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THEORY OF RISK MEASURES

By

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1. INTRODUCTION

1.1 THEORY OF RISK

According to Henry Markovitz in 1952 the expected attribution of investment is determined by a diagram of danger-return that the danger is calculated by a line of formal distributions.

In contradiction up to 1950 shaped perception that the more basic element of survival of enterprise was the return of capital, the continuously evolving financing environment and mainly the instability of purchases from 1970 and afterwards led the economic factors to the establishment of line of rules.

This legal framework that is even in effect today with small additions, includes 3 parts:

1. The minimal requirement of capital that should allocate some credit institution so that it copes with the likely danger

2. Process of monitoring of - evaluation

3. [discipline] of market

1.2 MANAGEMENT OF RISK

The objective objectives of management of risk are:

1. The possessed increase of financing attribution of credit institution

2. The guarantee that in a concrete crisis the credit institution will not suffer important losses

The management of risk includes the following stages:

1. Comprehension of risks

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2. Measurement of risks

3. Control

4. Interpretation knowledge of risks

The more basic category of risk management is the measurement of risk that becomes with the following methods:

1. Indicative

2. Measurement with factor

3. Measurement based on the distribution of loss

4. Measurement based on script

1.3 CATEGORIES OF RISK

1. Risk of market that is the loss from change in the value of marketable financial elements

2. Functional risk that is fixed as loss by failed internal processes uncommon exterior makes.

3. Credit risk that results from breach of contracts of enterprises of private individuals with the credit institution.

4. Operational risk, that can involve big pecuniary loss and has the following categories:

1. Loss of control

2. Risk of fluidity mainly because bigger need for financing [ap] that it acquires the credit institution

3. Risk of models, because insufficiency of evaluation of exemplary models

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4. Systemic (risk of systems) from failed drawings systems that cost enough money

5. Risk [reputation] from loss of turnover because reduction of reliability of credit institution.

2. VaR (Value at risk)

The value of portfolio with given level of confidence a, is at risk fixed as the such smaller number l in order that the probability loss L exceeds l is smallest than 1 - a. That is to say: The a can be the 0,95or 0,99 and for interval 1 until 10 days.

Serious disadvantage of VaR is that it in the event does not provide for us information for the size of loss if the probability is smaller than 1 - a. The knowledge of ranks of VaR is important in order to we determine the number r, that is the minimal capital which if we add in the place with damage and invest with risk it would become acceptable.

For example, if a portfolio of stocks has a one-day 5% VaR of $1 million, there is a 0.05 probability that the portfolio will fall in value by more than $1 million over a one day period if there is no trading. Informally, a loss of $1 million or more on this portfolio is expected on 1 day out of 20 days (because of 5% probability). A loss which exceeds the VaR threshold is termed a "VaR break."

A risk measure is defined as a mapping from a set of random variables to the real numbers. This set of random variables represents portfolio returns. The common notation for a risk measure associated with a

random variable X is $\rho(X)$. A risk measure $\rho : \mathcal{L} \to \mathbb{R} \cup \{+\infty\}$ should have certain properties:

Normalized

$$\rho(0) = 0$$

Translative

$$\text{If } a \in \mathbb{R} \text{ and } Z \in \mathcal{L}, \text{ then } \rho(Z + a) = \rho(Z) - a$$

Monotone

$$\text{If } Z_1, Z_2 \in \mathcal{L} \text{ and } Z_1 \leq Z_2, \text{ then } \rho(Z_1) \leq \rho(Z_2)$$

2.1 Set-valued

In a situation with $\mathbb{R}^d$-valued portfolios such that risk can be measured in $m \leq d$of the assets, then a set of portfolios is the proper way to depict risk. Set-valued risk measures are useful for markets.

A set-valued risk measure is a function $R : L_d^p \to \mathbb{F}_M$, where $L_d^p$is a d-dimensional, $\mathbb{F}_M = \{D \subseteq M : D = cl(D + K_M)\}$, and $K_M = K \cap M$where $K$is a constant and $M$is the set of portfolios of the $m$reference assets. R must have the following properties:

Normalized

$$K_M \subseteq R(0) \text{ and } R(0) \cap -\text{int}K_M = \emptyset$$

Translative in M

$$\forall X \in L_d^p, \forall u \in M : R(X + u1) = R(X) - u$$

Monotone

$$\forall X_2 - X_1 \in L_d^p(K) \Rightarrow R(X_2) \supseteq R(X_1)$$

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2.2 Variance

Variance (or standard deviation) is **not** a risk measure. This can be seen since it has neither the translation property nor monotonicity. That is, $Var(X+a) = Var(X) \neq Var(X) - a$ for all $a \in \mathbb{R}$, and a simple counterexample for monotonicity can be found. The standard deviation is a deviation risk measure.

2.3 Relation to acceptance set

There is a one-to-one correspondence between an acceptance set and a corresponding risk measure. As defined below it can be shown that $R_{A_R}(X) = R(X)$ and $A_{R_A} = A$.[4]

2.4 Risk measure to acceptance set

- If ρ is a (scalar) risk measure then $A_\rho = \{X \in L^p : \rho(X) \leq 0\}$ is an acceptance set.
- If R is a set-valued risk measure then $A_R = \{X \in L_d^p : 0 \in R(X)\}$ is an acceptance set.

2.5 Acceptance set to risk measure

- If A is an acceptance set (in 1-d) then $\rho_A(X) = \inf\{u \in \mathbb{R} : X + u1 \in A\}$ defines a (scalar) risk measure.
- If A is an acceptance set then $R_A(X) = \{u \in M : X + u1 \in A\}$ is a set-valued risk measure.

