \tau^{n*}_t\left[w^{k*}_t n^{k*}_t + \frac{v^{w*}}{v^{k*}}w^{w*}_t n^{w*}_t \overline{g}^{w*}_t\right] + \tau^{l*}_t + d^*_t$$

Similarly, in each period, one of $\{\tau^{c*}_t, \tau^{k*}_t, \tau^{n*}_t, \overline{g}^{c*}_t, \overline{g}^{i*}_t, \overline{g}^{w*}_t, \tau^{l*}_t, \lambda^*_t, d^*_t\}$ needs to follow residually to satisfy the government budget constraint in the foreign country. We also assume, except otherwise said, that this role is played by the end-of period total public debt divided by the number of foreign capitalists, d^*_t.

Similarly, the stock of foreign public infrastructure divided by the number of foreign private firms (which is equal to the number of foreign capitalists as we have assumed in Subsection 2.1), k^{g*}_t, evolves according to:

$$\overline{g}^{i*}_t = k^{g*}_t - (1-\delta^{g*})k^{g*}_{t-1} + \frac{\xi^{g*}}{2}\left(\frac{k^{g*}_t}{k^{g*}_{t-1}} - 1\right)^2 k^{g*}_{t-1} \tag{87}$$

where $0 \leq \delta^{g*} \leq 1$ is the depreciation rate of foreign public infrastructure stock and $\xi^{g*} \geq 0$ is a parameter capturing adjustment costs related to foreign public infrastructure stock.

2.7 World financial intermediary

We use a simple and popular model of financial frictions (see Cúrdia and Woodford (2010 and 2011), Benigno et al.(2014) and Philippopoulos et al.(2017)). In our model there is a financial intermediary or bank that intermediates between international lenders and international borrowers. This bank is located at home country and its role is limited to traditional banking meaning that receives deposits from lenders and lends the funds to borrowers.

The bank aims at maximizing its profits. These profits are defined as revenues, net of transaction or monitoring cost, minus costs. Specifically, the revenues of the bank comes from lending foreign government and foreign capitalists, $N^k f^{g*}_t - N^{k*} f^{k*}_t$, at the rate Q^*_t. The transaction (or monitoring) costs are assumed to have a quadratic form in the volume of loans, $N^k f^{g*}_t -$

$N^{k*} f_t^{k*}$. Except from the above operational costs (transaction or monitoring costs), the bank faces cost coming from collecting funds from domestic government and domestic capitalists, $N^k f_t^k - N^{k*} f_t^g$, at the rate Q_t.[27] Thus, the profit of bank divided by the number of domestic capitalists is written in real terms (details are in Appendix G):[28]

$$\widetilde{\pi}_t \equiv Q_{t-1}^* \left[\frac{P_{t-1}}{P_t}(f_{t-1}^{g*} - f_{t-1}^{k*}) - \frac{\psi}{2} \frac{P_{t-1}^H}{P_t^H} \frac{P_t^H}{P_t} (f_{t-1}^{g*} - f_{t-1}^{k*})^2 \right] - Q_{t-1} S_t \frac{P_t^*}{P_t} \frac{P_{t-1}^*}{P_t^*} \left(f_{t-1}^k - f_{t-1}^g \right) \tag{88}$$

where $\frac{\psi}{2} \frac{P_{t-1}^H}{P_t^H} \frac{P_t^H}{P_t} (f_{t-1}^{g*} - f_{t-1}^{k*})^2$ is real transaction cost divided by the number of foreign capitalists and $\psi \geq 0$ is a cost parameter in international borrowing (see Subsection 3.1 below for its value). On the RHS, the terms in the brackets are the revenues, net of transactions, while the last term (outside the brackets) is the cost of borrowing, that is payments to the savers.[29]

At each t, the bank chooses the volume of its loan, $N^k f_{t-1}^{g*} - N^{k*} f_{t-1}^{k*}$, taking Q_{t-1} and Q_{t-1}^* as given. The optimality condition is (details are in Appendix G):[30]

$$Q_{t-1}^* = \frac{Q_{t-1} \frac{S_t}{S_{t-1}}}{1 - \psi \frac{P_{t-1}^H}{P_{t-1}} (f_{t-1}^{g*} - f_{t-1}^{k*})} \tag{89}$$

where, in a currency union, $S_t \equiv 1$; thus, $Q_t^* > Q_t$ which means that borrowers pay a sovereign premium.

Notice that the Eq.(89) is rather intuitive and compatible with several empirical studies. In particular, this equation expresses a positive relation between the interest rate at which a country borrows from the international

[27] Here, f_t^k is each domestic capitalist's real foreign assets denominated in foreign currency, and f_t^g is real domestic public debt held by each foreign capitalist in the domestic country; similarly in the foreign country. Thus, if $N^k f_t^k - N^{k*} f_t^g$ is positive, it denotes net foreign assets in the home country and, then, $N^k f_t^{g*} - N^{k*} f_t^{k*}$ will be positive denoting net foreign liabilities in the foreign country. In equilibrium, we would have $(f_t^{g*} - f_t^{k*}) + \frac{S_t P_t^*}{P_t}(f_t^g - f_t^k) = 0$ meaning that the international assets of one country equals the international liabilities of the other country. Again, recall that the number of domestic capitalists equals that of foreign capitalists. Appendix G provides details.

[28] Recall that the number of domestic capitalists equals that of foreign capitalists.

[29] As in Cúrdia and Woodford (2010 and 2011), any resources consumed by the bank for the monitoring of its financial operations will be part of the domestic aggregate demand for the composite good (details are in Appendices D and G).

[30] Recall that the number of domestic capitalists equals that of foreign capitalists.

market and its total (private and public) foreign debt. In other words, the worse is a country's total foreign debt, the higher is the sovereign interest rate premium this country faces in international lending.

2.8 Monetary and fiscal policy

We now specify policy rules of the monetary and fiscal policy instruments.

2.8.1 Single monetary policy rule in a monetary union

In a flexible exchange rates regime, the exchange rate would be an endogenous variable and the two countries' nominal interest rate, R_t and R_t^*, could be free to be set independently by national monetary authorities, say, to follow national Taylor-type rules. Moving to a monetary union regime, like eurozone, we could assume that only one of the nominal interest rates, say R_t, can follow a Taylor-type rule, while R_t^* is an endogenous variable replacing the exchange rate which becomes an exogenous policy variable (see Galí and Monacelli, 2008, for a similar modelling).

Specifically, we assume a single monetary feedback policy rule of the following form:

$$
\begin{aligned}
log\left(\frac{R_t}{R}\right) = \phi_\pi \left[\tilde{\eta} log\left(\frac{\pi_t}{\pi}\right) + (1-\tilde{\eta}) log\left(\frac{\pi_t^*}{\pi^*}\right)\right] + \\
+ \phi_y \left[\tilde{\eta} log\left(\frac{y_t^H}{y^H}\right) + (1-\tilde{\eta}) log\left(\frac{y_t^{H*}}{y^{H*}}\right)\right]
\end{aligned} \tag{90}
$$

where π_t and π_t^* are the gross inflation rate of CPI in domestic and foreign country respectively, which are defined as $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and $\pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$, $\phi_\pi \geq 0$ and $\phi_y \geq 0$ are feedback monetary policy coefficients on price inflation and on output gap respectively, $0 \leq \tilde{\eta} \leq 1$ is the political weight given to the domestic country relative to the foreign country, variables without time subscripts denote values of the corresponding variables in a new reformed steady state and, again, a star as superscript denotes the counterpart of a variable in the foreign country.

2.8.2 National fiscal policy rules

Each country can follow its fiscal policy independently. National fiscal authorities in each country implement fiscal policy following simple feedback policy rules. This means that fiscal policy instruments react to easily observable endogenous macroeconomic indicators. In particular, in each country, we allow all the main spending-tax policy instruments, namely, the ratio of real government spending on private goods and services to real GDP, defined as s_t^g, the ratio of real government spending on investment to real GDP, defined as s_t^i, the ratio of real public wage bill to real GDP, defined as s_t^w, and the tax rates on consumption, capital income and labor income, τ_t^c, τ_t^k and τ_t^n respectively, to react to the public debt-to-GDP ratio as a deviation from a target value according to the following simple linear rules (see e.g. Schmitt-Grohé and Uribe, 2007, and Philippopoulos et al., 2017, for similar rules):

$$s_t^g = s^g - \gamma_l^g (l_{t-1} - l) \tag{91}$$

$$s_t^i = s^i - \gamma_l^i (l_{t-1} - l) \tag{92}$$

$$s_t^w = s^w - \gamma_l^w (l_{t-1} - l) \tag{93}$$

$$\tau_t^c = \tau^c + \gamma_l^c (l_{t-1} - l) \tag{94}$$

$$\tau_t^k = \tau^k + \gamma_l^k (l_{t-1} - l) \tag{95}$$

$$\tau_t^n = \tau^n + \gamma_l^n (l_{t-1} - l) \tag{96}$$

where l_{t-1} is the end-of-period government liabilities as share of GDP at $t-1$ (defined below), γ_l^q for $q = g, i, w, c, k, n$, are respectively feedback fiscal policy coefficients on public liabilities gap, and variables without time subscripts (i.e. $s^g, s^i, s^w, \tau^c, \tau^k, \tau^n, l$) denote values of the corresponding variable in the new reformed steady state. For example, as further discussed in Section 4 below, the public debt burden target, l, can be set to a value less than in the data (this will be the case of debt consolidation where fiscal policy systematically brings public debt down over time).

From the budget constraint of the domestic government, domestic public liabilities as share of GDP at the end of period t-1 expressed in real terms are:

$$l_{t-1} \equiv \frac{R_{t-1}\lambda_{t-1}D_{t-1} + Q_{t-1}\frac{S_t}{S_{t-1}}(1-\lambda_{t-1})D_{t-1}}{P_{t-1}^H y_{t-1}^H} \tag{97}$$

Fiscal policy in the foreign country is modelled similarly.

2.9 Exogenous variables

In this subsection, we will define the exogenous variables. Regarding the exogenously set policy instruments, we set the nominal exchange rate S_t at 1 (under fixed exchange rates) at all t and the fraction of domestic public debt held by domestic capitalists, λ_t, at its data average value at all t. We also assume that the lump-sum transfers as share of GDP, s_t^l, remain constant over time at their status quo steady state value(see Subsection 3.2 below). As we present in detail in the Appendix H.3, instead of working with nominal exchange rate, we can work with gross rate of exchange rate depreciation, defined as $\epsilon_t \equiv \frac{S_t}{S_{t-1}}$, and assume that its value remains constant at 1. Finally, the TFP, A_t, remains constant over time and equal to 1. Again, exogenous variables of the foreign country are determined similarly.

2.10 Final Equilibrium system

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of households maximizes utility; (ii) every private firm maximizes profit; (iii) the state-owned enterprise produces public goods and services; (iv) the world financial intermediary maximizes profit; (v) all constraints, including the government budget constraint and the balance of payments, are satisfied; and (vi) all markets clear, including the international asset market; (vii) policy instruments follow feedback rules.

This equilibrium system is presented in detail in Appendix H. It consists of 64 equations in 64 endogenous variables, $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, f_t^k, Q_t, y_t^H, y_t^g, mc_t, \widetilde{\omega}_t^k, \upsilon_t, \widetilde{\pi}_t^k, \pi_t, \pi_t^H, \tau\tau_t, d_t, l_t, c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*}, c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{H*}, y_t^{g*}, mc_t^*, \widetilde{\omega}_t^{k*}, \pi_t^*, \pi_t^{H*}, d_t^*, l_t^*]$, and 13 feedback policy rules in 13 policy instruments, $[R_t, s_t^g, s_t^i, s_t^w, \tau_t^c, \tau_t^k, \tau_t^n, s_t^{g*}, s_t^{i*}, s_t^{w*}, \tau_t^{c*}, \tau_t^{k*}, \tau_t^{n*}]$. This is for given the exogenous variables, $[\epsilon_t, \lambda_t, \lambda_t^*, A_t, A_t^*, s_t^l, s_t^{l*}]$, as defined in Subsection 2.9, the values of the feedback (monetary and fiscal) policy coefficients in the policy rules as defined in Subsection 2.8 and initial conditions for the state variables.

3 Data, parameterization and status quo steady state

In this section we solve the model numerically employing commonly used parameter values and fiscal data from Germany and Italy over the period 2001-2011. We choose this time period because 2001 is the year that the euro introduced and 2012 is the year when Italy began fiscal consolidation efforts (see e.g. EMU-Public Finances (2015) by the European Commission).

3.1 Parameters and fiscal policy variables

The model is solved numerically using parameter values and fiscal data averages as listed in Tables 1 and 2 respectively. The time unit is meant to be a year. The two countries are heterogeneous with respect to their discount factors (see β and β^* in Table 1), fiscal policy variables (see fiscal policy instruments in Table 2) and some parameters related to public sector (see $v^b, v^{b*}, \theta_g, \theta_g^*$ in Table 1). In all other respects, the two countries are assumed to be symmetric.

3.1.1 Structural parameters

The key parameters of the model, that capture the net foreign asset/debt position of these two countries, are the different discount factors in two countries, β and β^*, and the cost parameter in international borrowing, ψ. Specifically, the values of discount factors, the values of β and β^*, coming from the Euler equations of government bonds in the two countries at the steady state, $\beta Q/\pi = 1$ and $\beta^* Q^*/\pi^* = 1$, where Q/π and Q^*/π^* are the real interest rates in the two countries. According to the data over the period under consideration, the real interest rate in Germany, Q/π, is lower than that in Italy, Q^*/π^*, implying that the Germans are more patient than the Italians since the above Euler equations will give $\beta = 0.9833 > \beta^* = 0.9780$. As for the cost parameter in international borrowing, ψ, we set its value so that, from the optimality condition of the bank at the steady state, Eq.(89) at the steady state,[31] we match the Italy's net foreign debt liabilities, $\frac{P^H}{P}(N^k f^{g*} - N^{k*} f^{k*})$ (which are equal to Germany's net foreign assets).

Some parameter values related to public sector change across countries. In particular, the percentage of public employees in Germany and Italy are

[31] Given that at steady state all variables remain constant over time, time subscripts are eliminated and the Eq.(89) is reduced to $Q^* = \frac{Q}{1-\psi \frac{P^H}{P}(f^{g*}-f^{k*})}$.

set at values 0.16 and 0.2 respectively, which are close to the data. Furthermore, in Germany the share of private goods in public production, θ_g, is defined as $\frac{s^g}{s^g+s^w}$ and takes the value 0.3, while in Italy this parameter, θ_g^*, is defined in a similar way, $\frac{s^{g*}}{s^{g*}+s^{w*}}$, and takes the value 0.35.

All other parameter values are common for both countries and set at commonly used parameter values in related studies. Let us briefly discuss some of them. First of all, we set the neutral value of 0.5 at the political weight variable, $\tilde{\eta}$. In addition, θ_k and θ_k^*, which stand for the output elasticity of public infrastructure are both set at 0.05, as in Baxter and King, 1993.[32] The parameters θ and θ^*, standing for capitalists' labor efficiency parameters in each country, are set so that we obtain a reasonable value for the ratio of capitalists' wage to private workers' wage, $\frac{w^k}{w^w}$ and $\frac{w^{k*}}{w^{w*}}$, which, in our model, equals to 1.7 in Germany and 1.6 in Italy. We set Rotemberg's price adjustments cost parameters, ϕ^P and ϕ^{P*}, at 1.56 which correspond to a probability approximately 20 per cent a firm not to be able to reset its price each year in a Calvo pricing model (see e.g.Keen and Wang, 2007).

We report that our main results are robust to changes in these parameter values (these results are available upon request). Thus, although our numerical experiments below are not meant to provide a rigorous quantitative study, they illustrate the qualitative dynamic features of the model in a realistic way.

3.1.2 Fiscal policy

Regarding fiscal policy variables in two countries, we set the steady state values of government spending-to-GDP ratios (i.e. public consumption, public investment and public wage bill as shares of GDP) and the tax rates(on consumption, capital and labor) at their data averages in each country over 2001-2011.[33] In particular, as a measure of s^g and s^{g*}, which are found in the state-owned enterprise production function and typically thought of as part of total public spending on consuming private goods and services, we use the associated data. Furthermore, we use data of public investment as share of GDP as a measure of s^i and s^{i*} and of public wage bill as share of GDP as a measure of s^w and s^{w*}. Measure of s^l and s^{l*}, that captures lump-sum transfer payments as share of GDP, follows residually from each country's

[32]Leeper et al., 2010, report that there is a lack of consensus on the productivity of public capital in the literature and, consequently, in their experiments they assigned to this parameter the value 0.05 as in Baxter and King, 1993.

[33]This is the so-called status quo steady state (see Subsection 3.2 below).

government budget constraint. It is also worth to mention that lump-sum taxes/transfers are distributed to each class of households in each country according to their shares in the population. As tax rates, τ^c, τ^{c*}, τ^k, τ^{k*}, τ^n and τ^{n*} we use the associated effective tax rates (or what Eurostat calls implicit tax rates).

Regarding policy instruments along the transition, they can react to deviations of endogenous macroeconomic indicators from their steady state values.[34] As for the fiscal (tax-spending) policy instruments along the transition, they can respond to the inherited public debt as a deviation from its steady state value, where this reaction is quantified by the coefficients in the feedback policy rules (see e.g. $\gamma_l^q = 0$, where $q = g, i, w, c, n, k$, in the domestic country's fiscal policy rules, Eqs.(91)-(96). Similarly for the foreign country's case.). In our experiments we use only one fiscal instrument at a time in each country to respond to debt imbalances by setting the associated feedback policy coefficient on debt gap at 0.1 (e.g. $\gamma_l^g = 0.1$ and $\gamma_l^{k*} = 0.1$),[35] while we switch off the feedback policy coefficient on debt gap of other fiscal policy instruments. In all cases studied, other things equal, the above fiscal policy can guarantee a unique transition path.

As for monetary policy, we assume a Taylor-type rule (see Eq.(90)) for the nominal interest rate of the monetary union that aggressively react to each country's inflation meaning that the associated feedback policy coefficients (ϕ_π and ϕ_π^*) are set both at 1.5, while the feedback monetary policy coefficients on output gap (ϕ_y and ϕ_y^*) are both set at 0.5. Aggressive reaction of nominal interest rate on each country's inflation can guarantee determinacy. We report that our main results are robust to changes in these values.

3.2 Steady state solution in the status quo model

Table 3 reports the steady state solution when parameters and policy instruments are set as the values in Tables 1 and 2. Note that, since policy instruments react to deviations of macroeconomic indicators from their

[34] Since policy instruments react to deviations of macroeconomic indicators from their steady state values, feedback policy coefficients do not play any role in steady state solutions. Also, recall that "money is neutral" in the long run, so that the monetary policy regime also do not matter to the real economy at the steady state.

[35] These values are close to those found by optimized policy rules in related studies (see e.g. Schmitt-Grohé and Uribe, 2007, and Philippopoulos et al., 2017). They are also consistent with calibrated or estimated values by previous research(see e.g. Leeper et al., 2010, Forni et al., 2010, Coenen et al., 2012, Cogan et al., 2013, Erceg and Lindé, 2013).

Table 1: Parameter values

Parameter	Germany	Italy	Description
v^k, v^{k*}	0.20	0.20	share of capitalists in population
v^w, v^{w*}	0.64	0.60	share of private workers in population
v^b, v^{b*}	0.16	0.20	share of public employees in population
α, α^*	0.3	0.3	share of private physical capital in production
θ_k, θ_k^*	0.05	0.05	output elasticity of public infrastructure
θ, θ^*	0.26	0.26	labor efficiency parameter of capitalist
θ_g, ${\theta^*}_g$	0.30	0.35	share of private goods in public production
β, β^*	0.9833	0.9780	time discount factor
v, v^*	0.5	0.5	home goods bias in consumption
μ, μ^*	3.42	3.42	money demand elasticity in utility
δ, δ^*	0.1	0.1	private physical capital depreciation rate
δ^g, δ^{g*}	0.1	0.1	public physical capital depreciation rate
ϕ^P, ϕ^{P*}	1.56	1.56	Rotemberg's price adjustments cost parameter
ϕ, ϕ^*	6	6	price elasticity of demand
η, η^*	1	1	inverse of Frisch labor supply elasticity
σ, σ^*	1	1	inverse of elasticity of substitution in consumption
ζ, ζ^*	1	1	inverse of elasticity of public consumption in utility
$\tilde{\eta}$	0.5	0.5	political weight in union-wide policies
ψ	0.072	-	cost parameter in international borrowing
χ_m, χ_m^*	0.001	0.001	preference parameter related to real money balances
χ_n, χ_n^*	5	5	preference parameter related to work effort
χ_g, χ_g^*	0.1	0.1	preference parameter related to public spending
ξ, ξ^*	0.01	0.01	adjustment cost parameter of private physical capital
ξ^g, ξ^{g*}	0.01	0.01	adjustment cost parameter of public physical capital
A, A^*	1	1	TFP level

Table 2: Fiscal policy variables (data averages over 2001-2011)

Variable	Germany	Italy	Description
τ^c, τ^{c*}	0.19	0.18	consumption tax rate
τ^k, τ^{k*}	0.20	0.31	capital income tax rate
τ^n, τ^{n*}	0.38	0.42	labor income tax rate
s^g, s^{g*}	0.19	0.19	government consumption spending as share of GDP
s^i, s^{i*}	0.02	0.03	government investment spending as share of GDP
s^w, s^{w*}	0.08	0.10	public wage bill as share of GDP
λ, λ^*	0.52	0.61	share of one country's public debt held by this country's agents

Note: The data source is Eurostat.

steady state values, feedback policy coefficients do not play any role in steady state. In this steady state, which is called the status quo steady state, the debt-to-GDP ratio and fiscal policy instruments in both countries are set as in the data averages over 2001-2011, while lump-sum transfer payments as share of GDP play the role of the residually determined public financing variable in both countries. This steady state solution will serve as a point of departure to study various policy experiments. That is, in what follows, we will depart from this solution to study the implications of various policy reforms.

4 Description of policy experiments and solution strategy

The way we model public debt consolidation is similar to that of previous chapters. Nevertheless, it is repeated here for the reader's convenience. In our main thought experiment, the role of fiscal policy in both countries is to improve either resource allocation or "equality" by bringing its public debt-to-GDP ratio down over time. This is typically called "debt consolidation" in the related literature (see e.g.Wren-Lewis, 2010). Specifically, in our main thought experiment, fiscal policy in both countries (as domestic country defined to be Germany, while as foreign country defined to be Italy) is defined as follows: (a) In the new reformed steady state, each country's output share of public debt is exogenously set at a target value lower than their status quo steady state solution (in fact, the public debt-to-GDP ratio is set at 60% in Germany and 90% in Italy from around 68% and 110% respectively. Recall that the latter were the status quo steady state values in

Table 3: ”Status quo” steady state solution

Variables	Description	Home	Foreign
y^H, y^{H*}	output	0.8952	0.7863
c^k, c^{k*}	consumption of capitalist	0.2142	0.2064
c^w, c^{w*}	consumption of private worker	0.0824	0.0843
c^b, c^{b*}	consumption of public employee	0.0660	0.0680
n^k, n^{k*}	labor of capitalist	0.2492	0.2451
n^w, n^{w*}	labor of worker	0.3788	0.3736
k^k, k^{k*}	private physical capital	1.5226	1.1045
w^k, w^{k*}	real wage rate of capitalist	0.5163	0.5136
w^w, w^{w*}	real wage rate of worker	0.3020	0.3196
r^k, r^{k*}	real return to private physical capital	0.1470	0.1780
$Q^* - Q$	interest rate premium		0.0055
$-s^l, -s^{l*}$	lump-sum transfers as share of GDP	0.1639	0.1802
$\frac{c^k+\frac{v^w}{v^k}c^w+\frac{v^b}{v^k}c^b}{y^H\tau\tau^{v-1}}, \frac{c^{k*}+\frac{v^{w*}}{v^{k*}}c^{w*}+\frac{v^{b*}}{v^{k*}}c^{b*}}{y^{H*}\tau\tau^{1-v^*}}$	total consumption as share of GDP	0.6257	0.6351
$\frac{k}{y^H}, \frac{k^*}{y^{H*}}$	private physical capital as share of GDP	1.7009	1.4045
$\frac{d}{y^H\tau\tau^{v-1}}, \frac{d^*}{y^{H*}\tau\tau^{1-v^*}}$	total public debt as share of GDP	0.6861	1.08
$\frac{\frac{(1-\lambda)d}{\tau\tau^{v-1}}-f^k\tau\tau^{v^*}}{y_t^H}, \frac{\frac{(1-\lambda^*)d^*}{\tau\tau^{1-v^*}}-f^{k*}\tau\tau^{-v}}{y_t^{H*}}$	total country’s foreign debt as share of GDP	-0.0930	0.0950
y^k, y^{k*}	income of capitalist	0.3584	0.3230
y^w, y^{w*}	income of private worker	0.0824	0.0843
y^b, y^{b*}	income of public employee	0.0660	0.0680

Note: Parameters and policy variables are as in Tables 1 and 2.

Subsection 3.2.).

