https://doi.org/10.26219/heal.aueb.6210

ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ
ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

SCHOOL OF INFORMATION SCIENCES & TECHNOLOGY

DEPARTMENT OF STATISTICS

POSTGRADUATE PROGRAM

EXAMINING THE EVOLUTION OF EARLY ADULT MORTALITY

By

Eleni Kelli

A THESIS
Submitted to the Department of Statistics
of the Athens University of Economics and Business
in partial fulfilment of the requirements for
the degree of Master of Science in Statistics

Athens, Greece
September 2015

https://doi.org/10.26219/heal.aueb.6210

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ΣΧΟΛΗ ΕΠΙΣΤΗΜΩΝ & ΤΕΧΝΟΛΟΓΙΑΣ ΤΗΣ ΠΛΗΡΟΦΟΡΙΑΣ

ΤΜΗΜΑ ΣΤΑΤΙΣΤΙΚΗΣ

ΜΕΤΑΠΤΥΧΙΑΚΟ

ΔΙΑΧΡΟΝΙΚΗ ΕΞΕΛΙΞΗ ΤΗΣ ΘΝΗΣΙΜΟΤΗΤΑΣ ΝΕΑΡΩΝ ΕΝΗΛΙΚΩΝ

Ελένη Κέλλη

ΔΙΑΤΡΙΒΗ

Που υποβλήθηκε στο Τμήμα Στατιστικής
του Οικονομικού Πανεπιστημίου Αθηνών
ως μέρος των απαιτήσεων για την απόκτηση
Μεταπτυχιακού Διπλώματος Ειδίκευσης στη Στατιστική

Αθήνα
Σεπτέμβριος 2015

https://doi.org/10.26219/heal.aueb.6210

DEDICATION

This thesis is heartily dedicated to my father who was always proud of me.

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ACKNOWLEDGEMENTS

Foremost, I wish to express my sincere thanks to my supervisor, Professor in the Department of Statistics, Dr. Anastasia Kostaki, for her guidance, understanding and willingness to help me whenever needed during the preparation of this thesis. I would also like to thank my family for the encouragement and the support that they provided me.

https://doi.org/10.26219/heal.aueb.6210

https://doi.org/10.26219/heal.aueb.6210

VITA

My name is Eleni Kelli. I was born and grew up in Megara. In 2006 I graduated from the 2nd High School of Megara. From 2006 to 2011 I was a student at the Department of Mathematics in the National and Kapodistrian University of Athens. In October 2013, I was admitted in the Master Program in Statistics, full time, in the Athens University of Economics and Business. This thesis presents the completion of my studies at a Master level.

https://doi.org/10.26219/heal.aueb.6210

https://doi.org/10.26219/heal.aueb.6210

ABSTRACT

Eleni Kelli

Examining Early Adult Mortality

September 2015

A common problem faced by demographers is the estimation of the age-specific mortality pattern when the data is given in age groups. In this study we present a method that one can adopt when comes up against this problem i.e. the need to expand an abridged life table to a complete one. Additionally, we try to capture the changes in age patterns of mortality of several populations over the years. For this purpose, we consider abridged data sets for selected countries. Then we apply the expanding technique for estimating the age-specific probabilities of dying. The main tool of this technique is the Heligman and Pollard formula, which is convenient and effective for studying separately young, middle and old age patterns of mortality. This advantage allows us to examine the early adult mortality and how it has changed over the time.

https://doi.org/10.26219/heal.aueb.6210

https://doi.org/10.26219/heal.aueb.6210

ΠΕΡΙΛΗΨΗ

Ελένη Κέλλη

Εξέταση της Θνησιμότητας Νεαρών Ενηλίκων

Σεπτέμβριος 2015

Ένα σύνηθες πρόβλημα που αντιμετωπίζουν οι δημογράφοι είναι η εκτίμηση του κατά ηλικία μοτίβου θνησιμότητας όταν τα δεδομένα που έχουν στη διάθεση τους αφορούν ομάδες ηλικιών. Στην παρούσα εργασία παρουσιάζεται μια μέθοδος η οποία μας δίνει λύση σε αυτό το πρόβλημα, δηλαδή μας επιτρέπει να εκτιμήσουμε τον αναλυτικό πίνακα επιβίωσης του υπό-ανάλυση πληθυσμού από τον αντίστοιχο συνεπτυγμένο πίνακα. Επιπλέον, γίνεται μια προσπάθεια να εξεταστούν οι αλλαγές στη θνησιμότητα διαφόρων πληθυσμών με το πέρασμα των χρόνων. Για το σκοπό αυτό, θεωρούμε συνεπτυγμένα σύνολα δεδομένων για τις επιλεγμένες χώρες. Έπειτα, εφαρμόζουμε την προαναφερθείσα μέθοδο για να εκτιμήσουμε τις πιθανότητες θανάτου κάθε ηλικίας. Το βασικό εργαλείο αυτής της τεχνικής είναι η φόρμουλα των Heligman και Pollard, η οποία αποτελεί αναλυτικό υπόδειγμα για την περιγραφή της θνησιμότητας του συνολικού διαστήματος ζωής. Δηλαδή, μελετά ξεχωριστά τη θνησιμότητα της βρεφικής και παιδικής ηλικίας , την θνησιμότητα ηλικιών 12 έως 35 περίπου , η οποία αντανακλά την θνησιμότητα ατυχημάτων (accident hump) που επιδρά σε αυτές τις ηλικίες, και τη θνησιμότητα μετά τα 35 περίπου. Αυτό το πλεονέκτημα μας επιτρέπει να εξετάσουμε τη θνησιμότητα των νεαρών ενηλίκων και το πώς έχει αλλάξει με την πάροδο του χρόνου.

https://doi.org/10.26219/heal.aueb.6210

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Chapter 1

INTRODUCTION

The two general factors that form the physical notion of a human population are fertility and mortality. These two factors are analyzed extensively by demographers as they attract their interest. In demography, a construction of a life table is a method of completely describing the survivorship of a population as subjected to the risk of death. Life tables describe the effect of mortality and are needed for calculating life expectancy. A "mortality law", a mathematical expression for the graduation of the age pattern of mortality, has been object of great attention since the deployment of the first life tables by John Graunt (1662) and Edmund Halley (1693). Such an expression is useful for describing the mortality of a population and it provides the basis for helpful projections. Using life expectancy as an indicator of health inequalities has many advantages. Firstly, it is an all-age measure of mortality that does not require the use of a standard population. Moreover, it can easily be compared between areas, including other countries and it is understood by the majority of people.

As far as life tables are concerned, they can easily be constructed as the only required data are the population and the number of deaths. All other series of columns of data and the expectation of life can be calculated from these. There are abridged life tables and complete life tables. An abridged life table contains most often the five-year probabilities of dying except for the first five years of life which are usually presented in two intervals, $[0,1)$ and $[1,5)$, whereas a complete life table contains annual data. What has demographers, biostatisticians and actuaries concerned for a number of years is to estimate a

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complete life table when data is given in age groups. The main reason is that an abridged life table apart from being easier and more convenient to use, is more often available because of the phenomenon of "age heaping", caused by age misstatements in data registration. For example, misstatements such as the preference of ages ending in multiples of five cause age heaps. Numerous tests have been devised to ascertain which age grouping minimizes greater the observed errors. It is commonly believed that these systematic errors can be eliminated by grouping the data in five-year age groups, except the first age (age at birth) which presents alone (i.e. 0, 1-4,5-9,10-14 etc.). Another reason is that there are countries with incomplete and unstable documentation of vital statistics and as a result it is impossible to compute a complete life table for them.

Although it is easier to find out an abridged life table, the construction of a complete life table given the abridged one is very important. Several techniques concerning the expansion of an abridged life table to a complete one are proposed in the literature. This study provides an expanding method which was first presented by Kostaki (1987, 1991). For the application of this method, Kostaki (1987, 1991) has used the eight-parametric model of Heligman and Pollard (1980). All the analyses were performed using the United Nations software package for demographic measurement (1988a), namely MORTPAK, which uses the eight-parameter formula of Heligman and Pollard (1980) too. The UNABR procedure of the MORTPAK package is used in order to get estimates for the parameters of the model and to produce the single-year probabilities of dying.

The main purpose of this thesis is to estimate the one-year probabilities of dying, so that we can determine the age-specific mortality curves for several populations and how they have changed over the time. In this way, we will be able to examine and emphasizing the early adult mortality. For that purpose, we consider five-year empirical mortality data of Australia, Bulgaria, Canada, France, Russia and the Unites States of America for the years 1955 till 2009. Our investigation was based on population mortality experience from the Human Mortality Database. We depart from the abridged q_x- values of each population and then we apply our expanding technique for estimating the age-

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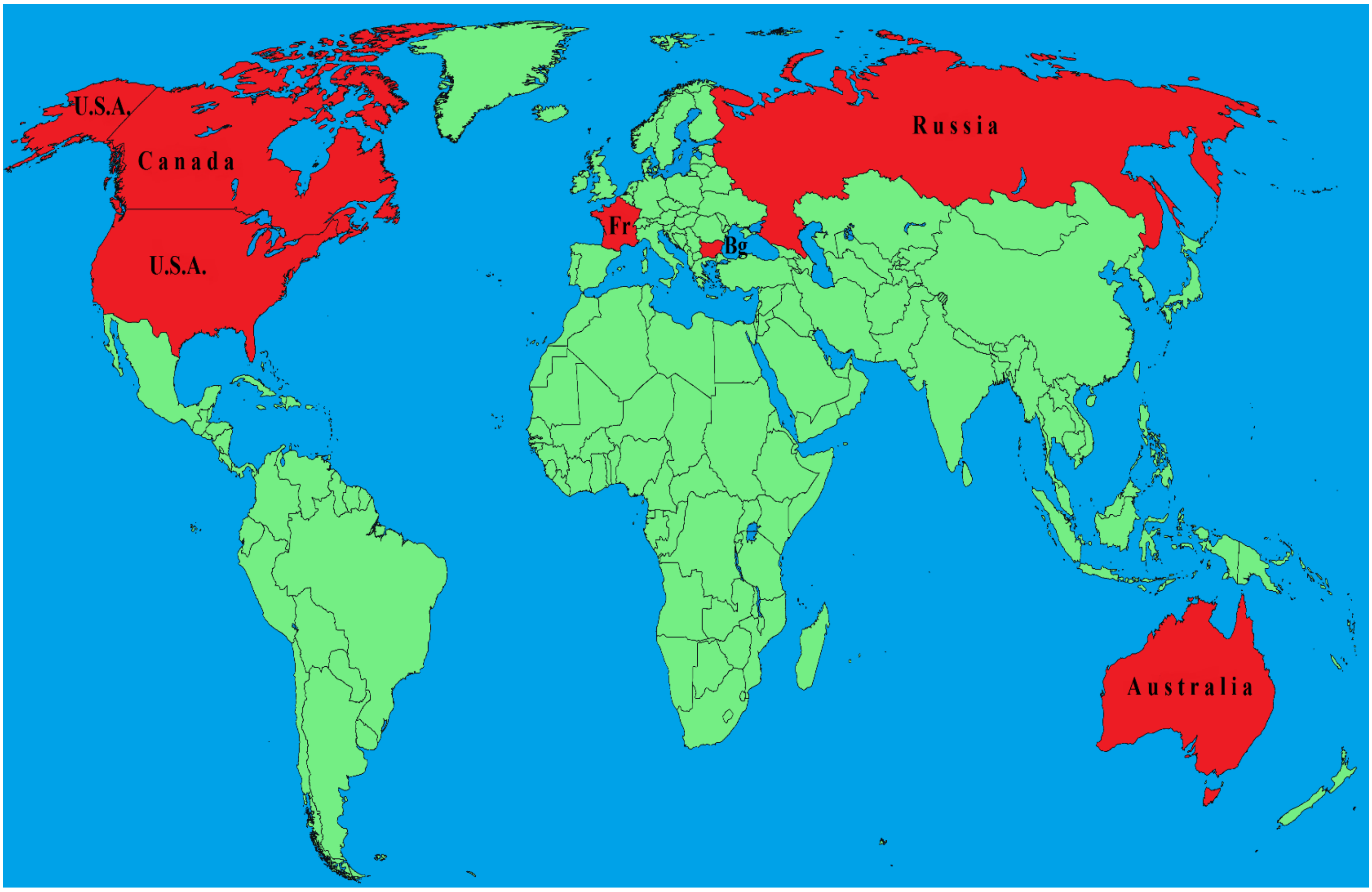
specific probabilities of dying. The world map with the selected countries in red is illustrated bellow.



Next chapter is devoted to a presentation of useful concepts and formulae of the theory of life tables. In the same chapter, we are also discussing the phenomenon of the "accident hump". Our expanding technique and its procedure for estimating the one-year probabilities of dying are presented in chapter 3. Then, in chapter 4 we present graphically the results of our calculations for both sexes of each country separately and we discuss them in detail. Finally, chapter 5 provides some concluding remarks.

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Chapter 2

BACKGROUND

A life table is a statistical model for measuring mortality (or any other type of "exit") experiences of a population. In practice, it is a table which shows the varying chances of dying as a function of age. Its main purpose is the presentation of the age specific mortality rates (m_x) of a population. Departing from these, we can derive and include in the table some statistics, such as the probability of surviving any particular year of age, the remaining life expectancy for people at different ages and the proportion of the original birth cohort still alive. Life tables are used not only by demographers, but also by vital statisticians, medical researchers, insurance companies and actuary departments. When they are used in biology, the age specific fertility rates are also included in the calculations. It is worth pointing out that they are usually constructed separately for males and females because of their substantially different mortality rates. The book "Natural and Political Observations Made upon the Bills of Mortality", which had been published in London in 1962, presented the first life table.

Life tables can be distinguished in two general types, based on the methods used to compute the age-specific probabilities of dying.

- Generation or Cohort Life Tables, which represent the mortality experience over the entire lifetime of individuals born in the same year and are followed until all members in that cohort die. Such tables are quite rare as for their construction it is needed to assemble mortality data over a long period of time. Although they are practical only in "closed"

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populations (with no migration), they are more suitable for projecting a population into the future and for analyzing the generational trends in mortality.

- Current or Period Life Tables, which represent the mortality rates for each age at some point in time. So, this kind of tables reflect the age-specific probabilities of death calculated by using the number of deaths and population size in the current year. A period life table is composed when it is desirable to describe current mortality conditions and it is also useful since the calculation of a cohort one is impractical.

Another distinguish of life tables is the following:

- Complete Life Tables, which present the age-specific mortality experience by single year of age.

- Abridged Life Tables, which usually contain data by 5- or 10- year age intervals except for the first year of life (age 0) which presents separately.

The construction of a life table is very useful not only for studying mortality but also for other several reasons. It summarizes the mortality experience of a population and information about life expectancy or longevity. It can also be used to summarize any duration variable, the duration of marriage for example. A typical life table contains several columns but it can easily be constructed as the only required data are the population and the number of deaths. Starting by getting the age-specific death rates ${}_nm_x$ for any value x in [x, x+n) from the death registrations, we can approach the probabilities of dying ${}_nq_x$. All other series of columns of data and the expectation of life can be calculated from these.

For the computation of a period life table we consider initially a notional cohort l_0, which is called the radix of the table (usually set at 100.000 or 10.000). The basic functions that are needed to compute an abridged life table are the following.

- ${}_nq_x$, which is the probability of dying during the age interval [x, x+n), i.e. the probability of an individual of age x to die before getting to the age x+n.
- ${}_np_x$, which is the probability of surviving through the interval [x, x+n), with ${}_np_x = 1 - {}_nq_x$. This is a useful quantity to calculate although it is not shown in the life table.
- l_x, which denotes the number of people alive at the beginning of the age interval, i.e. at exact age x.
- s_x, which is the probability of surviving in the age interval [0, x]. In other words, it is the proportion of individuals alive at the age of x, with

$$s_x = \frac{l_x}{l_0}$$

This function is not usually concluded in the life table but it is important for demographic and biostatistical analysis.

- ${}_nd_x$, which indicates the number of individuals that die in the age interval [x, x+n) and equals to ${}_nd_x = l_x - l_{x+n}$ or alternatively ${}_nd_x = l_x \cdot {}_nq_x$.
As everybody eventually dies, the sum of the number of deaths in all age intervals is equal to the radix of the life table, i.e.:

$$\sum_{x=0} {}_nd_x = l_0$$

- ${}_nL_x$, which is the number of years of life that the cohort experience within the age interval [x, x+n). It is assumed that L_x is the number of individuals alive at the middle of the age interval. Thus,

$${}_nL_x = n \cdot l_{x+n+} + \frac{n}{2} \cdot d_x$$

and hence that

$${}_nd_x = l_x - l_{x+n}$$

L_x may be defined as

$${}_nL_x = \frac{n}{2}(l_x + l_{x+n}).$$

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If the age will be considered as a continuous variable we will have,

$$_nL_x = \int_x^{x+n} l(t)dt = \int_0^n l(x+t)dt \approx n(\frac{l_x + l_{x+n}}{2}).$$

- T_x, which defines the total number of years left for all individuals to live after attaining age x. For its calculation we should sum the L_x values as follows:

$$T_x = \sum_{i \geq x} L_i$$

and also,

$$T_x = {}_nL_x + T_{x+n}.$$

In the continuous case we have,

$$T_x = \int_0^{w-x} l(x+t)dt,$$

with w being the age that nobody reaches (w-1, the greater age).

- $e_x{}^0$, which is called the life expectancy for an individual of age x, i.e. it represents the average number of years of life remaining for an individual alive at the beginning of age interval x.
This is the most interesting column of a life table and is calculated as

$$e_x{}^0 = \frac{T_x}{l_x} .$$

And in the continuous case,

$$e_x{}^0 = \frac{1}{l_x} \int_0^{w-x} l(x+t)dt .$$

A complete life table is also derived from the same formulae. It only requires to equate n=1. It is sometimes necessary to construct a life table by using the age-specific death rates $_nm_x$. This means that the values of $_nq_x$ had to be derived in order to produce the life table from a given radix. We present here

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some useful relations of the mortality measures $_nq_x$ and $_nm_x$ which are used in several expanding methods (for example see Pollard 1989).