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2.6 Relation with deviation risk measure

There is a one-to-one relationship between a deviation risk measure D and an expectation-bounded risk measure ρ where for any $X \in \mathcal{L}^2$

- $D(X) = \rho(X - \mathbb{E}[X])$
- $\rho(X) = D(X) - \mathbb{E}[X]$.

ρ is called expectation bounded if it satisfies $\rho(X) > \mathbb{E}[-X]$ for any nonconstant X and $\rho(X) = \mathbb{E}[-X]$ for any constant X.

VaR has four main uses in finance: risk management, financial control, financial reporting and computing regulatory capital. VaR is sometimes used in non-financial applications as well.

Important related ideas are economic capital, backtesting, stress testing, expected shortfall, and tail conditional expectation.

Common parameters for VaR are 1% and 5% probabilities and one day and two week horizons, although other combinations are in use.

The reason for assuming normal markets and no trading, and to restricting loss to things measured in daily accounts, is to make the loss observable. In some extreme financial events it can be impossible to determine losses, either because market prices are unavailable or because the loss-bearing institution breaks up. Some longer-term consequences of disasters, such as lawsuits, loss of market confidence and employee morale and impairment of brand names can take a long time to play out, and may be hard to allocate among specific prior decisions. VaR marks the boundary between normal days and extreme events. Institutions can lose far more

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than the VaR amount; all that can be said is that they will not do so very often.

The probability level is about equally often specified as one minus the probability of a VaR break, so that the VaR in the example above would be called a one-day 95% VaR instead of one-day 5% VaR. This generally does not lead to confusion because the probability of VaR breaks is almost always small, certainly less than 0.5.

Although it virtually always represents a loss, VaR is conventionally reported as a positive number. A negative VaR would imply the portfolio has a high probability of making a profit, for example a one-day 5% VaR of negative $1 million implies the portfolio has a 95% chance of making more than $1 million over the next day.

Another inconsistency is that VaR is sometimes taken to refer to profit-and-loss at the end of the period, and sometimes as the maximum loss at any point during the period. The original definition was the latter, but in the early 1990s when VaR was aggregated across trading desks and time zones, end-of-day valuation was the only reliable number so the former became the *de facto* definition. As people began using multiday VaRs in the second half of the 1990s, they almost always estimated the distribution at the end of the period only. It is also easier theoretically to deal with a point-in-time estimate versus a maximum over an interval. Therefore the end-of-period definition is the most common both in theory and practice today.

2.7 Varieties of VaR

The definition of VaR is nonconstructive; it specifies a property VaR must have, but not how to compute VaR. Moreover, there is wide scope

for interpretation in the definition. This has led to two broad types of VaR, one used primarily in risk management and the other primarily for risk measurement. The distinction is not sharp, however, and hybrid versions are typically used in financial control, financial reporting and computing regulatory capital.

To a risk manager, VaR is a system, not a number. The system is run periodically (usually daily) and the published number is compared to the computed price movement in opening positions over the time horizon. There is never any subsequent adjustment to the published VaR, and there is no distinction between VaR breaks caused by input errors (including Information Technology breakdowns, fraud and rogue trading), computation errors (including failure to produce a VaR on time) and market movements.

A frequentist claim is made, that the long-term frequency of VaR breaks will equal the specified probability, within the limits of sampling error, and that the VaR breaks will be independent in time and independent of the level of VaR. This claim is validated by a backtest, a comparison of published VaRs to actual price movements. In this interpretation, many different systems could produce VaRs with equally good backtests, but wide disagreements on daily VaR values.

For risk measurement a number is needed, not a system.

A Bayesian probability claim is made, that given the information and beliefs at the time, the subjective probability of a VaR break was the specified level. VaR is adjusted after the fact to correct errors in inputs and computation, but not to incorporate information unavailable at the time of computation. In this context, "backtest" has a different meaning.

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Rather than comparing published VaRs to actual market movements over the period of time the system has been in operation, VaR is retroactively computed on scrubbed data over as long a period as data are available and deemed relevant. The same position data and pricing models are used for computing the VaR as determining the price movements.

Although some of the sources listed here treat only one kind of VaR as legitimate, most of the recent ones seem to agree that risk management VaR is superior for making short-term and tactical decisions today, while risk measurement VaR should be used for understanding the past, and making medium term and strategic decisions for the future. When VaR is used for financial control or financial reporting it should incorporate elements of both. For example, if a trading desk is held to a VaR limit, that is both a risk-management rule for deciding what risks to allow today, and an input into the risk measurement computation of the desk's risk-adjusted return at the end of the reporting period.

2.8 VaR in Governance

VaR can also be applied to governance of endowments, trusts, and pension plans. Essentially trustees adopt portfolio Values-at-Risk metrics for the entire pooled account and the diversified parts individually managed. Instead of probability estimates they simply define maximum levels of acceptable loss for each. Doing so provides an easy metric for oversight and adds accountability as managers are then directed to manage, but with the additional constraint to avoid losses within a defined risk parameter. VaR utilized in this manner adds relevance as well as an easy way to monitor risk measurement control far more intuitive than Standard Deviation of Return. Use of VaR in this context, as well as a worthwhile critique on board governance practices as it

relates to investment management oversight in general can be found in *Best Practices in Governance.*

2.9 Mathematical definition

As we say before, given a confidence level $\alpha \in (0,1)$, the VaR of the portfolio at the confidence level a is given by the smallest number $l$such that the probability that the loss Lexceeds $l$is at most $(1-\alpha)$.

