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Acceptance Sampling Theory and Applications

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Abstract

Acceptance Sampling is one the most important components of Statistical Quality Control. Its value and importance have been well known since World War II, when the United States used such techniques to ensure the quality of its ammunition. The aim of the present thesis is to analyze the main category of Statistical Quality Control which is Acceptance Sampling. Also, at the end of the thesis, an application is presented regarding Acceptance Sampling in the R language.

At first, we provide a historical review of Quality Control, highlighting important historical figures who have effectively contributed to research, development and establishment of Statistical Quality Control methods worldwide. Reference is also made to the components of Statistical Quality Control and Acceptance Sampling.

Then, a detailed description of the categories of Acceptance Sampling plans is made and the most important Sampling Schemes or sampling plans are presented for each category. The first category is Attribute Sampling plans, which are used if the item inspection leads to a binary result, conforming or nonconforming. Then we present the types of operating characteristic (OC) curves and their use to estimate the probability that a lot submitted with a certain fraction defective will be either accepted or rejected. Then, the types of Sampling plans are presented in detail, which are Single, Double, Multiple and Sequential Sampling plans, while reference is made to Rectification Sampling which serves in situations where the manufacturer wishes to know the average level of quality that is likely to result at a given stage of manufacturing operations. The second category of Acceptance Sampling plans is Variables Sampling plans which are used when actual quantitative information can be measured on sampled items, rather than simply classifying them as conforming or nonconforming. There is detailed presentation of k-Method and M-Method methods, each one analyzed with Standard Deviation known and unknown and how both Lower Specification Limit and Upper Specification Limit are calculated for each method separately.

Also, we describe the available Sampling Schemes, which are Attribute Sampling Schemes and Variables Sampling Schemes. In the Attribute Sampling Schemes category belong the MIL-STD-105E Sampling Scheme and Derivatives and the MIL-STD-1916. Then, there is a presentation of the MIL-STD-414 Sampling Scheme and Derivatives and the Continuous Sampling - CSP-1 which belong to the Variables Sampling Schemes category.

Then, in order to implement the application in the next Chapter, the R language was used with the AQLSchemesinR library. In this application, the customer performs sampling inspection in 12 lots that are partially submitted by the supplier over a period of 12 weeks, and the goal is to calculate how many will be accepted by them for different AQL values. We generated 12 random samples, each of which is N = 2000 size, from a Bernoulli distribution, and with double sampling and the sampling scheme MIL-STD-105E, we observe how many lots will eventually be accepted and how many will be returned to the supplier. In order to get as close as possible to the real number of lots that will be accepted, in this application we repeat 1000 times the previous process, and then we find how many lots on average will be accepted in these 12 weeks, for each value of AQL.

Finally, an epilogue is made regarding the importance and breadth of Acceptance Sampling worldwide based on what has been extensively analyzed in the previous Chapters, while at the same time we mention that the research so far in specialized fields of Acceptance Sampling, although satisfactory, can be further expanded.

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Acknowledgments

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In addition, I would like to thank all my postgraduate teachers, because it is through their guidance that I have acquired the necessary knowledge background in statistics and programming so that I managed to conduct my research.

Finally, I would like to thank my family for their love and moral support but also for their optimism during my studies. I would like to dedicate this dissertation to my parents. Through their hard work over the years, they created the opportunity for me and my siblings to do our studies.

Chapter 1 Introduction

The quality and price of a product have always been very important factors for its purchase. Most people prefer to spend more money to ensure the quality of the goods they buy. All consumers, as well as us, evaluate the products we buy by testing their quality. Both individuals and companies evaluate the quality of the products they buy. A defective product may not harm the former but may be detrimental to the reputation and revenue of the latter. For example, an automobile manufacturer with a large percentage of faulty brakes, will face serious economic consequences, its reputation will be damaged and possibly the board members may face legal consequences if negligence is found. Therefore, it is necessary to find ways to control the quality assurance of the produced goods, with the most established method being Statistical Quality Control (SQC).

1.1 Early History

Quality control has been known to everyone since the time of the creation of industries. According to Rumane (2013), the inclusion of process in quality practices took place from the very beginning of the twentieth century. Back then, AT&T Bell Laboratories began systematic inspection and control of products and materials. The Institute for Quality Assurance in England was established under the name “Union of Technical Inspectors” in 1919. About thirteen years after the systematic inspection of products, quality department is established in AT&T Bell Laboratories.

W. A. Shewhart played an important role in the study and research of Statistical Quality Control. In 1924, he introduced the control diagrams to AT&T Bell Laboratories and seven years later he published the journal Economic Control of Quality of Manufactured Product. An example of such diagrams is shown in Figures 1 and 2. According to Montgomery (2009), W. A. Shewhart began teaching statistical methods of production and control diagrams at the University of London the following year. After receiving an invitation from W.E. Deming, W. A. Shewhart goes to A.S. Department of Agriculture Graduate School to teach in seminars on control charts, having already achieved a career in this field. In the 1940 U.S. Census, some sampling techniques were used for the first time. These techniques had been developed by Deming.

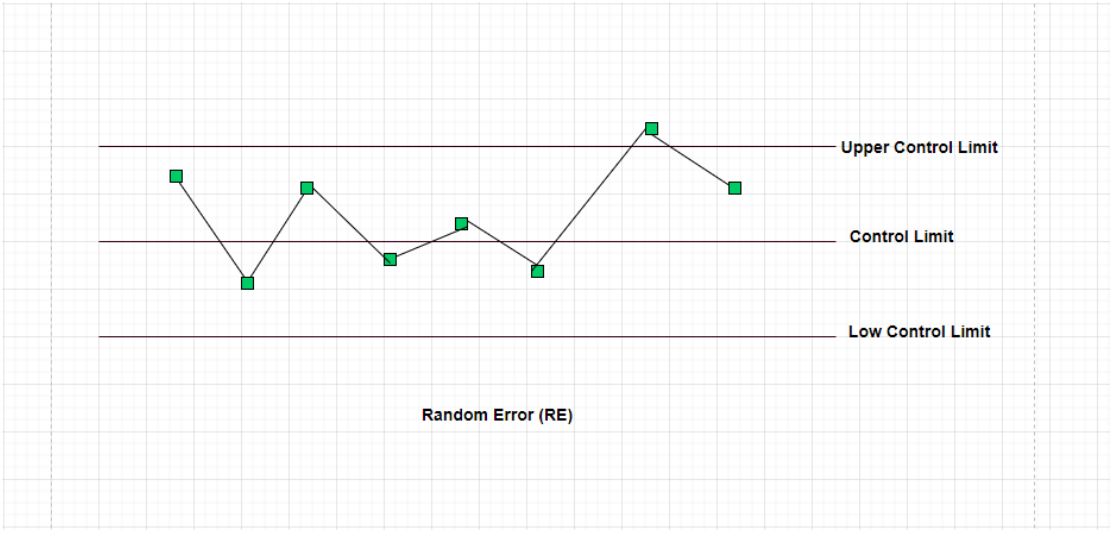

Figure 1 First example on Shewhart control chart

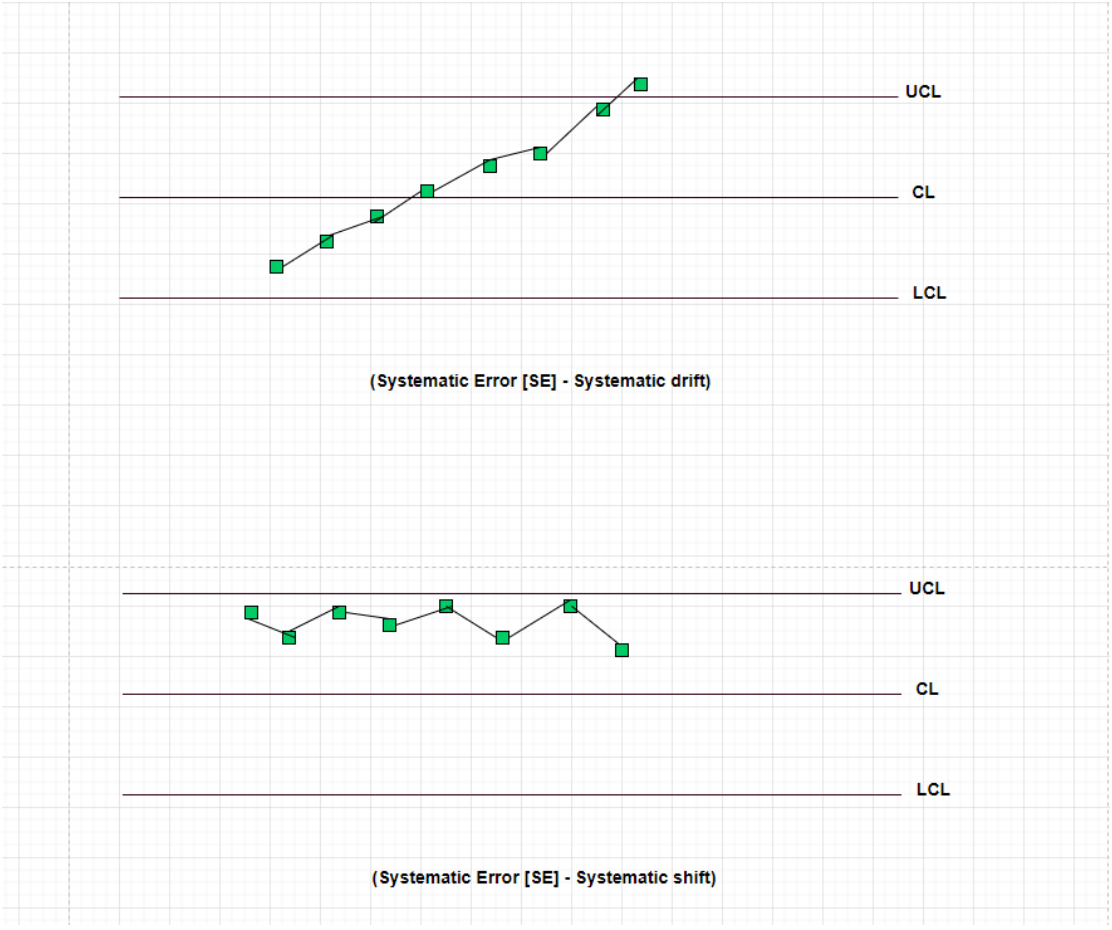

Figure 2 A second example on Shewhart control chart

According to "Timeline - W. Edwards Deming" (n.d.), Deming was sent to Japan by the Economic and Scientific Section of the War Department. The international domination of the japanese industry is largely due to him, as a quality revolution took place in Japan after World War II. Deming emphasized statistical diligence and control and tried to reduce variability in the production process.

1.2 Statistical Quality Control during WWII

The military has contributed in two ways to the adoption of statistical quality control methods by industry. On the one hand, the army's procurement services applied statistically derived sampling and inspection methods quite early. The second was that the War Department requested that an educational program should be implemented for a large part of the industrial personnel.

A sampling scheme based on an acceptable quality level (AQL) was employed by the Ordnance Sampling Inspection tables. The scheme assumed that there would be a continuing stream of lots submitted by a supplier. If the AQL was higher than the supplier's quality level, then there would be automatically tightened inspection to the scheme, which would lead to increased costs for the supplier due to the lots that would be returned to them as rejected. This scheme encouraged suppliers to improve quality (Burr, 2004).

Mainly suppliers of military equipment and others took part, in 1940, in an extensive training program organized by the army. Also, the American Standards Association developed American War Standards Z1.1-1941 and Guide for Quality Control Z1.2-1941, Control Chart Method of Analyzing Data-1941, and the Control Chart Method of Controlling Quality during Production Z1.3-1942 at the request of the War Department. All these played important role in the american control chart practice and contributed to subsequent training courses that took place at Stanford by Holbrook Working, E.L. Grant, and W. Edwards Deming.

In 1942, at Stanford University, representatives of the war industries and procurement agencies of the armed services attended an intensive course on Statistical Quality Control. Its duration was ten days and it contributed, along with more courses, to one’s of the foundations of the American Society for Quality Control creation ("Market conditions and international trade in semiconductors field...", 1980). The success of the program and the proposal of Dr. Walter A. Shewhart on the need for federal aid to be given to US war industries in developing statistical quality control applications, have led to the establishment of a nationwide program by the Office of Production and Research and Development (OPRD) of the War Production Board. The program provided immediate assistance to institutions for specific quality control problems and conducted intensive courses in high ranking executives of war industry.

This met the following needs:

1. Training of senior business executives on the benefits and importance of SPC,
2. Training of workers in the industries involved in quality control,
3. Advice on specific problems,
4. Training of individuals who would undertake the training of others,
5. Publication of papers related to the subject.

The main responsibility of the OPRD program was to train instructors. Experienced professors of statistics were used for this purpose who, through OPRD training, expanded their knowledge in specific techniques and theories related to Statistical Quality Control, became familiar with practical applications and learned

useful educational techniques. The courses, therefore, would be given to key quality control personnel from industry, by the university professors within the universities. This plan was implemented with administrative assistance and grants from the Engineering, Science and Management War Training Program (ESMWT). It was funded by the U.S. Office of Education.

Many of those previously trained, therefore trained staff working at their factory. OPRD argued that local teams should be set up, so that industries could train their staff. In this way, information and experiences could be more easily exchanged between neighboring establishments, which led to the creation of many regional quality control societies.

The publications of the American Standards Association and articles in engineering and technical journals, led to the creation of a literature on Statistical Quality Control. All of the above led to the wider acceptance of statistical quality control techniques during the war years. The factories made the great turnaround from civilian to military production due to the assurance of the quality and efficiency of the cost of manufactured goods. For example, the production of military aircraft rose to 85,000 in 1943 from 6,000 in 1940.

According to Boorstin (1974), at the conclusion of the War in 1946, seventeen of the local quality control societies formed during the war, organized themselves into the American Society for Quality Control (ASQC). This society was renamed as the «American Society for Quality» (ASQ), in order emphasize the fact that Quality is essential. An interesting fact is that an exhibit which preserves the memory of W.E. Deming's Red Bead Experiment (a teaching tool) stands outside the board room of ASQ in Milwaukee Wisconsin. It was used during the war effort, to show managers the futility of the standard reaction to common causes of variation.

The MIL-STD-105 Attributes Sampling Scheme was a result of the creation,the development and the use -even after the war- of sampling tables and sampling schemes for military procurement. It was later revised as 105B, 105C, 105D, and 105E. Additionally, Variables Sampling Schemes were created, developed and eventually resulted in MIL-STD-414.

