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The Non-Linear Impact of Debt on Economic Growth: The Empirics

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Contents

Abstract

This essay surveys part of the empirical literature dealing with the non-linear relationship between debt and growth and thresholds effects.

In recent years, particularly after the financial crisis, many researchers studied the possibility of a non-linear relationship and threshold effects in the debt-growth relation and their empirical work has identified various thresholds.

We present and analyze the data, methodology, model, robustness checks and results of the papers studying the non-linear debt-growth nexus published over the years 2010 to 2015. Their researches are subject to a number of conceptual and methodological issues that are also analysed. The papers are presented following a way that is considered a logical sequence which is based on the separation of the research's results among 4 improvised categories. The categories regard the existence of the non-linear relationship and the existence and level of thresholds.

We start by setting a benchmark study for the debt-growth relationship, as relevant literature also does, which is unarguably the influential research of Reinhart and Rogoff (2010). Reinhart and Rogoff used descriptive statistics and argued that the relationship between debt and growth is non-linear with a particular threshold of 90% of debt-to-GDP ratio beyond which GDP growth is dramatically worsen. The 90% debt threshold has gained much attention in the academic press and policy debates calling it as a "tipping point". But further revelations found faults with the descriptive analysis carried out by Reinhart-Rogoff. These faults are demonstrated in the work of Herndon et al. (2013).

The first category regards the researches following Reinhart-Rogoff that confirmed the existence of the 90% debt threshold through formal econometric approaches. Three of these analyses that found to be popular in the literature were the analyses of Cecchetti, Mohanty and Zampolli (2012), Checherita-Westphal and Rother (2012) and Kumar and Woo (2010). Through different methodologies and various robustness checks these authors identified and formally established a robust negative non-linear relationship and popularized even more the 90% debt threshold.

The second category comprehends the papers that decompose one element of the previous results. In this section the researchers ended up finding a robust negative non-linear link but with the debt-to-GDP thresholds lying elsewhere than the 90 percent level. Sulikova et al. (2015) found that relationship between the debt-to-GDP increase and GDP growth is determined by an inverted U-shaped curve with the peak at 64% debt-to-GDP ratio. The analysis of Caner, Grennes and Koehler-Geib (2010) also confirmed the negative non-linear relationship with a tipping point at 77% for the full sample of their data coverage. Elmeskov and Sutherland (2012) suggest that there may exist two debt thresholds above which the impact on GDP growth becomes more important. The lower debt threshold is estimated at 45 percent level and the higher at 66 percent level of GDP.

The completely opposite perspective, of course, has been advanced by those who disputed the idea of an explicit debt threshold beyond which debt largely hurts growth, if any such a

threshold exists in the first place. In the third category there are papers raising serious concerns whereby nonlinear effects do exist weakening the case for a common debt-to-GDP threshold. If nonlinearities exist, they probably are more complex and difficult to model than what has been initially assumed. We concentrated on the work of Égert (2013), Pescatori, Sandri and Simon (2014) and Eberhardt (2013).

The main way of investigating the debt thresholds has been to look exclusively for threshold effects of public debt on growth when debt is above or below a certain public debt threshold level. But, why would we believe a priori that the effect of public debt on growth is characterized only be excessive levels of debt? There is theoretical evidence that the effect of debt on growth might depends on other factors. In this fourth section we will analyse Kourtellos, Stengos and Tan (2013,b) which found that the impact of debt on growth is determined by democracy.

1. Introduction

1.1 An introduction to the main issue

High public debt appears to be connected with lower economic growth. But, which exactly is the relationship between debt and growth? Is it nonlinear? If yes, where is the particular debt threshold above which future growth is impaired? These are excessively valuable policy questions given the important role of government debt in the macroeconomic environment in developed and emerging market economies. The more recent increase in public debt has given rise to questions as to whether it is starting to cross a level beyond which it may decreases economic growth. In this essay we present and analyze a part of the relevant literature which examined empirically whether a non-linear impact of debt on economic growth exists and where the thresholds lie. The dispute about the debt-growth relationship has been animated by an increasing set of empirical researchers which differentiate in several ways (datasets, econometric issues, methodology, and results).

In the last years, the relevant literature in principal focused on emerging economies investigating the external debt levels besides sovereign debt. But it was soon after the 2008 economic and financial crisis that made the researchers' interest turn into developed economies and more specifically into the public debt-growth association in the member states of the euro area in the context of the sovereign debt crisis of the Eurozone. The reason for that was that before the global economic crisis public debt was not a serious and uncontrollable problem in developed countries; there where an abundance of liquidity on the markets, external financing opportunities at relative low prices, and indebted countries, with only a few exceptions, had much better credit ratings. Moreover, until recently most of the empirical studies relied on linear estimation frameworks. Only lately the focus has been shifting to non-linear threshold analyses. Due to the public debt crisis a new wave of literature has been sparked raising new questions concerning the public debt economic growth relationship.

Before proceeding, the definition of "public debt" in what follows must be interpreted. The majority of the papers refer to "public debt" as the gross government debt which captures the stock of outstanding government debt. Even though gross debt is not a good indicator of a government's financial situation, we found only rare cases where the net debt is used. Gathering net debt data is challenging, and therefore the gross government debt data are preferred. Nevertheless, even gross debt data may be subject to measurement errors. The researchers analyzing the debt-growth nexus employ decompositions of debt which are either "general government debt" or "central government debt". These measures seem to be easier to handle and more appropriate in order to extract policy implications. General government debt measures the consolidated debt of the general sector of government which includes the central government debt but also debt of the social security administrations and subnational governments.

As for its uses, government debt issuance can help smooth taxes when confronted with varying expenditures and by that smooth consumption through the lifetime of an individual currently alive but also across generations. On the presupposition that the future generations are going to be wealthier –as they are going to have more human capital and more productive technology- a shift of resources from future to current generations can increase society's intertemporal welfare. Rising taxes in order to fund higher current consumption may be delayed by rising public debt, yet without necessarily jeopardizing growth. Public debt issuance is definitely encouraging when it is used to finance public investments (health, education, public defense, technology and R&D), where under specific prerequisites public debt can play a productive role. In this way, financial deepening and rising debt is appropriate. It could lead to improvements in economic activity. Countries would not be able to prosper if they could not borrow and GDP would be more volatile than beneficial.

Indisputably, government debt increases do not always improve welfare. Before the 20th century, the rate of public debt accumulation had been, in general terms, sluggish and mainly noticed in war periods. But, during the last decades, public debt has been rising in greater rates, accompanying the increase of the government sector. For many industrial countries, the general government expenditures rose dramatically, during the 20th century. Parallel to this, governments have been borrowing in order to deal with business cycle fluctuations. Yet, while this should be a "healthy" reason for increasing debt levels, as it targets to boost economic activity and/or decrease unemployment, it is not always used in the way that is efficient for the economy. The noticeable government debt expansion in developed economies may be also attributed to the revenue pool problem: the individuals that make the most from additional spending are different from the ones that pay the extra cost of funding it. Furthermore, from political perspective, resorting to debt issuance is more desirable than raising taxes which generates discomfort to the society and political cost to the politicians.

The financial and economic crisis prompted by the unwinding US subprime mortgage market resulted in deep economic recession in many countries of the world. Governments and central banks reacted to the Great Recession by firing heavy artillery: fiscal and monetary policy expansion, unprecedented in size and in the way they were co-ordinated across countries, were swiftly enacted in advanced and emerging markets, and banking sector bailouts prevented the collapse of the financial system. While these actions certainly helped smooth the cycle, discretionary fiscal loosening and banking sector bail-outs contributed to a large extent to a sharp increase in many countries' public debt-to-GDP ratio.

The sharp increase in public debt has led to doubts concerning the fiscal sustainability and the subsequent economic impact. Academic and policy debates began questioning the degree to which public debt is going to have harmful consequences on capital accumulation, as well as productivity, and decrease economic growth.

Accumulating of debt may expose a country to danger. While debt levels increase, the country's probability of not being able to repay –and default- increases for given shocks. For example a decline in economic activity or interest rates increase may hit the economy. And then, all of a sudden, a highly indebted country may be considered as uncreditworthy. Afterwards, consumption and investment drop. If the decrease is large, it might lead to defaults, deficient demand and unemployment. These bad outcomes are higher, the higher the level of debt. So, accumulating debt might result to higher GDP volatility, higher financial fragility and is possibly connected to reducing average economic growth.

Higher debt loads might impact growth through more increased costs of capital or through higher distortionary taxes, inflation or larger volatility in policy. A higher cost of capital is likely to reduce the capital-to-labour ratio and hence productivity. Higher costs of capital affect the intensity of capital in production practically resulting in a level shift in potential output and as a consequence to growth rate effects over some finite period only. The impact on economic growth could me more durable to the degree that higher costs of capital result to fewer investment in research and development (literature expects that impacts of debt via R&D should be accrued through TFP). Tax increases that are necessary in order to service public debt decrease disposable income and saving and so reduce private investment. Rising public debt may also increase long-term rates significantly which in turn hurts productive public investment and equally importantly private investment by increasing the cost of capital.

While literature broadly supports the view that the increasing public debt is harmful for economic growth (especially in the long-term), there are related issues that are rather controversial. On many further issues researchers have not reached a consensus.

First of all, in the related literature, there is no consensus about the way of impact of public debt on the economic growth. There are many findings indicating that public debt can affect the economic growth either linear or nonlinear. Possibly this effect is non-linear in the sense that it becomes relevant only after a particular threshold has been reached. Empirical literature has discovered diverse thresholds in the debt-growth nexus.

Second, though high levels of public debt are likely to have negative impact on growth, the negative correlation does not imply causation. The negative impact might run from the economic growth to the public debt as a decrease in the economic activity is accompanied by an increase in the public debt-to-GDP ratio.

A final controversial element is whether the nonlinearities and thresholds are broadly the same in each country or whether they are country-specific. The standard empirical approach interprets non-linearities by adopting pooled models, thus imposing homogeneity across countries. But there is literature indicating that public debt affects growth in a different way across countries and time. There are many theoretical and empirical reasons for taking as granted that the debt thresholds and the debt-growth long-run relationship differentiate across countries. For example, debt may affect economic growth differently in low-income countries, because of less developed domestic financial markets, a different degree of openness and different institutions. More specifically, the production technology is different for every country, so if debt thresholds exist, it is reasonable to assume that debt starts hurting an economy on different levels between countries that vary by their productivity. Second, the ability of an economy to sustain high levels of debt without hurting growth is determined by characteristics typical of any such economy as their macroeconomic and institutional environment and structural weaknesses. These are characteristics often unobserved or difficult to measure. Third, the composition of debt (short-term versus long-term, foreign versus domestic debt etc), the manner that it has been accumulated and what it has financed also differentiate across countries and naturally affect the growth performance differently. Reinhart and Rogoff support for example that debts accumulated through war periods tend to be less detrimental for growth than debts accumulated in peace time. Consequently, the imposition of homogeneity across countries and time constitutes a weakness of this issue.

1.2 A brief review of the theoretical literature on the debt-growth relationship

We turn to a brief report of what theoretical analyses suggest.

Dating back to the middle of the 20th century, three distinctive perspectives end up to different directions of the effect of debt on growth. The neoclassical school stated that as public debt increases it should have a negative impact on economic growth, while on the contrary the Keynesian economists' view was that its effect is positive (during economic recession). The Ricardian equivalence proposition assumed the effect to be neutral or irrelevant.

The first part of the theoretical-perspective analysis affects the existence of the negative link between public debt and growth. There are researchers that claim that the public debt to GDP ratio and economic growth relationship is probable characterized by a negative correlation. The simplest connection between debt and growth is the public debt theory developed by Robert Barro (1979). Under the assumption that taxes in the end need to increase to reach debt sustainability, the distortionary impact of this will possibly decrease potential output. Government spending may also be implemented which also produce contractionary effects.

Elmendorf and Mankiw (1999) express various theoretical explanations which can support a negative long-run relationship between public debt and growth. In their "conventional view of public debt" government expenditures are assumed to be fixed. Researchers evaluate what happens if the taxes are temporarily reduced and the expenditures are then financed by issuing debt. The Ricardian Equivalence does not apply which means that public debt impact real variables. They argue that in the short-run output is demand-determined and fiscal deficits (or higher public debts) have a positive impact on disposable income, aggregate demand and overall output. This positive short-run effect of budget deficits (and higher debt) is probably large when the output is far from capacity. In the long-run however the relation becomes negative. Higher fiscal deficits will affect public savings negatively, and this loss will not be entirely counterbalanced by an increase in private savings. This results to the reduction of national savings, and then lower total investment, either domestically or abroad. The lower domestic investment will have a negative effect on GDP, as it will lead to a smaller capital stock, higher interest rates, lower labor productivity and wages. Lower foreign investment will affect adversely the foreign capital income and thus lowers the country's future GNP.

According to Cochrane (2011a, 2011b) the negative effect of public debt could be much larger if high public debt increases uncertainty or leads to expectations of future confiscation, possibly through inflation and financial repression. To this extent, higher debt may have a negative impact on growth even in the short-run.

Analyses based on standard overlapping generation models of growth, find that public debt reduces savings and capital accumulation through higher interest rates, leading to a decline in investments and thus weakening economic growth. In one of these researches, Modigliani (1961) refers to the debt burden, as the reduction in the aggregate stock of private capital, which will lead to a reduction in the flow of goods and services for future generations. Debt is a burden, because it crowds out capital. Diamond (1965) distinguish between external and internal debt. Beyond the debt burden effect of Modigliani (1961), he points out the reduction of the available lifetime consumption of the individuals from higher taxes required in order to finance the interest payments either of external or internal debt. Further, by reducing the disposable income of the individual, taxes reduce his savings and thus the capital stock. Internal debt produces one extra effect as it reduces capital stock emerging from the substitution of government debt for physical capital in individual portfolios.

In endogenous growth models public debt has generally a negative effect on long-run growth (Barro, 1990, Saint-Paul 1992). According to Saint-Paul (1992), standard growth theory supports that a government debt expansion caused by a fiscal deficit ends up in a permanent growth decrease in the endogenous growth model.

More recent standard endogenous growth models with variations also conclude to the existence of a negative relation between debt and growth. For example, Futagami et al. (2008) present an endogenous growth model with productive public spending and public debt where the government is not allowed to raise the debt to GDP ratio beyond a certain critical value. For that model, it turns out that the balanced growth rate is the higher the smaller the public debt to GDP ratio is.

Greiner (2011) argues that the effect of debt on growth depends on the presence of rigidities in the economy. Greiner presents an endogenous growth model with no rigidities and elastic labour supply where ongoing growth results from positive externalities of private capital. In that model public debt and economic growth are negatively correlated, too. It must also be pointed out that it is not the standard crowding-out mechanism that generates this outcome because, as the debt ratio rises, the primary surplus rises, too, to guarantee sustainability. Consequently, for a fixed tax rate public spending declines, thus, preventing a crowding-out of private investment. Rather, the decline in the growth rate, as a consequence of higher public debt, is due to the fact that higher public debt leads to a lower shadow price of savings which reduces labour supply and investment. On the case where wage rigidities and unemployment exist, public debt does not affect the allocation of resources and may rather impact growth positively, if it is driven to productive investments.

The second part of our theoretical-perspective investigation concerns the existence of non-linearity in the debt–growth nexus.

Sutherland (1997) and Perotti (1999) pointed out the asymmetric effects of fiscal policy which could motivate a non-linear effect of public debt on output growth in advanced economies.

Non-linearities and threshold effects might also appear if there is a critical value of fiscal sustainability: if debt is in elevated levels, it might destroy investment, because investors think that more taxes will be implemented to service the pre-existing debt (Krugman, 1988; Aguiar et al., 2009). However, it is possible that this argument does not apply to advanced countries where the individuals that hold debt are mainly residents (and therefore there is not an external transfer problem).

Alternatively, as debt levels rise with respect to GDP, creditors would ask for higher interest rates to compensate the risk of default and this effect would increase the cost of financing, constraining investment (Greenlaw et al., 2013).

Checherita-Westphal, Hughes Hallett, and Rother (2012) develop a theoretical model in which, over the business cycle, debt can only be issued to finance public investment and the optimal level of public debt is determined by the public to private capital ratio that maximizes economic growth. With such a set-up, they show that the level of debt that maximizes economic growth is a function of the output elasticity of the capital stock. However, Greiner (2012) shows that the results of Checherita-Westphal, Hughes Hallett, and Rother (2012) are influenced by their supposition that the deficit equals public investment at each point in time. Greiner (2012) supports that in a framework like this, debt is totally irrelevant and the debt-growth non-linear relationship is determined by the growth-maximizing tax rate.

If a critical value beyond which government debt unexpectedly turns unsustainable exists, it might be a possible explanation for the existence of non-linearities (Ghosh, Kim, Mendoza, Ostry, and Qureshi, 2013, provide a formal model). Yet, literature does not provide a theoretical model where such a critical value is integrated in a growth framework.

High debt levels might also set limits to the potential of a government to carry out countercyclical policies, which results to a more variable output and decrease economic activity. However, the relationship between debt and the ability of carrying out countercyclical policies is more likely to depend on the composition of public debt than on the level of public debt itself. This suggests that countries with different debt structures and monetary arrangements are likely to start facing problems at very different levels of debt.

This essay follows the empirical literature dealing with the non-linear relationship between debt and growth and thresholds effects. We should note that most of these researches before investigating whether non-linearities exist or not, first they try to establish the existence of a negative *linear* relationship between these two variables. Their exact sequence is followed and so the methodologies and the results of their linear specifications will also be presented. These will also be reported for comparison reasons to the parameters' estimations from the non-linear impact of debt on growth.

2. Model specification and methodological issues

2.1 Growth Equation: The Impact of Debt on Growth

The most usual method which is encountered in the literature in order to find the correlation between debt and growth is the estimation of alternative versions of the dynamic growth model:

$$GROWTH_{i,t-(t-n)} = \alpha ln(GDP)_{i,t-n} + \beta DEBT_{i,t-n} + \gamma X_{i,t-n} + \tau_t + \eta_i + \epsilon_{i,t} \qquad \textbf{(2.1)}$$

By including in growth regression (2.1) various terms connected with debt (e.g. dummy variables or quadratic terms) researchers try then to identify if there is a non-linear relationship between debt and growth. This empirical specification is derived from the neoclassical growth model of Solow, in which per capita income growth depends on the initial level of physical and human capital, converging to its steady state rate slowly over time. In turn, the steady state depends positively on the saving rate and negatively on the growth rate of the labor force, in addition to a number of parameters describing the technology and the preferences of the country.

The dependent variable is GDP growth of a country *i* over period *t-n* and *t.* The independent variables include the initial level of GDP, the ratio of public debt over GDP, and a vector of explanatory variables X which differentiates across studies. According to different researches studying the determinants of long-run growth, the control variables included in X have been identified to be significantly and robustly correlated with long-term growth (e.g., Barro and Sala-i-Martin, 2004, Sala-i-Martin et al., 2004, Aghion and Durlauf, 2005 among others).

Based on what the augmented Solow model implies, the rate of population growth, the ratio of investment-to-GDP and a measure of the stock of human capital are the most common explanatory variables included in the growth regression (Mankiew, Romer, and Weil, 1992). Here, it is also worth referring to the relevant study of Sala-i-Martin et al. (2004) on the robustness of 67 explanatory variables as growth determinants by employing comprehensive cross-country growth regressions. They ended up finding 18 explanatory variables to have high posterior inclusion probabilities. This means that each one of these variables has high marginal contribution to the explanatory power of the regression model, comparing to models that does not include them and so regarded as being significantly and robustly correlated with long-term growth. Not all of these 18 variables were economic variables. The economic variables were: the initial level of real per capita GDP, primary school enrollment, the initial government consumption share, trade openness, and the relative price of investment. The rest are regional variables (Africa, East Asia, Latin America), and variations of socio-political factors (including religious and ethnic variables). Most studies analyzed below include in their growth regressions the main suggested robust economic variables with some differentiations.

The initial level of per capita GDP is being used to capture the “catch-up-effect” or conditional convergence of the economy to its steady state. The growth regression (2.1) is considered a dynamic fixed-effects panel data model due to the inclusion of the log real GDP among the regressors. Thus, in general the estimates produced by the least squares dummy estimator may be biased, inconsistent and usually inefficient. Instead, literature proposes a number of alternative estimators in order to overcome this. These are the instrumental variable estimation (IV) (Anderson and Hsiao, 1981) or Generalised method of moments (Arellano and Bond, 1991; Arellano and Bover, 1995).

However, instrumental variable and GMM approaches yield consistent estimates when the cross-sectional dimension is large (and the time-series dimension small) and that makes them inappropriate for application to the typical macroeconomic datasets with moderate N and T. The difference and system GMM estimators were constructed for micro datasets and they do not fit perfectly to macroeconomic data (Bond, 2002).

The system GMM approach (Arellano and Bover, 1995 and Blundell and Bond, 1998) uses suitable lagged levels and lagged first differences of the regressors as their instruments. It constitutes a very familiar and popular procedure, and it is largely used in the estimations of the papers discussed in this essay. However, IV estimators are in principle less efficient than Ordinary Least Squares (OLS) as instrumental variables may only be weakly correlated with the instrumented variables. The instruments used by the GMM approach are supposed to be strong, yet this is not tested. Furthermore, Bazzi and Clemens (2013) argue that while many lags can be employed, this fails to address the weak instrument problem and may result to spurious results. More specifically, instrument proliferation, particularly in the present of highly persistent endogenous explanatory variables (as in the case of debt ratios), possibly weakens the validity of internal instruments in the system estimator (Roodman, 2009).

So, estimates through IV and GMM may also be biased and tests of hypothesis may have low power (Stock et al., 2002).

In additional, in order to estimate the Growth Equation (2.1), the length of the growth episode (*n*) needs to be selected and this choice causes several tradeoffs.

If *n* is chosen to equal 1, this refers to annual GDP growth and while this makes the number of observations the maximum possible, it is likely to produce estimates entirely influenced by business cycle fluctuations. Equally importantly, it may lead the estimates to be subject to endogeneity since growth will be just one year forward with respect to debt.

Setting *n* equal to five weakens these problems. This procedure gives the coefficient of the current level of debt (and the other explanatory variables) when regressed to the 5-year forward GDP growth rate and allows eliminating cyclical and other short-term effects that blur estimations produced by using annual growth rates. However, this approach decreases the observations (that may create problems to small databases) but more importantly it constitutes arbitrary the selection of the first and last observations used. Another common practice broadly used in the growth literature is that of five-year or three-year overlapping averages of the output growth rate. Here, lies the cost of introducing autocorrelation in the model. The use of forward GDP growth rates with respect to debt still does not solve the endogeneity problem of debt.

The majority of the researchers try to check pre-determined levels of debt as thresholds in the growth regressions while some other seek to determine the thresholds endogenously.

2.2 Country and time fixed effects

In panel data regressions, country-specific fixed effects, η_i, as well as time-specific fixed effects, τ_t are included in growth equation (2.1). They have considerable advantages.

In some papers, introducing within-country or time effects in the panel data regressions is mandatory due to the relatively small number of databases, which excludes a sensible analysis of cross-country differences.

In particular, time-specific fixed effects are meant to capture common effects or common shocks across countries that have taken place under the time of the examination. For example economic and monetary regime changes, like the formation of the European Union as well as global business cycle conditions that may have an impact on all countries and so on.

Country-specific fixed effects are being used because they allow measuring to what degree a change in one factor affects growth within a country.

They control for unobservable country specific characteristics, for differences between countries, assuming that the heterogeneity is stable over time. They capture economic and social characteristics, such as legal, institutional and cultural diversifications across countries. These characteristics cannot be measured, but if they remain constant over time their impact can be captured by country-specific fixed effects.

2.3 Endogeneity of debt

A variety of sources of bias concerning the estimation of growth equation have to be examined and resolved in order for the estimated parameters to be consistent.

The literature supports that in the growth equation government debt suffers from endogeneity and so significantly biased parameters are produced. While it is proven that public debt is negatively associated with economic growth, this does not necessarily indicate that debt slows down growth. This is the problem of reverse causation or simultaneity bias between debt and growth resulting in endogeneity bias. The correlation between public debt and growth *does not imply that causality goes from debt to growth*. The exact way of the causality is something very important for researchers to clarify but in practice extremely difficult. The correlation found between these variables could be explained as low or negative economic growth forcing governments to raise the levels of debt (Reinhart, Reinhart, and Rogoff, 2012). In other words, there seems to be possible that while high debt affects growth adversely, slow growth -for factors that are not connected to debt-could also result to high debt. In addition, endogeneity may stem from the possibility that government debt and growth are jointly determined by a third variable, and so the observed correlation could be due to this third factor. An obvious example for this is war. We conclude that debt is strongly likely to be endogenous, so literature tries to address this issue with several techniques.

Panizza and Presbitero (2012) present the endogeneity problem by using a simple bivariate model. In this model growth (*G*) is a function of debt (*D*), and debt is a function of growth:

$$G = a + bD + u, \qquad (2.2)$$

$$D = m + kG + v. \qquad (2.3)$$

The OLS estimator of *b* is then given by:

$$\hat{b} = \frac{b\sigma_v^2 + k\sigma_u^2}{\sigma_v^2 + k^2\sigma_u^2}, \qquad (2.4)$$

And the bias of the OLS estimates is:

$$E(\hat{b}) - b = \frac{k(1 - bk)}{\sigma_v^2/\sigma_u^2 + k^2}. \qquad (2.5)$$

According to equation (2.4), if *k*=0, it means that the OLS is unbiased and is the case where debt does not suffer from endogeneity. According to stability: *bk* <1, so if *k*<0 (which is the possible case), OLS estimates are negatively biased.

The literature deals with this problem in several ways, one of which is by employing the moving averages of the GDP growth (lagged debt). This technique although mitigates the endogeneity problem does not resolve it completely. It may be the case that if a country expects low economic growth and applies counter-cyclical expansionary fiscal policy, this causes the debt-to-GDP ratios at time *t* to increase. In period (*t*+1; *t*+*n*) the economy slows down but the negative debt-growth correlation that appears, should not be interpreted as public debt having a causal impact on growth in the future. Instead of this, the use of 10-

year growth that has been proposed further reduces endogeneity, but it is not useful when the cross-sectional dimension of the dataset is small.

Other authors defeat the endogeneity by employing the Instrumented Variables (IV), system and difference Generalised Method of Moments (GMM) estimators. But as discussed in a subsection above, IV and GMM techniques are not completely appropriate when used to the typical macroeconomic datasets causing rather inconsistent estimates. Therefore, IV and GMM estimations although widely used in several papers to solve the endogeneity of debt and the problem of reverse causality, their results should be interpreted with caution. The exact way of the causality between debt and growth could be correctly assessed by the identification of an external instrumental variable that has a direct impact on debt and indirect on economic growth.

2.4 Other types of Bias

In addition to the endogeneity bias of debt, other types of bias can occur in a number of different ways in panel growth regressions and lead to inconsistent estimates. It is important for researchers to be aware of these and find ways to minimize bias; a few common examples follow.

There exist the omitted-variables bias (heterogeneity bias) stemming from the correlation between country-specific fixed effects and the regressors which can produce inconsistent estimates of pooled OLS and BE (between estimator). Furthermore, classical measurements errors affecting the independent variables, which makes pooled OLS, BE and FE estimator inconsistent. Also, the dynamic panel bias produces inconsistent estimates to the FE estimator. It comes from the correlation between $y_{i,t-\tau}$ and ν_i in the presence of lagged dependent variable because $y_{i,t-\tau}$ is endogenous to the fixed effects (ν_i) in the error term. In the FE, the fixed effects (ν_i) are eliminated via within-transformation, but there is now a correlation between the transformed lagged dependent variable and the transformed error term, causing the FE to be inconsistent and biased downward.

Selecting anyone of the above estimators entails a trade-off between the biases. Analytically, the BE estimator reduces the magnitude of measurement error through time averaging of the regressors, yet is incapable of addressing the omitted-variable bias. Pooled OLS and BE are subject to omitted-variables bias and measurement errors. Fixed effects fight the omitted-variables bias through controlling for fixed-effects. In comparison to the BE and OLS, Fixed effects worsen the measurement error bias.

On the one hand, the dynamic panel GMM estimator deals many of the biases discussed above but on the other hand they may suffer from the weak instruments problem (Roodman,2009 and Bazzi and Clemens, 2009). The SGMM is commonly more robust to weak instruments compared to the difference GMM, but remains subject to weak instrument biases. Therefore, it is not easy to come to a conclusion on which technique produces the least total bias among all these sources of bias. Hauk and Wacziarg (2009) used Monte Carlo simulations to evaluate the bias properties of the above mentioned estimators. They concluded that BE has the best performance (i.e. yields the least total bias) among the four estimators on each of the estimated coefficients in the growth regressions when potential heterogeneity bias but also a variety of measurement error problems coexist. However, the most of papers below employ the SGMM estimator.

3. The Path-breaking Study of Reinhart and Rogoff

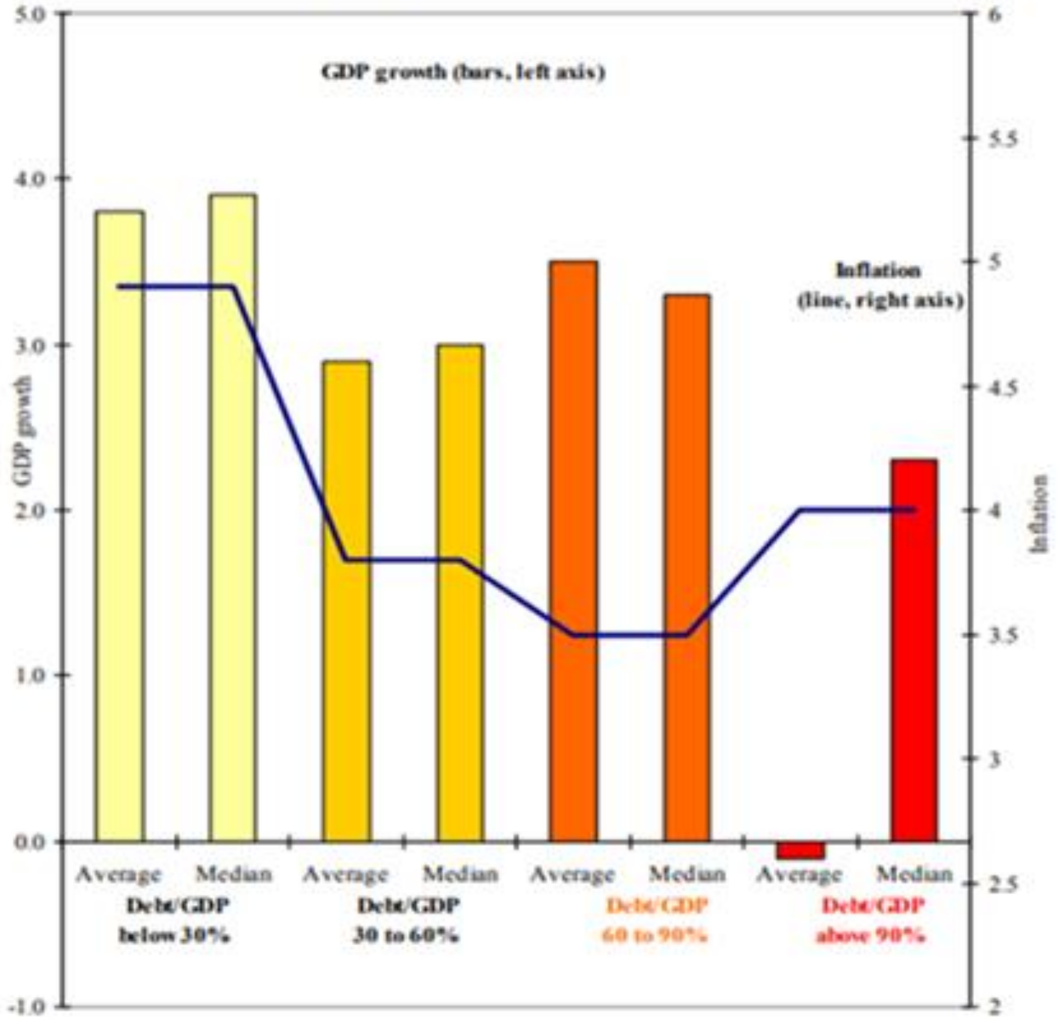
A good starting point for investigating the debt-growth relationship is unarguably the influential research of Reinhart and Rogoff, 2010 (hereafter RR) which has been pointed out by those who wanted to rationalize the policy of austerity agenda on both sides of the Atlantic. Although, they use simple descriptive statistics rather than formal econometric procedures, a vast majority of the relevant literature considers their work as a benchmark study. In their analysis, they gathered annual observations for 44 advanced and emerging economies for the time period 1946-2009, in order to study economic growth at different levels of government debt. RR found evidence of strong negative non-linear effects of high public debt on economic growth when public debt ratio exceeds 90%.

