https://doi.org/10.26219/heal.aueb.6403

ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

DEPARTMENT OF STATISTICS

POSTGRADUATE PROGRAM

ANALYSIS AND COMPARISON OF THE GREEK PARLIAMENTARY ELECTORAL SYSTEMS OF THE PERIOD 1974-1999

By

Aikaterini G. Kalogirou

A THESIS
Submitted to the Department of Statistics
of the Athens University of Economics and Business
in partial fulfilment of the requirements for
the degree of Master of Science in Statistics

Athens, Greece
2000

ISBN : 960-7929-35-7

https://doi.org/10.26219/heal.aueb.6403

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ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ΤΜΗΜΑ ΣΤΑΤΙΣΤΙΚΗΣ

ΑΝΑΛΥΣΗ ΚΑΙ ΣΥΓΚΡΙΣΗ ΤΩΝ ΕΛΛΗΝΙΚΩΝ ΕΚΛΟΓΙΚΩΝ ΣΥΣΤΗΜΑΤΩΝ ΠΟΥ ΕΦΑΡΜΟΣΤΗΚΑΝ ΣΤΙΣ ΒΟΥΛΕΥΤΙΚΕΣ ΕΚΛΟΓΕΣ ΚΑΤΑ ΤΗΝ ΠΕΡΙΟΔΟ 1974-1999

Αικατερίνη Καλογήρου

ΔΙΑΤΡΙΒΗ

Που υποβλήθηκε στο Τμήμα Στατιστικής

του Οικονομικού Πανεπιστημίου Αθηνών

ως μέρος των απαιτήσεων για την απόκτηση

Μεταπτυχιακού Διπλώματος Ειδίκευσης στη Στατιστική

Αθήνα

Σεπτέμβριος 2000

https://doi.org/10.26219/heal.aueb.6403

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ANALYSIS AND COMPARISON OF THE GREEK PARLIAMENTARY ELECTORAL SYSTEMS OF THE PERIOD 1974-1999

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September 2000

ACKNOWLEDGEMENTS

I would like to thank my supervisor, Professor J. Panaretos for suggesting the theme of my thesis, for providing helpful comments, and also for his support during my research.

https://doi.org/10.26219/heal.aueb.6403

VITA

I was born in Agrinio in 1973. In 1991 I graduated from the 6th General High School of Patras. In 1992 I was accepted in the Department of Mathematics of the University of Patras and graduated in 1996 with a B.Sc. degree in Mathematics. During the academic year 1996-1997 I attended a computer seminar organized by the Greek Mathematical Association. Since September 1997 I am an MSc student in the Department of Statistics of Athens University of Economics and Business.

https://doi.org/10.26219/heal.aueb.6403

ABSTRACT

Aikaterini Kalogirou

ANALYSIS AND COMPARISON OF THE GREEK PARLIAMENTARY ELECTORAL SYSTEMS OF THE PERIOD 1974-1999

September 2000

The most fundamental element of representative democracy is the Electoral System, as it translates vote totals into parliamentary seats. An important topic is the choice of the electoral system, which will be applied in the elections, because parliamentary seats distributed to political parties differ when a different system is used.

In Greece the last 25 years five different electoral systems have been applied in the Parliamentary Elections. The aim of this thesis is to describe and analyse, in detail, the operation of Greek Parliamentary Electoral systems of the period 1974-1999 and to present the rules of the allocation of the seats. Furthermore our purpose is to evaluate and compare these systems. In particular, we provide and implement a quantitative analysis of Greek electoral systems, using some measures of disproportionality. Disproportionality is the deviation of the parties' seat shares from their vote shares. For the evaluation of the Greek systems we apply the Rae, Loosemore-Hanby, Least Square, Adjusted Loosemore-Hanby, Lijphart, Saint-Lague, d'Hont and the Regression index.

https://doi.org/10.26219/heal.aueb.6403

ΠΕΡΙΛΗΨΗ

Αικατερίνη Καλογήρου

ΑΝΑΛΥΣΗ ΚΑΙ ΣΥΓΚΡΙΣΗ ΤΩΝ ΕΛΛΗΝΙΚΩΝ ΚΟΙΝΟΒΟΥΛΕΥΤΙΚΩΝ ΕΚΛΟΓΙΚΩΝ ΣΥΣΤΗΜΑΤΩΝ ΤΗΣ ΠΕΡΙΟΔΟΥ 1974-1999

Σεπτέμβριος 2000

Το Εκλογικό σύστημα αποτελεί το θεμέλιο της αντιπροσωπευτικής δημοκρατίας, καθώς μετατρέπει τους ψήφους σε κοινοβουλευτικές έδρες. Ένα σημαντικό πρόβλημα αποτελεί η επιλογή του εκλογικού συστήματος, το οποίο θα εφαρμοστεί στις εκλογές, καθώς το πλήθος των κοινοβουλευτικών εδρών, οι οποίες κατανέμονται στα πολιτικά κόμματα, διαφέρει όταν χρησιμοποιούνται διαφορετικά εκλογικά συστήματα.

Στην Ελλάδα τα 25 τελευταία χρόνια έχουν εφαρμοστεί πέντε διαφορετικά εκλογικά συστήματα στις κοινοβουλευτικές εκλογές. Ο σκοπός της διατριβής αυτής είναι να περιγράψει και να αναλύσει την λειτουργία των Ελληνικών Κοινοβουλευτικών Εκλογικών Συστημάτων της περιόδου 1974-1999, καθώς επίσης και να παρουσιάσει τους κανόνες της κατανομής των εκλογικών εδρών. Επιπλέον, ο σκοπός μας είναι να αξιολογήσουμε και να συγκρίνουμε τα συστήματα αυτά. Πιο συγκεκριμένα, παρέχουμε μια ποσοτική ανάλυση των Ελληνικών εκλογικών συστημάτων, χρησιμοποιώντας ορισμένους Δείκτες Δυσαναλογικότητας (Measures of Disproportionality). Η δυσαναλογικότητα αναφέρεται στην απόκλιση του μεριδίου των εδρών από το μερίδιο των ψήφων, των πολιτικών κομμάτων. Για να αξιολογήσουμε τα Ελληνικά συστήματα εφαρμόζουμε τους εξής δείκτες: Rae, Loosemore-Hanby, Least Square, Adjusted Loosemore-Hanby, Lijphart, Saint-Lague, d'Hont και Regression.

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ABBREVATIONS

P.R.	Proportional Representation
F.P.T.P.	First Past The Post
L.R.	Largest Remainders
L.V.	Limited Vote
L-H	Loosemore-Hanby
LSq	Least Square
L-H adj.	Adjusted Loosemore-Hanby
S-L	Saint-Lague

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Chapter 1

Introduction

In all communities, democracy necessarily means representative democracy in which elected officials make decisions on behalf of the people. People express their will and chose their leaders or representatives by voting. The indispensable task of the election of the representatives, in democracies, is performed by the electoral system. The electoral system is the set of the methods that are used in the elections, for the translation of vote totals into representative seats in the Parliament. Because of the central role elections play in so many modern states, the influence of electoral systems on politics, and vice versa, has long been a justifiable concern of the discipline of political science. Taagepera and Shugart (1989) give a general statement of the role that elections and electoral systems play in political life. They describe the electoral systems and their characteristics and present an empirical analysis of the electoral systems. Also, Lijphart (1994) analyse the operation and the political consequences of electoral systems, particularly the degree of proportionality of their translation of votes into seats and their effects on party systems. The emphasis is given to the electoral systems that have been used in the European democracies.

The study of electoral systems and their links to political issues and regime stability leads itself to quantitative analysis. Unlike many areas of social sciences, our data are already provided in a hard quantitative form: numbers of votes and numbers of seats. Systematic quantitative studies of electoral systems have been relatively rare. The studies in the field have been limited to particular countries, promoting particular types of systems.

The aim of this study is to describe and analyse the operation of Greek Parliamentary

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Electoral systems. In particular, we provide a quantitative study of Greek systems of the period 1974-1999. We compare these systems, with respect to the degree of the proportionality that they achieve, when they translate votes into seats.

Reviewing the electoral systems literature, Lijphart and Gibberd (1985) noted the absence of one serious attempt to solve the problem of how to measure proportionality or disproportionality. There are many ways to measure inequality, see, for example Allison (1978), but because they exceed a certain level of complexity, the results they provide became difficult to interpret. Gallagher (1991) presents and reviews the main Proportional Representation methods and discuses the principles underlying each of them. He also presents measures of disproportionality, suggests some new indices and applies these measures to competitive elections of the period 1979-1989. Lijphart (1994) has also applied some indices of disproportionality in a study of twenty-seven democracies of the period 1945-1990.

Electoral systems are creatures of politics and can be altered for political reasons. Even when an electoral system is stable, it is affected by a large number of issues such as socio-economic, religious, political and ethnic factors. Greek Electoral systems and their political and socio-economic aspects have been analysed and described by many researchers, under different points of view; see, for example, Dretakis (1982, 1984, 1986), Pantelis and Triantafillou (1985), Nikolakopoulos (1985, 1989), Dimitras (1991), among several others.