Mathematically, if $L$is the loss of a portfolio, then $\mathrm{VaR}_\alpha(L)$ is the level α-quantile, i.e.

$$\mathrm{VaR}_\alpha(L) = \inf\{l \in \mathbb{R} : P(L > l) \leq 1-\alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\}.$$

The left equality is a definition of VaR The right equality assumes an underlying probability distribution, which makes it true only for parametric VaR Risk managers typically assume that some fraction of the bad events will have undefined losses, either because markets are closed or illiquid, or because the entity bearing the loss breaks apart or loses the ability to compute accounts. Therefore, they do not accept results based on the assumption of a well-defined probability distribution. Nassim Taleb has labeled this assumption, "charlatanism."

On the other hand, many academics prefer to assume a well-defined distribution, albeit usually one with fat tails. This point has probably caused more contention among VaR theorists than any other.

Value of Risks can also be written as a distortion risk measure given by the distortion function $g(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1-\alpha \\ 1 & \text{if } 1-\alpha \leq x \leq 1 \end{cases}$.[15][16]

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2.10 Risk measure and risk metric

The term "VaR" is used both for a risk measure and a risk metric. This sometimes leads to confusion. Sources earlier than 1995 usually emphasize the risk measure, later sources are more likely to emphasize the metric.

The VaR risk measure defines risk as mark-to-market loss on a fixed portfolio over a fixed time horizon, assuming normal markets. There are many alternative risk measures in finance. Instead of mark-to-market, which uses market prices to define loss, loss is often defined as change in fundamental value. For example, if an institution holds a loan that declines in market price because interest rates go up, but has no change in cash flows or credit quality, some systems do not recognize a loss. Or we could try to incorporate the economic cost of things not measured in daily financial statements, such as loss of market confidence or employee morale, impairment of brand names or lawsuits.

Rather than assuming a fixed portfolio over a fixed time horizon, some risk measures incorporate the effect of expected trading (such as a stop loss order) and consider the expected holding period of positions. Finally, some risk measures adjust for the possible effects of abnormal markets, rather than excluding them from the computation.

The VaR risk metric summarizes the distribution of possible losses by a quantile, a point with a specified probability of greater losses.

Common alternative metrics are standard deviation, mean absolute deviation, expected shortfall and downside risk.

2.11 VaR risk management

Supporters of VaR-based risk management claim the first and possibly greatest benefit of VaR is the improvement in systems and modeling it forces on an institution. In 1997, Philippe Jorion wrote:

The greatest benefit of VAR lies in the imposition of a structured methodology for critically thinking about risk. Institutions that go through the process of computing their VAR are forced to confront their exposure to financial risks and to set up a proper risk management function. Thus the process of getting to VAR may be as important as the number itself.

Publishing a daily number, on-time and with specified statistical properties holds every part of a trading organization to a high objective standard. Robust backup systems and default assumptions must be implemented. Positions that are reported, modeled or priced incorrectly stand out, as do data feeds that are inaccurate or late and systems that are too-frequently down. Anything that affects profit and loss that is left out of other reports will show up either in inflated VaR or excessive VaR breaks. "A risk-taking institution that *does not* compute VaR might escape disaster, but an institution that *cannot* compute VaR will not."

The second claimed benefit of VaR is that it separates risk into two regimes. Inside the VaR limit, conventional statistical methods are reliable. Relatively short-term and specific data can be used for analysis. Probability estimates are meaningful, because there are enough data to test them. In a sense, there is no true risk because you have a sum of many independent observations with a left bound on the outcome. A casino doesn't worry about whether red or black will come up on the next roulette spin. Risk managers encourage productive risk-taking in this

regime, because there is little true cost. People tend to worry too much about these risks, because they happen frequently, and not enough about what might happen on the worst days.

Outside the VaR limit, all bets are off. Risk should be analyzed with stress testing based on long-term and broad market data. Probability statements are no longer meaningful. Knowing the distribution of losses beyond the VaR point is both impossible and useless. The risk manager should concentrate instead on making sure good plans are in place to limit the loss if possible, and to survive the loss if not.

One specific system uses three regimes.

One to three times VaR are normal occurrences. You expect periodic VaR breaks. The loss distribution typically has fat tails, and you might get more than one break in a short period of time. Moreover, markets may be abnormal and trading may exacerbate losses, and you may take losses not measured in daily marks such as lawsuits, loss of employee morale and market confidence and impairment of brand names. So an institution that can't deal with three times VaR losses as routine events probably won't survive long enough to put a VaR system in place.

1. Three to ten times VaR is the range for stress testing. Institutions should be confident they have examined all the foreseeable events that will cause losses in this range, and are prepared to survive them. These events are too rare to estimate probabilities reliably, so risk/return calculations are useless.
2. Foreseeable events should not cause losses beyond ten times VaR. If they do they should be hedged or insured, or the business plan should be changed to avoid them, or VaR should be increased. It's

> hard to run a business if foreseeable losses are orders of magnitude larger than very large everyday losses. It's hard to plan for these events, because they are out of scale with daily experience. Of course there will be unforeseeable losses more than ten times VaR, but it's pointless to anticipate them, you can't know much about them and it results in needless worrying. Better to hope that the discipline of preparing for all foreseeable three-to-ten times VaR losses will improve chances for surviving the unforeseen and larger losses that inevitably occur.

"A risk manager has two jobs: make people take more risk the 99% of the time it is safe to do so, and survive the other 1% of the time. VaR is the border."