As already mentioned, one issue that has to be taken into account concerns the way the meaning of "debt" is interpreted. The reason for this is because some authors regard debt differently. When RR used "public debt "they refer to gross central government debt. Their work starts by splitting the 44 countries into 2 sub-groups, the one included 20 selected advanced economies (OECD countries) and the other 24 emerging economies. Then, in each sub-group, they split the sample into four categories: (i) country-years for which public debt/GDP is below 30 percent; (ii) country-years for which public debt /GDP is between 30 and 60 percent; (iii) country-years for which public debt/GDP is between 60 and 90 percent; and (iv) country-years for which public debt/GDP is above 90 percent. The authors suggest that the four categories standing for low, medium-low, medium-high and high debt levels are based on the interpretation of much of the literature and policy discussion on what is considered low, high etc debt levels. However, the four thresholds would later be criticized for being arbitrarily chosen and vague. An additional important problem that received a lot of –fair-criticism is that RR checked only the correlations between debt and growth without accounting for any other determinants of growth. They also did not attend to the reverse causality problem.

Figure 3.1 presents the results of their computations for the sample of the 20 developed countries. The bars show average and median real GDP growth for each of the four debt categories. The computations indicate that no explicit link and also no large differences exist between GDP growth and debt/GDP ratios for the first three debt categories. On the contrary, average and median GDP growth are to a considerable extent lower for the years that debt/GDP ratios lies above the 90% threshold. A significant fall of 1.5 percentage points in median growth rates of GDP is identified when comparing the low debt groups (debt below 30 percent of GDP) and high debt groups (debt above 90 percent of GDP). But when considering the average GDP growth between debt levels below 30% and debt levels above 90% the result is remarkable, that is: roughly 4 percentage points. Specifically, the years that OECD countries have debt levels above 90% of GDP, the average GDP is -0.1% (roughly 3.6% lower than the average GDP growth of the previous debt space) and the median price is 2.5% (roughly 1% lower than that of the previous space). The use of lagged debt to somehow address reverse causality made no great differences to the results.

RR seek to enhance the validity of their results so then they used a dataset that comprises the same selected OECD countries and years covering one to two centuries between the time period 1790-2009. The results are illustrated in Table 3.1. The longer time observations yield surprisingly similar results. Average economic growth for debt levels above the 90% threshold is 1.7%, while the other 3 debt spaces are associated with mean growth ranging from 3% to 3.7%. Median growth lies between 3.9% and 2.8% for the first three lower debt categories but drops to 1.9% for the country-years debt is above 90%. The authors point out two things here. First, that the high-growth rates and high-debt levels of some countries are related to years following the World War II. Second, that there is substantial variation across countries, for example Australia and New Zealand do not undergo a serious decrease in growth while facing high debt levels.

Figure 3.1 Government Debt, Growth and Inflation: Selected Advanced Economies, 1946-2009



Notes: Central government debt includes domestic and external public debts. The 20 advanced economies included are Australia, Austria, Belgium, Canada,Denmark, Finland,France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, the United Kingdom, and the United States.
Sources: International Monetary Fund, *World Economic Outlook*, OECD, World Bank, *Global Development Finance*, and Reinhart and Rogoff (2009b)

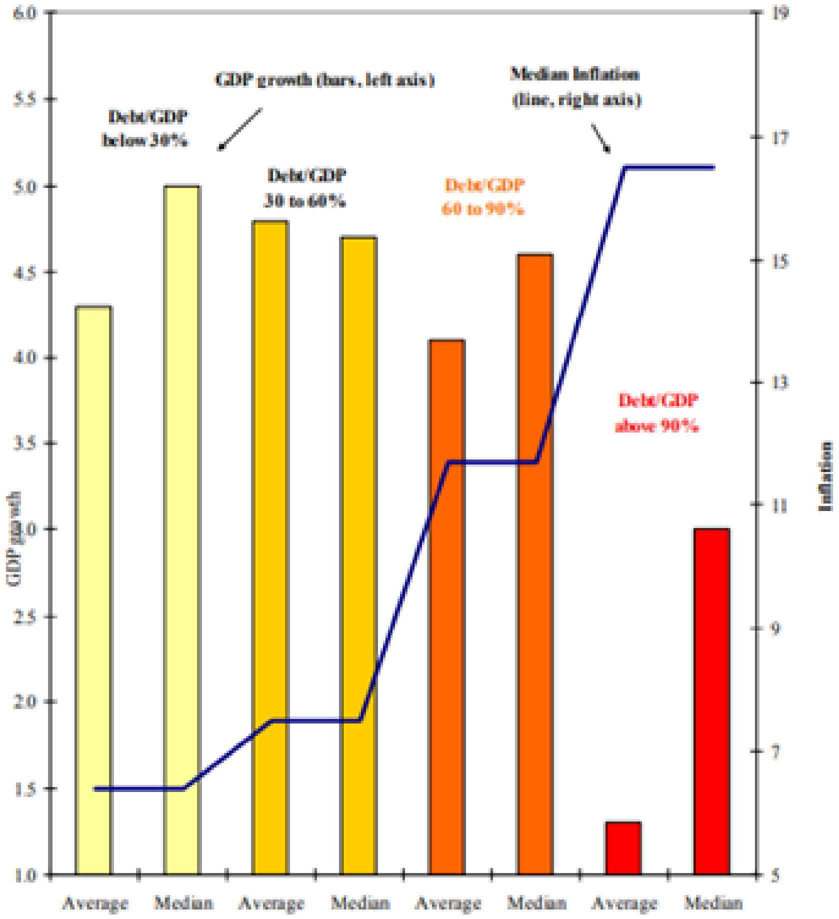
Table 3.1 Real GDP Growth as the Level of Government Debt Varies: Selected Advanced Economies, 1790-2009 1790-2009 (annual percent change)

Country	Period	Central (Federal) government debt/ GDP Below 30 percent	30 to 60 percent	60 to 90 percent	90 percent and above
Australia	1902-2009	3.1	4.1	2.3	4.6
Austria	1880-2009	4.3	3.0	2.3	n.a.
Belgium	1835-2009	3.0	2.6	2.1	3.3
Canada	1925-2009	2.0	4.5	3.0	2.2
Denmark	1880-2009	3.1	1.7	2.4	n.a.
Finland	1913-2009	3.2	3.0	4.3	1.9
France	1880-2009	4.9	2.7	2.8	2.3
Germany	1880-2009	3.6	0.9	n.a.	n.a.
Greece	1884-2009	4.0	***0.3***	***4.8***	2.5
Ireland	1949-2009	4.4	4.5	4.0	2.4
Italy	1880-2009	***5.4***	***4.9***	1.9	0.7
Japan	1885-2009	4.9	3.7	3.9	0.7
Netherlands	1880-2009	4.0	2.8	2.4	2.0
New Zealand	1932-2009	2.5	2.9	3.9	**3.6**
Norway	1880-2009	2.9	4.4	n.a.	n.a.
Portugal	1851-2009	4.8	2.5	1.4	n.a.
Spain	1850-2009	***1.6***	3.3	***1.3***	2.2
Sweden	1880-2009	2.9	2.9	2.7	n.a.
United Kingdom	1830-2009	2.5	2.2	2.1	1.8
United States	1790-2009	4.0	3.4	3.3	***-1.8***
Average		**3.7**	**3.0**	**3.4**	**1.7**
Median		**3.9**	**3.1**	**2.8**	**1.9**
Number of observations = **2,317**		866	654	445	352

Notes: An n.a. denotes no observations were recorded for that particular debt range. There are missing observations, most notably during World War I and II years; further details are provided in the data appendices to Reinhart and Rogoff (2009) and are available from the authors. Minimum and maximum values for each debt range are shown in ***bolded italics.***
Sources: There are many sources, among the more prominent are: International Monetary Fund, *World Economic Outlook*, OECD, World Bank, *Global Development Finance*. Extensive other sources are cited Reinhart and Rogoff (2009).

The same computations were applied to a dataset of 24 emerging market economies for the periods 1946-2009 and 1900-2009. The results are presented in Figure 3.2 and Table 3.2 and are not remarkably different than those of the advanced economies. The difference in median growth between the lowest and highest debt category is larger than that of the advanced economies (roughly 2.1 percentage points). Mean GDP growth for the years 1946-2009 and debt levels below 90% of GDP lies around 4.5% while the value for debt levels above 90% falls markedly at 1.25%. For the longer time period, 1900-2009, the average and median GDP growth is still around 4%-4.5% for low, medium-low and medium-high debt levels. A decline appears for the median growth at high debt levels (median GDP growth is 2.9%) and it is even sharper for the mean growth (1.0%). Again, RR cannot identify any clear shape that forms the impact of public debt on growth below the 90% ratio.

Figure 3.2 Public Debt, Growth, and Inflation: Selected Emerging Markets, 1946-2009



Notes: The 24 emerging market countries included are Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Ecuador, El Salvador, Ghana, India, Indonesia, Kenya, Korea, Malaysia, Mexico, Nigeria, Peru, Philippines, Singapore, South Africa, Sri Lanka, Thailand, Turkey, Uruguay, and Venezuela. The number of observations for the four debt groups are: 502 for debt/GDP below 30%; 385 for debt/GDP 30 to 60%; 145 observations for debt/GDP 60 to 90%; and 110 for debt/GDP above 90%. There are a total of 1142 annual observations.

Sources: International Monetary Fund, *World Economic Outlook*, World Bank, *Global Development Finance*, and Reinhart and Rogoff (2009b) and sources cited therein.

Table 3.2 Real GDP Growth as the Level of Government Debt Varies: Selected Emerging Market Economies, 1900-2009 (annual percent change)

Country	Period	Central (Federal) government debt/ GDP Below 30 percent	30 to 60 percent	60 to 90 percent	90 percent and above
Argentina	1900-2009	4.3	2.7	3.6	0.5
Bolivia	1950-2009	***0.7***	5.2	3.7	3.9
Brazil	1980-2009	3.2	2.3	2.6	2.3
Chile	1900-2009	4.0	***1.0***	7.5	-4.5
Colombia	1923-2009	4.3	3.0	n.a.	n.a.
Costa Rica	1950-2009	***6.9***	5.0	3.4	3.0
Ecuador	1939-2009	5.3	5.0	3.2.	1.5
El Salvador	1939-2009	3.6	2.6	n.a.	n.a.
Ghana	1952-2009	n.a.	4.6	4.7	1.9
India	1950-2009	4.2.	4.9	n.a.	n.a.
Indonesia	1972-2009	6.6	6.3	***-0.1***	3.1
Kenya	1963-2009	6.3	4.2	2.3	1.2
Malaysia	1955-2009	2.0	6.2	6.9	***5.5***
Mexico	1917-2009	4.1	3.4	1.2.	-0.7
Nigeria	1990-2009	5.4	***10.6***	***11.2***	2.6.
Peru	1917-2009	4.3	2.9	2.7	n.a.
Philippines	1950-2009	5.0	3.8	5.1	n.a.
Singapore	1969-2009	n.a.	9.5	8.2	4.0.
South Africa	1950-2009	2.0	3.5	n.a.	n.a.
Sri Lanka	1950-2009	3.3	3.7	4.2	5.0
Thailand	1950-2009	6.1	6.6	n.a.	n.a.
Turkey	1933-2009	5.4	3.7	3.2	-6.4
Uruguay	1935-2009	2.1	3.1	3.2	0.0
Venezuela	1921-2009	6.5	4.1	3.2	***-6.5***
Average		**4.3**	**4.1**	**4.2**	**1.0**
Median		**4.5**	**4.4**	**4.5**	**2.9**
Number of observations = **1,397**		686	450	148	113

Notes: An n.a. denotes no observations were recorded for that particular debt range. There are missing observations for some years details are provided in the data appendices to Reinhart and Rogoff (2009) and are available from the authors. Minimum and maximum values for each debt range are shown in ***bolded italics.***

Sources: There are many sources, among the more prominent are: International Monetary Fund, *World Economic Outlook*, OECD, World Bank, *Global Development Finance*. Extensive other sources are cited Reinhart and Rogoff (2009).

Their report includes computing the above same statistics for total gross external debt[1] and only for selected emerging market economies for the period 1970-2009. They argue that emerging economies rely heavily on external borrowing so it might have a meaning investigating the debt-growth relationship and thresholds based now only on external debt data. They add that external debt is almost entirely denominated in a foreign currency. As one can see in Figure 3.3, the threshold where average growth deteriorates is now lower than that resulted from the total public debt measure. Median and mean economic growth start declining from debt levels between 60% and 90% of GDP and become slightly negative for the threshold where debt reaches 90% of GDP and more. There are no long time observations on total gross external debt for advanced economies (which issue most of external debt in their own currency). So they cannot identify whether advanced economies demonstrate similar thresholds; they speculate that in this case the thresholds are higher.

As a concluding remark of their report RR suggest that in advanced but also in emerging economies high public debt to GDP ratio (in particular 90%) is correlated with low economic growth but no clear debt-growth connection seems to exist when debt-to-GDP ratios are lower than 90 percent. Thus, countries should avoid adopting loose fiscal policies that increase the public debt levels, as this may lead them towards high debt ratio regimes that consequently damage growth. However, they recognize that debt thresholds are importantly country-specific and highlight the necessity of further investigation of their existence.

In the light of these results, public policy debates were influenced followed by an abundance of academic publications on the management of government debt and fiscal policy more broadly. Official institutions, including the OECD and the EU Commission appear to have espoused the 90% threshold in policy recommendations considering it as a danger boundary. At the same time, economists and policymakers began discussions doubting the magic threshold of 90% in the public debt ratio that countries should avoid to cross .There were many arguments that this result rests on shaky foundations and so it is not suitable to form policy.

First of all RR do not allow for any dynamic relation between growth and debt. They just demonstrate statistical measures for the combinations of debt levels and contemporaneous economic growth with no inclusion of the impact of time which is what academic research suggests. Of course, the problem remains with the introduction of lagged debt levels.

Secondly, their research ends up finding a correlation between public debt and growth but it is not clear at all if there is a causal relationship between them. RR just assume this causal relationship going from debt to growth when they interpret the correlation they find. Paul Krugman asks for example, how much of this correlation remains if someone examines only the cases that causation runs from debt to poor growth, rather than what possibly could be spurious or reversed. Practically, in their research there is no evidence of causality at any form. Theory suggests that slow economic growth result in budget deficits which are financed with debt issuance.

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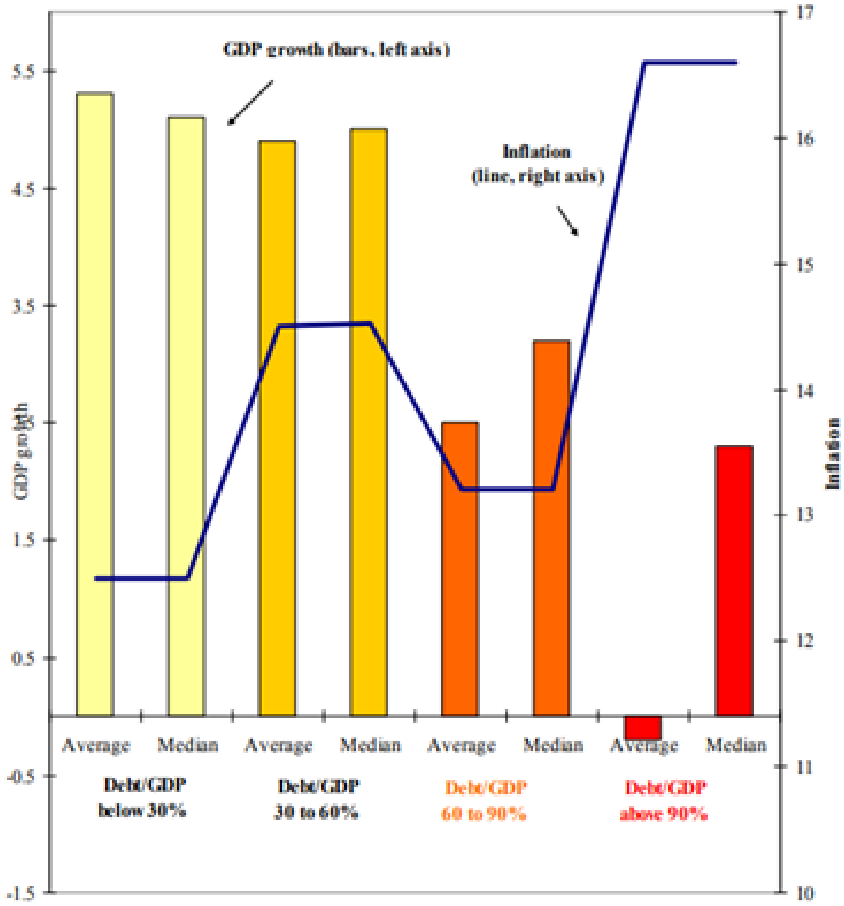
[1]Total gross external debt includes the external debts of all branches of government as well as private debt that is issued by domestic private entities under a foreign jurisdiction.

Figure 3.3 External Debt, Growth, and Inflation: Selected Emerging Markets, 1970-2009



Notes: External debt includes public and private debts. The 20 emerging market countries included are Argentina, Bolivia, Brazil, Chile, China, Colombia, Egypt, India, Indonesia, Korea, Malaysia, Mexico, Nigeria, Peru, Philippines, South Africa, Thailand, Turkey, Uruguay, and Venezuela. The number of observations for the four debt groups are: 252 for debt/GDP below 30%; 309 for debt/GDP 30 to 60%; 120 observations for debt/GDP 60 to 90%; and 74 for debt/GDP above 90%. There is a total of 755 annual observations.

Sources: International Monetary Fund, *World Economic Outlook*, World Bank, *Global Development Finance*, and Reinhart and Rogoff (2009b) and sources cited therein.

So, the contemporaneous link RR analysis found , even if it was a durable correlation, (which is not) may as well run in the opposite direction and so capture causal relationship going from slow growth to high debt.

The same case remains even with lagged debt levels because expectations of slowdown can possible generate counter-cyclical expansionary fiscal policy, leading to a higher public debt-to-GDP ratio the coming year. According to these claims, there is no compelling evidence for policy makers to take into consideration the 90% debt threshold and use it for rapid fiscal retrenchments.

The paper from Herndon et al (2013) cast further serious doubts on the stylized fact of RR findings as some of the calculations presented seem to be flawed. They used the same dataset RR used and discovered substantial mistakes in their calculations leading them to reject the main RR finding that economic growth declines sharply above the 90% debt threshold. Herndon et al. replicated RR calculations and found coding errors, selective exclusion of available data and unconventional weighting of summary statistics and after correcting all of these they concluded that the 90 % threshold effect vanishes. The replications focus only on the mean values of the twenty developed markets for 1946-2009. The errors made relatively small effects on measured average real GDP in the three lower public debt/GDP categories. GDP growth in the lowest public debt/GDP category is roughly 4 percent per year and in the next two categories is around 3 percent per year with or without correcting the errors. On the contrary, the errors had an extremely large effect on the average real growth of the country-years included in the 90% debt/GDP category. RR found this average GDP growth to be -0.1%, while when Herndon et al. properly computed the average they found it was actually 2.2%. So, the GDP growth at high levels of debt is not substantially different than when debt/GDP ratios are lower. The errors presented below and the corrections of them refer to the category of 90% debt/GDP.

First of all, there have been unjustified exclusions in the time coverage of observations even if these were publicly available. RR selectively excluded three countries: Australia (1946-1950), New Zealand (1946-1949) and Canada (1946-1950) which all of them would have contributed to the 90% debt/ratio measures. The 4 observations of New Zealand made a great impact to the average growth when they were included properly, because of the values of their respective GDP growth rates. The aforementioned GDP growth rates were: 7.7, 11.9, -9.9 and 10.8%. In additional to this mistake, Herndon et al. detected a coding error in the RR worksheet. It seems that RR omitted from the average GDP growth the values for the 25 years of Belgium, which also drop in the 90% debt/GDP category. Both of these mistakes accounted for dropping out 39 observations from the 90% category measures. Only 71 country-years were estimated by RR instead of the proper 110.

Furthermore, Herndon et al. have been skeptical about RR's methodology of equally weighted average of every country in each debt group as they find it arbitrary and unsupportable. RR assigns each country-year observation in one of the 4 debt ratios group. For each country in every group they average the GDP growth and so find the country averages. In the end, in each debt group they average the GDP growth of the country averages. That is like they weight each country equally, instead of weight each country-year equally (in each debt group). There might be an argument for this: that there exists possible within-countries serially correlated relationships, and so by RR methodology, this is taken into consideration as every country-year does not contribute proportional information in the GDP growth ratio. But, Herndon et al. although recognize this problem, they see additional problems stemming from equally country weighted averages in the final RR calculations. For example, they say that there are 19 observations for Greece or UK in the 90% category with an average GDP growth 2.9% and 2.4% respectively. New Zealand's single observation in the same category contributed with the value of -7.6%. They conclude that there is no reasonable argument to accept that Greece's or UK's almost 20 years performance is of equal weighted importance with a single performance of New Zealand's. Further justifications from the authors are requested in order to accept this argument. In their replications they include the excluded observations, correct the miscalculations and present the results in Table 3.3.

In the first column of Table 3.3 appear the correct observations of years for each country while in the second and third columns are the (lesser) observations RR included. In the next two columns are the weights for each country, the one Herndon et al. claiming to be correct, which is country-years weight and the other are RR's country weight. The last two columns show the corrected GDP growth averages of Herndon et al. and those of RR. In total, actual

Table 3.3: Years and real GDP growth with public debt/GDP above 90 percent, by country

	Count of years with public debt/GDP above 90 percent			Weights		GDP Growth	
	Correct	RR Exclusion of early years for Australia, Canada, and New Zealand	RR Spreadsheet error excluding Australia, Austria Belgium, Canada and, Denmark	Country-Years	RR	Correct	RR
Australia 1946–50	5	0	0	4.5	0.0	3.8	
Belgium 1947, 1984–2005, 2008–09	25	25	0	22.7	0.0	2.6	
Canada 1946–50	5	0	0	4.5	0.0	3.0	
Greece 1991–2009	19	19	19	17.3	14.3	2.9	2.9
Ireland 1983–89	7	7	7	6.4	14.3	2.4	2.4
Italy 1993–01,2009	10	10	10	9.1	14.3	1.0	1.0
Japan 1999–2009	11	11	11	10.0	14.3	0.7	0.7
New Zealand 1946–49,1951	5	1	5	4.5	14.3	2.6	−7.9
UK 1946–64	19	19	19	17.3	14.3	2.4	2.4
US 1946–49	4	4	4	3.6	14.3	−2.0	−2.0
	Count of country-years and countries with public debt/GDP above 90 percent			Average GDP Growth			
Country-Years	110	96	75	Country-year weights and correct GDP growth data		2.2	
Countries	10	8	7	RR equal weights and RR GDP growth data			−0.1

Notes. Years that each country spent in the highest debt/GDP category are listed. The Years columns show the count of years that each country spent in the highest debt/GDP category. The Correct column uses all available data for 1946–2009. The Exclusion column excludes available early years of data for Australia (1946–1950), Canada (1946–1950), and New Zealand (1946–1949). The Spreadsheet column reflects the spreadsheet error that omits Australia, Austria, Belgium, Canada, and Denmark. The Weights columns show the alternative weightings to compute average real GDP growth. The Year column shows weights proportional to country-years. The RR weight column shows equal weighting of country averages. GDP shows real average GDP growth for each country in the years in which it appeared in the highest debt/GDP category for all available years. The value of -7.9 for New Zealand in parentheses reflects both the exclusion of 1946–1949 and a transcription error of −7.6 to −7.9.
Source: Authors' calculations from working spreadsheet provided by RR.

average real growth is +2.2% per year in contrast to -0.1% of RR. So, the deterioration of mean growth from the previous debt ratio category, which Herndon et al. correctly measured to the value 3.2 % (than 3.4% of RR), is now only 1% (than 3.5% of RR).

In the context of the corrected measurements Herndon et al. seek to re-examine the non-linearity in the debt-growth relationship, which was an important component of RR's findings. They follow two procedures. In the first, they define the fourth category similar to the lower three categories spanning the same 30 percentage points -that is, the 90-120 percent public debt/GDP category- and add one more- that is for debt/GDP ratios above 120%. When classifying the data from the same working spreadsheet of RR, average GDP growth for the 90-120% category was computed 2.4 percent, fairly close to the 3.2 percent of the previous category. In the new category the average GDP growth was 1.6 percent. So, in the both cases average growth declines but that is far from falling off a nonlinear cliff. This contradicts the nonlinear response of growth when debt accumulations reach the 90% debt ratio.

The second procedure to check for nonlinearities regards computing the scatterplot of all country-years GDP growth plotted against public debt/GDP ratio and calculate a locally fitted regression function. In this case, the scatterplot displays a non-linearity but this appears in the change in the debt/GDP ratio from 0 to 30 percent where average growth decreases sharply. This disputes another one case made by RR, which is that no clear link of debt-growth relationship seems to exist below the 90% debt threshold.

As a concluding remark, Herndon et al. argued that RR in their calculations have clearly underestimated the average growth GDP for the highly indebted countries. The aggregate of their mistakes transformed the reality of a moderate decline in average GDP growth rates for countries with debt/GDP over 90% into a fake image of dramatic decline in GDP growth. Herndon et al. conclude that significant errors in RR calculations reduced the measured average GDP growth of countries in the high public debt category and so there is no clear evidence of a major decline in GDP growth above 90% of GDP, just a modest decrease. They also suggested that the austerity agenda that RR's findings had provoked should be reassessed.

The aware of the tension of the RR analysis and their controversial results sparked a new wave of researches with the aspiration to evaluate whether the RR results demonstrate the same robustness when a number of specifications changed. For example, if debt thresholds were endogenously estimated rather than arbitrarily chosen, if determinants of growth were included in a formal econometric structure, and if instruments for public debt were used to evaluate its causal effect on GDP growth rate. The researchers emphasized that formal econometric techniques (as the threshold regression framework) were undoubtedly necessary in order to deal with problems such as the reverse causality problem but more importantly to identify correctly the possible threshold effects and nature of the nonlinearity in both the short- and long-run debt-growth relation. We turn now to survey the part of the empirical literature that used econometric procedures to properly investigate if debt thresholds exist and their levels.

4. Confirmation of the 90% Debt Threshold

One problem of Reinhart and Rogoff's analysis to begin with, is that they inflicted the 90% debt threshold instead of having it, at first, formally tested. But, given to what has just been supported, this seemingly arbitrary threshold should had been searched and tested through more sophisticated techniques. In this section, results of papers running multivariate panel regressions will be presented. The results of these studies confirm a robust negative non-linear relationship between debt and growth with a threshold of around 90% just like RR found.

Reinhart and Rogoff (2010) and Herndon et al. (2013) made use of central government debt. But, drawing policy conclusions built on central government debt may be inaccurate. As a matter of fact, the consolidated debt of the government sector (general government debt) is more relevant to draw inferences on fiscal policy. This interpretation of debt is being used in what follows.

Cecchetti, Mohanty and Zampolli (2012) examined the debt-economic growth relationship by using a dataset of 18 OECD countries from 1982 to 2006. The countries are: United States, Japan, Germany, United Kingdom, France, Italy, Canada, Australia, Austria, Belgium, Denmark, Finland, Greece, Netherlands, Norway, Portugal, Spain and Sweden. First, they establish the presence of a robust linear negative correlation between public (government) debt and growth by estimating a growth equation. Cecchetti et al (2012) also examine the impact on growth of private sector, nonfinancial corporate and household debt, but this is something that will not be analyzed here as we are concern only about the impact of government debt on growth. Then they check for the existence of non-linearities. They confirm the existence of non-linearities through a non-dynamic panel threshold methodology that checks pre-determined thresholds using dummy variables. They find thresholds of around 85% beyond which government debt is bad for growth. The panel threshold methodology they applied is based on Hansen likelihood ratio (1999). As already mentioned the impact of debt on growth will be -in the majority of the papers- examined through specification and estimation of alternative versions of the growth equation.

In Cecchetti, Mohanty and Zampolli (2012) the growth equation for country *i* is:

$$\overline{g}_{i,t+1,t+k} = \phi y_{i,t} + \beta' X_{i,t} + \mu_i + \gamma_t + \varepsilon_{i,t,t+k}, \qquad (4.1)$$

where

$$\overline{g}_{i,t+1,t+k} = \frac{1}{k}\sum_{j=t+1}^{t+k} g_{i,j} \cong \frac{1}{k}\left(y_{i,t+k} - y_{i,t}\right) \qquad (4.2)$$

is the k-year forward average of annual per capita growth rates between years *t+1* and *t+k* (*k=5*) and *y* is the log of real per capita GDP. They use overlapping observations in order to mitigate the effects of business cycle fluctuations, which constitute a common practice as we have already referred to in the beginning. They also use them to bypass the arbitrary construction of five-year, non-overlapping blocks. Country-specific fixed effects as well as time-specific fixed effects are included.

Due to the dynamic nature of the growth regression LSDV in general produces biased estimations. However, researchers cite that according to Monte Carlo simulations, even for classic moderate macroeconomical dataset the LSDV estimator performs as well as or better than the alternative proposed estimation methods of IV estimators (Judson and Owen, 1999). Researchers' dataset has T=25 and N=18, therefore they use the LSDV estimator.

They employ a robust technique to calculate the standard errors of the coefficient estimates due to the overlapping nature of the per capita growth, which introduces a moving-average process to the errors (specifically, they employ Huber and White sandwich estimator).

The list of regressors includes:

- Public debt-to-GDP ratio
- gross saving (public and private) as a share of GDP;
- population growth;
- the number of years spent in secondary education, a proxy for the level of human capital;
- the (total) dependency ratio as a measure of population structure and aging;
- openness to trade, measured by the absolute sum of exports and imports over GDP;
- CPI inflation, a measure of macroeconomic stability;
- the ratio of liquid liabilities to GDP, as a measure of financial development; and
- a control for banking crises taking the value of zero if in the subsequent five years (as identified by Reinhart and Rogoff, 2008) there is no banking crisis, and the value of 1/5, 2/5, and so forth, if a banking crisis occurs in one, two, etc. of the subsequent five years.