In Greece, the last 25 years five different electoral systems have been applied in the Parliamentary Elections. Our purpose is to describe and analyse these electoral systems in detail, and to present the rules of the allocation of the seats at each distribution, for all the systems under consideration. We comment the rules, the characteristics and differences between the electoral systems. In particular, **we provide and implement a quantitative analysis** of Greek electoral systems of the period 1974-1999. We compare and evaluate the Greek Electoral systems [1974, 1977 (1981), 1985, 1989, 1993 (1996)] using some measures of disproportionality. Disproportionality is the deviation of the parties' seat shares from their vote shares. In the category of Proportional Representation systems (PR systems), where the Greek electoral systems we analyse, can be thought of, there are two broad categories of measures of disproportionality. The first concentrates on the absolute difference between parties' seats and votes, and the second focuses on the ratio between the parties' seats and votes. For the evaluation of the

Greek systems we apply the Rae, Loosemore-Hanby, Least Square, Adjusted Loosemore-Hanby, Lijphart indices, which belong to the first category, and the Sainte-Langue and the d'Hondt indices, which belong to the second category. We also apply the Regression index, which is a satisfactory measure of big-parties' bias. As far as we know, such an analysis has not been applied for the Greek Electoral systems. We implement the Greek systems using the data set of the parliamentary elections of 1996. In order to achieve more accurate results, we use a large number of data sets generated by introducing noise (error) to the initial dataset. Sensitivity analysis is performed, by analysing all the systems and the measures of disproportionality using 9 and 13 major districts, for the generated datasets.

The thesis is organized as follows. Electoral systems and their categories are presented in chapter two. Greek electoral systems are described and their characteristics and properties are presented in chapter three. In chapter four the measures of disproportionality are described, and the dataset we use together with the various generated datasets are described in chapter five. In chapter six, we apply the Greek electoral systems and the measures of disproportionality, in the parliamentary elections of 1996 and also in the generated datasets. We also, present the results of the elections for the European Parliament of 1999, in Greece, under the current system. Finally, we conclude in chapter seven with a brief discuss of the main results of this study.

https://doi.org/10.26219/heal.aueb.6403

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Chapter 2

Electoral Systems

As it has already been mentioned in the introduction, the main purpose of this study is to analyze the political effects of the various electoral systems applied in Greece, the last 25 years. For this purpose, a detailed description of all these systems is given in the next chapter. Before we study the Greek electoral systems, we give a general description of the various systems applied in parliamentary elections, in different countries, all over the world. Almost all electoral system experts agree that the two most important features of the electoral systems are a) the electoral formula, which is the method that is used for the translation of vote totals into representative seats in the parliament and b) the district magnitude, which is the total number of seats given to each district (the geographical regions that are used for the distribution of the seats). According to these features the electoral systems can be distinguished into three main types, and of course a large number of subtypes (See, Lijphart (1994), Gallagher (1992)).

1. **Majoritarian formulas**, with main subtypes

    - Plurality
    - Two-Ballot Majority-Plurality systems
    - Alternative Vote

2. **Proportional Representation systems (PR systems)**, classified further into

    - Largest Remainders (Quota systems)

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- Highest Averages (Divisor systems)
- Single Transferable Vote

3. **Intermediate systems**

- Semi-Proportional systems
- Reinforced PR
- Mixture of Majoritarian and PR.

2.1 Majoritarian formulas

The most common system of this category is the **Plurality**. It is also called First-Past-The-Post (FPTP) or Relative Majority method. According to this method, in each single member district, each voter can cast one vote and the candidate with the most votes wins. In two member districts voters give two votes, and the two candidates with the most votes win, and so on. Countries which have used plurality systems are Canada, India, New Zealand, United Kingdom and the United states.

The French Fifth Republic provides one instance of **Two-Ballot Majority-Plurality** system. According to this system, a majority is required for the election in the first ballot (first round of the elections). In that case majority means absolute majority which is more than half of the valid votes. If none wins in the first ballot, a second ballot is conducted and the candidate with the most votes wins, even if in the case of plurality of votes. In the second ballot can participate more than two candidates. In fact, what has happened in France is that the weakest candidates were forced to withdraw. We must distinguish this system from the Majority-Runoff, where the second ballot is restricted only to the two top parties. Majority-Runoff has been used for presidential elections in France, Portugal and Austria.

The third system in this category is the **Alternative Vote**. According to this system voters are asked to give their preference among all the alternatives. If a political party receives an absolute majority of the first preferences, is elected. If not, the weakest alternative is eliminated and its ballots are given to the rest of the candidates, according to the voters second preferences. The process continues until a majority winner emerges. For example, suppose that there are 4

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candidates A, B, C, D receiving 41, 29, 17 and 13 per cent of the voter's first preferences. Since none has received the majority of the first preferences, D is eliminated. Let us further assume that the candidates received 41, 29, 30 per cent of the second voter's preferences. In this case, B is eliminated, and the third round is a contest between A and C. Thus, one of these two will be the winner.

The most striking characteristic of all these systems is that they usually use single member districts or districts with magnitude close to one. Now, in most countries, which use majoritarian systems, only single-member districts have survived. For example, single member districts have survived in the United Kingdom after 1945, in India after 1957 and both in Canada and the United States after 1968. All the majoritarian systems make difficult for small parties to gain representation, because they need to win majorities or pluralities of votes in the electoral districts, unless of course in case of geographical concentration. Thus, we understand that majoritarian systems tend to favor the large parties. District magnitudes larger than one tend to reinforce the above phenomenon.

2.2 Proportional Representation systems

Proportional Representation (PR) systems are the most common electoral systems. The first type of them, the **Largest Remainders** or **Quota System** entails the calculation of a quota based on the number of the available seats and the number of votes cast. Each party is awarded as many seats as it has full quotas. If this leaves some seats unlocated, each party's 'remainder' is calculated as follows: The number of votes, that the party has already used to gain the seats, is subtracted from the total votes. The unlocated seats are awarded to the parties that present the largest remainders of votes. Different types of quota lead to different Largest Remainders methods, and the most common are the following:

- *Hare or Natural quota*, which is equal to the total number of valid votes cast (v) divided by the number of the available seats (s), in the district. $Hare\ quota = v/s$.

- *Droop quota or Hagenbach-Bischoff*, which is equal to the number of valid votes cast (v) divided by the number of the available seats (s) plus one, in the district. $Droop\ quota = v/(s+1)$.

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- *Imperiali quota*, which is equal to the number of valid votes cast (v) divided by the number of the available seats (s) plus two, in the district. $Imperiali\ quota = v/(s+2)$.

The following example clarifies the application of the Largest Remainders method (see, Gallagher (1992)). Suppose that the total number of votes is 100000 and that there are 5 seats to be allocated to 3 parties A, B and C. Each one of these parties receives 60000, 28000, 12000 votes. Droop quota is equal to $100000/(5+1) = 16667$. The allocation of seats using this quota is shown in the Table 1.

Table 1: Allocation of seats by Largest Remainders method using Droop quota.

Party	*Votes*	*seats by quota*	*remainder*	*seats by remainder*	*Total seats*
A	60000	3	10000	0	3
B	28000	1	11333	0	1
C	12000	0	12000	1	1
Total	100000	4	33333	1	5

Hare quota is equal to $100000/5 = 20000$, and the allocation of the seats remains the same, because the seats obtained by each party using this quota is 3, 1 and 0 respectively. The remainders of votes are 0, 8000 and 12000 respectively. Thus, the last seat would be obtained again by the third party. In this example, the two methods give the same results, but in general the results are different. This will be studied extensively in later chapters, because both Droop and Hare quota have been used by the Greek electoral systems we analyse. If the Imperiali method was used, the quota would be $100000/(5+2) = 14286$. In this case, the seats gained by each party using the quota are 4, 1, and 0 respectively. Hence, all seats are allocated without the need to consider the remainders of votes. Sometimes, the number of seats obtained by the quota is larger than the number of the available seats. In Italy, Imperiali is replaced by Droop quota when the above phenomenon occurs. It is obvious that a large number of LR methods can be invented, as a large number of different quotas can be found. For example, one might

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use the ratio $v/(s+3)$ or $v/(s+0.5)$. However, most of them can not be used for the allocation of the seats because some of them award too many seats and other too few seats. Suppose that, in the example described above, we use a quota that is equal to 15000 or lower. Then, all seats would be awarded without the use of the remainders of votes. If, however we use a quota of more than 30000, only one seat would be awarded in the initial distribution of the seats.