(b) In this new reformed steady state, since each country's public debt has been reduced and, thus, fiscal space has been created relative to status quo, one of public spending categories can be increased or one of the tax rates can be cut, following residually to close the government budget constraint. This is known as the long-term fiscal gain from debt consolidation.

(c) Along the transition to the new reformed steady state, in each country, one of the national tax-spending policy instruments is allowed to react to deviations from policy targets as discussed in Subsection 3.1 above. Given that the new debt policy target is set at a value lower than in the status quo in both countries (i.e. we depart from around 68% and 110% in Germany and Italy respectively, and end up to say 60% and 90% respectively), this requires lower public spending and/or higher tax rates (in our experiments, as we have said, one of public spending categories is cut or one of the tax rates rises), during the early phase of the transition period. This is known as the short-term fiscal pain of debt consolidation.

This intertemporal tradeoff, between short-term fiscal pain and medium-term fiscal gain, also implies that the implications of debt consolidation depend heavily on the mix of public financing policy instruments used, namely, which policy instrument adjusts endogenously to accommodate the exogenous change in fiscal policy (see also e.g. Leeper et al., 2010, and Davig and Leeper, 2011). Specifically, these implications depend both on which policy instrument bears the cost of adjustment in the early period of fiscal pain and on which policy instrument is anticipated to reap the benefit, once debt consolidation has been achieved. In the policy experiments we consider below, we will experiment with fiscal policy mixes, which means that the fiscal authority of each country is allowed to use an instrument in the transition and perhaps a different one in the new steady state.

In our main thought experiment, the model is solved numerically with the Matlab toolbox (programs are available upon request) using the following solution strategy: i) First-order approximate solutions are computed around the associated new steady state and saddle path stability is checked. ii) The feedback monetary and fiscal policy coefficients of the instruments used along the transition path - by the single monetary authority as well as by each national fiscal authority - are set as the values discussed above in Subsection 3.1[36] iii) Then, we will depart from the status quo steady state solution

[36] Along this transition, regarding public debt consolidation, we experiment with one fiscal policy instrument at a time in each country, which means that we allow in each country only one national fiscal policy instrument to react to its policy target, while switch off all other instruments(Details are in Subsection 3.1).

and compute the equilibrium transition path as we travel towards a new reformed steady state (policy reforms are defined above) by setting as initial values of the predetermined variables their steady state values in the status quo economy (see Table 3). Transition dynamics from the status quo steady state to a new steady state will be driven by debt consolidation policies in both countries.

5 Results

As expected, had tax-spending policy in both countries remained unchanged as in the data averages over 2001-2011, the model would be dynamically unstable. In other words, some type of fiscal reaction in each country (spending cuts and/or tax rises) to public debt imbalances was necessary for restoring dynamic stability.

In the policy experiments considered below, we will study the implications of debt consolidation when both countries take consolidation measures. To evaluate the implications of debt consolidation, we need to compare them to a reference regime. As a reference regime, we will use the case without debt consolidation in both countries, other things equal(i.e. the status quo steady state). In all cases, we will study aggregate and distributional implications both in steady state and along the transition. Regarding aggregate outcomes, we will look, for instance, at aggregate(or per capita) output. Regarding distribution, we will compute the net income of the representative worker and public employee relative to that of the capitalist. In the transition, we will work with present values of the above variables. All these variables values are compared to their respective values had we remained in the status quo permanently.

5.1 Aggregate results(efficiency)

5.1.1 Steady state results

We start with comparison of steady state solutions. Recall that in the SQ steady state, in both countries, public debt-to-GDP ratio and fiscal policy instruments were set as in the data and lump-sum transfers followed residually, while, in the reformed steady state, the public debt-to-GDP ratio is ad hoc cut[37] at 90% in Italy and at 60% in Germany so that one of the fiscal

[37] In the reformed steady, the value of lump-sum transfers remains as in its status quo steady state in both countries.

policy instruments in each country follows residually from the government budget constraint meaning that in the domestic country, and similarly for the foreign country, one of the s^g, s^i, s^w is allowed to rise or one of the τ^k, τ^n, τ^c is allowed to be cut.

Table 4 reports the value of aggregate output in the domestic country in each new reformed steady state depending on what the residual fiscal policy instrument is (columns). Similarly, Table 5 reports values of aggregate output in the foreign country in each new reformed steady state depending on what the residual fiscal policy instrument is(columns).

The conclusions that can be made by the Tables 4 and 5 are:

1) Debt consolidation in Germany (country with solid public finances) is productive relative to status quo (SQ) in the new reformed steady state under all cases studied. In terms of efficiency, the best way of using the fiscal space created by debt consolidation is to finance higher public investment spending.

2) Debt consolidation in Italy (high-debt country) is also productive relative to status quo (SQ) in the new reformed steady state under all cases studied. In terms of efficiency, the best way of using the fiscal space created by debt consolidation is to finance higher public investment spending.

Summing up, in terms of aggregate economy, our numerical results imply that it is better for both countries to finance an increase in public investment, once debt has been reduced.

5.1.2 Transition results

Tables 6 and 7 report the present value of aggregate output in the domestic and the foreign country respectively t periods after debt consolidation starts taking place in both countries depending on what the adjusting instrument is in the transition to a new reformed steady state. Both tables correspond to the case in which both countries in the reformed steady state use their fiscal space created by debt consolidation to rise public investment. Every row of these tables shows the present (discounted) value of aggregate output over different time horizons depending on what instrument adjusts to bring public debt down in the transition.

Inspection of the results in Tables 6 and 7 implies that if the criterion is aggregate, or per capita, output, the best policy mix for both countries is to use the fiscal space created by debt consolidation to finance a rise in public investment spending (s^i and s^{i*}) and, during the early period of fiscal pain, to use public consumption spending cuts (s^g and s^{g*}) to bring public debt down.

In the transition to the new reformed steady state, the above policy mix, when followed by both countries, is productive (see Tables 6 and 7) for both countries. As said, all this is relative to the status quo.

5.2 Distributional implications (equity)

In this subsection, we study the distributional implications of debt consolidation as measured by the net income of private workers and public employees relative to that of capitalists.[38] All this vis-à-vis the status quo.

I start with the study of the most efficient policy mix for both countries where public consumption spending cuts (s^g and s^{g*}) are used to bring public debt down during the early period of fiscal pain and, once debt has been reduced, the fiscal space created by debt consolidation is used to finance a rise in public investment spending (s^i and s^{i*}). Tables 16 and 17 report values of $\frac{\widetilde{y}_t^w}{\widetilde{y}_t^k}$ and $\frac{\widetilde{y}_t^b}{\widetilde{y}_t^k}$ (relative net incomes), where $\widetilde{y}_t^k$, $\widetilde{y}_t^w$ and $\widetilde{y}_t^b$ denote the present value of the capitalist's, private worker's and public employee's net income respectively in the domestic country t periods after debt consolidation starts taking place in both countries. Every entry of Tables 16 and 17 represents the value of $\frac{\widetilde{y}_t^w}{\widetilde{y}_t^k}$ and $\frac{\widetilde{y}_t^b}{\widetilde{y}_t^k}$ respectively over different t periods (columns) depending on what fiscal policy instrument (rows) is used for debt reduction by the domestic country during the transition. Similarly, Tables 18 and 19 report the values of $\frac{\widetilde{y}_t^{w*}}{\widetilde{y}_t^{k*}}$ and $\frac{\widetilde{y}_t^{b*}}{\widetilde{y}_t^{k*}}$ (relative net incomes), where a star denotes the counterpart of a variable in the foreign country. Every entry of Tables 18 and 19 represents the value of $\frac{\widetilde{y}_t^{w*}}{\widetilde{y}_t^{k*}}$ and $\frac{\widetilde{y}_t^{b*}}{\widetilde{y}_t^{k*}}$ respectively over different t periods (columns) depending on what fiscal policy instrument (rows) is used for debt reduction by the foreign country during the transition.

Inspection of the results in Tables 16, 17, 18 and 19 implies that the most

[38] In the domestic country, the net income of the capitalist is defined as $y_t^k \equiv -\tau_t^c c_t^k + (1-\tau_t^k)\left[r_t^k \tau\tau_t^{v-1} k_{t-1}^k + \widetilde{\omega}_t{}^k\right] + (1-\tau_t^n) w_t^k n_t^k + (Q_{t-1}-1)\tau\tau_t^{v+v^*-1}\frac{1}{\pi_t^*} f_{t-1}^k + (R_{t-1}-1)\frac{1}{\pi_t}\lambda_{t-1} d_{t-1} - v^k s_t^l y_t^H \tau\tau_t^{v-1} + \widetilde{\pi}_t{}^k$, of the private worker is defined as $y_t^w \equiv -\tau_t^c c_t^w + (1-\tau_t^n) w_t^w n_t^w - v^k s_t^l y_t^H \tau\tau_t^{v-1}$ and of the public employee is defined as $y_t^b \equiv -\tau_t^c c_t^b + (1-\tau_t^n)\frac{v^k}{v^b} s_t^w \tau\tau_t^{v-1} y_t^H - v^k s_t^l y_t^H \tau\tau_t^{v-1}$. In the foreign country, the net income of the capitalist is defined as $y_t^{k*} \equiv -\tau_t^{c*} c_t^{k*} + (1-\tau_t^{k*})\left[r_t^{k*} \tau\tau_t^{1-v^*} k_{t-1}^{k*} + \widetilde{\omega}_t{}^{k*}\right] + (1-\tau_t^{n*}) w_t^{k*} n_t^{k*} + (Q_{t-1}^*-1)\tau\tau_t^{1-v-v^*}\frac{1}{\pi_t} f_{t-1}^{k*} + (R_{t-1}^*-1)\frac{1}{\pi_t^*}\lambda_{t-1}^* d_{t-1}^* - v^{k*} s_t^{l*} y_t^{H*} \tau\tau_t^{1-v^*}$, of the private worker is defined as $y_t^{w*} \equiv -\tau_t^{c*} c_t^{w*} + (1-\tau_t^{n*}) w_t^{w*} n_t^{w*} - v^{k*} s_t^{l*} y_t^{H*} \tau\tau_t^{1-v^*}$ and of the public employee is defined as $y_t^{b*} \equiv -\tau_t^{c*} c_t^{b*} + (1-\tau_t^{n*})\frac{v^{k*}}{v^{b*}} s_t^{w*} \tau\tau_t^{1-v^*} y_t^{H*} - v^{k*} s_t^{l*} y_t^{H*} \tau\tau_t^{1-v^*}$.

Table 4: Output (GDP) in steady state(SS)

Residual instrument in the domestic economy	τ^k	τ^n	τ^c	s^g	s^i	s^w
GDP in the domestic economy	0.8972	0.8965	0.8958	0.8958	0.8997	0.8958

Steady state value of the output in status quo (SQ) for the domestic economy is 0.8952.
Note: The above values are barely affected by other country's choice of fiscal policy instrument in the new steady state.

Table 5: Output (GDP) in steady state(SS)

Residual instrument in the foreign economy	τ^{k*}	τ^{n*}	τ^{c*}	s^{g*}	s^{i*}	s^{w*}
GDP in the foreign economy	0.7919	0.7896	0.7879	0.7879	0.7946	0.7879

Steady state value of the output in status quo (SQ) for the foreign economy is 0.7863.
Note: The above values are barely affected by other country's choice of fiscal policy instrument in the new steady state.

Table 6: Present value of output (**GDP**) in the domestic economy over different t periods, $\widetilde{y}_t$, after debt consolidation starts taking place, **when this economy uses the fiscal space created by debt consolidation to finance a rise in public investment spending (**s^i**)**.

Adj. Instr.	$\widetilde{y}_5$	$\widetilde{y}_{10}$	$\widetilde{y}_{20}$	$\widetilde{y}_{40}$	$\widetilde{y}_{60}$	$\widetilde{y}_{80}$	$\widetilde{y}_\infty$
τ^k	4.3343	8.3271	15.3682	26.3528	34.2040	39.8120	51.9288
τ^n	4.3258	8.3236	15.3695	26.3595	34.2124	39.8208	51.9376
τ^c	4.3428	8.3408	15.3897	26.3832	34.2365	39.8448	51.9616
s^g	4.3463	8.3475	15.3990	26.3953	34.2496	39.8581	51.9750
s^i	4.3190	8.2948	15.3245	26.3156	34.1710	39.7798	51.8967
s^w	4.3413	8.3393	15.3884	26.3819	34.2353	39.8436	51.9604
SQ	4.3288	8.3081	15.3285	26.2736	34.0889	39.6692	51.7251

Note: Every row represents a different case depending on what fiscal policy instrument is used by the domestic country to bring debt down during the early period of fiscal pain. At the same time, in this case studied, we assume that the foreign country uses the fiscal space created by debt consolidation to rise public investment spending and, during the early period of fiscal pain, uses public consumption spending cuts to bring public debt down.

Table 7: Present value of output (**GDP**) in the foreign economy over different t periods, $\widetilde{y}^*_t$, after debt consolidation starts taking place, **when this economy uses the fiscal space created by debt consolidation to finance a rise in public investment spending** (s^{i*}).

Adj. Instr.	$\widetilde{y}^*_5$	$\widetilde{y}^*_{10}$	$\widetilde{y}^*_{20}$	$\widetilde{y}^*_{40}$	$\widetilde{y}^*_{60}$	$\widetilde{y}^*_{80}$	$\widetilde{y}^*_{\infty}$
τ^{k*}	3.7532	7.1267	12.8621	21.1528	26.4768	29.8909	35.5527
τ^{n*}	3.7390	7.1218	12.8734	21.1803	26.5084	29.9231	35.5849
τ^{c*}	3.7733	7.1647	12.9216	21.2306	26.5584	29.9730	35.6348
s^{g*}	3.7785	7.1763	12.9418	21.2566	26.5847	29.9991	35.6608
s^{i*}	3.7460	7.1072	12.8334	21.1422	26.4730	29.8876	35.5493
s^{w*}	3.7705	7.1598	12.9153	21.2235	26.5515	29.9662	35.6280
SQ	3.7625	7.1289	12.8360	21.0623	26.3344	29.7132	35.3158

Note: Every row represents a different case depending on what fiscal policy instrument is used by the foreign country to bring debt down during the early period of fiscal pain. At the same time, in this case studied, we assume that the domestic country uses the fiscal space created by debt consolidation to rise public investment spending and, during the early period of fiscal pain, uses public consumption spending cuts to bring public debt down.

Table 8: Incomes in steady state (SS) when the fiscal space created by debt consolidation is used by both economies to finance a rise in their public investment spending (s^i and s^{i*}).

	Domestic economy	Foreign economy
Capitalist's income	0.3595(0.3584)	0.3232 (0.3230)
Private worker's income	0.0829 (0.0824)	0.0851 (0.0843)
Public employee's income	0.0664 (0.0660)	0.0686 (0.0680)

Note: In brackets they are reported the associated SQ steady state values.

Table 9: Incomes in steady state (SS) when the fiscal space created by debt consolidation is used by both economies to cut their labor tax rate (τ^n and τ^{n*}).

	Domestic economy	Foreign economy
Capitalist's income	0.3586 (0.3584)	0.3216 (0.3230)
Private worker's income	0.0829 (0.0824)	0.0851 (0.0843)
Public employee's income	0.0664 (0.0660)	0.0686 (0.0680)

Note: In brackets they are reported the associated SQ steady state values.

Table 10: Values of the income of the private worker and public employee relative to that of the capitalist in steady state (SS) when the fiscal space created by debt consolidation is used by both economies to finance a rise in their public investment spending (s^i and s^{i*}).

	Domestic economy	Foreign economy
Private worker's net income relative to capitalist's net income $\left(\frac{y^w}{y^k} or \frac{y^{w*}}{y^{k*}}\right)$	0.2306 (0.2299)	0.2632 (0.2609)
Public employee's net income relative to capitalist's net income $\left(\frac{y^b}{y^k} or \frac{y^{b*}}{y^{k*}}\right)$	0.1848 (0.1842)	0.2123 (0.2104)

Note: In brackets they are reported the associated SQ steady state values.

efficient policy mix, when followed by both countries, also promotes equity in both countries. Namely, in both countries, the present values of the net income of the private worker and the public employee relative to that of the capitalist rise in all cases and over all time horizons as a result of debt consolidation. And, as said, all this relative to status quo.

Except for the above case, I have studied several alternative scenarios. I have experimented with several policy mixes, but I prefer to focus on the case in which both countries rise their tax rate on capital (τ^k and τ^{k*}) to bring public debt down during the early period of fiscal pain and, once debt has been reduced, use the fiscal space created by debt consolidation to finance a cut in their tax rate on labor (τ^n and τ^{n*}). This is a particularly popular policy politically in terms of redistribution in favor of the "poor". Every entry of Tables 20 and 21 represents the value of $\frac{\widetilde{y}_t^w}{\widetilde{y}_t^k}$ and $\frac{\widetilde{y}_t^b}{\widetilde{y}_t^k}$ respectively over different t periods (columns) depending on what fiscal policy instrument (rows) is used for debt reduction by the domestic country during the transition. Similarly, every entry of Tables 22 and 23 represents the value of $\frac{\widetilde{y}_t^{w*}}{\widetilde{y}_t^{k*}}$ and $\frac{\widetilde{y}_t^{b*}}{\widetilde{y}_t^{k*}}$ respectively over different t periods (columns) depending on what fiscal policy instrument (rows) is used for debt reduction by the foreign country during the transition.

Inspection of the results of Tables 20, 21, 22 and 23 implies that a better way to improve equity in both countries is to rise their capital tax rate to bring public debt down, during the early period of fiscal pain, and, once

Table 11: Values of the income of the private worker and public employee relative to that of the capitalist in steady state (SS) when the fiscal space created by debt consolidation is used by both economies to cut their labor tax rate (τ^n and τ^{n*}).

	Domestic economy	Foreign economy
Private worker's net income relative to capitalist's net income $\left(\frac{y^w}{y^k} or \frac{y^{w*}}{y^{k*}}\right)$	0.2311 (0.2299)	0.2646 (0.2609)
Public employee's net income relative to capitalist's net income $\left(\frac{y^b}{y^k} or \frac{y^{b*}}{y^{k*}}\right)$	0.1851 (0.1842)	0.2132 (0.2104)

Note: In brackets they are reported the associated SQ steady state values.

debt has been reduced, to use the fiscal space created by debt consolidation to finance a cut in their tax rate on labor. As said, all this relative to status quo. However, although the present values of the net income of the private worker and the public employee relative to that of the capitalist in both countries rise more than in Tables 16, 17, 18 and 19 (i.e. this policy mix helps the poor more relative to the most efficient case mentioned above), this policy mix is less efficient at aggregate level. In addition, the present values of the capitalist's, private worker's and public employee's net income in both countries fall as the same happens to the aggregate output.

In sum, the most efficient policy mix, in which both countries use the fiscal space created by debt consolidation to finance a rise in their public investment spending and, during the transition, use their public consumption spending cuts to bring public debt down, promotes equity. Nevertheless, after experimenting with several policy mixes, a better way to promote equity is both countries to use the fiscal space created by debt consolidation to finance a cut in their tax rate on labor and, during the early period of fiscal pain, to use their tax rate on capital to bring public debt down.

6 Closing the chapter and possible extensions

In this chapter was built and solved numerically a new Keynesian D(S)GE model of a two-country world economy that forms a monetary union. In this model the fiscal authorities of each country were engaged in public debt

Table 12: Present values of household j's income in the domestic economy over various t periods after debt consolidation starts taking place, $\widetilde{y}^j_t$, **when this economy uses the fiscal space created by debt consolidation to finance a rise in public investment spending (s^i) and, during the transition, brings debt down through public consumption spending cuts (s^g).**

	$\widetilde{y}^j_5$	$\widetilde{y}^j_{10}$	$\widetilde{y}^j_{20}$	$\widetilde{y}^j_{40}$	$\widetilde{y}^j_{60}$	$\widetilde{y}^j_{80}$	$\widetilde{y}^j_\infty$
PVs of domestic capitalist k's income ($j = k$)	1.7410	3.3368	6.1535	10.5470	13.6853	15.9264	20.768
	(1.7331)	(3.3263)	(6.1371)	(10.5192)	(13.6482)	(15.8824)	(20.709)
PVs of domestic private worker w's income ($j = w$)	0.4020	0.7715	1.4216	2.4350	3.1588	3.6756	4.792
	(0.3985)	(0.7649)	(1.4112)	(2.4188)	(3.1383)	(3.6521)	(4.762)
PVs of domestic public employee b's income ($j = b$)	0.3220	0.6180	1.1389	1.9507	2.5306	2.9446	3.839
	(0.3193)	(0.6128)	(1.1305)	(1.9378)	(2.5142)	(2.9258)	(3.815)

Note: (i) In this case studied, we assume that the foreign country uses the fiscal space created by debt consolidation to rise public investment spending and, during the early period of fiscal pain, uses public consumption cuts to bring public debt down. (ii) Results without debt consolidation are in parentheses.

Table 13: Present values of household j^*'s income in the foreign economy over various t periods after debt consolidation starts taking place, $\widetilde{y}_t^{j*}$, **when this economy uses the fiscal space created by debt consolidation to finance a rise in public investment spending (s^{i*}) and, during the transition, brings debt down through public consumption spending cuts(s^{g*}).**

	$\widetilde{y}_5^{j*}$	$\widetilde{y}_{10}^{j*}$	$\widetilde{y}_{20}^{j*}$	$\widetilde{y}_{40}^{j*}$	$\widetilde{y}_{60}^{j*}$	$\widetilde{y}_{80}^{j*}$	$\widetilde{y}_{\infty}^{j*}$
PVs of foreign capitalist k's income ($j = k$)	1.5373 (1.5453)	2.9123 (2.9279)	5.2519 (5.2719)	8.6313 (8.6505)	10.7979 (10.8158)	12.1865 (12.2035)	14.489 (14.505)
PVs of foreign private worker w's income ($j = w$)	0.4037 (0.4032)	0.7669 (0.7640)	1.3835 (1.3755)	2.2735 (2.2571)	2.8439 (2.8221)	3.2095 (3.1841)	3.816 (3.785)
PVs of foreign public employee b's income ($j = b$)	0.3255 (0.3252)	0.6184 (0.6161)	1.1157 (1.1094)	1.8335 (1.8203)	2.2936 (2.2760)	2.5884 (2.5680)	3.077 (3.052)

Notes: (i) In this case studied, we assume that the domestic country uses the fiscal space created by debt consolidation to rise public investment spending and, during the early period of fiscal pain, uses public consumption cuts to bring public debt down. (ii) Results without debt consolidation are in parentheses.

Table 14: Present values of household j's income in the domestic economy over various t periods after debt consolidation starts taking place, $\widetilde{y}_t^j$, **when this economy uses the fiscal space created by debt consolidation to cut the tax rate on labor (τ^n) and, during the transition, brings debt down through a rise in the tax rate on capital (τ^k).**

	$\widetilde{y}_5^j$	$\widetilde{y}_{10}^j$	$\widetilde{y}_{20}^j$	$\widetilde{y}_{40}^j$	$\widetilde{y}_{60}^j$	$\widetilde{y}_{80}^j$	$\widetilde{y}_\infty^j$
PVs of domestic capitalist k's income ($j = k$)	1.7223	3.3109	6.1204	10.5071	13.6406	15.8776	20.7081
	(1.7331)	(3.3263)	(6.1371)	(10.5192)	(13.6482)	(15.8824)	(20.7092
PVs of domestic private worker w's income ($j = w$)	0.3984	0.7664	1.4171	2.4304	3.1540	3.6706	4.7865
	(0.3985)	(0.7649)	(1.4112)	(2.4188)	(3.1383)	(3.6521)	(4.7620
PVs of domestic public employee b's income ($j = b$)	0.3191	0.6139	1.1350	1.9467	2.5263	2.9400	3.8338
	(0.3193)	(0.6128)	(1.1305)	(1.9378)	(2.5142)	(2.9258)	(3.8150

Note: (i) In this case studied, we assume that the foreign country uses the fiscal space created by debt consolidation to cut the tax rate on labor and, during the early period of fiscal pain, uses capital taxes to bring public debt down. (ii) Results without debt consolidation are in parentheses.

Table 15: Present values of household j^*'s income in the foreign economy over various t periods after debt consolidation starts taking place, $\widetilde{y}_t^{j*}$, **when this economy uses the fiscal space created by debt consolidation to cut the tax rate on labor (τ^{n*}) and, during the transition, brings debt down through a rise in the tax rate on capital (τ^{k*}).**

	$\widetilde{y}_5^{j*}$	$\widetilde{y}_{10}^{j*}$	$\widetilde{y}_{20}^{j*}$	$\widetilde{y}_{40}^{j*}$	$\widetilde{y}_{60}^{j*}$	$\widetilde{y}_{80}^{j*}$	$\widetilde{y}_{\infty}^{j*}$
PVs of foreign capitalist k's income ($j = k$)	1.5182 (1.5453)	2.8826 (2.9279)	5.2013 (5.2719)	8.5551 (8.6505)	10.7086 (10.8158)	12.0896 (12.2035)	14.3806 (14.5045)
PVs of foreign private worker w's income ($j = w$)	0.4037 (0.4032)	0.7667 (0.7640)	1.3839 (1.3755)	2.2736 (2.2571)	2.8439 (2.8221)	3.2095 (3.1841)	3.8158 (3.7845)
PVs of foreign public employee b's income ($j = b$)	0.3253 (0.3252)	0.6179 (0.6161)	1.1153 (1.1094)	1.8323 (1.8203)	2.2920 (2.2760)	2.5866 (2.5680)	3.0753 (3.0522)

Notes: (i) In this case studied, we assume that the domestic country uses the fiscal space created by debt consolidation to cut the tax rate on labor and, during the early period of fiscal pain, uses capital taxes to bring public debt down. (ii) Results without debt consolidation are in parentheses.