Assuming that $_na_x$ is the expected number of years that an individual of age x will live in the age interval [x, x+n) with $0< {_na_x}<n$. Then,

$$_nf_x = \frac{_na_x}{n} \quad , \qquad 0< {_nf_x}<1$$

is the expected percent of years lived by such individual in the same age interval. So, the number of l_{x+n} people and each person which dies in the age interval [x, x+n) will have total contribution of a number of $n\cdot l_{x+n}$ and $_na_x\cdot {_nd_x}$ years of life respectively. As a result, the number of years of life that the cohort experience within the age interval [x, x+n) is:

$$\begin{aligned} _nL_x &= n\cdot l_{x+n} + {_na_x}\cdot{_nd_x} = \\ &= n\cdot (l_x - {_nd_x}) + n\cdot {_nf_x}\cdot {_nd_x} = \\ &= n\cdot [l_x - {_nd_x}(1- {_nf_x})] \end{aligned}$$

Taking the previous into account, the mortality ratio $_nm_x$, takes the following form,

$$_nm_x = \frac{_nd_x}{_nL_x} = \frac{_nd_x}{n\cdot [l_x - {_nd_x}(1-{_nf_x})]} = \frac{l_x\cdot {_nq_x}}{n\cdot l_x[1-{_nq_x}(1-{_nf_x})]} \Rightarrow$$

$$\Rightarrow {_nm_x} = \frac{_nq_x}{n[1-{_nq_x}(1-{_nf_x})]}$$

and

$$_nq_x = \frac{n\cdot {_nm_x}}{1+n(1-{_nf_x})\cdot {_nm_x}} \ .$$

In case that we have uniform distribution of deaths in each age interval, i.e. $_na_x = \frac{n}{2}$ and $_nf_x = \frac{1}{2}$, we will have

$$_nm_x = \frac{_nq_x}{n(1-\frac{1}{2}\,{_nq_x})} \qquad \text{and} \qquad {_nq_x} = \frac{_nm_x}{\frac{1}{2}(1+\frac{n}{2}\cdot {_nm_x})} \ .$$

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Another very useful mortality function, which is one of the most important concepts in formal demography, is the force of mortality. We refer to a theoretical measure, denoted by μ_x, which describes the behaviour of a mortality rate, ${}_nm_x$, over an infinitely small duration n. This is called “Force of Mortality” or “Intensity of Mortality” or “Hazard Function” and is used in some certain expanding methods (see Pollard 1989). Mathematically we have,

$$\mu_x = \lim_{n \to 0} {}_nm_x$$

or alternatively, if the number of survivors at each age x, i.e. l_x, is a continuous function of age x, we can represent the force of mortality as the ratio of the rate of decrease of l_x at that age to the value of l_x. So,

$$\mu_x = -\frac{dl(x)}{dx}\frac{1}{l(x)} \ .$$

We should note that the minus sign is to make μ_x positive since l_x is a decreasing function of age. Moreover, by the theory of differential calculus we have:

$$\frac{d}{dx}(ln\big(l(x)\big) = \frac{1}{f(x)}\frac{d}{dx}\big(f(x)\big)\,.$$

Thus, we can write the force of mortality as

$$\mu_x = -\frac{dln\big(l(x)\big)}{dx}.$$

Integrating both sides over some range [0, n) results in

$$\int_0^n \mu_{x+t}dt = \int_0^n -\frac{dln\big(l(x+t)\big)}{dx}dt = -\,ln\big(l(x+n)\big) - ln\big(l(x)\big) =$$

$$= -l\,n\left(\frac{l(x+n)}{l(x)}\right).$$

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Now, since ${}_np_x=\frac{l(x+n)}{l(x)}$, we have from the previous,

$${}_np_x= exp(-\int_0^n \mu_{x+t} dt) .$$

Assuming stable intense of mortality for age interval [0, n), and for n=1 we get,

$$p_x = \exp(-\mu_x) \Rightarrow \mu_x = -lnp_x .$$

By the last expression we can produce μ_x-values for each age x in order to be included in a life table (see Pollard 1989).

In this study, we focus on the early adult mortality. It is observed that as the hormone production of individuals during puberty, especially of males, reaches its highest level, the probability of dying increases dramatically. The reason is that the release of the hormone testosterone spurs them to a risky and aggressive behaviour. This is a widely known phenomenon that is called the "accident hump". This phenomenon except for human beings, happens also among apes. The "accident hump" is a statistical bump in the mortality graphs of the adult population, i.e. a spike in the mortality rates. Early results find out an accident hump in mortality curves since the mid-18th century. The accident hump and changes in its timing are objects of great attention for several reasons. If we focus on this, we can get a sense of the realism of future forecasts or insights into changes in human biology and development over time. Moreover, it is said that the accident hump is an indicator of the male biological clock. As the risk conditions (for example the introduction of the automobile) and the risk-taking behaviour have changed over time, a shift of the curve to younger ages seems to have occurred. So, in case that is fixed or getting younger, many other life cycle events are being delayed.

Several studies have been made in order to examine more causes of young adults' deaths except of their risky behaviour. William H. Dietz (1998) ended up that obesity present in childhood or adolescence increase the probability of adult morbidity and mortality. In young men and women who were obese during adolescence, the cardiovascular disease and diabetes appear more often. In males who were overweight during puberty, all-cause mortality

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and especially mortality from cardiovascular disease or colon cancer have been increased. Furthermore, according to the World Health Organization, the main cause of illness and disability among adolescents is depression and suicide is the third cause of death. Moreover, major concerns in many countries that are leading to causes of death among early adults, are the harmful drinking, violence and unintentional injuries. Last but not least, it has been estimated that the HIV deaths are increasing among young people.

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Chapter 3

METHOD ANALYSIS

Actuaries and demographers are usually in need of the age specific mortality rates for solving many demographic problems. For that reason, smoothing mortality curves are useful in contrast with those which are jagged or irregular along the life span or sometimes sparse in small populations. Several methods have appeared in the literature for the estimation of the age specific mortality pattern from grouped data. A suggested solution is to apply an interpolation formula or a graduation procedure to the observed data. However, this solution is not preferred because of the systematic fluctuations, except of the random ones, that the data may contain. In addition, based on the data, the rates for the older people are not always reliable due to age misreporting and death sparseness. So, demographers abridge the data, in order to eliminate systematic errors and produce the unobserved but real grouped data, from which they try to estimate the age specific data.

We mention here methods that have been suggested in the literature as tools of the expansion of an abridged life table to a complete one. There are some techniques, such as those that are presented by King (1914) and Beers (1944), that are the application of some general interpolation formula. An interpolation method that is still very actual is the six-point Lagrangean formula, which is applied to the survivor's function l_x of an abridged life table. This method is presented in Abramowitz and Stegun (1965) and it forms the only expansion technique proposed in standard textbooks as Elandt-Johnson

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and Johnson (1980), and Namboodiri (1991). In addition, Valaoras (1984) has presented in detail a method which was initially presented by Reed. Another method which requires five-year age grouping is proposed by J.Pollard (1989). Kostaki (1987) developed a method which utilizes a parametric model of mortality, and it has also developed independently within the MORTPAK and MORTPAK-LITE software packages published by the United Nations (1988a, 1988b). Not only in Kostaki (1987) but also in the MORTPAK package the main tool is the eight-parameter formula proposed by Heligman and Pollard (1980). Moreover, Kostaki (1991) has made a final adjustment to the results of this expanding technique. A new technique presented some years later by Kostaki (2000), is a nonparametric one, in the sense that it does not require the use of a parametric model. This method relates the target abridged life table with a standard complete one. Great literature referred to spline interpolation (see e.g. Wegman and Wright (1983) of Hsieh (1991)) in a case of an osculatory technique, should also attract our interest.

Parametric models offer usually an efficient means of considering the amount of information to be specified as a set of assumptions which are imposed by a set of parameters and functions. The use of parametric models compared to non-parametric ones have a lot of advantages. Firstly, they have the abilities to derive smooth curves and to interpolate or extrapolate if necessary. Secondly, they contain interpretable parameters with easily compared values and they are suitable for summarizing large amounts of data. In addition, they can easily capture and reflect trends. Last but not least, parametric models are very useful as they make easier the comparison over time of the shape and the intensity of age-specific mortality curves (Debón, Montes, and Sala 2005; Congdon 1993; Rogers and Gard 1991; Hartmann 1987). A “Life Table” contains the information required for a parametric model which is adopted to represent a mortality schedule. A mortality schedule will normally exhibit a great shift of mortality following birth age, a sudden drop (ages 10 to 15), followed by another sudden shift, known as “accident hump” (for ages 20 to 40). Finally, after recalling to a minimum level, it will rise at an increasing pace until the last years of life.

For over a century, statisticians and demographers were searching for a “mortality law”. Gompertz was the first who tried to represent mortality via a

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parametric model (see, Pollard (1991)), which was used initially for modelling the elderly, but it was also adopted by the earlier adult ages (see, Pollard (1989)). In 1872, Thiele attempted to represent mortality at all ages by combining three functions, each one representing a different part of mortality schedule. In the same sense, Heligman and Pollard (1980) set out the most successful parametric model that is applicable across the entire age range and is presented here. Heligman and Pollard (1980), Hartmann (1983), Fofar and Smith (1987), Kostaki (1992, 2000), Mode and Busby (1982), Rogers (1986) and Congdon (1993) used classical non-linear least squares procedure to the empirical law. However, over parameterization is a concern with the Heligman and Pollard law and this can lead to numerical instabilities, as Rogers (1986) and Congdon (1993) noticed. Fixing the values of one or two parameters to a feasible constant can resolve this problem, as they suggested.

This thesis provides an application of the eight-parameter Heligman and Pollard (HP) empirical law as it is significant to use a parametric equation to smooth the data for estimating the mortality pattern and predicting the rates. Heligman and Pollard (1980) have made a recent attempt to represent mortality over the entire age range by a single analytical expression. They considered that it is convenient to divide the causes of death into three classes, namely those which affect infant and early ages, early and middle adult life and later ages, i.e. the ages greater than 40. So, they proposed a nonlinear model of eight parameters which consists of three nonlinear curves and represents the odds of mortality as a parametric function of age x and is given by the formula:

$$\frac{q_x}{1-q_x} = A^{(x+B)^C} + Dexp(-E(\ln(x/F))^2) + GH^x \quad ,$$

where the right hand side of the above mathematical formula is interpreted as $A^{B^C} + G \; for \; x = 0$. In this mathematical function, q_x is the modelled probability that someone who has reached the age of x will die before reaching the age of x+1. Therefore, the formula utilizes a parametric model that represents the odds that an individual of age x will die before becoming x+1. The positive eight parameters A, B, C, D, E, F, G and H have their own demographic interpretations and need to be estimated. For providing adequate

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starting values for the parameters the UNABR procedure of the MORTPAK package is used.

The first term of this expression, a rapidly declining exponential function, seeks to capture the fall in mortality at the infant and younger ages (high in the first year of life and then decline until the age of 10-13). This component of mortality contains the following three parameters:

A, that is approximately equal to q_1 (mortality at age one)
B, that affects the level of infant mortality
and C, that measures the rate of mortality decline during childhood (the higher the value of C, the faster the mortality decreases with an increasing age).

The second term, which is a function similar to the lognormal density, reflects the middle life mortality. More specifically, it reflects a commonly observed increase in mortality, especially due to accidents among males and the accident plus maternal mortality among females. This additional mortality forms the widely known "accident hump" on the natural curve. The accident appears generally between ages 10 and 40 and has three parameters:

D, which maps onto the intensity of the "accident hump"
E, which represents the spread of the "accident hump"
and F, which indicates the location of the "accident hump" on the age axis.

The last term, is a geometrically increasing Gompertz exponential function, that describes the exponential rise in mortality in the older ages, which means ages greater than 40 years old. It generally represents the ageing or deterioration of the body, so it is often called senescent mortality. In this term,

G represents the initial level of older age mortality
and H denotes the rate of increase of that mortality.

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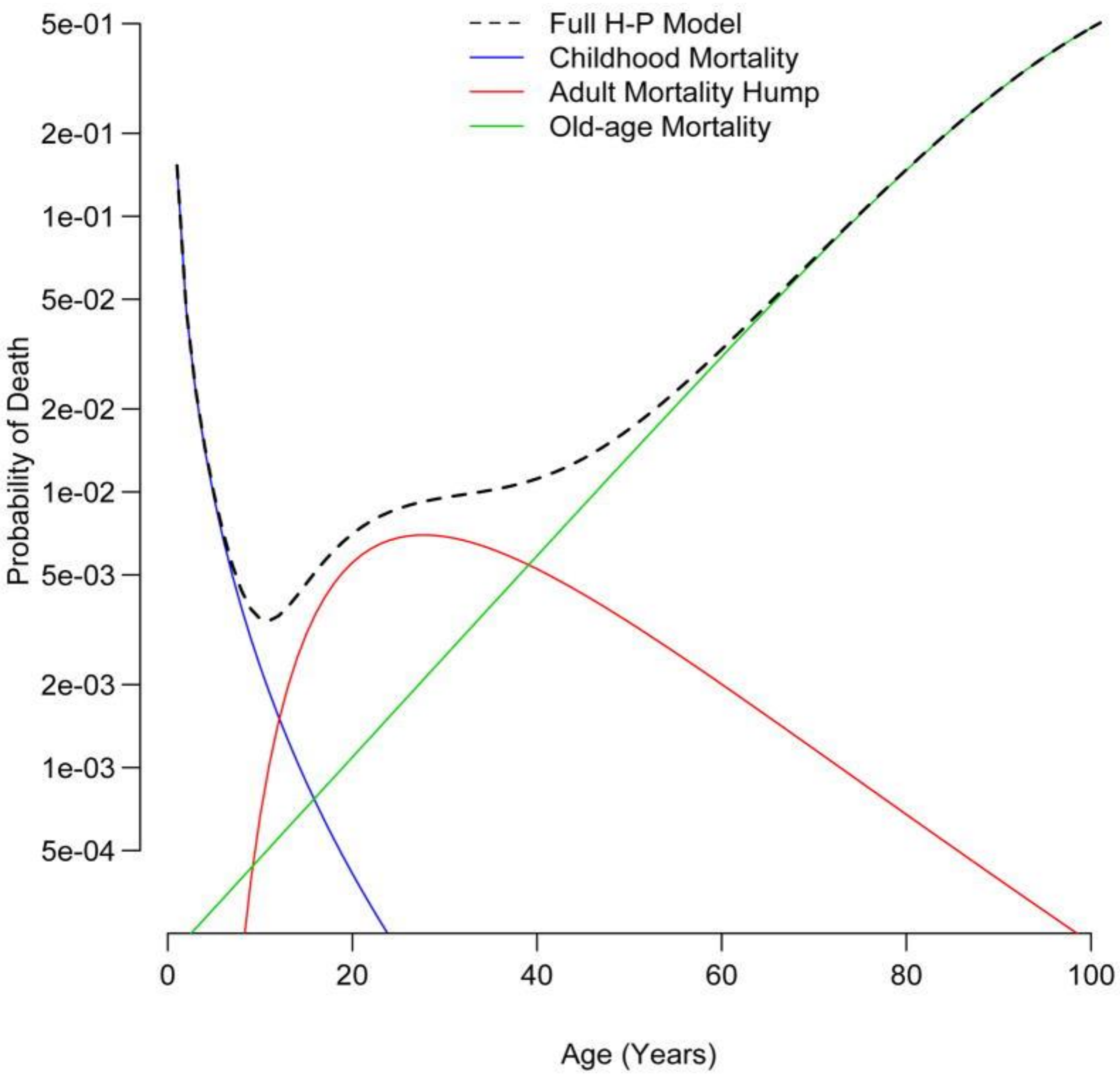
The three terms of this model control three separate additive components corresponding to the above mentioned age ranges of the mortality schedule i.e. child mortality, young adult mortality and senescent mortality. The following figure represents each of the components and the sum of them, i.e. the final curve (David J. Sharrow, Samuel J. Clark, Mark A. Collinson, Kathleen Kahn,Stephen M. Tollman (2013)).

Decomposition of the Heligman and Pollard model.



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The Heligman and Pollard (H&P) empirical law has been applied to many mortality data across all ages, for example, in Australia (Heligman and Pollard 1980), in the U.S.A. (Mode and Busby 1982), in Sweden (Hartmann 1987). It is shown that this model provides a satisfactory way for fitting the pattern of mortality, although attempting to fit many parameters simultaneously by a nonlinear algorithm can lead to inefficient model solutions. This formula has also been proved very efficient after its application to Swedish and Greek data (Kostaki 1985).

In order to expand an abridged life table to a complete one the parameters of the Heligman and Pollard model can be estimated by least squares. In this way, we can fit the H&P formula to the empirical q_x values in a complete life table. We seek for those parameter values which minimize usually the following sum of squares,

$$\sum_x \left(\frac{\hat{q}_x}{q_x} - 1\right)^2$$

where $\hat{q}_x$ is the estimated probability that the person who has reached age x will die before reaching the age of x+1, and q_x is the empirical observed probability value. The quantity to minimize has been initially proposed by Heligman and Pollard in 1980.

Letting *F(x; C)* to denote the right hand side of the H&P expression, with C being the notation for the parameters of the model, i.e. C=(A,B,C,D,E,F,G,H) , our model will become

$$\frac{q_x}{1-q_x} = F(x;C).$$

Then, for the one year odds of dying, we obtain

$$q_x = \frac{F(x;C)}{1+F(x;C)} = G(x;C)$$

and hence the relation

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$$ {}_nq_x = 1 - \prod_{i=0}^{n-1}(1 - q_{x+i}) $$

the model for the death probabilities in the abridged life table becomes

$$ {}_nq_x = 1 - \prod_{i=0}^{n-1}\big(1 - G(x+i;C)\big) = {}_nG(x;C) $$

where ${}_nG(x;C)$ is a complicated function of C, x and n.

Thus, for estimating C, the sum of squares to be minimized has to be taken as

$$ \sum_x\left(\frac{{}_nG(x;C)}{{}_nq_x} - 1\right)^2 $$

where the summation is over all relevant values of x. Then by inserting this C into the function ${}_nG(x;C)$, the one-year probabilities of dying q_x can be calculated.