2.12 VaR risk measurement

The VaR risk measure is a popular way to aggregate risk across an institution. Individual business units have risk measures such as duration for a fixed income portfolio or beta for an equity business. These cannot be combined in a meaningful way. It is also difficult to aggregate results available at different times, such as positions marked in different time zones, or a high frequency trading desk with a business holding relatively illiquid positions. But since every business contributes to profit and loss in an additive fashion, and many financial businesses mark-to-market daily, it is natural to define firm-wide risk using the distribution of possible losses at a fixed point in the future.

In risk measurement, VaR is usually reported alongside other risk metrics such as standard deviation, expected shortfall and "greeks" (partial derivatives of portfolio value with respect to market factors). VaR is a

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distribution-free metric, that is it does not depend on assumptions about the probability distribution of future gains and losses. The probability level is chosen deep enough in the left tail of the loss distribution to be relevant for risk decisions, but not so deep as to be difficult to estimate with accuracy.

2.13 Computational methods

VaR can be estimated either parametrically (for example, variance-covariance VaR or delta-gamma VaR) or nonparametrically (for examples, historical simulation VaR or resampled VaR). Nonparametric methods of VaR estimation are discussed in Markovich and Novak.

A McKinsey report published in May 2012 estimated that 85% of large banks were using historical simulation. The other 15% used Monte Carlo methods.

3. History of VaR

The problem of risk measurement is an old one in statistics, economics and finance. Financial risk management has been a concern of regulators and financial executives for a long time as well. Retrospective analysis has found some VaR-like concepts in this history. But VaR did not emerge as a distinct concept until the late 1980s. The triggering event was the stock market crash of 1987. This was the first major financial crisis in which a lot of academically-trained quants were in high enough positions to worry about firm-wide survival.

The crash was so unlikely given standard statistical models, that it called the entire basis of quant finance into question. A reconsideration of history led some quants to decide there were recurring crises, about one

or two per decade, that overwhelmed the statistical assumptions embedded in models used for trading, investment management and derivative pricing. These affected many markets at once, including ones that were usually not correlated, and seldom had discernible economic cause or warning (although after-the-fact explanations were plentiful).[21] Much later, they were named "Black Swans" by Nassim Taleb and the concept extended far beyond finance.

If these events were included in quantitative analysis they dominated results and led to strategies that did not work day to day. If these events were excluded, the profits made in between "Black Swans" could be much smaller than the losses suffered in the crisis. Institutions could fail as a result.

VaR was developed as a systematic way to segregate extreme events, which are studied qualitatively over long-term history and broad market events, from everyday price movements, which are studied quantitatively using short-term data in specific markets. It was hoped that "Black Swans" would be preceded by increases in estimated VaR or increased frequency of VaR breaks, in at least some markets. The extent to which this has proven to be true is controversial.

Abnormal markets and trading were excluded from the VaR estimate in order to make it observable. It is not always possible to define loss if, for example, markets are closed as after 9/11, or severely illiquid, as happened several times in 2008. Losses can also be hard to define if the risk-bearing institution fails or breaks up. A measure that depends on traders taking certain actions, and avoiding other actions, can lead to self reference.

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This is risk management VaR. It was well established in quantitative trading groups at several financial institutions, notably Bankers Trust, before 1990, although neither the name nor the definition had been standardized. There was no effort to aggregate VaRs across trading desks.

The financial events of the early 1990s found many firms in trouble because the same underlying bet had been made at many places in the firm, in non-obvious ways. Since many trading desks already computed risk management VaR, and it was the only common risk measure that could be both defined for all businesses and aggregated without strong assumptions, it was the natural choice for reporting firmwide risk. J. P. Morgan CEO Dennis Weatherstone famously called for a "4:15 report" that combined all firm risk on one page, available within 15 minutes of the market close.

Risk measurement VaR was developed for this purpose. Development was most extensive at J. P. Morgan, which published the methodology and gave free access to estimates of the necessary underlying parameters in 1994. This was the first time VaR had been exposed beyond a relatively small group of quants. Two years later, the methodology was spun off into an independent for-profit business now part of RiskMetrics Group.

In 1997, the U.S. Securities and Exchange Commission ruled that public corporations must disclose quantitative information about their derivatives activity. Major banks and dealers chose to implement the rule by including VaR information in the notes to their financial statements.

Worldwide adoption of the Basel II Accord, beginning in 1999 and nearing completion today, gave further impetus to the use of VaR. VaR is

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the preferred measure of market risk, and concepts similar to VaR are used in other parts of the accord

4. Criticism

VaR has been controversial since it moved from trading desks into the public eye in 1994. A famous 1997 debate between Nassim Taleb and Philippe Jorion set out some of the major points of contention. Taleb claimed VaR:

1. Ignored 2,500 years of experience in favor of untested models built by non-traders
2. Was charlatanism because it claimed to estimate the risks of rare events, which is impossible
3. Gave false confidence
4. Would be exploited by traders

More recently David Einhorn and Aaron Brown debated VaR in Global Association of Risk Professionals Review Einhorn compared VaR to

"an airbag that works all the time, except when you have a car accident." He further charged that VaR:

1. Led to excessive risk-taking and leverage at financial institutions
2. Focused on the manageable risks near the center of the distribution and ignored the tails
3. Created an incentive to take "excessive but remote risks"
4. Was "potentially catastrophic when its use creates a false sense of security among senior executives and watchdogs."