Table 16: Value of the $\frac{\widetilde{y}_t^w}{\widetilde{y}_t^k}$ over different time horizons (t) after debt consolidation starts taking place **when the domestic country uses the fiscal space created by debt consolidation to finance a rise in public investment spending (s^i).**

Adj. Instr.	$\frac{\widetilde{y}_5^w}{\widetilde{y}_5^k}$	$\frac{\widetilde{y}_{10}^w}{\widetilde{y}_{10}^k}$	$\frac{\widetilde{y}_{20}^w}{\widetilde{y}_{20}^k}$	$\frac{\widetilde{y}_{40}^w}{\widetilde{y}_{40}^k}$	$\frac{\widetilde{y}_{60}^w}{\widetilde{y}_{60}^k}$	$\frac{\widetilde{y}_{80}^w}{\widetilde{y}_{80}^k}$	$\frac{\widetilde{y}_{\infty}^w}{\widetilde{y}_{\infty}^k}$
τ^k	0.2316	0.2317	0.2315	0.2313	0.2311	0.2311	0.2310
τ^n	0.2286	0.2299	0.2301	0.2302	0.2303	0.2303	0.2304
τ^c	0.2301	0.2306	0.2306	0.2305	0.2305	0.2305	0.2305
s^g	0.2309	0.2312	0.2310	0.2309	0.2308	0.2308	0.2307
s^i	0.2298	0.2306	0.2308	0.2307	0.2307	0.2307	0.2307
s^w	0.2303	0.2309	0.2308	0.2307	0.2307	0.2307	0.2307
SQ	0.2299	0.2299	0.2299	0.2299	0.2299	0.2299	0.2299

Notes:

(i) $\frac{\widetilde{y}_t^w}{\widetilde{y}_t^k}$ stands for the ratio of $\widetilde{y}_t^w$ to $\widetilde{y}_t^k$, where $\widetilde{y}_t^w$ and $\widetilde{y}_t^k$ denote the present value of the net income of the private worker and the capitalist respectively in the domestic economy for the next t periods after fiscal consolidation starts taking place.
(ii) Every row presents a different case depending on what instrument is used by the domestic country to bring debt down during the early period of fiscal pain.
(iii) In this case studied, we assume that the foreign country uses the fiscal space created by debt consolidation to finance higher public investment spending and, during the early period of fiscal pain, uses public consumption spending cuts to bring public debt down.

Table 17: Value of the $\frac{\widetilde{y}_t^b}{\widetilde{y}_t^k}$ over different time horizons (t) after debt consolidation starts taking place **when the domestic country uses the fiscal space created by debt consolidation to finance a rise in public investment spending (s^i).**

Adj. Instr.	$\frac{\widetilde{y}_5^b}{\widetilde{y}_5^k}$	$\frac{\widetilde{y}_{10}^b}{\widetilde{y}_{10}^k}$	$\frac{\widetilde{y}_{20}^b}{\widetilde{y}_{20}^k}$	$\frac{\widetilde{y}_{40}^b}{\widetilde{y}_{40}^k}$	$\frac{\widetilde{y}_{60}^b}{\widetilde{y}_{60}^k}$	$\frac{\widetilde{y}_{80}^b}{\widetilde{y}_{80}^k}$	$\frac{\widetilde{y}_{\infty}^b}{\widetilde{y}_{\infty}^k}$
τ^k	0.1855	0.1856	0.1855	0.1853	0.1852	0.1851	0.1850
τ^n	0.1833	0.1843	0.1844	0.1845	0.1845	0.1846	0.1846
τ^c	0.1843	0.1847	0.1847	0.1847	0.1847	0.1847	0.1847
s^g	0.1850	0.1852	0.1851	0.1850	0.1849	0.1849	0.1849
s^i	0.1841	0.1847	0.1849	0.1849	0.1848	0.1848	0.1848
s^w	0.1736	0.1774	0.1790	0.1801	0.1808	0.1813	0.1821
SQ	0.1842	0.1842	0.1842	0.1842	0.1842	0.1842	0.1842

Notes:

(i) $\frac{\widetilde{y}_t^b}{\widetilde{y}_t^k}$ stands for the ratio of $\widetilde{y}_t^b$ to $\widetilde{y}_t^k$, where $\widetilde{y}_t^b$ and $\widetilde{y}_t^k$ denote the present value of the net income of the public employee and the capitalist respectively in the domestic economy for the next t periods after fiscal consolidation starts taking place.
(ii) Every row presents a different case depending on what instrument is used by the domestic country to bring debt down during the early period of fiscal pain.
(iii) In this case studied, we assume that the foreign country uses the fiscal space created by debt consolidation to finance a rise in public investment spending and, during the early period of fiscal pain, uses public consumption spending cuts to bring public debt down.

Table 18: Value of the $\frac{\widetilde{y}_t^{w*}}{\widetilde{y}_t^{k*}}$ over different time horizons (t) after debt consolidation starts taking place **when the foreign country uses the fiscal space created by debt consolidation to finance a rise in public investment spending (s^{i*}).**

Adj. Instr.	$\frac{\widetilde{y}_5^{w*}}{\widetilde{y}_5^{k*}}$	$\frac{\widetilde{y}_{10}^{w*}}{\widetilde{y}_{10}^{k*}}$	$\frac{\widetilde{y}_{20}^{w*}}{\widetilde{y}_{20}^{k*}}$	$\frac{\widetilde{y}_{40}^{w*}}{\widetilde{y}_{40}^{k*}}$	$\frac{\widetilde{y}_{60}^{w*}}{\widetilde{y}_{60}^{k*}}$	$\frac{\widetilde{y}_{80}^{w*}}{\widetilde{y}_{80}^{k*}}$	$\frac{\widetilde{y}_{\infty}^{w*}}{\widetilde{y}_{\infty}^{k*}}$
τ^{k*}	0.2642	0.2647	0.2646	0.2643	0.2641	0.2640	0.2639
τ^{n*}	0.2553	0.2586	0.2603	0.2613	0.2617	0.2619	0.2621
τ^{c*}	0.2597	0.2611	0.2617	0.2621	0.2623	0.2624	0.2626
s^{g*}	0.2626	0.2633	0.2634	0.2634	0.2634	0.2634	0.2633
s^{i*}	0.2602	0.2617	0.2627	0.2630	0.2631	0.2631	0.2631
s^{w*}	0.2603	0.2616	0.2622	0.2625	0.2627	0.2627	0.2628
SQ	0.2609	0.2609	0.2609	0.2609	0.2609	0.2609	0.2609

Notes:

(i) $\frac{\widetilde{y}_t^{w*}}{\widetilde{y}_t^{k*}}$ stands for the ratio of $\widetilde{y}_t^{w*}$ to $\widetilde{y}_t^{k*}$, where $\widetilde{y}_t^{w*}$ and $\widetilde{y}_t^{k*}$ denote the present value of the net income of the private worker and the capitalist respectively in the foreign economy for the next t periods after fiscal consolidation starts taking place.
(ii) Every row presents a different case depending on what instrument is used by the foreign country to bring debt down during the early period of fiscal pain.
(iii) In this case studied, we assume that the domestic country uses the fiscal space created by debt consolidation to rise public investment spending and, during the early period of fiscal pain, uses public consumption spending cuts to bring public debt down.

Table 19: Value of the $\frac{\widetilde{y}_t^{b*}}{\widetilde{y}_t^{k*}}$ over different time horizons (t) after debt consolidation starts taking place **when the foreign country uses the fiscal space created by debt consolidation to finance a rise in public investment spending (s^{i*}).**

Adj. Instr.	$\frac{\widetilde{y}_5^{b*}}{\widetilde{y}_5^{k*}}$	$\frac{\widetilde{y}_{10}^{b*}}{\widetilde{y}_{10}^{k*}}$	$\frac{\widetilde{y}_{20}^{b*}}{\widetilde{y}_{20}^{k*}}$	$\frac{\widetilde{y}_{40}^{b*}}{\widetilde{y}_{40}^{k*}}$	$\frac{\widetilde{y}_{60}^{b*}}{\widetilde{y}_{60}^{k*}}$	$\frac{\widetilde{y}_{80}^{b*}}{\widetilde{y}_{80}^{k*}}$	$\frac{\widetilde{y}_{\infty}^{b*}}{\widetilde{y}_{\infty}^{k*}}$
τ^{k*}	0.2130	0.2134	0.2134	0.2131	0.2130	0.2129	0.2128
τ^{n*}	0.2063	0.2088	0.2102	0.2109	0.2112	0.2113	0.2115
τ^{c*}	0.2094	0.2105	0.2111	0.2114	0.2116	0.2116	0.2117
s^{g*}	0.2117	0.2124	0.2124	0.2124	0.2124	0.2124	0.2124
s^{i*}	0.2098	0.2111	0.2119	0.2121	0.2122	0.2122	0.2122
s^{w*}	0.1882	0.1933	0.1976	0.2015	0.2033	0.2042	0.2055
SQ	0.2104	0.2104	0.2104	0.2104	0.2104	0.2104	0.2104

Notes:

(i) $\frac{\widetilde{y}_t^{b*}}{\widetilde{y}_t^{k*}}$ stands for the ratio of $\widetilde{y}_t^{b*}$ to $\widetilde{y}_t^{k*}$, where $\widetilde{y}_t^{b*}$ and $\widetilde{y}_t^{k*}$ denote the present value of the net income of the public employee and the capitalist respectively in the foreign economy for the next t periods after fiscal consolidation starts taking place.
(ii) Every row presents a different case depending on what instrument is used by the foreign country to bring debt down during the early period of fiscal pain.
(iii) In this case studied, we assume that the domestic country uses the fiscal space created by debt consolidation to rise public investment spending and, during the early period of fiscal pain, uses public consumption spending cuts to bring public debt down.

Table 20: Value of the $\frac{\widetilde{y}_t^w}{\widetilde{y}_t^k}$ over different time horizons (t) after debt consolidation starts taking place **when the domestic country uses the fiscal space created by debt consolidation to cut the tax rate on labor (τ^n).**

Adj. Instr.	$\frac{\widetilde{y}_5^w}{\widetilde{y}_5^k}$	$\frac{\widetilde{y}_{10}^w}{\widetilde{y}_{10}^k}$	$\frac{\widetilde{y}_{20}^w}{\widetilde{y}_{20}^k}$	$\frac{\widetilde{y}_{40}^w}{\widetilde{y}_{40}^k}$	$\frac{\widetilde{y}_{60}^w}{\widetilde{y}_{60}^k}$	$\frac{\widetilde{y}_{80}^w}{\widetilde{y}_{80}^k}$	$\frac{\widetilde{y}_{\infty}^w}{\widetilde{y}_{\infty}^k}$
τ^k	0.2313	0.2315	0.2315	0.2313	0.2312	0.2312	0.2311
τ^n	0.2270	0.2285	0.2298	0.2304	0.2307	0.2307	0.2308
τ^c	0.2292	0.2297	0.2303	0.2306	0.2308	0.2308	0.2309
s^g	0.2304	0.2306	0.2309	0.2309	0.2310	0.2310	0.2310
s^i	0.2289	0.2299	0.2309	0.2310	0.2310	0.2310	0.2310
s^w	0.2295	0.2300	0.2306	0.2308	0.2309	0.2309	0.2309
SQ	0.2299	0.2299	0.2299	0.2299	0.2299	0.2299	0.2299

Notes:

(i) $\frac{\widetilde{y}_t^w}{\widetilde{y}_t^k}$ stands for the ratio of $\widetilde{y}_t^w$ to $\widetilde{y}_t^k$, where $\widetilde{y}_t^w$ and $\widetilde{y}_t^k$ denote the present value of the net income of the private worker and the capitalist respectively in the domestic economy for the next t periods after fiscal consolidation starts taking place.
(ii) Every row presents a different case depending on what instrument is used by the domestic country to bring debt down during the early period of fiscal pain.
(iii) In this case studied, we assume that the foreign country uses the fiscal space created by debt consolidation to cut the tax rate on labor and, during the early period of fiscal pain, rise the tax rate on capital to bring public debt down.

Table 21: Value of the $\frac{\widetilde{y}_t^b}{\widetilde{y}_t^k}$ over different time horizons (t) after debt consolidation starts taking place **when the domestic country uses the fiscal space created by debt consolidation to cut the tax rate on labor (τ^n).**

Adj. Instr.	$\frac{\widetilde{y}_5^b}{\widetilde{y}_5^k}$	$\frac{\widetilde{y}_{10}^b}{\widetilde{y}_{10}^k}$	$\frac{\widetilde{y}_{20}^b}{\widetilde{y}_{20}^k}$	$\frac{\widetilde{y}_{40}^b}{\widetilde{y}_{40}^k}$	$\frac{\widetilde{y}_{60}^b}{\widetilde{y}_{60}^k}$	$\frac{\widetilde{y}_{80}^b}{\widetilde{y}_{80}^k}$	$\frac{\widetilde{y}_{\infty}^b}{\widetilde{y}_{\infty}^k}$
τ^k	0.1853	0.1854	0.1855	0.1853	0.1852	0.1852	0.1851
τ^n	0.1820	0.1832	0.1842	0.1846	0.1848	0.1848	0.1849
τ^c	0.1836	0.1840	0.1844	0.1847	0.1848	0.1849	0.1849
s^g	0.1846	0.1847	0.1849	0.1850	0.1850	0.1850	0.1850
s^i	0.1834	0.1841	0.1849	0.1850	0.1850	0.1850	0.1850
s^w	0.1690	0.1720	0.1769	0.1807	0.1822	0.1828	0.1835
SQ	0.1842	0.1842	0.1842	0.1842	0.1842	0.1842	0.1842

Notes:

(i) $\frac{\widetilde{y}_t^b}{\widetilde{y}_t^k}$ stands for the ratio of $\widetilde{y}_t^b$ to $\widetilde{y}_t^k$, where $\widetilde{y}_t^b$ and $\widetilde{y}_t^k$ denote the present value of the net income of the public employee and the capitalist respectively in the domestic economy for the next t periods after fiscal consolidation starts taking place.
(ii) Every row presents a different case depending on what instrument is used by the domestic country to bring debt down during the early period of fiscal pain.
(iii) In this case studied, we assume that the foreign country uses the fiscal space created by debt consolidation to cut the tax rate on labor and, during the early period of fiscal pain, rise the tax rate on capital to bring public debt down.

Table 22: Value of the $\frac{\widetilde{y}_t^{w*}}{\widetilde{y}_t^{k*}}$ over different time horizons (t) after debt consolidation starts taking place **when the foreign country uses the fiscal space created by debt consolidation to cut the tax rate on labor** (τ^{n*}).

Adj. Instr.	$\frac{\widetilde{y}_5^{w*}}{\widetilde{y}_5^{k*}}$	$\frac{\widetilde{y}_{10}^{w*}}{\widetilde{y}_{10}^{k*}}$	$\frac{\widetilde{y}_{20}^{w*}}{\widetilde{y}_{20}^{k*}}$	$\frac{\widetilde{y}_{40}^{w*}}{\widetilde{y}_{40}^{k*}}$	$\frac{\widetilde{y}_{60}^{w*}}{\widetilde{y}_{60}^{k*}}$	$\frac{\widetilde{y}_{80}^{w*}}{\widetilde{y}_{80}^{k*}}$	$\frac{\widetilde{y}_{\infty}^{w*}}{\widetilde{y}_{\infty}^{k*}}$
τ^{k*}	0.2659	0.2660	0.2661	0.2658	0.2656	0.2655	0.2653
τ^{n*}	0.2565	0.2592	0.2612	0.2624	0.2628	0.2630	0.2633
τ^{c*}	0.2613	0.2620	0.2627	0.2633	0.2635	0.2636	0.2638
s^{g*}	0.2648	0.2648	0.2647	0.2647	0.2647	0.2647	0.2646
s^{i*}	0.2613	0.2625	0.2639	0.2643	0.2644	0.2644	0.2644
s^{w*}	0.2620	0.2626	0.2633	0.2637	0.2639	0.2640	0.2641
SQ	0.2609	0.2609	0.2609	0.2609	0.2609	0.2609	0.2609

Notes:

(i) $\frac{\widetilde{y}_t^{w*}}{\widetilde{y}_t^{k*}}$ stands for the ratio of $\widetilde{y}_t^{w*}$ to $\widetilde{y}_t^{k*}$, where $\widetilde{y}_t^{w*}$ and $\widetilde{y}_t^{k*}$ denote the present value of the net income of the private worker and the capitalist respectively in the foreign economy for the next t periods after fiscal consolidation starts taking place.
(ii) Every row presents a different case depending on what instrument is used by the foreign country to bring debt down during the early period of fiscal pain.
(iii) In this case studied, we assume that the domestic country uses the fiscal space created by debt consolidation to cut the tax rate on labor and, during the early period of fiscal pain, rise the tax rate on capital to bring public debt down.

Table 23: Value of the $\frac{\widetilde{y}_t^{b*}}{\widetilde{y}_t^{k*}}$ over different time horizons (t) after debt consolidation starts taking place **when the foreign country uses the fiscal space created by debt consolidation to cut the tax rate on labor** (τ^{n*}).

Adj. Instr.	$\frac{\widetilde{y}_5^{b*}}{\widetilde{y}_5^{k*}}$	$\frac{\widetilde{y}_{10}^{b*}}{\widetilde{y}_{10}^{k*}}$	$\frac{\widetilde{y}_{20}^{b*}}{\widetilde{y}_{20}^{k*}}$	$\frac{\widetilde{y}_{40}^{b*}}{\widetilde{y}_{40}^{k*}}$	$\frac{\widetilde{y}_{60}^{b*}}{\widetilde{y}_{60}^{k*}}$	$\frac{\widetilde{y}_{80}^{b*}}{\widetilde{y}_{80}^{k*}}$	$\frac{\widetilde{y}_{\infty}^{b*}}{\widetilde{y}_{\infty}^{k*}}$
τ^{k*}	0.2143	0.2144	0.2144	0.2142	0.2140	0.2140	0.2138
τ^{n*}	0.2071	0.2092	0.2108	0.2116	0.2119	0.2121	0.2123
τ^{c*}	0.2106	0.2112	0.2117	0.2122	0.2124	0.2125	0.2126
s^{g*}	0.2134	0.2134	0.2133	0.2133	0.2133	0.2133	0.2133
s^{i*}	0.2106	0.2116	0.2127	0.2130	0.2131	0.2131	0.2131
s^{w*}	0.1886	0.1919	0.1964	0.2009	0.2031	0.2042	0.2056
SQ	0.2104	0.2104	0.2104	0.2104	0.2104	0.2104	0.2104

Notes:

(i) $\frac{\widetilde{y}_t^{b*}}{\widetilde{y}_t^{k*}}$ stands for the ratio of $\widetilde{y}_t^{b*}$ to $\widetilde{y}_t^{k*}$, where $\widetilde{y}_t^{b*}$ and $\widetilde{y}_t^{k*}$ denote the present value of the net income of the public employee and the capitalist respectively in the foreign economy for the next t periods after fiscal consolidation starts taking place.
(ii) Every row presents a different case depending on what instrument is used by the foreign country to bring debt down during the early period of fiscal pain.
(iii) In this case studied, we assume that the domestic country uses the fiscal space created by debt consolidation to cut the tax rate on labor and, during the early period of fiscal pain, rise the tax rate on capital to bring public debt down.

reduction over time. The emphasis was on the aggregate and distributional implications of debt consolidation, where income heterogeneity in each country, and hence distribution, had to do with the distinction between ”capitalists”, ”private workers” and ”public employees”. Since the results have already been written in the introduction, here I just mention some possible extensions. First, it would be interesting to examine what happens when policy is chosen optimally, with both cooperative and non-cooperative rules. Second, it would be interesting to allow for deviations from rational expectations (see e.g. Angeletos et al., 2018).

Appendix A Households as capitalists

This appendix provides details and the solution of domestic capitalists' optimization problem. Each domestic capitalist $k = 1, 2, ..., N^k$ solves an inter-temporal problem, in which he acts competitively to maximize discounted expected lifetime utility, and an intra-temporal one, in which he minimizes consumption expenditures.

A.1 Domestic capitalists' optimization problem

Inter-temporal problem: Each domestic capitalist k acts competitively to maximize discounted expected lifetime utility:

$$E_o \sum_{t=0}^{\infty} \beta^t U\left(c_t^k, n_t^k, m_t^k, y_t^g\right) \tag{98}$$

where c_t^k is k's consumption bundle at t as defined below in the intra-temporal problem, Eq.(105), n_t^k is k's hours of work at t, m_t^k is k's end-of-period real money balances at t, y_t^g are public goods and services at t divided by the number of domestic capitalists implying that the per capita public goods and services are defined as $v^k y_t^g$, E_o is the rational expectations operator conditional on the current period information set and $0 < \beta < 1$ is the time preference rate.

In our numerical solutions, we use a utility function of the form (see also e.g. Gali 2008):

$$U\left(c_t^k, n_t^k, m_t^k, y_t^g\right) = \left[\frac{(c_t^k)^{1-\sigma}}{1-\sigma} - x_n \frac{(n_t^k)^{1+\eta}}{1+\eta} + x_m \frac{(m_t^k)^{1-\mu}}{1-\mu} + x_g \frac{(v^k y_t^g)^{1-\zeta}}{1-\zeta}\right] \tag{99}$$

where x_n, x_m, x_g, σ, η, μ, ζ are standard preference parameters.

The budget constraint of each domestic capitalist k (written in real terms) is:

$$(1+\tau_t^c)c_t^k + \frac{P_t^H}{P_t}x_t^k + \frac{S_t P_t^*}{P_t}f_t^k + b_t^k + m_t^k = (1-\tau_t^k)\left[r_t^k \frac{P_t^H}{P_t}k_{t-1}^k + \widetilde{\omega_t}^k\right] + (1-\tau_t^n)w_t^k n_t^k + \tag{100}$$

$$+ Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}f_{t-1}^k + R_{t-1}\frac{P_{t-1}}{P_t}b_{t-1}^k +$$

$$+ \frac{P_{t-1}}{P_t}m_{t-1}^k - \tau_t^{l,k} + \widetilde{\pi_t}^k$$

where x_t^k is k's real private investment at t, f_t^k is the real value of k's end-of-period internationally traded assets at t denominated in foreign currency (if negative, it denotes foreign private debt), b_t^k is k's end-of-period real domestic government bonds at t, r_{t-1}^k is the gross real return to inherited private physical capital between $t-1$ and t, k_t^k is k's end-of-period private physical capital at t, $\widetilde{\omega_t}^k$ is k's real dividends paid by domestic private firms at t, w_t^k is domestic capitalists' real wage rate at t, Q_{t-1} is the gross nominal return to international assets between t-1 and t, $R_{t-1} \geq 1$ is the gross nominal return to domestic government bonds between $t-1$ and t, $\tau_t^{l,k}$ are real lump-sum taxes/transfers to each k from the government at t, $\widetilde{\pi_t}^k$ are profits distributed in a lump-sum fashion to each k by the financial intermediary (see below) at t, S_t is the nominal exchange rate (where an increase in S_t implies a depreciation), $0 \leq \tau_t^c \leq 1$ is the tax rate on consumption at t, $0 \leq \tau_t^k \leq 1$ is the tax rate on capital income at t, $0 \leq \tau_t^n \leq 1$ is the tax rate on labor income at t, P_t is the domestic consumer price index (CPI) and P_t^H is the price index of home tradables. Small letters denote real variables e.g. $f_t^k \equiv \frac{F_t^k}{P_t^*}$, $b_t^k \equiv \frac{B_t^k}{P_t}$, $\widetilde{\omega_t}^k \equiv \frac{\widetilde{\Omega_t}^k}{P_t}$, $\widetilde{\pi_t}^k \equiv \frac{\widetilde{\Pi_t}^k}{P_t}$, $w_t^k \equiv \frac{W_t^k}{P_t}$,$\tau_t^{l.k} \equiv \frac{T_t^{l,k}}{P_t}$. Also, letters with a star as superscript denote the counterpart of a variable in the rest-of-the world, e.g. P_t^* stands for the consumer price index (CPI) abroad.

The motion of private physical capital for each domestic capitalist k is:

$$k_t^k = (1-\delta)k_{t-1}^k + x_t^k - \frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k} - 1\right)^2 k_{t-1}^k \tag{101}$$

where $0 < \delta < 1$ is the depreciation rate of domestic private physical capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to domestic private physical capital.