Highest Averages Methods operate on a different principle from that of Largest Remainders. A party receives a seat according to its original number of votes and the number of seats it has already won. Each time a party receives a seat, progressively larger numbers divide its original vote total. Seats are, successively, allocated to the party with the 'highest average' at each step. The variation between the methods lies in the sequence of numbers employed as divisors. One allocation rule, which uses divisors, is the **Sainte-Lague**, introduced by the French mathematician Sainte-Lague. This method employs the sequence of 1, 3, 5, 7, etc.; the n^{th} divisor equals $2n-1$. For the first seat each party bids its total number of votes, because the first divisor is 1. For the second seat the party which obtained the first seat bids the one third (1/3) of its total number of votes, because the second divisor is 3, and the other parties bid their total number of votes. In order to clarify this method we use the same example with the Largest Remainders method. Suppose that, the total number of votes is 100000 and that there are 5 seats to be allocated to 3 parties A, B and C. Each party receives 60000, 28000 and 12000 votes respectively. The first seat is allocated to party A as it has the highest number of total votes. Its 'average' is then reduced by dividing its vote total with 3, so it bids 20000 votes while B and C bid 28000 and 12000 respectively. The second seat is allocated to B, and its 'average' is then reduced by dividing its vote total with 3, so it bids 9333 votes. Party A receives the third seat because it bids 20000 votes, while B bids 9333 and C bids 12000 votes. The fourth and the fifth seats are allocated to parties A and C, because their bids are 12000, and thus each one of these parties obtain one seat. The results are summarized in Table 2.

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Table 2: Allocation of seats by Sainte-Lague Highest Averages method

Party	*Votes* (v)	*Votes divided by 1st divisor*	*Votes divided by 2nd divisor*	*Votes divided by 3rd divisor*	*Total Seats*
A	60000	60000(1)	20000(3)	12000(5)	3
B	28000	28000(2)	9333		1
C	12000	12000 (4)			1
Total	100000				5

Note: The numbers in brackets after the parties' vote totals indicate the award of a seat; Thus party A gains the first seat, party B the second, party A the third and parties A and C gain the fourth and the fifth seat.

Simply, each seat is given to the party with the highest value of $v_i/(2s_i + 1)$, where v_i is the total number of votes for the i^{th} party and s_i is the number of seats received *so far* by the i^{th} party. In fact the effect of this method is to help small parties, see Gallagher (1991).

There is also the modified Sainte-Lague which uses the divisors 1.4, 3, 5, etc. The replacement of the first divisor by 1.4 reduces the opportunity the small parties to receive a seat, which Sainte-Lague method gives to small parties. The modified method tends to help middle-sized parties.

One frequently used highest averages procedure is the d' **Hondt** rule introduced by Victor d' Hondt. According to this rule, each seat is given to the party with the highest value of $v_i/(s_i + 1)$, where v_i is the total number of votes for the i^{th} party and s_i is the number of seats received *so far* by the i^{th} party. Thus, the sequence of the divisors it uses is 1, 2, 3, 4, ... and it is the least proportional among the highest averages methods as it favors the large parties, see, for example Taagepera and Shugart (1989). Its operation is illustrated in Table 3, which relates to the case where the total number of votes is 100000, the total number of seats to be allocated is 5 and 3 parties A, B and C share these seats, each of them received 60000, 28000 and 12000 votes respectively.

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Table 3: Allocation of seats by d' Hondt Highest Averages method.

Party	*Votes* (v)	*Votes divided by 1st divisor*	*Votes divided by 2nd divisor*	*Votes divided by 3rd divisor*	*Votes divided by 4th divisor*	*Total seats*
A	60000	60000(1)	30000(2)	20000(4)	15000(5)	4
B	28000	28000(3)	14000			1
C	12000	12000				0
Total	100000					5

Note: The numbers in brackets after the parties' vote totals indicate the award of a seat. Thus, the first and the second seat is obtained by A, the third by B, and so on.

Given that A has won 60 percent of the votes, the d'Hondt formula gives to this party 4 seats. One might think that 3 seats must be awarded, if a 'fair' formula is applied. The reason is that since some disproportionality is unavoidable, one (or more) party (ies) must be overrepresented and one (or more) party (ies) must be underrepresented. This formula tries to minimize the overrepresentation of the most overrepresented party. Thus, A's index of representation (%seats divided by %votes) is 80%/60% = 1.33, and 0.71 and 0, for the second and the third party respectively. If, instead, B was awarded the fifth seat, its index of representation would be 40%/28%=1.48 and 1 and 0 for the first and the third party respectively. If C received the fifth seat, its index of representation would be 20%/12%=1.67 and 1 and 0.71 for the first and the second party respectively. For d' Hondt formula is less undesirable overrepresenting A than overrepresenting either B or C.

Under **Single Transferable Vote** (STV), the voter is faced with a ballot paper containing the names of all candidates and ranks them in order of preference. Candidates whose first preference votes amount to or exceed the quota (usually the Droop quota) are elected at once. If there are unfilled seats they are distributed to the other candidates using the following procedure. The 'surplus' votes of the elected candidates, i.e. the number of votes that the elected candidates have in excess of the quota, are distributed to the other candidates in proportion to the second preferences marked on them. If there are still vacancies, the lowest placed candidate is eliminated and the votes are transferred to the other candidates, again according the second preferences marked on them. If the candidate awarded the second preference on a transferred ballot paper can not receive it, by having already been either elected or eliminated, the paper is transferred according to the third preference, or the fourth if the third ranked candidate is unable to receive it, and so on.

2.3 Intermediate categories

These are systems that do not fit either the majoritarian or the PR categories, like for example the Semi-Proportional systems applied in Japan. The rule is the **Limited Vote** (LV); according to it, voters cast their votes for individual candidates, as in the plurality systems, and the candidate with the highest number of votes wins. However, unlike plurality systems voters vote for fewer candidates than there are seats to be filled in the district. This is the reason why the formula is called limited vote. The more 'limited' votes each voter has, the more LV deviates from plurality and the more it resembles PR. **Reinforced PR systems**, also do not fit either the majoritarian or the PR categories perfectly. Greek electoral systems (1974-1985) belong to this category and will be described in detail in the next chapter. Finally, in some countries we meet a mixture of the Majoritarian and the PR categories, like the French system of 1951 and 1956.

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Chapter 3

Greek electoral systems applied in the Parliamentary elections after the hunte (1974-1999).

3.1 General Description

Greek Electoral systems do not fit either the majoritarian or the PR categories and most of the analysed systems (1874, 1977, 1981, 1985) are referred to as **Reinforced PR** ('enishimeni analogiki'). However, these systems can be regarded as sufficiently similar to PR and for this reason Lijphart (1994) includes them in his comparative analysis of all PR systems. All of them are **list** systems. In 'list PR' systems voters may or may not be allowed to express a preference to a particular candidate or candidates within the list. In Greece, in some cases, the purest form of list PR systems, which is the 'closed list', is used. It is pure because voters choose only the party they prefer, making no choice to individual candidates. In that case, each party submits a list of candidates prior to the election. The seats the party wins are distributed in rank of the fixed list. Thus, if there are seven seats to be filled, each party will ordinarily submit a list of seven candidates. If one party wins three seats, it elects the top three candidates of the list. In case of an 'open list' system, voters select a party and then, if they wish, they express a preference to a particular candidate, or candidates, within the list. Vote totals are

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translated into parliamentary seats occupied by the deputies. The parliament, in all the cases studied, consists of 300 deputies. The distribution of the 288 seats takes place in three steps: the primary, secondary, and tertiary distribution of seats, with the only exception of the 1989 system. A last step followed is for the allocation of 12 additional seats occupied by the State Deputies.

3.2 Description of the Electoral Systems

This section deals with the study of the Electoral Systems applied in the Greek Parliamentary Elections of 1974, 1977, 1981, 1985, 1989, 1993 and 1996. First of all, a detailed description of each formula is given. Then, each formula is expressed in mathematical relations in the form of pseudo-algorithm. Before the description of the systems some terms must be defined.

Lower districts ('nomoi', or 'elassones eklogikes periferies'). The geographical regions in which the state is divided, for the primary distribution of the seats.

Major districts ('meizones eklogikes periferies'). The geographical regions in which the state is divided, for the secondary distribution of the seats.

Higher districts. The geographical regions in which the state is divided, for the tertiary distribution of the seats.

District magnitude. It is the number of the available seats in the corresponding district.

Quota. As it has already been mentioned, in the case of the Largest Remainders PR systems, the quota is the ratio in which the distribution of the seats is based. This ratio involves the number of the seats and the number of the votes, and it depends on the method that is used (e.g. Hare, Droop) and on the district where it is applied (Lower, Major or Higher). In Greece, the term 'electoral measure' is used.

3.2.1 1974 Electoral formula[1]

Primary Distribution of Seats.