In their baseline estimations, they find that public debt has a consistently significant negative linear impact on future growth. A 10 percentage point increase in the debt-to-GDP ratio is associated with an 17 basis point reduction (when the crisis variable is included) in future average growth and this is considered a big impact. Results of the growth regression including the crisis variable are shown in Table 4.1. Without the crisis variable are shown in Table 4.2.

Table 4.1 Growth Regressions with Debt and Crisis Variable

VARIABLES	(3) growth05
National gross saving to GDP	0.0430 (0.027)
Change in population	-0.4183 (0.385)
Schooling	0.0054*** (0.001)
Log of real per capita GDP	-0.1865*** (0.014)
Trade openness	0.0341** (0.012)
Inflation rate	-0.0176 (0.016)
Total dependency ratio	-0.2195*** (0.041)
Liquid liabilities to GDP	1.2387*** (0.387)
Banking crisis	-0.0135*** (0.003)
Total nonfinancial debt	
Government debt	-0.0167*** (0.005)
Private debt	
Corporate debt	
Household debt	
Constant	1.9714*** (0.155)
Observations	383
R-squared	0.772

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

Table 4.2 Growth Regressions without Crisis Variable

VARIABLES	(3) growth05
National gross saving to GDP	0.0414 (0.029)
Change in population	-0.1800 (0.281)
Schooling	0.0054*** (0.002)
Log of real per capita GDP	-0.1916*** (0.020)
Trade openness	0.0437*** (0.015)
Inflation rate	-0.0282 (0.024)
Total dependency ratio	-0.1870*** (0.052)
Liquid liabilities to GDP	0.5382 (0.809)
Total nonfinancial debt	
Government debt	-0.0164** (0.007)
Private debt	
Corporate debt	
Household debt	
Constant	2.0059*** (0.212)
Observations	383
R-squared	0.705

Robust standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1.

So far, Cecchetti, Mohanty and Zampolli, tried to identify and establish a negative (linear) link for the debt and growth relationship. Next, they proceed to investigate the existence of non-linearities and threshold effects through the following model (4.3):

$$\bar{g}_{i,t+1,t+k} = -\phi y_{i,t} + \beta' X_{i,t} + \lambda_{-} d_{i,t} I(d_{i,t} < \tau) + \lambda_{+} d_{i,t} I\left(d_{i,t} \geq \tau\right) + \mu_i + \gamma_t + \varepsilon_{i,t,t+k}, \qquad (4.3)$$

This is a Panel Threshold Regression model (PTR) developed by Hansen (1996, 1999, 2000). Threshold Regression Models are widely used in the majority of the literature when trying to identify thresholds, as they are simple and considered to perform better than alternative models that have been employed to estimate nonlinear functions.

These models classify observations into stochastic processes depending on whether the observed value of the threshold variable lies above (or below) a specific threshold value.

Finally, they allow the identification of the significance of the threshold level, as well as the coefficients and the significance of the different regimes.

Equation (4.3) is identical to equation (4.1) plus the I (.) indicator term. This term is set equal to 1 if each debt variable is below a specified debt threshold and 0 in any other case. The indicator variable separates the debt variable into two, creating two different debt regimes. In doing so, the impact on growth is different depending on whether debt is below or above the specified threshold. The investigation of threshold is being done by including in the regression (4.3) one debt variable at a time and estimating it for all the values of debt-to GDP ratio included in the dataset. Next, they choose the one that yields the minimum value of the sum of squared residuals. Cecchetti et al. used Hansen's (1999) likelihood ratio statistics to assess the statistical significance of the estimated threshold. The LR statistic is calculated as the difference between the sum of squared residuals of the model for a generic value of the threshold and the sum of squared residuals corresponding to the estimated threshold (scaled by the variance of the sample residuals).

The threshold effects which they found to have adverse impact on economic activity are shown in Table 4.3. When they control for crises, they find that the point estimate of the debt threshold level is 96 percent of GDP. Below this threshold a 10 percentage point increase in the debt-to-GDP ratio causes the average subsequent GDP growth to diminish by a 7 basis point. The coefficient is not statistically significant. Above the 96 percent debt threshold, if the debt-to-GDP increases by a ten percentage points then GDP growth diminishes by a 14 basis point and the coefficient is statistically significant.

When they do not include the crises dummy variable the estimated public debt threshold is 84% of GDP. Below this, a 10 percentage point increase leads to a seven basis point reduction in growth rate and the coefficient is statistically insignificant. When the debt-to-GDP ratio lies above 84% a ten percentage point increase in the debt-to-GDP ratio reduces the economic activity by 13 basis point and the coefficient is statistically significant at 10 percent level.

Table 4.3 Threshold effects

	Threshold estimate	Coefficients		
Government debt				
Controlling for crises	96%	<96% -0.0065 (0.232)	>=96% -0.0138*** (0.004)	
Not controlling for crises	84%	<84% -0.0074 (0.382)	>=84% -0.0133* (0.057)	

Notes: Reported threshold estimates are obtained by minimizing the sum of squared residuals in text equation (4.3). Reported coefficients are for the marginal impact of debt on the five-year forward average per capita growth rate from estimating text equation (4.3). Numbers in parentheses are asymptotic p-values for the test that the coefficient estimate is equal to zero computed using standard errors estimated using the Huber-White sandwich estimator. */**/*** indicate coefficient estimates significantly different from zero at the 10/5/1 percent levels.

These results are very close with the RR's results as they confirm that the correlation between debt and growth is non-linear and public debt starts having a negative effect on growth when debt reaches 84% of GDP (96% in the case where they control for banking crises). They also confirm RR's result that debt has no consistent impact on growth below of a threshold around 90% as the coefficients of debt below the thresholds are not statistically significant in both cases.

The authors claim that high debt is clearly negatively linked with growth. When public debt reaches 84% of GDP, further increases in debt may start to have an important negative effect on trend growth. They suggest that authorities should aim at keeping their debt levels well below the estimated thresholds, since they never know when an extraordinary shock is going to hit their economies.

Cecchetti et al. (2011) conclude that when debt levels are low, this can facilitate the economy to grow or stabilize. Yet, beyond a certain level it increases volatility and highly indebted countries should focus not only at stabilizing their debt but also at reducing it to sufficiently low levels that do not retard growth.

The findings of Cecchetti et al. (2011) concerning the non-linear relationship between debt-growth should be interpreted with some caution. Hansen's panel threshold regression technique (Hansen, 1999) applies to a static model with iid errors but it might not to a dynamic model with heteroskedastic errors such as the dynamic growth model. A further problem is that that Hansen LR's statistics is not suitable for testing for the presence of a threshold. It only tests the statistical significance of an estimated threshold assuming that such a threshold exists. Moreover, we should keep in mind that these thresholds are predetermined in this research and not endogenously determined. Lastly, the identification of the non-linear relationship might be more complex than what a simple one-threshold model implies. Therefore their results should not be interpreted as evidence for the presence of non-linearities.

We turn now to another research, that of Checherita-Westphal and Rother (2012). They examined the debt-growth relationship on 12 euro-area countries, namely, Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, and Spain over the period 1970-2008. Paying exclusive attention to the old euro area countries allows examining countries which demonstrate certain policy features not encountered in the rest of the developed countries and possibly provides more relevant results for the sovereign debt crisis. They check for thresholds and non-linearities by incorporating a quadratic expression in debt in the growth equation. By employing this quadratic functional form they achieve to determine the thresholds endogenously in contrast to the previous analysis. Checherita-Westphal and Rother (2012) conclude that there is a robust negative non-linear effect of public debt on per capita GDP growth rate and that their relation is described as a concave function (inverted U-shape). The turning point, beyond which the debt-to-GDP ratio has a harmful effect on long-run growth, is found at about 90-100% of GDP. Their findings validate RR results.

The estimation equation is:

$$g_{it+k} = \alpha + \beta \ln(GDP/cap)_{it} + \gamma_1 debt\_sq_{it} + \gamma_2 debt_{it} + \delta\, saving/gfcf_{it} + \varphi\, pop.growth_{it} + other\ controls\ (fiscal;\ openness;\ interest\ rate) + \mu_i + v_t + \varepsilon_{it} \quad (4.4)$$

where

g_{it+k} : the growth rate of GDP per capita, k = 1 or 5 (three different measures are used in the empirical estimation: annual growth rate g_{it+1} ; 5-year cumulative overlapping growth rate $g_{it\ /\ t+5}$, where t takes annual values; and 5-year cumulative non-overlapping growth rate g_{it+5} , where t takes the values at the start of each half-decade);

$ln(GDP/cap)_{it}$: natural logarithm of the initial level of GDP per capita

$debt_{it}$: gross government debt as a share of GDP

$saving/gfcf_{it}$: saving or investment rate (proxied as gross fixed capital formation) as a share of GDP (the variables are used in the empirical estimation in aggregated terms, total national saving/investment rate, as well as on a disaggregated basis, as public and private saving/ investment rate)

other controls => see description below

μ_i : country fixed effects

v_t : time fixed effects

ε_{it} : the error term.

The use of a quadratic equation in debt is included in order to check for the non-linear impact of government debt on growth. The inclusion of a linear debt term yields no considerable results.

The other control variables that are used in the estimation of the growth equation are: (i) fiscal indicators (i.e. a proxy for the average tax rate and the government balance, both in cyclically adjusted terms) to allow more extensively for the possibility of fiscal policy affecting economic growth; (ii) the long-term (sovereign) real interest rate, capturing the impact of inflation and the effects of the fiscal-monetary policy mix; (iii) indicators for the openness of the economy and external competitiveness (such as the sum of export and import shares in GDP; terms of trade growth rate; real effective exchange rate REER) to expand the model beyond a closed-economy form.

Data description and sources are shown in Table 4.4.

The estimation method is panel fixed-effects corrected for heteroscedasticity and autocorrelation up to order 2 (for the annual growth rate and the cumulative 5-year non-overlapping growth rate) or 5 (for the cumulative 5-year overlapping growth rate)(Table 4.5).

The panel fixed-effects estimators are being used to show the direct effect of debt on growth considering the limited cross-sectional size of the data compared to the time dimension. The inclusion of debt variable in linear form in the growth equation does not produce considerable results. The fit of the regression is smaller and the significance of other control variables is in general unchanged.

The strong potential problem of endogeneity, especially reverse causality, is being solved by using various instrumental variable estimation methods (Table 4.6). The estimators used in their paper are either 2-SLS (two-stage least squares) or GMM estimators. With GMM they also correct for the possible heteroscedasticity and autocorrelation in the error structure by using the consistent estimator.

The debt variable for every country is being instrumented through either its time lags (up to the 5^{th} lag) or the average of the debt levels of the other countries. There is high correlation between these two instruments and the debt-to-GDP ratio, which can be verified by the first stage statistics such as Shea partial R-square. But instrumenting the debt-to-GDP ratio by the lagged terms of the regressors poses a problem as the debt series are highly persistent. So, for each country i and year t, they also make use of the average public debt-to-GDP ratio of the other 11 countries as instrument, which is considered as efficient instrument. It is uncorrelated with the growth rate to the degree that someone supposes that there is no robust relation between debt levels in other euro area countries and per-capita GDP growth rate in one particular country. The authors further assume that the use of 1 or 5 lagged years of the explanatory variables with respect to the dependent variable mitigates the endogeneity problem.

Table 4.4
Data description and sources

Variable abbrev.	Variable name/description	Source
debt	Gross government debt (% GDP)	AMECO
gov_bal	Government budget balance (% of GDP)	AMECO
gov_primary_bal	Government budget primary balance (excl. interest payments; % of GDP)	AMECO
gov_cab	Cyclically adjusted gov. balance (% of GDP at market prices)	AMECO
gov_rev_ca	Cyclically adjusted gov. revenue (% of GDP at market prices)	AMECO
GDP_cap	GDP at 2000 market prices per head of population (1000 euro)	AMECO
potentialGDP	Potential gross domestic product at 2000 market prices (bill. EUR)	AMECO
trendGDP	Trend gross domestic product at 2000 market prices (bill. EUR)	AMECO
pop.growth	Total population—growth rate	AMECO
openness	Calculated as sum of exports and imports (% of GDP)	AMECO
CA_bal	Current account balance (% GDP)	AMECO
gfcf_total	Gross fixed capital formation: total economy (% GDP)	AMECO
gfcf_gov	Gross fixed capital formation: general government (% GDP)	AMECO
gfcf_priv	Gross fixed capital formation: private sector (% GDP)	AMECO
saving_total	Gross national saving: total economy (%GDP)	AMECO
saving_pub	Gross saving: general government (% GDP)	AMECO
saving_priv	Gross saving: private sector (% GDP)	AMECO
reer	Real effective exchange rate, based on ULC, relative to rest 23 industrial countries	AMECO
LT_nom_i	Nominal long-term (LT) interest rates, sovereign (mostly central government LT bond yields)	AMECO
LT_real_i	Real long-term interest rates, sovereign; deflator: GDP at market prices	AMECO
ST_nom_i	Nominal short-term (ST) interest rates (3M-EURIBOR after 1999)	AMECO
ST_real_i	Real short-term interest rates; deflator: GDP at market prices	AMECO
inflation (GDPdefl.)	Annual rate of change in GDP deflator at market prices	AMECO
output_gap	Gap between actual and trend GDP at 2000 market prices/trend GDP	AMECO
old_dep_ratio	Age dependency ratio, old (% of population over 65 in working-age population)	WDI
young_dep_ratio	Age dependency ratio, young (% of population under 15 in working-age population)	WDI
credit_priv	Domestic credit to private sector (% of GDP)	WDI
TFP_g	Growth rate of Total Factor Productivity (TFP), calculated based on *TFP_index* (2000 = 100)	AMECO

Note: Sources of basic data are the European Commission's AMECO database and the World Bank's World Development Indicators (WDI). In the regression tables presented in the paper, the following symbols are also annexed to several variables: ln (natural logarithm of variable); *_sq* (square term of variable); *_g* (annual growth rate of variable); *_5yg* (5-year overlapping growth rate of variable).

Table 4.5
Fixed effects (FE) models

Variables	Annual growth rate		Cumulative 5-year overlapping growth rate		Cumulative 5-year non-overlapping growth rate	
	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**
debt	0.1198***	0.1291***	0.5236***	0.4066**	0.6462***	0.5032**
	(.0410)	(.0412)	(.1294)	(.1649)	(.1396)	(.2095)
debt_sq	−0.0006***	−0.0006***	−0.0025***	−0.0020***	−0.0031***	−0.0026**
	(.0001)	(.0002)	(.0006)	(.0008)	(.0007)	(.0011)
gov_rev_ca	−0.0511	−0.094	−0.3112	−0.1906	−0.3297	−0.2227
gov_cab	0.2484***	0.3083***	1.1756***	0.9392**	1.3120***	1.1244
ln(GDP/cap)	0.5403	−0.0036	−9.704	−20.5168**	−7.9847	−14.2007
pop.growth	−1.3086***	−1.1997**	−3.6186***	−2.7678*	−3.6534**	−3.9632*
gfcf_total	0.0087	-	-	-		-
gfcf_gov	-	0.3988***	0.6483	-	0.4319	-
gfcf_priv	-	−0.047	−0.4018**	-	−0.4034*	-
saving_pub	-	-	-	−0.4572	-	−0.4535
saving_priv	-	-	-	0.0601		0.0266
openness	0.0307**	0.0341**	0.1975***	0.1643***	0.1622*	0.1338
LT_real_i	0.0234	0.0017	0.1974	−0.1935	0.0758	−0.3489
Year dummies	Included (38) (1971–2008)	Included (38) (1971–2008)	Included (36) (1971–2006)	Included (36) (1971–2006)	Included (7) (1975–2005)	Included (7) (1975–2005)
Country dummies	Included (11)	Included (11)	Included (11)	Included (11)	Included (11)	Included (11)
cons	−0.8206	−0.0238	37.1960**	61.3611**	33.9344	46.2412
N	412	406	382	314	81	68
R2-within	0.62	0.62	0.73	0.76	0.84	0.85
AR correction	lag(2)	lag(2)	lag(5)	lag(5)	lag(2)	lag(2)
debt turning point	97.8	103.1	104.5	99.9	104.6	98.2
95% CI bootstrap						
normal-based CI	(75; 120)	(83; 124)	(94; 114)	(85; 114)	-	-
percentile CI	(73; 116)	(81; 123)	(93; 114)	(83; 112)	(77; 149)	-
bias-corrected CI	(70; 114)	(79; 121)	(72; 114)	(87; 114)	(73; 138)	-
95% CI nlcom	(76; 118)	(82; 123)	(86; 122)	(80; 118)	(85; 123)	(75; 121)

Note: The dependent variable is the economic growth rate(annual;cumulative5-yearoverlapping,and,respectively,cumulative5-yearnon- overlapping). The abbreviations for the explanatory variables are explained in Table 4.4.Countries included in the analysis:Austria,Belgium,Finland,France,Germany,Greece,Ireland,Italy,Luxembourg,Netherlands,Portugal,andSpain.The table shows the estimated coefficients and their significance level(*10%; **5%, ***1%). For the main variable of interest–government debt–and its square, standard errors (SE) are also shown in parentheses. The confidence intervals (CI) of the debt turning point are generated through bootstrapping based on (i) a normal distribution; (ii) percentile distribution; (iii) bias-corrected distribution. Where CI are not shown, the bootstrapping procedure rendered unstable CI or an estimation of the bootstrapped SE was not possible due to ‘‘lack of observations’’.The nlcom command implement the delta method to calculate CI.

Table 4.6
Instrumental variable (IVREG) models

Variables	Annual growth rate		Cumulative 5-year overlapping growth rate			Cumulative 5-year non-overlapping growth rate		
	Model 1	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**	**Model 8**
Instruments/ Estimator	Avg. gov debt(*n*−*i*) 2SLS	L(1/5).gov debt 2SLS	Avg. gov debt(*n*−*i*) 2SLS	L(1/5).gov debt GMM	Avg. gov debt (*n*−*i*) GMM (h.a.)	Avg. gov debt(*n*−*i*) 2SLS	GMM	L(1/2).gov debt 2SLS
debt	0.0535* (.0291)	0.1194*** (.0374)	0.3141*** (.0831)	0.7552*** (0.1168)	1.4400*** (.4928)	0.4104*** (.1575)	0.4104** (.1684)	0.7959*** (.2918)
debt_sq	−0.0003** (.0001)	−0.0006*** (.0002)	−0.0016*** (.0003)	−0.0038*** (0.0005)	−0.0076*** (.0027)	−0.0022*** (.0007)	−0.0022*** (.0005)	−0.0039*** (.0013)
gov_rev_ca	0.0687	0.0089	−0.1081	−0.5657***	−0.9871	−0.1544	−0.1544	−0.6953*
gov_cab	0.1830**	0.2320***	0.8760***	1.2778***	1.2469**	1.1135**	1.1135*	1.1392*
ln(GDP/cap)	0.1817	2.2802	−23.693***	−13.2096**	−45.744***	−4.0697***	−4.0697***	−2.8750**
pop.growth	−1.8422***	−1.7004***	−3.0004***	−2.0007***	2.401	−3.9632***	−4.0596***	−2.8196**
saving_pub	−0.0178	−0.0071	−0.4773*	−0.4700*	0.2328	−0.5302	−0.5302	−0.2405
saving_priv	0.1469***	0.1168**	0.1082	−0.0897	−0.3331	0.0695	0.0695	−0.3459
openness	0.0201***	0.0169**	0.1642***	0.1374***	0.1433**	0.1344***	0.1344**	0.1192***
LT_real_i	−0.2295***	−0.2175***	−0.2353	−0.2353	0.3721	−0.3714	−0.3714	−0.4636
Year dummies	Included (38) (1971–2008)	Included (38) (1971–2008)	Included (36) (1971–2006)	Included (36) (1971–2006)	–	Included (7) (1975–2005)	Included (7) (1975–2005)	Included (7) (1975–2005)
Country dummies	Included (11)	Included (11)	Included (11)	Included (11)	Included (11)	Included (11)	Included (11)	Included (11)
_cons	−4.529	−8.7638*	67.8248***	55.1877***	140.0319***	54.6590**	54.6590**	35.5593
N	338	319	314	314	314	68	68	59
R2-adj	0.67	0.67	0.76	0.881	0.21	0.77	0.77	0.81
Shea partial *R*-sq	0.89	0.64	0.88	0.65	–	0.86	0.86	0.33
debt turning point	81.2	93.1	96.7	99.8	94.5	95.2	95.2	99.8
95% CI nlcom	(49; 113)	(77; 108)	(80; 112)	(93; 106)	(83; 105)	(70; 119)	(72; 118)	(84; 118)

Note: The variable *debt* is instrumented for each country through either its time lags (up to the 5th lag; *L(1/5).gov debt*) or through the average of the debt levels of the other countries in the sample, *Avg. gov debt(n−i)*.
Models 1, 2, 3, 6, 8 are estimated through 2SLS (two-stage least squares); Models 4 and 7 through the heteroskedastic-efficient two-step GMM (generalised method of moments) estimator; Model 5 through GMM with heteroskedasticity and autocorrelation-consistent statistics.

In this paper the authors examine both the short-term and long-term impact of debt on growth by using the annual GDP growth rate and the 5-year specification respectively.

The results across all models from Tables 4.5 and 4.6 indicate a highly statistically significant non-linear relationship between government debt ratio and per-capita-GDP growth rate for the 12 euro area countries. The debt-to-GDP turning point of this concave relationship lies between 90 and 100% on average across all models. This means that above this threshold additional debt will have a considerable negative effect on economic growth. The turning point is the debt ratio that optimizes the quadratic equation, i.e., it is computed by: Debt turning point= - (1/2)(coef(debt)/coef(debt_sq)), where coef() denotes the regression coefficients (only if statistically significant) of variables debt and debt_sq obtained from the estimation of a given model.

Checherita-Westphal and Rother (2012) check the robustness of these results, namely the inverted U-shape relationship of debt and growth and the 90-100% debt-to-GDP turning point through several econometric robustness tests. First, they exclude from the dataset, one by one, the debt outliers which are Luxembourg (lowest average of the sample: 9.6% of GDP) and then Belgium (highest average of the sample: 97.8% of GDP) Then all the other countries. Table 4.7 shows the results which remain mostly unaltered. The coefficient of debt and debt squared continue to be statistically significant with the same sign, validating that the inverse U-shaped form is a strong result. Also, the debt turning point remains broadly the same across each and every test with excluded country. The same results are revealed when they employ the 5-year averages.

Table 4.7

Results with restricted cross-country sample, excluding one country at a time (Model 1 FE, dependent variable: annual growth rate).

country excluded	none	LU	BE	IT	IE	GR	NL	AT	PT	FR	DE	ES	FI
gov_debt	0.1198***	0.1264***	0.1337***	0.1062**	0.1008**	0.0954**	0.1327***	0.1365***	0.1143***	0.1190***	0.1508***	0.1159***	0.0793**
gov_debt_sq	−0.0006***	−0.0006***	−0.0008***	−0.0005**	−0.0004**	−0.0006***	−0.0007***	−0.0007***	−0.0006***	−0.0006***	−0.0008***	−0.0006***	−0.0004***
ln_GDP_cap	−0.0511	−0.0151	−0.0466	0.1891	−2.4699	−0.5444	1.1631	1.5436	1.1736	0.5082	0.9313	0.7044	−1.1733
gov_ca_rev	0.2484***	−0.0164	−0.0381	−0.0337	−0.0357	−0.0483	−0.0321	−0.0614	−0.0378	−0.0488	−0.1009*	−0.0673	−0.0165
gov_cab	0.5403	0.2015***	0.2866***	0.2710***	0.1228*	0.2139***	0.2638***	0.2530***	0.2353***	0.2445***	0.3009***	0.2466***	0.2553***
pop_growth	−1.3086***	−1.2303**	−1.3904***	−1.2274**	−1.2608**	−1.4223***	−1.1586**	−1.3710**	−1.2693**	−1.3157***	−0.8814**	−1.3810**	−1.6539***
gfcf_total	0.0087	0.0402	0.0267	−0.0053	0.0198	0.0058	0.0086	0.0202	−0.0099	0.0027	−0.0251	−0.0359	0.1188
openess	0.0307**	0.0755***	0.0318**	0.0288**	0.0184	0.0363**	0.0313**	0.0273*	0.0253*	0.0314**	0.0278*	0.0301**	0.0321**
LT_real_i	0.0234	0.0293	0.0328	0.0124	−0.0104	−0.0298	0.0254	0.0189	0.0274	0.021	0.0343	0.016	0.0537
Year dummies	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included	Included
Country dummies	Included (11)	Included (10)	Included (10)	Included (10)	Included (10)	Included (10)	Included (10)	Included (10)	Included (10)	Included (10)	Included (10)	Included (10)	Included (10)
_cons	−0.8206	−4.2657	−0.6174	0.51	7.2514	2.2558	−3.4833	−2.0735	−2.0194	−0.6817	−0.068	0.7639	0.6789
N	412	387	373	373	374	379	378	373	388	380	373	381	373
debt − turning point	97.8	103	88	104	114	86	99	98	97	96	99	98	92

Note: Country excluded shown in the table header.

Later, they eliminate group of countries. They start excluding the group of the two lowest averaged indebted countries (Luxembourg and Finland), continue with the group of the two highest averaged indebted (Belgium and Italy) and end with the combination of the lowest and highest averaged indebted (Luxembourg and Belgium). The quadratic relation is still robust across all these combinations of excluded countries and the turning point does not demonstrate significant differences.

They also conduct robustness tests excluding years and time periods (Table 4.8). First, the EMU period is excluded (i.e., the period 1999-2008). This can be done alternatively by excluding the time fixed effects and including a dummy variable that receives the value 1 for the years 1999-2008 (2001-2008 for Greece) and 0 otherwise. The concave relationship continues to be robust and the debt-turning point decreses only about 6 percentage points.

Next, they rule out the 70s; 70s and 80s; and in the end 70s, 80s and nearly all of the 90s (thus letting only the EMU period) as shown in Table 4.8. When they eliminate most of the 90s, the coefficients of debt and debt squared are no more statistical significant. It is possibly happening because of the smaller sample size.

Then they continue with robustness checks to the polynomial functional forms. They begin by setting powers higher than one-in increments of 0.2- and examine polynomial degrees up to power 3. The results still do not change across the different polynomial forms as the concave formation stays robust and the debt-to-GDP turning point still lies between 90% and 100% of GDP. When they experimented with lower powers, they obtained somewhat higher debt turning points and the other way round. Under the quadratic specification with the fixed-effects model the debt turning point is found to be 97.8% of GDP. The polynomial specification of the power of 1.2 indicates the turning point to be at 103.9%, while the power of 3 produces a turning point of 92.7%. The introduction of more than two debt terms in the regression does not return considerable results.

One further robustness test is investigating the effect of the debt-to-GDP on potential/trend GDP growth. The potential and trend GDP is constructed on the European Commission's methodology; the authors use annual data and 5-year averages rates. The advantages of using trend/potential growth are: (i) more precise capture of the long-term impact and avoidance of cyclical fluctuation, (ii) mitigation of endogeneity and particularly of reverse causality, (iii) testing the robustness of the debt turning point.

Table: 4.8
Results with restricted time sample, start year indicated (Model 1 FE, dependent variable :cumulative 5-year overlapping growth rate)

start period	1970			1980	1990	1999
	whole period	whole period/EMU dummy	1970 – 1998			
gov_debt	0.5112***	0.4062***	0.3742***	0.6812***	0.3782***	0.0712
gov_debt_sq	−0.0025***	−0.0021***	−0.0020***	−0.0035***	−0.0020***	−0.0006
ln_GDP_cap	−8.6239	−19.1990***	−25.0763***	−10.2743	−24.6240***	−79.3019***
gov_ca_rev	−0.2288	−0.2165*	−0.1195	−0.4490**	0.2112	−0.3636*
gov_cab	1.0548***	1.0775***	1.1013***	1.1537***	0.4044**	0.153
pop_growth	−4.0147***	−2.9737**	−1.216	−2.8937**	−3.9689***	−5.2554***
gfcf_total	−0.2503	−0.5704**	−0.8644***	−0.178	−0.0813	−0.176
openess	0.1785***	0.1426***	0.2392***	0.1548***	0.1010***	0.0609
LT_real_i	0.2664*	0.124	0.0448	0.2905	−0.7193**	−0.3465*
Year dummies	Included (38) (1971 – 2008)	EMU dummy only −6.2967***	Included (28) (1971 – 1998)	Included (28) (1981 – 2008)	Included (18) (1991 – 2008)	Included (10) (1999 – 2008)
Country dummies	Included (11)	Included (11)	Included (11)	Included (11)	Included (11)	Included (11)
_cons	32.8661**	71.3831***	85.6420***	17.6008	51.9371	282.0510***
N	388	388	292	312	202	96
R2 – within	0.73	0.53	0.72	0.75	0.87	0.94
debt – turning point	101	95	94	98	96	–

Note: Start year for the restricted time sample period shown. For the whole period, i.e., start year 1970, three models are shown: (i) baseline model with yearly dummies; (ii) model excluding yearly dummies and including an EMU dummy, which takes the value 1 starting with 1999 (2001 for Greece) and zero otherwise; (iii) model excluding the EMU period. For the next three models, the time period considered starts, respectively, in 1980, 1990 and 1999.

Using the same instrumental variable models as those for the growth rate of real per capita GDP the same robust results are obtained. The concave relationship remains robust with both debt terms highly significant and with debt turning point varying close to the same value. We note that we should be careful with these results because there could possibly be introduced distortions in the models as the potential GDP growth is only an estimated variable.

The simple average of the debt turning point across all models is 97%. Including only the instrumental variable models the turning point yields a value of 94%. These results are 95% and 91% when the potential and trend GDP growth rate are accounted for. The statistical confidence interval around the debt turning point is computed to begin at 70% of GDP.

Interesting part of the research is the investigation of the channels for the impact of public debt on growth. Based on the literature, Checherita-Westphal and Rother try to identify the effect debt makes on: (i) private saving and private investment (gross fixed capital formation) rate; (ii) public investment (gross fixed capital formation) rate; (iii) total factor productivity (TFP); and (iv) sovereign long-term nominal and real interest rates. Each of these candidate channels is estimated individually.

The regression equation for the investigation of the channel of the private saving ratio is presented below:

$$saving\_priv = \alpha_0 + \alpha_1 L.saving\_priv + \gamma_1 debt\_sq + \gamma_2 debt + other\ control\ variables\ (initial\ level\ GDP/cap;\ economic\ growth\ rate;\ population\ growth;\ taxrate; credit\text{-}to\text{-}GDP\ ratio;\ old\ and\ young\ people\ dependency\ ratio;\ LT\ interest\ rates;\ openness) + \mu_i + v_t + \varepsilon_{it} \quad (4.5)$$

The dynamic nature of the panel model is justified by the possibility that the private saving rate is highly persistent. For the investigation of the private and public investment channels a similar model is applied.

Except from the lagged private saving and the debt variable, the other control variables are the common determinants of private saving encountered in the literature (Masson et al., 1998). These extra determinants are: (i) the level of income per capita; (ii) demographic shifts and structure as proxied by the growth rate of the population and the ratio of the non-working age population to the working age population, split between old and young dependency ratio; (iii) the level of taxation (proxied by total government revenue as a share of GDP); (iv) the depth of the financial system and other financial indicators, as proxied by the share of domestic private credit-to-GDP and the long-term interest rate; (v) indicators of openness of the economy to capture the possibility of foreign saving inflows or outflows.