1.3 Statistical Quality Control in Post-War Japan

During the war, many companies that previously produced goods for public consumption were forced to produce defense equipment. After the WWII, they went back to their previous production, but in many cases, they stopped applying statistical quality control methods in the manufacture of non-military goods. Several women who had worked in these industries during the war and knew the technical use of SPC were replaced by veterans who were uneducated in these matters.

Industry in Europe was destroyed, and the demand for american manufactured goods exceeded the supply. Because there was so much demand, the industry managers in the U.S. did not understand that there had to be an improvement in quality. As there has been an increase in U.S. economy in the 1950s, there was a sense in the industries

that everything was going well. At the same time, Japan was receiving aid from the U.S. occupation forces to rebuild its industry. W. E. Deming was sent to Japan to assist in planning the 1951 Japan Census at the request of General Douglas McArthur. The Japanese Society of Scientists and Engineers (JUSE) invited Deming to speak with them about SPC due to his expertise in quality control techniques and his compassion for the plight of the Japanese.

At that time, the reputation of Japanese construction was almost completely destroyed and the "Made in Japan" label was synonymous with junk. JUSE members became interested in Shewhart's ideas and asked someone to advise them on how they could rebuild their industry. This man was Deming, who trained a lot of people in Japanese industry, university professors, engineers and technicians in charge of Statistical Quality Control. Despite the widespread use of these techniques in Japan, in the United States these were used only during the war and then they were abandoned, as mentioned earlier. Deming didn't like that, and that's why in his meetings with industry executives he noted that everyone should follow the principle of quality improvement, reduced waste and rework, improved productivity, bigger market share, staying in business and job creation. Deming though, had the following 14 points:

1. Continuous improvement of products and services.
2. This new philosophy should be adopted.
3. Inspection should not have the role of achieving quality.
4. No buying at lowest price.
5. Improve every process.
6. Training on the job should be instituted.
7. Leadership should be instituted too.
8. No fear.
9. No barriers between staff departments.
10. No slogans.
11. No numerical goals.
12. People should be encouraged to feel proud of their work.
13. Education and self-improvement should be the goal.
14. Everyone in the company should work for the goal.

The Japanese manufacturers immediately applied the techniques and philosophy supported by Deming and other American quality specialists. Improved quality, along with lower cost of goods, allowed the Japanese to create new international markets for Japanese products, especially cars and consumer electronics. Japan has risen from the ashes of war to one of the largest economies in the world. Due to the fact that the executives of japanese war industries applied the techniques taught to them by Deming, the Japanese industry recovered rapidly, and this resulted in the Japanese creating international markets for almost all of their products. From junk, japanese products have become synonymous with reliability and high quality.

Deming did not want to accept royalties on the published transcripts of the lectures he gave in 1950. Then, the proceeds were used by the Japanese Union of Scientists and Engineers (JUSE) board to establish the Deming prize. This is symbolized by a silver medal depicting the Deming's profile. Each year, it is awarded to a company with a significant contribution to the statistical application and to an individual for significant research in statistical theory. In 1950, Deming predicted that

japanese products gained respect in worldwide markets, something that was true. In addition, Japanese began to contribute new insights to the body of knowledge regarding SQC.

Kaoru Ishikawa, who was a recipient of the Deming Prize, had the idea of quality circles. Based on this, the foreman and the employees met to apply problem-solving tools in their own process, and this was the beginning of participative management. Ishikawa also wrote the Guide to Quality Control which was translated into English and identified the key quality tools shown by Christensen et al. (2013), as well as other books on quality control. Genichi Taguchi developed the idea for off-line quality control, where products and processes are designed to be insensitive to common sources of variation outside the design engineers' control. The rise in the price of oil from $3 to $12 a barrel in 1973, due to the Arab Oil Embargo, increased demand for small japanese cars that had low fuel consumption. In the United States, smaller horsepower cars, in addition to being more economical in cost, have also been found to be more reliable mechanically. This led to the downturn in the US automotive industry until 1979, the dismissal of many workers and the closure of many factories. On the occasion of the NBC Documentary "If Japan Can, Why Can't We" (1979), several top executives of the american automotive industry were motivated to apply the quality technologies that Japan had been implementing for several years.

1.4 Re-emergence of Statistical Quality Control in U.S. and the world

Quality goals gained a place next to financial and marketing goals around 1980 and beyond. For example, the slogan "Quality is Job 1" became popular as it was used by the Ford Motor Company. They also coordinated training programs for both their suppliers and their employees.

As this tactic was followed by other industries, there was a shift in the U.S. from meeting manufacturing specifications to customer satisfaction, which can be said that it was a quality revolution. Several companies in various sectors, such as the health sector, the banking sector, customer service companies etc., adopted Statistical Quality Control as espoused by Deming. This had a significant effect on the recovery of American products such as electronics, steel, automobiles, etc. Thus, the same philosophy began to be embodied in a global movement.

The Quality Assurance Systems Standards (ISO 9000), according to "ISO 9000" (n.d.), were published in 1987 by the International Organization for Standardization. This meant the acceptance of the systems approach to producing Quality across the globe. European Free Trade Association (EFTA), requires certification of compliance to these standards for companies to take part in. The Malcolm Baldrige National Quality Award was established by the U.S. Congress in 1988. It was named after the late secretary of commerce and it is similar to the Deming prize in Japan. With that, the U.S. government recognized the need to have a competitive economy by producing quality products and services.

However, the practices of statistical quality control have changed over the years for various reasons. For example, the United States Department of Defense, in an effort to cut costs, replaced military standards for sampling inspection with civilian standards. Thus, ANSI / ASQC Z1.4 replaced the MIL-STD-105E attribute sampling inspection tables. This standard is best used for transactions within the U.S., while ISO 2851-1 is the international standard.

ANSI / ASQC Z1.9 is the civilian standard that replaced the MIL-STD-414 variables sampling plans. In this standard, the inspection levels coincide with the Z1.4 plans for attributes and common switching rules are adopted. ISO 3951-1 is the -widespread- international version where also the inspection levels are matched to the ISO 2851-1 attribute plans very closely.

Computer use has also revolutionized technical methodologies for quality control. Until 1963, mechanical or electro-mechanical calculators dominated, so statisticians and engineers formed the sampling inspection tables and Shewhart's control charts with the tools available at the time.

1.5 Components of Statistical Quality Control

A set of statistical data analysis methods compose Statistical Quality Control. We divide this set into three subsets, as follows:

- Design of Experiments
- Statistical Process Control
- Acceptance Sampling

Each subset contains statistical methods which correspond to different phases of the production process.

Design of Experiments contains the statistical techniques by which the effect of various variables on the quality parameters of the product can be ascertained and therefore, the optimal design of the production process depends significantly on this.

Statistical Process Control contains the statistical techniques that are necessary for control in the manufacturing process.

Acceptance Sampling contains the statistical techniques on the basis of which we reject or accept a batch of materials from a supplier.

1.5.1 Acceptance Sampling

Acceptance Sampling works as follows: The supplier provides the consumer (usually a business) with some batches or lots of the product. Using a sampling plan, the consumer inspects a random sample of chosen units regarding a qualitative characteristic that they have. Depending on the quality level that the consumer finds after the inspection, they must decide, based on the quality of the lot, whether they will accept or reject it. The rejected lot is subject to rework or it is returned to the producer.

In many cases, however, the supplier applies Acceptance Sampling to its production in order to check if their products meet the quality standards. With

Acceptance Sampling, we are able to implement an intermediate solution so as to avoid both the 100% inspection of the products (which costs a lot of time and money), but also to buy products of dubious quality without inspection at all. So, we apply Acceptance Sampling when:

a) if the inspection leads to product's desctruction (e.g. the quality of the photographic films, the measurement of the fragility of glass, etc.),

b) it is technically impossible to do 100% inspection or it costs a lot of time and money,

(c) the supplier has a quality history that is neither excellent (so we cannot avoid to inspect their products) nor unacceptable (so that a 100% inspection would be required).

The main advantages of Acceptance Sampling over 100% inspection are:

(a) lower cost,

(b) less time needed,

(c) smaller number of inspected units get destroyed during inspection,

(d) applicated easier when a catastrophic audit occurs,

(e) less personnel involved in the inspection process.

Some of the disadvantages of Acceptance Sampling are:

(a) there is always a risk that a high-quality batch will be rejected (type I error), or a low-quality batch will be accepted (type II error).

(b) rules are needed to be set for its implementation,

(c) we receive less information regarding the lot's quality.

Depending on the type of quality attribute we use to determine the quality of each unit, we classify Acceptance Sampling into two categories, the Attribute Sampling and the Variables Sampling category. In the first category, each unit of the lot is classified as defective or non-defective. This depends on whether the value of the quality attribute does not match or matches with the product's specifications, respectively. In the second category we have a continuous measurement of the quality of the product. Based on this measurement, we classify each unit as defective or non-defective. This depends on whether the value of the quality feature is located outside or within the specifications of the unit, respectively.

We can distinguish between single, double, and multiple sampling plans. In the first category, depending on the information we draw from a sample, we decide whether to accept the lot or not. The double and multiple sampling plans give us -optionally- the possibility to examine additional samples from the same lot, in order to make a decision on whether or not to accept that lot.

Consumers, however, usually do not just want to inspect a single lot. That is why there are plenty of sampling systems or schemes with many employees and predefined rules for the sampling plans used. One of them is the MIL-STD-105E sampling system.

However, in order for the consumer and the supplier to effectively implement Acceptance Sampling, they must:

(a) inspect batches that are homogeneous., i.e. made by the same machine / operators / materials, etc.,

(b) inspect batches that are large in size. Usually, the size of sample does not increase in full proportion to the batch size (Montgomery, 2009), and for this reason there is an advantage in resources and working hours,

c) use safe methods for their packaging and transport.

Chapter 2 Acceptance Sampling Plans

The quality and reliability of manufactured goods are highly dependent on the quality of component parts. If the quality of component parts is low, the quality and/or reliability of the end assembly will also be low. While some component parts are produced in house, many are procured from outside suppliers; the final quality is, therefore, highly dependent on them. In response to stiff competition, Ford Motor Company adopted procedural requirements for their suppliers in the early 1980s to ensure the quality of incoming component parts. They demanded that all their suppliers show that their production processes were in a state of statistical control with a capability index greater than 1.5. Because Ford Motor Company bought such a large quantity of component parts from their suppliers, they were able to make this demand. Smaller manufacturing companies may not have enough influence to make similar demands of their suppliers. However, by internal use of acceptance sampling procedures, they can be sure that the quality level of their incoming parts will be close to an agreed upon level.

We tend to treat Acceptance Sampling as an activity related to receiving inspection. However, it is not its only use. Acceptance sampling can also be used by the producer to inspect their own product at various stages of production. Any lots that are rejected will be re-worked or scrapped, while any that are accepted, will be processed in the next steps if any, as reported by Kadry (2018).

It is very important to understand that the purpose of acceptance sampling is not to estimate the quality index of the lot but whether or not we will reject the lot. Due to its structure, acceptance sampling does not provide quality control, but only the answer as to whether or not the lot is acceptable. Its purpose is to ensure, as a control tool, that the process meets our quality requirements.

2.1 Attribute Sampling Plans

In case the item inspection results in conforming or nonconforming items (binary result), then we use attribute sampling plans. An inspection is performed on a random sample of the batch when we use a sampling plan. When a very high number of nonconforming items is discovered in the sample of inspected components results, the batch is returned to the supplier (just as a defective product is returned to the store by the customer). When there are many nonconforming items in a batch or many nonconforming records in a period of time, every item in the lot may be inspected thoroughly. On the other hand, the lot is accepted without problems if the number of nonconforming items discovered in the sample is small.

In case we do not inspect more than one samples in the batch, there is always a probability bigger than zero that there are nonconforming items in the lot, despite the fact that every item in the sample is acceptable. If there is an agreement though between

the customer and the supplier regarding the maximum proportion of nonconforming items in the batch, then an attribute sampling plan can be used to reject the batches in which the proportion of defective items inspected exceeds the agreed rate. Thus, with the sampling plan there can be a benefit both for the customer (by maximizing the probability of rejecting batches with higher proportion of defective items than the agreed one) and for the supplier (by maximizing the probability of accepting batches with lower proportion of defective items than the agreed one).

The Operating Characteristic Curve (OC) is an important measure of the performance of an acceptance sampling plan, as stated by Dumičic et al. (2006). This curve shows the probability of accepting the lot versus the lot fraction defective, i.e. the discriminatory power of the sampling plan. That is, it shows the probability that a lot will be accepted or not. In the next Figure, we see a relevant example:

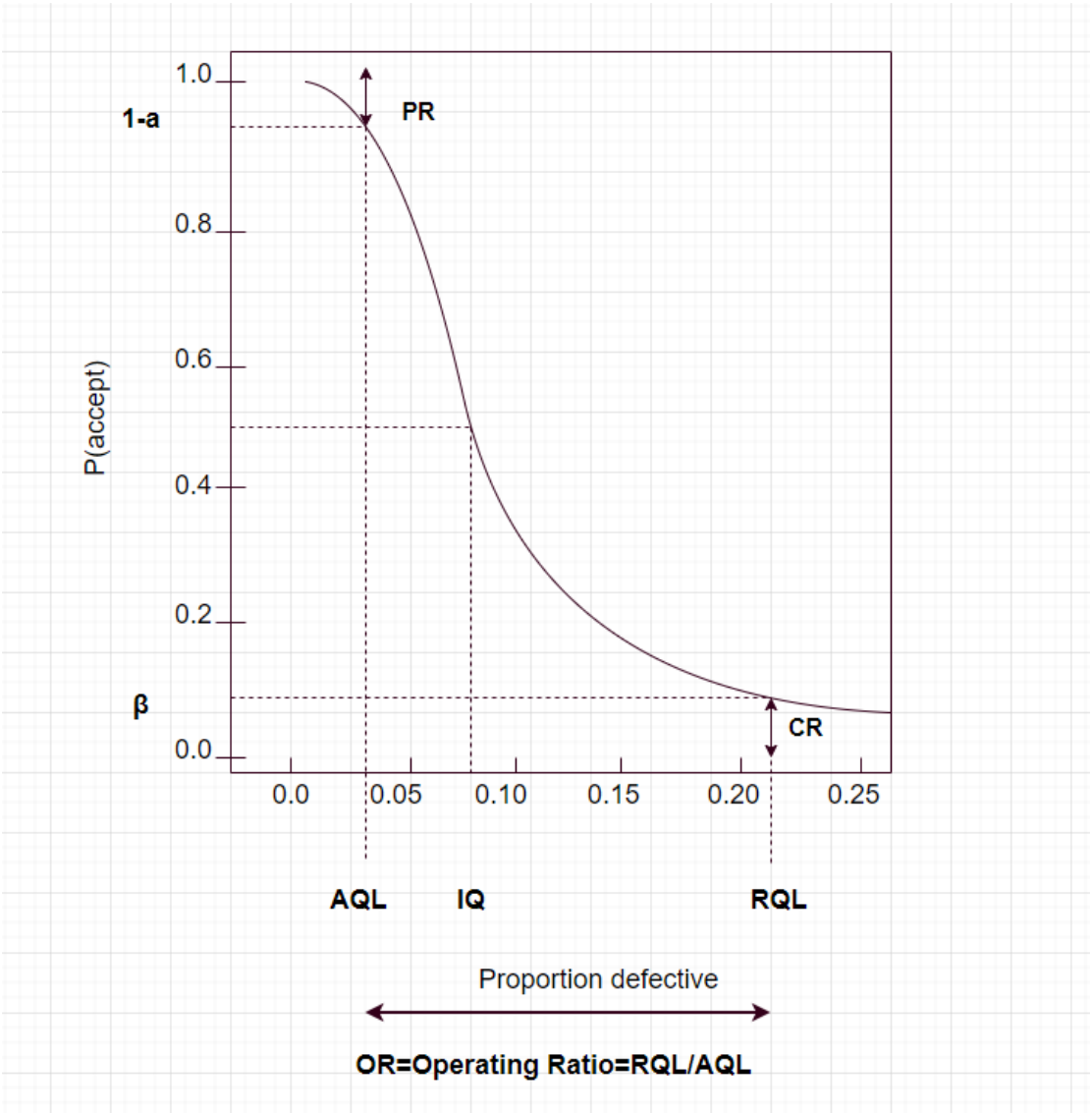

Figure 3 Operating Characteristic Curve

In Figure 3, the agreed upon maximum proportion of nonconforming items in a batch is represented by the AQL (Acceptance Quality Level). The probability that a lot with the AQL proportion nonconforming is accepted, is represented by $1-a$. Also, $PR=a$ represents the probability of rejecting a lot with AQL proportion nonconforming (supplier's risk). With IQ we define the indifference quality level in

which 50% of batches are rejected. RQL is the Rejectable Quality Level, which is something the customer decides on. In this case there is a probability b that the lot will be accepted. The standard values of $a = PR = 0.05$ and $b = CR = 0.10$ are provided from Schilling & Neubauer (2017).

Customers prefer a steeper OC curve with a smaller ratio of the RQL to the AQL. In this case, there is a smaller probability of accepting any batch with greater than the AQL proportion nonconforming. The supplier in this case is motivated to send batches that will not have proportion nonconforming greater than the AQL. In Figure 4 we see the ideal OC curve when using 100% inspection. We now accept lots with proportion nonconforming less than the AQL and otherwise reject them. In a sampling plan, as the fraction items sampled increases, the curve for the ideal case will be approached by the OC curve for that plan.

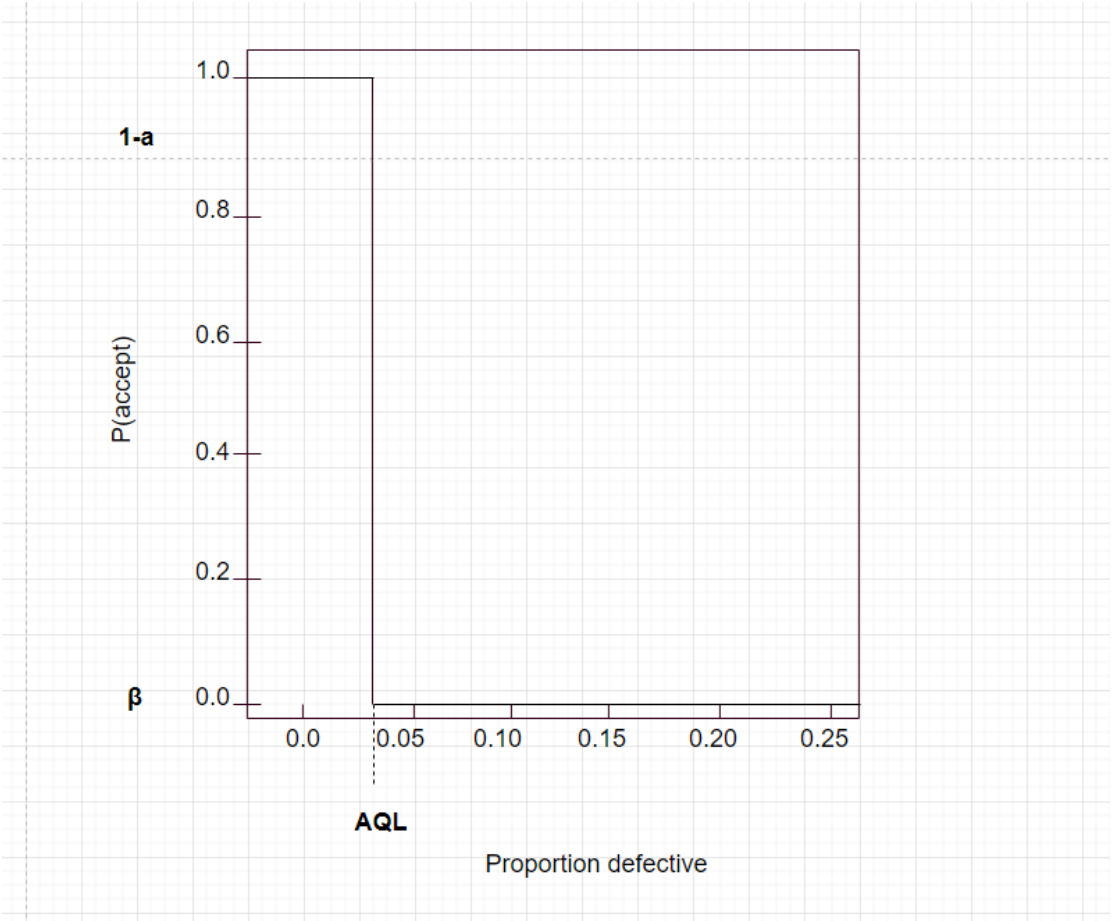

Figure 4 Customer's ideal Operating Characteristic Curve (OC curve)

Two types of Operating Characteristic Curves exist (Taylor, 1992), the type A and the type B curves.

Montgomery (2009), supports that Type-A Operating Characteristic Curves are used to calculate probabilities of accepting an isolated lot of finite size. Suppose that N is the lot size, n is the sample size, and c is the acceptance number. The hypergeometric distribution is the distribution of the number of nonconforming items in the sample. Therefore, it turns out that this is the probability of accepting a lot of size N with D nonconforming items:

$$P_a = \sum_{i=0}^{c} \frac{\binom{D}{i}\binom{N-D}{n-i}}{\binom{N}{n}}$$

Type-B Operating Characteristic Curves: In this case, we are under the assumption either that the samples come from a large lot or that we sample from a stream of randomly selected lots from a process. The exact probability distribution for calculating the probability of lot acceptance is the binomial distribution. Therefore, it turns out that this is the probability of accepting a lot of size N, sample size n, with d nonconforming items and c the max accepted number of nonconforming items:

$$P_a = \sum_{d=0}^{c} \frac{n!}{d!\,(n-d)!} p^d (1-p)^{n-d}$$

2.1.1 Single Sampling Plans

Suppose we inspect a lot of size N. The acceptance number is c and the sample size is n. Since if we assume that the size of the lot is $N = 10000$ and d is the number of nonconforming items, then we say that from a lot with size $N = 10000$, we inspect (i.e. we take a sample) $n = 89$ items. We also specify in our example that $c = 2$. The lot will be accepted if the number of nonconforming items d is less than or equal to $c = 2$. Correspondingly, the lot will be rejected if d is greater than $c = 2$. Therefore, each unit of the sample will be either conforming or nonconforming based on one or more quality features we are inspecting. If one or more features are not covered by the technical specifications of the unit we are inspecting, then this unit is called a defective unit. Montgomery (2009) considers that when the lot is sentenced based on the information contained in a sample of size n, then we say that we are performing the "single-sampling plan" process.

We can obtain single sampling plans such as the MIL-STD-105E from published tables which are indexed by the lot size and AQL. These tables, by agreement of the customer and the supplier, are used especially in cases where a continuous stream of lots is purchased.

In order to inspect isolated lots, we can construct custom derived sampling plans. In order to determine the size n of the sample and the acceptance number c, analytic procedures have been developed such that the probability of a lot acceptance with the AQL proportion nonconforming will be as close as possible to $1-a$, and the probability of a lot acceptance with RQL proportion nonconforming will be as close as possible to β.

If we construct a sampling plan, such that if there are lots with fraction defective p_1 and lots with fraction defective p_2, the probabilities of acceptance are $1-a$ and b respectively. Montgomery (2009), proposed that the solution with sample size n and acceptance number c, since we always assume that binomial sampling with type-B Operating Characteristic curve is appropriate, is:

$$1 - a = \sum_{d=0}^{c} \frac{n!}{d!\,(n-d)!} {p_1}^d (1-p_1)^{n-d}$$

$$\beta = \sum_{d=0}^{c} \frac{n!}{d!\,(n-d)!} {p_2}^d (1-p_2)^{n-d}$$

2.1.2 Double, Multiple and Sequential Sampling Plans

Hazewinkel (1997), states that there are some single sampling plans for attributes extensions such as double sampling plans, multiple-sampling plans, and sequential-sampling plans.

Double-Sampling Plans

The process by which a second sample is required for the lot to be sentenced or when it is desirable to give questionable lots a second chance, is called "double-sampling plan". If we cannot decide on the rejection or acceptance of the batch based on the results of the first sample, then we inspect a second sample. For the double-sampling plan, the following applies:

n_1 = the first sample sample size

c_1 = the first sample acceptance number

n_2 = the second sample sample size

c_2 = the both samples acceptance number

The double sampling plan is as follows: If there are c_1 or less nonconforming items in the n_1 size sample, then we accept the lot. The lot is rejected if the sample contains r_1 or more (where $r_1 \geq c_1$+2) nonconforming items. A second sample of size n_2 is obtained only if the number of nonconforming items in the first sample is between c_1+1 and r_1−1. The lot in this case is accepted only if the sum of the number of nonconforming items in the first and second sample is less than or equal to c_2.

Based on this approach, the average sample number for a double sampling plan becomes less than the sample size for a single sampling plan. This is the comparative advantage of this method, since, of course, the same producer and consumer risk points apply.

If we use the double sampling plan, we have the same protection as consumers by taking as a first sample a smaller sample than we would use with the single sampling plan. This applies whether the lot is accepted or rejected, and it means that we have the same protection at a lower inspection cost. By the term "curtailment" of the second sample, we mean the potentiality of rejecting a lot without complete inspection of the second sample. However, since single and double sampling plans can be selected so

that they have the same Operating Characteristic curves, there is no real advantage to using the double sampling plan.

However, there are two disadvantages to the double sampling plan: First and foremost, if a second sample curtailment is not applied, more inspection may be needed than in the single sampling plan, which eliminates any financial advantage we would have. Secondly, due to the complexity of the process, inspection errors might occur. In addition, the additional cost of storing and handling the inspected lots should be taken into account, since more time is needed complete the inspection of the second sample.

The sample size in the case of double-sampling will be either n_1 or $n_1 + n2$, depending on whether the lot, after the inspection of the first sample, is accepted or rejected. The Average Sample Number (ASN) for the double sample plan will be the following function:

$$ASN = n_1 + n_2 \times P(c_1 < x_1 < r_1)$$

as the probability of acceptance or rejection after the inspection of the first sample is affected by the number of nonconforming items in the lot, where x_1 is the number of nonconforming items of the first sample. At the next Figure, we present, for the double sampling plan, the ASN curves for complete and curtailed inspection with $n_1 = 60, c_1 = 2, n_2 = 120, c_2 = 3$ and the average sample number to be used in a single-sampling plan with $n = 89, c = 2$.

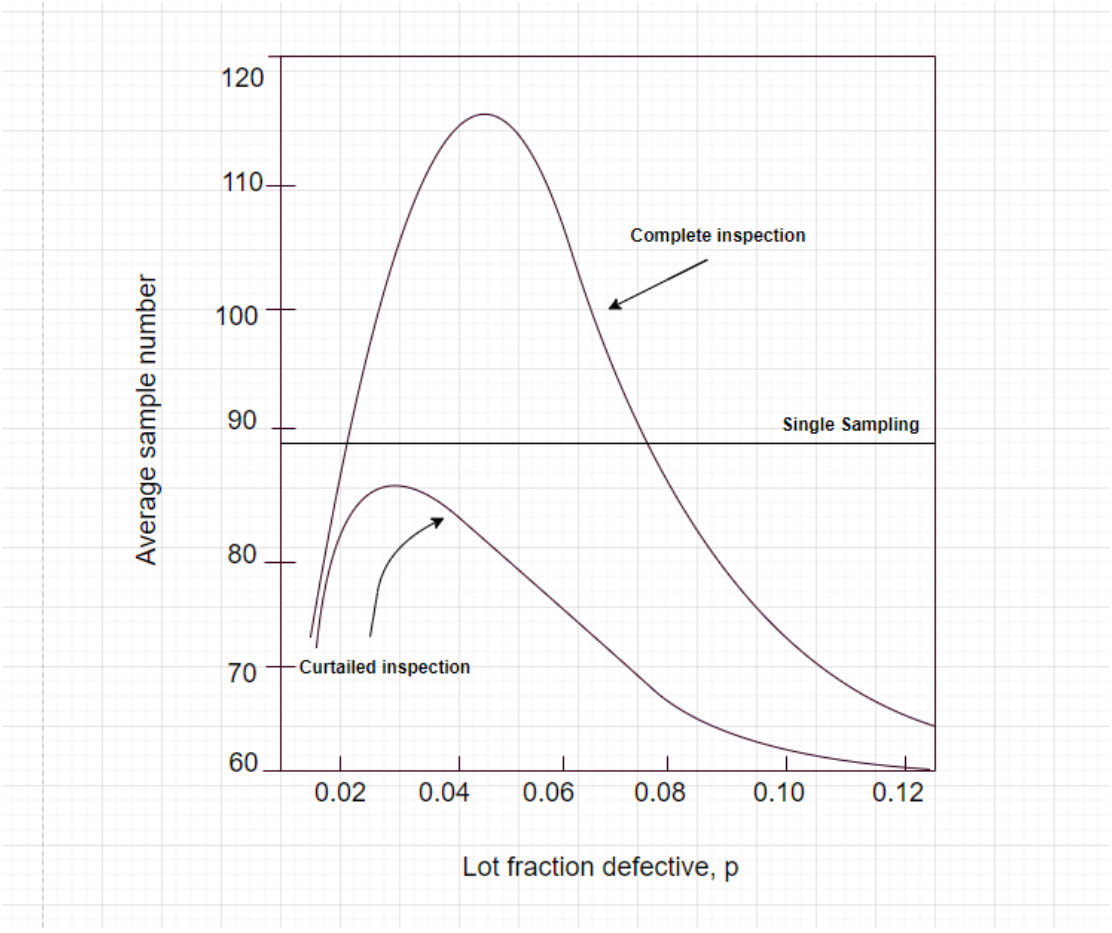

Figure 5 Comparison of different sample sizes (single sampling plan and double sampling plan)