The state is divided in 56 lower districts which are almost the same as the 52 geographical-administrative districts ('nomoi') of the country. In fact only the prefecture of Attica is divided

[1]**This electoral system is described, in details, in the Greek Government Gazette (1974).**

https://doi.org/10.26219/heal.aueb.6403

in 5 districts and the prefecture of Thessaloniki in two districts, due to overpopulation, while the other remain the same.[2] The seats are distributed, in each lower district, among all alternatives: independent candidates, single parties and cartels of two or more parties. The distribution is done according to the total number of votes. For this purpose, the total number of the valid votes, in each lower district, is divided by the district magnitude. The integer part of this ratio is known as **Hare** quota. This quota is applied in each district: the total number of valid votes, for each alternative, in each lower district, is divided by the quota. Parties are given as many seats they have won quotas. Thus, a party takes as many seats as the integer part of its total votes divided by the quota. In case of an independent candidate, he takes one seat only if his total number of valid votes is greater or equal to the quota. In districts with magnitude equal to one the seat is given to the party with the highest total of votes in this district (relative majority). Any remaining available seats are distributed in the following step.

Secondary Distribution of Seats.

The distribution is carried out in 9 **Major Districts.** Each one of them consists of four, five or even ten[3] lower districts. Only some parties are allowed to obtain seats in this distribution. A party takes part in the allocation of the seats only if it satisfies the following conditions:

- Single parties with a percentage of total votes greater or equal to 17%.
- Cartels of two parties with a percentage of total votes greater or equal to 25%.
- Cartels of more than two parties with a percentage of total votes greater or equal to 30%.

[2]The 56 lower districts are those of 1) A' disrict of Athens, 2) B' district of Athens, 3) A' district of Peireas, 4) B' district of Peireas, 5) The remaining of Attiki, 6)Biotias, 7) Evias, 8) Fthiotidas, 9) Fokidas, 10) Argolidas, 11) Arkadias, 12) Korinthias, 13) Lakonias, 14) Messinias, 15) Etolias and Akarnanias 16)Ahaias 17) Evritanias, 18) Zakinthou, 19) Ilias, 20) kefallinias, 21) Artas, 22)Thesprotias, 23) Ioanninon, 24) Kerkiras, 25) Lefkadas, 26)Prevezas, 27) Grevenon, 28) Karditsas, 29) Kozanis 30)Larissas, 31) Magnisias, 32) Trikalon, 33) Imathias, 34) Kastorias, 35) kilkis, 36) Pellas, 37) Pierias, 38)Serron, 39) Florinas, 40) Halkidikis, 41) A' disrict of Thessaloniki, 42) B' district of Thessaloniki, 43)Dramas, 44)Evrou, 45) Kavalas, 46)Ksanthis, 47) Rodapis, 48) Dodekanissou, 49) Kikladon, 50) Lesvou, 51)Samou, 52) Hiou, 53) Irakliou, 54) Lasithiou, 55) Rethimnis, 56) Hanion.

[3]The 9 Major districts are those of 1) A' disrict of Athens, B' district of Athens, A' district of Peireas, B' district of Peireas, The remaining of Attiki, Biotias, Evias, Fthiotidas, Fokidas, 2) Argolidas, Arkadias, Korinthias, Lakonias, Messinias, 3) Etolias and Akarnanias, Ahaias, Evritanias, Zakinthou, Ilias, kefallinias, 4) Artas, Thesprotias, Ioanninon, Kerkiras, Lefkadas, Prevezas, 5) Grevenon, Karditsas, Kozanis, Larissas, Magnisias, Trikalon, 6) Imathias, Kastorias, kilkis, Pellas, Pierias, Serron, Florinas, Halkidikis, A' disrict of Thessaloniki, B' district of Thessaloniki, 7)Dramas, Evrou, Kavalas, Ksanthis, Rodapis, 8) Dodekanissou, Kikladon, Lesvou, Samou, Hiou, 9) Irakliou, Lasithiou, Rethimnis, Haniou.

https://doi.org/10.26219/heal.aueb.6403

- Independent candidates are not included.

If only one party satisfies the above conditions, a second party (either a single party or a cartel of more than two parties) is allowed to take part in this distribution. It is the party i with the maximum value of the ratio r_i. The ratio r_i, for the party i, is defined as the percentage of its total votes divided by a number which describes the nature of the party. This number takes the value zero for the independent candidates, one for single parties, and λ for cartels of λ parties, (λ=2,3..).

If none of the parties satisfies the above conditions, two parties (either a single party or a cartel of more than two parties) are allowed to take part in this distribution. These two parties have the maximum value of the difference of the real percentage of their votes, in the nation, and the legal admissible percentage for the participation in this distribution (17% for single parties, 25% for cartels of two parties and 30% for cartels of three or more parties).

The remaining available seats, after the primary distribution, are aggregated in each respective major district. The distribution of the remaining seats in the major districts is done according to a new quota. It is defined as the ratio of the total votes for the parties taking place in this distribution, in each major district, divided by the respective remaining seats. Then the integer part of this ratio is taken. Thus, the Hare quota is used again adjusted to the major districts and to the available seats. Parties are given as many seats as they have won quotas.

So far, it is defined the way that the parties receive the seats in the major districts. We do not only want to know the number of the seats each party wins, in the major districts, but also how these seats are distributed to the parties in the lower districts. The electoral system includes the procedure in which the seats of major districts are allocated to the lower districts. This procedure is described below:

Allocation of seats of Major districts to Lower districts

First of all, the number of the remaining available seats of the primary distribution is computed, in each lower district. Then, the total number of votes is divided by the respective number of the remaining available seats, in each lower district, for the parties taking place in this distribution. The integer part of this ratio is the Hare quota, which is defined in each lower district. Each party takes as many seats, in a district, as the times the quota is contained in the party's total valid votes of this district. Simply, the parties are given as many seats as they have

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won quotas. The remaining undisposed seats are given according to the following procedure. Each available seat, in a lower district, is given to the party with the largest remainder of the quotient which is defined by the party's valid votes divided by the quota of this district.

In the lower districts with magnitude equal to two, if there is only one remaining undisposed seat from the primary distribution, the one and only seat is given to the party that has already taken the first seat in this district, in the primary distribution, only if the total number of valid votes of this party, in this district, divided by two is greater than the total valid votes of each one of the remaining parties, in this lower district.

If with the above mentioned procedure, some parties occupy more seats than they have to according to the secondary distribution, surplus seats are subtracted. The subtraction is done with the use of the following ratio: The total of valid votes for each party, for each lower district, is divided by the number of seats that has already been allocated to this party from the primary and the secondary distribution. The seats that are in excess, of each party, are subtracted from this lower district where the above ratio is the smallest. Each subtracted seat is added in the same lower district to the party that needs this seat according to the secondary distribution of seats. If there are more than two parties that need this seat, it is given to the party with the highest ratio. In each case of equal total votes between parties, in the allocation or substraction of seats, the selection of the party is done randomly. Seats are not subtracted when they have been allocated a) according to the quota, b)in districts with magnitude equal to two.

In fact this procedure can be omitted, as we are interested in the final result of the elections, because the final result of the system is not affected by this procedure. It is not affected because, this procedure is done in such a way that the total number of seats, for each party, in the major districts, is equal to the total number of seats, for each party, in the respective lower districts.

There is a special case for the lower districts with magnitude equal to two. If only one seat is allocated to a party in the primary distribution also the second seat is given to the same party only if half of its total votes is greater than the total votes of each one of the remaining parties.

https://doi.org/10.26219/heal.aueb.6403

Tertiary Distribution of Seats.

The parties which participate in this distribution are those who took part in the secondary distribution. The distribution of the seats is done throughout the state. Hence, the **Higher district** consists of the entire state. The quota is defined as the ratio of the total votes, in the nation, of the parties participating in this distribution, divided by the available number of seats. Then, the integer part of the ratio is taken. Thus, the Hare quota is used, adjusted to the entire state and to the available seats. Parties are given as many seats as they have won quotas. If there are still undisposed seats they are given to the party with the highest percentage of votes in the nation.

Distribution of Seats of State Deputies.

In this distribution of the 12 seats, all the parties that participated in the secondary and the tertiary distribution are allowed to participate. A new quota is used for the distribution of the seats. It is defined as the ratio of the total votes, in the nation, of the parties participating in this distribution, divided by 12. Parties are given as many seats as they have won quotas. Any remaining seats are given to the parties with the *largest remainders of votes*. The remainders are the votes that have not been used for the allocation of seats when the quota is used. For example, if the quota Q is used for the seats distribution, with $Q = V \ / \ S$, the remainder of the votes is equal to $U = V - SQ$.

3.2.2 1977 and 1981 Electoral formula[4]

The national elections of 1977 and 1981 were carried out with the exact same system. The distribution of the seats is exactly the same with the 1974 system except for the primary distribution. **Droop** quota is used for the seats allocation in the lower districts, instead of Hare quota. It is defined as the ratio of the total number of valid votes, in each lower district, for all the alternatives (single parties, cartels of parties, independent candidates) divided by the district magnitude, which is the total number of votes, plus one. Parties are given as many seats as they have won quotas. In Greece, this formula is called 'plus one' ('sin ena').

[4] This electoral system is described, in details, in the Government Gazette (1981).

3.2.3 1985 Electoral formula[5]

Primary Distribution of Seats.