Therefore, in the inter-temporal problem, each domestic capitalist k chooses $\{c_t^k, x_t^k, n_t^k, m_t^k, b_t^k, f_t^k, k_t^k\}_{t=0}^{\infty}$ to maximize Eqs.(98) and (99) subject to

Eqs.(100) and (101), by taking as given prices $\{r_t^k, w_t^k, Q_t, R_t, P_t, P_t^H, P_t^*\}_{t=0}^{\infty}$, dividends $\{\widetilde{\omega_t}^k\}_{t=0}^{\infty}$, profits $\{\widetilde{\pi_t}^k\}_{t=0}^{\infty}$, policy variables $\{S_t, \tau_t^c, \tau_t^n, \tau_t^k, \tau_t^{l,k}\}_{t=0}^{\infty}$, and initial conditions, $\{m_{-1}^k, b_{-1}^k, k_{-1}^k, f_{-1}^k\}$.

Intra-temporal problem: Each domestic capitalist k minimizes the following total consumption expenditure:

$$P_t c_t^k = P_t^H c_t^{k,H} + P_t^F c_t^{k,F} \tag{102}$$

where P_t^F is the price index of foreign tradables (expressed in domestic currency), $c_t^{k,H}$ is the composite domestic private good consisting of h varieties consumed by domestic capitalist k as defined below (see Eq.(106)) and $c_t^{k,F}$ is the composite imported private good consisting of f varieties consumed by domestic capitalist k as defined below(see Eq.(107)).

Each domestic capitalist k's total consumption expenditure is split into total expenditure on private home goods and private foreign goods respectively as follows:[39]

$$P_t^H c_t^{k,H} = \sum_{h=1}^{N^k} P_t^H(h) c_t^{k,H}(h) \tag{103}$$

$$P_t^F c_t^{k,F} = \sum_{f=1}^{N^{k^*}} P_t^F(f) c_t^{k,F}(f) \tag{104}$$

where $c_t^{k,H}(h)$ denotes the quantity of each variety h produced by domestic private firm h and consumed by each domestic capitalist k, $c_t^{k,F}(f)$ denotes the quantity of each imported variety f produced by foreign private firm f and consumed by each domestic capitalist k, $P_t^H(h)$ is the price of variety h produced at home and $P_t^F(f)$ is the price of variety f produced abroad expressed in domestic currency.

The consumption bundle of k is defined as:

$$c_t^k = \frac{\left(c_t^{k,H}\right)^v \left(c_t^{k,F}\right)^{1-v}}{v^v (1-v)^{1-v}} \tag{105}$$

where v is the degree of preference for domestic private goods (if $v > 1/2$, there is a home bias).

[39] Recall that, in Subsection 2.1 in the main text, we have assumed that the number of domestic firms(and, consequently, that of domestic varieties) equals that of domestic capitalists as well as that the number of foreign firms(and, consequently, of imported varieties) equals that of foreign capitalists.

Using a Dixit-Stiglitz aggregator, the composite domestic private good consumed by each k, $c_t^{k,H}$, consists of h varieties and is given by:[40]

$$c_t^{k,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{k,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{106}$$

where $\phi > 0$ is the elasticity of substitution across private good varieties produced in the domestic country.

Similarly, using a Dixit-Stiglitz aggregator, the composite imported private good consumed by each k, $c_t^{k,F}$, consists of f varieties and is given by:[41]

$$c_t^{k,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{k,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{107}$$

Therefore, in the intra-temporal problem, each domestic capitalist k chooses $\{c_t^{k,H}, c_t^{k,F}\}$ to minimize its total consumption expenditure, Eq.(102), subject to its consumption bundle, Eq.(105), by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^k. Next, each domestic capitalist k chooses $\{c_t^{k,H}(h), c_t^{k,F}(f)\}$ to minimize the sum of its consumption expenditure on private home goods and private foreign goods, sum of Eqs.(103) and (104), subject to the composite private domestic good and the composite private foreign good consisting of varieties, Eqs.(106) and (107), by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{k,H}$ and $c_t^{k,F}$.

A.2 Domestic capitalists' optimality conditions

Each domestic capitalist k acts competitively taking prices and policy as given.

Inter-temporal problem: The first order conditions include the k's budget constraint, Eq.(100), the law of motion of domestic private physical capital, Eq.(101), and:

[40] Recall that, in Subsection 2.1 in the main text, we have assumed that the number of domestic firms (and, consequently, of domestic varieties) equals that of domestic capitalists.

[41] Recall that, in Subsection 2.1 in the main text, we have assumed that the number of foreign firms (and, consequently, of imported varieties) equals that of foreign capitalists.

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}\frac{P_t^H}{P_t}\left[1+\xi\left(\frac{k_t^k}{k_{t-1}^k}-1\right)\right]=\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_{t+1}^H}{P_{t+1}}\times \tag{108}$$

$$\times\left[(1-\delta)+(1-\tau_{t+1}^k)r_{t+1}^k-\frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k}-1\right)^2+\xi\left(\frac{k_{t+1}^k}{k_t^k}-1\right)\frac{k_{t+1}^k}{k_t^k}\right]$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}S_t\frac{P_t^*}{P_t}=\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}Q_tS_{t+1}\frac{P_{t+1}^*}{P_{t+1}}\frac{P_t^*}{P_{t+1}^*} \tag{109}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}=\beta R_t\frac{P_t}{P_{t+1}}\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{110}$$

$$x_n(n_t^k)^{\eta}=(c_t^k)^{-\sigma}\frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^k \tag{111}$$

$$x_m(m_t^k)^{-\mu}=\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}-\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_t}{P_{t+1}} \tag{112}$$

Eqs.(108), (109) and (110) are respectively the Euler equations of domestic private physical capital, internationally traded assets and domestic government bonds, Eq.(111) is the optimality condition for work hours and Eq.(112) is the optimality condition for real money balances.

Intra-temporal problem: The first order conditions include the consumption bundle of k, Eq.(105), and:

$$\frac{c_t^{k,H}}{c_t^{k,F}}=\frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{113}$$

$$c_t^{k,H}(h)=\frac{c_t^{k,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{114}$$

$$c_t^{k,F}(f)=\frac{c_t^{k,F}}{N^{k*}}\left(\frac{P_t^F}{P_t^F(f)}\right)^{\phi} \tag{115}$$

Eq.(113) is the optimality condition for sharing the total consumption between domestic and imported private goods, Eqs.(114) and (115) are demand equations of domestic capitalist for varieties produced at home and abroad respectively.

Plugging Eqs.(114) and (115) into Eqs.(106) and (107) respectively, we get the following relations for price indexes:

$$P_t^H = \left\{ \sum_{h=1}^{N^k} \frac{1}{N^k} [P_t^H(h)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{116}$$

$$P_t^F = \left\{ \sum_{f=1}^{N^{k*}} \frac{1}{N^{k*}} [P_t^F(f)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{117}$$

Yet, Eqs.(102), (105) and (113) imply the following relation for consumer price index(CPI):

$$P_t = (P_t^H)^v (P_t^F)^{1-v} \tag{118}$$

Appendix B Households as private workers

This appendix provides details and the solution of domestic private worker w's optimization problem. Each domestic private worker $w = 1, 2, ..., N^w$ solves an inter-temporal problem, in which he acts competitively to maximize discounted expected lifetime utility, and an intra-temporal one, in which he minimizes consumption expenditures.

B.1 Domestic private workers' problem

Inter-temporal problem: Domestic private workers have the same utility function as domestic capitalists (see Eqs.(98) and (99)). Each domestic private worker w acts competitively to maximize discounted expected lifetime utility taking prices and policy as given.

The budget constraint of each domestic private worker w is in real terms:

$$(1 + \tau_t^c) c_t^w + m_t^w = (1 - \tau_t^n) w_t^w n_t^w + \frac{P_{t-1}}{P_t} m_{t-1}^w - \tau_t^{l,w} \tag{119}$$

where c_t^w is w's consumption bundle at t as defined below in the intra-temporal problem, Eq.(123), m_t^w is w's end-of-period real money balances at t, n_t^w is w's hours of work at t, w_t^w is domestic private workers' real wage rate at t and $\tau_t^{l,w}$ are real lump-sum taxes/transfers to each w from the government at t. Again small letters denote real variables, e.g. $w_t^w \equiv \frac{W_t^w}{P_t}$, $\tau_t^{l,w} \equiv \frac{T_t^{l,w}}{P_t}$.

Therefore, in the inter-temporal problem, each domestic private worker w chooses $\{c_t^w, n_t^w, m_t^w\}_{t=0}^{\infty}$ to maximize Eqs.(98) and (99) for w, subject to the budget constraint, Eq.(119), by taking as given prices $\{w_t^w, P_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,w}\}_{t=0}^{\infty}$, and initial conditions, m_{-1}^w.

Intra-temporal problem: Each domestic private worker w minimizes the following total consumption expenditure:

$$P_t c_t^w = P_t^H c_t^{w,H} + P_t^F c_t^{w,F} \tag{120}$$

where $c_t^{w,H}$ is the composite domestic private good consisting of h varieties consumed by domestic private worker w as defined below (see Eq.(124)) and $c_t^{w,F}$ is the composite imported private good consisting of f varieties consumed by domestic private worker w as defined below (see Eq.(125)).

Each domestic private worker w's total consumption expenditure is split into total expenditure on private home goods and private foreign goods respectively as follows:[42]

$$P_t^H c_t^{w,H} = \sum_{h=1}^{N^k} P_t^H(h) c_t^{w,H}(h) \tag{121}$$

$$P_t^F c_t^{w,F} = \sum_{f=1}^{N^{k*}} P_t^F(f) c_t^{w,F}(f) \tag{122}$$

where $c_t^{w,H}(h)$ denotes the quantity of each variety h produced by domestic private firm h and consumed by each domestic private worker w and $c_t^{w,F}(f)$ denotes the quantity of each imported variety f produced by foreign private firm f and consumed by each domestic private worker w.

The consumption bundle of w is defined as:

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v (1-v)^{1-v}} \tag{123}$$

[42]Recall that, in Subsection 2.1 in the main text, we have assumed that the number of domestic firms(and, consequently, that of domestic varieties) equals that of domestic capitalists as well as that the number of foreign firms(and, consequently, of imported varieties) equals that of foreign capitalists.

Using a Dixit-Stiglitz aggregator, the composite domestic private good consumed by each w, $c_t^{w,H}$, consists of h varieties and is given by:[43]

$$c_t^{w,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{w,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{124}$$

Similarly, using a Dixit-Stiglitz aggregator, the composite imported private good consumed by each w, $c_t^{w,F}$, consists of f varieties and is given by:[44]

$$c_t^{w,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{w,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{125}$$

Therefore, in the intra-temporal problem, each domestic private worker w chooses $\{c_t^{w,H}, c_t^{w,F}\}$ to minimize its total consumption expenditure, Eq.(120), subject to its consumption bundle, Eq.(123), by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^w. Next, each domestic private worker w chooses $\{c_t^{w,H}(h), c_t^{w,F}(f)\}$ to minimize the sum of its consumption expenditure on private home goods and private foreign goods, sum of Eqs.(121) and (122), subject to the composite domestic private good and the composite foreign private good consisting of varieties, Eqs.(124) and (125), by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{w,H}$ and $c_t^{w,F}$.

B.2 Domestic private workers' optimality conditions

Each domestic private worker w acts competitively taking as given prices and policy.

Inter-temporal problem: The first order conditions include the w's budget constraint, Eq.(119), and:

$$\frac{(c_t^w)^{-\sigma}}{x_n (n_t^w)^{\eta}} = \frac{1+\tau_t^c}{(1-\tau_t^n) w_t^w} \tag{126}$$

$$\frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} = \beta \frac{P_t}{P_{t+1}} \left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m (m_t^w)^{-\mu} \tag{127}$$

[43]Recall that, in Subsection 2.1 in the main text, we have assumed that the number of domestic firms (and, consequently, of domestic varieties) equals that of domestic capitalists.
[44]Recall that, in Subsection 2.1 in the main text, we have assumed that the number of foreign firms (and, consequently, of imported varieties) equals that of foreign capitalists.

Eqs.(126) and (127) are the optimality conditions for work hours and real money balances respectively.

Intra-temporal problem: The first order conditions include the consumption bundle of w, Eq.(123), and:

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{128}$$

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{129}$$

$$c_t^{w,F}(f) = \frac{c_t^{w,F}}{N^{k^*}}\left(\frac{P_t^F}{P_t^F(f)}\right)^{\phi} \tag{130}$$

Eq.(128) is the optimality condition for sharing the total consumption between domestic and imported private goods, Eqs.(129) and (130) are demand equations of domestic private worker for varieties of private goods produced at home and abroad respectively.

Plugging Eqs.(129) and (130) into Eqs.(124) and (125) respectively, we get the following relations for price indexes:

$$P_t^H = \left\{\sum_{h=1}^{N^k}\frac{1}{N^k}[P_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{131}$$

$$P_t^F = \left\{\sum_{f=1}^{N^{k^*}}\frac{1}{N^{k^*}}[P_t^F(f)]^{1-\phi}\right\}^{\frac{1}{1-\phi}} \tag{132}$$

which, as expected, coincide with Eqs.(116) and (117) respectively derived from domestic capitalist's solution.

Yet, Eqs.(120), (123) and (128) imply the following relation for consumer price index(CPI):

$$P_t = (P_t^H)^v(P_t^F)^{1-v} \tag{133}$$

which, as expected, coincides with Eq.(118) derived from domestic capitalist's solution.

Appendix C Households as public employees

This appendix provides details and the solution of domestic public employees' optimization problem. Each domestic public employee $b = 1, 2, ..., N^b$ solves an inter-temporal problem, in which he acts competitively to maximize discounted expected lifetime utility, and an intra-temporal one, in which he minimizes consumption expenditures.

C.1 Domestic public employees' optimization problem

Inter-temporal problem: Domestic public employees have the same utility function as domestic capitalists (see Eqs.(98) and (99)). Each domestic public employee b acts competitively to maximize discounted expected lifetime utility taking prices and policy as given.

The budget constraint of each domestic public employee b in real terms is:

$$(1 + \tau_t^c)c_t^b + m_t^b = (1 - \tau_t^n)w_t^b n_t^b + \frac{P_{t-1}}{P_t} m_{t-1}^b - \tau_t^{l,b} \tag{134}$$

where c_t^b is b's consumption bundle at t as defined below in the intra-temporal problem below, $Eq.$(139), m_t^b is b's end-of-period real money balances at t, n_t^b is b's hours of work at t, w_t^b is public employees' real wage rate at t and $\tau_t^{l,b}$ are real lump-sum taxes/transfers to each b from the government at t. Again small letters denote real variables, e.g. $w_t^b \equiv \frac{W_t^b}{P_t}$, $\tau_t^{l,b} \equiv \frac{T_t^{l,b}}{P_t}$.

Assuming that the domestic government exogeneously determines the total domestic public wage bill in real terms divided by the number of domestic capitalists, defined as $\overline{g}_t^w = \frac{v^b}{v^k} w_t^b n_t^b$, we can rewrite the budget constraint of b as follows:

$$(1 + \tau_t^c)c_t^b + m_t^b = (1 - \tau_t^n)\frac{v^k}{v^b}\overline{g}_t^w + \frac{P_{t-1}}{P_t} m_{t-1}^b - \tau_t^{l,b} \tag{135}$$

Therefore, in the inter-temporal problem, each domestic public employee b chooses $\{c_t^b, m_t^b\}_{t=0}^{\infty}$ to maximize Eqs.(98) and (99) for b, subject to its budget constraint, Eq.(135), by taking as given prices $\{P_t\}_{t=0}^{\infty}$, policy variables $\{\tau_t^c, \tau_t^n, \tau_t^{l,b}, \overline{g}_t^w\}_{t=0}^{\infty}$, and initial conditions, m_{-1}^b.

Intra-temporal problem: Each b minimizes the following total consumption expenditure:

$$P_t c_t^b = P_t^H c_t^{b,H} + P_t^F c_t^{b,F} \tag{136}$$

where $c_t^{b,H}$ is the composite domestic private good consisting of h varieties consumed by domestic public employee b as defined below (see Eq.(140)) and $c_t^{b,F}$ is the composite imported private good consisting of f varieties consumed by domestic public employee b as defined below (see Eq.(141)).

Each domestic public employee b's total consumption expenditure is split into total expenditure on private home goods and private foreign goods respectively as follows:[45]

$$P_t^H c_t^{b,H} = \sum_{h=1}^{N^k} P_t^H(h) c_t^{b,H}(h) \tag{137}$$

$$P_t^F c_t^{b,F} = \sum_{f=1}^{N^{k^*}} P_t^F(f) c_t^{b,F}(f) \tag{138}$$

where $c_t^{b,H}(h)$ denotes the quantity of each variety h produced by domestic private firm h and consumed by each domestic public employee b and $c_t^{b,F}(f)$ denotes the quantity of each imported variety f produced by foreign private firm f and consumed by each domestic public employee b.

The consumption bundle of b is defined as:

$$c_t^b = \frac{\left(c_t^{b,H}\right)^v \left(c_t^{b,F}\right)^{1-v}}{v^v (1-v)^{1-v}} \tag{139}$$

Using a Dixit-Stiglitz aggregator, the composite domestic private good consumed by each b, $c_t^{b,H}$, consists of h varieties and is given by:[46]

$$c_t^{b,H} = \left[\sum_{h=1}^{N^k} \left(\frac{1}{N^k} \right)^{\frac{1}{\phi}} [c_t^{b,H}(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{140}$$

Similarly, using a Dixit-Stiglitz aggregator, the composite imported private good consumed by each b, $c_t^{b,F}$, consists of f varieties and is given by:[47]

[45]Recall that, in Subsection 2.1 in the main text, we have assumed that the number of domestic firms(and, consequently, that of domestic varieties) equals that of domestic capitalists as well as that the number of foreign firms(and, consequently, of imported varieties) equals that of foreign capitalists.

[46]Recall that, in Subsection 2.1 in the main text, we have assumed that the number of domestic firms (and, consequently, of domestic varieties) equals that of domestic capitalists.

[47]Recall that, in Subsection 2.1 in the main text, we have assumed that the number of foreign firms (and, consequently, of imported varieties) equals that of foreign capitalists.

$$c_t^{b,F} = \left[\sum_{f=1}^{N^{k^*}} \left(\frac{1}{N^{k^*}} \right)^{\frac{1}{\phi}} [c_t^{b,F}(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{141}$$

Therefore, in the intra-temporal problem, each domestic public employee b chooses $\{c_t^{b,H}, c_t^{b,F}\}$ to minimize its total consumption expenditure, Eq.(136), subject to its consumption bundle, Eq.(139), by taking as given prices, $\{P_t^H, P_t^F\}$, and consumption bundle, c_t^b. Next, each domestic public employee b chooses $\{c_t^{b,H}(h), c_t^{b,F}(f)\}$ to minimize the sum of its consumption expenditure on private home goods and private foreign goods, sum of Eqs.(137) and (138), subject to the composite domestic private good and the composite foreign private good consisting of varieties, Eqs.(140) and (141), by taking as given prices, $\{P_t^H(h), P_t^F(f)\}$, and consumption bundles, $c_t^{b,H}$ and $c_t^{b,F}$.

C.2 Domestic public employees' optimality conditions

Each domestic public employee b acts competitively taking as given prices and policy.

Inter-temporal problem: The first order conditions include the b's budget constraint, Eq.(135), and:

$$\frac{(c_t^b)^{-\sigma}}{1+\tau_t^c} = \beta \frac{P_t}{P_{t+1}} \left[\frac{(c_{t+1}^b)^{-\sigma}}{1+\tau_{t+1}^c} \right] + x_m (m_t^b)^{-\mu} \tag{142}$$

Eq.(142) is the optimality condition for real money balances.

Intra-temporal problem: The first order conditions include the consumption bundle of b, Eq.(139), and:

$$\frac{c_t^{b,H}}{c_t^{b,F}} = \frac{v}{1-v} \frac{P_t^F}{P_t^H} \tag{143}$$

$$c_t^{b,H}(h) = \frac{c_t^{b,H}}{N^k} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{144}$$

$$c_t^{b,F}(f) = \frac{c_t^{b,F}}{N^{k^*}} \left(\frac{P_t^F}{P_t^F(f)} \right)^{\phi} \tag{145}$$

Eq.(143) is the optimality condition for sharing the total consumption between domestic and imported private products, Eqs.(144) and (145) are demand equations of public employee for varieties of private goods produced at home and abroad respectively.

Plugging Eqs.(144) and (145) into Eqs.(140) and (141) respectively, we get the following relations for price indexes:

$$P_t^H = \left\{ \sum_{h=1}^{N^k} \frac{1}{N^k} [P_t^H(h)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{146}$$

$$P_t^F = \left\{ \sum_{f=1}^{N^{k^*}} \frac{1}{N^{k^*}} [P_t^F(f)]^{1-\phi} \right\}^{\frac{1}{1-\phi}} \tag{147}$$

which, as expected, coincide with Eqs.(116) and (117) respectively derived from domestic capitalist's solution (as well as coincide with Eqs.(131) and (132) respectively derived from domestic private worker's solution).

Yet, Eqs.(136), (139) and (143) imply the following relation for consumer price index(CPI):

$$P_t = (P_t^H)^v (P_t^F)^{1-v} \tag{148}$$

which, as expected, coincides with Eq.(118) derived from domestic capitalist's solution (and also coincides with Eq.(133) derived from domestic private worker's solution).

Appendix D Domestic private firms

This appendix provides details and the solution of private firms' optimization problem in the domestic country. There are $h = 1, 2, ..., N^k$ domestic private firms. Each firm h produces a differentiated good of variety h under monopolistic competition and Rotemberg-type nominal price rigidities (see Bi et al., 2013).

D.1 Demand for the domestic private firm h's product

Each domestic private firm h faces demand for its product, $y_t^{H,d}(h)$. The latter comes from domestic households' private consumption and investment, $c_t^H(h)$ and $x_t(h)$, where $c_t^H(h) \equiv \sum\limits_{k=1}^{N^k} c_t^{k,H}(h) + \sum\limits_{w=1}^{N^w} c_t^{w,H}(h) + \sum\limits_{b=1}^{N^b} c_t^{b,H}(h)$

and $x_t(h) \equiv \sum_{k=1}^{N^k} x_t^k(h)$, from the domestic state-owned enterprise's use of private goods as inputs in its production function, denoted as $g_t^c(h)$, from the domestic government's investment, $g_t^i(h)$, from the financial intermediary which is located in the domestic country, denoted as $\widetilde{\upsilon}_t(h)$,[48] and from foreign households' consumption of the domestic private goods, $c_t^{F^*}(h)$, where $c_t^{F^*}(h) \equiv \sum_{k^*=1}^{N^{k^*}} c_t^{k,F^*}(h) + \sum_{w^*=1}^{N^{w^*}} c_t^{w,F^*}(h) + \sum_{b^*=1}^{N^{b^*}} c_t^{b,F^*}(h)$, with a star, again, we denote the counterpart of a variable in the foreign country. Thus, aggregate demand for each variety h is:

$$y_t^{H,d}(h) = \left[c_t^H(h) + x_t(h) + g_t^c(h) + g_t^i(h) + \widetilde{\upsilon}_t(h) + c_t^{F^*}(h)\right] \tag{149}$$

Aggregate demand for each variety h is associated with production of domestic private firm h according to the following relation:

$$y_t^{H,d}(h) = y_t^H(h) \times \left\{1 - \frac{\phi^P}{2}\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right]^2 \frac{y_t^H}{y_t^H(h)}\right\} \tag{150}$$

where $y_t^H(h)$ stands for the production of domestic private firm h, y_t^H stands for the total private output in the domestic country divided by the number of domestic capitalists, π^H stands for the steady state value of the gross domestic goods inflation rate and $\phi^P \geq 0$ is a parameter which determines the degree of nominal price rigidity. The term in the brackets captures the Rotemberg-type pricing cost and reflects the discrepancy between production and demand as one expected in a Rotemberg-type fashion.