Multiple Sampling Plans

According to Aft (1997), the logic of double sampling plans is extended with multiple sampling plans. Instead of giving batches of questionable quality a second chance by inspecting a second sample (double sampling), we not only get a second sample, but we have the ability to give many chances to the lot by inspecting many samples. With multiple sampling plans we have the advantage that usually the required samples are smaller than those of single and double sampling plans, which reduces the cost of the process. However, Montgomery (2009) claims that there is more complexity with this type of plans. Multiple-sampling plans can be presented in tabular form, as shown in the next Figure:

Sample	Sample Size	Cum. Samp. Size	Acc. Number	Rej. Number
1	n_1	n1	c_1	r_1
2	n_2	$n_1 + n_2$	c_2	r_2
⋮	⋮	⋮	⋮	⋮
k	n_k	$n_1 + n_2 + \ldots + n_k$	c_k	$r_k = c_k + 1$

Figure 6 A multiple sampling plan example

With this type of plan, we accept the lot if, at any stage of sampling, the number of defective items is less than or equal to the acceptance number, while we reject the lot if, at any stage of sampling, the number of defective items is equal to or exceeds the rejection number; otherwise the next sample is taken. That is, the lot is accepted if $x_1 \leq c_1$, where x_1 is the number of nonconforming items found in the first sample. Otherwise, the lot is rejected if $x_1 \geq r_1$ and if $c_1 < x_1 < r_1$, another sample is taken, etc.

Sequential Sampling Plans

The logic of the concept of double sampling and multiple sampling plans is extended with sequential sampling plans (Montgomery, 2009). In this case, the number of samples is allowed to be determined entirely by the results of the sampling process and in practice we obtain a sequence of samples from the lot. This process can theoretically be continued until the lot is 100% inspected but we usually truncate sequential sampling plans when the number inspected is three times the number that would have been inspected using a corresponding single sampling plan. We have group sequential sampling if the sample size inspected at each stage is greater than one. We have item-by-item sequential sampling if the size of the sample inspected at each stage

is one. This technique was based on the developed by Wald (1947), Sequential Probability Ratio Test (SPRT).

2.1.3 Rectification Sampling

Montgomery (2009) supports that in case of rejected lots, corrective action is required from the acceptance sampling programs. In this case we do 100% inspection or screening of rejected lots and, the nonconforming items are subject to either rework or sent back to the supplier or replaced from a stock of known good items. Since the final quality of the outgoing product is affected by the inspection activity, these sampling programs are called rectifying inspection programs. Suppose that the incoming lots to the inspection activity have a fraction defective p_0. If any lots get rejected, they get screened and the final fraction defective will be zero. However, those that are accepted have a fraction defective p_0. This means that the outgoing lots from the inspection activity have an average fraction defective in the stream of outgoing lots p_1 as it is a mixture of lots with fraction defective p_0 and fraction defective zero. This p_1 is always less than p_0. Thus, with the rectifying inspection program we achieve the "correction" of lot quality. In the following Figure, we see an example:

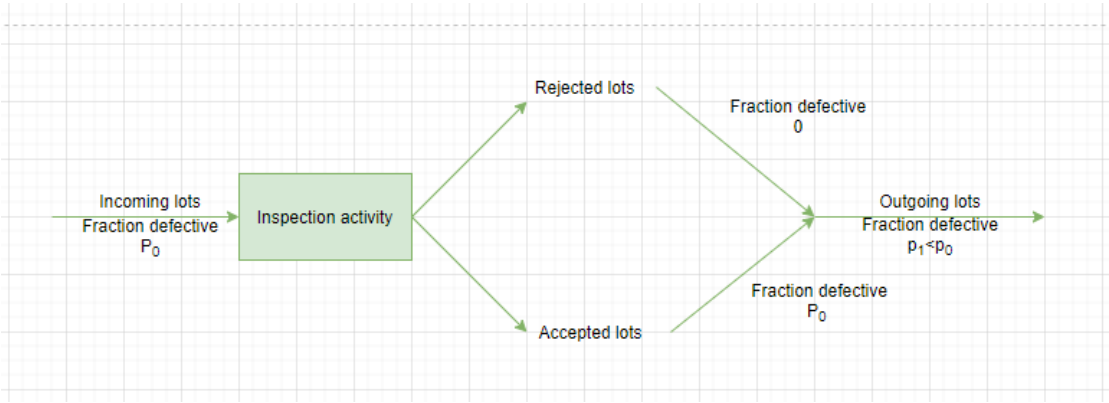

Figure 7 rectification sampling process

When the manufacturer wants to know the average quality level at each stage of the manufacturing operations, the rectifying inspection programs are used. This means that they are used either at receiving inspection, in process inspection of semi-finished products, or at final inspection of finished products. In case of in-plant usage, the objective is to have the assurance on whether the quality of the material meets the required technical specifications, so that it can be used in the next stages of the production process.

There are two options in the case of lots that are not accepted. Montgomery (2009), supports that this means either the rejected lots are returned to the supplier in order to screen and rework them, or these procedures are performed by the consumer, if possible. The first option is preferable as the supplier is responsible for the nonconforming items due to their fault, which can lead to an improvement in their overall manufacturing processes in order to prevent this from happening again. However, the second solution is often chosen, so that there is no delay due to the strict schedules that may exist during the production of the products.

If we inspect an ongoing stream of batches, the OC or probability that we accept a batch by a single sampling plan for attributes is given by the following formula (Binomial Distribution):

$$P_a = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i}$$

where the sample size is n, the acceptance number is c and the probability that the supplier's process produces a nonconforming item is p. The next formula is the Average Outgoing Quality (AOQ):

$$AOQ = \frac{P_a p(N-n)}{N}$$

where the batch size is N. During the supplier's process, the above function calculates the probability of a nonconforming (AQL). The maximum value of AOQ is the AOQL or Average Outgoing Quality Limit.

Furthermore, according to Montgomery (2009), the average outgoing quality is usually used to evaluate a rectifying sampling plan. This is the average value of lot quality that would be obtained, from a process with fraction defective p, over a long sequence of lots, i.e. it is defined as the quality in the lot resulting from the application of rectifying inspection.

In case we have a single sampling plan with rectification, either n or N is the number of items inspected, and the average total inspection (ATI) we require is

$$ATI = n + (1 - P_a)(N - n).$$

In case we have a double sampling plan with rectification,

$$ASN = n_1(P_{a1} + P_{r1}) + n_2(1 - P_{a1} - P_{r1})$$

$$ATI = n_1 P_{a1} + (n_1 + n_2)P_{a2} + N(1 - P_{a1} - P_{a2})$$

where the sample size for the first sample is n_1, the sample size for the second sample is n_2, the probability of accepting on the first sample is P_{a1}, and the probability of accepting on the second sample is P_{a2}, which is given by

$$\sum_{i=c_1+1}^{r_1-1} P(x_1 = i)P(x_2 \leq c_2 - x_1)$$

where x_1 and x_2 are the number nonconforming on the first and second sample respectively.

Ching-Ho Yen et al. (2020) proposed a more complicated but also advantageous rectification sampling plan. Based on the one-sided PCI (abbreviation of the process capability index), they proposed a quality cost model of repetitive sampling for the development of a rectifying acceptance sampling plan. What these indices achieve, is that they provide numerical measures regarding the process performance in the manufacturing industry. Essentially, they establish the relationship between the actual process performance and manufacturing specifications, and so, the proposed model minimizes the total quality cost (TQC) of sentencing a batch, including inspection costs, internal failure costs, and external failure costs. These indicators dominate the recent research in statistical and quality assurance.

2.2 Variables Sampling Plans

When measuring actual quantitative information on sampled items, instead classifying them as conforming or nonconforming, we can use variables sampling plans.

Variable sampling plans have the advantage that due to the fact that the measurements contain more information, we can use a smaller sample size than is required by an attributes sampling plan to obtain the same operating-characteristic curve (Wu & Liu, 2013). Thus, less sampling would be required for a variables sampling plan that has the same protection as an attribute sampling plan. But there is likely to be a higher cost per observation in terms of measurement data required by a variables sampling plan than the collection of attributes data. However, this increased cost is offset by the reduction in the sample size obtained. Suppose the following example: A sample of size 100 items is required by an attributes sampling plan as opposed to the equivalent variables sampling plan which requires a sample of size 65 items. If we assume that there is a lower cost of measurement data by $1,61$ times the cost of measuring the observations on an attributes scale, we conclude that the variables sampling plan will be more economical in terms of sampling costs. Inspection costs are reduced when destructive testing is used, as variables sampling is useful in reducing this cost.

Secondly, we must take into account the advantage that attributes data usually provide less information about the manufacturing process or the lot than measurement data as, in general, the item is more easily classified as defective or non-defective by numerical measurements of quality characteristics.

In addition, there may be significant advantages in switching to variables measurement as very large sample sizes are required by attribute sampling plans when acceptable quality levels are too small. This means that the variables sampling becomes very tempting as many manufacturers begin to highlight allowable numbers of defective parts per million.

However, variables sampling plans have disadvantages. One of them is that we must know the distribution of the quality characteristic in advance. In addition, the hypothesis that the distribution of the quality characteristic is normal, is used by most standard variables acceptance-sampling plans. If we employ a plan based on the normal assumption, it carries a risk of experiencing very serious departures from the advertised risks of rejecting or accepting lots, if the quality characteristic is not normally distributed (Montgomery, 2009).

It is also important to note that, with variables plans, we assume that the measurement error is small compared to the specification limits, when we obtain quantitative information from sampled items. Otherwise, for the lot acceptance criterion, type I and type II errors will both be inflated. By using a Gauge Repeatability and Reproducibility Study or Gauge R&R study by Burdick et al. (2005), we estimate the measurement error and compare it to the specification limits.

For each quality characteristic being inspected, a separate sampling plan must be employed due to the use of the variables sampling plan. This is a problem, as there must be four separate variables inspection sampling plans if, for example, an item is

inspected for four quality characteristics. In the case of attribute sampling, only an attributes sampling plan could be used.

Still, what is not often the case, is that the lot could be rejected in a variables sampling plan even though there are no defective items in the actual sample. This is something that both suppliers and consumers want to avoid as it might be extremely costly for both.

Lawson (2020), suggests that we can develop the acceptance criteria for a variables sampling plan with two different methods. The first method is the k-Method. With this method, we compare the standardized difference between the specification limit and the mean of the measurements made on each sampled item. That is why we define an acceptance constant k. M-method is the second method. With this, we can compare the maximum allowable proportion M to the estimated proportion of items out of specifications in the batch. With both methods, the same results are yielded when there is only one specification limit (USL or LSL). However, we must use the M-method in all but special circumstances when there is both a USL and a LSL.

2.2.1 The k-Method

2.2.1.1 Lower Specification Limit

The number of samples (n) and an acceptance constant (k) must be determined so as a variables sampling plan to be defined. We would accept a batch if $\frac{\bar{x}-LSL}{\sigma} > k$, where the sample average of the measurements from a sample is $\bar{x}$ and the standard deviation of the measurements is σ. We assume that we know the mean and standard deviation from past experience. According to the statistical theory of hypothesis testing, accepting the batch would be tantamount to failing to not accept the null hypothesis H_0 : $\mu \geq \mu_{AQL}$ in favor of the alternative H_α : $\mu < \mu_{AQL}$. When we assume that the measurements are normally distributed with a Lower Specification Limit (LSL), we can visualize the AQL and the RQL, in terms of proportion of items, below the Lower Specification Limit, as the area which is under the normal curve to the left of the LSL (depicted in Figure 8).

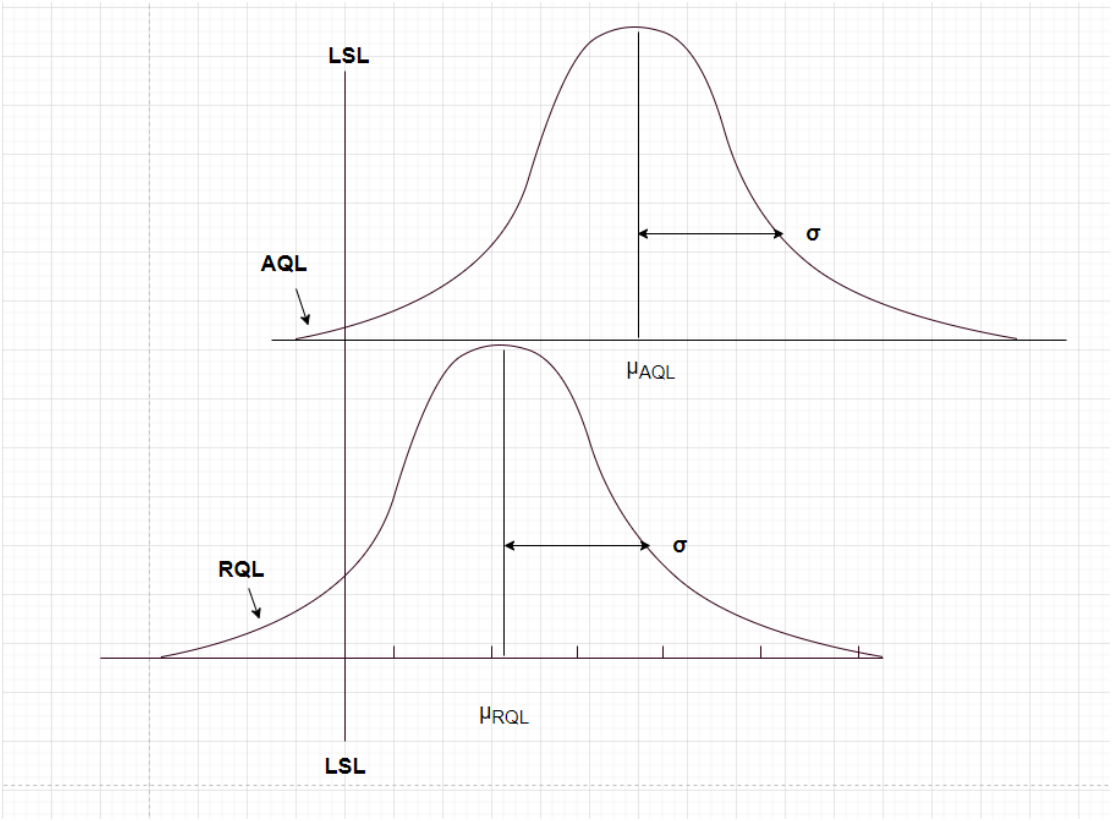

Figure 8 AQL and RQL (variables sampling plan)

Based on the above Figure, we see that if the mean of the distribution is μ_{AQL}, then AQL is the proportion of defective items and if the mean of the distribution is μ_{RQL}, then RQL is the proportion of defective items.