Table 4.9
Dynamic models for private saving rate, private investment, public investment and total factor productivity (TFP) growth rate

Explanatory/dependent variables	Private saving	Private gross fixed capital formation	Public gross fixed capital formation	TFP growth rate
gov_debt	0.1270*** (0.0246)	−0.0216 (0.0183)	0.0140 (0.0092)	0.1486*** (0.0475)
gov_debt_sq	−0.0007*** (0.0002)	0.0001 (0.0001)	−0.0002** (0.0001)	−0.0007** (0.0003)
LDV	0.3740***	0.6490***	0.7814***	0.1881
GDP_cap_g	0.2923***	0.2203***	0.0180*	−0.4761***
gov_ca_rev	0.0257	−0.1816***	–	–
pop_growth	0.9432***	–	–	−0.2178
credit_priv	−0.0171	−0.0076	–	−0.0102
old_dep_ratio	−0.4826*	–	–	−0.0538
young_dep_ratio	0.1002	–	–	−0.0416
LT_real_i	0.0434	−0.0994***	0.0166	0.0461
Openness	−0.0421**	0.0062	0.0041**	–
gfcf^_Lo	–	−0.5881***	−0.0467***	–
*gfcf^*	–	0.3339***	0.0705**	–
gov_bal	–	–	0.0315***	–
ToT_g	–	–	–	−0.0588***
Year dummies	Included (38) (1971–2008)	Included (38) (1971–2008)	Included (38) (1971–2008)	Included (38) (1971–2008)
_cons	19.4582***	13.7089***	−0.3749	−0.3362
N	313	313	393	389
Instruments DV	Lag(2/6)	Lag(2/6)	Lag(2/6)	Lag(2/6)
ABond test for AR in first-differenced errors				
AR(1) test; *p*-value	0.0144	0.0140	0.0258	0.0037
AR (2) test; *p*-value	0.4210	0.2852	0.1639	0.5674
debt turning point	92.8	–	46.9	109.3
95% CI nlcom	(53; 132)	–	(19; 75)	(52; 167)

Notes: All panel models are dynamic, estimated using the GMM Arellano-Bond estimator. All regression models use annual data. *DV* denotes the dependent variable. When DV is measured at time *t* (private saving rate, private and public investment) then the explanatory variables including debt and debt squared are lagged one year. When DV is a growth rate (calculated for period *t*+1, i.e., the TFP growth rate), then the explanatory variables are measured at time *t*. *gfcf^* denotes the fact that *gfcg_gov* is used as an explanatory variable for the private investment model, while *gfcg_priv* is used for the public investment model in order to capture crowding-out effects between sectors. The suffix _Lo denotes the fact that the *gfcf^* variable is measured at time *t* (its coefficient capturing a contemporaneous effect).

Table 4.9 provides the results, where there seems to be a non-linear impact of public debt on private saving that mimics the impact found for per capita growth. But overall the estimates indicate that the debt-to-GDP turning point is found on lower levels, between 82% and 93%. The authors explain the impact of debt on private saving as the outcome of private agents anticipating inflationary pressures in the financial markets and/or transfer capital abroad.

By investigating with the similar way the private investment, they could not discover any direct effect of public debt on it. The debt variables were found mostly insignificant. The estimation equation:

$$gfcf\_priv = \alpha_0 + \alpha_1 L.gfcf\_priv + \gamma_1 debt\_sq + \gamma_2 debt + other\ controls\ (public\ investment;\ economic\ growth\ rate;\ initial\ level\ of\ GDP/cap;\ tax\ rate;\ private\ credit-to-GDP\ ratio;\ LT\ interest\ rates;\ openness\ indicators) + \mu_i + v_t + \varepsilon_{it} \quad (4.6)$$

The public investment (government gross fixed capital formation) was also examined.

$$gfcf\_gov = \alpha_0 + \alpha_1 L.gfcf\_gov + \gamma_1 debt\_sq + \gamma_2 debt + other\ controls\ (private\ investment;\ economic\ growth\ rate;\ initial\ level\ of\ GDP/cap;\ gov.budget\ balance;\ LT\ interest\ rates;\ openness\ indicators) + \mu_i + v_t + \varepsilon_{it} \quad (4.7)$$

The results unveil a concave relationship between public debt and public investment, robust across a variety of models. The turning point varies from 47% to 70% of GDP. Beyond this threshold, the negative effects mean that, in their consolidation efforts, countries might reduce resources distributed for public investment, including maintenance of public infrastructure.

Turning, next, to the impact of debt on total factor productivity:

$$TFP = \alpha_0 + \alpha_1 L.TFP + \gamma_1 debt\_sq + \gamma_2 debt + other\ controls\ (lagged\ economic\ growth\ rate;\ population\ growth\ rate;\ old\ and\ young\ dependency\ ratio;\ private\ credit-to-GDP\ ratio;\ LT\ interest\ rates;\ openness\ indicators) + \mu_i + v_t + \varepsilon_{it} \quad (4.8)$$

The estimations indicate the similar impact of debt on TFP namely through a concave relationship, with the turning point of debt being at 100% of GDP.

At the end, they searched the possible impact of public debt on long-term (LT) sovereign interest rates.

$$LT\_nom\_i = \alpha_0 + \alpha_1 ST\_nom\_i + \gamma_1 debt\_sq + \gamma_2 debt + other\ controls\ (inflation\ rate;\ gov.primary\ balance;\ lagged\ economic\ growth\ rate;\ output\ gap;\ external\ balance\ and\ openness\ indicators) + \mu_i + v_t + \varepsilon_{it} \quad (4.9)$$

$$LT\_real\_i = \alpha_0 + \alpha_1 ST\_real\_i + \gamma_1 debt\_sq + \gamma_2 debt + other\ controls\ (gov.primary\ balance;\ lagged\ economic\ growth\ rate;\ output\ gap;\ external\ balance\ and\ openness\ indicators) + \mu_i + v_t + \varepsilon_{it} \quad (4.10)$$

They discovered that the level of public debt ratio –either in linear or quadratic forms- is insignificant when controlling for the long-term interest rates.

According to their findings, the authors conclude that the possible channels through which debt impacts growth are private saving, public investment and TFP.

We go back to the way Checherita-Westphal and Rother handled the debt-growth relationship, to refer to Panizza and Presbitero's (2013) criticisms of the paper's results. First, they are skeptical about the polynomial specification, as they find it arbitrary. They also think that the inverted U-shape relationship between debt and growth is easily affected by extreme values and that a hump-shaped relationship may be an outcome produced by a very small number of observations. They suggested that Checherita-Westphal and Rother (2012) should have conducted semi-parametric estimations to strongly support the quadratic relationship or have checked if the presence of a U-shaped relationship is supported by the Sasabuchi-Lind-Mehlum test (Lind and Mehlum, 2010).

They also wonder whether the techniques employed so as to correct for the endogeneity were efficient. The instrument of the average debt ratio of the other countries should satisfy certain criteria that are in fact difficult to defend. In order to be a valid instrument there must be no robust relation between debt levels in other euro area countries and the per-capita GDP growth rate in one particular country. If it is correct that debt in a specific country has a deleterious impact on growth arguing that debt levels in other euro-area countries have no effect on growth in the excluded country is similar as suggesting that GDP in the euro area (without a specific country) must have no impact on the GDP growth of the excluded country. However, this supposition is difficult to support.

Moreover, the estimations of the models with the instrumental variables in Checherita-Westphal and Rother's (2012) analysis are very close to the OLS (panel fixed effects) estimations. The same problem is revealed if someone compares the estimations of their GMM techniques with those of OLS (see Table 4.5 and 4.6). These indicate either that debt is not endogenous or that these techniques fail to solve the endogeneity problem and so perhaps fail to identify correctly the debt thresholds.

Kumar and Woo (2010) is another one paper that employs the multivariate growth regression to study the long-term non-linear impact of debt on growth. Their research focuses on 38 advanced and developing countries over the period 1970-2007. Their empirical findings, based on a variety of estimation techniques, indicate a strong linear inverse relation between initial debt and subsequent growth. Specifically, if the debt-to-GDP ratio rises by ten percentage point the annual real per capita GDP growth diminishes by 0.2 percentage points per year in the sample of developing and advanced economies with the effect being lower in advanced economies. Regarding the non-linear relationship, a spline regression is used as it has the ability of integrating one or more knots (Marsh and Cormier, 2002). They consider two externally-imposed thresholds at 30% and 90% debt levels by using dummy variables. They discover a nonlinear link when debt is quite high (above 90% of GDP). Only in that level a significant negative impact on growth for both advanced and developing countries identified.

The baseline panel growth regression is as follows:

$$y_{i,t} - y_{i,t-\tau} = \alpha y_{i,t-\tau} + \mathbf{X}_{i,t-\tau}\beta + \gamma Z_{i,t-\tau} + \eta_t + \nu_i + \varepsilon_{i,t}, \quad (4.11)$$

The dataset includes the years 1970-2007 which are divided to 8 non-overlapping five years periods.

The term y stands for the logarithm of real per capita GDP; country-specific fixed effect and time-fixed effect are included; $X_{i,t-\tau}$ integrates a set of economic variables analyzed below; $Z_{i,t-\tau}$ denotes the initial government debt (in percent of GDP).

The vector X includes: human capital, to reflect the notion that countries with an abundance of it are more likely to have a greater ability to attract investors, absorb ideas from the rest of the world, and engage in innovation activities. Government consumption, initial trade openness, liquid liabilities, initial inflation as measured by CPI inflation, and terms of trade growth rates are also included. The vector X further comprises the fiscal deficit and a measurement of banking crisis episodes. The latter reflects discoveries that connect banking crises with simultaneously increases in public debt. Banking crises tend to also be accompanied by lower GDP rates.

The estimation uses initial level of debt to avoid the reverse causality problem. That is, as already supported; low economic activity may result to high debt burden, instead of high debt reduces growth. However, the authors know that this method does not effectively address the endogeneity problem since growth and debt might be both defined by a third factor. The system GMM approach of Arellano and Bover (1995) and Blundell and Bond (1998) is considered to resolve the endogeneity in this paper.

The 38 advanced and emerging economies included in the panel regression are:

Table 4.10

Country	Country
Australia	Japan
Austria	Korea
Belgium	Malaysia
Brazil	Mexico
Canada	Netherlands
Chile	Pakistan
China	Peru
Colombia	Philippines
Czech Republic 1/	Poland
Denmark	Portugal
Egypt	Russian Federation 1/
France	Slovak Republic 1/
Germany	South Africa
Greece	Spain
Hong Kong	Sweden
Hungary	Switzerland
India	Turkey
Indonesia	United Kingdom
Italy	United States

This paper pays particular attention to a variety of estimation methodologies in order to (i) address for the many sources of biases, (ii) conduct the comparison of statistical significance of the coefficients between various methodologies and (iii) ensure robust results for the estimated coefficients. The methodologies are pooled OLS, between estimator (BE), fixed effects (FE) panel regression, and system GMM (SGMM) dynamic panel regression.

The results from the baseline estimation are in Table 4.11 .

Table 4.11

Baseline Panel Regression—Growth and Initial Government Debt, 1970 –2007 (Five-year Period Panel)

Sample: Advanced and Emerging Economies, Dependent Variable: Real per Capita GDP Growth

Explanatory Variables	(1) BE	(2) Pooled OLS	(3) FE	(4) SGMM	(5) Pooled OLS	(6) FE	(7) SGMM
Initial per capita real GDP	-2.616***	-2.257***	-3.598***	-2.555***	-2.187***	-4.506***	-2.823***
	(-6.66)	(-3.26)	(-3.03)	(-3.04)	(-2.74)	(-3.31)	(-3.33)
Initial years of schooling	4.246***	2.965***	5.622***	4.333*	2.863***	4.138**	4.161**
	(4.58)	(2.96)	(3.66)	(1.70)	(2.72)	(2.34)	(2.12)
Initial inflation rate	0.931	-2.351***	-2.571***	-3.062**	-2.234***	-2.467***	-2.296
	(0.47)	(-3.81)	(-4.65)	(-2.27)	(-3.49)	(-6.93)	(-1.43)
Initial government size	0.1**	0.086**	0.125	0.113	0.087**	0.012	0.168
	(2.45)	(2.30)	(1.41)	(0.99)	(2.29)	(0.15)	(1.20)
Initial trade openness	0.002	0.001	0.024	-0.006	-0.001	0.020	-0.004
	(0.39)	(0.18)	(1.71)	(-1.14)	(-0.25)	(1.47)	(-0.71)
Initial financial depth	0.024***	0.018***	-0.001	0.033***	0.019***	0.006	0.026***
	(2.98)	(2.76)	(-0.07)	(2.98)	(2.87)	(0.71)	(2.72)
Terms of trade growth	0.111*	-0.015	0.011	-0.024	-0.019	-0.003	-0.025
	(1.67)	(-0.64)	(0.41)	(-0.97)	(-0.88)	(-0.14)	(-0.96)
Banking crisis	-1.143	-0.819**	-0.782***	-1.196*	-0.728**	-0.673**	-1.519
	(-0.85)	(-2.50)	(-3.62)	(-1.91)	(-2.27)	(-2.64)	(-1.42)
Fiscal deficit	0.012	-0.048***	-0.051***	-0.056***	-0.044***	-0.037***	-0.036*
	(0.44)	(-4.89)	(-4.60)	(-3.42)	(-4.91)	(-4.63)	(-1.78)
Government debt, initial	-0.026***	-0.020***	-0.019***	-0.029***	-0.018***	-0.004	-0.020**
	(-3.04)	(-3.64)	(-3.23)	(-3.24)	(-2.66)	(-0.79)	(-2.49)
Arellano-Bond AR(2) test p-value 1/				0.64			0.12
Hansen J-statistics (p-value) 2/				0.28			0.26
Number of observations	166	166	166	166	166	166	166
R^2	0.78	0.55	0.4		0.66	0.60	
Time-fixed effects	N/A	No	No	No	Yes	Yes	Yes

Note: Heteroskedasticity and country-specific autocorrelation consistent *t-statistics* are in parentheses. Time dummies are not reported.
Levels of significance: *** 1 percent, ** 5 percent, * 10 percent. In the OLS regressions, dummies for OECD, Asia, Latin America, and sub-Saharan Africa are also included in each regression (not reported to save space). FE refers to the fixed effects panel regressions and BE is the between estimator.
For the dynamic panel estimation, a two-step system GMM (SGMM) with the Windmeijer's finite-sample correction for the two-step covariance matrix.
1/ The null hypothesis is that the first-differenced errors exhibit no second-order serial correlation.
2/ The null hypothesis is that the instruments used are not correlated with the residuals.

Government debt is found to be negative under all the four estimation techniques and statistical significant. The values vary within -0.019 and -0.29. In Columns 2-4 time-fixed effects are not included. The authors claim that global factors may simultaneously affect both domestic growth and public debt and for that reason possibly bias the results toward finding a stronger relationship between debt and growth.

Debt under pooled OLS and SGMM is still significant but with a smaller (absolute) value, when time-fixed effects are included. It is thus implied that as the initial debt-to-GDP rises by a 10 percentage point GDP growth per capita is reduced by 0.2 percent.

It is clear by the FE estimations of initial debt that the introduction of time-fixed effects alters the results significantly. The FE estimator regularly captures the within-country variations in contrast to cross-sectional variations of the other estimators. Thus, it is normal that the within-country variation is at a great degree diminished with the inclusion of time-fixed effects.

The OLS estimator is possibly biased upwards while FE estimators downwards. The consistent GMM should lie between the two (Bond, 2002). Consistency of SGMM is tested through 2 tests proposed by Arellano and Bover (1995) and Blunedell and Bond (1998). The first is a Hansen J-test that tests if the instruments are not correlated with the residuals. The second (Arellano-Bond) tests the hypothesis that the error term $\varepsilon_{i,t}$ is not serially correlated and find that the null hypothesis cannot be rejected.

Several robustness checks are conducted to test the validity of the results. In one of these checks, they considered additional variables in the regression such as population size, domestic investment and a measure of fiscal volatility. The results remain broadly unaltered (Table 4.12).

Table 4.12

Robustness Checks—Additional Variables: Advanced and Emerging Economies Dependent Variable: Real per Capita GDP Growth

Explanatory Variables	(1) BE	(2) Pooled OLS	(3) FE	(4) SGMM	(5) BE	(6) Pooled OLS	(7) FE	(8) SGMM	(9) BE	(10) Pooled OLS	(11) FE	(12) SGMM
Initial per capita real GDP	-2.254***	-2.029***	-4.128***	-3.198**	-2.8***	-2.773***	-3.975**	-3.207***	-2.611**	-2.233***	-4.617***	-2.495***
	(-4.51)	(-2.84)	(-3.61)	(-2.27)	(-7.34)	(-3.12)	(-2.31)	(-2.95)	(-5.66)	(-2.79)	(-3.28)	(-2.85)
Initial years of schooling	4.053***	2.711***	1.335	5.232**	3.920***	3.221***	3.992**	4.85**	4.247***	2.752**	3.876**	3.463
	(4.33)	(2.77)	(0.55)	(2.25)	(4.41)	(3.17)	(2.17)	(2.03)	(4.49)	(2.63)	(2.21)	(1.17)
Initial inflation rate	1.423	-2.050***	-2.601***	-8.396	0.782	-2.434***	-2.478***	-3.477	0.931	-2.112***	-2.40***	-3.572
	(0.70)	(-3.42)	(-6.70)	(-1.45)	(0.41)	(-3.73)	(-6.57)	(-0.89)	(0.46)	(-3.21)	(-5.95)	(-1.57)
Initial government size	0.087**	0.085**	0.029	0.234	0.111***	0.089**	-0.026	0.078	0.1**	0.083**	0.012	0.146
	(2.10)	(2.47)	(0.33)	(1.50)	(2.86)	(2.42)	(-0.30)	(0.73)	(2.33)	(2.18)	(0.15)	(1.19)
Initial trade openness	0.006	0.003	0.034**	-0.013	0.004	0.001	0.019	-0.004	0.002	0.0002	0.021	0.005
	(1.06)	(0.76)	(2.52)	(-0.85)	(0.90)	(0.15)	(1.22)	(-0.55)	(0.30)	(0.06)	(1.46)	(0.67)
Initial financial depth	0.017*	0.016**	0.007	0.028	0.023***	0.019***	0.005	0.014	0.024**	0.017**	0.005	0.019*
	(1.75)	(2.59)	(0.87)	(1.18)	(3.11)	(3.32)	(0.63)	(1.23)	(2.74)	(2.58)	(0.68)	(1.66)
Terms of trade growth	0.127*	-0.015	-0.010	-0.048	0.169**	-0.014	-0.004	-0.014	0.112	-0.018	-0.003	-0.038*
	(1.88)	(-0.70)	(-0.40)	(-1.36)	(2.46)	(-0.65)	(-0.16)	(-0.40)	(1.59)	(-0.80)	(-0.11)	(-1.66)
Banking crisis	-1.587	-0.816**	-0.512*	-0.521	-0.697	-0.678*	-0.606**	-2.020	-1.139	-0.709**	-0.648**	-0.968
	(-1.14)	(-2.68)	(-1.93)	(-0.35)	(-0.54)	(-1.94)	(-2.17)	(-1.11)	(-0.82)	(-2.25)	(-2.61)	(-1.25)
Fiscal deficit	0.022	-0.042***	-0.035***	-0.050**	0.002	-0.046***	-0.038***	-0.045*	0.012	-0.042**	-0.036***	-0.042**
	(0.80)	(-4.71)	(-4.65)	(-2.02)	(0.07)	(-5.29)	(-4.61)	(-1.74)	(0.42)	(-4.69)	(-4.36)	(-2.29)
Initial government debt	-0.025***	-0.018***	0.001	-0.023***	-0.019**	-0.015**	-0.007	-0.018**	-0.026***	-0.017**	-0.004	-0.019*
	(-2.98)	(-2.94)	(0.16)	(-2.84)	(-2.29)	(-2.94)	(-1.00)	(-2.59)	(-2.97)	(-2.67)	(-0.72)	(-2.01)
Initial population size	0.244	0.233	6.861**	-0.294								
	(1.16)	(1.43)	(2.30)	(-0.34)								
Initial investment					0.065**	0.052*	-0.045	0.063				
					(2.07)	(1.97)	(-0.86)	(1.02)				
Fiscal volatility									0.009	-0.229	-0.114	-0.081
									(0.02)	(-1.11)	(-0.52)	(-0.25)
Arellano-Bond AR(2) test p-value 1/				0.12				0.20				0.16
Hansen J-statistics (p-value) 2/				0.66				0.53				0.91
Number of observations	166	166	166	166	166	166	166	166	166	166	166	166
R^2	0.79	0.66	0.63		0.81	0.67	0.61		0.78	0.66	0.61	
Time-fixed effects	N/A	Yes	Yes	Yes	N/A	Yes	Yes	Yes	N/A	Yes	Yes	Yes

Note: Heteroskedasticity and country-specific autocorrelation consistent *t-statistics* are in parentheses. Time dummies are not reported.
Levels of significance: *** 1 percent, ** 5 percent, * 10 percent. In the OLS regressions, dummies for OECD, Asia, Latin America, and sub-Saharan Africa are also included in each regression (not reported to save space). FE refers to the fixed effects panel regressions and BE is the between estimator.
For the dynamic panel estimation, a two-step system GMM (SGMM) with the Windmeijer's finite-sample correction for the two-step covariance matrix.
1/ The null hypothesis is that the first-differenced errors exhibit no second-order serial correlation.
2/ The null hypothesis is that the instruments used are not correlated with the residuals.

In columns 1-4 where they include the log of initial population, debt is negative and significant except for the FE estimator. Columns 5-8 include the initial domestic investment and the coefficients remain of the same sign and statistically significant at 5 percent level (again the FE estimator excluded). Finally, in columns 9-12 a measure of fiscal spending volatility is included. Aggressive fiscal policy that is not being undertaken to mitigate business cycle fluctuations may induce instability and lower growth as supported by Fatas and Mihov (2003). Simultaneously, these fiscal policies may induce large debt buildup. In the context of these, they check whether too much fiscal discretion is a hidden cause for the negative debt-growth correlation. If that is right, the coefficient of initial debt must turn to insignificant or at least get smaller, when fiscal volatility is included. Yet, the results do not consent to this. The fiscal volatility estimations are not significant and also change size across estimations. On the other hand, the coefficients of initial debt continue being significant and their value is in general the same with the baseline estimations.

Next, they continue to explore the nonlinearities. They include in the model interaction terms between initial debt and dummy variables to test three ranges of debt: "Dum_30" for debt below 30 percent of GDP; "Dum_30–90" for debt between 30 and 90 percent of GDP; and "Dum_90" for debt over 90 percent of GDP. The model is as follows:

$$GROWTHi,t-(t-4)=\alpha ln(GDP)_{i,t-4}+\beta_1 DEBT_{i,t-4}\times D_{30} +\beta_2 DEBT_{i,t-4}\times D_{30-90}+\beta_3 DEBT_{i,t-4} \times D_{90}+\gamma X_{i,t-4}+\tau_t+\eta_i+\varepsilon_{i,t} \quad (4.12)$$

Table 4.13

Panel Regression—Different Levels of Initial Debt and Advanced vs. Emerging Economies Dependent Variable: Real per Capita GDP Growth

Explanatory Variables	(1) BE	(2) Pooled OLS	(3) FE	(4) SGMM	(5) BE	(6) Pooled OLS	(7) FE	(8) SGMM
Initial per capita real GDP	-3.684***	-2.369***	-4.486***	-2.367**	-3.052***	-2.750***	-4.405***	-3.331**
	(-3.06)	(-3.39)	(-3.52)	(-2.42)	(-5.64)	(-3.33)	(-3.43)	(-2.55)
Initial years of schooling	5.634***	2.931***	4.001**	3.882	4.109***	2.776***	3.823**	3.837*
	(3.75)	(3.17)	(2.25)	(1.61)	(4.43)	(2.89)	(2.24)	(1.88)
Initial inflation rate	-2.564***	-2.469***	-2.312***	-2.073	-0.271	-2.802***	-2.336***	-6.258**
	(-4.44)	(-3.82)	(-5.96)	(-1.55)	(-0.12)	(-3.32)	(-5.81)	(-2.21)
Initial government size	0.122	0.097***	-0.014	0.104	0.091**	0.067*	0.013	0.213**
	(1.35)	(2.83)	(-0.17)	(1.01)	(2.22)	(1.83)	(0.16)	(2.10)
Initial trade openness	0.025*	-0.0004	0.022*	-0.001	0.002	-0.001	0.020	-0.004
	(1.73)	(-0.14)	(1.67)	(-0.17)	(0.55)	(-0.16)	(1.47)	(-0.75)
Initial financial depth	-0.001	0.019***	0.006	0.014*	0.024***	0.019***	0.005	0.031***
	(-0.06)	(3.37)	(0.76)	(1.68)	(3.05)	(3.25)	(0.58)	(3.89)
Terms of trade growth	0.012	-0.019	-0.006	-0.011	0.126*	-0.012	-0.002	-0.02
	(0.44)	(-0.77)	(-0.27)	(-0.36)	(1.88)	(-0.54)	(-0.07)	(-0.88)
Banking crisis	-0.755***	-0.678**	-0.655**	-0.689	-0.996	-0.711**	-0.66**	-0.903
	(-3.23)	(-2.05)	(-2.39)	(-0.57)	(-0.74)	(-2.23)	(-2.58)	(-0.86)
Fiscal deficit	-0.051***	-0.045***	-0.035***	-0.036	0.004	-0.048***	-0.034***	-0.054***
	(-4.44)	(-4.80)	(-4.14)	(-1.58)	(0.15)	(-5.07)	(-3.91)	(-3.48)
Initial debt*Dum_30	-0.003	-0.001	0.001	0.031				
	(-0.11)	(-0.02)	(0.03)	(0.82)				
Initial debt*Dum_30-90	-0.015	-0.025**	0.005	-0.018				
	(-1.56)	(-2.61)	(0.59)	(-1.24)				
Initial debt*Dum_90	-0.017**	-0.016***	-0.002	-0.018*				
	(-2.67)	(-2.93)	(-0.30)	(-1.78)				
Initial debt*Dum_Advanced					-0.021**	-0.014***	-0.005	-0.017*
					(-2.27)	(-2.77)	(-0.94)	(-1.74)
Initial debt*Dum_Emerging					-0.036***	-0.034**	0.007	-0.041*
					(-2.90)	(-2.64)	(0.51)	(-1.65)
Arellano-Bond AR(2) test p-value 1/				0.22				0.14
Hansen J-statistics (p-value) 2/				0.68				0.74
Number of observations	166	166	166	166	166	166	166	166
R^2	0.6	0.68	0.61		0.79	0.67	0.61	
Time-fixed effects	N/A	Yes	Yes	Yes	N/A	Yes	Yes	Yes

Note: Heteroskedasticity and country-specific autocorrelation consistent *t-statistics* are in parentheses. Time dummies are not reported.
Levels of significance: *** 1 percent, ** 5 percent, * 10 percent. In the OLS regressions, dummies for OECD, Asia, Latin America, and sub-Saharan Africa are also included in each regression (not reported to save space). FE refers to the fixed effects panel regressions and BE is the between estimator.
For the dynamic panel estimation, a two-step system GMM (SGMM) with the Windmeijer's finite-sample correction for the two-step covariance matrix.
1/ The null hypothesis is that the first-differenced errors exhibit no second-order serial correlation.
2/ The null hypothesis is that the instruments used are not correlated with the residuals.

The coefficients of the low debt levels are not statistically significant and turn positive across FE and SGMM, as shown in Table 4.13 (Columns 1-4). Under the OLS, the coefficient of medium level of debt is -0.025 and significant at 5 percent. But, this coefficient is insignificant in all the other estimations. The high-debt coefficients are negative and significant under BE, OLS and SGMM and vary between -0.016 and -0.018. This means, that a 10 percentage points increase in debt-to-GDP after the level of 90%, decreases subsequent growth by 0.16 to 0.18 percentage points.

Kumar and Woo (2010) seek also to analyze possible differences between advanced and emerging market economies. They find evidence of smaller negative impact of initial debt on growth in advanced compared to emerging economies. Results shown in Table 4.13 (Columns 5-8).

They also wanted to explore the effect on growth as debt-to-GDP increases by a 10 percent, (or by any particular increase) given that debt initially lies at different levels (Table 4.14). Initially debt ratios are gathered in 4 groups: <30%; 30-60%; 60-90%; and >90%. Then they calculated the mean debt ratio for each group and in Row 2 the average estimated coefficient (from BE, OLS and SGMM) of interaction terms for every group. The impact on growth of a 10 percent increase in debt is found by the multiplication of the first two Rows by 10 percent. According to the findings, the higher the initial level of debt, the higher the adverse effect of a ten percent increase on growth. This applies not only for a 10 percent increase but for every specific proportionate rise in the debt to GDP ratio. In the examined case, a 10 percent increase in the debt ratio in countries with debt ratio above 90 percent is accompanied with a decline in growth of around 0.19 percent. The same increase applied to the 30-60 percent group is accompanied with a decrease in growth of around 0.11 percent.

We note that this findings do not change if debt groups differ and also that they are very close to those estimated in columns 1-4 of Table 4.13.

Table 4.14

Impact on Real per Capital GDP Growth of a 10 Percent Increase in the Debt-to-GDP Ratio

	Initial Debt Ratios (in percent of GDP)			
	<30	30-60	60-90	>90
(1) Sample average of Initial Debt/GDP	15.8	45.1	70.3	111.9
(2) Regression coefficient, average 1/	0.022	-0.025	-0.023	-0.017
(3) Growth impact of 10 percent increase in Debt/GDP from sample average 2/	0.04	-0.11	-0.16	-0.19

1/ Average of the esimates (from BE, OLS, SGMM) on the coefficients of interaction terms between initial debt-to-GDP and dummy variables for four categories of levels of initial debt-to-GDP (below 30 percent, between 30 and 60 percent; between 60 and 90 percent; above 90 percent of GDP) for the 1970-2007 period. The results need to be interepreted with caution because the coefficients of low debt level (initial debt*Dum_30) are not statistically different from zero. Also, the statistical significance of coefficients of other interaction terms varies across estimations.

2/ This estimate of growth impact of 10 percent increase in debt ratio is obtained as the product of the regression coefficient (Row 2) and 10 percent of the sample average debt ratios (Row 1).

As Kumar and Woo (2010) experiment with various estimation techniques, they claim that the system GMM helps them to resolve endogeneity. Yet, the findings of their linear effects have been called into question, largely because system GMM estimations are close to those received with pooled OLS regressions. To see this, check columns 2 and 4 and 5 and 7 of Table 4.11. In particular, system GMM estimates have a greater value than the OLS indicating that debt is not subject to endogeneity or system GMM does not address this issue. Against this background, it has been already argued in the 'Methodological Issues' subsection, that the GMM estimators have been proved inappropriate in solving the endogeneity problem and are generally not suitable for the macroeconomic datasets.

The SGMM and OLS estimations concerning the non-linear relationship have also some problems. In Table 4.13 Column 4, the coefficients of medium initial debt (30-90%) and high initial debt (over 90%) are identical and both equal to -0.018. They only differ in their statistical significance; β_2 is insignificant and has a t-statistics of 1.24 (with a p-value of 0.22) and β_3 is marginally significant with a t-statistics of 1.78 (with a p-value of 0.08). A t-test of $\beta_2=\beta_3$ will not reject the null of equality, which means that the existence of a non-linear impact of debt on growth is rejected. The OLS regressions (Column 2) complicates the non-linearity hypothesis even more as both β_2 and β_3 are statistically significant and $|\beta_2|>|\beta_3|$.