Since we have:

$$c_t^{k,H}(h) = \frac{c_t^{k,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{151}$$

$$c_t^{w,H}(h) = \frac{c_t^{w,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{152}$$

$$c_t^{b,H}(h) = \frac{c_t^{b,H}}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{153}$$

[48] See also e.g.Cúrdia and Woodford, 2010 and 2011, for a similar modelling where any resources consumed by banks for monitoring of financial operations will be part of the aggregate demand; the latter are modelled below. That is, the model requires banks use real resources in the period in which the loan is originated.

$$x_t^k(h) = \frac{x_t^k}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{154}$$

$$g_t^c(h) = \frac{N^k\overline{g}_t^c}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{155}$$

$$g_t^i(h) = \frac{N^k\overline{g}_t^i}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{156}$$

$$\widetilde{\upsilon}_t(h) = \frac{\widetilde{\upsilon}_t}{N^k}\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{157}$$

$$c_t^{k,F^*}(h) = \frac{c_t^{k,F^*}}{N^{k^*}}\left(\frac{P_t^{F^*}}{P_t^{F^*}(h)}\right)^{\phi} \tag{158}$$

$$c_t^{w,F^*}(h) = \frac{c_t^{w,F^*}}{N^{k^*}}\left(\frac{P_t^{F^*}}{P_t^{F^*}(h)}\right)^{\phi} \tag{159}$$

$$c_t^{b,F^*}(h) = \frac{c_t^{b,F^*}}{N^{k^*}}\left(\frac{P_t^{F^*}}{P_t^{F^*}(h)}\right)^{\phi} \tag{160}$$

we can rewrite the relation (149) as:

$$y_t^{H,d}(h) = \frac{1}{N^k}\left[c_t^H + x_t + N^k\overline{g}_t^c + N^k\overline{g}_t^i + \widetilde{\upsilon}_t + c_t^{F^*}\right] \times \left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi} \tag{161}$$

where $c_t^H \equiv \sum\limits_{k=1}^{N^k} c_t^{k,H} + \sum\limits_{w=1}^{N^w} c_t^{w,H} + \sum\limits_{b=1}^{N^b} c_t^{b,H}$ is total consumption of domestic private home goods by domestic households, $x_t \equiv \sum\limits_{k=1}^{N^k} x_t^k$ is total domestic private investment, $N^k\overline{g}_t^c$ denotes total domestic goods and services of private sector that are used by the domestic state-owned enterprise for the production of total domestic public goods and services, $N^k\overline{g}_t^i$ denotes domestic public infrastructure investment, $\widetilde{\upsilon}_t$ denotes total resources consumed by the financial intermediary and $c_t^{F^*} \equiv \sum\limits_{k^*=1}^{N^{k^*}} c_t^{k,F^*} + \sum\limits_{w^*=1}^{N^{w^*}} c_t^{w,F^*} + \sum\limits_{b^*=1}^{N^{b^*}} c_t^{b,F^*}$ is total consumption of domestic private goods by foreign households (i.e. domestic country's exports). Also notice that the law of one price implies that in Eqs.(158), (159) and (160):

$$\frac{P_t^{F^*}}{P_t^{F^*}(h)} = \frac{\frac{P_t^H}{S_t}}{\frac{P_t^H(h)}{S_t}} = \frac{P_t^H}{P_t^H(h)} \tag{162}$$

and recall that the number of domestic capitalists equals that of foreign capitalists(see Subsection 2.1 in the main text).

Since domestic aggregate demand, $N^k y_t^{H,d}$, is:

$$N^k y_t^{H,d} = \left[c_t^H + x_t + N^k \overline{g}_t^c + N^k \overline{g}_t^i + \widetilde{\upsilon}_t + c_t^{F^*} \right] \tag{163}$$

then domestic aggregate demand for each variety h is rewritten as:

$$y_t^{H,d}(h) = y_t^{H,d} \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{164}$$

Another equivalent expression of demand for each variety h in terms of private production follows:

$$y_t^H(h) = y_t^H \left(\frac{P_t^H}{P_t^H(h)} \right)^{\phi} \tag{165}$$

where

$$y_t^{H,d} \equiv y_t^H \times \left\{ 1 - \frac{\phi^P}{2} \left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1 \right]^2 \frac{y_t^H}{y_t^H(h)} \right\}$$

Notice that in the domestic private firm h's optimization problem below we should use Eq.(164) as an expression for demand of each variety h. However, it is more convenient for someone to work with Eq.(165) instead of Eq.(164).

D.1.1 Domestic private firms' optimization problem

Nominal profits of each domestic private firm h are defined as:

$$P_t \widetilde{\omega}_t(h) = P_t^H(h) y_t^H(h) - P_t^H r_t^k k_{t-1}(h) - W_t^w n_t^w(h) - W_t^k n_t^k(h) - \frac{\phi^P}{2} \left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1 \right)^2 P_t^H y_t^H \tag{166}$$

where $k_{t-1}(h)$ denotes the private physical capital input chosen by domestic private firm h, $n_t^w(h)$ denotes domestic private workers' labor input chosen by domestic private firm h and $n_t^k(h)$ denotes the domestic capitalists' labor input chosen by domestic private firm h. The quadratic cost that the domestic private firm h faces once it changes the price of its product is proportional to the aggregate domestic private output divided by the number of domestic private firms (which is equal to the number of domestic capitalists as we have said in Subsection 2.1 in the main text).[49]

All domestic private firms use the same technology represented by the production function(similar to e.g. Hornstein et al., 2005, and Baxter and King, 1993):

$$y_t^H(h) = A_t \left\{ [k_{t-1}(h)]^{\alpha} \left[\{n_t^k(h)\}^{\theta} \{n_t^w(h)^{1-\theta}\} \right]^{1-\alpha} \right\} \left(k_{t-1}^g \right)^{\theta_k} \tag{167}$$

where A_t is an exogenous TFP, k_{t-1}^g denotes the stock of domestic public infrastructure divided by the number of domestic capitalists which is common for all domestic private firms, $0 < \alpha < 1$ is the share of domestic private physical capital, $0 < \theta_k < 1$ is the output elasticity of domestic public infrastructure for domestic private firm h and $0 < \theta < 1$ the labor efficiency parameter of domestic capitalist. We assume a positive θ_k, which implies that the production function has increasing returns to scale with respect to all inputs, as in Baxter and King, 1993. Notice that we keep CRS over private inputs.

Profit maximization of domestic private firm h is also subject to the demand for its product as derived above, Eq.(164). But as we have mentioned before, instead of using Eq.(164), we can equivalently use Eq.(165).

Each domestic private firm h chooses its price, $P_t^H(h)$, and its inputs, $k_t(h)$, $n_t^k(h)$, $n_t^w(h)$, to maximize discounted expected lifetime real dividends, $\max E_o \sum\limits_{t=0}^{\infty} \Xi_{0,0+t} \widetilde{\omega_t}(h)$, subject to Eq.(165) and its production function, Eq.(167). The objective function of h in real terms is given by:

$$\max E_o \sum_{t=0}^{\infty} \Xi_{0,0+t} \left[\frac{P_t^H(h)}{P_t} y_t^H(h) - \frac{P_t^H}{P_t} r_t^k k_{t-1}(h) - w_t^w n_t^w(h) - w_t^k n_t^k(h) - \frac{\phi^P}{2} \left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1 \right)^2 \frac{P_t^H y_t^H}{P_t} \right] \tag{168}$$

[49]This specification of Rotemberg-type cost is similar to that of Bi et al., 2013. Here, working with summations instead of integrals, we should have a pricing cost which is proportional to the aggregate domestic private output divided by the number of private firms. With this modification, we can derive the same NK Philips curve as Bi at al, 2013.

where $\Xi_{0,0+t}$ is a stochastic discount factor taken as given by the domestic private firm h. This is defined as $\Xi_{0,0+t} = \prod_{i=0}^{t-1} \left\{ \frac{1}{R_i} \right\} = \beta^t \prod_{i=0}^{t-1} \left[\left(\frac{P_i}{P_{i+1}} \right) \left(\frac{1+\tau_i^c}{1+\tau_{i+1}^c} \right) \left(\frac{c_{i+1}^k}{c_i^k} \right)^{-\sigma} \right]$ and arises from the Euler of domestic government bonds.

D.1.2 Domestic private firms' optimality conditions

Following the related literature, instead of solving the above problem, we follow a two-step procedure. We first solve a cost minimization problem, where each domestic private firm h minimizes its cost by choosing factors of production given technology and prices. The solution will give a minimum real cost function, which is a function of factor prices and output produced by the domestic private firm. In turn, given this cost function, we solve the dynamic profit maximization problem of each domestic private firm h by choosing its price.

Cost minimization problem: In the first stage, we solve a static cost minimization problem, where each h minimizes its cost by choosing its factors of production, $k_t(h)$, $n_t^k(h)$, $n_t^w(h)$, subject to its production function, Eq.(167), given technology and prices. The cost function is defined in real terms as follows:

$$\min \widetilde{\psi} = \left[\frac{P_t^H}{P_t} r_t^k k_{t-1}(h) + w_t^w n_t^w(h) + w_t^k n_t^k(h) \right] \tag{169}$$

The solution to the cost minimization problem gives the following input demand functions:

$$\frac{P_t^H}{P_t} r_t^k k_{t-1}(h) = mc_t \alpha y_t^H(h) \tag{170}$$

$$w_t^k n_t^k(h) = mc_t \theta (1-\alpha) y_t^H(h) \tag{171}$$

$$w_t^w n_t^w(h) = mc_t (1-\theta)(1-\alpha) y_t^H(h) \tag{172}$$

where $mc_t \equiv \widetilde{\psi}'(y_t^H(h))$, since, by definition, the real marginal cost is the derivative of the associated minimum real cost function, $\widetilde{\psi}(y_t^H(h))$, with respect to $y_t^H(h)$.

Summing up the three above equations, it arises the following relation for the associated minimum cost function of h in real terms:

$$\widetilde{\psi}(y_t^H(h)) = mc_t y_t^H(h) \tag{173}$$

Where the real marginal cost, mc_t, it can be shown that equals:

$$mc_t = \frac{1}{A_t(k_{t-1}^g)^{\theta_k}}\left[\frac{P_t^H}{P_t}\frac{r_t^k}{\alpha}\right]^{\alpha}\left[\left\{\frac{w_t^k}{\theta(1-\alpha)}\right\}^{\theta}\times\left\{\frac{w_t^w}{(1-\theta)(1-\alpha)}\right\}^{1-\theta}\right]^{1-\alpha} \tag{174}$$

implying that mc_t is common for all domestic private firms since it only depends on prices, parameters, the stock of domestic public infrastructure divided by the number of domestic capitalists and technology which are common for all domestic private firms.

Profit maximization: The solution to the cost minimization problem will give a minimum real cost function, Eq.(173), which is a function of prices and output produced by the domestic private firm. In turn, given this cost function, we solve the dynamic maximization problem of h by choosing its price. Specifically, in the second stage, h chooses its price, $P_t^H(h)$, to maximize the discounted expected lifetime real profits:

$$\max E_o\sum_{t=0}^{\infty}\Xi_{0,0+t}\left[\frac{P_t^H(h)}{P_t}y_t^H(h)-\widetilde{\psi}(y_t^H(h))-\frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right)^2\frac{P_t^H y_t^H}{P_t}\right] \tag{175}$$

The above profit maximization is subject to the Eq.(165), which is equivalent to the demand equation that the monopolistically competitive domestic private firm h faces, Eq.(164).

The first order condition gives:

$$(1-\phi)\frac{P_t^H(h)}{P_t}y_t^H(h)+\phi mc_t y_t^H(h)-\phi^P\left[\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H}-1\right]\frac{P_t^H}{P_t}\frac{y_t^H P_t^H(h)}{P_{t-1}^H(h)\pi^H} =$$
$$\beta\phi^P\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1-\frac{P_{t+1}^H(h)}{P_t^H(h)\pi^H}\right]\frac{P_{t+1}^H(h)}{P_t^H(h)\pi^H}\frac{P_{t+1}^H}{P_{t+1}}y_{t+1}^H \tag{176}$$

Thus, the behavior of h is summarized by Eqs.(170), (171), (172) and (176).

Since all domestic private firms solve the identical problem, they all set the same price, $P_t^H(h)$, which implies through the Eq.(116) (or the identical Eqs.(131) and (146)) that $P_t^H(h) = P_t^H$.

Appendix E Government budget constraint

This Appendix presents the government budget constraint in some detail. We start by presenting the domestic government's budget constraint in nominal terms:

$$
\begin{aligned}
&N^{k*}Q_{t-1}S_tF^g_{t-1} + N^kR_{t-1}B_{t-1} + N^kP^H_t\overline{g}^c_t + N^kP^H_t\overline{g}^i_t + N^kP_t\overline{g}^w_t + N^kM_{t-1} = \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (177)\\
&= N^kM_t + \tau^c_t\left[P^H_tc^H_t + P^F_tc^F_t\right] + \tau^k_t\left[N^kr^k_tP^H_tk^k_{t-1} + N^kP_t\widetilde{\omega}^k_t\right] + \\
&+ \tau^n_t\left[N^kW^k_tn^k_t + N^wW^w_tn^w_t + N^kP_t\overline{g}^w_t\right] + \\
&+ \left[N^kT^{l,k}_t + N^wT^{l,w}_t + N^bT^{l,b}_t\right] + N^kB_t + N^{k*}S_tF^g_t
\end{aligned}
$$

where F^g_t is the end-of-period domestic nominal public debt held by each foreign agent at t and expressed in foreign currency, B_t is the end-of-period domestic nominal public debt held by each domestic capitalist at t, $\overline{g}^w_t$ is the total domestic public wage bill in real terms at t divided by the number of domestic capitalists, M_t is the end-of-period stock of nominal money balances at t divided by the number of domestic capitalists and $c^F_t \equiv \sum\limits_{k=1}^{N^k} c^{k,F}_t + \sum\limits_{w=1}^{N^w} c^{w,F}_t + \sum\limits_{b=1}^{N^b} c^{b,F}_t$. The rest of the variables have been defined above.

Then, dividing by the current domestic CPI, P_t, and the constant number of domestic capitalists, N^k, we get the government budget constraint in real terms:[50]

$$
\begin{aligned}
&Q_{t-1}\frac{S_tP^*_t}{P_t}\frac{P^*_{t-1}}{P^*_t}f^g_{t-1} + R_{t-1}\frac{P_{t-1}}{P_t}b_{t-1} + \frac{P^H_t}{P_t}\overline{g}^c_t + \frac{P^H_t}{P_t}\overline{g}^i_t + \overline{g}^w_t + \frac{P_{t-1}}{P_t}m_{t-1} = \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (178)\\
&= m_t + \tau^c_t\left[\frac{P^H_t}{P_t}\left(c^{k,H}_t + \frac{v^w}{v^k}c^{w,H}_t + \frac{v^b}{v^k}c^{b,H}_t\right) + \frac{P^F_t}{P_t}\left(c^{k,F}_t + \frac{v^w}{v^k}c^{w,F}_t + \frac{v^b}{v^k}c^{b,F}_t\right)\right] + \tau^k_t\left[r^k_t\frac{P^H_t}{P_t}k^k_{t-1} + \widetilde{\omega}^k_t\right] + \\
&\qquad + \tau^n_t\left[w^k_tn^k_t + \frac{v^w}{v^k}w^w_tn^w_t + \overline{g}^w_t\right] + \tau^l_t + b_t + S_t\frac{P^*_t}{P_t}f^g_t
\end{aligned}
$$

where small letters denote real values e.g. $f^g_t \equiv \frac{F^g_t}{P^*_t}$, $b_t \equiv \frac{B_t}{P_t}$, $m_t \equiv \frac{M_t}{P_t}$ and $\tau^l_t \equiv \left[\tau^{l,k}_t + \frac{v^w}{v^k}\tau^{l,w}_t + \frac{v^b}{v^k}\tau^{l,b}_t\right]$. All other variables have been defined above.

[50] I have aggregated over all agents, divided by the total number of agents and, in turn, divided all terms by v^k.

For convenience, let $N^k D_t \equiv N^k B_t + N^{k*} S_t F_t^g$ denote the total nominal public debt issued by the domestic government. This debt can be held either by a domestic private agent(capitalist), $B_t \equiv \lambda_t D_t$, or by a foreign private agent(capitalist),[51] $S_t F_t^g \equiv (1-\lambda_t) D_t$, where $0 \leq \lambda_t \leq 1$.[52] Then, the above government budget constraint is rewritten as:

$$Q_{t-1} \frac{S_t P_t^*}{P_t} \frac{P_{t-1}^*}{P_t^*} \frac{P_{t-1}}{S_{t-1} P_{t-1}^*} (1-\lambda_{t-1}) d_{t-1} + R_{t-1} \frac{P_{t-1}}{P_t} \lambda_{t-1} d_{t-1} + \frac{P_t^H}{P_t} \overline{g}_t^c + \frac{P_t^H}{P_t} \overline{g}_t^i + \overline{g}_t^w + \frac{P_{t-1}}{P_t} m_{t-1} = \tag{179}$$

$$= m_t + \tau_t^c \left[\frac{P_t^H}{P_t} \left(c_t^{k,H} + \frac{v^w}{v^k} c_t^{w,H} + \frac{v^b}{v^k} c_t^{b,H} \right) + \frac{P_t^F}{P_t} \left(c_t^{k,F} + \frac{v^w}{v^k} c_t^{w,F} + \frac{v^b}{v^k} c_t^{b,F} \right) \right] + \tau_t^k \left[r_t^k \frac{P_t^H}{P_t} k_{t-1}^k + \widetilde{\omega_t}^k \right] +$$
$$+ \tau_t^n \left[w_t^k n_t^k + \frac{v^w}{v^k} w_t^w n_t^w + \overline{g}_t^w \right] + \tau_t^l + d_t$$

where $d_t \equiv \frac{D_t}{P_t}$.

Therefore, as in e.g. Alesina et al., 2002, we include four main types of government spending (purchases of goods and services from the private sector, public investment in infrastructure, public wages, and transfers to individuals). We also include three main types of taxes (taxes on consumption, labor and capital income).

In each period, one of $\{\tau_t^c, \tau_t^k, \tau_t^n, \overline{g}_t^c, \overline{g}_t^i, \overline{g}_t^w, \tau_t^l, \lambda_t, d_t\}$ needs to follow residually to satisfy the government budget constraint in the domestic country.

Here, we model public infrastructure as a stock variable assuming that it accumulates like private physical capital (see also e.g. Fischer and Turnovsky, 1998). Hence, the stock of domestic public infrastructure divided by the number of domestic private firms (which is equal to the number of domestic capitalists as we have assumed in Subsection 2.1 in the main text), k_t^g, evolves according to:

---

[51] Recall that the number of domestic capitalists equals that of foreign capitalists as we have assumed in Subsection 2.1 in the main text.

[52] Public debt differs from foreign debt. The end-of-period total domestic public debt divided by the number of domestic capitalists(Recall that the number of domestic capitalists is equal to the number of foreign capitalists as we have assumed in Subsection 2.1 in the main text), written in nominal terms, is $N^k D_t = N^k B_t + S_t N^{k*} F_t^g$, where $B_t = \lambda_t D_t$ is domestic government bonds held by each domestic capitalist and $S_t F_t^g = (1-\lambda_t) D_t$ denotes domestic government bonds held by each foreign investor. On the other hand, the country's end-of-period net foreign debt divided by the number of domestic capitalists, written in nominal terms, is $S_t (N^{k*} F_t^g - N^k F_t^k) = (1-\lambda_t) N^k D_t - S_t N^k F_t^k$, where F_t^k is foreign assets held by each domestic capitalists (if negative, it denotes liabilities).

$$\overline{g}^i_t = k^g_t - (1-\delta^g)k^g_{t-1} + \frac{\xi^g}{2}\left(\frac{k^g_t}{k^g_{t-1}} - 1\right)^2 k^g_{t-1} \tag{180}$$

where $0 \leq \delta^g \leq 1$ is the depreciation rate of domestic public infrastructure and $\xi^g \geq 0$ is a parameter capturing adjustment costs related to domestic public infrastructure stock.

Similarly, the government budget constraint in real terms[53] in the foreign country is:

$$\begin{aligned} Q^*_{t-1}\frac{P_t}{S_tP^*_t}\frac{P_{t-1}}{P_t}f^{g*}_{t-1} + R^*_{t-1}\frac{P^*_{t-1}}{P^*_t}b^*_{t-1} + \frac{P^{H*}_t}{P^*_t}\overline{g}^{c*}_t + \frac{P^{H*}_t}{P^*_t}\overline{g}^{i*}_t + \overline{g}^{w*}_t + \frac{P^*_{t-1}}{P^*_t}m^*_{t-1} = \\ = m^*_t + \tau^{c*}_t\left[\frac{P^{H*}_t}{P^*_t}\left(c^{k,H*}_t + \frac{v^{w*}}{v^{k*}}c^{w,H*}_t + \frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right) + \frac{P^{F*}_t}{P^*_t}\left(c^{k,F*}_t + \frac{v^{w*}}{v^{k*}}c^{w,F*}_t + \frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right)\right] + \\ +\tau^{k*}_t\left[r^{k*}_t\frac{P^{H*}_t}{P^*_t}k^{k*}_{t-1} + \widetilde{\omega}^{k*}_t\right] + \tau^{n*}_t\left[w^{k*}_t n^{k*}_t + \frac{v^{w*}}{v^{k*}}w^{w*}_t n^{w*}_t + \overline{g}^{w*}_t\right] + \tau^{l*}_t + b^*_t + \frac{P_t}{P^*_tS_t}f^{g*}_t \end{aligned} \tag{181}$$

As we have mentioned, a star denotes the counterpart of a variable or a parameter in the foreign country.

Let D^*_t denote the total nominal foreign public debt in foreign currency divided by the number of foreign capitalists. This can be held either by a foreign private agent(foreign capitalist), $B^*_t = \lambda^*_t D^*_t$, or by a domestic private agent(domestic capitalist), $\frac{F^{g*}_t}{S_t} = (1-\lambda^*_t)D^*_t$. Then, we have:

$$\begin{aligned} Q^*_{t-1}\frac{P_t}{S_tP^*_t}\frac{P_{t-1}}{P_t}\frac{S_{t-1}P^*_{t-1}}{P_{t-1}}(1-\lambda^*_{t-1})d^*_{t-1} + R^*_{t-1}\frac{P^*_{t-1}}{P^*_t}\lambda^*_{t-1}d^*_{t-1} + \frac{P^{H*}_t}{P^*_t}\overline{g}^{c*}_t + \frac{P^{H*}_t}{P^*_t}\overline{g}^{i*}_t + \overline{g}^{w*}_t + \frac{P^*_{t-1}}{P^*_t}m^*_{t-1} = \\ = m^*_t + \tau^{c*}_t\left[\frac{P^{H*}_t}{P^*_t}\left(c^{k,H*}_t + \frac{v^{w*}}{v^{k*}}c^{w,H*}_t + \frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right) + \frac{P^{F*}_t}{P^*_t}\left(c^{k,F*}_t + \frac{v^{w*}}{v^{k*}}c^{w,F*}_t + \frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right)\right] + \\ +\tau^{k*}_t\left[r^{k*}_t\frac{P^{H*}_t}{P^*_t}k^{k*}_{t-1} + \widetilde{\omega}^{k*}_t\right] + \tau^{n*}_t\left[w^{k*}_t n^{k*}_t + \frac{v^{w*}}{v^{k*}}w^{w*}_t n^{w*}_t + \overline{g}^{w*}_t\right] + \tau^{l*}_t + d^*_t \end{aligned} \tag{182}$$

In each period, one of $\{\tau^{c*}_t, \tau^{k*}_t, \tau^{n*}_t, \overline{g}^{c*}_t, \overline{g}^{i*}_t, \overline{g}^{w*}_t, \tau^{l*}_t, \lambda^*_t, d^*_t\}$ needs to follow residually to satisfy the government budget constraint in the foreign country.

[53] I have aggregated over all agents, divided by the total number of agents and, in turn, divided all terms by v^{k*}.

The stock of foreign public infrastructure divided by the number of foreign firms (which is equal to the number of foreign capitalists as we have said in Subsection 2.1 in the main text), k_t^{g*}, evolves according to:

$$\overline{g}_t^{i*} = k_t^{g*} - (1-\delta^{g*})k_{t-1}^{g*} + \frac{\xi^{g*}}{2}\left(\frac{k_t^{g*}}{k_{t-1}^{g*}} - 1\right)^2 k_{t-1}^{g*} \tag{183}$$

where $0 \leq \delta^{g*} \leq 1$ is the depreciation rate of foreign public infrastructure and $\xi^{g*} \geq 0$ is a parameter capturing adjustment costs related to foreign public infrastructure stock.

Appendix F The state-owned enterprise

Following most of the related literature,[54] we assume that total domestic public goods and services, $N^k y_t^g$, are produced using goods and services purchased from the domestic private sector, $N^k \overline{g}_t^c$, and total domestic public employment, l_t^g. In particular, following e.g. Linnemann (2009) and Economides et al.(2013, 2014), we use a Cobb-Douglas production function of the form:

$$N^k y_t^g = A\left(N^k \overline{g}_t^c\right)^{\theta_g}\left(l_t^g\right)^{1-\theta_g} \tag{184}$$

where $0 \leq \theta_g \leq 1$ is a technology parameter. Notice that both domestic private and domestic public good production face the same TFP; this is because we do not want our results to be driven by exogenous factors. The total cost of domestic public production, $N^k \overline{g}_t^c + w_t^b l_t^g$, is financed by the domestic government through taxes and bonds (see the budget constraint of the domestic government, Eq.(179), above).

Similarly, we assume that in the foreign country total public goods and services, $N^{k*} y_t^{g*}$, are produced using goods and services purchased from the private sector, $N^{k*} \overline{g}_t^{c*}$, and total public employment, l_t^{g*}. In particular, following e.g.Linnemann (2009) and Economides et al.(2013, 2014), we use a Cobb-Douglas production function in total terms of the form:

$$N^{k*} y_t^{g*} = A_t^*\left(N^{k*} \overline{g}_t^{c*}\right)^{\theta_g^*}\left(l_t^{g*}\right)^{1-\theta_g^*} \tag{185}$$

[54]See Economides et al.,2014, for details and a review of the literature on the production function of public goods. Again, see Economides et al., 2014, for the role of public capital in public good production functions.

where $0 \leq \theta_g^* \leq 1$ is a technology parameter. Notice that we assume that in the foreign country both private and public good production face the same TFP; this is because we do not want our results to be driven by exogenous factors. The total cost of foreign public production, $N^{k*}\overline{g}_t^{c*} + w_t^{b*} l_t^{g*}$, is financed by the foreign government through taxes and bonds (see the budget constraint of the foreign government, Eq.(182), above).