It can be considered as a drawback that the one-year probabilities of dying

$$ {}_n\hat{q}_x = 1 - \prod_{i=0}^{n-1}(1 - \hat{q}_{x+i}) $$

that are constructed by the above procedure, will not be exactly the same with ${}_nq_x$. Taking that into consideration, Kostaki (1991) presented an additional adjustment of the results of this technique in which recommended choosing

$$ \widehat{q'}_{x+i} = 1 - (1 - \hat{q}_x)^K $$

where $$ K = \frac{\ln(1 - {}_nq_x)}{\sum_{i=0}^{n-1}(1 - \hat{q}_{x+i})} $$

so as to make

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$$ {}_{n}q_{x} = 1 - \prod_{i=0}^{n-1} (1 - \hat{q}'_{x+i}) \ . $$

A few years later, Kostaki (2000) presented a non-parametric expanding technique. According to that, a complete life table (T) is used as a standard and its $q^{(T)}(x)$ one year probabilities are related to the abridged ${}_{n}q_{x}$ −values that need to be expanded. Lets assume that the force of mortality, $\mu(x)$, underlying the original abridged life table in each n-year age interval [x,x+1] , is a constant multiple of the one underlying the standard life table in the same age interval, $\mu^{(T)}(x)$, i.e.,

$$ \mu(x) = {}_{n}K_{x} \cdot \mu^{(T)}(x) \ . $$

So, having the values ${}_{n}q_{x}$ and $q^{(T)}{}_{x}$ we can produce the estimates for the one-year probabilities underlying the original life table by using the following equation

$$ q_{x+i} = 1 - (1 - q^{(T)}{}_{x+i})^{\,{}_{n}K_{x}} $$

where q_{x+i} are the one-year probabilities, i=0,1,….,n-1 and the constant ${}_{n}K_{x}$ is given by

$$ {}_{n}K_{x} = \frac{\ln(1 - {}_{n}q_{x})}{\sum_{i=0}^{n-1} \left(1 - q^{(T)}{}_{x+i}\right)} \ . $$

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Chapter 4

RESULTS- DISCUSSION

The main purpose of this study is to examine the shape of the age-specific mortality curves for Australia, Bulgaria, Canada, France, Russia and the United States over the recent years. Biologists were the first who studied the shapes of survivorship curves (Deevey 1947, Pearl and Doering 1923, Pearl and Mina 1935). A survivorship curve is a graph showing the number or proportion of individuals surviving to each age for a given species or group (e.g. males or females). Survivorship curves can be constructed for a given cohort based on a life table. Our investigation was based on population mortality experience from the Human Mortality Database. The probability of dying to any age with a logarithmic scale is plotted on the y-axis, while the age is plotted on the x-axis. The pattern of mortality observed in most of the figures, which are represented in this chapter is typical of developed countries. Mortality rates during the first year of life are relatively high for both males and females primarily due to congenital abnormalities and perinatal conditions. After the first year of life, rapidly dropping mortality rates are observed, as a result of the increasing capacity to fend disease and the limited exposure to life threatening situations. Around age 10, mortality rates reach a minimum. Moreover, in this study we focus on the existence of a more or less clear peak in the late teens and early twenties in the following figures, which is a phenomenon known as “the accident hump”. Then, the mortality is rising continuously until the age of 80. In this chapter, we will represent each

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country's mortality curves separately for females and males, and how they have changed over the time. We are going to discuss them in detail and possibly explain the early adult peak in mortality.

AUSTRALIA

As far as Australian females are concerned, figure 1 shows that the mortality rates have fallen at virtually all ages, apart from the mortality of early adults, i.e. the accident hump, which is intensified strongly over time. We can easily observe that it is more severe in 1965-1969 and in 1975-1979 than in 1955-1959, that it does not even exist. Although the development of the Australian male accident hump is remarkable since 1955, as it is shown in figure 2, the mortality curves are declining more smoothly at all ages over the time. The exception is at ages around 20, where the mortality rates of 1975-1979 are a bit higher than those of 1965-1969. A noticeable point is that the absolute rates of mortality of males and females are quite similar, except those of the adolescent females which are being less than those of their peer males in each time period. Generally, over the course of the twentieth century, mortality rates in Australia have shown essential improvements at all ages. Several investigations have been conducted for the mortality rates of the Australian people and the main causes of their death, such as that of Pollard (1996) or as that of Richard Taylor, Milton Lewis and John Powles (1998). All of them conclude also to the continuing decline of the mortality during last century. This may happens due to less industrialization, lower population density and better nutrition seeing that Australia do not show war-related conditions in this mortality decline. Last but not least, Pollard (1996) underlines the appearance of AIDS-related deaths which occur predominantly in the age range 25-40 and a disturbing increase in suicides of males of all ages in 1992 compared with those of 1982.

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Figure 1: Mortality q(x) curves of Australian females.

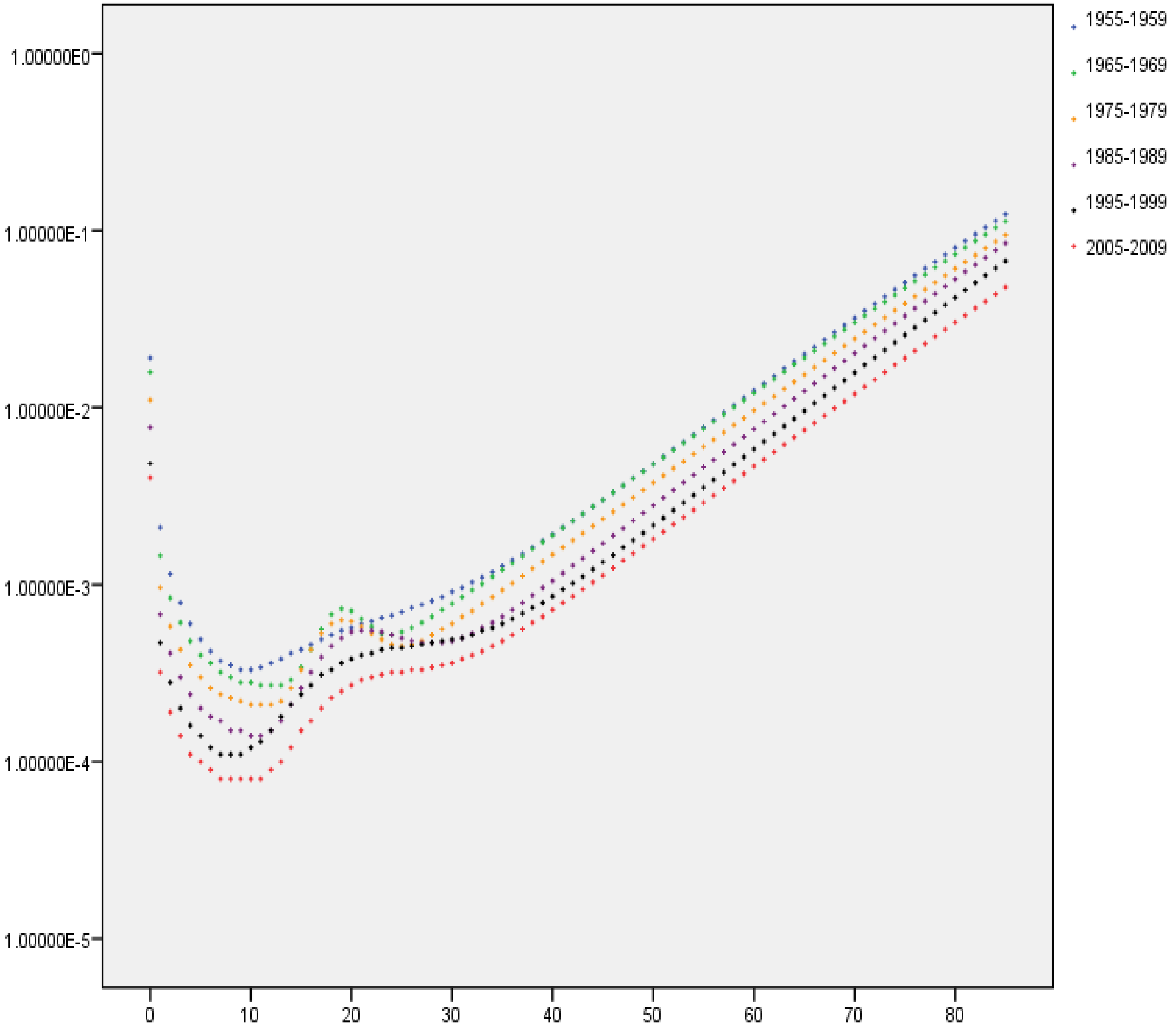

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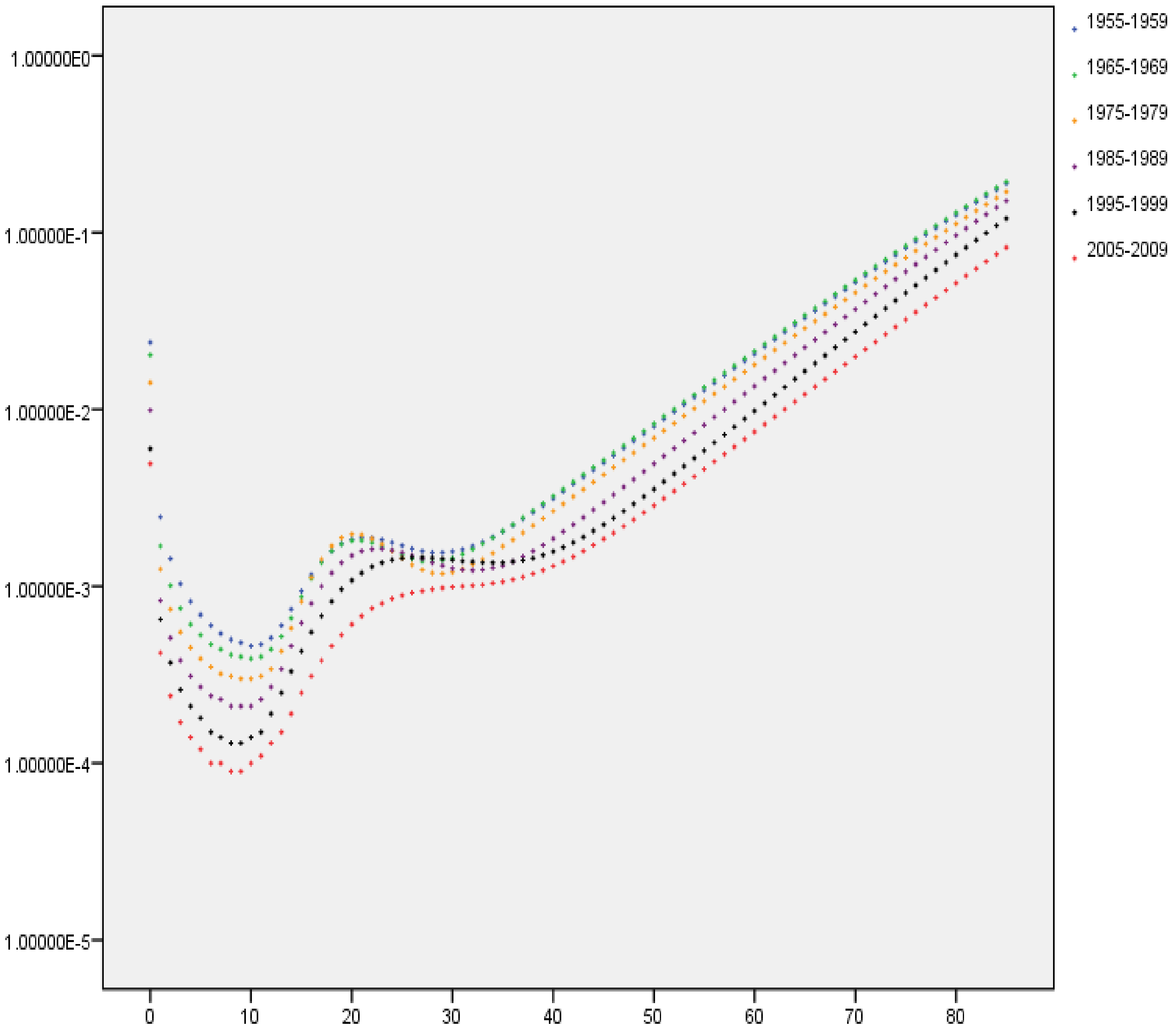
Figure 2: Mortality q(x) curves of Australian males.



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BULGARIA

The following figures represent the mortality curves for the population of Bulgaria. We can obviously see that the rates of mortality of Bulgarian females have fallen in the last years (figure 3). Moreover, only in 2005-2009 a small accident hump is appeared in contrast with the previous years that it did not exist. However, in the mortality curves of Bulgarian males over the last decades (figure 4), we can observe that the existence of an accident hump is intensified over the last years. A remarkable point is that since 1985, the mortality rates from the thirties to the eighties have increased and have exceeded even those of 1955-1959. However, in 2005-2009 a little progress in mortality rates has been made and this is encouraging. The striking aspect of Bulgarian mortality patterns, is on one hand the substantially higher mortality level for males, especially during adult ages, and on the other hand the differential increase in the force of mortality by age. It is shown that this country has experienced a dramatic increase in mortality rates, being among the highest rates in Europe, due to coronary heart disease and stroke. It is also proved that the mortality of unmarried men in Bulgaria is very high and it is thought that this happens due to the rise of their unemployment and the worsening of their health behaviours (i.e. drinking alcohol and smoking a lot). Another example that may explain the increase in male mortality over the decades is that, among 15 European countries, Bulgaria ranked first among men in cardiovascular disease mortality between 1970 and 1992. Generally, in Bulgaria the health situation continued to worsen up to the late 90s.

Figure 3: Mortality q(x) curves of Bulgarian females.

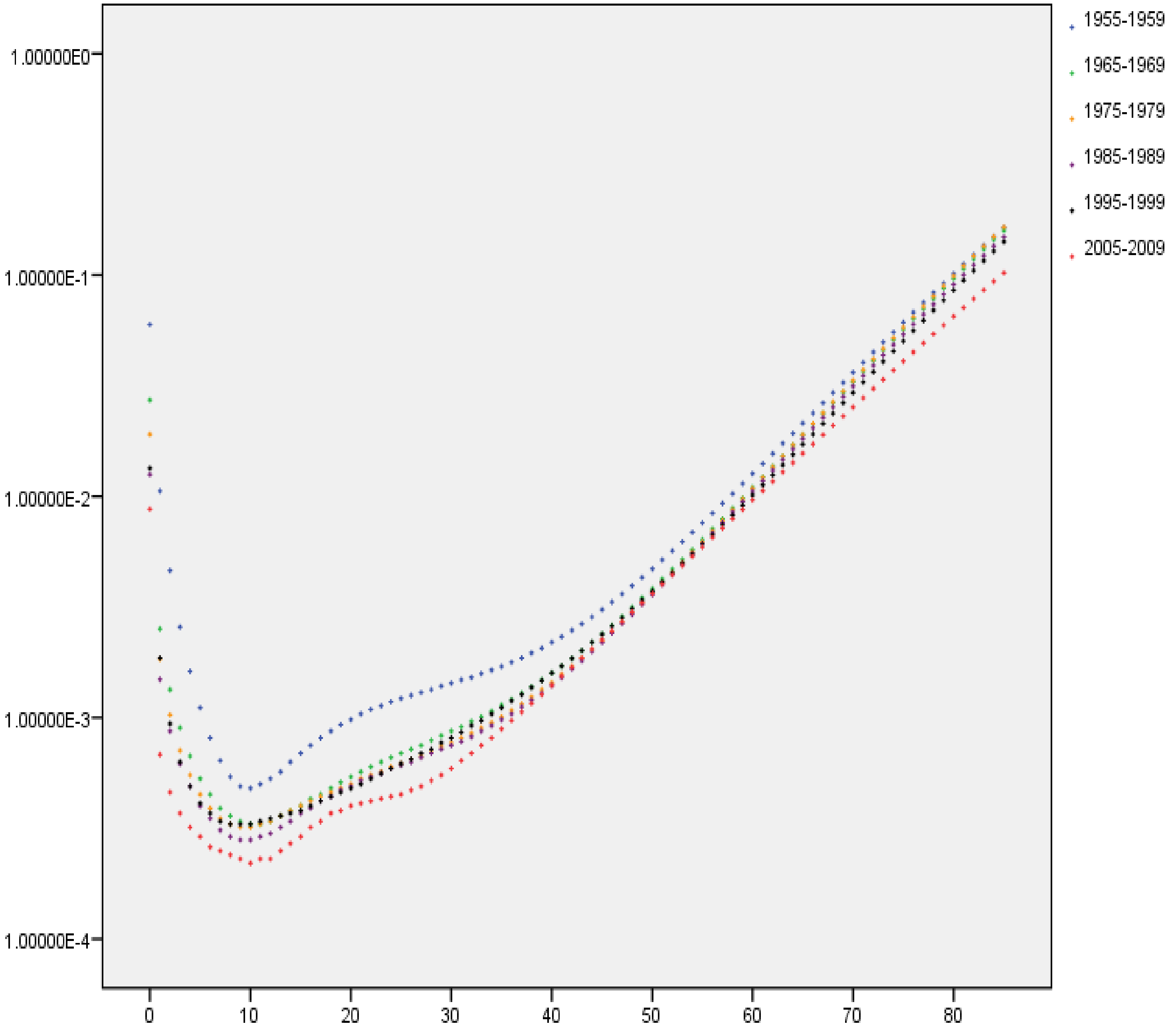

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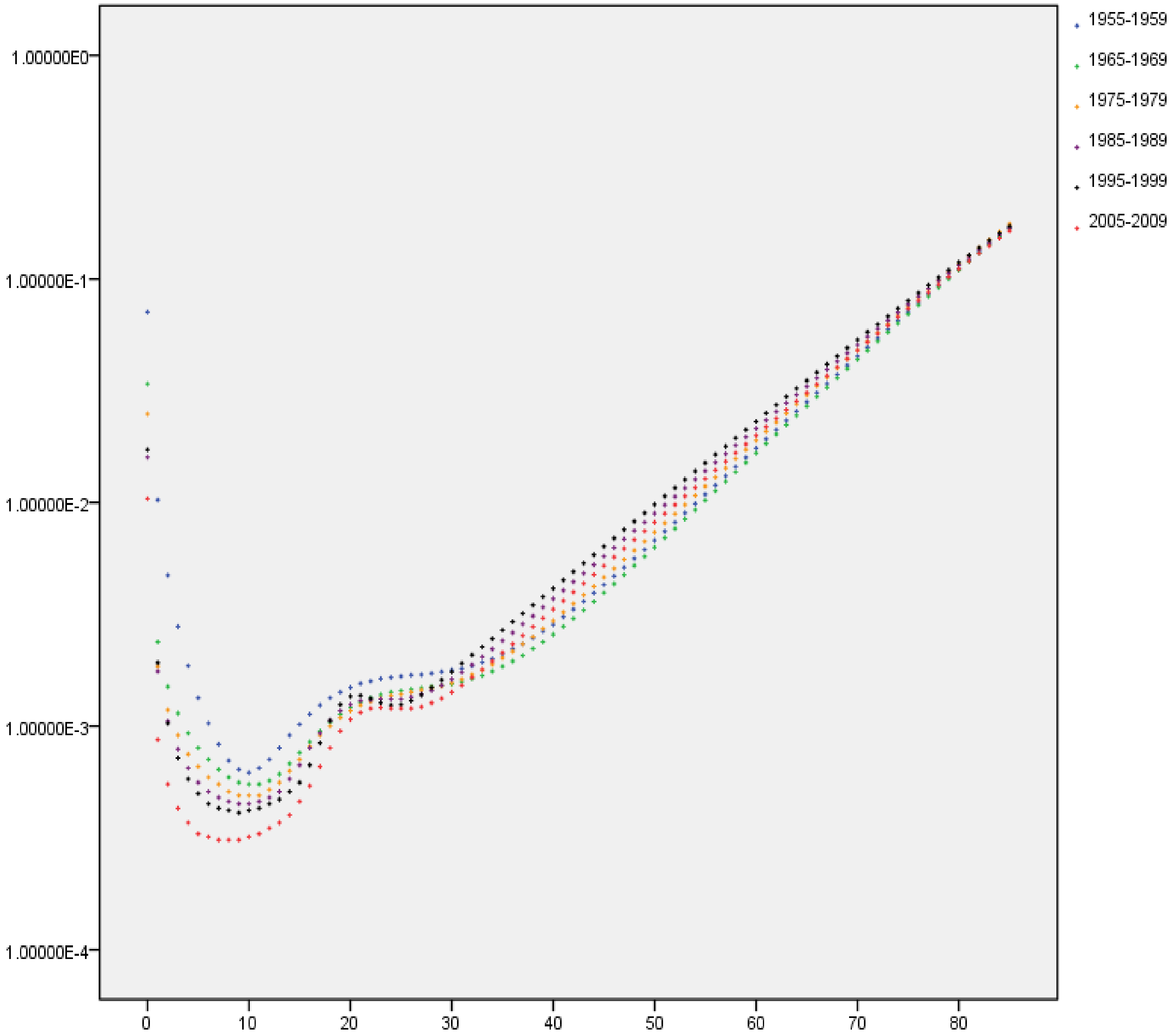
Figure 4: Mortality q(x) curves of Bulgarian males.