The state is divided in 56 lower districts which are almost the same as the 52 geographical-administrative districts of the country. Some of them are divided in more districts due to overpopulation. e.g. Attica is divided in 5 districts. In fact, the 56 lower districts are exactly the same as the previous systems. The seats are distributed, in each lower district, among all alternatives: independent candidates, single parties and cartels of two or more parties. The distribution is done according to the total number of votes. For this purpose, the distribution of seats for the single parties and the cartels of more than two parties is done in the following way: In districts with magnitude equal to one, and those are the districts with only one available seat, the one and only seat is given by the use of the *plurality rule.* It is also called *relative majority* or *first past the post.* According to this rule the seat is given to the party with the most valid votes, whether or not that party has an absolute majority (50 percent plus one) of the votes cast. In case that more than one parties have the same total number of valid votes, the seat is given to one of them randomly.

In districts with magnitude greater or equal to two, and those are the multi-member districts, seats are given according to *Droop* quota. In each lower district Droop quota is computed. That is, the total number of the valid votes, in each district, divided by the district magnitude increased by one ('sin ena'). Each party occupies as many seats, in a district, as the times the quota is contained in the party's total valid votes of this district. Simply, the parties are given as many seats as they have won quotas.

The distribution of the seats for independent candidates is done in the following way: An independent candidate takes one seat in a lower district only if its total valid votes, in this district, is greater or equal to the droop quota.

If the previous procedure gives in some districts more seats than available, the seats that are in excess are subtracted according to the smallest remainders. The remainders are the seats that have not been used for the allocation of the seats. In a district the surplus seat is subtracted from the party of the smallest remainder. It is the remainder of the division of

[5]The Greek Parliament vote this electoral system the January of 1985. See, Government Gazette (1985).

its total valid votes by the district's magnitude. In case two or more parties have the same remainder, the selection of the party is done randomly. Any remaining seats are distributed in the following step.

Secondary Distribution of Seats.

The distribution is carried out in 9 **Major districts.** Each one of them comes from the aggregation of four, five or even ten lower districts. Single parties and cartels of more than two parties take part in the allocation of the seats, while independent candidates are excluded.

The distribution of the remaining seats of the primary distribution is done, in each major district, in the following way: The available seats from the primary distribution are aggregated in each respective major district. A new quota is defined in each major district. It is the ratio of the total votes, for the parties taking place in this distribution, divided by the respective remaining seats in each major district (Hare). Each party takes as many seats in a district as the times the quota is contained in the party's total valid votes of this district. Simply, parties are given as many seats as they have won quotas.

The above procedure gives the number of seats that are allocated to each party according to the secondary distribution of seats. In this procedure the seats are allocated to major districts. Thus, the number of the seats that each party obtains is known, in each major district. The remaining seats are allocated to parties in the following stage, the tertiary distribution of seats. For the realization of this step, the secondary distribution of seats in the lower districts, is needed. In simple words we have to know not only the number of the seats obtained by each party, in the major districts, but also the number of the seats obtained by each party,in the lower districts. For this purpose I perform the procedure for the allocation of seats of major districts to lower districts.

Allocation of seats of Major districts to Lower districts

First of all, the number of the remaining available seats of the primary distribution is computed, in each lower district. Then the total number of votes, in each lower district, for the parties taking place in this distribution, is divided by the respective number of the remaining available seats. The integer part of this ratio is the Hare quota which is defined in each lower district. Each party takes as many seats, in a district, as many times the quota is contained in the party's total valid votes of this district. Simply, parties are given as many seats as they have

https://doi.org/10.26219/heal.aueb.6403

won quotas. The remaining undisposed seats are given according to the following procedure. Each available seat, in a lower district, is given to the party with the largest remainder of the quotient which is defined by the party's valid votes divided by the quota of this district.

In the lower districts with magnitude equal to two, if there is only one remaining undisposed seat from the primary distribution, the one and only seat is given to the party that has already occupied the first seat in this district, in the primary distribution, only if the total number of valid votes of this party in this district divided by two is greater than the total valid votes of each one of the remaining parties in this lower district.

If with the above procedure, some parties occupy more seats than they have to according to the secondary distribution, surplus seats are subtracted.. The subtraction is done with the use of the following ratio: Total valid votes of each party, for each lower district, is divided by the number of seats that has already been allocated to this party from the primary and the secondary distribution. Seats that are in excess, of each party, are subtracted from this lower district where the above ratio is the smallest. Each subtracted seat is added in the same lower district to the party that needs this seat according to the secondary distribution of seats. If there are more than two parties that need this seat, it is given to the party with the largest ratio. In each case of equal total votes between parties, in the allocation or substraction of seats, the selection of the party is done randomly. The seats are not subtacted when they have been allocated a) according to the quota, b)in districts with magnitude equal to two.

In fact this procedure can be omitted, when we are interested in the final result of the elections, because the final result of the system is not affected by this procedure. It is not affected because this procedure is done in such a way that the total number of seats, for each party in the major districts, is equal to the total number of seats for each party in the respective lower districts.

There is a special case for the lower districts with magnitude equal to two. If only one seat is allocated to a party in the primary distribution also the second seat is given to the same party only if half of its total votes is greater than the total votes of each one of the remaining parties.

Tertiary Distribution of Seats.

The parties which participate in this distribution are also the same who participated in

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the secondary distribution. Thus, independent candidates are excluded. This distribution is done throughout the state which means that the **Higher district** consists of the entire state. The remaining undisposed seats of the primary and the secondary distribution is computed in each lower district. The remaining seats, in each lower district, are given to the party that has also the *plurality* (relative majority) of its total votes in this lower district only if this party has the *plurality* of the total valid votes throughout the state. For the remaining seats the ratio of the total votes in the nation of the parties participating in this distribution divided by the remaining number of seats, is computed. Parties are given as many seats as they have won quotas[6]. The remaining seats are given to the party that has the *plurality* of total votes throughout the state. Any remaining seats are given to the party with the highest percentage of votes in the nation.

Distribution of Seats of State Deputies.

The parties which participate in this distribution are those who took part in the secondary and also the tertiary distribution. Thus, again independent candidates are excluded. The quota is defined as the ratio of the total votes in the nation of the parties participating in this distribution divided by 12 (Hare quota). Parties are given as many seats as the have won quotas. For the remaining seats the following procedure is followed. Let s_i be the number of seats allocated to i with the use of the quota. Any remaining seats are given to the parties according to the following ratio: total valid votes of each party is divided by $s_i + 1$. The first remaining undisposed seat is given to the party with the largest ratio $v_i/(s_i + 1)$. The second is given to the party with the next largest ratio, where s_i are the total seats that the party has already gained. The procedure continues until all remaining seats are given to the parties. (d' Hondt formula)

[6]The electoral system includes the procedure with which those seats are allocated to lower districts. This procedure can be excluded as the next stage is the distribution of seats throughout the state and thus the number of seats allocated to each party, in each lower district, is not needed. Furthermore, this procedure does not affect the final result of the electoral formula.

https://doi.org/10.26219/heal.aueb.6403

3.2.4 1989 Electoral formula[7]

Primary Distribution of Seats.

The state is divided in 56 lower districts which are almost the same as the 52 geographical-administrative districts, but some of them are divided in more districts due to overpopulation. The seats are distributed in each lower district among all alternatives: independent candidates, single parties and cartels of two or more parties. The distribution is done according to the total number of votes. For this purpose, the distribution of seats for single parties and cartels of more than two parties is done in the following way: In districts with magnitude equal to one, and those are the districts with only one available seat, the one and only seat is given by the use of the *plurality rule.* According to this rule the seat is given to the party with the most valid votes, whether or not that party has an absolute majority (50 percent plus one) of the votes cast. In case there are more than one parties that have the same total number of valid votes, the seat is given to one of them randomly.

In districts with magnitude greater or equal to two, and those are the multi-member districts, seats are given according to *Droop* quota. In each lower district Droop quota is computed. That is the total number of the valid votes, in each district, divided by the district magnitude increased by one ('sin ena'). The total number of valid votes for each party, for each district, is divided by the quota of the district. Each party occupies as many seats in a district as many times the quota is contained in the party's total valid votes of this district. Simply, the parties are given as many seats as they have won quotas.

The distribution of the seats for independent candidates is done in the following way: An independent candidate occupies one seat in a lower district only if its total valid votes in this district is greater or equal to the Droop quota of the district.

If the previous procedure offers, in some districts, more seats than available, seats in excess are subtracted according to the smallest remainders. The remainders are the seats that have not been used for the allocation of the seats. In a district the 'surplus' seat is subtracted from the party with the smallest remainder. It is the remainder of the division of its total valid votes by the district's magnitude. In case two or more parties have the same remainder, the selection

[7] The Greek Parliament vote this electoral system on the 31th of March on 1989. See, Government Gazette (1989).

of the party is done randomly. Any remaining seats are distributed in the following step.

Secondary Distribution of Seats.

The distribution is carried out in 13 Major Districts [8]. Each one of them consists of two, three or even six lower districts. Single parties and cartels of more than two parties are allowed to take part in the allocation of the seats, in this stage, while independent candidates are excluded.