Appendix G World financial intermediary

The total profit, net of transactions, of the international financial intermediary, which is made from loans between $t-1$ and t, is defined in nominal terms as:[55]

$$Q_{t-1}^* \left[(N^k F_{t-1}^{g*} - N^{k*} F_{t-1}^{k*}) - \frac{\psi}{2} N^{k*} P_{t-1}^H (f_{t-1}^{g*} - f_{t-1}^{k*})^2 \right] - Q_{t-1} S_t \left(N^k F_{t-1}^k - N^{k*} F_{t-1}^g \right) \tag{186}$$

where F_t^k are nominal international assets (in foreign currency) held by each domestic private agent (domestic capitalist), F_t^{k*} are nominal international assets (in domestic currency) held by each foreign private agent(foreign capitalist) and ψ is a cost parameter in international borrowing and $\frac{\psi}{2} N^{k*} P_{t-1}^H (f_{t-1}^{g*} - f_{t-1}^{k*})^2$ is nominal transaction cost divided by the number of foreign capitalists. $\left(N^k F_{t-1}^k - N^{k*} F_{t-1}^g \right)$ denotes the position of the domestic country in the world financial market, while $\left(N^{k*} F_{t-1}^{k*} - N^k F_{t-1}^{g*} \right)$ denotes the position of the foreign country. If they are positive (respectively negative), they denote net asset (respectively liability). Notice that the real resources used by the bank are assumed to be consumed the time at which the interest payments/income are repaid/received, namely at time t, rather than the time at which the loan contract was originated, namely at time $t-1$.

Then, dividing by the current domestic CPI, P_t, and by the constant size of domestic capitalists in the population, N^k, the profit of bank received by each domestic capitalist is defined in real terms as:

[55] Thus, at the beginning of period t, agents carry over assets and liabilities from period $t-1$. Borrowers honor their preexisting obligations to lenders. In particular, in the international capital market, where transactions take place via the bank, the bank receives interest income from borrowers and pays off the lenders. The latter is the interest payments that the bank promised at $t-1$ to pay at t. The bank also pays the monitoring cost associated with these transactions.

$$\widetilde{\pi}_t \equiv Q^*_{t-1}\left[\frac{P_{t-1}}{P_t}(f^{g*}_{t-1}-f^{k*}_{t-1})-\frac{\psi}{2}\frac{P^H_{t-1}}{P^H_t}\frac{P^H_t}{P_t}(f^{g*}_{t-1}-f^{k*}_{t-1})^2\right]-Q_{t-1}S_t\frac{P^*_t}{P_t}\frac{P^*_{t-1}}{P^*_t}\left(f^k_{t-1}-f^g_{t-1}\right) \tag{187}$$

Recall that we have assumed in Subsection 2.1 in the main text that the number of domestic capitalists equals that of foreign capitalists.

Since, in equilibrium, international borrowing equals international lending at every t, namely $N^kF^{g*}_t - N^{k*}F^{k*}_t = S_t(N^kF^k_t - N^{k*}F^g_t)$ in nominal terms, or $f^{g*}_t - f^{k*}_t = S_t\frac{P^*_t}{P_t}(f^k_t - f^g_t)$ in real terms,[56] the above profit function can be rewritten as:

$$\widetilde{\pi}_t = Q^*_{t-1}\left[\frac{P_{t-1}}{P_t}(f^{g*}_{t-1}-f^{k*}_{t-1})-\frac{\psi}{2}\frac{P^H_{t-1}}{P^H_t}\frac{P^H_t}{P_t}(f^{g*}_{t-1}-f^{k*}_{t-1})^2\right]-Q_{t-1}\frac{S_t}{S_{t-1}}\frac{P_{t-1}}{P_t}\left(f^{g*}_{t-1}-f^{k*}_{t-1}\right) \tag{188}$$

If the volume of the loan divided by the number of domestic capitalists,[57] $(f^{g*}_{t-1}-f^{k*}_{t-1})$, is chosen optimally by the financial intermediary, the first-order condition is:

$$Q^*_{t-1} = \frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1-\psi\frac{P^H_{t-1}}{P_{t-1}}(f^{g*}_{t-1}-f^{k*}_{t-1})} \tag{189}$$

In what follows, we define $Q^*_{t-1}\frac{\psi}{2}\frac{P^H_{t-1}}{P^H_t}(f^{g*}_{t-1}-f^{k*}_{t-1})^2 \equiv \upsilon_t$, which is the same term arising from the above domestic aggregate demand in real terms if we divide by the number of domestic agents and, then, divide by the the percentage of domestic capitalists in population, v^k. That is, υ_t is the same term in the equation
$y^{H,d}_t = \left[\left(c^{k,H}_t + \frac{v^w}{v^k}c^{w,H}_t + \frac{v^b}{v^k}c^{b,H}_t\right) + x^k_t + \overline{g}^c_t + \overline{g}^i_t + \upsilon_t + \left(c^{k,F*}_t + \frac{v^{w*}}{v^{k*}}c^{w,F*}_t + \frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right)\right]$.
Recall also the assumptions $N^k = N^{k*}$, $N^w = N^{w*}$ and $N^b = N^{b*}$ we have made in Subsection 2.1 in the main text as well as the assumption that $S_t \equiv 1$ at all t in a currency union regime.

[56] Recall that the number of domestic capitalists equals that of foreign capitalists.

[57] Recall that the number of domestic capitalists equals that of foreign capitalists.

Appendix H Equilibrium in the status quo economy

This Appendix presents in some detail the status quo equilibrium system, given feedback policy coefficients. We will work in steps.

H.1 Market clearing conditions and the balance of payments

In the domestic economy, the market-clearing conditions in the capital markets, the labor markets, the money market, the domestic government bond market and the domestic dividend market are respectively (and similarly in the foreign country):[58]

$$\sum_{k=1}^{N^k} k_{t-1}^k = \sum_{h=1}^{N^k} k_{t-1}(h)$$

$$\sum_{k=1}^{N^k} k_{t-1}^g = \sum_{h=1}^{N^k} k_{t-1}^g(h)$$

$$\left(since \sum_{k=1}^{N^k} \overline{g}_t^i = \sum_{h=1}^{N^k} \overline{g}_t^i(h) \right)$$

$$\sum_{k=1}^{N^k} n_t^k = \sum_{h=1}^{N^k} n_t^k(h)$$

$$\sum_{w=1}^{N^w} n_t^w = \sum_{h=1}^{N^k} n_t^w(h)$$

$$\sum_{b=1}^{N^b} n_t^b = l_t^g$$

$$(\text{setting } n_t^b \equiv 1)$$

[58]Recall that we have assumed in Subsection 2.1 in the main text that the number of domestic capitalists equals that of domestic firms.

$$\sum_{k=1}^{N^k} m_t^k + \sum_{w=1}^{N^w} m_t^w + \sum_{b=1}^{N^b} m_t^b = N^k m_t$$

$$\sum_{k=1}^{N^k} b_t^k = N^k b_t$$

$$\sum_{k=1}^{N^k} \omega_t^k = \sum_{h=1}^{N^k} \widetilde{\omega_t}(h)$$

The market-clearing condition for the profits made by the international financial intermediary (these profits are distributed to capitalists in the domestic economy who bear the associated transaction costs) is:

$$\sum_{k=1}^{N^k} \widetilde{\pi}_t^k = N^k \widetilde{\pi}_t$$

Regarding the balance of payments in each country, this is obtained by adding the constraints of households, firms and the government. Then, the balance of payments in real terms in the domestic country[59] is:

$$\frac{P_t^H}{P_t}\left\{\left[c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H} + \frac{v^b}{v^k}c_t^{b,H}\right] + x_t^k + \overline{g}_t^c + \overline{g}_t^i\right\} + \frac{P_t^F}{P_t}\left[c_t^{k,F} + \frac{v^w}{v^k}c_t^{w,F} + \frac{v^b}{v^k}c_t^{b,F}\right] = \tag{190}$$

$$S_t\frac{P_t^*}{P_t}(f_t^g - f_t^k) - Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}(f_{t-1}^g - f_{t-1}^k) + \widetilde{\pi}_t^k + \frac{1}{N^k}\sum_{h=1}^{N^k}\frac{P_t^H(h)y_t^H(h)}{P_t} -$$

$$-\frac{1}{N^k}\sum_{h=1}^{N^k}\left[\frac{\phi^P}{2}\left(\frac{P_t^H(h)}{P_{t-1}^H(h)\pi^H} - 1\right)^2\frac{P_t^H}{P_t}y_t^H\right]$$

where are variables have been defined above.

It can be shown, by solving the Eq.(165), $y_t^H(h) = y_t^H\left(\frac{P_t^H}{P_t^H(h)}\right)^{\phi}$, with respect to $P_t^H(h)^{-\phi}$, and plugging this term into the domestic price index, $P_t^H =$

[59]I have aggregated over all agents, divided by the total number of agents and, in turn, divided all terms by v^k.

$\left\{\sum_{h=1}^{N^k}\frac{1}{N^k}[P_t^H(h)]^{1-\phi}\right\}^{\frac{1}{1-\phi}}$, that $N^kP_t^Hy_t^H=\sum_{h=1}^{N^k}P_t^H(h)y_t^H(h)$. Furthermore, due to symmetry in private firms' problem, price of every private firm h, $P_t^H(h)$, will be common for all firms and this, given the relation for domestic price index above, implies $P_t^H(h)=P_t^H$. Hence, the term of the third line and the last term of the second line on the RHS of the balance of payments above can be written as $\frac{\phi^P}{2}\left(\frac{P_t^H}{P_{t-1}^H\pi^H}-1\right)^2\frac{P_t^H}{P_t}y_t^H$ and $\frac{P_t^H}{P_t}y_t^H$ respectively. Therefore, the balance of payments in the domestic economy is:

$$\frac{P_t^H}{P_t}\left\{\left[c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right]+x_t^k+\overline{g}_t^c+\overline{g}_t^i-y_t^H\left[1-\frac{\phi^P}{2}\left(\frac{P_t^H}{P_{t-1}^H\pi^H}-1\right)^2\right]\right\}+ \tag{191}$$
$$+\frac{P_t^F}{P_t}\left[c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right]=S_t\frac{P_t^*}{P_t}(f_t^g-f_t^k)-Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}(f_{t-1}^g-f_{t-1}^k)+\widetilde{\pi}_t^k$$

where, recall that the resources used by the financial intermediary, $\upsilon_t\equiv Q_{t-1}^*\frac{\psi}{2}\frac{P_{t-1}^H}{P_t^H}(f_{t-1}^{g*}-f_{t-1}^{k*})^2$, are paid by the domestic country, so that
$y_t^H\left[1-\frac{\phi^P}{2}\left(\frac{P_t^H}{P_{t-1}^H\pi^H}-1\right)^2\right]=\left[\left(c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right)+x_t^k+\overline{g}_t^c+\overline{g}_t^i+\upsilon_t+\left(c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right)\right]$
(See also Appendix D) and where (from Appendix G and market clearing conditions in Appendix H) $\widetilde{\pi}_t^k\equiv Q_{t-1}^*\frac{P_{t-1}}{P_t}(f_{t-1}^{g*}-f_{t-1}^{k*})-\frac{P_t^H}{P_t}\upsilon_t-Q_{t-1}S_t\frac{P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\left(f_{t-1}^k-f_{t-1}^g\right)$.
Using these two relations, the balance of payments becomes
$\frac{P_t^H}{P_t}\left\{\left[c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right]+x_t^k+\overline{g}_t^c+\overline{g}_t^i+\upsilon_t-y_t^H\left[1-\frac{\phi^P}{2}\left(\frac{P_t^H}{P_{t-1}^H\pi^H}-1\right)^2\right]\right\}=-\frac{P_t^H}{P_t}[c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}]$ so that the terms $\frac{P_t^F}{P_t}\left[c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right]-\frac{P_t^H}{P_t}\left[c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right]$ on the LHS is the trade balance.

Working similarly, we get the balance of payments in the foreign country:

$$\frac{P_t^{H*}}{P_t^*}\left\{\left[c_t^{k,H*}+\frac{v^{w*}}{v^{k*}}c_t^{w,H*}+\frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right]+x_t^{k*}+\overline{g}_t^{c*}+\overline{g}_t^{i*}-y_t^{H*}\left[1-\frac{\phi^{P*}}{2}\left(\frac{P_t^{H*}}{P_{t-1}^{H*}\pi^{H*}}-1\right)^2\right]\right\}+ \tag{192}$$
$$+\frac{P_t^{F*}}{P_t^*}\left[c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right]=\frac{1}{S_t}\frac{P_t}{P_t^*}(f_t^{g*}-f_t^{k*})-Q_{t-1}^*\frac{P_t}{S_tP_t^*}\frac{P_{t-1}}{P_t}(f_{t-1}^{g*}-f_{t-1}^{k*})$$

where now

$$y_t^{H*}\left[1-\frac{\phi^{P*}}{2}\left(\frac{P_t^{H*}}{P_{t-1}^{H*}\pi^{H*}}-1\right)^2\right]=\left[\left(c_t^{k,H*}+\frac{v^{w*}}{v^{k*}}c_t^{w,H*}+\frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right)+x_t^{k*}+\bar{g}_t^{c*}+\bar{g}_t^{i*}+\left(c_t^{k,F}+\frac{v^{w}}{v^{k}}c_t^{w,F}+\frac{v^{b}}{v^{k}}c_t^{b,F}\right)\right].$$

Finally, as said in Appendix G above, the market clearing condition in the market of internationally traded assets is (written in real terms):

$$f_t^{g*}-f_t^{k*}=S_t\frac{P_t^*}{P_t}(f_t^k-f_t^g)$$

which means that net foreign liabilities in the foreign country (the LHS) are equal to net foreign assets in the domestic country (the RHS).

H.2 Decentralized equilibrium (given policy)

We now combine all the above to solve for a Decentralized Equilibrium (DE) for any feasible policy. The DE is defined to be a sequence of allocations, prices and policies such that: (i) every type of households maximizes utility; (ii) every private firm maximizes profit; (iii) the state-owned enterprise produces public goods and services; (iv) the world financial intermediary maximizes profit; (v) all constraints, including the government budget constraint and the balance of payments, are satisfied; and (vi) all markets clear, including the international asset market.

The DE is summarized by the following conditions:[60]

$$x_n(n_t^k)^\eta(c_t^k)^\sigma=\frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^k \tag{D1}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}S_t\frac{P_t^*}{P_t}=\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}Q_tS_{t+1}\frac{P_{t+1}^*}{P_{t+1}}\frac{P_t^*}{P_{t+1}^*} \tag{D2}$$

$$\begin{aligned}\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}\frac{P_t^H}{P_t}\left[1+\xi\left(\frac{k_t^k}{k_{t-1}^k}-1\right)\right]=\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\frac{P_{t+1}^H}{P_{t+1}}\times\\ \times\left[(1-\delta)+(1-\tau_{t+1}^k)r_{t+1}^k-\frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k}-1\right)^2+\xi\left(\frac{k_{t+1}^k}{k_t^k}-1\right)\frac{k_{t+1}^k}{k_t^k}\right]\end{aligned} \tag{D3}$$

[60] I have aggregated over all agents, divided by the total number of agents and, in turn, divided all terms by v^k.

$$x_m (m_t^k)^{-\mu} = \frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} - \beta \frac{P_t}{P_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D4}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)} = \beta R_t \frac{P_t}{P_{t+1}} \frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D5}$$

$$k_t^k = (1-\delta) k_{t-1}^k + x_t^k - \frac{\xi}{2} \left(\frac{k_t^k}{k_{t-1}^k} - 1 \right)^2 k_{t-1}^k \tag{D6}$$

$$\frac{c_t^{k,H}}{c_t^{k,F}} = \frac{v}{1-v} \frac{P_t^F}{P_t^H} \tag{D7}$$

$$c_t^k = \frac{\left(c_t^{k,H}\right)^v \left(c_t^{k,F}\right)^{1-v}}{v^v (1-v)^{1-v}} \tag{D8}$$

$$\left[\left(c_t^{k,H} + \frac{v^w}{v^k} c_t^{w,H} + \frac{v^b}{v^k} c_t^{b,H} \right) + x_t^k + \overline{g}_t^c + \overline{g}_t^i + \upsilon_t + \left(c_t^{k,F*} + \frac{v^{w*}}{v^{k*}} c_t^{w,F*} + \frac{v^{b*}}{v^{k*}} c_t^{b,F*} \right) \right] =$$

$$= y_t^H \left[1 - \frac{\phi^P}{2} \left(\frac{P_t^H}{P_{t-1}^H \pi^H} - 1 \right)^2 \right] \tag{D9}$$

$$x_n (n_t^w)^{\eta} (c_t^w)^{\sigma} = \frac{(1-\tau_t^n)}{(1+\tau_t^c)} w_t^w \tag{D10}$$

$$x_m (m_t^w)^{-\mu} = \frac{(c_t^w)^{-\sigma}}{1+\tau_t^c} - \beta \frac{P_t}{P_{t+1}} \left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c} \right] \tag{D11}$$

$$\frac{c_t^{w,H}}{c_t^{w,F}} = \frac{v}{1-v} \frac{P_t^F}{P_t^H} \tag{D12}$$

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v (1-v)^{1-v}} \tag{D13}$$

$$(1+\tau_t^c) c_t^w + m_t^w = \frac{P_{t-1}}{P_t} m_{t-1}^w + (1-\tau_t^n) w_t^w n_t^w - v^k \tau_t^l \tag{D14}$$

$$x_m (m_t^b)^{-\mu} = \frac{(c_t^b)^{-\sigma}}{1+\tau_t^c} - \beta \frac{P_t}{P_{t+1}} \left[\frac{(c_{t+1}^b)^{-\sigma}}{1+\tau_{t+1}^c} \right] \tag{D15}$$

$$\frac{c_t^{b,H}}{c_t^{b,F}} = \frac{v}{1-v}\frac{P_t^F}{P_t^H} \tag{D16}$$

$$c_t^b = \frac{\left(c_t^{b,H}\right)^v \left(c_t^{b,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D17}$$

$$(1+\tau_t^c)c_t^b + m_t^b = \frac{P_{t-1}}{P_t}m_{t-1}^b + (1-\tau_t^n)\frac{v^k}{v^b}\overline{g}_t^w - v^k\tau_t^l \tag{D18}$$

$$\frac{P_t^H}{P_t}r_t^k k_{t-1}^k = mc_t \alpha y_t^H \tag{D19}$$

$$w_t^k n_t^k = mc_t \theta(1-\alpha)y_t^H \tag{D20}$$

$$\frac{v^w}{v^k}w_t^w n_t^w = mc_t(1-\theta)(1-\alpha)y_t^H \tag{D21}$$

$$y_t^H = A_t\left\{[k_{t-1}^k]^\alpha\left[\{n_t^k\}^\theta \times \left\{\frac{v^w}{v^k}n_t^w\right\}^{1-\theta}\right]^{1-\alpha}\right\}\left(k_{t-1}^g\right)^{\theta_k} \tag{D22}$$

$$\widetilde{\omega_t}^k = \frac{P_t^H}{P_t}y_t^H - mc_t y_t^H - \frac{\phi^P}{2}\left(\frac{P_t^H}{P_{t-1}^H\pi^H} - 1\right)^2\frac{P_t^H}{P_t}y_t^H \tag{D23}$$

$$Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}f_{t-1}^g + R_{t-1}\frac{P_{t-1}}{P_t}b_{t-1}^k + \frac{P_t^H}{P_t}\overline{g}_t^c + \frac{P_t^H}{P_t}\overline{g}_t^i + \overline{g}_t^w + \frac{P_{t-1}}{P_t}\left[m_{t-1}^k + \frac{v^w}{v^k}m_{t-1}^w + \frac{v^b}{v^k}m_{t-1}^b\right] = \tag{D24}$$

$$= \left[m_t^k + \frac{v^w}{v^k}m_t^w + \frac{v^b}{v^k}m_t^b\right] + \tau_t^c\left[\frac{P_t^H}{P_t}\left(c_t^{k,H} + \frac{v^w}{v^k}c_t^{w,H} + \frac{v^b}{v^k}c_t^{b,H}\right) + \frac{P_t^F}{P_t}\left(c_t^{k,F} + \frac{v^w}{v^k}c_t^{w,F} + \frac{v^b}{v^k}c_t^{b,F}\right)\right] +$$

$$+\tau_t^k\left[r_t^k\frac{P_t^H}{P_t}k_{t-1}^k + \widetilde{\omega_t}^k\right] + \tau_t^n\left[w_t^k n_t^k + \frac{v^w}{v^k}w_t^w n_t^w + \overline{g}_t^w\right] + \tau_t^l + b_t^k + S_t\frac{P_t^*}{P_t}f_t^g$$

$$
\frac{P_t^H}{P_t}\left\{\left[c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right]+x_t^k+\overline{g}_t^c+\overline{g}_t^i-y_t^H\left[1-\frac{\phi^P}{2}\left(\frac{P_t^H}{P_{t-1}^H\pi^H}-1\right)^2\right]\right\}+
$$

$$
+\frac{P_t^F}{P_t}\left[c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right]=S_t\frac{P_t^*}{P_t}(f_t^g-f_t^k)-Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}(f_{t-1}^g-f_{t-1}^k)+\widetilde{\pi}_t^k \tag{D25}
$$

$$
P_t=(P_t^H)^v(P_t^F)^{1-v} \tag{D26}
$$

$$
P_t^F=S_tP_t^{H*} \tag{D27}
$$

$$
(1-\phi)\frac{P_t^H}{P_t}y_t^H+\phi mc_ty_t^H-\phi^P\left[\frac{P_t^H}{P_{t-1}^H\pi^H}-1\right]\frac{P_t^H}{P_t}\frac{y_t^HP_t^H}{P_{t-1}^H\pi^H}=
$$

$$
\beta\phi^P\left[\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1-\frac{P_{t+1}^H}{P_t^H\pi^H}\right]\frac{P_{t+1}^H}{P_t^H\pi^H}\frac{P_{t+1}^H}{P_{t+1}}y_{t+1}^H \tag{D28}
$$

$$
\widetilde{\pi}_t^k\equiv Q_{t-1}^*\left[\frac{P_{t-1}}{P_t}(f_{t-1}^{g*}-f_{t-1}^{k*})-\frac{\psi}{2}\frac{P_{t-1}^H}{P_t^H}\frac{P_t^H}{P_t}(f_{t-1}^{g*}-f_{t-1}^{k*})^2\right]-Q_{t-1}S_t\frac{P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\left(f_{t-1}^k-f_{t-1}^g\right) \tag{D29}
$$

$$
Q_{t-1}^*=\frac{Q_{t-1}\frac{S_t}{S_{t-1}}}{1-\psi\frac{P_{t-1}^H}{P_{t-1}}(f_{t-1}^{g*}-f_{t-1}^{k*})} \tag{D30}
$$

$$
\upsilon_t\equiv Q_{t-1}^*\frac{\psi}{2}\frac{P_{t-1}^H}{P_t^H}(f_{t-1}^{g*}-f_{t-1}^{k*})^2 \tag{D31}
$$

$$
y_t^g=A\left(\overline{g}_t^c\right)^{\theta_g}\left(\frac{v^b}{v^k}\right)^{1-\theta_g} \tag{D32}
$$

$$
\overline{g}_t^i=k_t^g-(1-\delta^g)k_{t-1}^g+\frac{\xi^g}{2}\left(\frac{k_t^g}{k_{t-1}^g}-1\right)^2k_{t-1}^g \tag{D33}
$$

$$
x_n^*(n_t^{k*})^{\eta*}(c_t^{k*})^{\sigma*}=\frac{(1-\tau_t^{n*})}{(1+\tau_t^{c*})}w_t^{k*} \tag{D34}
$$

$$\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})}\frac{1}{S_t}\frac{P_t}{P_t^*}=\beta^*\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})}Q_t^*\frac{1}{S_{t+1}}\frac{P_{t+1}}{P_{t+1}^*}\frac{P_t}{P_{t+1}} \tag{D35}$$

$$\begin{aligned}\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})}\frac{P_t^{H*}}{P_t^*}\left[1+\xi^*\left(\frac{k_t^{k*}}{k_{t-1}^{k*}}-1\right)\right]=\beta^*\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})}\frac{P_{t+1}^{H*}}{P_{t+1}^*}\times\\ \times\left[(1-\delta^*)+(1-\tau_{t+1}^{k*})r_{t+1}^{k*}-\frac{\xi^*}{2}\left(\frac{k_{t+1}^{k*}}{k_t^{k*}}-1\right)^2+\xi^*\left(\frac{k_{t+1}^{k*}}{k_t^{k*}}-1\right)\frac{k_{t+1}^{k*}}{k_t^{k*}}\right]\end{aligned} \tag{D36}$$

$$x_m^*(m_t^{k*})^{-\mu*}=\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})}-\beta^*\frac{P_t^*}{P_{t+1}^*}\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})} \tag{D37}$$