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CANADA

The mortality curves of the Canadian females (figure 5) and the Canadian males (figure 6) are evolved normally over the last years besides the age range of the early adults. The 1965-1969 and the 1975-1979 curves presented the unexpected results because the mortality rates from the late teens to the early twenties are a bit higher respecting those of the time period 1955-1959. So, the accident hump is intensified since 1965 both in females and in males, whereas in females in 1955-1959 did not exist at all. In particular, the peak of the accident hump in 1965-1969 and in 1975-1979 is superior to the previous years. For this population, male mortality rates exceed female mortality rates at adult ages and the accident hump is slightly more spread out in males than in females. Afterwards the increase of male mortality by age is slowing down similar to the female pattern. Rogers and Hackenberg (1987) have shown that deaths in Canada become increasingly influenced by individual behaviours and new lifestyles. It is worth noting, that injury is a leading cause of death for not only children, but for all Canadians between the ages of one and 44. The main causes of injury of people aged 15-19 in Canada are motor vehicle traffic crash, poisoning, drowning and fall. Another factor that constitutes a negative influence on death rates in the age group 30-39 is the AIDS, which is also strongly related to a person's behaviour and lifestyle except in the case where it is acquired through blood transfusion. Taking into consideration the continuing reduction of the mortality rates at all ages since 1985, we can say that a new era in Canadian mortality may has set in.

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Figure 5: Mortality q(x) curves of Canadian females.

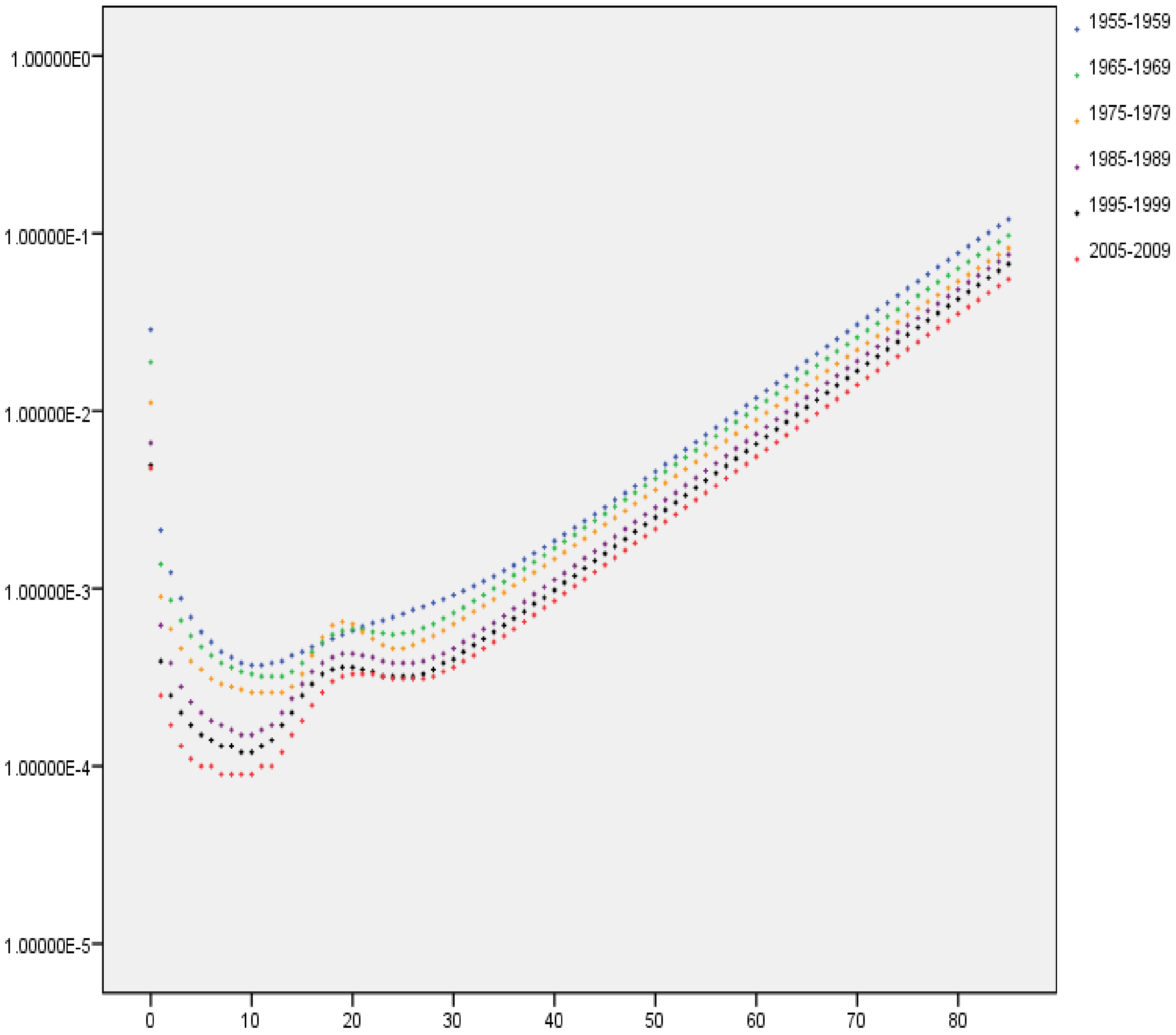

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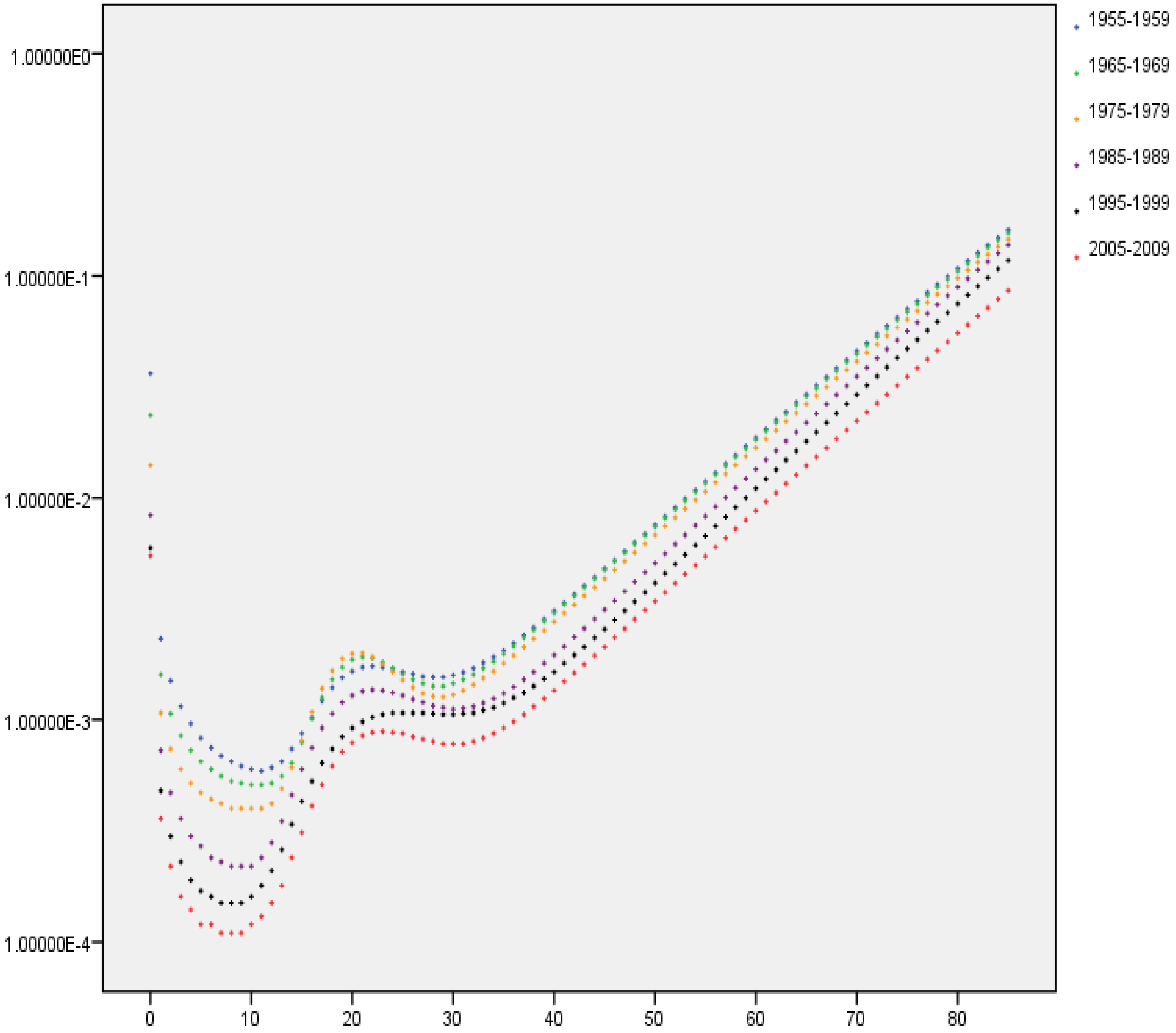
Figure 6: Mortality q(x) curves of Canadian males.



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FRANCE

An interesting presentation of mortality patterns over the years is that of French females (figure 7). The reason is that in 1955-1959 there is no accident hump, but 10 years later it appears. Although in the time periods 1985-1995 and 1995-1999 it disappears again, in 2005-2009 we can easily observe its strong intensification. Moreover, in 2005-2009 the mortality rates around individuals aged 20 and those from fifties to seventies are higher than ten years earlier. A similar phenomenon is presented in the males of the same country (figure 8), where in 1975-1979 there is a clear preponderance of the probabilities of dying of those around 20. So, the peak of the accident hump in 1975-1979 is higher than the previous years. The mortality curves of males are progressing smoothly at all other ages. France has repeatedly been identified as a country where average lifespan is very high and where compression of mortality has either stopped or slowed down substantially, even at older ages (Canudas-Romo 2008; Cheung, Robine, and Caselli 2008; Cheung and Robine 2007; Thatcher et al. 2010). It is approved that cancer mortality increased by 20% from 125,000 deaths in 1980 to 150,000 in 2000. Contrariwise, the road fatalities have decreased dramatically by 48% over the last 10 years. Séverine Arnold, Alexandre Boumezoued, Héloïse Labit Hardy, Nicole El Karoui (2015) having chosen to group the causes of death for the French population in 2008 in six categories: cancers, diseases of the circulatory system, diseases of the respiratory system, external causes, infectious and parasitic diseases and all the other causes brought together, they have shown that the main causes of death are cancers, diseases of circulatory system and the category other causes. In addition, they pointed out that cancer impacts more generally those aged above 40, while external causes despite the fact that they are less presented in the total deaths, embed the accident hump and impact principally ages around 25.

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Figure 7: Mortality q(x) curves of French females.

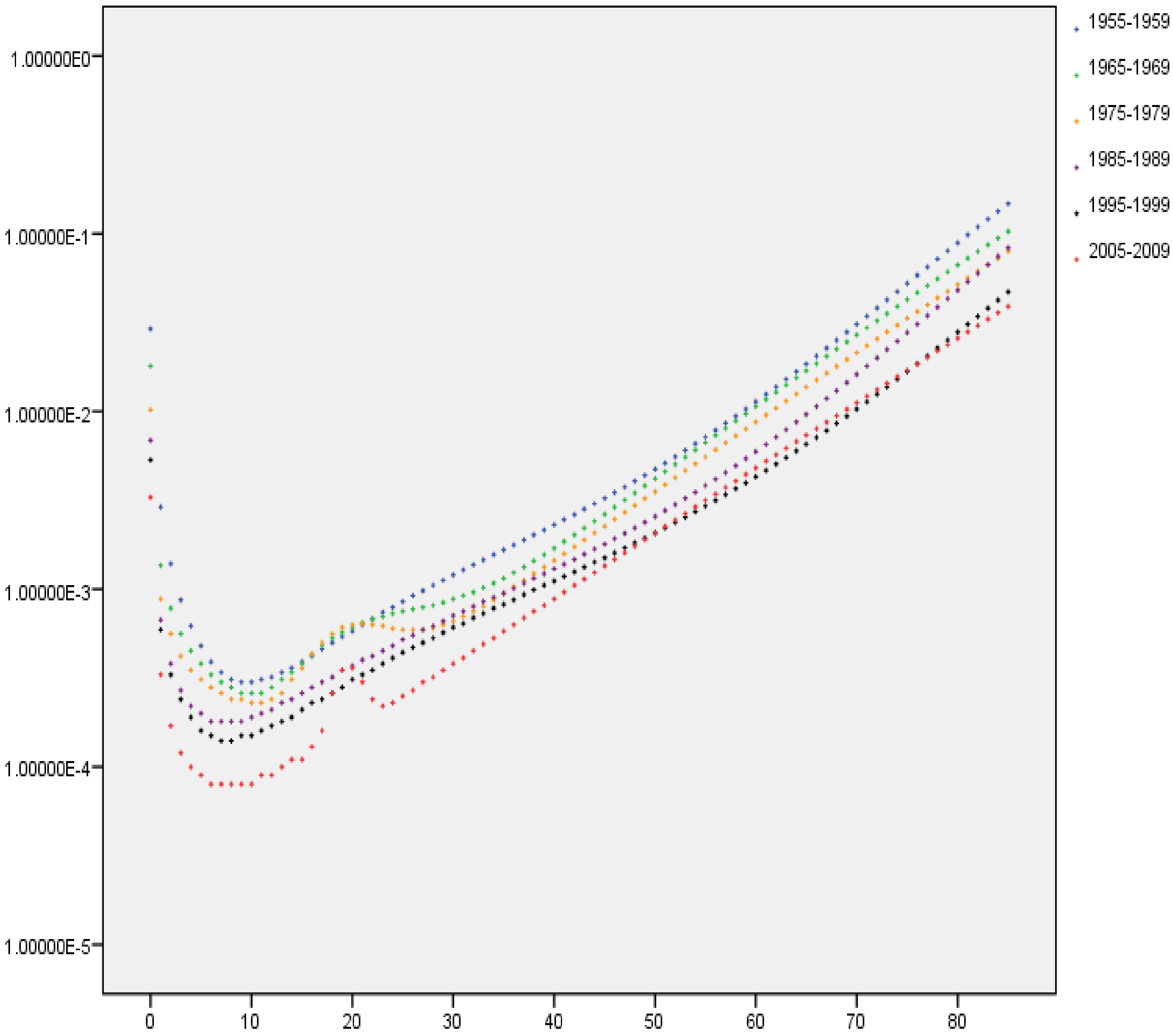

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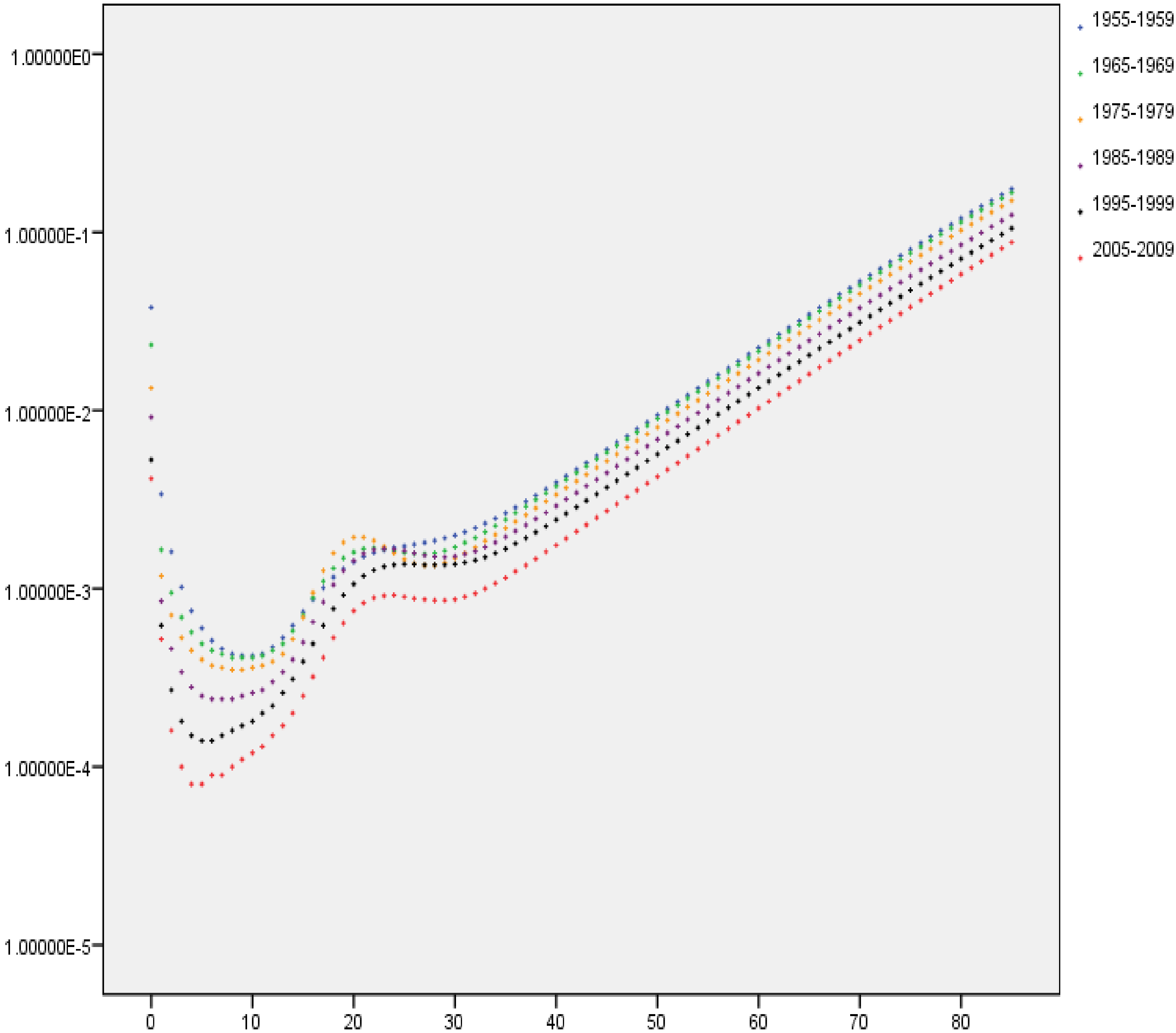
Figure 8: Mortality q(x) curves of French males.