New York Times reporter Joe Nocera wrote an extensive piece Risk Mismanagement on January 4, 2009 discussing the role VaR played in

the Financial crisis of 2007-2008. After interviewing risk managers (including several of the ones cited above) the article suggests that VaR was very useful to risk experts, but nevertheless exacerbated the crisis by giving false security to bank executives and regulators. A powerful tool for professional risk managers, VaR is portrayed as both easy to misunderstand, and dangerous when misunderstood.

Taleb, in 2009, testified in Congress asking for the banning of VaR on two arguments, the first that "tail risks are non-measurable" scientifically and the second is that for anchoring reasons VaR for leading to higher risk taking.

A common complaint among academics is that VaR is not subadditive. That means the VaR of a combined portfolio can be larger than the sum of the VaRs of its components. To a practising risk manager this makes sense. For example, the average bank branch in the United States is robbed about once every ten years. A single-branch bank has about 0.0004% chance of being robbed on a specific day, so the risk of robbery would not figure into one-day 1% VaR. It would not even be within an order of magnitude of that, so it is in the range where the institution should not worry about it, it should insure against it and take advice from insurers on precautions. The whole point of insurance is to aggregate risks that are beyond individual VaR limits, and bring them into a large enough portfolio to get statistical predictability. It does not pay for a one-branch bank to have a security expert on staff.

As institutions get more branches, the risk of a robbery on a specific day rises to within an order of magnitude of VaR.

At that point it makes sense for the institution to run internal stress tests and analyze the risk itself. It will spend less on insurance and more on in-house expertise. For a very large banking institution, robberies are a routine daily occurrence. Losses are part of the daily VaR calculation, and tracked statistically rather than case-by-case. A sizable in-house security department is in charge of prevention and control, the general risk manager just tracks the loss like any other cost of doing business.

As portfolios or institutions get larger, specific risks change from low-probability/low-predictability/high-impact to statistically predictable losses of low individual impact. That means they move from the range of far outside VaR, to be insured, to near outside VaR, to be analyzed case-by-case, to inside VaR, to be treated statistically.

Even VaR supporters generally agree there are common abuses of VaR:

1. Referring to VaR as a "worst-case" or "maximum tolerable" loss. In fact, you expect two or three losses per year that exceed one-day 1% VaR.
2. Making VaR control or VaR reduction the central concern of risk management. It is far more important to worry about what happens when losses exceed VaR.
3. Assuming plausible losses will be less than some multiple, often three, of VaR. The entire point of VaR is that losses can be extremely large, and sometimes impossible to define, once you get beyond the VaR point. To a risk manager, VaR is the level of losses at which you stop trying to guess what will happen next, and start preparing for anything.
4. Reporting a VaR that has not passed a backtest. Regardless of how VaR is computed, it should have produced the correct number of

breaks (within sampling error) in the past. A common specific violation of this is to report a VaR based on the unverified assumption that everything follows a multivariate normal distribution.

5. VaR, CVaR and EVaR

The VaR is not a coherent risk measure since it violates the sub-additivity property, which is

$$\text{If } X, Y \in \mathbf{L}, \text{ then } \rho(X+Y) \leq \rho(X) + \rho(Y).$$

However, it can be bounded by coherent risk measures like Conditional Value-at-Risk (CVaR) or entropic value at risk (EVaR). In fact, for $X \in \mathbf{L}_M+$(with $\mathbf{L}_M+$the set of all Borel measurable functions whose moment-generating function exists for all positive real values) we have

$$\mathrm{VaR}_{1-\alpha}(X) \leq \mathrm{CVaR}_{1-\alpha}(X) \leq \mathrm{EVaR}_{1-\alpha}(X),$$

where

$$\begin{aligned}
\mathrm{VaR}_{1-\alpha}(X) &:= \inf_{t \in \mathbf{R}} \{t : \Pr(X \leq t) \geq 1-\alpha\}, \\
\mathrm{CVaR}_{1-\alpha}(X) &:= \frac{1}{\alpha} \int_0^{\alpha} \mathrm{VaR}_{1-\gamma}(X) d\gamma, \\
\mathrm{EVaR}_{1-\alpha}(X) &:= \inf_{z>0} \{z^{-1} \ln(M_X(z)/\alpha)\},
\end{aligned}$$

in which $M_X(z)$is the moment-generating function of Xat z. In the above equations the variable X denotes the financial loss, rather than wealth as is typically the case