To sum up, the spline regression employed in this analysis must be examined with a critical eye. While more flexible than the quadratic function of Checherita-Westphal and Rother (2012), it is also arbitrary as the number and cutoff of the knots are selected to produce the maximum fit of the model. So, the findings of Kumar and Woo (2010) do not provide clear proof of a non-linear relation between debt and growth.

In spite of different sample periods, country coverage, control variables, modelling of the nonlinearity and choice of moment conditions for identification, the above researches broadly end up on the same findings. That is, when debt-to-GDP passes the 90% threshold, debt and growth are negatively linked. Yet, as Panizza and Presbitero (2013) claim, the results are sensitive to small changes and outliers in the dataset. More importantly, the test hypotheses on coefficients of the pairwise linear terms are rejected which finally constitutes rejection of the statistically significance of the change in the coefficient beyond the threshold.

5. Robust thresholds at different debt levels

The researches so far, confirming the Reinhart-Rogoff outcome of the 90% public debt level over which economic performance reduces considerably, argued in favor of debt reduction to boost long-run economic activity. But a different part of literature emerged calling the one-size-fits-all number of the 90% into question. The threshold can be lower or the non-linearity may alter depending on the various samples and specifications employed. In this section we discuss the papers which find a robust negative non-linear link between debt and growth but with the debt-to-GDP thresholds lying elsewhere than the 90 percent level.

Sulikova et al. (2015) use the dynamic panel data model that has the advantage of investigating the nonlinear effect of debt-to-GDP increase and decrease on economic growth at the same time. Through their model they find that debt-to-GDP decrease and GDP growth are linearly connected. Simultaneously, they find that the relationship between the debt-to-GDP increase and GDP growth is determined by an inverted U-shaped curve with the top at 64% debt-to-GDP ratio. They choose to analyze 13 EU countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Netherlands, Portugal, Spain, Sweden and United Kingdom in the time period: 1993-2013.

Sulikova et al. study the impact of the government debt on economic growth through the estimation of a growth equation of the type of an inverted U-shaped curve. Theory suggests that in the part of the inverted U-shaped curve that increases the multiplication effects of the government outcomes dominate and speeds up the economic activity. In the part that decreases public debt decelerates the economic activity. Yet, from a practical perspective we wonder whether the economy policy focused on either promoting economic growth or austerity policy to decrease the public debt does really trace the same trajectory and so, whether the estimated inverted U-shaped curve parameters are of the same values in both regimes. Therefore, we focused our research on revealing and quantification asymmetries between both the debt increase and debt reduction impacts on economic growth.

The independent variable is real GDP per capita growth. They focus exclusively on short term impact of debt on economic activity and so they use the annual growth rate. The list of regressors are : (i) log of real per capita GDP to preserve the convergence tendency; (ii) annual population growth to catch population driven economy growth; (iii) gross domestic savings as a prevailing financial source; (iv) gross fixed capital formation as a proxy for physical capital; (v) average length of total schooling (in years) as a human capital measure; (vi) age dependency ratio (percentage of working-age population) to catch the productivity of the labour force and financial burden evoked by ageing of the population; (vii) economy openness computed as (Import + Export)/GDP assuming to have a significantly positive effect on GDP growth in panel data growth models as estimated by Baum, Checherita-Westphal and Rother (2013); (viii) inflation given as Consumer Price Index (annual, %).

Given the existence of endogeneity of debt they make the estimation in 2 steps. They first fit the debt panel data by regressing on all available regressors lagged by 1 period and replaced the original debt panel by its fit. They do this to address for the endogeneity bias caused by reverse causation. Then, they estimate four kinds of panel regressions using dummy variables indicating both regimes of the (fitted) public debt increase/decrease:

(i) traditional Fixed Effects panel data model;
(ii) Fixed Effects model using instrumental variables to minimize potential endogeneity bias;
(iii) Fixed Effects Instrumental Variables model with lagged GDP growth to capture dynamics ;
(iv) Alternative Dynamic Instrumental Variables model using GDP gap and US growth as the additional variables instead of production function proxies given in previous models.

The growth equation is:

$$
\begin{aligned}
GROWTH_{it} &= \beta_1 GROWTH_{i,t-1} + \beta_2 POPgr_{i,t} + \beta_3 lGDP_{i,t-1} + \\
&+ \beta_4 DEBT_{i,t-1} D^M_{i,t-1} + \beta_5 DEBT_{i,t-1} D^P_{i,t-1} + \beta_6 DEBT^2_{i,t-1} D^P_{i,t-1} + \\
&+ \beta_7 D^P_{i,t-1} + \beta_8 GDS_{it} + \beta_9 GFCF_{i,t-1} + \beta_{10} AYTOA_{i,t-1} + \beta_{11} ADR_{i,t-1} + \\
&+ \beta_{12} OPEN_{i,t-1} + \beta_{13} INFL_{i,t-1} + \beta_{14} D^{08}_{i,t} + \beta_{15} D^{09}_{i,t} + u_{it}
\end{aligned} \tag{5.1}
$$

They try to address endogeneity by setting the explanatory variables lagged by one period relative to the explained GDP growth.

The dummy variables through which they distinguish the two regimes are defined below. $D^P_{(.)}$ signifies the increasing debt-to-GDP regime, while respectively $D^M_{(.)}$ signifies the decreasing debt-to-GDP regime.

D^P_{it} – a dummy variable; = 1 if $DEBT_{i,t} \geq DEBT_{i,t-1}$, = 0 otherwise,

D^M_{it} – a dummy variable; = 1 if $DEBT_{i,t} < DEBT_{i,t-1}$, = 0 otherwise,

D^{08}_{it} – a dummy variable; = 1 if $t = 2008$, = 0 otherwise,

D^{09}_{it} – a dummy variable; = 1 if $t = 2009$, = 0 otherwise.

The panel data model is estimated (Table 5.1).

In each kind of estimation method, the signs of the estimated coefficients of debt are as expected. As debt grows the debt-growth relationship is described by an inverted U-shaped curve, while as debt reduces the debt-growth relationship is described by a line that declines. So, the debt vs. economy growth relationship modelling is robust across all the modifications of the proposed panel regressions. In Figure 5.1, they present the graph of the evolutions of these two relationships.

Table 5.1

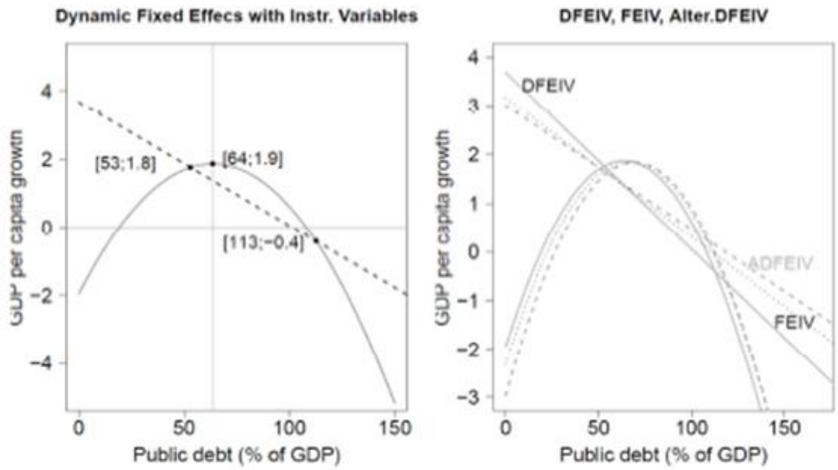
Parsimonial Models of Non-dynamic and Dynamic Panel Data Regressions

Dependent Variable: Real GDP per capita Growth (%)

Explanatory variable		**Fixed effects model (FE)**	**Fixed effects instrumental variables (FEIV)**	**Dynamic fixed effects instr. variables (DFEIV)**	**Alternative dynamic fixed effects instr. variables (Alter DFEIV)**
$GROWTH_{i,t-1}$	β_1	x	x	0.067443*	0.214234*
$POPgr_{i,t}$	β_2	not signif.	not signif.	0.63501*	0.697020**
$lGDP_{i,t-1}$	β_3	−8.1793***	−12.027***	−12.648***	−6.47463***
$DEBT_{i,t-1}D^{M}_{i,t-1}$	β_4	−0.012445**	−0.02857***	−0.036303***	−0.0254214***
$DEBT_{i,t-1}D^{P}_{i,t-1}$	β_5	0.14118***	0.12461***	0.11994***	0.1383345***
$DEBT^2_{i,t-1}D^{P}_{i,t-1}$	β_6	−0.0009706***	−0.000932***	−0.0009432***	−0.00099535***
$D^{P}_{i,t-1}$	β_7	−5.45682***	−5.4891***	−5.6319***	−5.976741***
$GDS_{i,t-1}$	β_8	0.250426***	0.35295***	0.34375***	0.4807214***
$GFCF_{i,t-1}$	β_9	not signif.	not signif.	−0.18505**	x
$AYTOA_{i,t-1}$	β_{10}	not signif.	0.47989**	0.5727***	x
$ADR_{i,t-1}$	β_{11}	−0.12437**	−0.12987**	−0.16707**	x
$OPEN_{i,t-1}$	β_{12}	not signif.	not signif.	not signif.	not signif.
$INFL_{i,t-1}$	β_{13}	−0.58144***	−0.61002***	−0.8069***	−0.462321***
$D^{08}_{i,t-1}$	β_{14}	−1.6369***	−1.7439***	−1.6223***	−1.7317444***
$D^{09}_{i,t-1}$	β_{15}	−4.9960***	−5.0301***	−4.4720***	−4.462321***
$USgrowth_{i,t-1}$	β_{16}	x	x	x	not signif.
$GAP_{i,t-1}$	β_{17}	x	x	x	−0.309951***
Adjusted R-squared		0,660	0.724	0.727	0.728
Pooling F test		F = 7.26***	F = 6.6269***	F = 8.9174***	F = 6.58***
Breusch-Pagan LM test (Chisq stat.)		Chisq = 199.04***	Chisq = 184.67***	Chisq = 155.20***	Chisq = 185.37***
Pessaran Cross correl. ation test (z stat.)		z = 8.9957***	z = 8.09***	z = 6.073**	z = 8.44***
Breusch-Godfrey Woodridge (Chisq st.)		Chisq = 46.96***	Chisq = 54.47***	Chisq = 41.06***	Chisq = 49.22***
Breusch Pagan heterosced. (BP stat.)		BP = 49.45***	BP = 50.11***	BP = 62.73***	BP = 47.10***

Notes: *** = 0.001, ** = 0.01, * = 0.05, = 0.1 denotes significance levels. Pooling F-test of the country specific dummies significance shows heterogeneity of the country data; Hausman test identified the Random effect model as providing the inconsistent estimations. Breusch-Pagan/LM (Breusch and Pagan, 1980) test and Pesaran cross-sectional dependence (Pesaran, 2004) test reveal significant cross-sectional dependence; the Breusch--Godfrey/Wooldridge test confirms the existence of serial correlation; the studentised Breusch-Pagan test reveals heteroscedasticity. The non-parametric method of Driscoll and Kraay (1998) was used for nonparametric covariance matrix estimator providing the heteroskedasticity and autocorrelation consistent standard errors robust to general forms of spatial and temporal dependence.

Figure 5.1
Illustration of Estimated Linear and Quadratic Functions for Public Debt vs. GDP Growth



Note: Dynamic Fixed Effects Instrumental Variables model (DFEIV) vs. Fixed Effects Instrumental Variables (FEIV) and Alternative Fixed Effects Instrumental Variables (Alter. DFEIV – model with alternative control variables); linear relation: in the case of previous public debt decrease; parabola: in the case of previous public debt increase.

Curve (parabola): $GDPgrowth = -1.9474 + 0.11994 PublicDebt - 0.0009432 (PublicDebt)^2$.

Line: $GDPgrowth = 4.2715 - 0.036303 PublicDebt$.

The good robustness of the results can also be demonstrated by the graphs as the locations and shapes of the curves are quite similar. The authors choose the DFEIV model to use for their conclusions as it is a predominant functional form in similar research works.

A linear relation is detected when we analyze the decreasing direction of debt-to-GDP. When we look the debt-to-GDP path as it increases, a non-linear relationship is being unveiled. The non-linear relationship is represented in the graph by an inverted U-shaped (parabola) form. Within the economic cycle, the debt-to-GDP vs. GDP growth data oscillate along the closed shape bordered by debt-to-GDP ratios given as the intersections of the line and parabola (i.e. 53% and 113% of the debt-to-GDP). The peak of the U-shaped curve is at 64% of the debt-to-GDP ratio indicating that if debt increases above this level it affects growth negatively. Increases in debt beyond the 113% level cause debt trap problems to a country. At that level, the parabola and the line start to diverge, as the parabola’s tail follows another direction towards large debts and negative economic growths. Even a consolidation of public finances (see the mutual positions of the line and parabola below the 113% level, Figure 5.1) is connected with negative economic growth and rather instability of the economy given by obvious line and parabola divergences. Although, if the country, having debt-to-GDP smaller than 108% (threshold given by the zero GDP growth), recognizes an abrupt decline in GDP growth even in the case of the expansionary fiscal policy, it is still possible to maintain the sustainable economy growth by performing austerity.

According to the authors the estimated line and parabola following the evolution of the relation between debt-to-GDP and GDP growth leads to some deductions. Firstly, as a country reduces its public debt, a downward line is formed by the data, i.e. debt is on a decreasing path and GDP growth increases. Secondly, as debt-to-GDP increases, the data reveal a threshold at 64% debt-to-GDP below which GDP increases. Beyond this, GDP growth starts to decline. Furthermore, below the 113% debt-to-GDP ratio, data freely oscillate along line and parabola. Yet, as debt rises to 113%, fiscal policy measures severely impair growth.

The analysis of Caner, Grennes and Koehler-Geib (2010) confirms the non-linear relationship by applying a Panel Threshold Methodology based on a yearly dataset and finds the threshold of the debt-to-GDP ratio where a positive relation becomes negative. The dataset draws from 101 developing and developed countries over 1980 to 2008. Their econometric studies point out that a country coverage matters substantially for the threshold effect. In contrast to the Reinhart-Rogoff 90% debt threshold, they find that the tipping point is at 77% for the full sample. The estimation for the developing economies is found lower at 64% debt-to-GDP where each additional percentage point in public debt ratio decreases annual real growth by 0.02 percentage points.

The dataset (1980-2008) is as follows:

Table 5.2

Countries covered

Economy type	*Countries*
Developing economy (75)	Algeria; Angola; Argentina; Bangladesh; Benin; Bolivia; Brazil; Bulgaria; Burkina Faso; Burundi; Cameroon; Chad; Chile; China; Colombia; Congo, Rep. of; Costa Rica; Côte d'Ivoire; Croatia; Dominican Republic; Ecuador; Egypt, Arab Rep. of; El Salvador; Estonia; Ethiopia; Ghana; Guatemala; Guinea; Haiti; Honduras; Hungary; India; Indonesia; Jamaica; Jordan; Kenya; Lao PDR; Latvia; Lebanon; Lithuania; Madagascar; Malaysia; Mali; Mexico; Morocco; Nicaragua; Niger; Nigeria; Pakistan; Panama; Papua New Guinea; Paraguay; Peru; the Philippines; Poland; Romania; Russian Federation; Rwanda; Senegal; Sierra Leone; Singapore; the Slovak Republic; Slovenia; South Africa; Sri Lanka; Tanzania; Thailand; Togo; Tunisia; Turkey; Uganda; Ukraine; Uruguay; Venezuela, R. B. de; Vietnam
Developed economy (26)	Australia; Austria; Belgium; Canada; the Czech Republic; Denmark; Finland; France; Germany; Greece; Iceland; Ireland; Italy; Japan; Korea, Rep. of; Portugal; the Netherlands; New Zealand; Norway; the Slovak Republic; Slovenia; Spain; Sweden; Switzerland; the United Kingdom; the United States

The methodology draws on the already mentioned threshold regression technique following Hansen (1996, 2000). The threshold regression model that has to be verified in order to estimate the threshold in the relationship between the long-run average public debt-to-GDP ratio and long-run average growth is the following:

$$Y_i = \beta_{0,1} 1_{\{X_i \le \lambda\}} + \beta_{0,2} 1_{\{X_i > \lambda\}} + \beta_{1,1} X_i 1_{\{X_i \le \lambda\}} + \beta_{1,2} X_i 1_{\{X_i > \lambda\}} + \beta_{2,1} W_i 1_{\{X_i \le \lambda\}} + \beta_{2,2} W_i 1_{\{X_i > \lambda\}} + u_i \quad (5.2)$$

Y stands for the long run average real growth rate and X the long run average public debt-to-GDP ratio. W represents the control variables: log of initial GDP per capita, trade openness and inflation. Public debt ratio is the general government gross debt to GDP.

The term 1 equals one if each one condition is satisfied, otherwise zero. The unknown threshold value λ and the coefficients $\beta_{0,1}$ through $\beta_{2,2}$ are estimated with the threshold LS method of Hansen (2000).

The existence of thresholds is tested through the null hypothesis that the slope coefficients and intercepts are the same in both regimes. It is tested by using the heteroscedasticity-consistent Lagrange multiplier test of Hansen (1996).

Caner et al. address endogeneity in their approach through this procedure: Estimations are repeated adding initial debt/GDP to control for omitted variables bias and reverse causality. The findings do not change qualitatively manifesting the same threshold values and also minor changes of the coefficients in both regimes.

The results for a set of 79 countries (initial GDP data were unavailable for 22 countries) suggest that the threshold level of the average long-run public debt-to-GDP ratio is 77.1 percent (Table 5.3).

Table 5.3
Threshold Regression, Two Regimes Based on Estimated Threshold Debt Level, Dependent Variable real average GDP growth

Variables	Regime 1: Debt>=77%		Regime 2: Debt <77%	
	Slope	**Std Error**	**Slope**	**Std Error**
Log Initial GDP/capita(1970)	0.00006*	0.00001	0.0002*	0.0001
Trade Openness	0.0454*	0.0078	-0.0007	0.0012
Inflation	0.0012	0.0007	-0.0244	0.0164
Debt/GDP	-0.0174*	0.0010	0.0653*	0.0128

Note: Note: R^2 for the first regime is 0.985, and for the second regime is 0.987. There are 12 countries in the first regime, and 67 in the second regime. The 95% confidence interval for the Debt/GDP in Regime 1 is [-0.0195, -0.0154], for the second regime [0.0402, 0.0905]. These are based on Likelihood ratio test in Hansen (2000). The 95% Confidence Interval for Threshold estimate is [0.770574, 0.770574]. * represents significance at 5% level by using standard normal critical values as in Hansen (2000).

Above this 77.1% level, one percentage point increase in the ratio of debt-to-GDP is associated with 0.0174 percentage points decrease in annual average real growth. Under this threshold, further increases in debt causes growth to increase. This result support the view that at low debt levels and as debt-to-GDP ratio rises (up to a point), credit constraints remain loose and the country has the potential to use the extra resources and devote them to fund more for investment.

Next, they perform the estimation for a group of 55 developing countries (reduced due to the lack of data on initial GDP) and find that the threshold differs between the developing group and the mixed set of 79. For the subsample of developing countries the debt threshold is 64% (Table 5.4). The absolute value of the impact of debt going over this threshold is somewhat larger than in the mixed set of countries (-0.020). The difference between the two groups implies that developing countries face growth rate problems at a lower debt to GDP levels.

Table 5.4
Threshcld Regression, Two Regimes Based on Estimated Threshold Debt Level, Developing Countries Sample, Dependent Variable real average GDP growth

Variables	**Regime 1 Debt>=64%**		**Regime 2 Debt<64%**	
	Slope	**Std Error**	**Slope**	**Std Error**
Log Initial GDP/capita(1970)	0.0249*	0.0015	0.0034	0.0024
Trade Openness	0.0002*	0.0001	-0.0015*	0.0007
Inflation	0.0008*	0.0004	-0.0086	0.0311
Debt/GDP	-0.0203*	0.0039	0.0739*	0.0093

Note: R^2 for the first regime is 0.98, and for the second regime is 0.98. There are 16 countries in the first regime, and 40 in the second regime. The 95% confidence interval for the Debt/GDP in Regime 1 is [-0.0312, -0.0088], for the second regime [0.0491, 0.0965]. These are based on Likelihood ratio test in Hansen (2000). The 95% Confidence Interval for Threshold estimate is [0.6335, 0.8524]. * represents significance at 5% level by using standard normal critical values as in Hansen (2000).

The authors then try to assess the cost on the GDP growth of persistent violations of debt threshold levels for each country. In this way, they wanted to see what the estimated coefficients imply with regard to quantitative impact of public debt on growth. In Table 5.5, they conclude that it is costly in terms of GDP growth if debt stays at elevated levels for an extended period of time. Nicaragua demonstrates the most severe impact as the average annual real growth rate could have been 4.7 percent higher had debt been at the 64 percent debt threshold for developing countries.

Table 5.5

Estimated Forgone Growth as a Result of Exceeding the Debt Threshold, by Country

Country	*How high growth could have been if the debt-to-GDP ratio had been at the threshold level (percent real average growth rate)*	*Annual percentage point loss in real GDP growth*	*Cumulated loss over 28 years (percentage point loss in real GDP growth)*
Angola	3.2	1.2	62.8
Belgium	2.7	0.6	18.4
Bolivia	2.4	0.1	1.6
Bulgaria	2.5	0.6	16.7
Burundi	2.6	0.8	24.3
Canada	3.1	0.4	11.6
Congo, Rep. of	5.0	1.0	32.7
Côte d'Ivoire	2.1	1.2	41.1
Croatia	1.5	0.2	6.0
Ecuador	3.0	0.1	1.5
Greece	2.2	0.0	0.5
Guinea	4.0	0.4	13.0
Hungary	1.8	0.1	3.2
Indonesia	6.8	1.3	45.2
Italy	2.1	0.4	10.9
Jamaica	2.0	0.2	5.1
Japan	2.9	0.6	18.6
Jordan	5.1	0.1	2.3
Lao PDR	6.8	0.8	33.0
Latvia	2.5	0.1	3.1
Lebanon	5.2	0.4	11.7
Madagascar	2.4	0.5	15.3
Mali	3.3	0.2	5.2
Nicaragua	6.6	4.7	264.6
Nigeria	3.4	0.2	4.7
Philippines	3.2	0.0	1.2
Sierra Leone	3.1	1.0	33.0
Singapore	7.3	0.4	13.0
Tanzania	5.0	0.2	6.3

Note: For developed economies a threshold of 77 percent public debt-to-GDP ratio is applied and for developing countries of 64 percent.

Caner et al. afterwards estimated simple pooled least squares regressions (with heteroscedasticity-corrected errors) for subsamples above and below the estimated threshold of Reinhart-Rogoff. They used the same 20 industrial countries, but general instead of central government debt and a shorter period. They run first a regression for the group of countries with debt levels of at least 90 percent (Table 5.6). The second group includes debt ratios below 90 percent. They found a regime switch as Reinhart-Rogoff suggested. However, when they repeat the two regressions for the 60% debt threshold, they also found a regime switch (Table 5.7). The difference between slope coefficients for the last case is modest in comparison to the 90 percent threshold. But a regime switch found also when considering the 60% debt threshold can be indicative that the Reinhart-Rogoff methodology does not deliver clear threshold levels. These results should be taken with some skepticism as there is an indisputable need for controlling for other determinants in the regressions.

Table 5.6
GDP Growth and Debt Ratio: 90% Threshold Level

Debt	Slope	Std Error	t test	p-value
>=90%	-0.0137	0.0065	-2.10	0.038
<90%	0.0012	0.0055	0.23	0.819

Table 5.7
GDP Growth and Debt Ratio: 60% Threshold Level

Debt	Slope	Std Error	t test	p-value
>=60%	-0.0091	0.0037	-2.43	0.016
<60%	-0.0057	0.0089	-0.03	0.519

Altogether, this paper provides an evidence for rejecting the 90% debt-to-GDP ratio above which the negative impact on growth appears in favor of the much lower 77 percent. Also, they found that the threshold value decreases to 64% as high-income countries are exempted from sample. The authors finally claim that as the debt ratio surpasses the threshold for a couple of years its long-term growth need not suffer. But if debt explosions keep them above the thresholds for an extended period, economic activity is severely hurt.

Elmeskov and Sutherland (2012) studied the impact of public debt on growth across the 12 countries by estimating a simple growth regression and over the time period 1965-2010. They used the OLS methodology and the Hansen (1999) bootstrap to identify the possible debt thresholds. The countries were: Austria, Belgium, Canada, France, United Kingdom, Italy, Japan, Korea, Netherlands, Norway, Sweden, and United States.

The results (Table 5.8) suggest that there may exist two debt thresholds above which the impact on GDP growth becomes more important. The lower debt threshold is estimated at 45 percent and it is statistical significant at 10 percent level and the higher at 66 percent of GDP and it is statistical significant at 5 percent level (estimated value: -0.1). The authors suggest that the threshold were quite robust as they exclude sequentially each country, estimate again the relationship and found the same results.

However, Elmeskov and Sutherland claim that their findings might be subjected to mistakes due to a number of estimation biases stemming from the dynamic nature of the model.

Table 5.8
Growth Regressions

Dependent variable: per capital real GDP growth

	Coefficient	Coefficient
log of real per capital GDP	-0.180 ***	-0.173 ***
Years of education	0.015 ***	0.014 ***
Population growth	-0.411 **	-0.356
Inflation	-0.051 **	-0.063 *
Openness ratio	0.015	0.014
Saving rate	0.002	0.002
Government gross financial liabilities	-0.040 **	
Gross financial liabilities < 45% of GDP		-0.040
Gross financial liabilities between lower and upper thresholds		-0.050 *
Gross financial liabilities > 66% of GDP		-0.100 **
Adjusted R-squard	0.490	0.523
Observations	96	96
P value for three regime model		0.01

Notes: *, ** and *** denote statistical significance at the 10%, 5% and 1% levels.

6. Doubting the non-linear relationship and threshold existence

Despite the robustness of the results of the previous empirical researches on the actual debt threshold, the matter of controversy might not be where this level lies but rather if any such a threshold exists in the first place. A negative non-linear relation and the threshold effects between public debt and growth should not be taken for granted. Indeed, nonlinear effects might as well not exist or if they exist they are possibly complicated and so modelling them might be more problematic than what has been considered so far. It is equally possible that nonlinear effects might be unstable and change over time, across countries, economic conditions or under any other unknown condition. The papers analyzed in this subsection come to different result than the papers so far. Here, the researchers cannot find robust evidence of the non-linear link between public debt and growth and argue that there is great uncertainty and ambiguity around this issue.

Égert (2013) uses the Reinhart and Rogoff (2010) dataset relying on descriptive statistics as well as on a formal econometric testing to see whether public debt has a negative non-linear effect on growth when debt exceeds 90% of GDP. By using descriptive statistics he found a weaker real GDP growth when the central government debt-to-GDP ratio passes the 30% threshold but no further considerable slowdown to growth for the debt-to-GDP ratios over 60% and 90% for the periods 1790-2009 and 1946-2009. He also studied the existence of nonlinearities and threshold effects for a group of 20 developed economies over the period 1790-2009 and 1946-2009 by using an endogenous threshold model. The estimated thresholds found generally much lower than 90 percent. According to his analysis, the negative association may set in at 20% of debt-to-GDP. More or larger thresholds seem to exist but their significance is largely unstable. Regarding the general government debt the threshold above which the negative link appears is at about 50%. Overall, he showed that the negative non-linear relationship is not very robust as it is extremely sensitive to small changes in data frequency, and changes in the assumptions on the minimum number of observations included in each regime. Égert concluded that the evidence for the presence of non-linearities and thresholds effects is not as strong as commonly thought.

At first, Égert, experimented with the same data and methodology of Reinhart and Rogoff (2010) and reproduced their results in order to see how debt and growth are correlated. There was one difference between the two data series: Égert's data keeps out Ireland and includes Switzerland. We remind that Reinhart and Rogoff used descriptive statistics and argued that high levels of debt, beyond 90%, have a detrimental effect on real GDP growth. Specifically, the mean value of GDP growth diminishes from more than 3% to -0.1% as public debt-to-GDP ratio increases from below 30% to over 90% for twenty advanced economies. However, as claimed by Herndon et al. (2013), the average annual growth rates were miscalculated (Table 6.1). Égert's (2013) calculations matched those of Herndon et al.: the economic growth is indeed lower if debt goes over the 90% threshold of GDP, though no dramatic drop in real GDP growth occurs at these levels.

Table 6.1
Real GDP growth and central government debt as a % of GDP, 1946-2009

		Level of central government debt (as a % of GDP)			
		x<30%	30%< x<60%	60%<x<90%	x>90%
Reinhart-Rogoff (2010)	**Average annual growth rates**	3.9	2.9	3.5	-0.1
Herndon et al (2013)	**Average annual growth rates**	4.0	3.0	3.0	1.9
Egert (2012)	**Average annual growth rates**	3.4	2.4	1.9	1.9
This paper	**Average annual growth rates**	4.3	3.2	3.2	2.2
	Average of 10-year average growth rates	4.3	2.6	3.1	3.4
		Lagged level of central government debt			
		x<30%	30%< x<60%	60%<x<90%	x>90%
	Average of annual growth rates	4.1	3.1	3.1	2.9
	Average of 10-year average growth rates	3.6	2.6	2.8	2.7

Égert also computed the 10-year non-overlapping averages for real GDP growth, as he claims that yearly data may be just too noisy to unveil the thruth. The averages of these multiyear averages, indicate that GDP growth does not drop at high levels of public debt. Lower growth appears when public debt is above 30% of GDP but then growth increases as debt increases. This also holds when the lagged level of central government debt is taken into account.

The Reinhart and Rogoff dataset allows computing the annual growth averages for larger periods for some countries. They are calculated for the years 1790-2009 (Table 6.2).

Table 6.2
Real GDP growth and central government debt as a % of GDP, 1790-2009

	Level of central government debt (as a % of GDP)			
	x<30%	30%< x<60%	60%<x<90%	x>90%
	Level of central government debt (as a % of GDP)			
1790-2010	4.0	3.1	2.5	2.2
1790-1939	3.7	2.9	1.9	2.3
	Level of lagged central government debt (as a % of GDP)			
1790-2010	3.7	3.0	2.6	2.7
1790-1939	3.2	2.7	2.1	2.6

Source: Égert's calculations based on the Reinhart-Rogoff dataset

Here, there seems to exist the same small negative correlation between debt and annual average growth for the years 1790-2009 as GDP growth diminishes continuously from 4% to 2.2% when government debt increases from below 30% to beyond 90% of GDP. Considering lagged government debt growth reduces from about 3.5% to below 3% with debt exceeding 60% of GDP. There is no large reduction beyond 90% of GDP. Basically, regarding the period 1790-1939 growth increases beyond the 90% threshold.

The calculations were made once again taken into account this time the general government debt as a measure for public debt and time period 1960-2009 (Table 6.3). Égert calculates the difference between the two public debt series and shows that it can be sometimes very large.