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RUSSIA

Russia's survival curves for both sexes are noteworthy. Regarding Russian females (figure 9) in 1985-1989 we notice for first time the so called "accident hump". But the most impressive finding is that after a decade the mortality rates have risen up at almost all ages above 15. Furthermore, in 2005-2009 we remark that the mortality rates at ages from middle-twenties to middle-forties are even higher. So, we can conclude that the probability of dying went up over the years. In figure 10, which refers to Russian males, it is also obvious that in the last decades the mortality has increased dramatically at almost all ages and it has been fluctuating. Although the mortality rates for young adult males are higher than those of the same aged females, at higher ages, this female advantage in survival diminishes, and male and female mortality rates converge. Amanda Nicholson, Martin Bobak, Michael Murphy, Richard Rose and Michael Marmot (2005) have concluded that the most common cause of death in Russia is cardiovascular disease. Nevertheless, their most striking result is that alcohol and mortality are tightly connected in this country especially for men, but this does not mean that it is not a major contributor to female mortality. Tamara Men, Paul Brennan, Paolo Boffetta, David Zaridze, director (2003) have proved that over the period 1992-2001, the rise in mortality in Russia was astonishing, reaching 2.5-3 million extra deaths in young and middle aged adults. They notice that the main reason of the fluctuations in mortality are from vascular disease and violent deaths (mainly suicides, homicides, unintentional poisoning by alcohol, and traffic incidents) among early adults in contrast with cancer mortality which changed a little over the examining period. Finally, it is important to mention that the Russian mortality increased more in 1998 due to the economic crisis.

Figure 9: Mortality q(x) curves of Russian females.

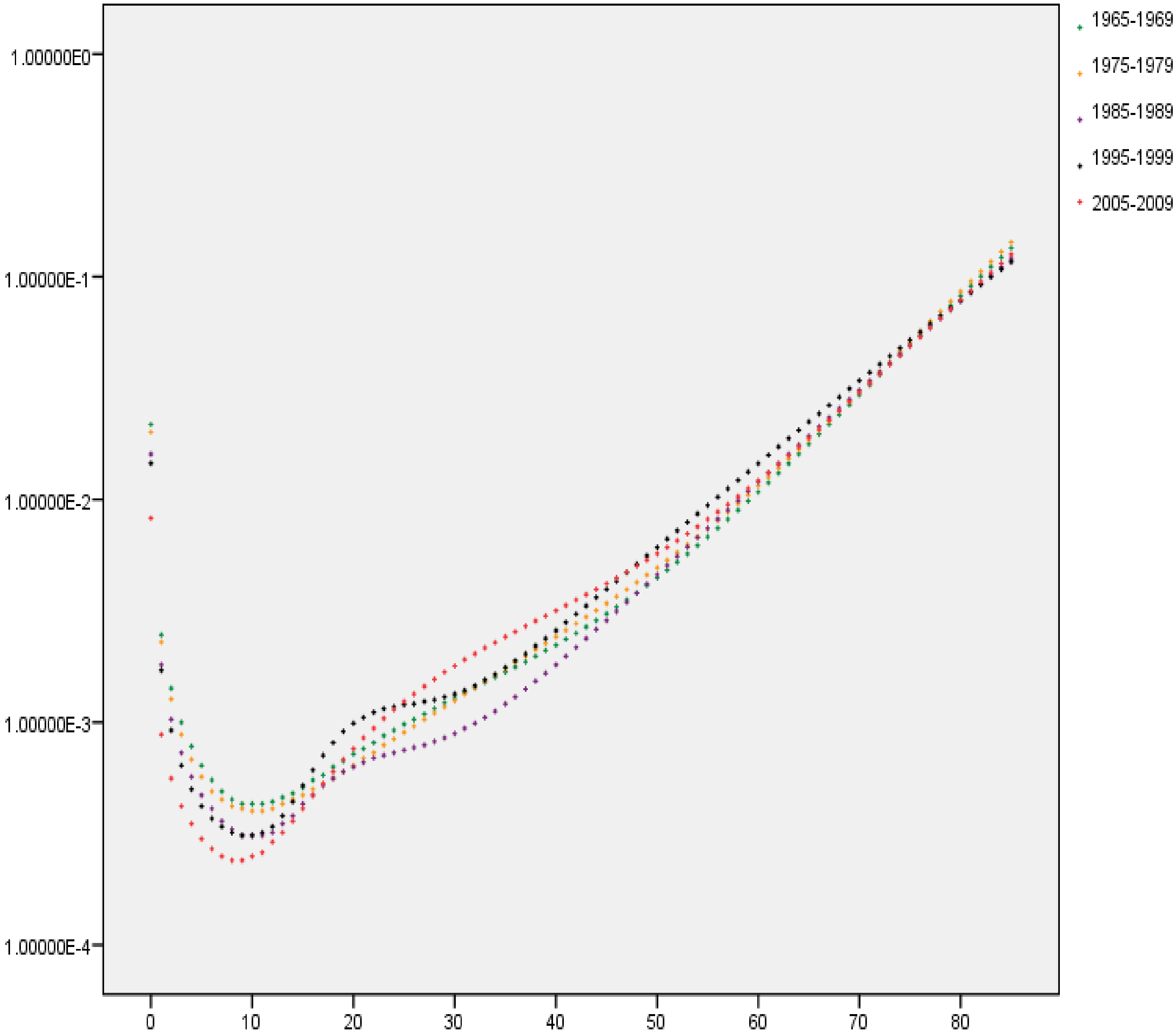

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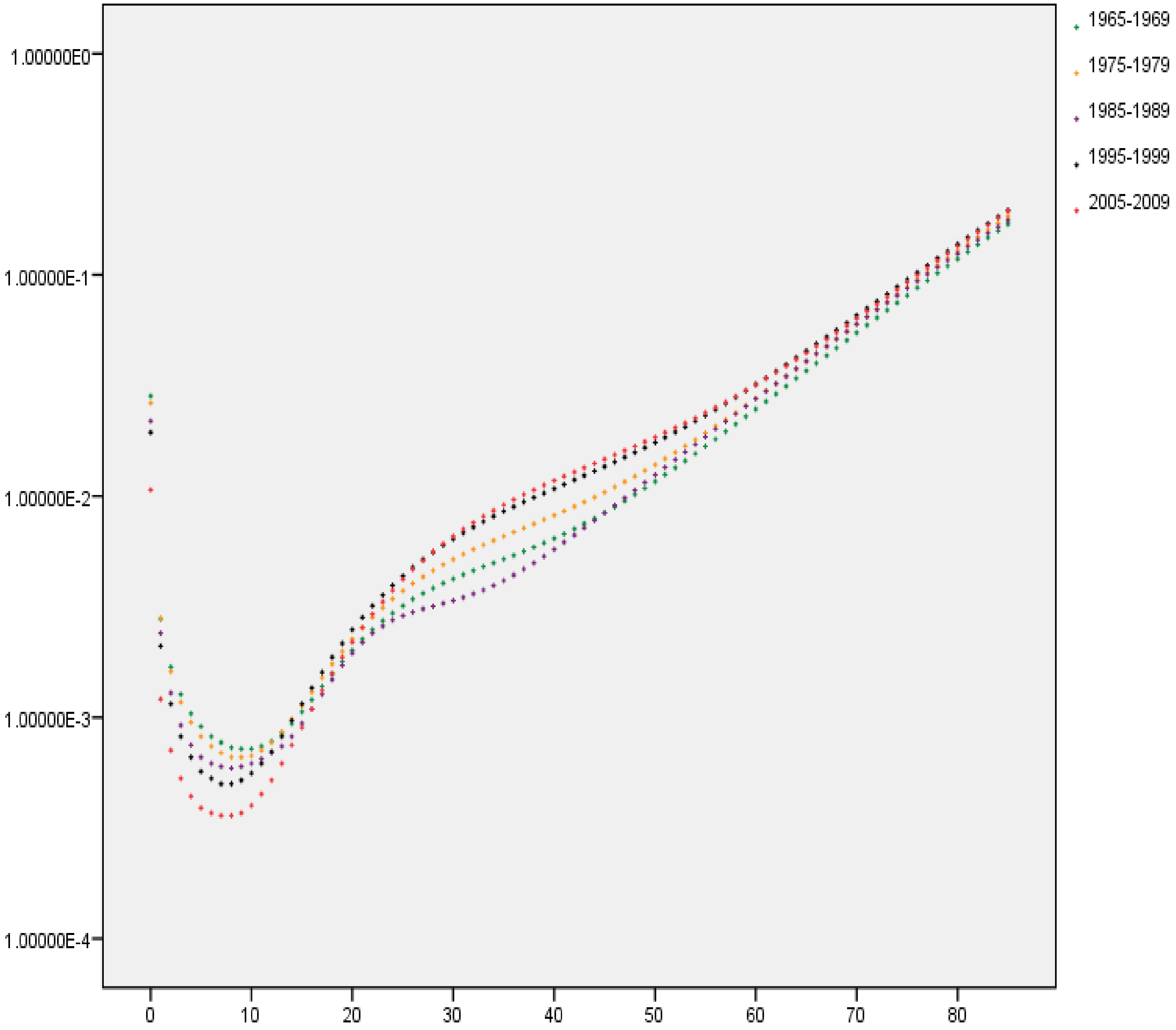
Figure 10: Mortality q(x) curves of Russian males.



USA

Observing the mortality curves of the US females (figure 11), we can see that there is no accident hump in the time period 1955-1959. The development of an accident hump starts in 1965-1969 curve and since then, the mortality curves are evolved as expected. Concerning the males of the same country (figure 12), we have to deal with a quite normal representation of the mortality at all ages over the examining periods. We should only notice that the mortality rates from the late teens to the late twenties are a bit higher in the time periods 1955-1959 and 1965-1969 respecting those of 1955-1959. Comparing these two figures, we can observe the appearance of the accident hump ten years earlier in males than in females, and also that its peak at all time periods is higher in males than in females. For both sexes of this country we notice a lowering of mortality rates at almost all ages over the years. The average life expectancy in the United States is 77.9 years, according to the National Center for Health Statistics. The main causes of death in the USA (in descending order) are heart disease, cancer, stroke, chronic lower respiratory diseases, accidents, Alzheimer's disease and diabetes, as the Centers for Disease Control and Prevention report. However, mortality from heart disease has reduced during the last three decades. We should also mention that the death rates among children 1 to 4 years old appear a great reduction (92 %) after the universal childhood vaccination against varicella. P.D. Sorlie, E. Backlund, and J.B. Keller (1995) concluded that employment status, income, education, occupation, race, and marital status are connected with mortality in the United States because they notice that mortality in people with lower incomes, lower level occupations, less education and in those who are not married is higher than in the others. Similarly, the mortality in Blacks is higher than in Whites less than 65 years old. Given the fact that the US mortality appears a continuing decline over the years, in the future we may see large increases in life expectancy at older ages.

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Figure 11: Mortality q(x) curves of US females.

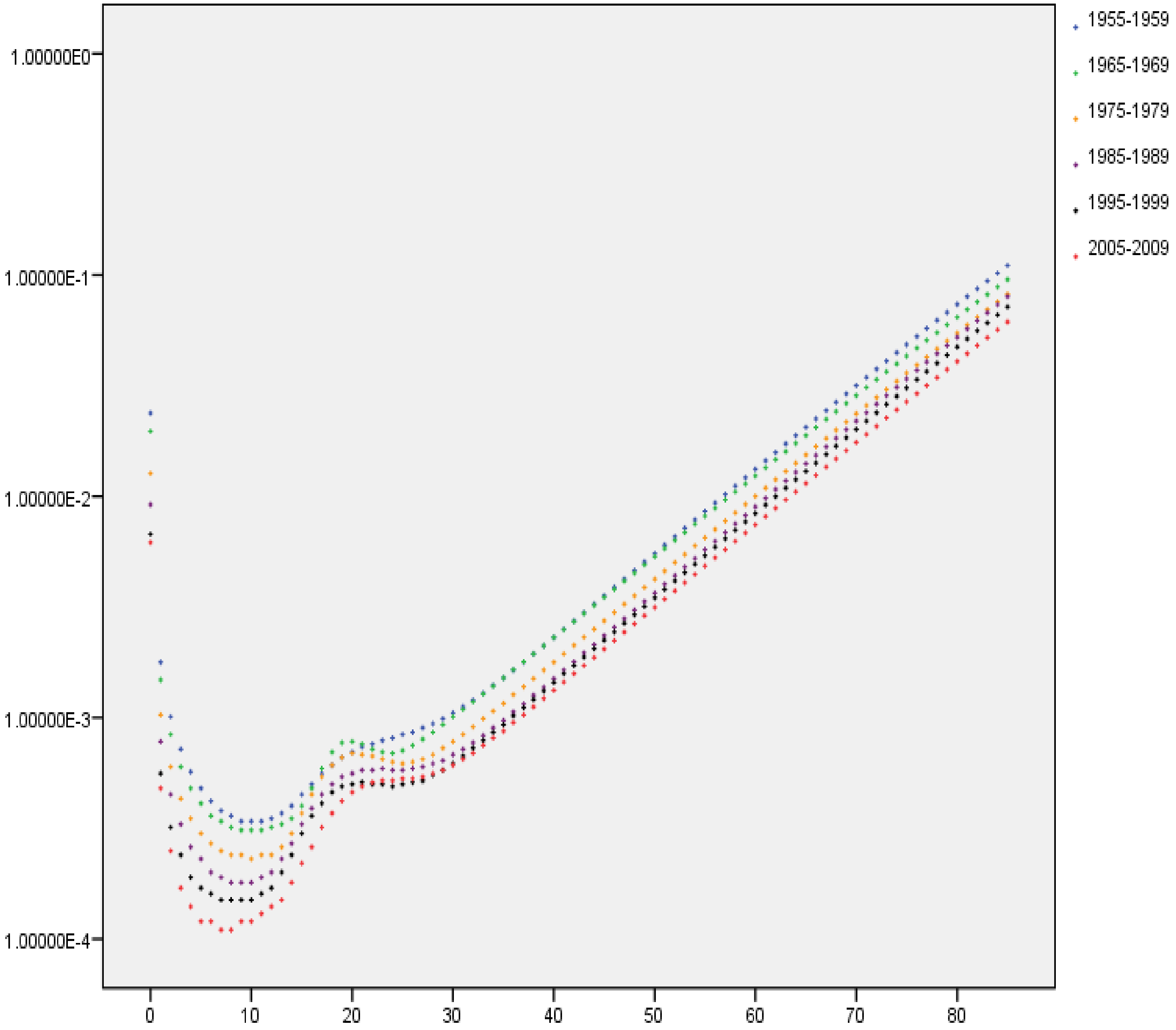

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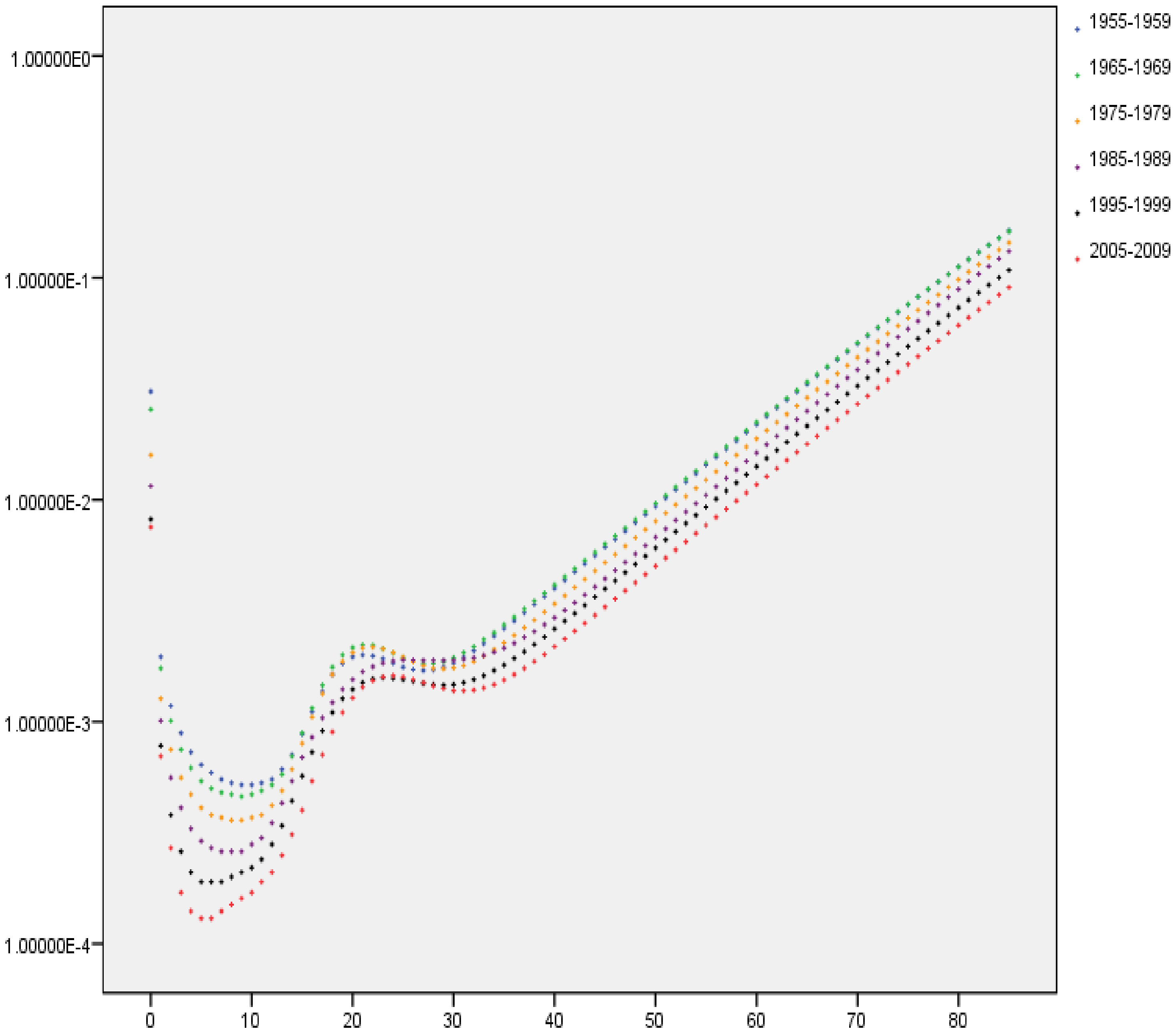
Figure 12: Mortality q(x) curves of US males.