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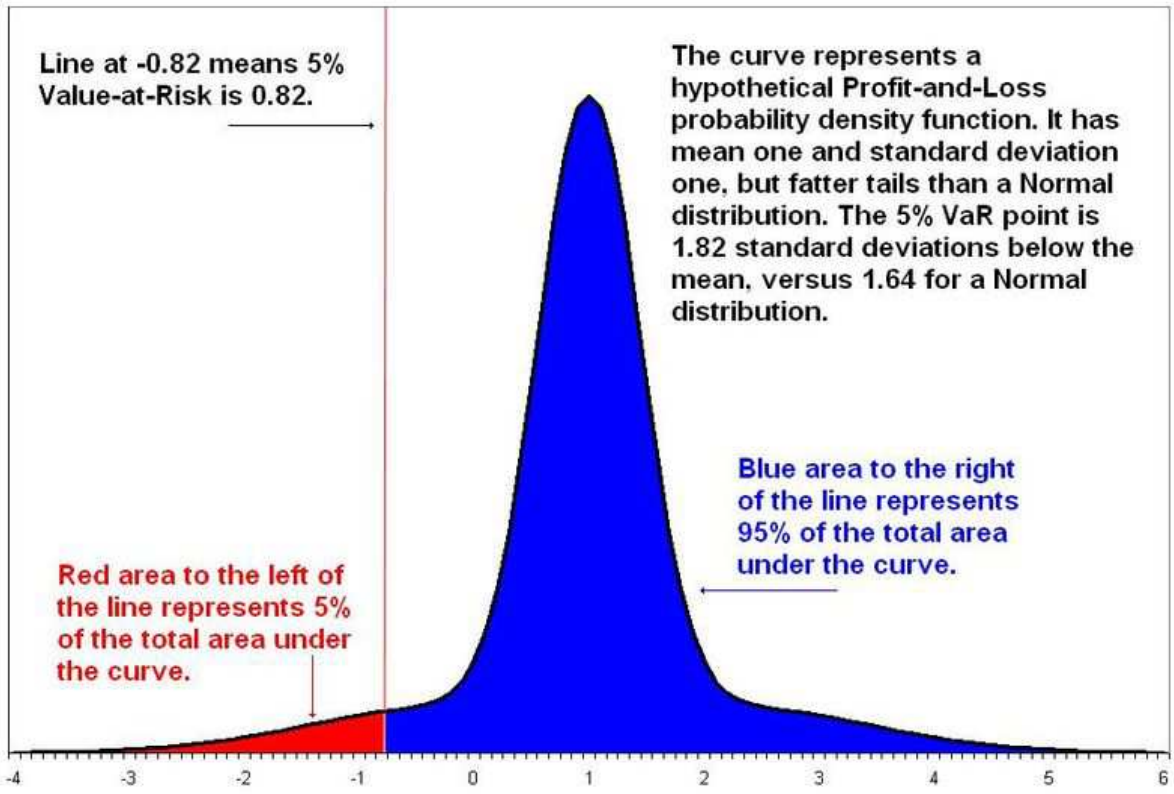
VaR diagram



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6. Stock market indices application

On October 29, 1929, known as black Tuesday, and the stock market in New York collapsed, losing 14 billion dollars that day. i.e., to have a benchmark, ten times more than the annual budget of the Federal Government. Until 1920, the stock market was a market for professionals. In the mid 1920s the market began to widen, resulting in more and more Americans to acquire shares of corporations.

Indeed, in the mid-1920s there was impressive growth and the index of the New York Stock Exchange grows up from 60 to 400 units. For average investors investing in equities was a decision with zero risk, putting their homes mortgage or putting their life savings or borrowers money from banks, without studying the fundamentals of companies. However, as supported by Machiavelli, "most people's big flaw is that the good weather did not contemplate" the storm ". That day the Dow Jones fell 12% and the market lost 14 billion. from its value. It is estimated that 140 billion. deposits disappeared, banks and financial firms collapsed, people committed suicide and millionaires businessmen came to become peddlers at street corners. From one moment to the other one-third of Americans found beneath the poverty line. Feature of the climate that prevailed at the time it was speculation and the lightness of the decisions of investors.

Studying similar subsequent financial crises we observe that the investor is exposed to many and varied risks. Therefore, the investor takes steps to avoid them in various ways. A way to spread their risks by the investor is the so-called risk diversification. Diversification in finance means the reduction of the risk of placing his money into various investments, creating a portfolio. Referring to the term "portfolio" refers to the total of the assets (stocks, bonds, real estate, currencies, precious metals, etc.) in the next few paragraphs will demonstrate the benefits of diversification with a specific application.

Traditionally, the analysis of investment in shares of companies listed or not in the stock market recognizes the special importance of performance and risk.

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6.1 Stock performance

Performance for an individual share is expressed by dividend yield, as well as capital gains or losses.

The stock's performance is expressed by the following formula:

$$R_{at} = \frac{D_{at} + (P_{at} - 1)}{P_{at} - 1}$$

Where:

Rat: a stock's performance in period t.

Dat: a Stock Dividend in period t.

Pat: the stock price a t at the end of the period.

Pat – 1: the stock price a at the beginning of period t.

6.2 Risk and uncertainty

Although the two words "risk" and "uncertainty" are used interchangeably, technically as concepts are different. The term ' risk ' means that he gets a decision knowing the possible consequences of his decision and the associated probabilities during the time it takes the decision. The term "uncertainty" means a situation about which the chance of possible decisions (outcomes) is not known.

The risk for an individual share is expressed in relation to the volatility of its stock that yields. The volatility of share yield is expressed as the variance (σ2) and standard deviation (σ). The use of the standard deviation (σ), which is the square root of the variance, has prevailed because it is expressed as a percentage and is therefore comparable. Therefore, the standard deviation is a measure of risk calculation (Bernstein, 1998). I.e. a share with a high standard deviation reflects high volatility, and therefore a high risk.

An additional measure to determine the risk of a stock is the coefficient of variation. The coefficient of variation is the ratio of the standard deviation to the mean.

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The coefficient of variation of a share is expressed by the following formula:

$$coefficientof\ \text{var}iation = \frac{s\tan darddeviation}{averageyield}$$

Stock = standard deviation coefficient of variation Average yield

This measure compares the scattering of a stock relative to its performance.

Application:

Whether or not the returns of stock below (tab. 1). We will calculate the odds, variances, standard deviations and the coefficients of variations of each one separately for the shares (a) and (b) we have:

Yield (a)

Is: (2.4% + 4.35% + 3.9% + 6.2% + 6.7% tax)/5 = 4.71% Similarly to share (b).