The distribution of the remaining available seats is done, in each major district, in the following way: The remaining valid votes for single parties and cartels of more than two paries, are aggregated, in each major district. The remaining votes (*remainders of votes*), for each party, are its votes, that have not been accounted for the seats allocation in the primary distribution. For example, if the quota Q is used for the seats allocation, in the primary distribution of seats, with $Q = V \ / \ S$, the remainder of the votes is equal to $U = V - SQ$. The sum of the remaining votes is divided by the respective available seats, in each major district. Then the integer part of the ratio is taken. Each party occupies, in a district, as many seats as the times that the ratio is contained in the party's total remaining valid votes of this district. If there are still available seats, they are given to the parties (single parties and cartels of more than two paries) according to their remaining valid votes from the above allocation and the one of the first distribution. This means that the party with the largest value of the remaining votes, which have not been accounted for the seats allocation, in the first step of this procedure, takes the first seat. The party with the second largest value occupies the second seat, and so on.

If there are single parties or cartels of more than two parties with a total percentage of valid votes greater or equal to 2%, they obtain at least 3 seats. In case of parties with a total percentage of valid votes smaller or equal to 2%, but not smaller than 1%, they obtain at least one seat. For that purpose, if there are single parties or cartels of more than two parties that have the right to obtain 3 seats and have not reached this number, the next procedure follows.

[8]The 13 Major districts are those of 1) A' disrict of Athens, B' district of Athens, A' district of Peireas, B' district of Peireas, The remaining of Attiki, 2) Biotias, Evias, Fthiotidas, Fokidas and Evritanias 3) Argolidas, Arkadias, Korinthias, Lakonias, Messinias, 4) Etolias and Akarnanias, Ahaias and Ilias 5) Zakinthou, , Kefallinias, Kerkiras, and Lefkadas,6) Artas, Thesprotias, Ioanninon, Prevezas, 7) Grevenon, Kastorias, Kozanis, Florinas 8)Karditsas, ,Larissas, Magnisias, Trikalon, 9) Imathias, , kilkis, Pellas, Pierias, Serron, , Halkidikis, A' disrict of Thessaloniki, B' district of Thessaloniki, 10)Dramas, Evrou, Kavalas, Ksanthis, Rodopis, 11) Dodekanissou and Kikladon 12) Lesvou, Samou, Hiou, 13) Irakliou, Lasithiou, Rethimnis, Haniou.

https://doi.org/10.26219/heal.aueb.6403

The distribution of seats to these parties is done according to the total percentages of votes, in major districts, such as: the party with the highest percentage of valid votes, in the district, obtains the first seat, the party with the second highest percentage of valid votes takes the second seat and so on. The procedure continues until they reach the number of three seats. This procedure is not applied to the parties that have already won one or two seats in the primary distribution, in the major districts. If there are single parties or cartels of more than two parties that have the right to obtain one seat and they have not obtained it yet, they take the seat in the major district, where they have gained the higher number of valid votes. If, there is a major district in which there are more parties, that have the right to obtain a seat, than seats, the available seats are given to parties according to the highest percentages of valid votes. The rest of the parties, obtain the seats that they have to in the other major districts, according to the largest percentages of valid votes.

If there are still available seats, they are distributed to the rest of the parties (single parties or cartels of more than two parties). The same procedure is followed as the one used for the allocation of seats in the secondary distribution. Thus, the sum of the remaining votes (remainders of votes) is divided by the number of the respective available seats, in each major district. The seats that have already been awarded to parties in the primary and the secondary distribution, so far, are not taken into account. Then the integer part of the ratio is taken. Each party occupies, in a district, as many seats as the times the ratio is contained in the party's total remaining valid votes of this district. The remaining seats are given to parties (single parties and cartels of more than two paries) according to their remaining valid votes.

So far, the allocation of seats, in the secondary distribution, is done to the parties in the major districts. Simply, so far, we know the number of seats, each party obtains, in each major district. The system also includes the procedure in which the allocation of the seats is done in the lower districts. This procedure will be omitted, because we interested in the final result of the system and not in the results of each district.

Distribution of Seats of State Deputies.

The parties which participate in this distribution are also those who took part in the secondary and the tertiary distribution. Thus, independent candidates are excluded. The quota is defined as the ratio of the total votes in the nation of the parties participating in this distribu-

https://doi.org/10.26219/heal.aueb.6403

tion divided by 12 (Hare quota). Parties are given as many seats as the have won quotas. For the remaining seats the following procedure is followed. Let s_i be the number of seats allocated to the party i with the use of the quota. Any remaining seats are given to the parties according to the following ratio: total valid votes of each party is divided by $s_i + 1$. The first remaining undisposed seat is given to the party with the largest ratio $v_i/(s_i + 1)$,where s_i are the total seats that the party has already gained. The second seat is given to the party with the next highest ratio. The procedure continues until all the remaining seats are given to parties. (d' Hondt formula)

3.2.5 1993 and 1996 Electoral formula[9]

Primary Distribution of Seats.

The state is divided in 56 lower districts which are the same as the lower districts used in the previous mentioned systems. The seats are distributed in each lower district among all alternatives: independent candidates, single parties and cartels of two or more parties. The distribution is done according to the total number of votes. For this purpose, the distribution of seats for single parties and cartels of more than two parties is done in the following way: In districts with magnitude equal to one, those are the districts with only one available seat, the one and only seat is given by the use of the *plurality rule.* According to this rule the seat is given to the party with the most valid votes, whether or not that party has an absolute majority (50 percent plus one) of the votes cast. In case more than one parties have the same total number of valid votes, the seat is given to one of them randomly.

In districts with magnitude greater or equal to two, and those are the multi-member districts, seats are given according to *Droop* quota. In each lower district Droop quota is computed. That is, the total number of the valid votes, in each district, divided by the district magnitude increased by one ('sin ena'). The total number of valid votes, for each party, for each district is divided by the quota of the district. Each party occupies as many seats in a district as many times the quota is contained in the party's total valid votes of this district. Simply, the parties are given as many seats as they have won quotas.

The distribution of the seats for independent candidates is done in the following way: An

[9]This electoral system is described, in details, in the Government Gazette (1993).

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independent candidate occupies one seat in a lower district only if its total valid votes in this district is greater or equal to the Droop quota of the district.

If the previous procedure gives, in some districts, more seats than the available, seats in excess are subtracted according to the smallest remainders. The remainders are the seats that have not been used for the allocation of the seats. In a district the 'surplus' seat is subtracted from the party with the smallest remainder. It is the remainder of the division of its total valid votes by the district's magnitude. In case two or more parties have the same remainder, the selection of the party is done randomly.

If there are parties (single parties, cartels of two or more parties or even independent candidates) with a percentage of valid votes smaller than the 3% of the total valid votes of all the parties throughout the state, then these parties, which might be single parties, cartels of two or more parties or even independent candidates, they are not allowed to gain a seat, in any district, in any distribution. This means that these parties do not occupy a seat; not only in the primary distribution but also in none of the following distributions. On the other hand, the parties with a percentage of valid votes greater or equal to 3% of the total valid votes of all the parties, throughout the state, they obtain a minimum number of seats. This number is the integer part of the 70% of the seats that correspond to the percentage of the party's valid votes, multiplied by 300. If there are parties that have obtained less seats than the number they are entitled to gain, then they are conferred the appropriate number of seats. These seats are taken from other parties according to the total number of seats: the first seat is removed from the party with the smallest percentage of seats, the second seat is removed from the next smallest party and so on. If there are two or more parties that are entitled to take seats, then the distribution is done according to the highest percentages of votes. The first seat is given to the party with the largest percentage of valid votes, the second seat is given to the next largest party, and so on. When a party gains a seat with the above procedure, it gains the seat in the district with the highest remainders of votes, with respect to the primary distribution of seats. The seats are not allocated to the parties, in the lower districts, where they have occupied a seat in the secondary distribution. In this case the entitled seat is allocated to the lower district with the next largest remainder. Any remaining seats are distributed in the following step.

Secondary Distribution of Seats.

The distribution is carried out in 13 **major districts,** which are exactly the same as the major districts of the previous mentioned system. As it was mentioned in the previous systems each one of these districts comes from the aggregation of two, three or even six lower districts. In this stage, single parties and cartels of more than two parties take part in the allocation of the seats, while independent candidates are excluded.

The distribution of the remaining seats is done, in each major district, according to the total votes: The total valid votes for single parties and cartels of more than two parties, are aggregated, in each major district. The sum of the total votes is divided by the respective available seats, in each major district. Then the integer part of the ratio is taken. Each party occupies, in a district, as many seats as the times the ratio is contained in the party's total valid votes of this district. The exact same procedure, as in the previous systems, is also used for the allocation of the seats in the lower districts.

Tertiary Distribution of Seats.