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Chapter 5

CONCLUDING REVIEW

In this study, we have reviewed a technique for expanding an abridged life table to a complete one. The technique that we used, provides a way to expand a life table through the estimation of the one-year probabilities of dying, q_x , when the abridged ${}_nq_x$ -values are available. This method requires the use of a parametric model that has been proposed by Heligman and Pollard in 1980. This model expresses the probability of dying as a function of age x, and it uses a three-term expression that covers the entire span of life. The eight parameters of the Heligman and Pollard formula govern the shape of the mortality curve. Additionally, we computed the single-year probabilities of dying by using the program UNABR of the MORTPAK software package, which was provided by the United Nations. The software in order to calculate the complete life tables, estimates our mortality model by the least-squares method.

In the present thesis, we have examined mortality data from six countries, namely, Australia, Bulgaria, Canada, France, Russia and the United States of America. By their life tables we have formed the mortality curves and we have examine a variety of questions. An important one is whether the survival curves of males and females are the same or different. In case that they are different we can notice which sex has a survival advantage. The sexual differences of the mortality curves, may be affected by social or political factors, such as a war. In the majority of our selected countries, males have higher mortality rates than females in most ages. Another question that we are able to answer by observing our graphs, is whether different historical periods

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have had different mortality curves and how they have changed over the time. We have also generally concluded, that the mortality rates of the individuals except those around 20 have shown substantial improvements at all ages over the years. Factors to consider for this, are mainly the improved health attention, health care and medical services. Moreover, the development of diagnostic procedures, improvements in cancer management and changes in the prevalence of risk factors, contribute also to increases in survival. The most noticeable results are those of Russia, that we have noticed an important increase in the mortality rates of most ages.

The Heligman&Pollard mortality model contains the so-called "accident hump", which reflects the slight rise in mortality from accidents occurring in the late teens and early twenties mid-century. Regarding the accident hump, we should note that it hits its maximum in the late phase of puberty. It is undoubtedly more remarkable for males and it has similar but less spectacular pattern for females. Lastly, a fact that need to be considered is that it has become greater and more intense in many countries, as their recent mortality experiences suggest.

https://doi.org/10.26219/heal.aueb.6210

APPENDICES

https://doi.org/10.26219/heal.aueb.6210

https://doi.org/10.26219/heal.aueb.6210

Table 1: Estimated $\widehat{q_x}$- values for Australian females, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	1910	1584	1106	771	484	402	45	301	301	235	171	134	113
1	210	146	96	68	47	32	46	331	331	258	189	147	124
2	115	84	58	41	28	19	47	363	363	283	208	162	137
3	79	61	43	30	20	14	48	399	398	311	230	178	150
4	60	48	35	24	16	11	49	438	436	342	254	196	165
5	49	40	30	20	14	10	50	481	479	376	280	216	181
6	42	36	26	18	12	9	51	529	525	413	310	238	199
7	37	32	24	17	11	8	52	582	576	453	342	263	219
8	35	30	23	15	11	8	53	640	632	498	378	290	240
9	33	28	22	15	11	8	54	704	693	547	417	320	264
10	33	28	21	14	12	8	55	774	761	601	460	354	290
11	34	27	21	14	13	8	56	851	834	660	508	391	319
12	36	27	21	15	15	9	57	936	915	725	561	431	351
13	38	27	22	17	18	10	58	1030	1004	797	620	477	385
14	41	29	26	21	21	12	59	1133	1101	875	684	527	424
15	43	34	33	26	24	15	60	1246	1208	962	756	582	466
16	46	43	43	32	27	17	61	1370	1325	1056	834	643	512
17	49	56	53	39	31	20	62	1507	1453	1160	921	710	562
18	52	68	60	45	33	23	63	1657	1593	1274	1017	785	618
19	55	73	63	50	36	25	64	1821	1746	1399	1123	867	679
20	57	71	62	54	38	27	65	2002	1914	1537	1239	957	746
21	60	64	58	55	40	29	66	2201	2098	1687	1368	1058	820
22	62	58	53	55	41	30	67	2418	2299	1852	1509	1168	901
23	65	53	49	54	43	31	68	2657	2519	2033	1665	1290	990
24	67	52	46	52	44	32	69	2919	2760	2231	1837	1425	1087
25	70	54	45	50	44	32	70	3205	3022	2447	2026	1573	1194
26	74	57	46	48	45	33	71	3519	3309	2685	2234	1736	1312
27	77	61	48	47	46	33	72	3862	3622	2944	2463	1917	1440
28	81	66	52	47	47	34	73	4237	3963	3228	2715	2115	1582
29	85	72	56	47	48	35	74	4647	4335	3539	2992	2334	1736
30	91	78	60	48	49	36	75	5094	4741	3878	3296	2574	1906
31	96	85	66	50	50	38	76	5582	5182	4248	3630	2839	2092
32	103	93	71	53	52	40	77	6114	5662	4651	3997	3130	2296
33	110	101	78	57	55	42	78	6693	6183	5091	4399	3450	2519
34	118	111	85	61	57	45	79	7322	6749	5571	4839	3802	2763
35	127	121	93	66	60	48	80	8006	7363	6092	5321	4187	3030
36	138	132	102	72	64	52	81	8747	8028	6659	5848	4610	3322
37	149	145	112	79	69	56	82	9550	8747	7275	6423	5073	3641
38	162	159	123	87	74	61	83	10418	9524	7942	7051	5580	3989
39	176	174	135	96	79	66	84	11355	10362	8665	7735	6135	4369
40	192	190	148	105	86	72							
41	210	209	162	116	94	79							
42	229	229	178	128	102	86							
43	251	251	195	141	111	94							
44	275	275	214	155	122	103							

https://doi.org/10.26219/heal.aueb.6210

Table 2: Estimated $\widehat{q_x}$- values for Australian males, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	2390	2031	1418	990	598	492	45	500	516	427	298	223	185
1	246	169	125	83	65	42	46	549	568	470	329	243	200
2	143	101	74	51	37	24	47	603	624	517	364	266	218
3	103	75	55	38	26	17	48	663	686	569	402	292	238
4	82	61	45	31	21	14	49	729	754	627	445	321	261
5	69	53	39	27	18	12	50	802	829	689	492	354	286
6	60	47	35	24	15	10	51	881	912	759	544	391	314
7	54	44	32	23	14	10	52	969	1002	835	602	432	345
8	50	41	31	21	13	9	53	1065	1102	918	666	477	379
9	48	40	30	21	13	9	54	1171	1211	1011	737	529	417
10	46	39	30	21	14	10	55	1287	1331	1112	816	585	459
11	47	40	31	23	15	11	56	1414	1463	1223	903	649	506
12	51	44	34	27	19	13	57	1554	1608	1345	999	719	557
13	60	52	43	34	25	15	58	1708	1767	1480	1106	797	614
14	74	66	58	46	33	19	59	1877	1941	1627	1223	884	677
15	94	87	82	62	43	25	60	2062	2132	1789	1353	980	747
16	116	111	112	80	55	31	61	2265	2342	1967	1497	1087	824
17	138	136	142	100	68	38	62	2487	2571	2163	1656	1205	908
18	158	158	169	119	82	46	63	2731	2823	2377	1831	1336	1002
19	174	173	188	136	96	53	64	2998	3098	2612	2024	1482	1105
20	184	181	197	149	108	61	65	3290	3400	2869	2238	1643	1218
21	188	182	195	158	119	68	66	3610	3729	3151	2473	1821	1343
22	187	177	186	162	129	75	67	3959	4090	3460	2732	2018	1481
23	183	168	173	163	136	80	68	4341	4483	3798	3018	2237	1633
24	177	159	158	160	141	85	69	4758	4913	4167	3333	2478	1800
25	170	150	144	155	144	89	70	5213	5381	4571	3679	2744	1984
26	163	143	132	149	146	92	71	5708	5891	5012	4060	3039	2186
27	158	139	124	143	146	94	72	6248	6446	5493	4478	3364	2409
28	155	138	119	136	145	96	73	6835	7050	6017	4938	3723	2653
29	155	140	118	131	143	98	74	7473	7706	6588	5442	4118	2922
30	157	144	120	126	141	99	75	8165	8417	7208	5994	4553	3217
31	161	152	125	124	139	100	76	8915	9187	7882	6598	5031	3540
32	168	162	133	123	138	101	77	9727	10021	8614	7258	5557	3895
33	178	174	142	124	136	102	78	10604	10920	9406	7979	6135	4284
34	190	188	154	127	136	104	79	11550	11890	10264	8765	6768	4710
35	204	205	168	131	136	106	80	12568	12933	11189	9619	7462	5176
36	221	223	183	138	138	109	81	13663	14054	12187	10548	8220	5685
37	240	244	201	147	140	113	82	14837	15254	13261	11555	9048	6241
38	262	268	220	158	144	118	83	16093	16537	14413	12644	9950	6847
39	286	294	242	171	150	123	84	17433	17906	15648	13820	10931	7508
40	314	323	266	186	157	130							
41	344	354	292	204	166	138							
42	377	389	321	223	177	147							
43	414	428	353	245	190	158							
44	455	470	388	270	206	171							

https://doi.org/10.26219/heal.aueb.6210

Table 3: Estimated $\widehat{q_x}$- values for Bulgarian females, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	5965	2723	1906	1253	1341	876	45	308	239	223	218	238	225
1	1057	252	184	149	186	68	46	333	261	245	241	259	247
2	462	134	103	87	94	46	47	362	286	270	266	283	272
3	257	90	71	62	63	37	48	394	315	299	293	310	300
4	162	67	55	49	49	32	49	430	347	330	325	340	330
5	111	53	45	40	41	29	50	470	382	366	360	373	364
6	81	45	39	35	37	26	51	516	423	406	400	410	401
7	64	39	35	31	34	25	52	567	468	452	444	451	442
8	54	36	33	29	33	24	53	624	518	503	494	497	487
9	49	34	32	28	33	23	54	688	575	560	550	548	536
10	48	33	32	28	33	22	55	759	639	624	612	606	591
11	50	33	33	29	34	23	56	840	711	696	682	670	651
12	53	34	34	30	35	23	57	929	791	778	760	742	718
13	57	36	36	32	36	25	58	1029	881	869	847	822	791
14	63	38	38	34	37	27	59	1141	982	971	945	911	872
15	69	40	40	37	38	29	60	1266	1095	1085	1054	1012	961
16	75	43	42	39	40	32	61	1405	1222	1214	1176	1123	1059
17	81	45	44	42	42	34	62	1560	1364	1358	1312	1248	1167
18	87	48	46	44	44	37	63	1733	1522	1520	1464	1388	1287
19	93	51	48	47	46	38	64	1926	1700	1701	1633	1544	1418
20	98	54	50	49	48	40	65	2140	1898	1903	1822	1718	1562
21	104	57	53	52	50	41	66	2378	2120	2130	2033	1912	1720
22	109	60	55	54	53	42	67	2644	2367	2383	2268	2129	1895
23	113	63	57	56	56	43	68	2938	2644	2666	2529	2371	2087
24	118	66	60	59	59	44	69	3265	2952	2982	2820	2641	2298
25	122	69	62	61	62	45	70	3627	3295	3335	3144	2942	2530
26	126	72	65	63	65	47	71	4029	3678	3728	3503	3276	2784
27	130	75	67	66	69	49	72	4474	4103	4167	3903	3649	3064
28	134	79	70	69	72	52	73	4966	4577	4654	4346	4063	3370
29	139	83	74	72	77	55	74	5509	5102	5196	4836	4523	3706
30	143	87	77	75	81	59	75	6109	5684	5798	5380	5034	4075
31	148	91	81	78	86	64	76	6770	6329	6464	5980	5600	4478
32	152	96	85	82	92	69	77	7497	7042	7202	6643	6226	4919
33	158	101	90	87	97	75	78	8296	7829	8017	7374	6919	5401
34	164	107	95	92	104	81	79	9172	8696	8916	8179	7684	5927
35	170	114	101	98	111	89	80	10130	9650	9905	9062	8527	6502
36	178	121	108	104	119	97	81	11177	10697	10991	10031	9454	7127
37	186	129	115	112	127	106	82	12318	11842	12180	11091	10471	7808
38	196	138	124	120	137	116	83	13557	13093	13479	12248	11585	8548
39	206	148	134	129	147	127	84	14900	14454	14893	13507	12801	9351
40	219	159	144	140	159	140							
41	232	171	157	152	171	154							
42	248	185	170	166	185	169							
43	265	201	186	181	201	186							
44	285	219	203	199	219	204							

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Table 4: Estimated $\widehat{q_x}$- values for Bulgarian males, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	7115	3391	2488	1593	1722	1040	45	429	395	463	575	636	521
1	1027	238	185	176	192	87	46	469	433	507	628	693	570
2	473	150	118	105	103	55	47	513	475	557	686	756	623
3	279	114	91	79	72	43	48	562	522	611	749	824	682
4	186	93	75	65	58	37	49	616	574	671	817	898	746
5	134	80	66	56	50	33	50	677	631	737	893	979	816
6	103	71	59	51	45	32	51	743	695	810	975	1067	892
7	83	64	55	48	43	31	52	816	765	890	1064	1163	976
8	70	59	51	46	42	31	53	898	843	978	1162	1267	1068
9	64	56	49	45	41	31	54	987	928	1075	1269	1380	1167
10	62	55	49	45	42	32	55	1086	1023	1181	1385	1504	1277
11	65	55	49	46	43	33	56	1194	1127	1298	1512	1638	1396
12	71	57	52	48	45	35	57	1314	1243	1427	1650	1785	1526
13	80	61	56	51	47	37	58	1446	1369	1568	1801	1944	1669
14	91	68	63	58	51	40	59	1590	1509	1723	1965	2116	1824
15	102	76	71	67	56	46	60	1750	1663	1893	2144	2304	1994
16	113	85	81	80	67	54	61	1925	1833	2079	2339	2509	2179
17	124	95	91	93	84	66	62	2118	2020	2284	2552	2730	2380
18	134	104	100	106	106	80	63	2329	2225	2508	2783	2971	2600
19	142	113	109	117	125	95	64	2561	2451	2754	3034	3232	2840
20	149	121	117	125	136	107	65	2816	2699	3023	3307	3516	3101
21	155	128	124	130	137	115	66	3096	2972	3318	3604	3823	3385
22	159	134	129	131	133	120	67	3402	3272	3640	3927	4157	3695
23	163	138	133	132	127	121	68	3738	3601	3993	4277	4517	4031
24	165	142	137	132	124	120	69	4105	3962	4378	4657	4908	4397
25	167	144	139	132	125	120	70	4507	4357	4799	5069	5330	4794
26	169	146	142	135	130	120	71	4946	4790	5258	5516	5787	5225
27	170	148	145	139	138	122	72	5425	5264	5758	5999	6280	5693
28	172	150	148	144	149	127	73	5949	5782	6303	6522	6812	6200
29	175	152	151	152	161	133	74	6519	6347	6895	7086	7385	6749
30	178	154	156	162	175	142	75	7139	6964	7539	7696	8003	7342
31	181	158	162	174	191	152	76	7814	7635	8238	8353	8668	7983
32	186	163	170	188	208	165	77	8547	8366	8995	9062	9382	8675
33	193	168	178	204	226	179	78	9342	9160	9814	9823	10148	9421
34	200	176	189	221	246	195	79	10202	10021	10700	10641	10970	10224
35	210	185	202	241	269	213	80	11132	10953	11655	11519	11849	11087
36	221	195	216	262	293	233	81	12135	11961	12683	12459	12788	12013
37	233	207	233	286	319	254	82	13215	13048	13787	13464	13791	13005
38	248	222	251	311	348	278	83	14376	14217	14972	14537	14858	14065
39	266	238	273	340	379	304	84	15620	15473	16239	15680	15993	15198
40	285	257	296	371	413	333							
41	308	279	323	405	450	364							
42	333	303	353	442	491	398							
43	362	330	386	482	535	435							
44	394	361	422	527	584	476							