Table 6.3
Real GDP growth and general (and central) government debt as a % of GDP, 1960-2009

	x<30%	30%< x<60%	60%<x<90%	x>90%
	Average annual real GDP growth rate			
General government debt	3.3	3.0	2.8	1.9
Lagged general government debt	2.8	2.9	2.9	2.0
Central government debt	3.3	2.8	2.3	2.0
Lagged central government debt	3.0	2.8	2.6	2.2
	Average of 10-year average real GDP growth rates			
General government debt	3.6	3.3	2.8	2.1
Lagged general government debt	2.8	2.8	2.2	1.9
Central government debt	3.5	3.1	2.1	2.1
Lagged central government debt	2.7	2.7	1.7	2.0

Average annual real GDP growth rates diminish gently from 3.3% to 2.8% with general government debt rising up to 90%, but then it falls to 1.9% as debt exceeds 90% of GDP. When growth is computed relative to the (one year) lagged general government debt, growth exhibits no correlation to the increasing of general government debt at lower debt levels (specifically up to 90%). At higher levels there is some small decline. The same repeats when employing 10-year averages of GDP growth. A smooth decline in growth occurs as the general government debt rises from one level to another for the lower debt levels and a slightly larger between the two last debt levels. When he uses lagged general debt, a one percentage point decrease in GDP growth is spread when moving from 30%-60% to above 90% of GDP.

Next, Égert (2013) attempts to discover the thresholds endogenously employing the Hansen (1999) testing procedure. He employs a simple bivariate threshold model, where the impact of debt on growth is based on the level of debt. At first, a linear model is compared to a two-regime model. Through the testing procedure, one of the two models is rejected. If the linear model is rejected, then afterwards the two-regime model against the three-regime model is tested.

The specifications are following:

$$\Delta y_t = \begin{cases} \alpha_1 + \beta_1 \cdot DEBT_t + \varepsilon_t & if \quad DEBT < T \\ \alpha_2 + \beta_2 \cdot DEBT_t + \varepsilon_t & if \quad DEBT \geq T \end{cases} \quad (6.1a)$$

$$\Delta y_t = \begin{cases} \alpha_1 + \beta_1 DEBT_t + \varepsilon_t & if \quad DEBT < T_1 \\ \alpha_2 + \beta_2 DEBT_t + \varepsilon_t & if \quad T_2 > DEBT \geq T_1 \\ \alpha_3 + \beta_3 DEBT_t + \varepsilon_t & if \quad DEBT \geq T_2 \end{cases} \quad (6.1b)$$

T stands for the debt threshold value in the two-regime specification, while T1, T2 are the lower and upper debt thresholds respectively in the three-regime specification. In order to detect the threshold variable they search the value that makes the SSR of the estimated model the minimum possible. This is accomplished through a grid search procedure with steps of 1% of the distribution starting at 20% until the 80% so as to assure that a large enough number of observations are included into each regime. But they also experiment with alternative parametrizations (30%, 10%, 5% and 1%).

The three-regime model is estimated based on two threshold values of the threshold variable that minimize the sum of squared residuals across the estimated models. The threshold from the two-regime model is held fixed and a grid search is used to identify the second threshold. We impose the restriction that the two thresholds should be separated at least by 10% of our sample observations. When the second threshold is identified, a backward grid search is performed to identify the first threshold as suggested by Hansen (1999).

Then he performs the sequential testing of the models. Hansen (1999) tests the null hypothesis of $\beta_1=\beta_2$ from equations (6.1a) by using a likelihood ratio test. Given that the likelihood ratio test statistic does not follow a standard asymptotic distribution as the threshold value is not identified under the null hypothesis, the distribution of the test statistic is obtained through bootstrapping with random draws with replacement. The bootstrap is carried out through 500 replications. If the likelihood ratio test statistic rejects the null hypothesis of the linear model against the two-regime model (on the basis of the bootstrapped critical values), the three-regime rather than the two-regime model is then analysed. This bootstrap procedure is applied to the two-regime and three-regime models.

In line with the literature, Égert uses lagged public debt-to-GDP in order to ensure that lagged debt influence (subsequent) growth rather than the other way round; specifically employing the central government debt at first. The estimations were made for the time period 1790-2009 and are presented in Table 6.4. These unveil a negative nonlinear relationship. However, it is not clear if this nonlinear association concerns two or three different regimes and the exact debt thresholds' values.

Table 6.4
Reinhart-Rogoff dataset, 1790-2009

		Minimum % of observations required in one regime				
		30%	**20%**	**10%**	**5%**	**1%**
		Nonlinear variable = lagged central government debt/GDP				
		Threshold variable = lagged central government debt/GDP				
Test of nonlinearity		Bootstrapped p-value				
H0: linear *vs.* H1: 2-regimes		**0.084**	**0.000**	**0.000**	**0.000**	**0.000**
H0: 2-regimes *vs.* H1: 3-regimes		0.184	0.248	**0.060**	**0.082**	**0.054**
Coefficients	Low debt	0.015	-0.022**	0.044	0.044	0.699
	Middle debt			-0.018**	-0.018**	-0.020**
	High debt	-0.006	-0.009**	-0.006*	-0.006*	-0.007**
Debt thresholds (%)	Threshold 1	27.72	71.99	14.27	14.27	4.40
	Threshold 2			94.27	94.27	94.27
No. of OBS		2177	2177	2177	2177	2177
		Nonlinear variable = lagged rate of growth in central government debt/GDP				
		Threshold variable = lagged central government debt/GDP				
Test of nonlinearity		Bootstrapped p-value				
H0: linear *vs.* H1: 2-regimes		**0.002**	**0.004**	**0.000**	**0.002**	**0.002**
H0: 2-regimes *vs.* H1: 3-regimes		**0.004**	**0.002**	**0.000**	**0.002**	**0.000**
Coefficients	Low debt	0.003	0.004	0.004	0.004	0.004
	Middle debt	-0.041**	-0.038**	-0.038**	-0.038**	-0.010**
	High debt	-0.007**	-0.007**	-0.007**	-0.007**	-0.242**
Debt thresholds (%)	Threshold 1	23.64	19.62	19.62	19.62	13.48
	Threshold 2	52.98	67.86	67.86	67.86	155.00
No. of OBS		2120	2120	2120	2120	2120

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively. The estimations are carried out with country fixed effects.

This is due to the minimum number of observations that are included in the outer regimes: a two-regime model is identified when 30% of the observations are allowed in one regime and the debt threshold lies at 30 percent of GDP. If the minimum number of observations falls to 1%, the model reveals the existence of three regimes with debt thresholds of 4% and 90%. Clearly, the lower the minimum number of observations in specific regimes, the higher the probability that a very low or very high threshold is picked. At the same time, the results might not be right because they will be more sensitive to outliers.

The coefficients are negative in the high-debt regimes (from -0.006 to -0.009) but they are lower than the negative coefficients estimated for the low and middle debt regimes (-0.018 to -0.022). This means that the negative relationship between debt and growth decreases as debt rises. Égert re-estimates the model using as independent variable the (lagged) rate of growth of the central government debt. The threshold variable remains the lagged public debt-to-GDP ratio. The new findings appear to be steadier. According to the bootstrapped p-value the linear specification and the two-regime model are always rejected in favor of the three-regime specification. The lowest threshold tends to lie at around 20% of GDP and the highest at approximately 60% of GDP, implying that in principal the government debt value over which GDP is related to lower growth is around 20%. Though, the estimations of the upper debt regimes exhibit in all -but one-case still smaller values than the ones corresponding to the middle debt regime; this clearly contradicts the findings of Reinhart and Rogoff. An exception appears when the minimum number of observations is set at 1%. The upper threshold rises to 150% of GDP and the related estimation is -0.242.

The estimations were reproduced for the period 1946-2009. The results, summarized in Table 6.5, are generally in agreement to the previous findings. In general, the negative relationship appears at the threshold of about 20% of GDP. There exist another one at about 60%. However, in the latter case the coefficients are higher below this threshold than above it. The estimations are again carried out through the growth rate of (lagged) government debt as a nonlinear variable. A debt threshold is identified at 20% of GDP. After this, a 1 percent increase in central government debt produces a 0.04 percentage point lower growth. The results also reveal one extra debt threshold, lying between 55%-130%, where the negative association is even larger.

The central government debt was then replaced by general government debt as it is a more relevant measure for policymakers. In this case the two-regime specification prevails over the linear model. Moreover, in a few instances the three-regime specification seems to be valid (Table 6.6).

Table 6.5
Reinhart-Rogoff dataset, 1946-2009, annual data

		Minimum % of observations required in one regime				
		30%	20%	10%	5%	1%
		Nonlinear variable = lagged central government debt/GDP				
		Threshold variable = lagged central government debt/GDP				
Test of nonlinearity		Bootstrapped p-value				
H0: linear *vs.* H1: 2-regimes		**0.016**	**0.000**	**0.000**	**0.002**	**0.000**
H0: 2-regimes *vs.* H1: 3-regimes		0.140	**0.054**	**0.006**	**0.000**	**0.000**
Coefficients	Low debt	0.025*	0.028	0.050	0.238**	0.238**
	Middle debt		-0.022**	-0.023**	0.047**	0.047**
	High debt	-0.011**	-0.012**	-0.013**	-0.007*	-0.007*
Debt thresholds (%)	Threshold 1	26.73	19.33	14.43	10.02	10.02
	Threshold 2		64.60	64.60	22.68	22.68
No. of OBS		1189	1189	1189	1189	1189
		Nonlinear variable = lagged rate of growth in central government debt/GDP				
		Threshold variable = lagged central government debt/GDP				
Test of nonlinearity		Bootstrapped p-value				
H0: linear *vs.* H1: 2-regimes		**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
H0: 2-regimes *vs.* H1: 3-regimes		0.220	0.194	**0.024**	0.108	**0.070**
Coefficients	Low debt	0.016	0.019	0.030**	0.019	0.018
	Middle debt			-0.026**		-0.039**
	High debt	-0.040**	-0.040**	-0.063**	-0.040**	-0.187**
Debt thresholds (%)	Threshold 1	24.54	21.14	13.25	21.14	21.14
	Threshold 2			55.11		126.53
No. of OBS		1164	1164	1164	1164	1164

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively. The estimations are carried out with country fixed effects.

Table 6.6
General government debt, 1960-2009

		Minimum % of observations required in one regime				
		30%	20%	10%	5%	1%
		Nonlinear variable = lagged general government debt/GDP				
		Threshold variable = lagged general government debt/GDP				
Test of nonlinearity		Bootstrapped p-value				
H0: linear *vs.* H1: 2-regimes		**0.026**	**0.016**	**0.000**	**0.000**	**0.002**
H0: 2-regimes *vs.* H1: 3-regimes		0.148	0.102	**0.092**	**0.008**	**0.010**
Coefficients	Low debt	-0.037**	-0.043**	-0.029*	0.077**	0.077**
	Middle debt			-0.009	0.006	0.006
	High debt	-0.022**	-0.022**	-0.021**	-0.012**	-0.012**
Debt thresholds (%)	Threshold 1	49.75	42.61	34.64	20.37	20.37
	Threshold 2			88.98	88.98	88.98
No. of OBS		687	687	687	687	687
		Nonlinear variable = lagged rate of growth in general government debt/GDP				
		Threshold variable = lagged geneal government debt/GDP				
Test of nonlinearity		Bootstrapped p-value				
H0: linear *vs.* H1: 2-regimes		**0.002**	**0.000**	**0.000**	**0.000**	**0.000**
H0: 2-regimes *vs.* H1: 3-regimes		0.442	0.292	0.136	**0.004**	**0.000**
Coefficients	Low debt	0.016	0.016	0.016	0.106**	0.123**
	Middle debt				-0.011	-0.012
	High debt	-0.063**	-0.063**	-0.063**	-0.075**	-0.075**
Debt thresholds (%)	Threshold 1	44.59	44.59	44.59	17.63	16.79
	Threshold 2				49.22	49.22
No. of OBS		666	666	666	666	666

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively. The estimations are carried out with country fixed effects.

In the upper case of Table 6.6 it seems that the thresholds are very susceptible to the parametrization of the threshold models and their values vary from 50% - 90% of GDP. A significant negative relationship appears at 45-50% of GDP when they employ the lagged rate of growth of debt as the nonlinear variable. Estimations on central government debt validate these findings although they are more uncertain (Table 6.7). The negative debt-growth correlation kicks in roughly in between 30% to 70%. Going beyond this, a one percent change in central government debt slows down growth by 0.04 – 0.12 percentage points.

So far, he has taken for granted that the debt-growth relation was homogenous across countries, which means that the same slope coefficients and debt thresholds apply for the 20 countries of the Reinhart-Rogoff dataset. But, this seems very restricting. It seems more possible that debt affects economic growth differently in each country. Thus, he also estimates country specific threshold models on the Reinhart-Rogoff dataset to assess possible country-specific nonlinearities.

Table 6.7

Central government debt, 1960-2009, annual data

		Minimum % of observations required in one regime				
		30%	20%	10%	5%	1%
Nonlinear variable = central government debt/GDP; Threshold variable = central government debt/GDP						
Test of nonlinearity		Bootstrapped p-value				
H0: linear vs. H1: 2-regimes		0.002	0.000	0.002	0.000	0.000
H0: 2-regimes vs. H1: 3-regimes		0.382	0.358	0.000	0.000	0.000
Coefficients	Low debt	-0.061**	-0.061**	0.124**	0.124**	0.124**
	Middle debt			0.002	0.002	0.002
	High debt	-0.033**	-0.033**	-0.022**	-0.022**	-0.022**
Debt thresholds (%)	Threshold 1	37.98	37.98	16.33	16.33	16.33
	Threshold 2			79.25	79.25	79.25
No. of OBS		708	708	708	708	708
Nonlinear variable = lagged central government debt/GDP; Threshold variable = lagged central government debt/GDP						
Test of nonlinearity		Bootstrapped p-value				
H0: linear vs. H1: 2-regimes		0.000	0.000	0.000	0.000	0.000
H0: 2-regimes vs. H1: 3-regimes		0.080	0.086	0.000	0.006	0.010
Coefficients	Low debt	-0.075**	-0.075**	0.113**	0.105**	0.105**
	Middle debt	-0.038**	-0.038**	0.011	0.008	0.008
	High debt	-0.027**	-0.027**	-0.011**	-0.014**	-0.014**
Debt thresholds (%)	Threshold 1	46.45	34.23	16.33	16.33	16.33
	Threshold 2	34.23	46.45	73.31	84.28	84.28
No. of OBS		687	687	687	687	687
Nonlinear variable = rate of growth in central government debt/GDP; Threshold variable = central government debt/GDP						
Test of nonlinearity		Bootstrapped p-value				
H0: linear vs. H1: 2-regimes		0.000	0.000	0.000	0.000	0.000
H0: 2-regimes vs. H1: 3-regimes		0.114	0.044	0.002	0.002	0.000
Coefficients	Low debt	-0.055**	-0.041**	-0.022	-0.022	-0.022
	Middle debt		-0.068**	-0.074**	-0.074**	-0.074**
	High debt	-0.148**	-0.188**	-0.276**	-0.276**	-0.276**
Debt thresholds (%)	Threshold 1	54.43	23.64	16.33	16.33	16.33
	Threshold 2		58.86	79.25	79.25	79.25
No. of OBS		687	687	687	687	687
Nonlinear variable = lagged rate of growth in central government debt/GDP; Threshold variable = lagged central government debt/GDP						
Test of nonlinearity		Bootstrapped p-value				
H0: linear vs. H1: 2-regimes		0.030	0.032	0.002	0.002	0.004
H0: 2-regimes vs. H1: 3-regimes		0.056	0.032	0.034	0.026	0.012
Coefficients	Low debt	-0.006	-0.006	0.038**	0.043**	0.043**
	Middle debt	0.039**	0.039**	-0.014	-0.014	-0.014
	High debt	-0.037**	-0.037**	-0.124**	-0.124**	-0.124**
Debt thresholds (%)	Threshold 1	34.84	34.84	14.08	13.23	13.23
	Threshold 2	49.56	49.56	73.31	73.31	73.31
No. of OBS		666	666	666	666	666

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively. The estimations are carried out with country fixed effects.

The results (Table 6.8) highlight a large amount of cross-country heterogeneity. First of all, a robust negative nonlinear relationship holds only for: Belgium, Finland, Germany and the United States. The negative relation appears at extremely low levels, roughly in less than 30% of GDP. Furthermore, not surprisingly, the negative values differentiate a lot across countries a lot: a small value in comparison to the other countries is detected in the case of Belgium while a larger for Germany and the US. Second, for the group of Austria, Canada, and Ireland, there is a large degree of uncertainty around the level of public debt beyond which the negative relationship sets in. This level lies between 30% and 70%, as it is sensitive to the minimum number of observations selected in one regime. Third, no nonlinear relationship is found in Australia and Spain. In these two countries, not even a negative linear relationship is found. Fourth, in Denmark, Italy and Japan, even though the presence of nonlinearity can be detected, this relation is positive in the high debt regime. Finally, in the rest of the countries, whether public debt has a negative or positive link with real GDP growth above a certain level of the central government debt-to-GDP ratio depends on the minimum number of observations required for individual regimes. These results show that the non-linearity between debt and growth is weak. The author claims that these instabilities may be due to nonlinear effects changing over time within countries and economic conditions.

Égert concluded that overall the negative non-linear relationship is extremely unstable. This means that the stylized fact that public debt beyond 90% considerably hurts economic growth is a statistical fallacy. His results show that the negative impact may occur at considerable low debt levels. If debt causes lower growth at low public debt levels, then this could be an argument for reexamination of some important fiscal policies such as the rather arbitrary Maastricht debt level of 60% of GDP.

However, in this paper exists one important problem that should be noted and taken into consideration: the simple correlations above may suffer from the omitted variable bias given that there are variables correlated with both debt and growth which are not included in the regressions.

Table 6.8

Country-specific results, 1790-2009

Nonlinear variable = lagged growth rate of central government debt/GDP

Threshold variable = lagged central government debt/GDP

	Test of nonlinearity (p-value)		Coefficients			Debt thresholds (%)		No obs
	lin vs 2reg	2reg vs. 3reg	low	middle	high	Low	High	
AUS	0.28	0.42	0.068*					140
AUT	**0.00**	**0.02**	0.027	0.093	-0.236**	18.94	58.15	104
BEL	**0.02**	0.46	0.061		-0.021*	29.36		153
CAN	**0.00**	**0.00**	0.096	-0.240**	-0.605**	45.36	55.42	74
DEU	**0.00**	**0.00**	0.074**	-0.002	-0.159**	13.43	24.09	57
DNK	**0.00**	1.00	-0.037		0.056**	40.53		83
ESP	0.55	0.29	0.001					127
FIN	**0.00**	**0.00**	0.020	-0.013	-0.113**	12.02	16.96	80
FRA	**0.00**	**0.00**	-0.060**	-0.006**	0.086**	35.33	70.44	92
GBR	0.33	0.11	0.036					161
GRC	**0.00**	**0.00**	0.059**	-0.270**	-0.003	24.57	110.74	105
IRL	**0.00**	1.00	-0.208**		-0.133**	27.84		62
ITA	**0.00**	0.13	-0.063		0.068**	28.58		107
JPN	**0.00**	**0.00**	0.116*	0.062	-0.053	19.41	54.08	96
NLD	**0.01**	**0.00**	-0.072**	0.075	-0.187**	47.87	70.69	100
NOR	**0.00**	**0.00**	-0.023	-0.022	0.087**	21.57	26.99	112
NZL	**0.00**	**0.00**	0.108	-0.064	-0.241**	37.11	53.45	67
PRT	**0.00**	**0.00**	-0.018	-0.085	0.082	54.27	70.12	87
SWE	**0.00**	1.00	0.099**		0.002	18.88		112
USA	**0.00**	**0.03**	0.000	0.035	-0.159**	17.25	33.73	201
Minimum % of observations required in one regime: 10%								
AUS	0.146	0.052	0.068*					140
AUT	**0.000**	0.744	0.027		-0.198**	18.94		104
BEL	**0.052**	0.278	0.089		-0.021*	18.33		153
CAN	**0.000**	**0.000**	0.096	-0.135	-0.476**	45.36	77.59	74
DEU	**0.000**	**0.000**	0.073**	-0.001	-0.164**	13.43	27.20	57
DNK	**0.000**	1.000	-0.037		0.056**	40.53		83
ESP	0.326	0.004	0.001					127
FIN	**0.000**	**0.000**	0.020	-0.013	-0.113**	12.02	16.96	80
FRA	**0.000**	**0.000**	-0.043**	-0.006**	-0.241*	70.44	105.10	92
GBR	**0.046**	0.644	0.153**		0.017	38.15		161
GRC	**0.000**	**0.000**	0.059**	-0.270**	-0.003	24.57	110.74	105
IRL	**0.000**	**0.000**	-0.148**	-0.060	-0.369**	65.50	77.17	62
ITA	**0.000**	0.512	-0.136		0.075**	27.57		107
JPN	**0.000**	**0.000**	0.005	-0.293**	0.096**	54.08	70.20	96
NLD	0.904	0.000	-0.113**					100
NOR	**0.006**	0.498	0.057		-0.010	22.34		112
NZL	**0.000**	**0.000**	-0.090	-0.358	0.061	53.45	94.51	67
PRT	**0.000**	**0.000**	-0.018	-0.140	0.088	54.27	71.69	87
SWE	**0.004**	**0.000**	0.171**	-0.067	0.031	15.62	57.07	112
USA	**0.002**	**0.042**	0.000	0.035	-0.159**	17.25	33.73	201

Note: * and ** denote statistical significance at the 10% and 5% levels, respectively. The estimations are carried out with country fixed effects.

Pescatori, Sandri and Simon (2014) used a novel empirical methodology and a sizeable dataset on advanced economies and concluded that there is no proof for any clear debt threshold above which medium-term growth prospects are dramatically hurt by a marginal increase in debt-to-GDP ratio.

The list of countries is given in Table 6.9. Public debt ratio is defined as gross government debt to GDP ratio.

Table 6.9
Data Coverage

	Real per capita GDP		Debt/GDP	
	Start	End	Start	End
Australia	1821	2011	1901	2010
Austria	1871	2011	1880	2010
Belgium	1847	2011	1880	2010
Canada	1871	2011	1870	2010
China,P.R.:Hong Kong	1951	2011	2001	2010
Cyprus			1970	2010
Czech Republic	1991	2011	1920	2010
Denmark	1821	2011	1880	2010
Estonia	1991	2011	1995	2010
Finland	1861	2011	1914	2010
France	1821	2011	1880	2010
Germany	1851	2011	1880	2010
Greece	1914	2011	1884	2010
Iceland			1951	2010
Ireland	1922	2011	1929	2010
Israel	1951	2011	1972	2011
Italy	1862	2011	1861	2010
Japan	1871	2011	1875	2012
Korea, Republic of	1912	2011	1958	2010
Luxembourg			1974	2010
Malta			1965	2010
Netherlands	1821	2011	1814	2010
New Zealand	1871	2011	1860	2010
Norway	1831	2011	1880	2010
Portugal	1866	2011	1851	2010
Singapore	1851	2011	1963	2010
Slovak Republic	1991	2011	1992	2010
Slovenia	1991	2011	1993	2010
Spain	1851	2011	1880	2010
Sweden	1821	2011	1800	2010
Switzerland	1851	2011	1899	2010
Taiwan Prov.of China	1902	2011	1997	2010
United Kingdom	1831	2011	1830	2010
United States	1871	2011	1791	2010

The authors look at the relationship between today's stock of debt over GDP, b_t, and GDP growth in the next h-years $g_{it}(h)=y_{t+h}/y_t$. They implement both short- and long-term episodes of GDP growth. By implementing long-term episodes they try to weaken the reverse causality effects induced to debt in the short run by economic booms or recessions.

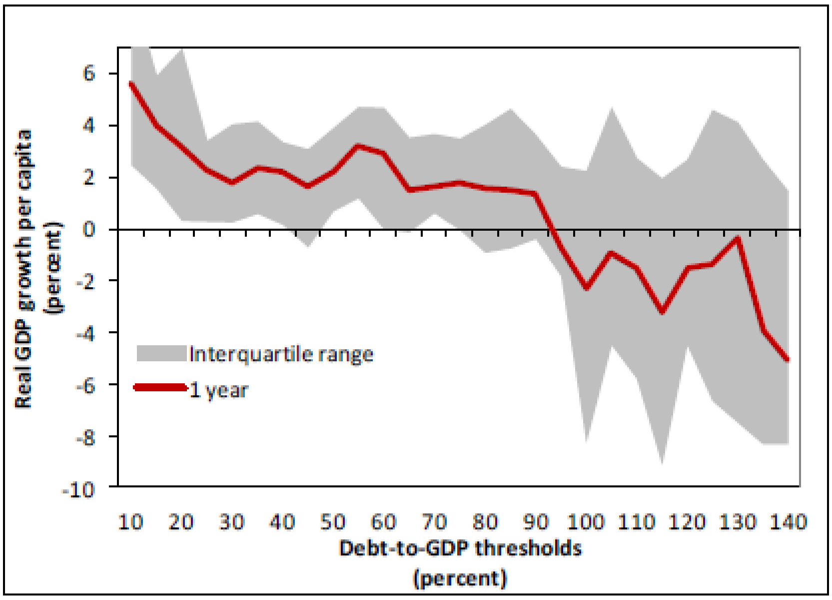
They check the episodes where debt increased over a threshold τ. The starting date of a raising debt episode is considered the first year in which the debt-to-GDP surpasses x percent, conditional on the ratio being below x percent in the last years. Then they look at the real GDP growth per capita over the following h years, where h belongs to [1, 5, 10, 15]. They allow countries to have multiple, but not overlapping, episodes. The actual conditions that have to been followed are described below:

$$b_{it} \geq \tau, b_{it-1} < \tau \text{ and } \nexists\ j \in [1,\ldots,h] \text{ s.t. } b_{it-j} \geq \tau, b_{it-j-1} < \tau. \qquad (6.2)$$

Some important features of their methodology are worth noted. First, their approach considers a broad range of debt thresholds. Second, they study the episodes of the way each country grows for a particular period of time no matter the final level that debt reaches and not only the time debt stays over a given level. By doing this, they avoid a particular truncation problem that appears if they consider when they define the end of an episode according to the level of debt. Concentrating only on the cases when debt exceeds a particular threshold is problematic. So, they concentrated on the time periods when after debt had risen beyond a particular threshold, then countries managed to diminish it. Of course, also on the periods when debt is beyond a particular threshold, which is the case usually included in the studies. Third, each country is represented only by very few episodes. This is a consequence of eliminating overlapping episodes and allowing that each episode starts when debt passes a particular threshold from below. These episodes are pooled together and weighted equally when computing averages. The methodology R&R (2010) had employed, concentrating on the contemporaneous debt-growth link had the outcome that some countries vastly outnumbered others. If these observations are weighted differently it may cause significantly different conclusions. Fourth, unlike the growth regressions used widely, their specification is advantageous as it does not impose a linear or any arbitrary polynomial specification.

The analysis starts by concentrating on the short-term debt-growth relation. Figure 6.1 presents the average real GDP growth rate per capita in the year after the debt-to-GDP ratio passes a particular threshold, i.e. h=1. The exclusion window for episodes is based upon h=15 for consistency reason as later they examine longer time-horizons. This does not change the result of a huge reduction in growth beyond the 90% threshold (Figure 6.1). The Figure reveals the same result as R&R (2010). GDP performance is considerably poor at the year after the debt-to-GDP ratio hikes beyond 90 percent. Specifically, GDP growth averages round 2 percent in countries having debt below 90 percent, and rolls to about -2 percent in countries whose debt ratio rises above that level. Simultaneously, the inter-quartile range across all episodes indicates the diversification of the GDP performance of the countries where debt increases beyond 90%. As already mentioned, when h is set equal to 1, the causal relation between debt and growth might not been unveiled. And so, the problem of reverse causation might remain in these results and blurs them.

Figure 6.1
Debt and Growth in the Short Run



The authors explain this as a country entering a state of distress beyond the 90% that leads economic activity to drop. At the same time it could be the case that debt expansions beyond this threshold are the consequence of an omitted variable that decreases GDP and tax revenues which subsequently further increases debt.

Also, it is noted, that the wide inter-quartile range indicates that the findings are somewhat sensitive and affected by outliers. The case of Japan is referred as a characteristic case. The debt-to-GDP ratio increases from 133 percent in 1943 to 204 percent in 1944, and the subsequent growth in 1945 was -50%. This results to a particular large decrease in the average growth for debt thresholds beyond 135 percent of GDP.

Then, Pescatori, et al., extend the horizon of analysis so as to attenuate the reverse causality bias and potential omitted variables problems. This further addresses the problem of outliers, such as Japan.

In Figure 6.2 one can see the growth performance of the same episodes over longer horizons of *h*= 5, 10, 15. Compared to the case where *h*=1, the growth performance gets particularly better even when measured at a 5-year horizon. The improvement is clearer if we notice the horizons of 10 and 15 years. But, the most interesting result is that now seems to be no obvious debt-to-GDP threshold beyond which growth decreases harshly, although higher debt is still related to lower growth. They find no proof of any threshold effects over these relatively longer time horizons. According to this, the prima facie case for debt thresholds fades away.

The absence of a long-run debt-growth link might be driven by the fact that the debt-to-GDP ratio drops quickly after crossing high levels. They tried to test this case; results are illustrated in Figure 6.3.

Figure 6.2

Debt and Growth over the Medium Run

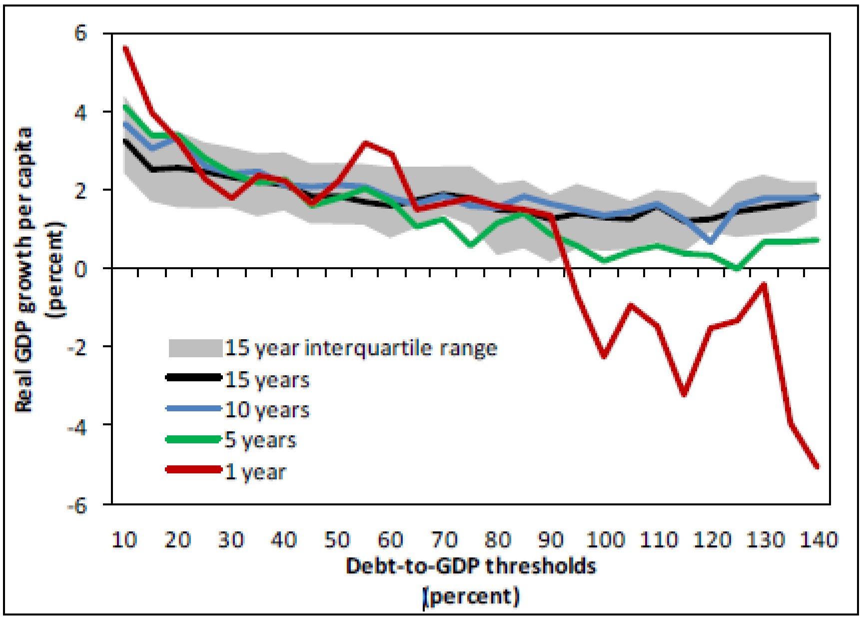
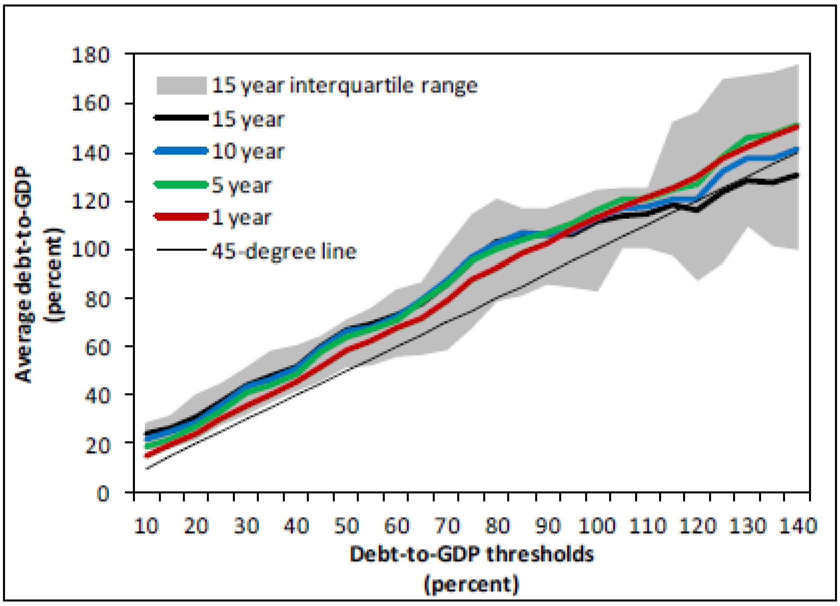


Figure 6.3

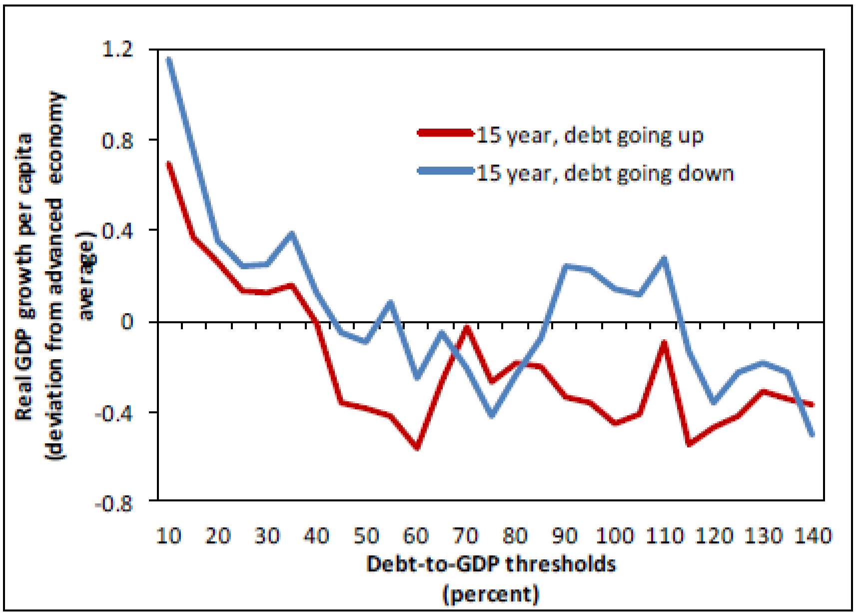
Debt Dynamics over the Medium Term



The Figure 6.3 reveals that the hypothesis is not right. The average-debt-to-GDP ratios during the 1 until 15 subsequent years is showed, for any given debt threshold on the horizontal axis. It is obvious that, whereas the debt ratio tends to reduce at particularly high levels, the process is extremely smooth.