The parties which participate in this distribution are those who also participated in the secondary and tertiary distribution. Thus independent candidates are excluded. This distribution is done throughout the state and consequently, the **Higher district** consists of the entire state. The remaining indisposed seats from the primary and the secondary distribution is computed by using the following quota: it is defined as the ratio of the total votes, in the nation, of the parties participating in this distribution divided by the remaining number of seats. Then, the integer part of the ratio, is taken. Thus, the Hare quota is used, adjusted to the entire state and to the remaining seats. The parties are given as many seats as the have won quotas. The procedure for the distribution of these seats in the lower districts follows. However, it can be omitted from our study, as it is not required neither for the following stages of the distribution, nor does it affect the final result. The indisposed seats, after the tertiary distribution, are awarded to the single party with the highest value of total valid votes throughout the state. There is a special case where these seats are allocated to a cartel of λ parties. This is when there is a cartel with an average percentage of votes for the λ parties greater than the votes percentage of the largest single party.

https://doi.org/10.26219/heal.aueb.6403

Distribution of Seats of State Deputies.

The parties which participate in this distribution are those who also participated in the secondary and tertiary distribution. Thus, independent candidates are excluded. The quota is defined as the ratio of the total votes, in the nation, of the parties participating in this distribution divided by 12 (Hare quota). Parties are given as many seats as the have won quotas. For the remaining seats the next procedure is followed. Let s_i be the number of seats allocated to party i with the use of the quota. Any remaining seats are given to the parties according to the following ratio: total valid votes for each party is divided by $s_i + 1$. The first remaining undisposed seat is given to the party with the highest ratio $v_i/(s_i+1)$. The next seat is given to the party with the next highest ratio, where s_i are the total seats that the party has already gained. The procedure continues until all the remaining seats are given to the parties. (d' Hondt formula).

3.2.6 Comments on the systems

After the description of the electoral systems we can summarize the following: The distribution of the 288 seats is done in three stages. The only exception is the 1989 system, in which the 288 seats were distributed in the primary and the secondary distribution. The 12 additional seats, obtained by the state deputies, are distributed by using a separate procedure. State deputies' seats were introduced in 1974 and were applied in all parliamentary electoral systems up to now. The first distribution is done in 56 lower districts, which are the same in all the cases studied. Although the lower districts remain the same, it does not happen the same to the lower districts magnitude, and consequently to the major districts magnitude. The number of the available seats, in each district, sometimes changes, due to population shifts. From 1926 up to 1985 the available seats in the major district of Athens doubled, while the available seats of the Agaio district has decreased by 1/3. For more details see Pantelis and Triantafillou (1985).

First district: in all systems, all parties, (including single parties, cartels and also independent candidates) are entitled to participate, which means that all of them are able to gain the seats. In fact, it is the only distribution where independent candidates are allowed to participate. Furthermore, each one of the independent candidates, is allowed to obtain at the very most one seat. In most cases, they do not obtain the number of votes that enables them to enter the parliament. We have to point out that in single-member districts, in all systems studied, the plurality rule is applied. The plurality (relative majority) rule in a single member district is the same as the application of PR d' Hondt, for this situation: the party with the highest value of $v_i/(s_i+1)$, with s_i the number of seats received *so far* by the i^{th} party. However, in a single member district s_i is equal to 0 for all competitive parties and hence the seat is received by the party with the highest votes percentage. The application of a quota like the Hare, in single member districts is almost impossible. For example, Hare quota is a single member district, would be Hare $= v/s = v/1 = v$. Thus, a party has to attract all voters, in order to obtain the one and only available seat. One might say that the small parties are favored by this regulation, as it gives them the opportunity to obtain seats, provided that they will achieve a relative majority. What happens in reality is that only large parties achieve this relative majority and especially the first party. In multi-member districts the Hare quota was applied only in 1974, while in all other cases the Droop quota was used. Hare divides total votes by the available seats, while Droop divides total votes by the available seats plus one. Thus, the value of Hare is greater than the value of Droop, when they are applied in the same number of votes and seats. Given that parties occupy as many seats as the times the quota is contained in the number of votes, quota is more easily covered from the parties (large or small), in case of Droop. In this case more seats are allocated in the primary distribution, so the remaining seats are less for the other stages. The application of Droop instead of Hare is more important in lower-member districts. In the case of a district with a large number of seats the additional 'one' in the quota, does not affect the quota a lot. In multi-member districts both v and s are large ($v =$ total votes, $s =$the available seats) and thus the two ratios v/s and $v/s+1$ do not differ a lot. The difference between the two ratios increases for small values of v and s. The systems after the 1985 include a procedure for the elimination of the 'surplus' seats. 'Surplus' seats appear when the distributed seats are more than the available. The procedure is based on the remainders of votes. The votes that have not been used in the allocation of the seats. It is a very rare case and it might happen only, in the case of Droop quota.

Secondary distribution: In all cases, independent candidates are excluded from this distribution. In the 1985, 1989 and 1993 systems all parties and cartels of parties were allowed to participate in the secondary distribution. Since all parties are allowed to participate, the quota is quite high and only the big parties manage to obtain seats. It is not obvious, but in fact, this regulation excludes from the secondary distribution all parties except the two largest. The previous systems imposed the threshold of 17% for single parties, 25% for cartels of two parties and 30% for cartels of more than two parties. Taking into account that Greece is a two-party system (there are two large parties), this threshold does not matter so much. Pantellis (1988) points out that the abolition of the 17% threshold in 1989 is only of phycological importance. The two large parties participate in this distribution. Furthermore, the participation of all parties in all distributions have a bias towards the second party and favors the first and the small parties; see Dimitras (1991). The distribution is done by using the Hare quota, in each major district. In the case of 1989 the procedure differs. It uses the remaining votes of the primary distribution. The remaining votes for each party are its votes that have not been accounted for the seats allocation, in the primary distribution. It is obvious that this method favors the small parties a lot. Dimitras (1991) points out that according to the 1989 system there was much less bias in favor of large parties. The procedure that has been used in the 1989 system for the secondary distribution is most proportional with respect to the procedure that the other systems use. This happens because large parties do not use all their votes in the secondary distribution and this means that they do not use the votes used in the primary allocation of the seats again. When all votes are used the quota is much larger than in the case that the remaining votes are used. However, a larger quota enables over-representation for large parties. Furthermore, small parties are more reinforced in the 1989 system because parties with at least one per cent of votes occupy at least one seat, and parties with at least 2% of votes take at least three seats. The secondary distribution, for 1974 up to 1895 systems, includes a special case for two-member districts. It was removed from the 1989 system but it was applied again in the 1993 electoral system. This is the following: If only one seat is allocated to a party in a primary distribution also the second seat is given to the same party only if half of its total votes is greater than the total votes for each one of the remaining parties. In fact this regulation is in a way the application of the PR d' Hondt in the two-member districts: the largest value of $v_i/(s_i+1)$ is used , with s_i the number of seats received *so far* by the i^{th} party. For the party i^* that received the first seat the value of $v_{i*}/(s_{i*}+1) = v_{i*}/(1+1) = v_{i*}/2$, while for the rest parties $v_i/(0+1) = v_i$. The seat is given to the party i^*only if $v_{i*}/2 > v_i, \forall\ i$. Such restrictions

https://doi.org/10.26219/heal.aueb.6403

reveal the sparsity of the appearance of this case. As Pantellis and Triantafillou (1985) point out this case in fact appeared only one time, in 1963 in the lower district of Samos. In his opinion, this regulation was introduced after the interference of E.R.E, the political party that won the seat, and after that none has changed it. Thus, its omitting will not cause different results.

Tertiary distribution: All systems include a tertiary distribution of seats with the exception of 1989 system. Hare quota is applied throughout the state while the remaining seats are distributed according to the plurality rule. The 1985 system differs, as seats are allocated in the lower districts according to the relative majority in both lower districts and the entire state. Thus this rule is closer to the majority systems than to PR systems. Consequently, it favors the first party more and eliminates the possibility that small parties will gain seats in this distribution, because the number of seats which are allocated in this distribution are also eliminated.

State deputies' seats: In all cases Hare quota is used, as a first step, as total votes are divided by 12. In the first three systems the undisposed seats are distributed according to the largest remainders, while after 1985 d' Hondt rule is applied.

As far as the thresholds are concerned, we have to point out the importance of the threshold of the 1993 system, which does not permit to small parties to gain a seat, in any district, in any distribution. These small parties are all parties with a total percentage of votes smaller than 3%. This means that these parties do not take a seat neither in the primary distribution nor in any of the following distributions. On the other hand, parties with a percentage of valid votes greater or equal to 3% of the total valid votes of all the parties, throughout the state, they obtain a minimum number of seats. This number is the integer part of the 70% of the seats that correspond to the percentage of a party's valid votes, multiplied by 300. These seats are eliminated from other parties according to the total number of seats. Thus, this regulation take seats from large parties and gives them to middle parties. The main rules of all systems are given in Table 4.

https://doi.org/10.26219/heal.aueb.6403

Table 4: The main rules, for all systems, for each one of the Primary (A), Secondary (B), Tertiary (C) and State Deputies (S. D.) distribution.