Table 5: Estimated $\widehat{q_x}$- values for Canadian females, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	2868	1881	1113	659	495	476	45	287	264	229	178	157	136
1	213	137	90	62	39	25	46	315	289	250	196	173	149
2	123	86	59	38	25	17	47	345	316	274	216	190	164
3	88	66	46	28	20	13	48	378	347	300	237	209	180
4	69	54	39	23	17	11	49	415	380	328	261	229	197
5	57	47	35	20	15	10	50	456	416	359	286	252	216
6	50	42	31	18	14	10	51	501	456	393	315	277	238
7	44	38	29	17	13	9	52	551	500	430	346	305	261
8	41	36	28	16	13	9	53	606	547	471	381	335	287
9	38	34	27	15	12	9	54	667	600	516	419	369	315
10	37	33	26	15	12	9	55	733	658	565	461	405	346
11	37	32	26	16	13	10	56	807	721	619	507	446	379
12	38	32	26	17	14	10	57	888	790	678	557	490	417
13	39	32	26	20	17	12	58	977	866	743	613	539	458
14	42	34	28	24	20	15	59	1075	950	813	674	593	502
15	44	38	33	29	25	18	60	1183	1041	891	741	652	552
16	47	44	42	34	29	22	61	1302	1141	976	815	717	606
17	49	50	53	38	33	26	62	1433	1251	1068	896	789	665
18	52	55	62	41	35	30	63	1577	1371	1170	985	867	730
19	55	58	65	43	36	32	64	1735	1502	1281	1083	954	802
20	58	59	63	43	36	33	65	1908	1646	1403	1191	1048	880
21	61	59	57	42	35	33	66	2099	1804	1536	1309	1153	966
22	64	57	52	41	34	33	67	2309	1976	1682	1439	1267	1061
23	66	56	48	39	32	32	68	2539	2164	1841	1581	1393	1164
24	69	55	46	38	32	31	69	2791	2370	2015	1737	1530	1278
25	72	56	46	38	32	31	70	3067	2595	2205	1909	1682	1402
26	76	57	48	38	32	31	71	3371	2841	2413	2097	1848	1539
27	79	60	51	39	33	31	72	3703	3110	2639	2303	2030	1688
28	83	63	54	41	35	32	73	4066	3403	2887	2529	2229	1852
29	87	68	58	43	38	34	74	4463	3723	3157	2776	2448	2031
30	92	73	63	46	40	36	75	4898	4071	3451	3047	2688	2227
31	97	78	68	50	44	39	76	5372	4451	3772	3343	2950	2442
32	103	85	74	54	48	42	77	5889	4864	4121	3667	3237	2677
33	110	92	80	59	52	46	78	6453	5313	4501	4022	3551	2933
34	117	100	87	64	57	50	79	7067	5802	4914	4408	3895	3214
35	126	109	95	70	62	54	80	7735	6332	5363	4831	4270	3520
36	135	119	104	77	68	59	81	8460	6908	5851	5291	4679	3855
37	146	129	113	84	74	65	82	9246	7531	6380	5793	5126	4220
38	158	141	123	93	82	71	83	10097	8206	6954	6339	5613	4617
39	171	154	134	102	89	78	84	11016	8936	7575	6933	6143	5051
40	185	169	147	112	98	85							
41	202	184	160	122	108	94							
42	220	201	175	134	118	103							
43	240	220	191	148	130	113							
44	262	241	209	162	143	124							

https://doi.org/10.26219/heal.aueb.6210

Table 6: Estimated $\widehat{q_x}$- values for Canadian males, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	3626	2360	1400	838	595	550	45	481	473	433	314	257	214
1	232	160	108	73	48	36	46	526	518	474	345	282	235
2	150	107	74	47	30	22	47	576	566	519	380	310	258
3	115	85	60	36	23	16	48	630	620	568	419	341	284
4	96	73	52	30	19	14	49	690	678	622	462	376	312
5	83	65	47	27	17	12	50	755	742	681	509	414	342
6	75	60	44	24	16	12	51	826	812	745	561	457	376
7	69	56	42	23	15	11	52	905	889	816	618	504	413
8	65	53	40	22	15	11	53	991	973	893	682	555	454
9	62	52	40	22	15	11	54	1085	1065	978	751	612	498
10	60	51	40	22	16	12	55	1188	1165	1071	828	675	547
11	59	51	40	24	18	13	56	1301	1275	1173	913	745	601
12	61	52	42	28	21	15	57	1424	1396	1284	1006	822	660
13	65	56	49	35	26	18	58	1559	1528	1406	1109	906	725
14	74	64	61	46	34	24	59	1707	1672	1539	1222	1000	797
15	87	79	81	60	43	31	60	1869	1829	1685	1346	1103	875
16	103	101	109	75	53	41	61	2045	2002	1844	1483	1216	961
17	122	126	139	92	64	51	62	2239	2190	2018	1634	1341	1055
18	140	152	167	107	74	62	63	2450	2395	2209	1799	1479	1158
19	155	173	189	120	84	72	64	2680	2620	2416	1982	1630	1271
20	166	187	199	129	92	79	65	2932	2864	2643	2182	1797	1396
21	173	192	200	135	98	85	66	3207	3131	2891	2402	1980	1532
22	175	190	192	137	103	88	67	3506	3423	3161	2644	2182	1681
23	173	182	179	136	106	89	68	3833	3740	3456	2910	2404	1844
24	170	172	165	133	108	88	69	4188	4085	3777	3201	2648	2024
25	165	161	151	129	108	87	70	4576	4461	4126	3520	2916	2220
26	161	152	140	124	108	84	71	4997	4870	4507	3870	3210	2434
27	157	146	132	120	108	82	72	5455	5315	4921	4253	3533	2669
28	156	142	128	116	107	80	73	5952	5797	5371	4673	3887	2926
29	156	142	127	114	106	78	74	6491	6321	5859	5131	4275	3207
30	159	146	130	112	106	78	75	7076	6888	6389	5632	4700	3513
31	164	152	136	113	107	78	76	7709	7502	6963	6179	5164	3848
32	171	160	144	115	108	80	77	8394	8167	7585	6774	5672	4214
33	181	171	154	119	111	83	78	9134	8884	8257	7423	6227	4612
34	192	184	166	125	114	87	79	9932	9658	8984	8129	6831	5046
35	206	199	180	132	119	92	80	10791	10492	9767	8895	7490	5519
36	222	216	195	141	126	98	81	11715	11389	10611	9726	8207	6033
37	241	235	213	152	133	106	82	12707	12352	11519	10625	8986	6591
38	261	255	232	165	142	115	83	13770	13384	12493	11597	9831	7197
39	284	279	253	180	153	125	84	14907	14488	13537	12645	10746	7855
40	309	304	277	196	165	136							
41	338	332	303	215	180	149							
42	368	362	331	236	196	163							
43	403	396	362	259	214	178							
44	440	433	396	285	234	195							

https://doi.org/10.26219/heal.aueb.6210

Table 7: Estimated $\widehat{q_x}$- values for French females, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	2917	1798	1020	687	532	328	45	324	264	226	179	150	135
1	289	136	88	67	59	33	46	349	289	247	192	160	147
2	139	78	56	38	33	17	47	375	317	270	206	171	160
3	87	56	42	27	24	12	48	405	347	296	221	182	174
4	62	45	35	22	19	10	49	437	381	323	238	194	190
5	48	38	31	20	16	9	50	473	418	354	256	207	206
6	39	33	28	18	15	8	51	512	459	387	277	221	225
7	34	30	26	18	14	8	52	556	504	424	299	237	245
8	31	28	24	18	14	8	53	604	553	464	324	254	266
9	30	26	24	18	15	8	54	657	607	507	351	273	290
10	30	26	23	19	15	8	55	717	667	555	382	293	315
11	31	26	23	20	16	9	56	783	732	608	416	315	343
12	32	28	24	21	17	9	57	857	804	665	453	340	373
13	34	31	26	23	18	10	58	939	882	728	495	367	406
14	36	34	31	24	19	11	59	1030	969	797	542	396	442
15	39	38	36	26	21	11	60	1132	1064	872	594	429	481
16	42	43	43	28	23	13	61	1246	1168	954	652	465	524
17	46	48	50	30	24	16	62	1373	1282	1044	717	505	570
18	50	53	56	32	26	26	63	1515	1407	1143	790	549	620
19	54	57	61	35	28	35	64	1673	1545	1250	872	598	675
20	58	61	63	37	31	36	65	1850	1695	1368	963	653	734
21	63	65	64	40	33	30	66	2047	1860	1497	1066	713	799
22	68	68	63	42	35	24	67	2268	2041	1637	1181	780	869
23	74	70	62	45	38	22	68	2514	2239	1790	1309	854	945
24	79	73	60	48	41	23	69	2789	2456	1958	1454	937	1028
25	85	75	59	52	44	25	70	3096	2693	2141	1615	1030	1118
26	92	77	59	55	47	27	71	3438	2952	2341	1797	1133	1216
27	98	79	59	59	50	30	72	3819	3236	2558	2000	1247	1323
28	105	81	60	62	53	32	73	4244	3545	2796	2228	1375	1438
29	112	84	63	66	57	35	74	4717	3884	3055	2484	1517	1564
30	120	88	66	71	61	38	75	5243	4253	3337	2771	1676	1700
31	128	92	70	75	64	41	76	5828	4655	3644	3091	1852	1848
32	137	96	75	80	69	45	77	6476	5094	3978	3451	2050	2009
33	146	102	80	85	73	49	78	7194	5571	4342	3852	2269	2183
34	156	108	87	90	78	53	79	7988	6091	4737	4301	2514	2372
35	166	115	94	95	82	58	80	8866	6655	5167	4803	2787	2577
36	177	124	103	101	87	63	81	9832	7268	5633	5362	3091	2799
37	189	133	112	108	93	69	82	10895	7932	6138	5985	3429	3039
38	202	144	122	115	99	75	83	12061	8652	6685	6678	3805	3300
39	215	156	133	122	105	81	84	13336	9430	7277	7448	4224	3582
40	230	170	145	130	111	88							
41	246	185	158	138	118	96							
42	263	202	173	147	125	105							
43	282	220	189	157	133	114							
44	302	241	207	168	142	124							

https://doi.org/10.26219/heal.aueb.6210

Table 8: Estimated $\widehat{q_x}$- values for French males, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	3776	2332	1336	914	528	414	45	605	580	520	446	370	273
1	339	165	118	85	62	52	46	660	633	568	486	403	298
2	161	95	71	46	27	16	47	720	691	620	530	439	326
3	102	69	53	34	18	10	48	786	754	676	578	478	356
4	75	57	45	28	15	8	49	858	823	738	629	521	389
5	60	49	40	25	14	8	50	937	899	805	686	568	425
6	51	45	37	24	14	9	51	1023	981	879	747	619	464
7	46	43	36	24	15	9	52	1116	1070	959	814	674	507
8	43	41	35	24	16	10	53	1219	1168	1046	887	734	554
9	42	41	35	25	17	11	54	1330	1274	1141	967	800	606
10	42	41	36	26	18	12	55	1452	1390	1245	1053	872	662
11	43	42	37	27	20	13	56	1585	1517	1358	1147	950	723
12	47	45	39	30	22	15	57	1729	1655	1481	1250	1035	790
13	53	49	43	34	26	17	58	1887	1805	1615	1361	1127	863
14	62	58	52	40	31	20	59	2058	1968	1761	1483	1227	942
15	74	71	69	50	39	25	60	2245	2146	1920	1614	1337	1029
16	87	89	95	65	49	32	61	2449	2340	2093	1758	1456	1124
17	101	110	126	84	62	41	62	2670	2551	2281	1914	1585	1227
18	116	130	158	105	77	53	63	2911	2780	2486	2083	1726	1340
19	129	148	182	126	92	64	64	3173	3029	2709	2267	1878	1463
20	141	160	194	143	106	75	65	3457	3300	2951	2467	2044	1597
21	151	167	194	157	118	83	66	3766	3594	3214	2684	2225	1743
22	159	169	186	165	127	89	67	4102	3913	3500	2920	2421	1902
23	165	167	172	167	133	91	68	4466	4260	3810	3176	2634	2076
24	170	163	158	166	136	92	69	4860	4636	4146	3453	2865	2265
25	173	159	146	163	137	90	70	5288	5043	4511	3754	3115	2471
26	177	157	138	158	137	88	71	5751	5483	4906	4079	3387	2694
27	181	156	135	154	136	87	72	6252	5960	5334	4432	3682	2938
28	186	158	135	151	136	86	73	6793	6476	5797	4813	4001	3203
29	192	163	140	150	136	86	74	7378	7033	6298	5226	4346	3491
30	199	171	148	152	137	87	75	8008	7633	6838	5671	4720	3804
31	208	181	158	156	140	90	76	8687	8281	7422	6153	5124	4143
32	219	193	170	162	144	94	77	9418	8978	8050	6672	5561	4512
33	232	208	185	171	150	100	78	10204	9728	8727	7232	6033	4911
34	248	225	201	182	158	107	79	11047	10532	9456	7834	6542	5344
35	265	244	218	195	167	115	80	11951	11396	10238	8483	7090	5813
36	285	266	238	210	179	125	81	12918	12320	11076	9180	7681	6320
37	308	289	259	227	192	135	82	13950	13308	11975	9927	8317	6868
38	334	315	283	246	207	147	83	15051	14362	12936	10729	9000	7460
39	362	344	309	267	224	161	84	16223	15484	13961	11586	9734	8099
40	393	375	337	291	243	175							
41	428	409	367	317	264	191							
42	466	446	401	345	287	209							
43	508	487	437	376	312	228							
44	554	532	477	409	339	250							

Table 9: Estimated $\widehat{q_x}$- values for Russian females, 1965-2009, multiplied by 100,000.

x	$\widehat{q_x}$ *65-69*	$\widehat{q_x}$ *75-79*	$\widehat{q_x}$ *85-89*	$\widehat{q_x}$ *95-99*	$\widehat{q_x}$ *05-09*	*x*	$\widehat{q_x}$ *65-69*	$\widehat{q_x}$ *75-79*	$\widehat{q_x}$ *85-89*	$\widehat{q_x}$ *95-99*	$\widehat{q_x}$ *05-09*
0	2175	2005	1600	1453	825	45	307	341	286	395	419
1	247	229	181	171	88	46	329	367	315	431	444
2	142	127	103	92	56	47	354	394	346	470	472
3	100	88	73	64	42	48	381	425	380	512	502
4	78	68	57	50	35	49	411	458	418	559	535
5	64	57	47	42	30	50	445	494	460	609	571
6	55	49	41	37	27	51	482	534	506	665	610
7	49	45	36	34	25	52	524	579	556	725	654
8	45	42	33	32	24	53	570	627	612	791	702
9	43	41	31	31	24	54	622	681	674	863	755
10	43	40	31	31	25	55	679	740	742	941	814
11	43	40	31	32	26	56	743	806	816	1026	879
12	44	41	32	34	29	57	815	879	898	1119	951
13	46	43	35	38	32	58	894	960	989	1220	1030
14	48	45	38	44	36	59	983	1050	1088	1331	1118
15	51	47	43	52	41	60	1081	1150	1197	1451	1216
16	55	50	47	61	47	61	1191	1261	1317	1582	1324
17	58	53	52	71	53	62	1314	1384	1449	1724	1444
18	63	56	56	81	60	63	1450	1522	1594	1880	1578
19	67	60	60	91	68	64	1603	1676	1754	2048	1725
20	72	64	63	99	76	65	1772	1847	1929	2232	1889
21	76	69	66	105	85	66	1961	2037	2121	2432	2070
22	81	73	69	111	94	67	2171	2250	2331	2649	2271
23	87	79	71	115	104	68	2404	2487	2563	2885	2494
24	92	84	73	117	114	69	2664	2752	2816	3141	2740
25	98	90	75	120	124	70	2952	3047	3094	3420	3013
26	103	96	77	121	134	71	3273	3376	3399	3722	3314
27	109	103	79	124	145	72	3629	3743	3732	4049	3647
28	115	110	82	126	156	73	4023	4151	4096	4404	4015
29	122	117	85	130	168	74	4460	4606	4495	4789	4421
30	129	125	89	134	179	75	4944	5111	4930	5205	4869
31	136	134	94	139	191	76	5478	5673	5405	5656	5362
32	143	143	99	146	203	77	6069	6297	5923	6143	5905
33	151	153	105	155	216	78	6720	6988	6487	6669	6502
34	159	163	112	164	228	79	7438	7754	7101	7237	7157
35	168	174	121	176	242	80	8227	8600	7769	7848	7874
36	177	186	130	189	255	81	9093	9533	8493	8507	8660
37	187	198	141	203	270	82	10041	10561	9278	9216	9518
38	198	212	153	220	285	83	11078	11691	10128	9977	10454
39	210	226	166	238	300	84	12209	12928	11046	10793	11473
40	222	242	181	258	317						
41	236	259	198	281	335						
42	251	277	217	306	354						
43	268	297	238	333	374						
44	286	318	261	363	395						