Next, they recalculate using a different measure for the GDP growth rate as a robustness check. The episodes they have checked happen throughout the 20th century. In that period mean growth varies considerably from lows during the 1930s to highs during the 1950s. So, the results may be misleading. Instead they compute for each episode the relative growth GDP measure: $g_{it}(h)$-$\bar{g}_t(h)$ rather than the absolute average growth rate. The term $\bar{g}_t(h)$ is the mean growth rates for all economies over the same episode and it holds that $\bar{g}_t(h)$ = (1/N)Σ y_{it+h}/y_{it}. Results are in Figure 6.4.

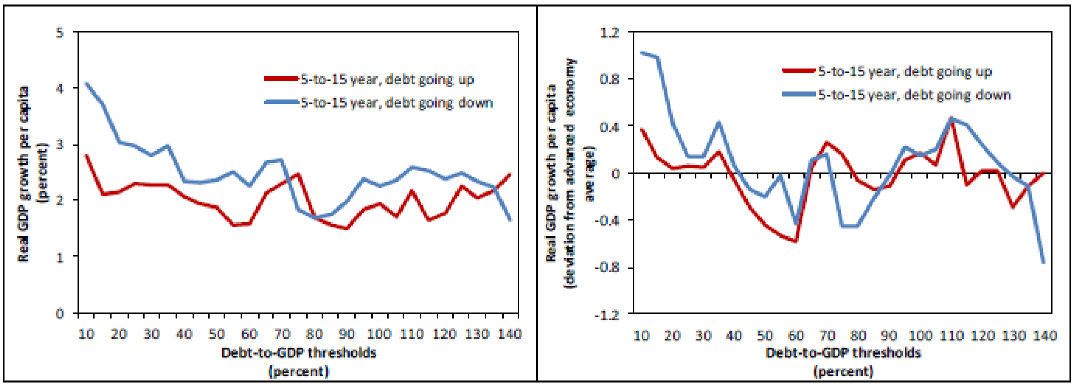
Figure 6.4
Relative Growth Performance



The computations show that highly indebted economies behave very similar to their peers. Excluding the lower debt layers, no great differences (in particular no more than 0.5% annually) exist. As in the previous case where the absolute simple growth rate was used, here it also clear that there is no unique debt threshold after which growth diminishes considerably. On the contrary, at higher debt levels the link is rather weak between debt and medium-term growth.

Next, as an additional robustness check, they repeat the computations for the growth performance from 5 -15 years after passing a specific debt-to-GDP value (Figure 6.5).

Figure 6.5
Growth Performance from 5 to 15 Years after Crossing Debt Thresholds



In this way, the first 5 years of each episode are eliminated as they could be contaminated by reverse causality. In their sample they find that growth (of one year) and the subsequent are little correlated.

But, when they regress growth at t+5 on growth at t the estimation is not statistically significant. So, as they remove the first 5 years, they exclude the automatic stabilizers impact and the serial correlation that may produce spurious correlations in the short-term. The left hand side of Figure 6.5 represents the average growth rates in absolute terms, $g_{it,\,t+5}(10)=y_{t+15}/y_{t+5}$. The right hand side uses the growth rates in respect to the average growth rate in advanced economies, $g_{it,\,t+5}(10)-\bar{g}_{t,\,t+5}(10)$.

The elimination of the first 5 years leads to an even more flat association as presented in the charts.

Pescatori, Sandri and Simon (2014) reached to the conclusion that no magic debt level exists beyond which growth performance in the medium-run is extremely impaired. In opposition to this, the link between high levels of debt and growth becomes weak if someone looks at any but the shortest-term period, particularly when considering the average growth performance of country peers.

The authors remind, however, that even by limiting the short-term reverse causality problems, their results are still subject to potential endogeneity as in previous empirical studies. The results cannot provide a formally established firm causality. Therefore, anyone should be careful before drawing policy implications.

Eberhardt (2013) employed time series methods to specify the existence of nonlinearities in the long-run debt-growth relationship from a new angle. He tried to do this by investigating whether linear or various nonlinear specifications of the debt-growth relation define 'long-run equilibrium relations'. His analysis comprehends 4 countries (United States, Great Britain, Sweden and Japan) covering a time period of over two centuries. They end up finding little indication for long-run nonlinearities between debt and growth, which implies that the equilibrium debt-growth relationship is different for each one of these countries. Thus, the argument for a common 90% or indeed any common debt-to-GDP threshold loses its power.

Eberhardt works with two specifications to model the hypothesized nonlinearity due to their demonstrated practicality: in addition to the linear model (Model 1) he uses polynomial specifications which include linear and squared (Model 2) or linear, squared and cubed (Model 3) debt-to-GDP terms (in logarithms). Secondly, he adopts piecewise linear specifications where the debt-to-GDP ratio (in levels) is divided into two variables made up of values below and above a specified threshold, which is treated as exogenous. He considers three threshold values for Great Britain: 90, 70 and 50 percent. For the United States and Japan he considers only the 50 percent threshold as there are not many observations above the two other thresholds. He does not apply the threshold model for Sweden, as its debt observations do not serve for this purpose.

The polynomial specifications are:

$$y_t = \alpha_0 + \varphi t + \phi_1 x_t + \varepsilon_t \qquad \textit{(Model 1)}$$

$$y_t = \alpha_0 + \varphi t + \phi_1 x_t + \phi_2 x_t^2 + \varepsilon_t \qquad \textit{(Model 2)}$$

$$y_t = \alpha_0 + \varphi t + \phi_1 x_t + \phi_2 x_t^2 + \phi_3 x_t^3 + \varepsilon_t \qquad \textit{(Model 3)}$$

The terms *y* is per capita GDP, *x* is the debt-to-GDP ratio (both in logarithms), α_0 is an intercept, *t* is a linear trend term with parameter φ and ε_t is white noise.

The threshold model specification is:

$$\begin{aligned} y_t = {} & \alpha_0 + \varphi t + \theta_1 X_t \times \mathbb{1}(X_t < \text{threshold}) \\ & + \theta_2 X_t \times \mathbb{1}(X_t \geq \text{threshold}) + \varepsilon_t \end{aligned} \qquad (6.3)$$

The public debt is defined as the total gross central government debt comprising domestic and external debt. For the Great Britain the series refer to net rather than gross central government debt. Data covers two different time periods; for the US, Britain and Sweden the series start in 1800, for Japan in 1872-all series end in 2010. The author conducts summability, balance and co-summability tests for an additional dataset of 23 countries – mainly OECD as well as some of emerging- (for various time coverages) as a robustness check.

In his analysis, he does not regard the direction of causation, justifying it as the causation not affecting the statistical validity of his findings. Ex post, having concluding that nonlinear (or linear) long-run relations do not exist, he claims that the standard empirical specifications of thresholds or polynomial functions analyzing the debt-growth nexus so far are extremely wrongly defined and the causal interpretation attributed to these studies is incorrect.

Before reporting the findings some methodological issues are necessary to be mentioned. No one doubts that the concepts of integration and cointegration have been and still are very useful in time series econometrics. In the existence of a non-linear model with integrated variables a number of difficulties arise when using conventional time series analysis. Which is the order of integration of these non-linear transformations? Such a question does not have a clear answer since the definitions of integrability do not properly apply. Integration is a linear concept. The order of integration is valid to characterize linear processes; but it is not appropriate for non-linear worlds.

Furthermore, defining multivariate non-linear models as balanced or not turns out to be rather complex. Unbalancedness is a characteristic of a misspecified model, a feature that more frequently appears when managing non-linear transformations of persistent variables. In linear structures, the integrability notion performs well dealing with balanced/unbalanced relations. Yet, in non-linear setups, the absence of a comprehensive quantitative measure complicates testing the balancedness of a postulated model.

For these reasons, extensions of the linear concepts of integration are required to generalize to non-linear setups. Gonzalo and Pitarakis (2006) faced these problems and were the first to introduce, in a very heuristic way, the idea of 'order of summability' of a stochastic process while dealing with threshold effects in co-integrating regressions.

Berenguer-Rico and Gonzalo (2013, 2014) further developed the notion of order of summability as: "a summary measure of the stochastic properties-such as persistence-of the times series without relying on linear structures". They showed that integrated time series are particular cases of summable processes, in the sense that the order of summability is a more general concept than the order of integration. Therefore, summability is a generalization of integrability. Subsample estimations construct confidence intervals to establish inference.

Furthermore, summability does not only characterize some properties of univariate time series, but also allows to easily study the balancedness of a postulated relationship —linear or not. And even more important, nonlinear long run equilibrium relationships between non-stationary time series can be properly defined. In the same way integration constitutes the first step to check balancedness of a linear relationship and to analyze co-integration, summability can be used to study non-linear long run relationships. Confidence intervals are constructed using subsample results. The confidence interval contains zero if the null hypothesis of balance holds; balancedness is a necessary but no sufficient condition for the existence of a long-run equilibrium relation. Once balancedness of a non-linear model is established, the analysis of non-linear long run relationships can be done using the concept of co-summability. Inference is accomplished as in the other testing procedures. Co-summability holds if the confidence interval includes zero.

Eberhardt investigates the evidence for long-run equilibrium debt-growth relationship by applying the summability, balance and co-summability testing to the polynomial and threshold specifications presented. First, he estimates the order of summability for all model variables and the results are presented in Table 6.10.

Table 6.10
Estimated Order of Summability

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
USA	1800	2010	-	211	ln(GDP pc)	0.652	1.490	2.329
					Δln(GDP pc)	-0.524	0.066	0.657
					ln(Debt/GDP)	0.551	1.082	1.613
					ln(Debt/GDP) squared	0.383	0.860	1.336
					ln(Debt/GDP) cubed	0.404	0.993	1.582
				168	Debt/GDP $<$ 50%	0.313	0.825	1.337
				43	Debt/GDP $\geq$ 50%	0.691	1.409	2.127
GBR	1800	2010	-	211	ln(GDP pc)	0.731	1.696	2.662
					Δln(GDP pc)	-0.444	0.126	0.695
					ln(Debt/GDP)	0.540	0.967	1.393
					ln(Debt/GDP) squared	0.509	0.948	1.386
					ln(Debt/GDP) cubed	0.475	0.931	1.387
				100	Debt/GDP $<$ 90%	0.511	1.062	1.613
				111	Debt/GDP $\geq$ 90%	0.405	0.936	1.467
				86	Debt/GDP $<$ 70%	0.428	1.200	1.972
				125	Debt/GDP $\geq$ 70%	0.465	0.923	1.381
				64	Debt/GDP $<$ 50%	0.447	1.068	1.689
				147	Debt/GDP $\geq$ 50%	0.459	0.898	1.336
SWE	1800	2010	-	211	ln(GDP pc)	0.361	0.904	1.334
					Δln(GDP pc)	-0.359	0.030	0.357
					ln(Debt/GDP)	0.637	1.624	2.603
					ln(Debt/GDP) squared	0.614	1.577	2.451
					ln(Debt/GDP) cubed	0.473	1.538	2.399
JPN	1872	2010	2	125	ln(GDP pc)	0.824	2.070	3.315
					Δln(GDP pc)	-0.687	-0.001	0.685
					ln(Debt/GDP)	0.420	1.099	1.778
					ln(Debt/GDP) squared	0.371	1.108	1.845
					ln(Debt/GDP) cubed	0.406	1.115	1.823
				77	Debt/GDP $<$ 50%	0.195	1.019	1.843
				48	Debt/GDP $\geq$ 50%	0.503	1.325	2.147

Notes: For the US and GBR we also provide summability estimates for data below and above various debt/GDP thresholds: for the former this is only feasible for a 50 percent debt/GDP threshold, whereas for the latter we can test 90, 70 and 50 percent. Obs reports the number of observations. CI low and up indicate the 95% confidence interval for the summability estimate $S(\delta)$ constructed from subsampling – shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted we allow for deterministic terms (constant and trend).

The hypothesis of summability of order zero is rejected in the cases of per capita GDP levels or in all of the debt variables cases where the confidence intervals do not include zero. On the contrary, in the case of the per capita GDP growth rates the order of summability is always very close to zero. The same results hold when he carries out the estimates for the larger set of 23 countries. The exact pattern is identified in 20 of these and so the hypothesis that the per capita GDP growth rate is summability of order zero cannot be rejected. For the equivalent levels series the null is rejected. In 25 of 27 countries, all three debt variables reject the S(0). Thus, that is evidence for the significant persistence of the data and raise concerns that the time series properties discussed are very important when one wants to analyze the long-run debt-growth nexus. In any regression that contains these variables Eberhardt claims that exist the risk of spurious results, unless the empirical models can be confirmed as balanced and co-summable.

Tests for balance and co-summability are conducted including as dependent variable the per capita GDP growth rates (Table 6.11). As Eberhardt interested to investigate the long-run connection he adopts the levels variable for income, instead of its growth rate. But for comprehensiveness he analyses as well summability and balance for the per capita GDP growth rate. The evidence from Table 6.11 reveal that none of these specifications constitutes balanced empirical equations as, regardless of the specification or country, the confidence intervals do not include zero. The support and approval of the 'growth' specification in the relevant literature is explained by the existence of a lagged level of per capita GDP as additional regressor. This quasi-error correction specification which provides estimates for a long-run levels relationship is however misspecified as a growth equation. According to these estimations, it is implied that growth rates and debt levels do not have the same order of summability, which is in line with expectations of nonstationary log levels and stationary first differences (growth rates).

Table 6.11
Balance and Co-Summability — Δln(GDP pc) specifications

	Start	End	Gaps	Obs	Nonlinearity	Balance			
						CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict
USA	1801	2010	-	210	-	-2.199	-1.403	-0.606	$S(\delta_y) \neq S(\delta_z)$
				$b = 15$	ln(Debt/GDP)²	-2.642	-1.622	-0.603	$S(\delta_y) \neq S(\delta_z)$
				$M = 196$	ln(Debt/GDP)³	-3.237	-2.037	-0.836	$S(\delta_y) \neq S(\delta_z)$
					Threshold	-3.024	-1.822	-0.620	$S(\delta_y) \neq S(\delta_z)$
GBR	1801	2010	-	210	-	-1.781	-0.944	-0.106	$S(\delta_y) \neq S(\delta_z)$
				$b = 15$	ln(Debt/GDP)²	-2.573	-1.458	-0.343	$S(\delta_y) \neq S(\delta_z)$
				$M = 196$	ln(Debt/GDP)³	-3.252	-1.903	-0.554	$S(\delta_y) \neq S(\delta_z)$
					Threshold	-3.416	-2.007	-0.597	$S(\delta_y) \neq S(\delta_z)$
SWE	1801	2010	-	210	-	-2.201	-1.457	-0.713	$S(\delta_y) \neq S(\delta_z)$
				$b = 15$	ln(Debt/GDP)²	-2.806	-1.884	-0.963	$S(\delta_y) \neq S(\delta_z)$
				$M = 196$	ln(Debt/GDP)³	-3.301	-2.218	-1.135	$S(\delta_y) \neq S(\delta_z)$
JPN	1873	2010	14	122	-	-1.917	-1.192	-0.467	$S(\delta_y) \neq S(\delta_z)$
				$b = 12$	ln(Debt/GDP)²	-2.682	-1.659	-0.636	$S(\delta_y) \neq S(\delta_z)$
				$M = 111$	ln(Debt/GDP)³	-3.297	-2.043	-0.789	$S(\delta_y) \neq S(\delta_z)$
					Threshold	-3.196	-1.960	-0.723	$S(\delta_y) \neq S(\delta_z)$

Notes: In all models we take the per capita GDP growth rate, Δln(GDP pc), as the dependent variable. See Table 2 for all other details. Since no model satisfies the balance test we do not carry out co-summability testing.

Table 6.12 gives the balance and co-summability tests when the dependent variable is the per capita GDP level. Unbalanced equations are formed only for the United States and two of its nonlinear specifications: the threshold model and the polynomial specification with linear, squared and cubed terms. In all other models balanced specifications cannot be rejected.

Table 6.12

Balance and Co-Summability — ln(GDP pc) specifications

						Balance				Co-Summability			
	Start	End	Gaps	Obs	Nonlinearity	CI low	$\hat{\delta}_y - \hat{\delta}_z$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1800	2010	-	211	-	-1.265	-0.507	0.252	$S(\delta_y) = S(\delta_z)$	0.467	1.050	1.633	$S(\delta_{\hat{e}_t}) \neq 0$
				$b = 16$	ln(Debt/GDP)^2	-1.611	-0.726	0.160	$S(\delta_y) = S(\delta_z)$	0.277	0.943	1.610	$S(\delta_{\hat{e}_t}) \neq 0$
				$M = 196$	ln(Debt/GDP)^3	-2.158	-1.145	-0.132	$S(\delta_y) \neq S(\delta_z)$				
					Threshold	-1.865	-0.942	-0.019	$S(\delta_y) \neq S(\delta_z)$				
GBR	1800	2010	-	211	-	-0.913	-0.178	0.558	$S(\delta_y) = S(\delta_z)$	0.664	1.202	1.739	$S(\delta_{\hat{e}_t}) \neq 0$
				$b = 16$	ln(Debt/GDP)^2	-1.705	-0.694	0.317	$S(\delta_y) = S(\delta_z)$	0.703	1.204	1.704	$S(\delta_{\hat{e}_t}) \neq 0$
				$M = 196$	ln(Debt/GDP)^3	-2.383	-1.137	0.109	$S(\delta_y) = S(\delta_z)$	0.706	1.203	1.700	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 90%‡	-2.509	-1.240	0.028	$S(\delta_y) = S(\delta_z)$	0.726	1.163	1.601	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 70%‡	-2.509	-1.240	0.028	$S(\delta_y) = S(\delta_z)$	0.720	1.175	1.629	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 50%‡	-2.509	-1.240	0.028	$S(\delta_y) = S(\delta_z)$	0.526	1.131	1.736	$S(\delta_{\hat{e}_t}) \neq 0$
SWE	1800	2010	-	211	-	-1.054	-0.350	0.354	$S(\delta_y) = S(\delta_z)$	0.793	1.577	2.362	$S(\delta_{\hat{e}_t}) \neq 0$
				$b = 16$	ln(Debt/GDP)^2	-1.660	-0.767	0.126	$S(\delta_y) = S(\delta_z)$	0.678	1.642	2.605	$S(\delta_{\hat{e}_t}) \neq 0$
				$M = 196$	ln(Debt/GDP)^3	-2.245	-1.108	0.028	$S(\delta_y) = S(\delta_z)$	0.716	1.636	2.556	$S(\delta_{\hat{e}_t}) \neq 0$
JPN	1872	2010	14	125	-	-1.408	-0.539	0.330	$S(\delta_y) = S(\delta_z)$	0.478	1.097	1.716	$S(\delta_{\hat{e}_t}) \neq 0$
				$b = 12$	ln(Debt/GDP)^2	-2.187	-1.009	0.169	$S(\delta_y) = S(\delta_z)$	0.262	0.864	1.465	$S(\delta_{\hat{e}_t}) \neq 0$
				$M = 114$	ln(Debt/GDP)^3	-2.778	-1.383	0.011	$S(\delta_y) = S(\delta_z)$	0.228	0.856	1.483	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold	-2.624	-1.264	0.095	$S(\delta_y) = S(\delta_z)$	1.186	2.261	3.336	$S(\delta_{\hat{e}_t}) \neq 0$

Notes: In all models we take the per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. Gaps indicates the number of missing observations. In all tests conducted we allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=)\hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=)0$ that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of the subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP). ‡ Results for balance do not differ across different threshold values since $X_t \mathbb{1}(X_t < \text{threshold}) + X_t \mathbb{1}(X_t \geq \text{threshold}) = X_t$.

Yet, in all cases where balanced equations had been found, the subsequent co-summability tests were all rejected. Furthermore, the rejection is not marginal, as all confidence intervals distance away from zero. Estimations for the extended dataset of 23 countries confirm these results. Only for one country, Uruguay, and for the specification with linear, squared and cubed debt terms the balancedness and co-summability is satisfied. These findings strongly support the notion that does not exist any nonlinear –or, for that matter, linear- specification in the dataset. These suggest that the debt-to-GDP ratio and the per capita income do not move together when investigating their long-run relationship and thus no causality exists between them.

The author claims that two characteristics of the selected time coverage may induce some problems while trying to identify the true relationship between debt and growth. The first problem is that the data may insufficiently capture the serious shocks of the two world wars and the recent global financial crisis faced by these countries over the last centuries. These events may unduly affect the empirical testing. The second problem is that the analysis focuses on time series for over two centuries, implying that the long-run equilibrium relationship is stable over this long time, which may as well not be the case.

Trying to deal with the above caveats, in what follows he makes the computations for a rolling window of sixty years instead of the full sample. First, he executes the balance and co-summability tests for this rolling window of sixty years for the 4 economies and the time horizon 1800-2010, which turn out to be 152 subsamples (66 for Japan). The author follows this procedure only for the polynomial specifications as the characteristics of the dataset prohibit practicing it on the other specifications. Country-specific time-varying results for this sub-sample analysis presented in graphical form (Figures 6.6a, 6.6b, 6.7a and 6.7b).

Figure 6.6a
Balance Testing (Sub-Samples)

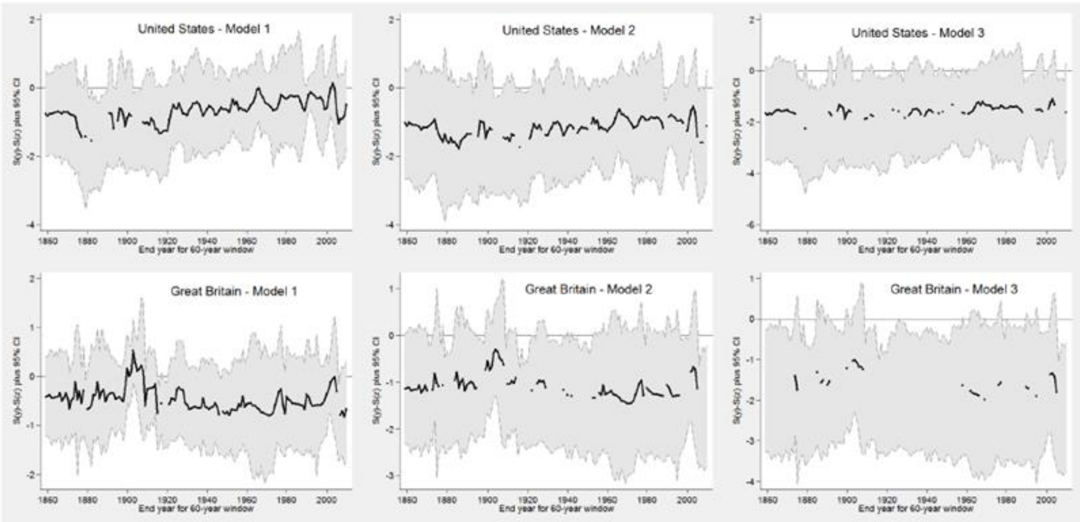
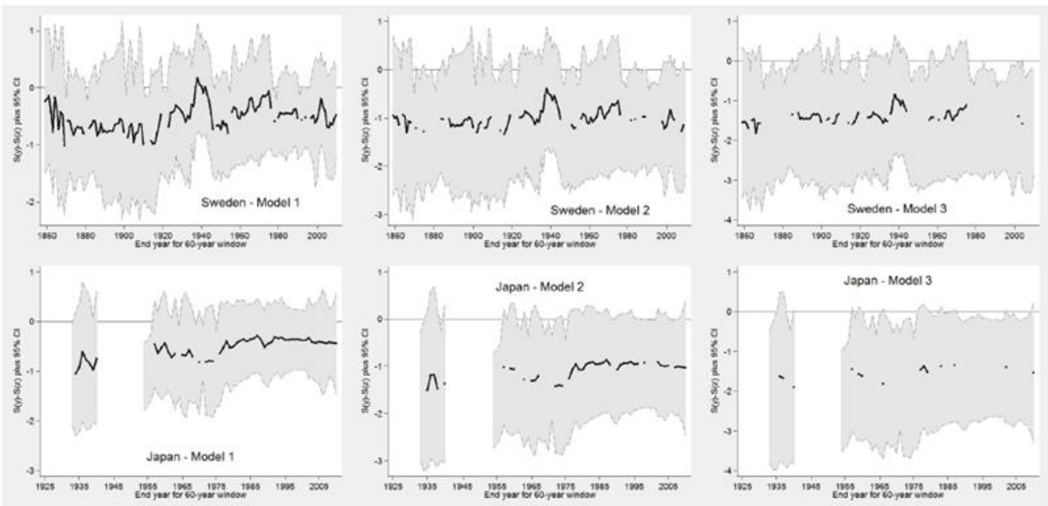


Figure 6.6b
Balance Testing (Sub-Samples)



Notes: The shaded areas represent the Bonferroni-corrected 95% Confidence Intervals for the Balance statistic computed in a moving window of 60-year time periods; the solid line represents the balance estimate for consecutive windows: we only plot this when balance cannot be rejected. The coverage of the data differs across countries: for the US, Great Britain and Sweden we have data from 1800-2010 (152 subsamples), for Japan from 1872-2010 (with gaps; 66 subsamples). Model 1 refers to a specification with linear debt terms only, Model 2 to a specification with linear and quadratic debt terms, Model 3 further includes a cubed debt term. The graphs capture both subsequent end years in which subsamples were balanced as well as 'isolated' years.

Figure 6.7a
Co-Summability Testing (Sub-Samples)

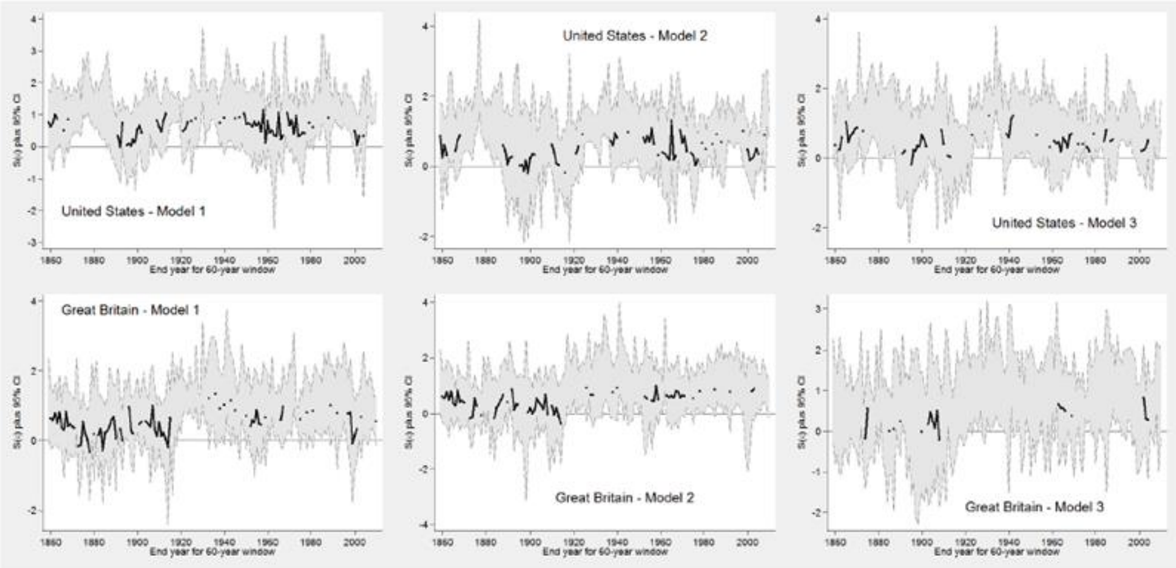
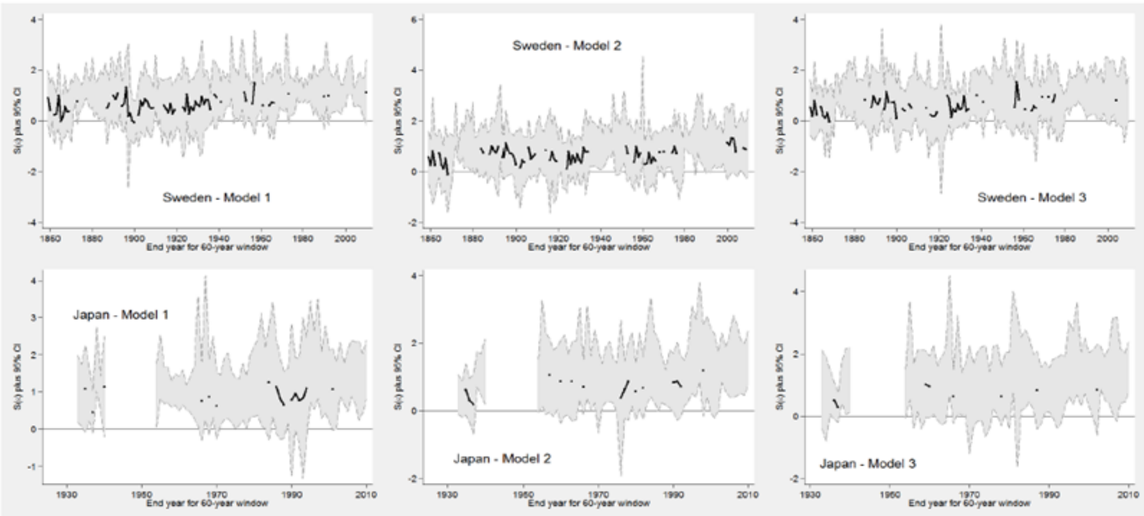


Figure 6.7b
Co-Summability Testing (Sub-Samples)



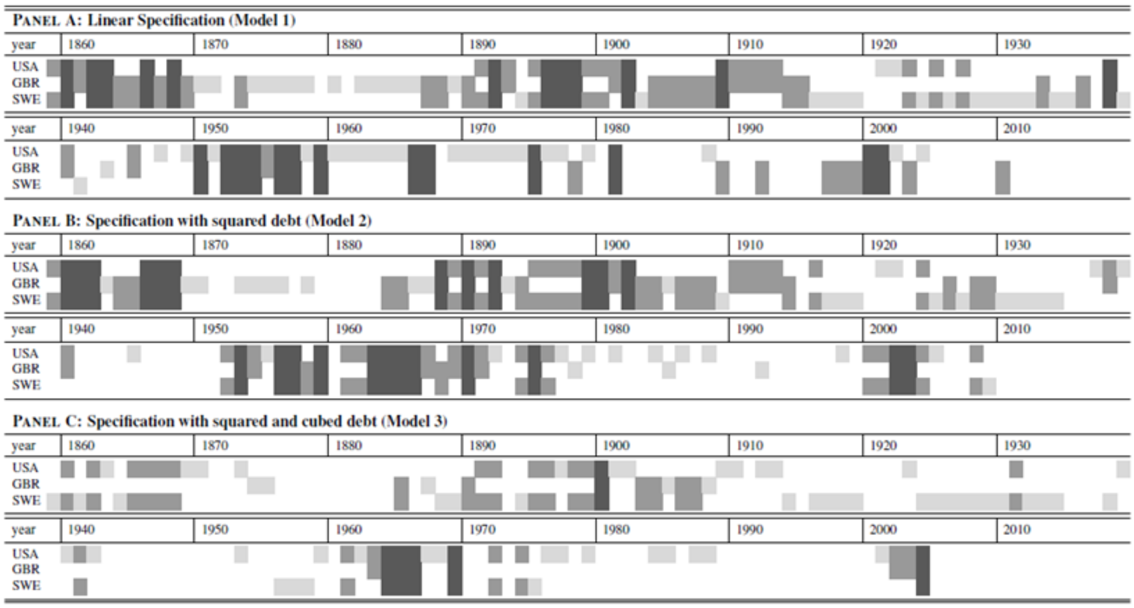
Notes: The shaded areas represent the Bonferroni-corrected 95% Confidence Intervals for the Co-Summability statistic computed in a moving window of 60-year time periods. The solid black line represents the computed Co-Summability statistic— this line is only shown if the prerequisite balance test could not reject this feature for the specific subsample. We allow for an intercept in the co-summability analysis. For further details see Figures 6.6a and 6.6b. The graphs capture both subsequent end years in which subsamples were balanced (line) as well as 'isolated' years (dots).