Yield b

Is: (+ 8.2% 9.10% + 7.10% + 6.40% + 5.30)/5 = 7,22%

Stock variation is: (2.40%-4.71%) and 2 + (4.35%-4.71%) and 2 + (3.90%-4.71%) and 2 + (8.2%-4.71%) and 2 + (8.70%-4.71%) and 2 = 0.0003

Stock variation b

Is: (8.2%-7,22 2%)) + (8.10%-7,22%) and 2 + (7.10%-7,22%) and 2 + (6.40%-7,22%) and 2 + (5.30%-7,22%) and 2 = 0.0002

Stock standard deviation s

They are: the square root of the variance of the stock (a), i.e.: 1,75%.

Stock b standard deviation

Is:

The square root of the variance of the stock b, namely: 1.49%. Stock volatility factor A

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Is: 1.75%/4.71% = 37,23%.

Stock volatility factor b

Is: 1.48%/% = 20,62 7,22%.

We have aggregated the following table 2.

Table 1:

Time	Stock variation A %	Stock variation B %
1	2,4	8,2
2	4,35	9,1
3	3,9	7,1
4	6,2	6,4
5	6,7	5,3

Table 2:

	Stock variation A %	Stock variation B %
STOCK VARIATION	4.71	7.22
VARIATION	0.0003	0.0002
STANDARD DEVIATION	1.75	1.49
CV	37.23	20.62

These measures risk identification refer to individual stocks.

6.3 What factors pose the risk.

The factors that create fluctuations in returns-stock prices are factors of risk. Some factors (external), such as changes in interest rates, the rate of inflation, the level of unemployment, exchange rates, can not be controlled by the company and affect a large number of shares. Some other factors (internal), as the company's bargaining power with its suppliers, competition within the industry, the research and development of new products and services, can be controlled largely by the company.

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These factors cannot be controlled by the company regarding sources of systemic risk. These factors that can be controlled by the company and who are particular about certain sectors or companies relate to non-systemic risk sources or specific risk.

Systemic risk refers to the proportion of the total variance of the stock yields due to economic, political and social factors. For example, when the economy moving toward recession and corporate profits shrink, then stock prices of awarded-NIEs are pushed down.

In a study done by Lonie, Power and Sinclair (1990, 1992) found that when lending rates increase dramatically, there is a big impact to the profit and loss account of companies with high leverage ratio. On the New York Stock Exchange averaged 50% of price volatility a particular company is affected (explained) by the variability of the indicator. That is, half the average risk for a common share on the systematic risk (Donald e. Fischer, Ronald j. Jordan 1991).

The non-systematic risk is the percentage of total risk that is unique to each company or industry. Factors relating to sources of this risk is consumer habits, the ability of the administration of workers ' strikes and Companies with high systemic risk tend sales, profits and stock prices to below the level of overall economic activity. These companies are companies that produce commodities and raw materials. For example, car companies and all companies related to automobile production, such as iron industries, industries that produce tires or glass industries. Companies with larger non-systemic risk are companies that produce non-durable consumer goods. For example, commodity suppliers (food, telecommunications, electricity, etc.) sales, profits and stock prices do not depend so much on economic activity.

6.4 Portfolio creation

An investor as usual maintains more than one asset (shares, bonds, mutual funds, etc.) therefore the investor no longer cares about the performance and the risk of the entire portfolio.

6.5 Portfolio performance

The yield of the portfolio is the weighted average of the individual assets that make up (e.g. stocks, bonds, mutual funds, term deposits, foreign exchange, precious metals, etc.)

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6.6 Portfolio risk

As mentioned, the risk is expressed by the measure of the standard deviation. The standard deviation of the portfolio is expressed by the following formula: portfolio Variance = x = 2A * VARAB σ2 + 2 * HA * HV * sat + Ch2B * s2B

Portfolio standard deviation = √ VAR if you consider the shares (a) and (b) of the above application as a portfolio by 40% of the money in stock and 60% in stock b, then return and risk will be:

6.7 Portfolio, performance shares (a & b)

Is: (4.71% x 40%) + (7,22% x 60%) = 6.22% Variance portfolio, shares (a & b) Is: (0.4) 2 * 2 + 0.0003 * 0.6 * 0.4 * (-0.02%) + (0.6) 2 * 0,0044 = 0.0002% Is portfolio standard deviation: 0.67%

6.8 Portfolio weighted standard deviation, equities (s & b)

Is: (1.75% * 0.4) + (1.49% * 0.6) = 1.59%

Aggregated we have the following table:

Portfolio performance	6.22%
Portfolio variance	0.0044%
Portfolio standard deviation	0.67%
Portfolio weighted standard deviation	1.59%

Note that the standard deviation of the portfolio (0.67 percent). is less (1.59 percent). compared to the weighted average of the individual standard deviations – risks. This is because adding shares in the portfolio to reduce the risk of the portfolio. The phenomenon of risk reduction which is obtained by adding the shares in portfolio risk diversification – called dispersion. Two approaches have been developed regarding the creation of a diversified portfolio. The first approach is based on a random selection of shares that make up the portfolio. I.e. the investor "does not put all the eggs in one basket," sharing the risk in many stocks, and thus the risk is much smaller than a portfolio with very few shares.

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Therefore the whole non-systematic risk disperse, leaving the systematic risk or market risk which are non-differentiated (Solnik1992). See figure.

O effective non-systematic risk reduction can be realized even in portfolios with a small number of shares, i.e. about 30 shares (Pike & Neale, 1993).

According to Evans, there is a steady and predictable relationship between the number of shares that are included in a portfolio and the level of risk spreading. Evans argues that the non-systematic risk decreases dramatically from adding to the stock portfolio of 8, while from the 9th until the 40th share the non-systematic risk reduction are marginal.