https://doi.org/10.26219/heal.aueb.6210

Table 10: Estimated $\widehat{q_x}$- values for Russian males, 1965-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	x	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	2834	2638	2184	1939	1068	45	841	1042	839	1359	1469
1	278	280	240	210	121	46	893	1099	908	1426	1536
2	169	161	129	115	71	47	952	1161	982	1497	1606
3	127	117	92	82	53	48	1016	1230	1063	1573	1681
4	104	95	75	66	44	49	1086	1304	1151	1656	1761
5	91	82	66	57	39	50	1164	1386	1246	1745	1846
6	82	74	62	53	37	51	1249	1476	1349	1842	1937
7	77	69	60	50	36	52	1342	1575	1461	1946	2035
8	73	66	59	50	36	53	1444	1682	1582	2060	2142
9	72	66	60	52	37	54	1556	1800	1713	2184	2257
10	72	67	62	56	40	55	1678	1929	1854	2319	2381
11	74	71	65	62	45	56	1811	2069	2007	2465	2517
12	78	77	69	70	52	57	1956	2223	2172	2625	2664
13	85	86	74	82	62	58	2115	2390	2351	2799	2825
14	94	98	82	97	75	59	2287	2573	2544	2988	3000
15	106	113	94	115	90	60	2474	2772	2753	3194	3191
16	120	130	109	136	109	61	2677	2988	2978	3418	3399
17	138	151	127	160	132	62	2898	3224	3221	3663	3626
18	157	174	148	187	158	63	3138	3479	3483	3928	3874
19	179	199	172	216	187	64	3398	3757	3765	4216	4144
20	201	226	195	249	219	65	3680	4059	4069	4530	4438
21	225	254	218	283	255	66	3984	4386	4397	4870	4759
22	249	283	240	319	293	67	4314	4740	4750	5239	5109
23	273	313	259	357	333	68	4671	5123	5130	5639	5489
24	296	343	275	395	375	69	5056	5537	5539	6073	5903
25	319	373	288	435	420	70	5472	5985	5978	6542	6353
26	342	403	299	476	466	71	5921	6468	6449	7048	6841
27	363	432	309	517	513	72	6404	6989	6955	7596	7371
28	384	461	318	558	561	73	6925	7550	7497	8186	7945
29	404	490	327	599	610	74	7485	8154	8078	8823	8567
30	423	518	337	641	659	75	8087	8803	8700	9507	9239
31	442	546	348	683	709	76	8733	9500	9364	10243	9964
32	461	574	361	725	759	77	9426	10247	10074	11033	10746
33	480	602	376	767	810	78	10168	11047	10832	11880	11587
34	499	630	394	810	861	79	10961	11902	11638	12785	12492
35	519	658	415	853	912	80	11809	12815	12497	13753	13462
36	540	688	440	896	963	81	12713	13788	13409	14784	14501
37	563	718	468	940	1015	82	13676	14823	14377	15882	15611
38	587	749	499	986	1067	83	14700	15922	15402	17048	16795
39	614	783	535	1032	1120	84	15787	17088	16487	18285	18054
40	643	818	574	1081	1174						
41	675	856	618	1131	1229						
42	711	897	666	1183	1286						
43	750	941	719	1239	1345						
44	793	989	776	1297	1406						

https://doi.org/10.26219/heal.aueb.6210

Table 11: Estimated $\widehat{q_x}$- values for US females, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09	*x*	$\widehat{q_x}$ 55-59	$\widehat{q_x}$ 65-69	$\widehat{q_x}$ 75-79	$\widehat{q_x}$ 85-89	$\widehat{q_x}$ 95-99	$\widehat{q_x}$ 05-09
0	2375	1967	1268	918	674	618	45	355	350	274	234	224	204
1	178	148	103	78	56	48	46	388	381	299	256	244	223
2	101	84	60	45	32	25	47	423	414	326	280	267	243
3	72	60	43	33	24	17	48	462	451	355	306	292	265
4	57	48	35	26	19	14	49	505	490	387	335	318	289
5	48	41	30	23	17	12	50	551	534	422	366	348	315
6	42	36	27	20	16	12	51	602	581	460	401	380	343
7	38	34	25	19	15	11	52	657	632	502	438	415	374
8	36	32	24	18	15	11	53	718	687	547	479	453	407
9	34	31	24	18	15	12	54	783	748	597	524	494	444
10	34	31	23	18	15	12	55	855	814	650	574	540	484
11	34	31	24	19	16	13	56	934	885	709	627	590	528
12	35	32	24	20	17	14	57	1020	963	773	686	644	575
13	37	33	26	23	20	15	58	1113	1048	843	751	703	627
14	40	35	30	27	24	18	59	1215	1139	919	821	767	683
15	45	40	37	33	30	22	60	1327	1239	1001	898	838	744
16	50	48	45	39	36	26	61	1448	1348	1092	982	915	811
17	56	59	54	45	41	32	62	1580	1466	1190	1074	998	884
18	61	70	61	50	46	37	63	1724	1594	1297	1174	1090	963
19	66	77	66	54	49	42	64	1881	1733	1413	1284	1190	1049
20	70	78	69	56	50	46	65	2053	1884	1540	1404	1298	1143
21	74	76	68	58	51	49	66	2239	2048	1678	1534	1417	1245
22	76	72	67	58	50	51	67	2442	2226	1828	1677	1546	1356
23	79	70	65	59	50	52	68	2663	2419	1991	1833	1687	1477
24	81	69	63	58	49	52	69	2903	2629	2168	2003	1840	1608
25	84	71	62	58	50	53	70	3164	2856	2361	2188	2007	1751
26	86	75	63	59	51	53	71	3448	3101	2570	2390	2188	1906
27	90	80	65	60	52	54	72	3756	3368	2798	2611	2386	2075
28	94	86	68	62	55	56	73	4091	3656	3045	2851	2601	2258
29	99	93	73	64	58	58	74	4454	3968	3313	3112	2835	2458
30	105	101	78	68	62	61	75	4848	4306	3604	3396	3090	2674
31	112	109	84	72	67	65	76	5275	4671	3920	3706	3366	2909
32	120	118	91	77	73	69	77	5737	5065	4261	4043	3666	3164
33	129	128	99	83	79	75	78	6237	5490	4632	4408	3992	3440
34	140	139	107	90	86	81	79	6777	5949	5032	4805	4346	3739
35	151	151	116	97	93	87	80	7361	6444	5466	5236	4729	4064
36	164	165	127	106	102	95	81	7990	6977	5934	5704	5145	4415
37	178	179	138	115	111	103	82	8668	7550	6440	6210	5594	4795
38	194	194	150	126	121	112	83	9398	8167	6986	6758	6081	5206
39	211	211	164	137	132	122	84	10183	8829	7574	7351	6607	5650
40	230	230	178	150	144	133							
41	251	250	194	164	158	145							
42	273	272	212	179	172	158							
43	298	296	231	196	188	172							
44	326	322	251	214	205	187							

 https://doi.org/10.26219/heal.aueb.6210

Table 12: Estimated $\widehat{q_x}$- values for US males, 1955-2009, multiplied by 100,000.

x	$\widehat{q_x}$ *55-59*	$\widehat{q_x}$ *65-69*	$\widehat{q_x}$ *75-79*	$\widehat{q_x}$ *85-89*	$\widehat{q_x}$ *95-99*	$\widehat{q_x}$ *05-09*	*x*	$\widehat{q_x}$ *55-59*	$\widehat{q_x}$ *65-69*	$\widehat{q_x}$ *75-79*	$\widehat{q_x}$ *85-89*	$\widehat{q_x}$ *95-99*	$\widehat{q_x}$ *05-09*
0	3073	2553	1592	1151	817	753	45	611	631	521	441	397	329
1	196	174	127	101	78	70	46	665	686	568	480	432	358
2	118	101	75	56	38	27	47	724	747	619	523	470	389
3	89	75	56	41	26	17	48	789	813	674	570	512	424
4	73	62	47	33	21	14	49	859	885	735	622	557	461
5	64	54	41	29	19	13	50	935	963	801	678	606	502
6	59	50	38	27	19	13	51	1018	1047	872	740	660	546
7	55	48	37	26	19	14	52	1109	1140	951	808	718	595
8	53	47	36	26	20	15	53	1208	1240	1036	882	782	647
9	52	46	36	26	21	16	54	1315	1349	1128	962	851	705
10	52	47	37	28	22	17	55	1432	1468	1229	1050	926	767
11	53	49	38	30	24	19	56	1559	1596	1339	1146	1008	834
12	55	52	42	35	28	21	57	1697	1736	1459	1251	1097	908
13	61	58	49	43	34	25	58	1847	1888	1589	1365	1193	988
14	71	70	61	54	44	31	59	2010	2053	1731	1490	1298	1075
15	88	89	80	69	57	40	60	2188	2232	1884	1626	1412	1169
16	111	115	105	85	73	54	61	2380	2426	2052	1774	1536	1272
17	137	146	134	104	91	71	62	2590	2637	2234	1935	1671	1384
18	163	176	163	122	110	90	63	2817	2866	2431	2110	1817	1505
19	183	200	187	140	127	110	64	3063	3113	2646	2302	1976	1637
20	196	216	205	155	140	128	65	3331	3382	2879	2510	2149	1780
21	200	222	215	168	150	143	66	3620	3673	3132	2736	2336	1935
22	198	221	217	177	156	153	67	3934	3987	3406	2982	2539	2104
23	192	213	213	184	158	159	68	4274	4328	3703	3249	2759	2286
24	184	204	205	188	157	161	69	4642	4696	4026	3540	2998	2485
25	176	194	195	190	155	159	70	5041	5094	4375	3855	3256	2700
26	172	187	186	190	152	155	71	5471	5523	4753	4198	3536	2933
27	170	183	179	189	149	150	72	5936	5987	5161	4569	3840	3186
28	171	183	174	189	147	145	73	6437	6486	5603	4972	4168	3459
29	176	187	173	189	146	141	74	6978	7025	6080	5408	4523	3756
30	184	194	175	189	147	138	75	7561	7604	6596	5879	4906	4076
31	195	205	179	191	150	138	76	8188	8227	7151	6390	5320	4423
32	209	218	187	194	155	139	77	8862	8896	7749	6941	5767	4797
33	225	234	198	199	161	142	78	9586	9614	8393	7536	6250	5202
34	243	252	211	206	170	147	79	10362	10383	9085	8177	6769	5639
35	263	273	227	215	180	154	80	11193	11206	9829	8868	7328	6110
36	285	296	245	226	193	163	81	12082	12085	10625	9612	7930	6618
37	310	322	265	240	207	174	82	13031	13024	11478	10410	8577	7164
38	337	350	287	255	223	187	83	14043	14023	12391	11266	9271	7752
39	367	380	312	274	241	201	84	15120	15086	13364	12184	10014	8384
40	399	413	340	294	262	218							
41	434	450	370	318	284	236							
42	473	489	403	344	308	256							
43	515	533	439	373	335	278							
44	561	580	478	405	365	302							

https://doi.org/10.26219/heal.aueb.6210

Table 13: Parameters A-H of the Heligman&Pollard model. Australia, 1955-2009

	A	B	C	D	E	F	G	H
				Females				
1955-59	0.00214	0.04615	0.14302	0.00023	3.02573	21.57772	0.00004	1.10149
1965-69	0.00144	0.02603	0.12487	0.00039	38.28148	18.89055	0.00004	1.09819
1975-79	0.00094	0.01842	0.10927	0.00037	23.34512	18.82708	0.00003	1.09971
1985-89	0.00067	0.02140	0.10549	0.00036	11.21810	20.64490	0.00002	1.10519
1995-99	0.00046	0.03000	0.10385	0.00026	3.95990	22.34390	0.00001	1.10588
2005-09	0.00031	0.02507	0.10321	0.00016	6.97476	21.97183	0.00002	1.09966
				Males				
1955-59	0.00244	0.02764	0.13263	0.00127	9.84718	20.56362	0.00007	1.10089
1965-69	0.00163	0.01561	0.11978	0.00124	12.48127	20.01631	0.00007	1.10081
1975-79	0.00121	0.02111	0.11797	0.00152	13.67978	19.99859	0.00005	1.10156
1985-89	0.00080	0.01458	0.10265	0.00129	8.05652	21.96981	0.00003	1.10794
1995-99	0.00065	0.04332	0.11461	0.00117	5.33086	24.91511	0.00002	1.11074
2005-09	0.00042	0.03739	0.11585	0.00065	5.03116	26.18976	0.00002	1.10432

https://doi.org/10.26219/heal.aueb.6210

Table 14: Parameters A-H of the Heligman&Pollard model. Bulgaria, 1955-2009

	A	B	C	D	E	F	G	H
				Females				
1955-59	0.01353	0.22175	0.28060	0.00097	1.53753	28.80784	0.00002	1.11662
1965-69	0.00258	0.03441	0.14924	0.00049	1.21689	34.77276	0.00001	1.12174
1975-79	0.00187	0.02962	0.13107	0.00039	1.17779	30.95080	0.00001	1.12338
1985-89	0.00152	0.03780	0.11995	0.00034	1.92962	26.99120	0.00001	1.11893
1995-99	0.00205	0.12391	0.17180	0.00000	-0.02842	0.00001	0.00001	1.12141
2005-09	0.00065	0.00379	0.07827	0.00012	9.22347	19.42274	0.00003	1.10367
				Males				
1955-59	0.01161	0.12123	0.24745	0.00109	3.48832	22.19410	0.00005	1.10257
1965-69	0.00234	0.00495	0.10956	0.00076	5.29455	23.25758	0.00004	1.10454
1975-79	0.00179	0.00593	0.10472	0.00064	6.25627	22.37821	0.00006	1.10106
1985-89	0.00169	0.03218	0.12564	0.00054	14.27451	20.26185	0.00010	1.09323
1995-99	0.00191	0.06716	0.15967	0.00059	35.37732	19.99968	0.00013	1.09092
2005-09	0.00079	0.01740	0.11014	0.00052	19.64029	21.27104	0.00009	1.09493

https://doi.org/10.26219/heal.aueb.6210

Table 15: Parameters A-H of the Heligman&Pollard model.
Canada, 1955-2009

	A	B	C	D	E	F	G	H
				Females				
1955-59	0.00211	0.01335	0.12753	0.00024	3.00983	23.91109	0.00003	1.10241
1965-69	0.00134	0.00709	0.10299	0.00023	19.79161	19.23456	0.00004	1.09767
1975-79	0.00087	0.00682	0.08995	0.00034	31.88367	18.80637	0.00004	1.09665
1985-89	0.00060	0.01948	0.09900	0.00024	12.34883	18.74961	0.00002	1.10068
1995-99	0.00037	0.00876	0.08401	0.00019	13.09099	18.43886	0.00002	1.10065
2005-09	0.00023	0.00384	0.08052	0.00018	13.01606	19.56718	0.00002	1.09883
				Males				
1955-59	0.00225	0.00334	0.10667	0.00100	10.88709	21.07412	0.00007	1.09706
1965-69	0.00152	0.00276	0.09311	0.00124	15.09093	20.67486	0.00007	1.09651
1975-79	0.00101	0.00363	0.08525	0.00143	15.16387	20.19265	0.00007	1.09670
1985-89	0.00070	0.01341	0.09680	0.00100	8.38431	21.31166	0.00004	1.10348
1995-99	0.00045	0.01439	0.09594	0.00075	6.15964	22.81393	0.00003	1.10427
2005-09	0.00033	0.01648	0.10501	0.00062	9.24069	21.92374	0.00003	1.09913

https://doi.org/10.26219/heal.aueb.6210

Table 16: Parameters A-H of the Heligman&Pollard model. France, 1955-2009

	A	B	C	D	E	F	G	H
				Females				
1955-59	0.00307	0.06306	0.17835	0.01852	0.22659	1143.409	0.00001	1.12497
1965-69	0.00133	0.01675	0.12193	0.00032	5.36335	22.51286	0.00004	1.09930
1975-79	0.00085	0.01276	0.09913	0.00033	12.30916	19.90838	0.00004	1.09560
1985-89	0.00069	0.03957	0.11729	0.000000003	-0.05153	0.00001	0.000003	1.12908
1995-99	0.00061	0.05177	0.11546	0.000000001	-0.05942	0.00001	0.000002	1.12300
2005-09	0.00033	0.09355	0.14203	0.00021	101.5594	19.46238	0.00003	1.08867
				Males				
1955-59	0.00349	0.06127	0.19532	0.00077	7.56187	22.09542	0.00011	1.09290
1965-69	0.00155	0.01790	0.13457	0.00092	14.11289	20.70227	0.00011	1.09250
1975-79	0.00109	0.02464	0.12300	0.00131	19.42064	20.11262	0.00010	1.09215
1985-89	0.00080	0.06166	0.14946	0.00102	13.15455	22.00708	0.00009	1.09043
1995-99	0.00081	0.27479	0.23465	0.00077	10.80346	22.94570	0.00008	1.09028
2005-09	0.00239	0.79564	0.40839	0.00053	14.23297	22.28477	0.00005	1.09314

https://doi.org/10.26219/heal.aueb.6210

Table 17: Parameters A-H of the Heligman&Pollard model.
Russia, 1965-2009

	A	B	C	D	E	F	G	H
				Females				
1965-69	0.00252	0.03327	0.13127	0.00132	0.82136	61.82808	0.00001	1.11753
1975-79	0.00237	0.04576	0.14011	0.000000002	-0.06105	0.00001	0.00001	1.12614
1985-89	0.00183	0.03984	0.13072	0.00031	5.26563	21.80263	0.00004	1.10190
1995-99	0.00173	0.06629	0.14981	0.00054	7.93577	22.09431	0.00008	1.09175
2005-09	0.00087	0.01692	0.09424	0.00233	1.29491	58.10025	0.00002	1.11222
				Males				
1965-69	0.00265	0.01998	0.12986	0.00218	3.67924	31.65680	0.00016	1.08787
1975-79	0.00270	0.03494	0.14466	0.00337	2.74902	36.40222	0.00017	1.08874
1985-89	0.00231	0.06807	0.17094	0.00119	8.80806	24.72432	0.00022	1.08430
1995-99	0.00204	0.05593	0.15609	0.00619	2.01594	45.50335	0.00015	1.09063
2005-09	0.00115	0.04858	0.13140	0.00884	1.99504	51.25500	0.00010	1.09499

https://doi.org/10.26219/heal.aueb.6210

Table 18: Parameters A-H of the Heligman&Pollard model.
USA, 1955-2009

	A	B	C	D	E	F	G	H
				Females				
1955-59	0.00174	0.01900	0.13356	0.00024	9.01499	20.81731	0.00006	1.09296
1965-69	0.00143	0.02251	0.13450	0.00033	31.93140	19.22226	0.00008	1.08891
1975-79	0.00099	0.02303	0.12174	0.00034	17.73896	19.48765	0.00005	1.09112
1985-89	0.00076	0.02807	0.11911	0.00029	9.91991	20.02491	0.00004	1.09465
1995-99	0.00053	0.03223	0.11934	0.00024	12.89640	19.23527	0.00004	1.09260
2005-09	0.00046	0.06043	0.14622	0.00023	12.30817	21.10629	0.00004	1.09044
				Males				
1955-59	0.00184	0.00875	0.12484	0.00116	15.35929	20.34962	0.00012	1.09036
1965-69	0.00161	0.01611	0.13556	0.00136	15.14054	20.50904	0.00013	1.08928
1975-79	0.00118	0.02253	0.12808	0.00145	12.51868	21.03892	0.00010	1.09071
1985-89	0.00096	0.04093	0.13812	0.00120	6.99990	22.93016	0.00008	1.09272
1995-99	0.00080	0.12252	0.18647	0.00100	9.77055	21.79841	0.00009	1.08917
2005-09	0.00098	0.26617	0.26236	0.00109	11.36528	22.84299	0.00007	1.08898

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