In Figures 6.6a and 6.6b, in each plot the (broken) line and dots represent that the model is balanced. In Figures 6.7a and 6.7b these signify balanced and co-summable specifications. The results from the balanced tests coincide with that of the full-sample. In all four economies, the majority of the linear models (Model 1) constitute balanced empirical models. This also holds broadly for the polynomial function with linear and squared debt terms (Model 2). The only exception appears when the cubed term is included (Model 3). In this case, Great Britain and Japan demonstrate a considerable number of unbalanced subsamples; Sweden as well but in a lower degree.

The results of balance and co-summability tests (Figures 6.7a and 6.7b) indicate that just an extremely small number of sub-samples of all country and specification cases are balanced and co-summable. So, these results overall confirm the full sample estimations. In general, while in certain consecutive sub-periods indications for long-run equilibrium relationship were found, none of the three models distinctly dominate the others in terms of balance and co-summability.

Next, he compares graphically the above balance and co-summability subsample results. In doing so, he unveils common features but also differences in the long-run equilibrium relationship across countries (Table 6.13). Panel A refers to the linear model (Model 1), Panels B and C to the polynomial specifications with (in addition) squared and cubed debt terms, respectively (Models 2 and 3). For each country a shaded cell indicates the sixty-year subsample ending in the year specified constituted a balanced and co-summable specification, while the intensity of the shading indicates whether this property occurred in one (lightest), two (intermediate) or all three (darkest) countries.

Table 6.13
Balance and Co-Summability — Cross-Country Comparison



Notes: Increasing shading indicates the number (0-3) of countries for which the sixty-year subsample ending in the year indicated has a balanced and co-summable specification.

It is obvious that (for each specification) the timing of the subperiods, in which co-movements between debt and income exist in the three countries, largely appears to differ across countries. Across all three models the proportion of concurrent episodes for one or two countries is roughly twice that respectively of episodes for all three countries, providing evidence of the heterogeneity in the long-run debt-growth relationship across these economies. Through this, Eberhardt supported that it is not correct to study countries in a pooled panel data model, as the standard approach in the empirical literature does, which is being done by imposing the parameter homogeneity in the debt-growth relation.

Finally, he estimates balance and co-summability tests for the final six sixty-year subsamples for the United States, Great Britain and Sweden (1946-2005, ..1951-2010), which benefits focusing in the post-WWII period while simultaneously analyzing subsamples eliminating the years of the global financial crisis (2008-2010). Results provide quite limited evidence for co-movements when restricting the sample to the post-WWII period. Furthermore, it does not become more straightforward if the inclusion or exclusion of the global crisis years produces different results for the full-sample analysis.

Overall, the sub-sample analysis results point to a distinct possibility that the full sample results are not extremely twisted by global shocks or structural breaks during the long period, as assumed earlier, given than just a small number of subsamples are found to be balanced and co-summable across all countries and specifications.

Through an alternative approach Eberhardt concludes that there is no indication for any long-run debt-growth relationship in the nonlinear, or even linear, specification for the economies. There exist only a small number of subperiods over the long time horizon for which co-movement between debt and income are identified. The evidence that no link seems to connect debt and growth implies that the popular policy issue of the necessity of fiscal adjustment in order to achieve long-run economic stability and sustainability loses its validity. Still, he does not support the notion that a high debt burden should not been taken under consideration by policymakers or that in the short-term debt does not hurt economic activity. He just emphasizes the absence of signs for nonlinearities, such as the famous 90% debt-to-GDP threshold, in the long-run relationship with growth, which leads to reject the need of aggressive austerity programs and government spending cuts to enhance growth.

The analysis points out an important issue that has not been given much attention in the literature. There is theoretical evidence that the non-linear relationship –if exists- might not hold for all countries in the same way. In this research, the very few confirmed cases of co-movements found to largely differ over time across countries. Existing literature finds extremely different results when moving away from full sample analysis in homogeneous parameter regression models to investigating sub-samples along geographic, institutional or income terms. The author suggests that analyses should concentrate on unveiling the possible heterogeneities across countries.

7. Heterogeneity in the debt-growth relationship

A major failure of the researchers' work has been the lack of paying sufficiently attention to the possible heterogeneous way in which debt impacts growth. Parameter heterogeneity refers to the notion that the data generating process that describes the cross-country growth process is not common for all observations. The researchers so far have been looking exclusively for threshold effects of debt on growth when government debt is above or below a specific predetermined value. Their work focuses and analyses only the debt-to-GDP ratio as threshold candidate, unjustifiably disregarding any other reasonable candidate thresholds. Beyond this, the only case that is being more frequently suggested is just that nonlinearities do not exist. But, why should we only consider that the effect of public debt on growth is characterized only be excessive levels of debt? Theory supports that different factors; e.g., a country's trade openness or institutional quality, are plausible sources of convergence clubs and therefore can be used as threshold variables to sort countries into multiple growth regimes in which countries obey the same growth model. As literature develops, researchers investigate the non-linearities that depend on other factors.

Kourtellos, Stengos and Tan (2013,b), among others, correctly observed that the impact of public debt on economic activity might be driven by factors such as trade openness or institutional quality and that researchers should investigate the possibility of parameter heterogeneity; otherwise they might end up with spurious results. Thus, they contributed to the literature by investigating different threshold variables than the usual debt-to-GDP ratio. By doing so, they simultaneously manage to avoid one conceptual problem of the literature which has been the testing of the hypothesis of the presence of a debt threshold against the alternative of no threshold. They tried to assess the strongest evidence for a particular factor from a large group of reasonable candidates, within the framework of threshold regression models, as being the most plausible threshold variable to characterize the heterogeneous effects of public debt on growth and by that means, therefore, assorting countries into multiple growth regimes. Specifically, by using the structural threshold regression model they developed (Kourtellos, Stengos, Tan, 2013, a), which addresses the problem of parameter heterogeneity, they found conclusive empirical evidence that the basic cause of heterogeneity is based on democracy, as a proxy for institutional quality. They discovered that there exists a critical level of democracy (low-democracy regime) under which, higher public debt is correlated with lower growth, ceteris paribus. On the contrary, the correlation between debt and growth in the high-democracy regime is not statistically significant.

The structural threshold regression model (STR) they employed allows to address parameter heterogeneity that characterizes cross-country growth data. Also, allows for endogeneity in the slope regressors X_{it} (i.e. endogeneity of regressors), in the threshold variable (i.e. endogeneity of thresholds) and also regime specific heteroscedasticity. The authors note that recent literature largely disregards the endogeneity issue of the threshold variable. But if the threshold variable is endogenous, the estimated parameters for the regime-specific partial effects may be inconsistent.

They view their model as a generalization of the simple threshold regression framework of Hansen (2000). They define the indicator function (7.1) assuming a threshold variable q_{it} (in the meaning of public debt) which organizes the observations into regimes:

$$I(q_i \leqslant \gamma) = \begin{cases} 1 & \text{iff} q_i \leqslant \gamma : \text{Regime1} \\ 0 & \text{iff} q_i > \gamma : \text{Regime2} \end{cases} \tag{7.1}$$

$$I(q_i > \gamma) = 1 - I(q_i \leqslant \gamma) \tag{7.2}$$

Furthermore, they assume that q_i can represent each one of the non-constant variables included in the set of growth determinants X_i. And also q_i is assumed to be endogenous so that the reduced form equation determining which regime applies takes the form:

$$q_i = \pi_q' Z_i + \upsilon_{qi}, \tag{7.3}$$

The authors are unaware of which observation belongs to each regime (i.e. they do not know the threshold value) but they can observe the threshold variable.

The model can be generalized to allow for two regimes:

$$g_i = \beta' X_i + \delta' X_i I(q_i \leqslant \gamma) + \kappa \lambda_i(\gamma) + \varepsilon_i, \tag{7.4}$$

where $E(\varepsilon_i—Z_i) = 0$.

The term $\lambda_i(\gamma)$ is a scalar variable that involves an inverse Mills ratio term for each regime in order to restore the conditional mean zero property of the errors. In particular, $\lambda_{it}(\gamma)$ is defined as follows:

$$\lambda_i(\gamma) = \lambda_{1i}(\gamma) I(q_i \leqslant \gamma) + \lambda_{2i}(\gamma) I(q_i > \gamma), \tag{7.5}$$

$$\lambda_1(\gamma - Z_i'\pi_q) = -\frac{\phi(\gamma - Z_i'\pi_q)}{\Phi(\gamma - Z_i'\pi_q)} \text{ and } \lambda_2(\gamma - Z_i'\pi_q) = \frac{\phi(\gamma - Z_i'\pi_q)}{1 - \Phi(\gamma - Z_i'\pi_q)}$$

The functions ϕ (.) and Φ (.) denote the normal pdf and cdf, respectively. The coefficients β are the coefficients of the second regime, that is $\beta=\beta_2$ and δ is the difference between the coefficients of regime 1, β_1 and regime 2, β_2; that is, $\delta=\beta_1-\beta_2$. The estimation of the threshold parameter is based on a concentrated least squares method while the slope coefficients are obtained through 2SLS or GMM.

In Equation (7.4), when $\delta=\kappa=0$ the linear growth model (here referred to as augmented Solow growth regression model) is formed:

$$g_i = \beta' X_i + e_i = \alpha_S' S_i + \alpha_d d_i + e_i, i = 1, 2, \ldots, N, \tag{7.6}$$

The panel dataset covers 10 years (1980-1989, 1990-1999, 2000-2009) and 82 countries. The three 10-year growth periods averages out business cycle effects. The dependent variable is the growth rate of real per capita GDP over the time interval. For each candidate threshold variable, the null hypothesis of a linear model, H_0: $\delta=0$, is tested against the alternative of a threshold. To achieve this, they employ the sup Wald test of Kourtellos et al. (2013, a). If the threshold parameter γ is not identified under the null that means that the particular candidate parameter produces threshold effects and then p-values are computed by a bootstrap method.

Table 7.1 provides an extensive presentation of the variables. Table 7.2 shows the results of a test of the existence of a threshold effect against the null of global linearity for each of the candidate threshold variables.

Table 7.1

Data Appendix

Variable	Description
Time trend	Time trend variable for the periods 1980-89, 1990-99 and 2000–2009.
Growth	Growth rate of real per capita GDP in chain series for the periods 1980–1989, 1990–1999 and 2000–2009. Source: PWT 7.0.
Initial Income	Logarithm of real per capita GDP in chain series at 1980, 1990, 2000. Lagged values correspond to 1975, 1985 and 1995. Source: PWT 7.0.
Population Growth Rates	Logarithm of average population growth rates plus 0.05 for the periods 1980–1989, 1990–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: PWT 7.0.
Investment	Logarithm of average ratios over each period of investment to real GDP per capita for the periods 1980–1989, 1990–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: PWT 7.0.
Schooling	Logarithm of average years of male secondary and tetriary school attainment (25+) in 1980, 1990, and 1999. Lagged values correspond to 1975, 1985 and 1995. Source: Barro and Lee (2000).
Debt	Public debt to GDP for the periods 1980–1989, 1990–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: IMF, Debt Database Fall 2011 Vintage
Government	Logarithm of average ratios for each period of government consumption to real GDP per capita for the periods 1975–1979, 1985–1989 and 1995–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: PWT 7.0
Inflation	Logarithm of average inflation plus 1 for the periods 1980–1989, 1990–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: Worldbank
Openness	Average ratios for each period of exports plus imports to real GDP per capita for the periods 1980–1989, 1990–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: PWT 7.0.
Life Expectancy	Log of average life expectancy at birth for the periods 1980–1989, 1990–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: World Bank
Fertility	Logarithm of the average total fertility rate (births per woman) in 1980–1989, 1990–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: World Bank.
Executive Constraints	A measure of the extent of institutionalized constraints on the decision making powers of chief executives. This variable ranges from one to seven where higher values equal a greater extent of institutionalized constraints on the power of chief executives. This variable is calculated as the average for the periods 1980–1989, 1990–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: Polity IV
Democracy	A measure of the extent of institutionalized democracy, presence of institutions and procedures, existence of institutionalized constraints on the exercise of power by the executive, and guarantee of civil liberties to all citizens. This variable ranges from one to ten where higher values equal a greater extent of institutionalized democracy. This variable is calculated as the average for the periods 1970–1979, 1980–1989, 1990–1999 and 2000–2009. Lagged values correspond to 1975–1979, 1985–1989 and 1995–1999. Source: Polity IV
Language	Measure of linguistic fractionalization based on data describing shares of languages spoken as mother tongues. Source: A. Alesina, A. Devleeschauwer, W. Easterly, S. Kurlat, and R. Wacziarg (2003).
Tropics	Percentage of land area classified as tropical and subtropical via the in Koeppen-Geiger system. Source: The Center for International Development at Harvard University
LCR100km	Percentage of a country's land area within 100km of an ice-free coast. Source: The Center for International Development at Harvard University
Eastern Religion	Eastern Religion share in 1970 and 2000 expressed as a fraction of the population. It includes Chinese Universists, Confucians, Neoreligionists, Shintos, and Zoroastrians (Parsis). Source: World Christian Encyclopedia

Table 7.2

Threshold tests. This table presents sup Wald tests for the null hypothesis that the linear Solow growth model augmented by the debt-gdp-ratio in Eq. (7.6) against the alternative hypothesis of the threshold model in Eq. (7.4). All models include constant and trend

Threshold variable	Sup Wald	Boot *p*-value
Initial income	53.575[c]	0.057
Schooling	43.1849[a]	0.002
Investments	30.1235[c]	0.067
Population growth	57.804[b]	0.015
Fertility	51.9421[b]	0.037
Life expectancy	81.4932[a]	0.000
Public debt	16.4969	0.517
Government	20.2704	0.29
Inflation	28.7319[c]	0.066
Openness	25.9372	0.131
Democracy	31.7114[c]	0.096
Executive constraints	21.1275	0.202
Tropics	42.1866[a]	0.006
LCR100km	21.5703	0.208
Language	20.2187	0.235

[a] Significance at 1%.
[b] Significance at 5%
[c] Significance at 10%.

Nine of the 15 potential candidates, Initial Income, Schooling, Investments, Population Growth, Fertility, Life Expectancy, Inflation, Tropics, and Democracy, ended up in rejecting the null. Surprisingly, there is not much indication that Public Debt is a good threshold variable for sample splitting, at least for this particular country dataset. However, the results confirm that there is strong evidence of parameter heterogeneity meaning that nonlinearities exist but may be based on many other threshold variables rather than the debt ratio.

Table 7.3 shows the estimate for the threshold value for each of the 9 threshold variables, the associated 90% confidence interval for the threshold value, the number of observations for each of the two regimes that come from splitting the sample according to each of these threshold variables, and the associated J statistic for the STR model using each of these threshold variables, respectively.

Table 7.3

Threshold tests. This table shows the point estimate of the threshold parameter along with the associated the 90% confidence interval, the sample size of two growth regimes, and the J statistic for the STR models that rejected the null of the linear model in Table 7.2

Threshold variable	Threshold estimate	90% Confidence interval	n^{low}	n^{High}	*J* statistic
Initial income	6.93585	[6.9258,7.4708]	36	210	2.3E−21
Schooling	0.95985	[0.4689,1.3219]	163	83	6.39E−21
Investments	2.729622	[2.7296,2.7476]	35	211	5.89E−22
Population growth	−2.87913	[−2.9211, −2.5471]	54	192	1.17E−18
Fertility	1.067608	[0.8776,1.2866]	109	137	1.7E−19
Life expectancy	3.97159	[3.9706,3.9716]	43	203	9.52E−22
Inflation	2.776564	[1.7656,2.8246]	192	54	3.38E−21
Democracy	4.599	[2.949,4.799]	90	156	1.58E−22
Tropics	0.443	[0,0.967]	129	117	9.57E−21

Each of the 9 threshold variables therefore constitutes a potential STR model for the data. The authors, so, have to identify the model that best fits the data, which is being accomplished though the J criterion. The J criterion receives the minimum value for the threshold variable Democracy. Therefore, the results from the STR model that splits the sample into a Low-Democracy regime (i.e. countries with Democracy scores below 4.5) and a High-Democracy regime (i.e., countries with Democracy scores above 4.5) are presented in Table 7.4.

Table 7.4

STR estimation. This table presents the estimation of the STR model of Kourtellos et al. (2013) using Democracy as a threshold variable. All variables are instrumented using their lagged values. It also presents the TR model of Hansen (2000) that ignores endogeneity. The last two columns report the GMM and LS results for the global estimation that ignores the presence of a threshold. The means of the variables are also reported for each regime.

Method	STR-GMM		TR-LS		Linear-GMM	Linear-LS
Threshold estimate	4.500		4.600			
90% Confidence interval	[2.949, 4.799]		[1.2, 5.6]			
J statistic	1.577E−22					
	Low	High	Low	High		
Initial income	0.0023	−0.0147[a]	0.0013	−0.0118[a]	−0.0047[b]	−0.0032[c]
	(0.0052)	(0.0031)	(0.0047)	(0.0023)	(0.0022)	(0.0019)
Schooling	0.0056	0.0083[c]	0.0047	0.0099[a]	0.0056[c]	0.0062[b]
	(0.0050)	(0.0046)	(0.0049)	(0.0037)	(0.0033)	(0.0029)
Investments	0.0060	−0.0042	0.0173[a]	0.0116[c]	0.0061	0.0187[a]
	(0.0069)	(0.0103)	(0.0057)	(0.0064)	(0.0051)	(0.0039)
Population growth	−0.0132	−0.0811[a]	0.0283	−0.0630[a]	−0.0554[a]	−0.0197[c]
	(0.0514)	(0.0237)	(0.0341)	(0.0144)	(0.0164)	(0.0109)
Public debt	−0.0109[a]	0.0040	−0.0121[a]	−0.0028	−0.0004	−0.0071[a]
	(0.0036)	(0.0045)	(0.0031)	(0.0032)	(0.0029)	(0.0022)
Const	−0.0571	−0.0680	0.0200	−0.0928[b]	−0.1227[a]	−0.0738[a]
	(0.1110)	(0.0490)	(0.0638)	(0.0363)	(0.0383)	(0.0282)
Trend	−0.0005	−0.0001	0.0040	0.0003	0.0020	0.0028
	(0.0040)	(0.0017)	(0.0033)	(0.0017)	(0.0020)	(0.0018)
IMR-kappa	−0.0063[a]					
	(0.0017)					
Number of obs	90	156	91	155		
Means						
Growth	0.0052	0.0187	0.0050	0.0189		
Public debt	0.8288	0.6656	0.8270	0.6656		
Initial Income	7.5315	8.9378	7.5454	8.9387		
Schooling	0.1268	0.8700	0.1314	0.8721		
Investments	2.9927	3.0768	2.9906	3.0786		
Population growth	−2.6149	−2.7671	−2.6178	−2.7664		
Democracy	1.1737	8.3786	1.2114	8.4030		

[a] Significance at 1%.
[b] Significance at 5%.
[c] Significance at 10%.

Table 7.5 presents the exact sample of countries that apply to each regime and the corresponding period as well as the Democracy scores.

The estimations from this STR model are quite remarkable indicating parameter heterogeneity in the meaning that the effect of debt on growth is based on democracy. Keeping everything else equal, higher public debt causes lower growth in countries where democracy is considered of low-quality. The public debt coefficient for this regime is negative and strongly significant at the 1% level. However, for countries with better quality institutions, i.e., countries in the High-Democracy regime, public debt found to have no significant effect on growth. The results also indicate that countries in the Low-Democracy regime tend to have, on average, higher public debt levels than those in the High-Democracy countries. The mean public debt level for countries in the Low-Democracy regime is around 0.8. So, the results implicitly reveal those in the existing literature that found that the countries exhibiting higher levels of debt are the same that are inclined to present larger negative growth impacts from higher debt levels. Yet, this analysis shows that the threshold parameter that determines the effect of debt on growth is the quality of institutions rather than the level of debt itself. Also, it is noted, that the Low-Democracy regime is also typical of lower growth and income relative to the High-Democracy regime.

Table 7.5

Low- and High-Democracy regimes countries. This table presents the countries marked as Low-Democracy countries (L) (i.e. countries with democracy scores less than or equal to 4.5) and High-Democracy countries (H) (i.e. countries with democracy scores greater than 4.5) for each period.

	1980–1989	1990–1999	2000–2009
Europe			
Austria	(H) 10	(H) 10	(H) 10
Belgium	(H) 10	(H) 10	(H) 9.4
Denmark	(H) 10	(H) 10	(H) 10
Finland	(H) 10	(H) 10	(H) 10
France	(H) 8.4	(H) 9	(H) 9
Greece	(H) 8.8	(H) 10	(H) 10
Ireland	(H) 10	(H) 10	(H) 10
Italy	(H) 10	(H) 10	(H) 10
Netherlands	(H) 10	(H) 10	(H) 10
Norway	(H) 10	(H) 10	(H) 10
Portugal	(H) 9.8	(H) 10	(H) 10
Spain	(H) 9.8	(H) 10	(H) 10
Sweden	(H) 10	(H) 10	(H) 10
United Kingdom	(H) 10	(H) 10	(H) 10
Offshoots			
Australia	(H) 10	(H) 10	(H) 10
Canada	(H) 10	(H) 10	(H) 10
New Zealand	(H) 10	(H) 10	(H) 10
United States	(H) 10	(H) 10	(H) 10
East Asia and the Pacific			
Indonesia	(L) 0	(L) 0.7	(H) 7.6
Japan	(H) 10	(H) 10	(H) 10
Korea Republic of	(L) 1.75	(H) 7.2	(H) 8
Malaysia	(H) 5	(L) 4.5	(L) 4.4
Papua New Guinea	(L) 4	(L) 4	(L) 4
Philippines	(L) 3	(H) 8	(H) 8
Thailand	(L) 3.2	(H) 7.7	(H) 6.6
Europe and Central Asia			
Turkey	(H) 5.7	(H) 8.7	(H) 8
Middle East and North Africa			
Algeria	(L) 0.1	(L) 0.7	(L) 2.2
Cyprus	(H) 10	(H) 10	(H) 10
Egypt	(L) 0	(L) 0	(L) 0.5
Iran	(L) 0	(L) 1.2	(L) 1.6
Israel	(H) 9	(H) 9.1	(H) 10
Morocco	(L) 0	(L) 0	(L) 0
Syria	(L) 0	(L) 0	(L) 0
Tunisia	(L) 0	(L) 0.7	(L) 1
South Asia			
Bangladesh	(L) 0	(H) 5.4	(H) 4.8
India	(H) 8	(H) 8.5	(H) 9
Nepal	(L) 1.8	(H) 5.2	(L) 4.5
Pakistan	(L) 1.6	(H) 7	(L) 1.2
Sri Lanka	(H) 6	(H) 6	(H) 6.6

	1980–1989	1990–1999	2000–2009
Latin America and the Caribbean			
Argentina	(H) 5.5	(H) 7.1	(H) 8
Bolivia	(H) 6.9	(H) 9	(H) 8.2
Brazil	(H) 4.7	(H) 8	(H) 8
Chile	(L) 1	(H) 8	(H) 9.4
Colombia	(H) 8	(H) 7.9	(H) 7
Costa Rica	(H) 10	(H) 10	(H) 10
Dominican Republic	(H) 6	(H) 6.6	(H) 8
Ecuador	(H) 8.6	(H) 8.9	(H) 5.8
Guatemala	(L) 2	(H) 5.6	(H) 8
Guyana	(L) 0	(H) 4.8	(H) 6
Honduras	(H) 5.6	(H) 6.1	(H) 7
Jamaica	(H) 10	(H) 9.3	(H) 9
Mexico	(L) 1.2	(L) 3.8	(H) 8
Nicaragua	(L) 0.625	(H) 7	(H) 8.3
Panama	(L) 0.8	(H) 8.6	(H) 9
Paraguay	(L) 0.3	(H) 6.1	(H) 7.9
Peru	(H) 7	(L) 3.9	(H) 9
Trinidad & Tobago	(H) 8.6	(H) 9.3	(H) 10
Uruguay	(H) 4.6	(H) 10	(H) 10
Venezuela	(H) 9	(H) 8.1	(H) 5.3
Sub-Saharan Africa			
Benin	(L) 0	(H) 6	(H) 6.4
Botswana	(H) 6.3	(H) 7.3	(H) 8
Burundi	(L) 0	(L) 0.25	(L) 4
Cameroon	(L) 0	(L) 0.8	(L) 1
Central African Republic	(L) 0	(L) 3.5	(L) 2.2
Congo Republic of	(L) 0	(L) 3.45	(L) 0
Cote d'Ivoire	(L) 0	(L) 0	(H) 5
Gabon	(L) 0	(L) 0	(L) 0.4
Gambia The	(H) 7.1	(L) 3.2	(L) 0
Ghana	(L) 0.6	(L) 1.675	(H) 7.2
Kenya	(L) 0	(L) 0.6	(H) 6.5
Lesotho	(L) 0	(H) 5.6	(H) 8
Malawi	(L) 0	(L) 3.6	(H) 5.9
Mali	(L) 0	(H) 5.825	(H) 6.8
Mauritania	(L) 0	(L) 0	(L) 0.4
Niger	(L) 0	(L) 4.4	(H) 5.9
Senegal	(L) 2	(L) 2	(H) 7.7
Sierra Leone	(L) 0	(L) 1.25	(H) 5.9
South Africa	(H) 7	(H) 8.33	(H) 9
Swaziland	(L) 0	(L) 0	(L) 0
Togo	(L) 0	(L) 0.83	(L) 1
Zambia	(L) 0	(L) 4.2	(H) 5.2
Zimbabwe	(L) 2.7	(L) 0.1	(L) 1.2

Kourtellos et al. (2013, b) found very little evidence for non-linearities regarding the debt-growth relationship when investigating a number of suggested threshold parameters, surprisingly public debt included. One threshold parameter was found to crucially influence non-linearities, and that is the quality of democracy. Further public debt increases result to lower growth if the measure of a country's institutions lies under a certain quality level. But, if the quality of institution is considered as high, then public debt is growth neutral.

8. Conclusion

Researches focusing on the effects caused by growing sovereign debt of developed countries on economic growth gained new momentum in the past few years. One half of the empirical studies concludes that the rising debt slows down the pace of economic growth while the other half of the analyses says this rule works only above a fix rate of the proportion of debt to GDP. All of them are calling for fiscal austerity measures especially after the massive increase of debt levels over the years following the financial crisis.

But according to the presented results, the issue turns out to be rather confusing, as there are empirical researches indicating that there is no clear evidence about the extent to which we should be concerned about the public debt increase and no clear evidence for the existence of debt thresholds.

Results have important policy implications. If debt thresholds exist, it drives policymakers in favor of pursuing fiscal austerity measures; not only at stabilizing debt but at reducing the debt burden in fear of 'dangerous' debt levels. The most vocal academic supporters of fiscal austerity point out to these panel studies in order to justify their policy recommendations.

If debt thresholds do not exist, a completely different policy approach could be followed. It would imply that priority should be increasing growth, instead of decreasing debt and, therefore, that much less short-term fiscal austerity is required.

Looking at the debt-growth nexus literature two characteristics become apparent. The empirical work that identified robust thresholds should be regarded with skepticism as researchers have not managed to correctly address the endogeneity problem of debt. This means that they may not just interpret to a different extent the negative impact of debt on growth but more importantly that they have not identified the right direction of the causality between the two key macroeconomic variables.

Second to notice was that in many of the cases where a negative non-linear relationship between debt and growth has been established, the thresholds were not robust across samples, specifications and estimation techniques. Heterogeneity is crucial and the aggregate non-linear relationship between debt and subsequent growth might be induced by very different country-specific patterns. Maybe the key is that researchers should recognize that thresholds are importantly country-specific and stop investigating common threshold effects across countries.

Summing up, is spite of the rhetoric already embraced by a number of governments, defining a causal relationship from public debt to growth as well as the potential nonlinearity between them is widely considered as unresolved or at best highly contentious empirical issues. It is remarkable that the majority of the papers analysed stressed the need to further examination of the mechanisms underlying the nonlinearities before making economic policy recommendations, particularly in the high-debt framework.

Turning back to the questions raised in the introduction of this essay we have to say that there is at present no clear agreement on the answer. It is puzzling to deduce a clear-cut relationship that links debt and growth, especially when considering the presence or the absence of nonlinearities. It will remain an issue of heated academic and political debate at the forefront of public policy debate as perhaps are going to remain many of the traditional debt management issues.

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