The second approach was developed by Markowitz (1965), which relies on the use of statistical tools, such as correlation analysis (correlation analysis) and Covariance (covariance). The correlation analysis calculate the relationship between two variables, such as, for example, the odds of a and b shares, and takes values from-1 to + 1. the formula is the following:

$$P_{A,B} = \frac{COV_{A,B}}{\sigma_A \sigma_B}$$

Where:

PA,B = correlation coefficient.

COVAB = Covariance of shares (a) and (b).

σA = standard deviation of a.

σB = standard deviation of the b.

correlation coefficient= -76,7%

The Markowitz proved that two perfectly negatively related stock returns, i.e. (PA, B =-1) and with the appropriate weighting in the portfolio, the whole risk, systematic and specific, can be differentiated.

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Namely, the moderate performance of a share may be offset by the good performance of a stock b. Practically, the perfect negative correlation is an extreme case. In the above application the correlation coefficient is:

or-0,7670.

correlation coefficient= -0,02%

This means that the fluctuations of the stock yields a explained in 76,70% from fluctuations in stock b. yields If the value of p is less than + 1, then the risk of the portfolio decreases. More portfolio risk is reduced if the correlation coefficient is negative. The covariance is equal to the product of deviations of returns of the shares by the average. The formula is the following:

CovA, B = PA,B * σA * σB

In the above application the covariance is: the rate «beta» essentially is the covariance of returns of a stock with the market returns, dividing by the variance of the returns of the market. It is therefore a measure of systematic risk of equity. Ie:

$$BETA = \frac{COV_{A,M}}{\sigma_{m^2}}$$

Where:

CovAm: the performance of the stock population covariance with the market yields set m.

in m2: variation of market.

When the coefficient "beta" is equal to the unit, this means that the systematic risk of equity is equal to the market risk. When it is greater than, e.g., 1.50, then for any change in the General index by 1% the share will vary by 1.5%. The participle in this case characterized as aggressive. When the unit is shortened, e.g. 0,50, for any change in the General Index by 1% then will vary by 0.5%. The participle in this case characterized as defensive. Aggressive share means that when the General index is on the rise, the share will give higher yields compared to the market. In the case of the fall of the General index, the share will record greater losses in the market. Defensive stock means that when the General index is on the rise,

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the share will give fewer returns, but also less damage in the case that the overall index is in decline.

6.9 E select portfolio

From the above brief analysis leads to the conclusion that three key factors influence in reducing the risk of a portfolio in comparison with the standard deviation of individual shares.

These factors are as follows:

(a) the degree of Association) (p), which must be less than the unit.

(b) the number of shares that includes the portfolio, and

(c) the ratio of individual shares in the portfolio in relation to their mutual correlations.

In recent years there have been several studies and experiments from other scientific fields, such as psychology, sociology, anthropology, and even of medicine, from which the conclusion was that the investor decides not based on "economic rationalism" but taking into account human frailties and emotions. For example, many people have invested in shares of a company listed on the stock market because they just heard that this company is "good" or, for example, gives good dividend or profit, without further to consider the fundamentals of the company or the historical data that stock yields or to gather information from published reports.

Therefore, the portfolio selection is a process that takes design and great care.

O general formula for determining the risk of a portfolio consisting of n elements is the following:

$$\sigma_\rho = (\Sigma^n_{i=1} \Sigma^n_{j=1} W_i W_j p_{ij} \sigma_i \sigma_j)^{1/2}$$

To determine the real danger therefore let's look at the example below.

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instance

Suppose there are two assets (shares in a public limited company and an apartment) having the following characteristics:

ASSETS	ANTICIPATED RETURN $E(R_i)$	DANGER σ_i
SHARE	30%	30%
FLAT	10%	10%

Suppose also that the two elements involved in equal shares to the value of the portfolio, W1 = W2 = 0,5
The expected return of the portfolio is equal to

$$\mathbf{E(R_p)= (0,5)(0,30) + (0,5)(0,10)= 0,20\ ή\ 20\%}$$

For the determination of the risk in the portfolio consists of only two elements have:

$$\mathbf{\sigma_\rho=(W^2_1\sigma^2_1+ W^2_2\sigma^2_2+2W_1W_2\sigma_1\sigma_2\rho_{12})^{1/2}}$$

With the data of example we have therefore:

$$\mathbf{\sigma_\rho=(0,5)^2(0,30)^2 + (0,5)^2(0,10)^2 + 2(0,5)(0,5)(0,30)(0,10)p_{12}=(0,025) + (0,15)p_{12}}$$

If between the two elements was perfect positive correlation with respect to their efficiencies (p12 = + 1), the risk of the portfolio would be to:

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$$\sigma_{\rho}=(0,025) + (0,015)(1)=0,04=0,20 \text{ ή } 20\%$$

Although p12 was again equal to 0.5, the risk of the portfolio would be equal to:

$$\sigma_{\rho}=(0,025) + (0,015)(0,5)=0,0325=0,18 \text{ ή } 18\%$$

and if there was a perfect negative correlation (p12 = -1) the risk would be much lower and equal to:

$$\sigma_{\rho}=(0,025) + (0,015)(-1)=0,01=0,10 \text{ ή } 10\%$$

It is evident from the above example, that other factors constant, the risk of the portfolio decreases the smaller the degree of correlation between all pairs of assets, its component.
Course to achieve an optimal portfolio depends not only on the existence of assets as possible lesser degrees of correlation between them, but also the combinations will be regarding participation rates overall portfolio value (Wi).

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