$$\frac{(c_t^{k*})^{-\sigma*}}{(1+\tau_t^{c*})}=\beta^*R_t^*\frac{P_t^*}{P_{t+1}^*}\frac{(c_{t+1}^{k*})^{-\sigma*}}{(1+\tau_{t+1}^{c*})} \tag{D38}$$

$$k_t^{k*}=(1-\delta^*)k_{t-1}^{k*}+x_t^{k*}-\frac{\xi^*}{2}\left(\frac{k_t^{k*}}{k_{t-1}^{k*}}-1\right)^2k_{t-1}^{k*} \tag{D39}$$

$$\frac{c_t^{k,H*}}{c_t^{k,F*}}=\frac{v^*}{1-v^*}\frac{P_t^{F*}}{P_t^{H*}} \tag{D40}$$

$$c_t^{k*}=\frac{\left(c_t^{k,H*}\right)^{v*}\left(c_t^{k,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D41}$$

$$\begin{aligned}\left[\left(c_t^{k,H*}+\frac{v^{w*}}{v^{k*}}c_t^{w,H*}+\frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right)+x_t^{k*}+\overline{g}_t^{c*}+\overline{g}_t^{i*}+\left(c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right)\right]=\\ y_t^{H*}\left[1-\frac{\phi^{P*}}{2}\left(\frac{P_t^{H*}}{P_{t-1}^{H*}\pi^{H*}}-1\right)^2\right]\end{aligned} \tag{D42}$$

$$x_n^*(n_t^{w*})^{\eta*}(c_t^{w*})^{\sigma*}=\frac{(1-\tau_t^{n*})}{(1+\tau_t^{c*})}w_t^{w*} \tag{D43}$$

$$x_m^*(m_t^{w*})^{-\mu*}=\frac{(c_t^{w*})^{-\sigma*}}{1+\tau_t^{c*}}-\beta^*\frac{P_t^*}{P_{t+1}^*}\left[\frac{(c_{t+1}^{w*})^{-\sigma*}}{1+\tau_{t+1}^{c*}}\right] \tag{D44}$$

$$\frac{c_t^{w,H*}}{c_t^{w,F*}} = \frac{v^*}{1-v^*}\frac{P_t^{F*}}{P_t^{H*}} \tag{D45}$$

$$c_t^{w*} = \frac{\left(c_t^{w,H*}\right)^{v*}\left(c_t^{w,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D46}$$

$$(1+\tau_t^{c*})c_t^{w*} + m_t^{w*} = \frac{P_{t-1}^*}{P_t^*}m_{t-1}^{w*} + (1-\tau_t^{n*})w_t^{w*}n_t^{w*} - v^{k*}\tau_t^{l*} \tag{D47}$$

$$x_m^*(m_t^{b*})^{-\mu*} = \frac{(c_t^{b*})^{-\sigma*}}{1+\tau_t^{c*}} - \beta^*\frac{P_t^*}{P_{t+1}^*}\left[\frac{(c_{t+1}^{b*})^{-\sigma*}}{1+\tau_{t+1}^{c*}}\right] \tag{D48}$$

$$\frac{c_t^{b,H*}}{c_t^{b,F*}} = \frac{v^*}{1-v^*}\frac{P_t^{F*}}{P_t^{H*}} \tag{D49}$$

$$c_t^{b*} = \frac{\left(c_t^{b,H*}\right)^{v*}\left(c_t^{b,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D50}$$

$$(1+\tau_t^{c*})c_t^{b*} + m_t^{b*} = \frac{P_{t-1}^*}{P_t^*}m_{t-1}^{b*} + (1-\tau_t^{n*})\frac{v^{k*}}{v^{b*}}\overline{g}_t^{w*} - v^{k*}\tau_t^{l*} \tag{D51}$$

$$\frac{P_t^{H*}}{P_t^*}r_t^{k*}k_{t-1}^{k*} = mc_t^*\alpha^* y_t^{H*} \tag{D52}$$

$$w_t^{k*}n_t^{k*} = mc_t^*\theta^*(1-\alpha^*)y_t^{H*} \tag{D53}$$

$$\frac{v^{w*}}{v^{k*}}w_t^{w*}n_t^{w*} = mc_t^*(1-\theta^*)(1-\alpha^*)y_t^{H*} \tag{D54}$$

$$y_t^{H*} = A_t^*\left\{[k_{t-1}^{k*}]^{\alpha^*}\left[\{n_t^{k*}\}^{\theta^*} \times \left\{\frac{v^{w*}}{v^{k*}}n_t^{w*}\right\}^{1-\theta^*}\right]^{1-\alpha^*}\right\}\left(k_{t-1}^{g*}\right)^{\theta_k^*} \tag{D55}$$

$$\widetilde{\omega}_t^{k*} = \frac{P_t^{H*}}{P_t^*}y_t^{H*} - mc_t^* y_t^{H*} - \frac{\phi^{P*}}{2}\left(\frac{P_t^{H*}}{P_{t-1}^{H*}\pi^{H*}} - 1\right)^2\frac{P_t^{H*}}{P_t^*}y_t^{H*} \tag{D56}$$

$$Q^*_{t-1}\frac{P_t}{S_tP^*_t}\frac{P_{t-1}}{P_t}f^{g*}_{t-1}+R^*_{t-1}\frac{P^*_{t-1}}{P^*_t}b^{k*}_{t-1}+\frac{P^{H*}_t}{P^*_t}\overline{g}^{c*}_t+\frac{P^{H*}_t}{P^*_t}\overline{g}^{i*}_t+\overline{g}^{w*}_t+\frac{P^*_{t-1}}{P^*_t}\left[m^{k*}_{t-1}+\frac{v^{w*}}{v^{k*}}m^{w*}_{t-1}+\frac{v^{b*}}{v^{k*}}m^{b*}_{t-1}\right]= \tag{D57}$$

$$=\left[m^{k*}_t+\frac{v^{w*}}{v^{k*}}m^{w*}_t+\frac{v^{b*}}{v^{k*}}m^{b*}_t\right]+\tau^{n*}_t\left[w^{k*}_tn^{k*}_t+\frac{v^{w*}}{v^{k*}}w^{w*}_tn^{w*}_t+\frac{v^{b*}}{v^{k*}}w^{b*}_tn^{b*}_t\right]+$$

$$+\tau^{c*}_t\left[\frac{P^{H*}_t}{P^*_t}\left(c^{k,H*}_t+\frac{v^{w*}}{v^{k*}}c^{w,H*}_t+\frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right)+\frac{P^{F*}_t}{P^*_t}\left(c^{k,F*}_t+\frac{v^{w*}}{v^{k*}}c^{w,F*}_t+\overline{g}^{w*}_t\right)\right]+$$

$$+\tau^{k*}_t\left[r^{k*}_t\frac{P^{H*}_t}{P^*_t}k^{k*}_{t-1}+\widetilde{\omega}^{k*}_t\right]+\tau^{l*}_t+b^{k*}_t+\frac{1}{S_t}\frac{P_t}{P^*_t}f^{g*}_t$$

$$\frac{P^{H*}_t}{P^*_t}\left\{\left[c^{k,H*}_t+\frac{v^{w*}}{v^{k*}}c^{w,H*}_t+\frac{v^{b*}}{v^{k*}}c^{b,H*}_t\right]+x^{k*}_t+\overline{g}^{c*}_t+\overline{g}^{i*}_t-y^{H*}_t\left[1-\frac{\phi^{P*}}{2}\left(\frac{P^{H*}_t}{P^{H*}_{t-1}\pi^{H*}}-1\right)^2\right]\right\}+ \tag{D58}$$

$$+\frac{P^{F*}_t}{P^*_t}\left[c^{k,F*}_t+\frac{v^{w*}}{v^{k*}}c^{w,F*}_t+\frac{v^{b*}}{v^{k*}}c^{b,F*}_t\right]=\frac{1}{S_t}\frac{P_t}{P^*_t}(f^{g*}_t-f^{k*}_t)-Q^*_{t-1}\frac{P_t}{S_tP^*_t}\frac{P_{t-1}}{P_t}(f^{g*}_{t-1}-f^{k*}_{t-1})$$

$$P^*_t=(P^{H*}_t)^{v*}(P^{F*}_t)^{1-v*} \tag{D59}$$

$$P^{F*}_t=\frac{P^H_t}{S_t} \tag{D60}$$

$$(1-\phi^*)\frac{P^{H*}_t}{P^*_t}y^{H*}_t+\phi^*mc^*_ty^{H*}_t-\phi^{P*}\left[\frac{P^{H*}_t}{P^{H*}_{t-1}\pi^{H*}}-1\right]\frac{P^{H*}_t}{P^*_t}\frac{y^{H*}_tP^{H*}_t}{P^{H*}_{t-1}\pi^{H*}}=$$

$$\beta^*\phi^{P*}\left[\left(\frac{P^*_t}{P^*_{t+1}}\right)\left(\frac{1+\tau^{c*}_t}{1+\tau^{c*}_{t+1}}\right)\left(\frac{c^{k*}_{t+1}}{c^{k*}_t}\right)^{-\sigma*}\right]\left[1-\frac{P^{H*}_{t+1}}{P^{H*}_t\pi^{H*}}\right]\frac{P^{H*}_{t+1}}{P^{H*}_t\pi^{H*}}\frac{P^{H*}_{t+1}}{P^*_{t+1}}y^{H*}_{t+1} \tag{D61}$$

$$y^{g*}_t=A^*(\overline{g}^{c*}_t)^{\theta^*_g}\left(\frac{v^{b*}}{v^{k*}}\right)^{1-\theta^*_g} \tag{D62}$$

$$\overline{g}^{i*}_t=k^{g*}_t-(1-\delta^{g*})k^{g*}_{t-1}+\frac{\xi^{g*}}{2}\left(\frac{k^{g*}_t}{k^{g*}_{t-1}}-1\right)^2k^{g*}_{t-1} \tag{D63}$$

Thus, we have a system of 63 equations [(D1)-(D63)] in the 63 following endogeneous variables

$$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, b_t^k,$$

$$f_t^k, Q_t, y_t^g, y_t^H, mc_t, \widetilde{\omega_t}^k, \upsilon_t, \widetilde{\pi_t}^k, P_t, P_t^H, P_t^F, c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*},$$

$$c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, b_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{g*}, y_t^{H*}, mc_t^*,$$

$$\widetilde{\omega_t}^{k*}, P_t^*, P_t^{H*}, P_t^{F*}]_{t=0}^{\infty}$$

Conclusively, the Decentralized Equilibrium is a sequence of

$$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, b_t^k,$$

$$f_t^k, Q_t, y_t^g, y_t^H, mc_t, \widetilde{\omega_t}^k, \upsilon_t, \widetilde{\pi_t}^k, P_t, P_t^H, P_t^F, c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*},$$

$$c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, b_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{g*}, y_t^{H*}, mc_t^*,$$

$$\widetilde{\omega_t}^{k*}, P_t^*, P_t^{H*}, P_t^{F*}]_{t=0}^{\infty}$$

satisfying the equations [(D1)-(D63)], given:
a) exogenous variables $[S_t, A_t, A_t^*]_{t=0}^{\infty}$,
b) initial conditions for state variables,
c) policy.

H.3 Transformed variables

We first express prices in rate form. We define 6 new variables, which are the gross domestic CPI inflation rate, $\pi_t \equiv \frac{P_t}{P_{t-1}}$, the gross foreign CPI inflation rate, $\pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$, the gross domestic goods inflation rate for the domestic economy, $\pi_t^H \equiv \frac{P_t^H}{P_{t-1}^H}$, the gross domestic goods inflation rate for the foreign economy, $\pi_t^{H*} \equiv \frac{P_t^{H*}}{P_{t-1}^{H*}}$, the gross rate of exchange rate depreciation, $\epsilon_t \equiv \frac{S_t}{S_{t-1}}$, and the terms of trade, $\tau\tau_t \equiv \frac{P_t^F}{P_t^H} = \frac{S_t P_t^{H*}}{P_t^H}$.[61] Eq.(D60) implies that $\frac{P_t^{F*}}{P_t^{H*}} = \frac{1}{\tau\tau_t}$. Plugging P_t^{F*} of the latter equation into other equations, this unknown eliminates and the system of equations is reduced by one equation. Yet, as we have shown in Appendix E for the domestic economy, we can replace f_t^g and b_t^k with $\frac{P_t}{S_t P_t^*}(1-\lambda_t)d_t$ and $\lambda_t d_t$ respectively. Similarly, we have shown in the same Appendix for the foreign economy that we can replace f_t^{g*} and b_t^{k*} with $\frac{S_t P_t^*}{P_t}(1-\lambda_t^*)d_t^*$ and $\lambda_t^* d_t^*$ respectively. Hence, in what follows, we use the 10 new variables π_t, π_t^*, π_t^H, π_t^{H*}, ϵ_t, $\tau\tau_t$, λ_t, d_t, λ_t^*, d_t^*, instead of the 11 variables P_t, P_t^*, P_t^H, P_t^{H*}, S_t, P_t^F, P_t^{F*}, f_t^g, b_t^k, f_t^{g*}, b_t^{k*}, meaning that the variables are reduced by one, as it happens with the number of equations.

Also, for convenience and comparison with the data, we express fiscal and public finance variables as shares of real GDP, $\frac{P_t^H}{P_t} y_t^H$. In particular, using the definitions above, the total public spending on goods and services purchased from the private sector in real terms divided by the number of capitalists, $\frac{P_t^H}{P_t}\overline{g}_t^c$, can be written as ratio of real GDP, as $\frac{P_t^H}{P_t}\overline{g}_t^c = s_t^g \frac{P_t^H}{P_t} y_t^H$, where s_t^g denotes the output share of government spending on private goods and services. The total public investment in infrastructure in real terms divided by the number of capitalists, $\frac{P_t^H}{P_t}\overline{g}_t^i$, can be written as ratio of real GDP, as $\frac{P_t^H}{P_t}\overline{g}_t^i = s_t^i \frac{P_t^H}{P_t} y_t^H$, where s_t^i denotes the output share of public investment. The total public wage bill in real terms divided by the number of capitalists, $\overline{g}_t^w$, can be written as ratio of real GDP, as $\overline{g}_t^w = s_t^w \frac{P_t^H}{P_t} y_t^H$, where s_t^w denotes the output share of total public wage bill. The total lump-sum taxes/transfers divided by the number of domestic capitalists in real terms, τ_t^l, can be written as ratio of real GDP, as $\tau_t^l = s_t^l \frac{P_t^H}{P_t} y_t^H$, where as s_t^l are defined the lump-sum taxes/transfers as share of output. We work similarly for the foreign country.

Finally, given the above, notice that we make use of the following equations:

$$\tau\tau_t = \frac{P_t^F}{P_t^H} = S_t \frac{P_t^{H*}}{P_t^H} = \frac{P_t^{H*}}{P_t^{F*}}$$

[61] Thus, $\frac{\tau\tau_t}{\tau\tau_{t-1}} = \frac{\frac{S_t}{S_{t-1}} \frac{P_t^{H*}}{P_{t-1}^{H*}}}{\frac{P_t^H}{P_{t-1}^H}} = \frac{\epsilon_t \pi_t^{H*}}{\pi_t^H}$.

$$\frac{P_t^H}{P_t} = \tau\tau_t^{v-1}$$

$$\frac{P_t^{H*}}{P_t^*} = \tau\tau_t^{1-v^*}$$

$$\frac{P_t^F}{P_t} = \tau\tau_t^{v}$$

$$\frac{P_t^{F*}}{P_t^*} = \tau\tau_t^{-v^*}$$

$$S_t\frac{P_t^*}{P_t} = \tau\tau_t^{v+v^*-1}$$

$$\overline{g}_t^c = s_t^g y_t^H$$

$$\overline{g}_t^i = s_t^i y_t^H$$

$$\overline{g}_t^w = s_t^w \tau\tau_t^{v-1} y_t^H$$

$$\tau_t^l = s_t^l \tau\tau_t^{v-1} y_t^H$$

H.4 Final equations

Using the above, we now present the final non-linear stochastic system (given feedback policy coefficients).

The domestic country is summarized by the following equations:

$$x_n(n_t^k)^\eta(c_t^k)^\sigma = \frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^k \tag{D1'}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}\tau\tau_t^{v+v^*-1} = \beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}Q_t\tau\tau_{t+1}^{v+v^*-1}\frac{1}{\pi_{t+1}^*} \tag{D2'}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}\tau\tau_t^{v-1}\left[1+\xi\left(\frac{k_t^k}{k_{t-1}^k}-1\right)\right]=\beta\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)}\tau\tau_{t+1}^{v-1}\times$$

$$\times\left[(1-\delta)+(1-\tau_{t+1}^k)r_{t+1}^k-\frac{\xi}{2}\left(\frac{k_{t+1}^k}{k_t^k}-1\right)^2+\xi\left(\frac{k_{t+1}^k}{k_t^k}-1\right)\frac{k_{t+1}^k}{k_t^k}\right] \tag{D3'}$$

$$x_m(m_t^k)^{-\mu}=\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}-\beta\frac{1}{\pi_{t+1}}\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D4'}$$

$$\frac{(c_t^k)^{-\sigma}}{(1+\tau_t^c)}=\beta R_t\frac{1}{\pi_{t+1}}\frac{(c_{t+1}^k)^{-\sigma}}{(1+\tau_{t+1}^c)} \tag{D5'}$$

$$k_t^k=(1-\delta)k_{t-1}^k+x_t^k-\frac{\xi}{2}\left(\frac{k_t^k}{k_{t-1}^k}-1\right)^2k_{t-1}^k \tag{D6'}$$

$$\frac{c_t^{k,H}}{c_t^{k,F}}=\frac{v}{1-v}\tau\tau_t \tag{D7'}$$

$$c_t^k=\frac{\left(c_t^{k,H}\right)^v\left(c_t^{k,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D8'}$$

$$\left[\left(c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right)+x_t^k+s_t^gy_t^H+s_t^iy_t^H+v_t+\left(c_t^{k,F*}+\frac{v^{w*}}{v^{k*}}c_t^{w,F*}+\frac{v^{b*}}{v^{k*}}c_t^{b,F*}\right)\right]= \tag{D9'}$$

$$=y_t^H\left[1-\frac{\phi^P}{2}\left(\frac{\pi_t^H}{\pi^H}-1\right)^2\right]$$

$$x_n(n_t^w)^\eta(c_t^w)^\sigma=\frac{(1-\tau_t^n)}{(1+\tau_t^c)}w_t^w \tag{D10'}$$

$$x_m(m_t^w)^{-\mu}=\frac{(c_t^w)^{-\sigma}}{1+\tau_t^c}-\beta\frac{1}{\pi_{t+1}}\left[\frac{(c_{t+1}^w)^{-\sigma}}{1+\tau_{t+1}^c}\right] \tag{D11'}$$

$$\frac{c_t^{w,H}}{c_t^{w,F}}=\frac{v}{1-v}\tau\tau_t \tag{D12'}$$

$$c_t^w = \frac{\left(c_t^{w,H}\right)^v \left(c_t^{w,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D13'}$$

$$(1+\tau_t^c)c_t^w + m_t^w = \frac{1}{\pi_t}m_{t-1}^w + (1-\tau_t^n)w_t^w n_t^w - v^k s_t^l y_t^H \tau\tau_t^{v-1} \tag{D14'}$$

$$x_m(m_t^b)^{-\mu} = \frac{(c_t^b)^{-\sigma}}{1+\tau_t^c} - \beta\frac{1}{\pi_{t+1}}\left[\frac{(c_{t+1}^b)^{-\sigma}}{1+\tau_{t+1}^c}\right] \tag{D15'}$$

$$\frac{c_t^{b,H}}{c_t^{b,F}} = \frac{v}{1-v}\tau\tau_t \tag{D16'}$$

$$c_t^b = \frac{\left(c_t^{b,H}\right)^v \left(c_t^{b,F}\right)^{1-v}}{v^v(1-v)^{1-v}} \tag{D17'}$$

$$(1+\tau_t^c)c_t^b + m_t^b = \frac{1}{\pi_t}m_{t-1}^b + (1-\tau_t^n)\frac{v^k}{v^b}s_t^w \tau\tau_t^{v-1} y_t^H - v^k s_t^l y_t^H \tau\tau_t^{v-1} \tag{D18'}$$

$$\tau\tau_t^{v-1} r_t^k k_{t-1}^k = mc_t \alpha y_t^H \tag{D19'}$$

$$w_t^k n_t^k = mc_t \theta(1-\alpha)y_t^H \tag{D20'}$$

$$\frac{v^w}{v^k}w_t^w n_t^w = mc_t(1-\theta)(1-\alpha)y_t^H \tag{D21'}$$

$$y_t^H = A_t\left\{[k_{t-1}^k]^\alpha\left[\{n_t^k\}^\theta \times \left\{\frac{v^w}{v^k}n_t^w\right\}^{1-\theta}\right]^{1-\alpha}\right\}\left(k_{t-1}^g\right)^{\theta_k} \tag{D22'}$$

$$\widetilde{\omega_t}^k = \tau\tau_t^{v-1}y_t^H - mc_t y_t^H - \frac{\phi^P}{2}\left(\frac{\pi_t^H}{\pi^H}-1\right)^2 \tau\tau_t^{v-1}y_t^H \tag{D23'}$$

$$Q_{t-1}\tau\tau_t^{v+v^*-1}\frac{1}{\pi_t^*}\tau\tau_{t-1}^{1-v-v^*}[1-\lambda_{t-1}]d_{t-1}+R_{t-1}\frac{1}{\pi_t}\lambda_{t-1}d_{t-1}+s_t^g y_t^H\tau\tau_t^{v-1}+$$

$$+s_t^i y_t^H\tau\tau_t^{v-1}+s_t^w y_t^H\tau\tau_t^{v-1}+\frac{1}{\pi_t}\left(m_{t-1}^k+\frac{v^w}{v^k}m_{t-1}^w+\frac{v^b}{v^k}m_{t-1}^b\right)= \tag{D24'}$$

$$=\left(m_t^k+\frac{v^w}{v^k}m_t^w+\frac{v^b}{v^k}m_t^b\right)+\tau_t^c\left[\tau\tau_t^{v-1}\left(c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right)+\right.$$

$$\left.+\tau\tau_t^v\left(c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right)\right]+\tau_t^k\left[r_t^k\tau\tau_t^{v-1}k_{t-1}^k+\widetilde{\omega}_t^k\right]+$$

$$+\tau_t^n\left[w_t^k n_t^k+\frac{v^w}{v^k}w_t^w n_t^w+s_t^w y_t^H\tau\tau_t^{v-1}\right]+s_t^l y_t^H\tau\tau_t^{v-1}+d_t$$

$$\tau\tau_t^{v-1}\left\{\left[c_t^{k,H}+\frac{v^w}{v^k}c_t^{w,H}+\frac{v^b}{v^k}c_t^{b,H}\right]+x_t^k+s_t^g y_t^H+s_t^i y_t^H-y_t^H\left[1-\frac{\phi^P}{2}\left(\frac{\pi_t^H}{\pi^H}-1\right)^2\right]\right\}+ \tag{D25'}$$

$$+\tau\tau_t^v\left[c_t^{k,F}+\frac{v^w}{v^k}c_t^{w,F}+\frac{v^b}{v^k}c_t^{b,F}\right]=\tau\tau_t^{v+v^*-1}(\tau\tau_t^{1-v-v^*}[1-\lambda_t]d_t-f_t^k)-$$

$$-Q_{t-1}\tau\tau_t^{v+v^*-1}\frac{1}{\pi_t^*}(\tau\tau_{t-1}^{1-v-v^*}[1-\lambda_{t-1}]d_{t-1}-f_{t-1}^k)+\widetilde{\pi}_t^k$$

$$\frac{\pi_t}{\pi_t^H}=\left(\frac{\tau\tau_t}{\tau\tau_{t-1}}\right)^{1-v} \tag{D26'}$$

$$(1-\phi)\tau\tau_t^{v-1}y_t^H+\phi mc_t y_t^H-\phi^P\left[\frac{\pi_t^H}{\pi^H}-1\right]\tau\tau_t^{v-1}\frac{y_t^H\pi_t^H}{\pi^H}=$$

$$\beta\phi^P\left[\left(\frac{1}{\pi_{t+1}}\right)\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)\left(\frac{c_{t+1}^k}{c_t^k}\right)^{-\sigma}\right]\left[1-\frac{\pi_{t+1}^H}{\pi^H}\right]\frac{\pi_{t+1}^H}{\pi^H}\tau\tau_{t+1}^{v-1}y_{t+1}^H \tag{D27'}$$

$$\widetilde{\pi}_t^k\equiv Q_{t-1}^*\left[\frac{1}{\pi_t}([1-\lambda_{t-1}^*]d_{t-1}^*\tau\tau_{t-1}^{v+v^*-1}-f_{t-1}^{k*})-\frac{\psi}{2}\frac{1}{\pi_t^H}\tau\tau_t^{v-1}([1-\lambda_{t-1}^*]d_{t-1}^*\tau\tau_{t-1}^{v+v^*-1}-f_{t-1}^{k*})^2\right]$$

$$-Q_{t-1}\tau\tau_t^{v+v^*-1}\frac{1}{\pi_t^*}\left(f_{t-1}^k-\tau\tau_{t-1}^{1-v-v^*}[1-\lambda_{t-1}]d_{t-1}\right) \tag{D28'}$$

$$Q^*_{t-1} = \frac{Q_{t-1}\epsilon_t}{1 - \psi\tau\tau^{v-1}_{t-1}([1-\lambda^*_{t-1}]d^*_{t-1}\tau\tau^{v+v^*-1}_{t-1} - f^{k*}_{t-1})} \tag{D29'}$$

$$v_t \equiv Q^*_{t-1}\frac{\psi}{2}\frac{1}{\pi^H_t}([1-\lambda^*_{t-1}]d^*_{t-1}\tau\tau^{v+v^*-1}_{t-1} - f^{k*}_{t-1})^2 \tag{D30'}$$

$$y^g_t = A\left(s^g_t y^H_t\right)^{\theta_g}\left(\frac{v^b}{v^k}\right)^{1-\theta_g} \tag{D31'}$$

$$s^i_t y^H_t = k^g_t - (1-\delta^g)k^g_{t-1} + \frac{\xi^g}{2}\left(\frac{k^g_t}{k^g_{t-1}} - 1\right)^2 k^g_{t-1} \tag{D32'}$$

$$l_{t-1} \equiv \frac{R_{t-1}\lambda_{t-1}d_{t-1}\tau\tau^{1-v}_{t-1} + Q_{t-1}\epsilon_t(1-\lambda_{t-1})d_{t-1}\tau\tau^{1-v}_{t-1}}{y^H_{t-1}} \tag{D33'}$$