If α is the producer's risk and β is the consumer's risk, then

$$\frac{P(\overline{x} - LSL)}{\sigma} > k \,|\, \mu = \mu_{AQL}) = 1 - \alpha$$

and

$$\frac{P(\overline{x} - LSL)}{\sigma} > k \,|\, \mu = \mu_{RQL}) = \beta$$

Thus, these are the steps for the conduction of the sampling plan on a batch of material:

Standard deviation known:

1. A random sample of n items from the batch should be taken
2. The critical characteristic x on each sampled item should be measured
3. The mean measurement $\bar{x}$ should be calculated
4. $(\bar{x} - LSL)/\sigma$ should be compared to the acceptance constant k =1.967411
5. If $(\bar{x} - LSL)/\sigma > k$, the batch should be accepted, otherwise rejected.

Standard deviation unknown:

1. A random sample of n items from the batch should be taken

2. The critical characteristic x on each sampled item should be measured

3. The mean measurement $\bar{x}$, and the sample standard deviation s should be calculated

4. $(\bar{x} - LSL)/\sigma$ should be compared to the acceptance constant k

5. If $(\bar{x} - LSL)/\sigma > k$, the batch should be accepted, otherwise rejected.

2.2.1.2 Upper Specification Limit

If there is an upper specification limit (USL), instead of a lower specification limit (LSL) and a variables sampling plan was required for this situation, then these are the steps for the conduction of the sampling plan on a batch of material:

Standard deviation known:

1. A random sample of n items from the batch should be taken

2. The critical characteristic x on each sampled item should be measured

3. The mean measurement $\bar{x}$ should be calculated

4.$(USL - \bar{x})/\sigma$ should be compared to the acceptance constant k $=1.967411$

5. If $(USL - \bar{x})/\sigma > k$, the batch should be accepted, otherwise rejected.

Standard deviation unknown:

1. A random sample of n items from the batch should be taken

2. The critical characteristic x on each sampled item should be measured

3. The mean measurement $\bar{x}$, and the sample standard deviation S should be calculated

4. $(USL - \bar{x})/\sigma$ should be compared to the acceptance constant k

5. If $(USL - \bar{x})/\sigma > k$, the batch should be accepted, otherwise rejected.

2.2.1.3 Upper and Lower Specification Limits

When an Upper Specification Limit (USL) and a Lower Specification Limit (LSL) exist, Schilling & Neubauer (2017) proposed a procedure which is quite simple and which we can apply to find out if we can use two single specification limit plans which are separated. This is:

1. $Zp^* = \dfrac{(LSL - USL)}{2\sigma}$ should be calculated

2. $p^* = P(Z < Z_p) where Z \sim N(0,1)$ should be calculated. That area is under the standard normal density to the left of Zp^*

3. If $2p^* \geq RQL$ then the batch should be rejected, because the a proportion outside the specification limits will be too high despite the fact that the distribution is centered between the specification limits

4. If $2p^* \leq AQL$ then two single specification sampling plans should be used (one for Lower Specification Limit and one for Upper Specification Limit).

5. If $AQL \le 2p^* \le RQL$, then we should use the M-method for LSL and USL (as described above).

When an upper (USL) and a lower specification limit (LSL) exist, and we do not know the standard deviation, we use the M-method (as described in the following session).

2.2.2 The M-method

2.2.2.1 Lower Specification Limit

Standard deviation known:

When we have variables sampling plan, we compare the maximum allowable proportion to the estimated proportion below the Lower Specification Limit, using M-Method. Thus, we must calculate and use the uniform minimum variance unbiased estimate of the proportion below the Lower Specification Limit which was developed by Lieberman & Resnikoff (1955). This is a function of the acceptance constant k which we use in the k-Method. If there is a LSL and we know the standard deviation, then the uniform minimum variance unbiased estimate of the proportion defective is calculated as follows:

$$P_L = \int_{Q_L}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

(area under the standard normal distribution to the right of $Q_L = Z_L \left(\sqrt{\frac{n}{n-1}}\right)$),

where $Z_L = (LSL - \bar{x})/\sigma$. This is the maximum allowable proportion of defective:

$$M = \int_{k\sqrt{\frac{n}{n-1}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt,$$

(area under the standard normal distribution to the right of $k\sqrt{n/n-1}$), where k is the acceptance constant which we use in the k-method.

Standard deviation unknown:

We use the symmetric standardized Beta distribution instead of the standard Normal distribution to calculate the uniform minimum variance unbiased estimate of the proportion defective. Next, we take a look at the standardized Beta CDF type:

$$B_x(a,b) = \frac{\Gamma(\alpha+b)}{\Gamma(a)\Gamma(b)} \int_0^x v^{\alpha-1}(1-v)^{b-1} dv,$$

where $0 \le x \le 1, a > 0$, and $b > 0$. When $a = b$, we can see that we have a symmetrical density function. In this case, the estimate of the proportion defective is

$$\widehat{p_L} = B_x(a,b),$$

where $a = b = \frac{n}{2} - 1$, and

$$x = \max\left(0, 0.5 - 0.5 Q_L \left(\sqrt{\frac{n}{n-1}}\right)\right)$$

and we substitute the standard deviation s of the sample for σ in the formula for

$$Q_L = \left(\bar{x} - LSL\right) / s$$

When we do not know the standard deviation, the maximum allowable proportion defective is as follows:

$$M = B_{B_M}\left(\frac{n-2}{2}, \frac{n-2}{2}\right),$$

where

$$B_M = 0.5\left(1 - k\frac{\sqrt{n}}{n-1}\right),$$

and the acceptance constant is k.

2.2.2.2 Upper Specification Limit

Standard deviation known:

When the standard deviation is known and a USL exists, then the acceptance criterion must change from $P_L < M$ to $P_u < M$, where

$$P_U = \int_{Q_U}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt,$$

which is practically the area under the standard normal distribution to the right of

$$Q_U = Z_U\left(\sqrt{\frac{n}{n-1}}\right) \text{ and}$$
$$Z_U = (USL - \bar{x})/\sigma.$$

Standard deviation unknown:

When a USL exists and we do not know the standard deviation, then the acceptance criterion is $\hat{P}_u < M$ where,

$$\widehat{p_U} = B_x(a, b),$$
$$a = b = \frac{n}{2} - 1,$$
$$x = \max\left(0, 0.5 - 0.5 Q_U \left(\sqrt{\frac{n}{n-1}}\right)\right),$$
$$Q_U = \frac{USL - \bar{x}}{s},$$

and M is the same as that we defined for the LSL.

2.2.2.3 Upper and Lower Specification Limits

Standard deviation known:

When both upper and lower specification limits exist and we know the standard deviation, this is the acceptance criterion: accept if

$$P = (P_L + P_U) < M$$

Where M is the same as the M in 2.2.2.1 and we know the standard deviation.

Standard deviation unknown:

When we do not know the standard deviation, this is the acceptance criterion: accept if

$$\hat{p} = \widehat{p_U} + \widehat{p_L} < M,$$

where M is the same as the M in 2.2.2.1 and we do not know the standard deviation.

Regarding resubmitted batches, a very interesting approach was proposed by Rao et al. (2019), who have developed a group acceptance sampling plan (GASP) for batch resubmission, thus achieving quality assurance of product lifetime, under the assumption that the product lifetime follows the exponentiated Fréchet distribution. The GASP parameters are determined by the satisfaction of the specified producer's and consumer's risks according to the number of testers and the experiment termination time. It was found, after the comparison between the ordinary group sampling plans and the proposed group sampling plans, that the latter have a lower sample number compared to the former.

Chapter 3 Sampling Schemes

During World War II, the development of Sampling Schemes flourished, as both by attributes and by variables, standard procedures were developed for use in the inspection process. The way in which sampling plans must be used, is specified by these overall strategies, the Sampling Schemes.

3.1 Attribute Sampling Schemes

3.1.1 The MIL-STD-105E Sampling Scheme and Derivatives

According to Mitra (2016), as part of the World War II effort, sampling inspection schemes were developed by the United States Military. This was because a munitions inspection system had to be set up that required less than 100% inspection. The MIL-STD-105A, B,…, E were variations of this system, with various types of improvements that were gradually incorporated over subsequent years. Its rapid spread worldwide led to contracts, government and non-government. Batch size and AQL played an important role in the schemes in MIL-STD and their applications were regarding a stream of batches. Later we will see that plans for normal sampling, tightened sampling, reduced sampling are included in them, with associated switching rules. For each batch size - AQL combination we can use single, double and multiple sampling plans with equivalent OC curves.

1995, however, was the year the army discontinued the MIL-STD-105E scheme's support. American Standards Institute (ANSI), the International Standards Organization (ISO) (which where civilian standards-writing organizations) and many others created their own variations, based on the MIL-STD-105E system. One of these variants is the American national standard called ANSI / ASQC Z1.4 standard, which was derived from the MIL-STD-105E scheme and was used mainly for domestic contracts. The central tables were not involved in the slightest changes that were incorporated in it. Figure 9 shows the rules for switching between normal sampling, tightened sampling and reduced sampling using the MIL-STD-105E standard. If plans and switching rules are used, we will have an OC curve that is closer to the ideal and in this way, suppliers will be encouraged not to provide batches with proportion nonconforming which is higher than the agreed AQL.

For international trading use, the ISO 2859 standard is widely used, with which the MIL-STD-105 ideas are modified in order to integrate the technological evolution.

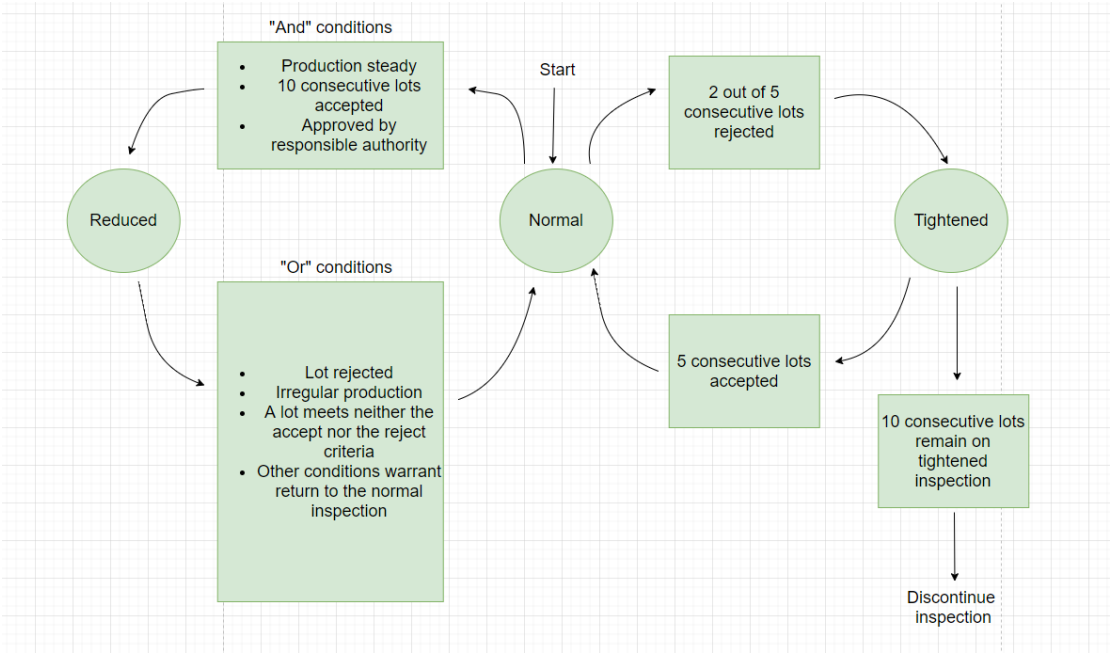

Figure 9 Switching rules between normal inspection, tightened inspection, and reduced inspection (MIL-STD-105E)

Figure 10 shows the switching rules required for the proper use of the plans. In this case, when the percent nonconforming is less than the stated AQL, then the supplier is protected against batch rejection while still, if the percent nonconforming is higher than AQL, then the customer is protected against batch acceptance. Of course, a prerequisite is that these rules are followed, otherwise the benefits are lost. The customer benefits by following the rules, as good quality allows inspection samples which are smaller in size.

According to Figure 10, we can calculate the switching score either by determination from a competent authority or starting at the beginning of the regular inspection. At the beginning, we set the score for switching to zero and we recalculate it after inspecting each subsequent batch in the original (regular) inspection.