Next, the foreign country is summarized by the following equations:

$$x^*_n(n^{k*}_t)^{\eta*}(c^{k*}_t)^{\sigma*} = \frac{(1-\tau^{n*}_t)}{(1+\tau^{c*}_t)}w^{k*}_t \tag{D34'}$$

$$\frac{(c^{k*}_t)^{-\sigma*}}{(1+\tau^{c*}_t)}\tau\tau^{1-v-v^*}_t = \beta^*\frac{(c^{k*}_{t+1})^{-\sigma*}}{(1+\tau^{c*}_{t+1})}Q^*_t\tau\tau^{1-v-v^*}_{t+1}\frac{1}{\pi_{t+1}} \tag{D35'}$$

$$\begin{aligned}\frac{(c^{k*}_t)^{-\sigma*}}{(1+\tau^{c*}_t)}\tau\tau^{1-v^*}_t\left[1+\xi^*\left(\frac{k^{k*}_t}{k^{k*}_{t-1}}-1\right)\right] &= \beta^*\frac{(c^{k*}_{t+1})^{-\sigma*}}{(1+\tau^{c*}_{t+1})}\tau\tau^{1-v^*}_{t+1}\times \\ \times\left[(1-\delta^*)+(1-\tau^{k*}_{t+1})r^{k*}_{t+1} - \frac{\xi^*}{2}\left(\frac{k^{k*}_{t+1}}{k^{k*}_t}-1\right)^2 + \xi^*\left(\frac{k^{k*}_{t+1}}{k^{k*}_t}-1\right)\frac{k^{k*}_{t+1}}{k^{k*}_t}\right]&\end{aligned} \tag{D36'}$$

$$x^*_m(m^{k*}_t)^{-\mu*} = \frac{(c^{k*}_t)^{-\sigma*}}{(1+\tau^{c*}_t)} - \beta^*\frac{1}{\pi^*_{t+1}}\frac{(c^{k*}_{t+1})^{-\sigma*}}{(1+\tau^{c*}_{t+1})} \tag{D37'}$$

$$\frac{(c^{k*}_t)^{-\sigma*}}{(1+\tau^{c*}_t)} = \beta^* R^*_t\frac{1}{\pi^*_{t+1}}\frac{(c^{k*}_{t+1})^{-\sigma*}}{(1+\tau^{c*}_{t+1})} \tag{D38'}$$

$$k^{k*}_t = (1-\delta^*)k^{k*}_{t-1} + x^{k*}_t - \frac{\xi^*}{2}\left(\frac{k^{k*}_t}{k^{k*}_{t-1}}-1\right)^2 k^{k*}_{t-1} \tag{D39'}$$

$$\frac{c_t^{k,H*}}{c_t^{k,F*}} = \frac{v^*}{1-v^*}\frac{1}{\tau\tau_t} \tag{D40'}$$

$$c_t^{k*} = \frac{\left(c_t^{k,H*}\right)^{v*}\left(c_t^{k,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D41'}$$

$$\left[\left(c_t^{k,H*} + \frac{v^{w*}}{v^{k*}}c_t^{w,H*} + \frac{v^{b*}}{v^{k*}}c_t^{b,H*}\right) + x_t^{k*} + s_t^{g*}y_t^{H*} + s_t^{i*}y_t^{H*} + \left(c_t^{k,F} + \frac{v^w}{v^k}c_t^{w,F} + \frac{v^b}{v^k}c_t^{b,F}\right)\right] =$$

$$= y_t^{H*}\left[1 - \frac{\phi^{P*}}{2}\left(\frac{\pi_t^{H*}}{\pi^{H*}} - 1\right)^2\right] \tag{D42'}$$

$$x_n^*(n_t^{w*})^{\eta*}(c_t^{w*})^{\sigma*} = \frac{(1-\tau_t^{n*})}{(1+\tau_t^{c*})}w_t^{w*} \tag{D43'}$$

$$x_m^*(m_t^{w*})^{-\mu*} = \frac{(c_t^{w*})^{-\sigma*}}{1+\tau_t^{c*}} - \beta^*\frac{1}{\pi_{t+1}^*}\left[\frac{(c_{t+1}^{w*})^{-\sigma*}}{1+\tau_{t+1}^{c*}}\right] \tag{D44'}$$

$$\frac{c_t^{w,H*}}{c_t^{w,F*}} = \frac{v^*}{1-v^*}\frac{1}{\tau\tau_t} \tag{D45'}$$

$$c_t^{w*} = \frac{\left(c_t^{w,H*}\right)^{v*}\left(c_t^{w,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D46'}$$

$$(1+\tau_t^{c*})c_t^{w*} + m_t^{w*} = \frac{1}{\pi_t^*}m_{t-1}^{w*} + (1-\tau_t^{n*})w_t^{w*}n_t^{w*} - v^{k*}s_t^{l*}y_t^{H*}\tau\tau_t^{1-v^*} \tag{D47'}$$

$$x_m^*(m_t^{b*})^{-\mu*} = \frac{(c_t^{b*})^{-\sigma*}}{1+\tau_t^{c*}} - \beta^*\frac{1}{\pi_{t+1}^*}\left[\frac{(c_{t+1}^{b*})^{-\sigma*}}{1+\tau_{t+1}^{c*}}\right] \tag{D48'}$$

$$\frac{c_t^{b,H*}}{c_t^{b,F*}} = \frac{v^*}{1-v^*}\frac{1}{\tau\tau_t} \tag{D49'}$$

$$c_t^{b*} = \frac{\left(c_t^{b,H*}\right)^{v*}\left(c_t^{b,F*}\right)^{1-v*}}{v^{*v^*}(1-v^*)^{1-v^*}} \tag{D50'}$$

$$(1+\tau_t^{c*})c_t^{b*} + m_t^{b*} = \frac{1}{\pi_t^*}m_{t-1}^{b*} + (1-\tau_t^{n*})\frac{v^{k*}}{v^{b*}}s_t^{w*}\tau\tau_t^{1-v^*}y_t^{H*} - v^{k*}s_t^{l*}y_t^{H*}\tau\tau_t^{1-v^*} \tag{D51'}$$

$$\tau\tau_t^{1-v^*} r_t^{k*} k_{t-1}^{k*} = mc_t^* \alpha^* y_t^{H*} \tag{D52'}$$

$$w_t^{k*} n_t^{k*} = mc_t^* \theta^* (1-\alpha^*) y_t^{H*} \tag{D53'}$$

$$\frac{v^{w*}}{v^{k*}} w_t^{w*} n_t^{w*} = mc_t^* (1-\theta^*)(1-\alpha^*) y_t^{H*} \tag{D54'}$$

$$y_t^{H*} = A_t^* \left\{ [k_{t-1}^{k*}]^{\alpha^*} \left[\{n_t^{k*}\}^{\theta^*} \times \left\{ \frac{v^{w*}}{v^{k*}} n_t^{w*} \right\}^{1-\theta^*} \right]^{1-\alpha^*} \right\} \left(k_{t-1}^{g*} \right)^{\theta_k^*} \tag{D55'}$$

$$\widetilde{\omega_t}^{k*} = \tau\tau_t^{1-v^*} y_t^{H*} - mc_t^* y_t^{H*} - \frac{\phi^{P*}}{2} \left(\frac{\pi_t^{H*}}{\pi^{H*}} - 1 \right)^2 \tau\tau_t^{1-v^*} y_t^{H*} \tag{D56'}$$

$$\begin{aligned}
& Q_{t-1}^* \tau\tau_t^{1-v-v^*} \frac{1}{\pi_t} [1-\lambda_{t-1}^*] d_{t-1}^* \tau\tau_{t-1}^{v+v^*-1} + R_{t-1}^* \frac{1}{\pi_t^*} \lambda_{t-1}^* d_{t-1}^* + s_t^{g*} y_t^{H*} \tau\tau_t^{1-v^*} + \\
& s_t^{i*} y_t^{H*} \tau\tau_t^{1-v^*} + s_t^{w*} y_t^{H*} \tau\tau_t^{1-v^*} + \frac{1}{\pi_t^*} \left(m_{t-1}^{k*} + \frac{v^{w*}}{v^{k*}} m_{t-1}^{w*} + \frac{v^{b*}}{v^{k*}} m_{t-1}^{b*} \right) = \\
& = \tau_t^{c*} \left[\tau\tau_t^{1-v^*} \left(c_t^{k,H*} + \frac{v^{w*}}{v^{k*}} c_t^{w,H*} + \frac{v^{b*}}{v^{k*}} c_t^{b,H*} \right) + \tau\tau_t^{-v^*} \left(c_t^{k,F*} + \frac{v^{w*}}{v^{k*}} c_t^{w,F*} + \frac{v^{b*}}{v^{k*}} c_t^{b,F*} \right) \right] + \\
& + \left(m_t^{k*} + \frac{v^{w*}}{v^{k*}} m_t^{w*} + \frac{v^{b*}}{v^{k*}} m_t^{b*} \right) + \tau_t^{k*} \left[r_t^{k*} \tau\tau_t^{1-v^*} k_{t-1}^{k*} + \widetilde{\omega_t}^{k*} \right] + \\
& + \tau_t^{n*} \left[w_t^{k*} n_t^{k*} + \frac{v^{w*}}{v^{k*}} w_t^{w*} n_t^{w*} + s_t^{w*} y_t^{H*} \tau\tau_t^{1-v^*} \right] + s_t^{l*} y_t^{H*} \tau\tau_t^{1-v^*} + d_t^*
\end{aligned} \tag{D57'}$$

$$\begin{aligned}
& \tau\tau_t^{1-v^*} \left\{ \left[c_t^{k,H*} + \frac{v^{w*}}{v^{k*}} c_t^{w,H*} + \frac{v^{b*}}{v^{k*}} c_t^{b,H*} \right] + x_t^{k*} + s_t^{g*} y_t^{H*} + s_t^{i*} y_t^{H*} - y_t^{H*} \left[1 - \frac{\phi^{P*}}{2} \left(\frac{\pi_t^{H*}}{\pi^{H*}} - 1 \right)^2 \right] \right\} + \\
& + \tau\tau_t^{-v^*} \left[c_t^{k,F*} + \frac{v^{w*}}{v^{k*}} c_t^{w,F*} + \frac{v^{b*}}{v^{k*}} c_t^{b,F*} \right] = \\
& = \tau\tau_t^{1-v-v^*} ([1-\lambda_t^*] d_t^* \tau\tau_t^{v+v^*-1} - f_t^{k*}) - Q_{t-1}^* \tau\tau_t^{1-v-v^*} \frac{1}{\pi_t} ([1-\lambda_{t-1}^*] d_{t-1}^* \tau\tau_{t-1}^{v+v^*-1} - f_{t-1}^{k*})
\end{aligned} \tag{D58'}$$

$$\frac{\tau\tau_t}{\tau\tau_{t-1}} = \epsilon_t \frac{\pi_t^{H*}}{\pi_t^{H}} \tag{D59'}$$

$$\frac{\pi_t^*}{\pi_t^{H*}} = \left(\frac{\tau\tau_t}{\tau\tau_{t-1}}\right)^{v^*-1} \tag{D60'}$$

$$\begin{aligned}(1-\phi^*)\tau\tau_t^{1-v^*} y_t^{H*} + \phi^* mc_t^* y_t^{H*} - \phi^{P*}\left[\frac{\pi_t^{H*}}{\pi^{H*}} - 1\right]\tau\tau_t^{1-v^*}\frac{y_t^{H*}\pi_t^{H*}}{\pi^{H*}} = \\ \beta^*\phi^{P*}\left[\left(\frac{1}{\pi_{t+1}^*}\right)\left(\frac{1+\tau_t^{c*}}{1+\tau_{t+1}^{c*}}\right)\left(\frac{c_{t+1}^{k*}}{c_t^{k*}}\right)^{-\sigma*}\right]\left[1-\frac{\pi_{t+1}^{H*}}{\pi^{H*}}\right]\frac{\pi_{t+1}^{H*}}{\pi^{H*}}\tau\tau_{t+1}^{1-v^*} y_{t+1}^{H*}\end{aligned} \tag{D61'}$$

$$y_t^{g*} = A^*\left(s_t^{g*} y_t^{H*}\right)^{\theta_g^*}\left(\frac{v^{b*}}{v^{k*}}\right)^{1-\theta_g^*} \tag{D62'}$$

$$s_t^{i*} y_t^{H*} = k_t^{g*} - (1-\delta^{g*})k_{t-1}^{g*} + \frac{\xi^{g*}}{2}\left(\frac{k_t^{g*}}{k_{t-1}^{g*}} - 1\right)^2 k_{t-1}^{g*} \tag{D63'}$$

$$l_{t-1}^* \equiv \frac{R_{t-1}^*\lambda_{t-1}^* d_{t-1}^* \tau\tau_{t-1}^{v^*-1} + Q_{t-1}^*\frac{1}{\epsilon_t}(1-\lambda_{t-1}^*)d_{t-1}^*\tau\tau_{t-1}^{v^*-1}}{y_{t-1}^{H*}} \tag{D64'}$$

We finally have the feedback monetary and fiscal policy rules:

$$\begin{aligned}log\left(\frac{R_t}{R}\right) = \phi_\pi\left[\tilde{\eta} log\left(\frac{\pi_t}{\pi}\right) + (1-\tilde{\eta}) log\left(\frac{\pi_t^*}{\pi^*}\right)\right] + \\ + \phi_y\left[\tilde{\eta} log\left(\frac{y_t^H}{y^H}\right) + (1-\tilde{\eta}) log\left(\frac{y_t^{H*}}{y^{H*}}\right)\right]\end{aligned} \tag{D65'}$$

$$s_t^g = s^g - \gamma_l^g\left(l_{t-1} - l\right) \tag{D66'}$$

$$s_t^i = s^i - \gamma_l^i\left(l_{t-1} - l\right) \tag{D67'}$$

$$s_t^w = s^w - \gamma_l^w\left(l_{t-1} - l\right) \tag{D68'}$$

$$\tau_t^c = \tau^c + \gamma_l^c\left(l_{t-1} - l\right) \tag{D69'}$$

$$\tau_t^k = \tau^k + \gamma_l^k (l_{t-1} - l) \tag{D70'}$$

$$\tau_t^n = \tau^n + \gamma_l^n (l_{t-1} - l) \tag{D71'}$$

$$s_t^{g*} = s^{g*} - \gamma_l^{g*} (l_{t-1}^* - l^*) \tag{D72'}$$

$$s_t^{i*} = s^{i*} - \gamma_l^{i*} (l_{t-1}^* - l^*) \tag{D73'}$$

$$s_t^{w*} = s^{w*} - \gamma_l^{w*} (l_{t-1}^* - l^*) \tag{D74'}$$

$$\tau_t^{c*} = \tau^{c*} + \gamma_l^{c*} (l_{t-1}^* - l^*) \tag{D75'}$$

$$\tau_t^{k*} = \tau^{k*} + \gamma_l^{k*} (l_{t-1}^* - l^*) \tag{D76'}$$

$$\tau_t^{n*} = \tau^{n*} + \gamma_l^{n*} (l_{t-1}^* - l^*) \tag{D77'}$$

Therefore, we have in total 64 equations in 64 endogenous variables and 13 feedback policy rules in 13 policy instruments. Specifically, we have for the home country 33 equations, Eqs.(D1')-(D33'), in 33 endogenous variables, which are $[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, f_t^k, Q_t, y_t^H, y_t^g, mc_t, \widetilde{\omega}_t^k, v_t, \widetilde{\pi}_t^k, \pi_t, \pi_t^H, \tau\tau_t, d_t, l_t]$, as well as 7 feedback policy rules, Eqs.(D65')-(D71') in 7 policy instruments which are $[R_t, s_t^g, s_t^i, s_t^w, \tau_t^c, \tau_t^k, \tau_t^n]$. Also, we have for the foreign country 31 equations, Eqs.(D34')-(D64'), in 31 endogenous variables, which are $[c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*}, c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{H*}, y_t^{g*}, mc_t^*, \widetilde{\omega}_t^{k*}, \pi_t^*, \pi_t^{H*}, d_t^*, l_t^*]$, as well as 6 feedback policy rules, Eqs.(D72')-(D77'), in 6 policy instruments, which are $[s_t^{g*}, s_t^{i*}, s_t^{w*}, \tau_t^{c*}, \tau_t^{k*}, \tau_t^{n*}]$. This is given the exogenous variables, $[\epsilon_t, \lambda_t, \lambda_t^*, A_t, A_t^*, s_t^l, s_t^{l*}]$, initial conditions for the state variables, $[k_{-1}^k, k_{-1}^g, f_{-1}^k, d_{-1}, R_{-1}, m_{-1}^k, m_{-1}^w, m_{-1}^b, k_{-1}^{k*}, k_{-1}^{g*}, f_{-1}^{k*}, d_{-1}^*, R_{-1}^*, Q_{-1}^*, m_{-1}^{k*}, m_{-1}^{w*}, m_{-1}^{b*}, \tau\tau_{-1}, l_{-1}, l_{-1}^*]$, and the values of the feedback (monetary and fiscal) policy coefficients in the policy rules.

Conclusively, we have a system of 64 equations, Eqs.(D1')-(D33') and Eqs.(D34')-(D64'), in the 64 following endogenous variables

$[c_t^k, c_t^{k,H}, c_t^{k,F}, c_t^w, c_t^{w,H}, c_t^{w,F}, c_t^b, c_t^{b,H}, c_t^{b,F}, n_t^k, w_t^k, n_t^w, w_t^w, m_t^k, m_t^w, m_t^b, r_t^k, k_t^k, k_t^g, x_t^k, f_t^k,$

$Q_t, y_t^H, y_t^g, mc_t, \widetilde{\omega}_t^k, v_t, \widetilde{\pi}_t^k, \pi_t, \pi_t^H, \tau\tau_t, d_t, l_t, c_t^{k*}, c_t^{k,H*}, c_t^{k,F*}, c_t^{w*}, c_t^{w,H*}, c_t^{w,F*}, c_t^{b*}, c_t^{b,H*}, c_t^{b,F*}, n_t^{k*}, w_t^{k*}, n_t^{w*}, w_t^{w*}, m_t^{k*}, m_t^{w*}, m_t^{b*}, r_t^{k*}, k_t^{k*}, k_t^{g*}, x_t^{k*}, R_t^*, f_t^{k*}, Q_t^*, y_t^{H*}, y_t^{g*}, mc_t^*, \widetilde{\omega}_t^{k*}, \pi_t^*, \pi_t^{H*}, d_t^*, l_t^*$] as well as 13 feedback policy rules, Eqs.(D65')-(D71') and Eqs.(D72')-(D77'), in the following 13 policy instruments [$R_t, s_t^g, s_t^i, s_t^w, \tau_t^c, \tau_t^k, \tau_t^n, s_t^{g*}, s_t^{i*}, s_t^{w*}, \tau_t^{c*}, \tau_t^{k*}, \tau_t^{n*}$].

References

[1] Angeletos, G., Collard, F., and Dellas, H. Quantifying Confidence. *Econometrica 86*, 5 (September 2018), 1689–1726.

[2] Baxter, M., and King, R. G. Fiscal Policy in General Equilibrium. *American Economic Review 83*, 3 (June 1993), 315–334.

[3] Benigno, P., Eggertsson, G. B., and Romei, F. Dynamic Debt Deleveraging and Optimal Monetary Policy. NBER Working Papers 20556, National Bureau of Economic Research, Inc, Oct. 2014.

[4] Bi, H., Leeper, E. M., and Leith, C. Uncertain fiscal consolidations. *The Economic Journal 123*, 566 (2013), F31–F63.

[5] Coenen, G., Erceg, C. J., Freedman, C., Furceri, D., Kumhof, M., Lalonde, R., Laxton, D., Lindé, J., Mourougane, A., Muir, D., Mursula, S., and d, C. Effects of Fiscal Stimulus in Structural Models. *American Economic Journal: Macroeconomics 4*, 1 (January 2012), 22–68.

[6] Coenen, G., Mohr, M., and Straub, R. Fiscal consolidation in the euro area: Long-run benefits and short-run costs. *Economic Modelling 25*, 5 (2008), 912–932.

[7] Cogan, J. F., Taylor, J. B., Wieland, V., and Wolters, M. H. Fiscal consolidation strategy. *Journal of Economic Dynamics and Control 37*, 2 (2013), 404 – 421. Fiscal Policy in the Aftermath of the Financial Crisis.

[8] Cúrdia, V., and Woodford, M. Conventional and unconventional monetary policy. *Review*, May (2010), 229–264.

[9] Cúrdia, V., and Woodford, M. The central-bank balance sheet as an instrument of monetarypolicy. *Journal of Monetary Economics 58*, 1 (January 2011), 54–79.

[10] Davig, T., and Leeper, E. M. Monetary-fiscal policy interactions and fiscal stimulus. *European Economic Review 55*, 2 (February 2011), 211–227.

[11] Economides, G., Papageorgiou, D., Philippopoulos, A., and Vassilatos, V. Smaller Public Sectors in the Euro Area: Aggregate and Distributional Implications. *CESifo Economic Studies 59*, 3 (September 2013), 536–558.

[12] Economides, G., Philippopoulos, A., and Vassilatos, V. Public, or private, providers of public goods? A dynamic general equilibrium study. *European Journal of Political Economy 36*, C (2014), 303–327.

[13] Erceg, C. J., and Lindé, J. Fiscal consolidation in a currency union: Spending cuts vs. tax hikes. *Journal of Economic Dynamics and Control 37*, 2 (2013), 422–445.

[14] EuropeanCommission. Public Finances in EMU, Brussels(2015).

[15] Fisher, W. H., and Turnovsky, S. J. Public Investment, Congestion, and Private Capital Accumulation. *Economic Journal 108*, 447 (March 1998), 399–413.

[16] Forni, L., Gerali, A., and Pisani, M. The macroeconomics of fiscal consolidations in euro area countries. *Journal of Economic Dynamics and Control 34*, 9 (2010), 1791–1812.

[17] Gali, J., et al. Introduction to monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework [monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework]. *Introductory Chapters* (2008).

[18] Gali, J., and Monacelli, T. Optimal monetary and fiscal policy in a currency union. *Journal of International Economics 76*, 1 (September 2008), 116–132.

[19] Hornstein, A., Krusell, P., and Violante, G. L. The Effects of Technical Change on Labor Market Inequalities. In *Handbook of Economic Growth*, P. Aghion and S. Durlauf, Eds., vol. 1 of *Handbook of Economic Growth*. Elsevier, July 2005, ch. 20, pp. 1275–1370.

[20] Leeper, E. M., Plante, M., and Traum, N. Dynamics of fiscal financing in the united states. *Journal of Econometrics 156*, 2 (2010), 304–321.

[21] Linnemann, L. Macroeconomic effects of shocks to public employment. *Journal of Macroeconomics 31*, 2 (June 2009), 252–267.

[22] Lombardo, G., and Vestin, D. Welfare implications of Calvo vs. Rotemberg-pricing assumptions. *Economics Letters 100*, 2 (August 2008), 275–279.

[23] Philippopoulos, A., Varthalitis, P., and Vassilatos, V. Fiscal consolidation and its cross-country effects. *Journal of Economic Dynamics and Control 83*, C (2017), 55–106.

[24] Schmitt-Grohé, S., and Uribe, M. Optimal simple and implementable monetary and fiscal rules. *Journal of monetary Economics 54*, 6 (2007), 1702–1725.

[25] Walsh, C. *Monetary Theory and Policy, Third Edition*, 3 ed., vol. 1. The MIT Press, 2010.

[26] Wickens, M. Macroeconomic theory. *A Dynamic General Equilibrium Approach. Princeton, NJ* (2008).

[27] Wren-Lewis, S. Macroeconomic policy in light of the credit crunch: the return of counter-cyclical fiscal policy? *Oxford Review of Economic Policy 26*, 1 (Spring 2010), 71–86.