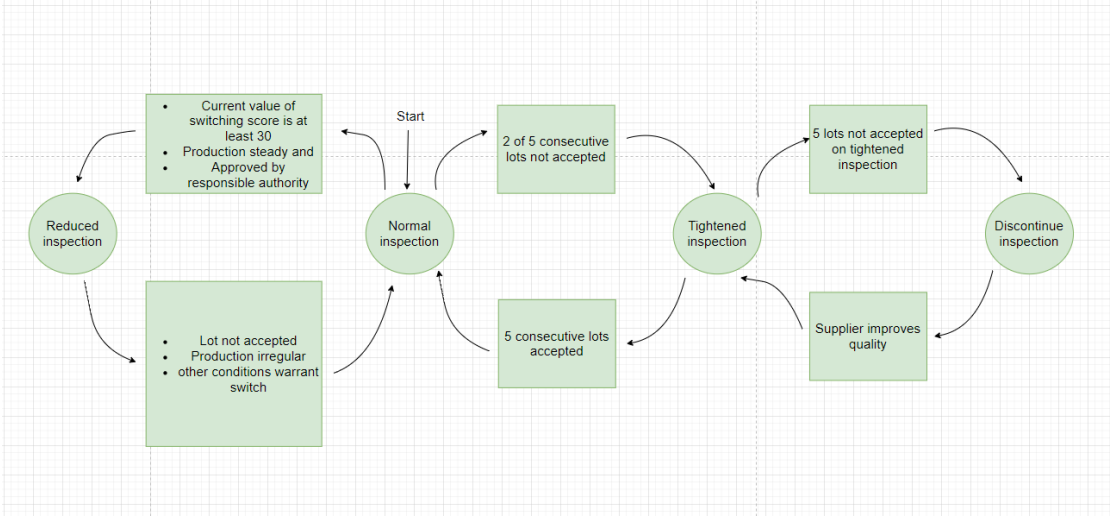

Figure 10 Rules for switching (ISO 2859)

a) Single sampling plans:

1) If the acceptance number is 2 or more and if the lot would have been accepted if the AQL had been one step higher, we add 3 to the switching score. Otherwise, the switching score must be reset to zero.

2) If the acceptance number is 0 or 1, and if the lot is accepted, we add 2 to the switching score. Otherwise, the switching score must be reset to zero.

b) Double or multiple sampling plans:

1) If we use a double sampling plan, and if we accept the lot after the first sample, we add 3 to the switching score. Otherwise the switching score must be reset to zero.

2) If we use a multiple sampling plan, and if we accept the lot by the third sample, we add 3 to the switching score. Otherwise the switching score must be reset to zero.

In order to make use of the tables from ANSI/ASQC Z1.4 or ISO 2859, we determine a code letter from a table, based on the inspection level and batch size. In Figure 11, we see an example of this table. The next step is to decide on the type of sampling (single, double or multiple) and the inspection level (normal, tightened or reduced). Finally, we obtain the acceptance-rejection numbers and sample sizes from the tables. There is a book by Christensen et al. (2013), which shows an example for the process by which this is done with the ANSI / ASQC Z1.4 tables (normal inspection).

	Special Inspection Levels				General Inspection Levels		
Lot or Batch size	S-1	S-2	S-3	S-4	I	II	III
2 to 8	A	A	A	A	A	A	B
9 to 15	A	A	A	A	A	B	C
16 to 25	A	A	B	B	B	C	D
26 to 50	A	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	C	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1200	C	C	E	F	G	J	K
1201 to 3200	C	D	E	G	H	K	L
3201 to 10000	C	D	F	G	J	L	M
10001 to 35000	C	D	F	H	K	M	N
35001 to 150000	D	E	G	J	L	N	P
150001 to 500000	D	E	G	J	M	P	Q
500001 and over	D	E	H	K	N	Q	R

Figure 11 MIL-STD-105E code letters for different sample sizes

It is worth noting that when we specify the numbers of acceptance and rejection, then a gap appears between them. In case no rejection number has been reached but we still exceed the acceptance number in a ANSI / ASQC Z1.4 (reduced inspection) plan,

then the batch must be accepted, but continue the process with normal inspection. Subsequently, the gaps that appeared by the specification of the acceptance and rejection numbers, were eliminated in a modification incorporated during the development of the international ISO 2859 derivative of MIL-STD-105E.

The effects of MIL-STD-105E were investigated by Stephens & Larson (1967). The ANSI/ASQC Standard Z1.4 and the ISO 2859 are also relevant. When we ignore the reduced plan (which can only be used after approval by the competent authority) and consider only the normal system and the tightened system, we can consider again the sampling scheme as a two state Markov chain (the two states are the normal inspection and the tightened inspection). The acceptance probability of this scheme or OC is given by the following formula:

$$Pr(accept) = \frac{aP_N + bP_T}{a + b},$$

where the acceptance probability under normal inspection is P_N and the acceptance probability under tightened inspection is P_T, and

$$a = \frac{2 - P_N^4}{(1 - P_N)(1 - P_N^4)}$$

$$b = \frac{1 - P_T^5}{(1 - P_T)P_T^5}$$

Additionally, $a/(a+b)$ is the probability of the steady state of being in the normal sampling state, and $b/(a+b)$ is the probability of the steady state of being in the tightened sampling state. For the normal-tightened scheme, the average sample number (ASN) is given by the following formula:

$$ASN = \frac{an_N + bn_T}{a + b}$$

The benefits of using the ANSI / ASQC Standard Z1.4 sampling scheme for attributes sampling can be understood in the case where we use a single sampling scheme for a continuous stream of batches consisting of batch sizes between 151 and 280. G is the code letter. If required AQL is 1%, the normal inspection plan is $n = 50$, with $c = 1$, and the tightened inspection plan is $n = 80$, with $c = 1$.

MIL-STD-1916

Schilling & Neubauer (2017) state that in 1996, the U.S. Department of Defense issued the MIL-STD-1916. This scheme is completely unique and does not come from the MIL-STD-105E. In addition to the guidelines on quality control charts and quality management procedures, this standard also includes variables sampling schemes and attribute sampling schemes. In MIL-STD-1916, attribute sampling schemes are zero nonconforming plans where the acceptance number $c = 0$ always. This makes sense, as those who use total quality management (TQM) and continuous process improvement (i.e. companies as well as the Department of Defense), want to avoid AQL-based acceptance sampling plans that allow a level of nonconformities different than zero.

3.1.2 Quick Switching Scheme

Romboski (1969), proposed a sampling scheme known as the QSS-1 quick switching scheme. The plan is as follows:

At this scheme, we have two acceptance sampling plans. A sample size n and acceptance number c_N define the first one, which is the normal inspection plan. Also, the same sample size n and a reduced acceptance number c_T define the second one, which is the tightened inspection plan. These switching rules bellow aply:

1. We use normal inspection plan at first.
2. We must switch to the tightened inspection plan from the normal inspection, after we reject a batch.
3. We must switch to the normal inspection plan from the tightened inspection, after we accept a batch.
4. We must follow rules 1-3 and do not skip making back and forth steps in alternations.

This can see this plan diagrammed in Figure 12:

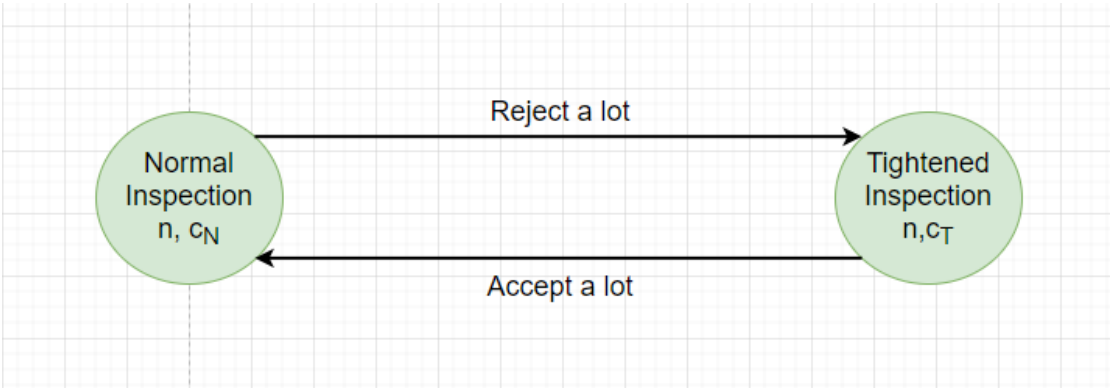

Figure 12 The Quick Switching System (QSS-1) by Romboski

Suppose that $n = 20, c_N = 1, c_T = 0$. Then, the Operating Characteristic curves are compared for the normal and tightened inspection plans in Figure 13:

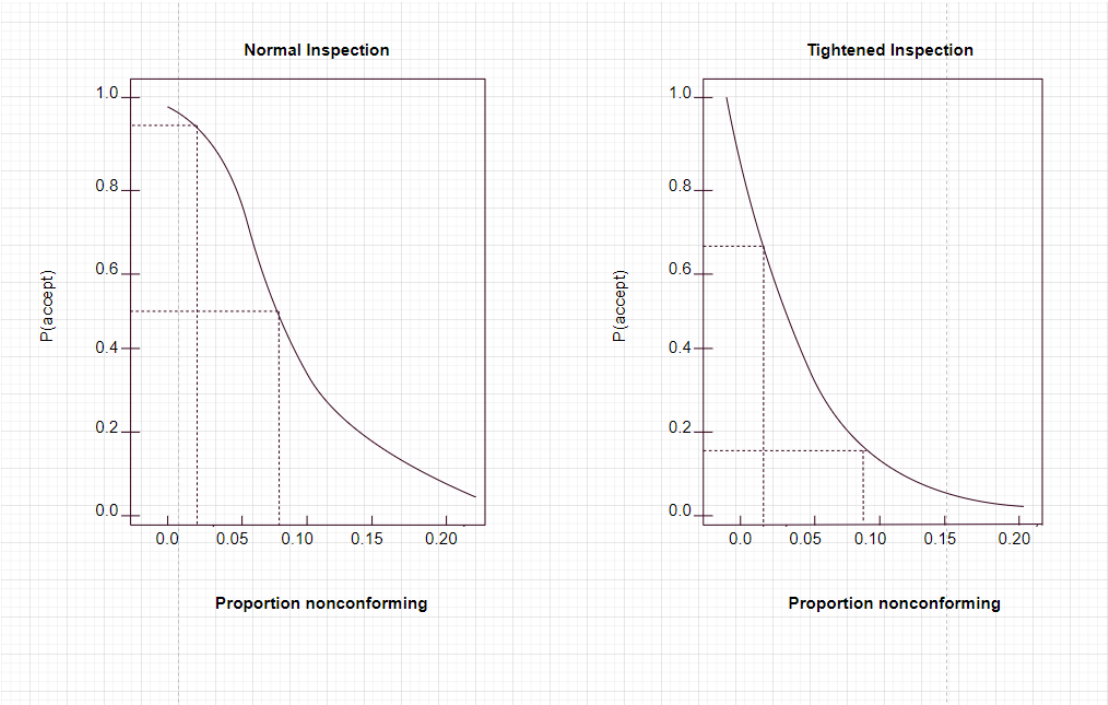

Figure 13 Normal plan and tightened plan Operating Characteristic Curves

As we can see, normal inspection plan, benefits the supplier and tightened inspection plan, benefits the consumer. This is because in the first case, there is at least 94% probability of lots' acceptance with 0.02 or less nonconforming. The customer, on the other hand, does not benefit, as the probability of a lot's rejection with 0.08 nonconforming is less than 50%. In the second case, however, there is a reversal in the results, as the probability of a lot's rejection with 0.08 nonconforming is more than 80%. However, it should be noted that, in the acceptance quality range, the Operating Characteristic curve is very steep in the case of tightened inspection. Also, the probability of a lot's rejection with 0.02 nonconforming is greater than 32%, which is something that does not benefit suppliers.

The solution to these is the quick switching scheme, which due to the use of switching rules (i.e. switching between normal and tightened inspection), benefits both customers and suppliers. If we refer to the ideal Operating Characteristic Curve of Figure 4, we will find that the Operating Characteristic Curve for the scheme will approach the ideal without there being an increase in the sample size. As we can see schematically, Operating Characteristic Curve for the scheme has a crucial point (shoulder) which represents a high probability of accepting a lot when there is a low percentage nonconforming. This is like the normal inspection plan's Operating Characteristic Curve and works for the benefit of the supplier. Then we see schematically, that the Operating Characteristic Curve for the scheme is dropped very steeply to the right of the Acceptance Quality Level (AQL), which reminds us of the tightened inspection plan's Operating Characteristic Curve and works in favor of the customer.

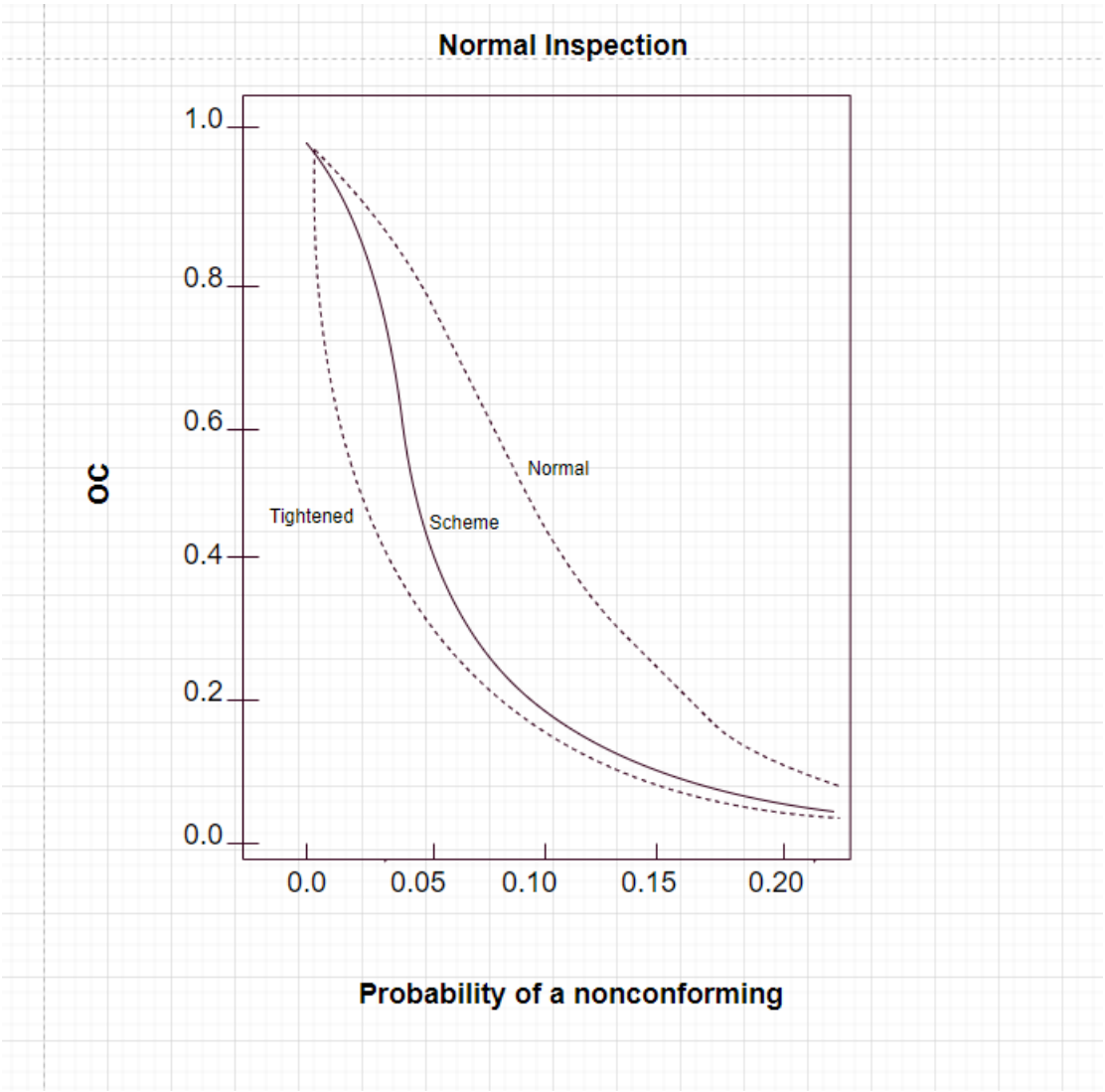

Figure 14 Normal and Tightened and Scheme Operating Characteristic Curves

If we define the first state as normal inspection and the second state as tightened inspection, then we can consider this scheme as a two - state Markov chain. Based on this, Romboski defined that the probability of accepting a batch by the scheme is:

$$P(accept) = \frac{P_T}{(1 - P_N) + P_T}$$

where P_N is the probability of accepting under normal inspection. By the Binomial Probability Distribution, the formula is:

$$P_N = \sum_{i=0}^{c_N} \binom{n}{i} p^i (1-p)^{n-i}$$

and p is the probability that the supplier produces a nonconforming item where P_T is the probability of accepting under tightened inspection. By the Binomial Probability Distribution, the formula is:

$$P_T = \sum_{i=0}^{c_T} \binom{n}{i} p^i (1-p)^{n-i}$$

Not only the QSS-1 scheme results in an OC curve that without increasing the sample size n is closer to the ideal curve, but we can also see additional benefit: With this plan, we are led to a reduced probability of acceptance (switching to a tightened plan), when a supplier constantly submits batches where the proportion nonconforming is higher than the allowable limit. This does not benefit the supplier though. Returning the batches to the supplier, due to the high cost involved in time and money, is an incentive to improve their production processes in order to avoid similar incidents in the future and thus reduce their proportion nonconforming. Of course, small customers may not always be able to demand from their suppliers to reduce proportion nonconforming (something that large corporate customers achieve more easily), however it is a given that the sampling scheme motivates suppliers to try to have small proportion nonconforming.

3.2 Variables Sampling Schemes

3.2.1 The MIL-STD-414 and Derivatives

Using as a basis the research of Lieberman & Resnikoff (1955), in 1957, the MIL-STD-414 standard for variables sampling inspection which was based on the Acceptance Quality Level (AQL), was issued by the U.S. Department of Defense. There was a great similarity between the OC performance, Acceptance Quality Levels and code letters of the MIL-STD-414 and the MIL-STD-105A-C. Of course, there were smaller sample sizes in the variables sampling plans. We can see the MIL-STD-414 content in the following Figure. Using the range (R) instead of the sample's standard deviation (s) (Range method - dark green boxes), we were able to reduce the required calculations, which was important at a time when there were no computers. Of course, these methods are no longer needed, as computer calculations have been fully automated. The need for the k-Method is eliminated by the M-method, which is used either for the single specification limits or for the double specification limits.

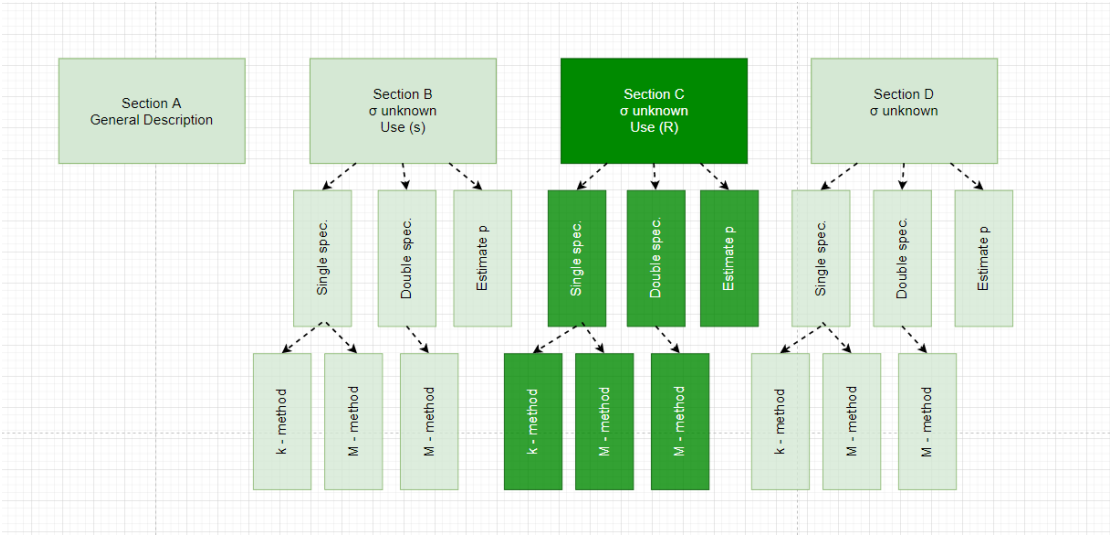

Figure 15 MIL-STD-414 content

Also, because we use the same code letters in this plan as in the MIL-STD-105E standard, but we assign them to different sample sizes, care must be taken not to make a mistake. Still, the batch size categories are not the same. In order to specify in each category the sample sizes, we use inspection level and batch size. There is always a provision for either normal inspection, tightened inspection, or reduced inspection. Additionally, at this standard we assume arbitrarily a normal distribution that specifies the quality characteristic of interest.

	Inspection Levels				
Lot size	I	II	III	IV	V
3 to 8	B	B	B	B	C
9 to 15	B	B	B	B	D
16 to 25	B	B	B	C	E
26 to 40	B	B	B	D	F
41 to 65	B	B	C	E	G
66 to 110	B	B	D	F	H
111 to 180	B	C	E	G	I
181 to 300	B	D	F	H	J
301 to 500	C	E	G	I	K
501 to 800	D	F	H	J	L
801 to 1300	E	G	I	K	L
1301 to 3200	F	H	J	L	M
3201 to 8000	G	I	L	M	N
8001 to 22000	H	J	M	N	O
22001 to 110000	I	K	N	O	P
110001 to 550000	I	K	O	P	Q
550001 and over	I	K	P	Q	Q

Figure 16 MIL-STD-414 code letters for different sample sizes

The MIL-STD-414 was marginalized due to the D and E versions of the MIL-STD-105. Eric Gascoigne, commander of the British Navy, however, "saved" this plan and, in 1980, the ANSI / ASQC Z1.9 (civilian standard) incorporated his ideas. It is now possible to switch between ANSI / ASQC Z1.9 (variables sampling plan) and ANSI / ASQC Z1.4 (attribute sampling plan) by keeping the same OC performance, provided we have the same Acceptance Quality Level and batch size inspection level. These plans are mainly used in the United States.

With these variables sampling schemes, we can sample a stream of batches submitted by a supplier. Normal sampling plans, tightened and reduced are included in them. In addition, they use the same switching rules as MIL-STD-105E (ANSI / ASQC-Z1.4) (Schilling & Neubauer, 2017). When using the standard, the customer and supplier must set an Acceptance Quality Level and comply with the switching rules, so that there is a bigger benefit and a higher protection level for both, than in the corresponding single sampling plan. This achieves an Operation Characteristic curve that has a steep scheme and approaches the ideal we saw in Figure 4. ISO 3951-1 is the international version of MIL-STD-414 and is used in international trade. A graphical criterion for double specification limits is used by this international ISO version, since the plans that use the range (R) (Range method - dark green boxes in Figure 15) have been dropped. With this criterion, and after calculating the $\bar{x}$ and the standard deviation s, we can plot the coordinates (x, s) to see if it is in the acceptance area.

3.2.2 Continuous Sampling - CSP-1

General Idea

Unlike the lot-by-lot plans we have seen so far, there are assembly processes which are characterized by great complexity and lead to batch formation which is unnatural. Thus, while we have assumed that individual batches sentencing is the purpose of the sampling plan, in practice, this assumption often does not exist.

In continuous production lines, we can form batches in two ways: In the first way, we can accumulate the production in the assembly line at predetermined times. This has the disadvantage that it requires space to store products and carries safety risks.

In the second way, we delineate specified production sections as batches. However, the downside here is that we may be led to recall products from processes or manufacturing stages when a 100% inspection needs to be made.

Therefore, in order to eliminate these disadvantages, sampling plans were developed for continuous production lines. These plans are the so-called “continuous sampling plans”, and it is possible when we perform sampling inspections, to alternate the inspections’ sequences. The process is as follows: Initially we start with 100% inspection until the clearance number is found, i.e. a number of units that is clear of nonconforming. Then, we can initiate the sampling inspection. Next, we move from sampling inspection to 100% inspection when we reach a predefined number of nonconforming items. We see that this is similar to rectifying inspection plans, as we do partial screening so that the quality of the product gets improved.

Procedure

Harold F. Dodge (1943), proposed continuous sampling plans for the first time. The CSP-1 was his initial plan and the procedure was as follows:

We first perform 100% inspection to all units. Then we stop when the clearance number has been achieved (i.e. there are i continual production units conforming). So, we stop the 100% inspection and we inspect a portion (f) from the production units. We can choose the sample either from the production line completely randomly, or by choosing one by one the units. In case a nonconforming item is found, we perform 100% inspection again and so on. Also note that the nonconforming items, either get replaced or reworked. In Figure 17, we can see the procedure for CSP-1.

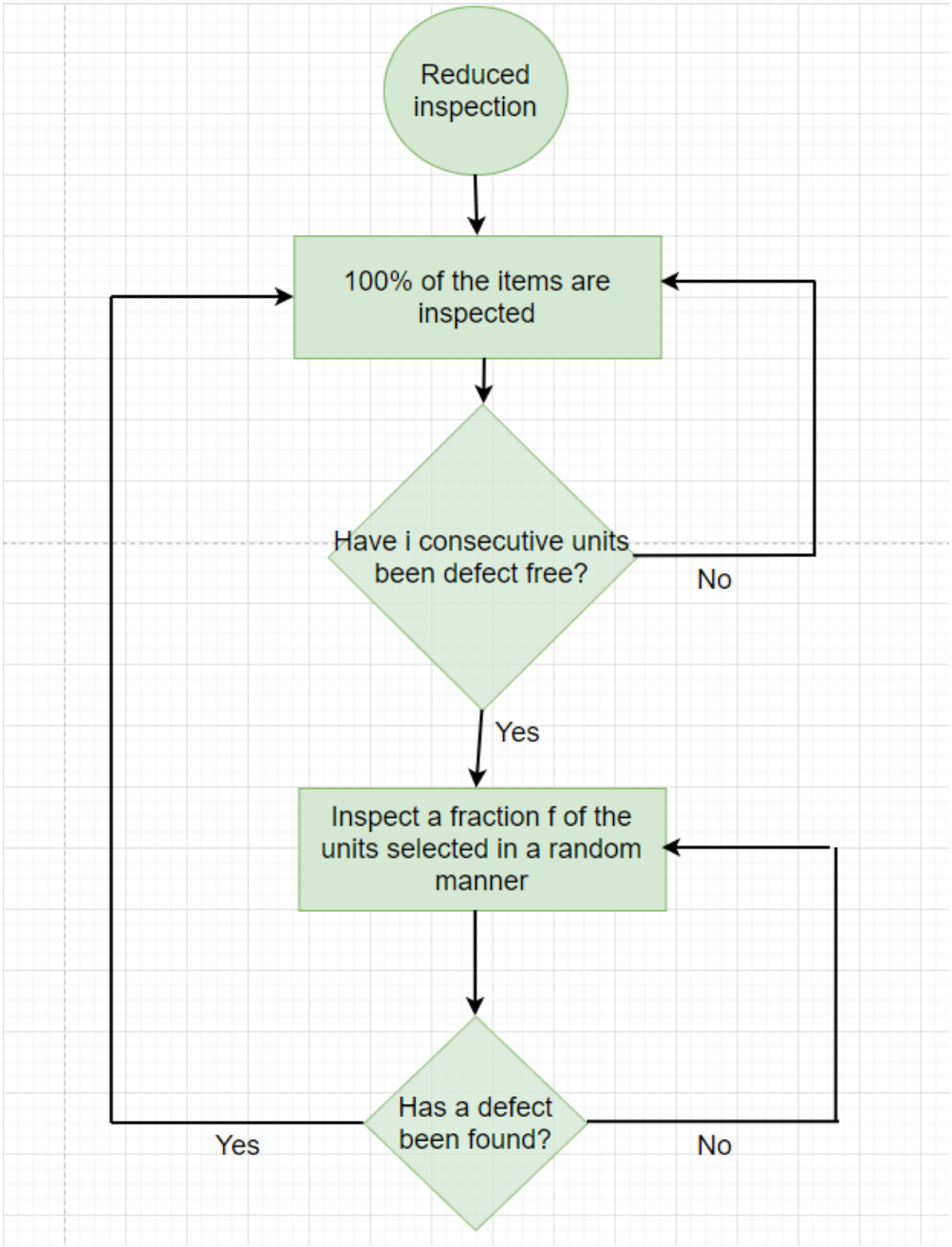

Figure 17 CSP-1 procedure

In each CSP-1 we can assign an overall AOQL value. The value of the AOQL is determined by the clearance number i and the sampling fraction f. We can be led to the same AOQL if we use different combinations of i and f. In Figure 18, we can see the various values of i and f.

	AOQL (%)														
f	0.018	0.033	0.046	0.074	0.113	0.143	0.198	0.33	0.53	0.79	1.22	1.90	2.90	4.94	7.12	11.46
1/2	1,540	840	600	375	245	194	140	84	53	36	23	15	10	6	5	3
1/3	2,550	1,390	1,000	620	405	321	232	140	87	59	38	25	16	10	7	5
1/4	3,340	1,820	1,310	810	530	420	303	182	113	76	49	32	21	13	9	6
1/5	3,960	2,160	1,550	965	630	498	360	217	135	91	58	38	25	15	11	7
1/7	4,950	2,700	1,940	1,205	790	623	450	270	168	113	73	47	31	18	13	8
1/10	6,050	3,300	2,370	1,470	965	762	550	335	207	138	89	57	38	22	16	10
1/15	7,390	4,030	2,890	1,800	1,180	930	672	410	255	170	108	70	46	27	19	12
1/25	9,110	4,970	3,570	2,215	1,450	1,147	828	500	315	210	134	86	57	33	23	14
1/50	11,730	6,400	4,590	2,855	1,870	1,477	1,067	640	400	270	175	110	72	42	29	18
1/100	14,320	7,810	5,600	3,485	2,305	1,820	1,302	790	500	330	215	135	89	52	36	22
1/200	17,420	9,500	6,810	4,235	2,760	2,178	1,583	950	590	400	255	165	106	62	43	26

Figure 18 Values of i for CSP-1 Plans

As we can see from Figure 18, we have the same AOQL value (0.79%) as a result of the two sampling plans with $i=59$ and $f=\frac{1}{3}$ or with $i=113$ and $f=\frac{1}{7}$.

During the process, we must define the basis for the selection of i and f which are affected, for example, by the workload of system operators and inspectors. Of course, we can solve issues such as these if we use quality assurance inspectors to carry out the sampling inspection and if we place on industrialization the burden of 100% inspection. However, care must be taken when selecting prices $f<\frac{1}{200}$, which can lead to a lack of protection against poor quality production. For this reason, it is emphasized to choose $f>\frac{1}{200}$.

Provided that our process is operating in control and if the occurrence of a nonconforming is followed by a 100% screening sequence, then the inspected units have an average number equal to

$$u=\frac{1-q^i}{pq^i}$$

where the fraction defective produced is p and $q=1-p$. Under the sampling inspection, the average number of units approved is equal to

$$v=\frac{1}{fp}$$

Here is the formula for the average fraction of total manufactured units (in the long run) which are inspected:

$$AFI = \frac{u + fv}{u + v}$$

Under the sampling procedure, the average fraction of passed manufactured units is equal to

$$P_a = \frac{v}{u + v}$$

If we plot P_a as a function of p, we obtain, for a continuous sampling plan, an OC curve. The difference between the categories of sampling plans is that in the lot-by-lot acceptance case, the Operating Characteristic curves represent the fraction of the passed batches with sampling inspection, while in the continuous sampling plans, the Operating Characteristic curves show the fraction of the passed units with sampling inspection. Note that the shape of the curve is mainly affected by i if we choose a value of f that is small or moderate.

Chapter 4 An application to simulated data

Suppose a former entrepreneur decides to produce face masks to protect people from the Covid-19 virus. A supermarket chain owner wants to supply the employees of his stores with appropriate masks for their protection from the virus but also for the protection of the customers. Due to the huge demand globally and the depletion of stocks, the owner of the supermarket chain decides to work with the former entrepreneur. The supplier, as a newcomer to this market, does not have any certification and is of questionable quality. However, the price at which the entrepreneur has the masks on the market is extremely tempting. After all, the deal between the supplier and a supermarket owner is that a sample of 2000 masks from a lot will be inspected each week for 3 months. Based on the results of the inspection, the decision will be made by the supermarket owner on whether he will receive the inspected lot the specific week the inspection was performed.

The supermarket chain owner decides that he needs to inspect the masks and chooses to do so, using MIL-STD-105E. The MIL-STD-105E sampling scheme is described in Figure 9. Specifically, he chooses a double sampling plan and also, he does not want to be too strict with the quality of the supplier since the price of the masks is very low and extremely advantageous. Therefore, the owner does not want to be in the situation of not having masks for the staff. Also, due to the fact that the supplier is new to the market, he cannot avoid inspecting the samples. We are going to demonstrate the MIL-STD-105E for three different values of AQL, in order to decide which is the best for our example. The AQL values that we are going use are 2.5%, 4% and 6.5%. Now imagine the case that the probability of making a non-conforming for the producer is 5%. Let's see how many times the supermarket owner will accept each lot, using the double sampling plan, if the estimated fraction defective for the producer is 5%, which is bigger than the AQL values of 2.5% and 4%. The random samples that the supplier submits every week to the supermarket owner, follow a Bernoulli distribution and each has a size N=2000. In order to get as close as possible to the real number of lots that will be accepted, in this application we repeat 1000 times the previous process, and then we find how many lots on average will be accepted in these 12 weeks, for each value of AQL.

At the beginning, we describe the full procedure for the AQL equal to 4%:

AQL 4%

In order to find the inspection process that the owner will follow, using the library AQLSchemes in the statistical package R, and with the command AADouble('Normal') we select:

- What is the inspection level? → II
- What is the lot size? → 1201-3200
- What is the AQL in percent nonconforming per 100 items? → 4.0

Then, we repeat this procedure for AADouble('Tightened') and AADouble('Reduced').

The output of the previous steps is:

	n	c	r
First	80	5	9
Second	80	12	13

Figure 19 Normal inspection (AQL=4%)

From Figure 19, the sampling plan under normal inspection is: Accept the first sample of 80 if there are 5 or less nonconforming, and reject if there are 9 or more nonconforming. If the number nonconforming in the first sample of 80 is 6, 7 or 8, take a second sample of 80 and accept if the combined total of nonconforming items in the two samples is 12 or less, otherwise reject.

	n	c	r
First	80	3	7
Second	80	11	12

Figure 20 Tightened inspection (AQL=4%)

From Figure 20, the sampling plan under tightened inspection is: Accept the first sample of 80 if there are 3 or less nonconforming, and reject if there are 7 or more nonconforming. If the number nonconforming in the first sample of 80 is 4, 5 or 6, take a second sample of 80 and accept if the combined total of nonconforming items in the two samples is 11 or less, otherwise reject.

	n	c	r
First	32	2	7
Second	32	6	9

Figure 21 Reduced inspection (AQL=4%)

From Figure 21, the sampling plan under reduced inspection is: Accept after the first sample of 32 if there are 2 or less nonconforming, and reject if there are 7 or more nonconforming. If the number nonconforming in the first sample of 32 is 3, 4, 5 or 6, take a second sample of 32 and accept if the combined total of nonconforming items in the two samples is 6 or less, otherwise reject.

The above sampling plan is as follows:

Using the sampling scheme MIL-STD-105E with AQL equal to 4%, 8.838 lots will be accepted on average from the 12 that will be submitted. (See the Appendix).

AQL 2.5%

Using the AQL value equal to 2.5%, which is tighter than the previous value of 4%, the outputs are the following:

	n	c	r
First	80	3	7
Second	80	8	9

Figure 22 Normal inspection (AQL=2,5%)

	N	c	r
First	80	2	5

Second	80	6	7

Figure 23Tightened inspection (AQL=2,5%)

	N	c	r
First	32	1	5
Second	32	4	7

Figure 24 Reduced inspection (AQL=2,5%)

Using the AQL value equal to 2.5%, we can see that only 3.889 lots will be accepted on average from the 12 that will be submitted (see the Appendix). It is interesting to note that under MIL-STD-105E plan, if 10 consecutive lots remain on tightened inspection, then the inspection is discontinued, and action should be taken at the supplier level to improve the quality of submitted lots.

AQL 6.5%

Lastly, if we use the AQL value equal to 6.5%, we have the following inspection rules:

	n	c	r
First	80	7	11
Second	80	18	19

Figure 25 Normal inspection (AQL=6,5%)

	N	c	r
First	80	6	10
Second	80	15	16

Figure 26 Tightened inspection (AQL=6,5%)

	N	c	r
First	32	3	8
Second	32	8	12

Figure 27 Reduced inspection (AQL=6,5%)

With this value of AQL, 11.445 lots will be accepted on average from the 12 that will be submitted. This fact makes us wonder whether the value 4% for the AQL is tight enough for this problem or the value 6.5% is too loose. But these two values seem the most reasonable, as the value 2.5% for the AQL is too tight.

Chapter 5 Conclusions

4.1 Discussion

In the previous Chapters, we described in detail the Acceptance Sampling and some of the most important and widely used plans and schemes of this statistical technique. We have seen that the development of new ideas and technologies based on acceptance of sampling is interrelated with the evolution in each production and acceptance line. Today, we can conclude that this technique has proven its value to consumers, suppliers and in general to the development of economic life and trade through a variety of examples. For example, a product of the acceptance sampling technique is the safety and quality certification given worldwide by the largest airlines.

Clearly, there are disadvantages to Acceptance Sampling, as with any other method. However, nothing remains unchanged, but, through research, testing and continuous improvements, sampling plans and sampling schemes are constantly being upgraded. We have seen this before, for example, in improving the cost and OC curve of single sampling plans, where they expanded to double sampling and multiple sampling plans. Clearly, any upgrade is likely to lead to a significant increase in the complexity of the method as well as several additional disadvantages, but that's the reason it needs to be evaluated and a decision has to be made as to whether the disadvantages are adequately offset by the potential benefits.

4.2 Future Work

Undoubtedly, the technology has a major role to play in further research and development of Acceptance Sampling, as well as statistics and science in general. Today, with various statistical packages such as R, Minitab, SAS, StatGraphics and SPSS, we can implement many different sampling plans. However, the research so far in specialized fields of Acceptance Sampling, although satisfactory, can be further expanded. Indicatively, areas that need further research and development are acceptance sampling plans for autocorrelated processes, multivariate sampling inspection problems, and the development of new sampling plans for the inspection of big data samples.

By comparing sampling plans or sampling schemes in real data applications, we can evaluate their contribution to the development of the industry, but also better understand them.

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Appendix

```
set.seed(10)
mil4=double(1000)
mil25=double(1000)
mil65=double(1000)
for (j in 1:1000){
### Data
samples<-matrix(NA,nrow = 12, ncol = 2000)
for (i in 1:12) {
 samples[i,]<- rbinom(2000,1,prob = 0.05)
}

### MIL STD 105E with AQL=4%
accept<-rep(0,12)

## start with normal inspection for the first 5 lots
for (i  in 1:5) {
 if (sum(samples[i,1:80])<=5){
  accept[i]<-1
 } else if (sum(samples[i,1:80]<=8) & sum(samples[i,1:80]>5) ) {
  if (sum(samples[i,1:160])<=12) {
   accept[i]<-1
  }
 }
}

### now continue the original sampling scheme

for (i in 6:10) {
 control<-(accept[i-1]+accept[i-2]+accept[i-3]+accept[i-4]+accept[i-5])/5
 if (control<=(3/5)) { ## tightened
  if (sum(samples[i,1:80])<=3){
   accept[i]<-1
  } else if (sum(samples[i,1:80]<=6) & sum(samples[i,1:80]>3) ) {
   if (sum(samples[i,1:160])<=11) {
    accept[i]<-1
   }
  }
 } else {  ## normal
  if (sum(samples[i,1:80])<=5){
```

```
   accept[i]<-1
  } else if (sum(samples[i,1:80]<=8) & sum(samples[i,1:80]>5) ) {
   if (sum(samples[i,1:160])<=12) {
    accept[i]<-1
   }
  }
  }
}

for (i  in 11:12) {
  if (sum(samples[i,1:80])<=5){
   accept[i]<-1
  } else if (sum(samples[i,1:80]<=8) & sum(samples[i,1:80]>5) ) {
   if (sum(samples[i,1:160])<=12) {
    accept[i]<-1
   }
  }
}

mil4[j]=sum(accept)



### MIL STD 105E with AQL=2.5%
accept<-rep(0,12)

## start with normal inspection for the first 5 lots
for (i  in 1:5) {
  if (sum(samples[i,1:80])<=3){
   accept[i]<-1
  } else if (sum(samples[i,1:80]<=6) & sum(samples[i,1:80]>3) ) {
   if (sum(samples[i,1:160])<=8) {
    accept[i]<-1
   }
  }
}



### now continue the original sampling scheme

for (i in 6:10) {
```

```
  control<-(accept[i-1]+accept[i-2]+accept[i-3]+accept[i-4]+accept[i-5])/5
  if (control<=(3/5)) { ## tightened
    if (sum(samples[i,1:80])<=2){
      accept[i]<-1
    } else if (sum(samples[i,1:80]<=4) & sum(samples[i,1:80]>2) ) {
      if (sum(samples[i,1:160])<=6) {
        accept[i]<-1
      }
    }
  } else {  ## normal
    if (sum(samples[i,1:80])<=3){
      accept[i]<-1
    } else if (sum(samples[i,1:80]<=6) & sum(samples[i,1:80]>3) ) {
      if (sum(samples[i,1:160])<=6) {
        accept[i]<-1
      }
    }
  }
}

for (i  in 11:12) {
  if (sum(samples[i,1:80])<=2){
    accept[i]<-1
  } else if (sum(samples[i,1:80]<=4) & sum(samples[i,1:80]>2) ) {
    if (sum(samples[i,1:160])<=6) {
      accept[i]<-1
    }
  }
}

mil25[j]=sum(accept)



### MIL STD 105E with AQL=6.5%
accept<-rep(0,12)
## start with normal inspection for the first 5 lots
for (i  in 1:5) {
  if (sum(samples[i,1:80])<=7){
    accept[i]<-1
  } else if (sum(samples[i,1:80]<11) & sum(samples[i,1:80]>7) ) {
```

```
   if (sum(samples[i,1:160])<=18) {
    accept[i]<-1
   }
  }
}

### now continue the original sampling scheme

for (i in 6:10) {
 control<-(accept[i-1]+accept[i-2]+accept[i-3]+accept[i-4]+accept[i-5])/5
 if (control<=(3/5)) { ## tightened
   if (sum(samples[i,1:80])<=6){
    accept[i]<-1
   } else if (sum(samples[i,1:80]<=9) & sum(samples[i,1:80]>6) ) {
    if (sum(samples[i,1:160])<=15) {
     accept[i]<-1
    }
   }
 } else {  ## normal
   if (sum(samples[i,1:80])<=7){
    accept[i]<-1
   } else if (sum(samples[i,1:80]<=10) & sum(samples[i,1:80]>7) ) {
    if (sum(samples[i,1:160])<=18) {
     accept[i]<-1
    }
   }
  }
}

for (i  in 11:12) {
 if (sum(samples[i,1:80])<=7){
   accept[i]<-1
 } else if (sum(samples[i,1:80]<=10) & sum(samples[i,1:80]>7) ) {
   if (sum(samples[i,1:160])<=18) {
    accept[i]<-1
   }
  }
}

mil65[j]=sum(accept)
```

```
}
mean(mil4) #8.838
mean(mil25) #3.889
mean(mil65) #11.445
```