	A		**B**	**C**	**S.D.**
	Magn. = 1	*Magn.* $\neq$ 1			
1974	*Rel. Maj.* (*d'Hondt*)	*Hare*	*Hare* (17%)	*Hare* *remaining* : 1^{st} *party*	*Hare* *L.R*
1981	*Rel. Maj.* (*d'Hondt*)	*Droop*	*Hare* (17%)	*Hare* *remaining* 1^{st} *party*	*Hare* *L.R.*
1985	*Rel.Maj.*	*Droop clause for 'surpus' seats*	*Hare*	*Rel.Maj.* (*lower* + *state*) *remaining* : 1^{st} *party*	*Hare* *d'Hondt*
1989	*Rel. Maj.*	*Droop clause for 'surpus' seats*	*based on remainders* (1%, 2%)		*Hare* *d'Hondt*
1993	*Rel. Maj.*	*Droop clause for 'surpus' seats*	*Hare* (3%, 70%)	*Hare*	*Hare* *d'Hondt*

https://doi.org/10.26219/heal.aueb.6403

3.3 Electoral Formulas' Algorithms

3.3.1 Notation

In this section, each electoral system is performed with mathematical relations in the form of pseudo-algorithm. For this purpose we introduce the following notations:

l, m, h: the lower, the major, and the higher electoral districts respectively.

v: the total number of the lower electoral districts.

u: the total number of the major electoral districts.

i: political parties.

p: the total number of political parties participating in the elections.

E_l: district magnitude of the lower district l.

E_m: district magnitude of the major district m.

E_{il}: the number of the seats occupied by the party i, in the lower district l.

E_{im}: the number of the seats occupied by the party i, in the major district m.

E_{ih}: the number of the seats occupied by the party i, in the higher district h.

E_{is}: the number of the seats occupied by the party i, throughout the state s.

Ψ_{il}: total votes of the party i, in the lower district l.

Ψ_{im}: total votes of the party i, in the major district m.

Ψ_l: total votes in the lower district l. ($\Psi_l=\sum_{i=1}^{p} \Psi_{il}$).

$\sum_{l} \Psi_l$: total votes in the nation.

R_a: the total number of the available seats of the primary distribution.

R_{ab}: the total number of the available seats of the primary and the secondary distribution.

R_c: the total number of the available seats of the tertiary distribution.

R_s: the total number of the available seats of the distribution of the state deputes.

P_i: Percentage of votes, for the party i, in the nation. ($P_i = \sum_{l=1}^{v} \Psi_{il} / \sum_{1=1}^{v} \Psi_l$).

P_{il}: Percentage of votes, for the party i, in the lower district l. ($P_{il} = \Psi_{il} / \sum_{i=1}^{p} \Psi_{il}$).

M_a, M_b, M_c, M_s: the quota of the primary, secondary, tertiary and the distribution of state deputies, respectively.

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Z_a, Z_b, Z_c, Z_s: the total number of seats given to parties in the primary, secondary, tertiary and the distribution of state deputies, respectively.

Z_{ia}, Z_{ib}, Z_{ic}, Z_{is}: The total number of seats, distributed to each party i, in the primary, secondary, tertiary and the distribution of state deputies respectively.

n_i: describes the nature of the party i ($n_i = 1$ represents single parties, $n_i = \lambda$ represents cartels of λ parties, (λ=2,3..) and $n_i = 0$ represents independent candidates).

I: the set of all parties participating in the secondary, tertiary and the distribution of state deputies, respectively.

T_i: total seats distributed to party i.

$\lfloor x \rfloor$: the lower integer part of x, where x is a real number.

3.3.2 1974 algorithm

Primary Distribution of Seats (a)

1. Calculate the Hare quota: $M_a = \left\lfloor \frac{\Psi_l}{E_l} \right\rfloor$, for each lower district l, $l = 1, 2, ..v$.
2. Computation of E_{il}:
   - if $E_l \neq 1$, then calculate the number of the seats occupied by the party i, in the lower district l: $E_{il} = \left\lfloor \frac{\Psi il}{Ma} \right\rfloor, i = 1, 2, ..p,\ l = 1, 2, ..v$
   - if $E_l = 1$, then we set $\Psi i^* l = \max_i \{\Psi_{il}\}$ and $E_{il} = 0$ if $i \neq i^*$ and $E_{il} = 1$ if $i = i^*$.
3. Calculate the total number of seats given to all parties $Z_a = \sum_{l=1}^{v} \sum_{i=1}^{p} E_{il}$.
4. Calculate the total number of seats given to each party i, $Z_{ia} = \sum_{l=1}^{v} E_{il}$.

Secondary Distribution of Seats (b)

1. First we define the set of parties i that consist I
   - if $n_i = 0 \Rightarrow i \notin I$
   - if $n_i = 1$ and $P_i \geq 17\% \Rightarrow i \in I$

https://doi.org/10.26219/heal.aueb.6403

- if $n_i = 2$ and $P_i \geq 25\% \Rightarrow i \in I$
- if $n_i > 2$ and $P_i \geq 30\% \Rightarrow i \in I$

2. If $|I| = 1$, then let i^* be the one and only political party such that $i^* \in I$. A second party i is included in I only if $\frac{P_i}{n_i} = \max_{i \neq i^*} \left\{ \frac{P_i}{n_i} \right\}$.
3. If $|I| = 0$, then define δ_i such that:
   - $\delta_i = 17 - P_i$ if $n_i = 1$
   - $\delta_i = 25 - P_i$ if $n_i = 2$
   - $\delta_i = 30 - P_i$ if $n_i > 2$.
   - The set $\left\{i', i^*\right\}$ is equal to I if and only if $\delta i^* = \max_i \{\delta i\}$ and $\delta i' = \max_{i \neq i^*} \{\delta i\}$.
4. Calculate the quota $M_b = \left\lfloor \frac{\sum_{i \in I} \Psi_{im}}{R_a} \right\rfloor$, $\forall m$, where $R_a = E_m - \sum_{i=1}^{p} E_{im}$ are the available seats in each major district m.
5. Calculate $E_{im} = \left\lfloor \frac{\Psi_{im}}{M_b} \right\rfloor$, $\forall\, i, m$.
6. Calculate $Z_b = \sum_{m=1}^{u} \sum_{i \in I} E_{im}$.
7. Calculate $Z_{bi} = \sum_{m=1}^{u} E_{im}$ for each party i, $i \in I$.

Tertiary Distribution of Seats (c)

1. Evaluate $R_{ab} = 288 - Z_a - Z_b$
2. Calculate the quota $M_c = \left\lfloor \frac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{R_{ab}} \right\rfloor$ and $E_{ih} = \left\lfloor \frac{\sum_{l=1}^{v} \Psi_{il}}{M_c} \right\rfloor$.
3. Calculate $Z_c = \sum_{i \in I} E_{ih}$ and $Z_{ic} = E_{ih}$.
4. Calculate $R_c = R_{ab} - \sum_{i \in I} E_{ih}$. If $R_c \neq 0$ we find the party i_0 such that $P_{i_0} = \max_i \{P_i\}$. We define $R_{ci} = R_c$ for $i = i_0$ and $R_{ci} = 0$ for $i \neq i_0$. Each party takes $Z_{ci} + R_{ci}$ seats.

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Distribution of the State Deputies' Seats (s)

1. Calculate quota $M_s = \left\lfloor \frac{\sum_{l=1}^{v}\sum_{i\in I}\Psi_{il}}{12} \right\rfloor$ and $E_{is} = \left\lfloor \frac{\sum_{l=1}^{v}\Psi_{il}}{M_s} \right\rfloor$.

2. Calculate the remaining seats $R_s = 12 - \sum_{i\in I} E_{is}$

   - if $R_s \neq 0$ the remaining available seats are given according to the largest remainders.
   - for the party i the remainder is given by $\sum_{l=1}^{v}\Psi_{il} - E_{is}\cdot M_s$.
   - find i^* such that $\{\Psi_{i^*l} - M_s * E_{i^*s}\} = \max\{\Psi_{il} - M_s * E_{is}\}, \forall i \neq i^*$ and then set $E_{i^*l} = E_{i^*l} + 1$,
   - each party i takes R_{is} seats. It is $Z_{is} = E_{is} + R_{is}$.

Finally, $T_i = Z_{ia} + Z_{ib} + Z_{ic} + R_{ic} + Z_{is}$.

3.3.3 1977 and 1981 algorithm

The only difference is in the quota of the primary distribution. Now Droop quota is used instead of Hare quota.

Droop quota: $M_a = \left\lfloor \frac{\Psi_l}{E_l + 1} \right\rfloor$.

3.3.4 1985 algorithm

Primary Distribution of Seats (a)

1. The distribution of the seats depends on the value of the n_i :

   - if $n_i > 0$ then

     (a) calculate the Hare quota: $M_a = \left\lfloor \frac{\Psi_l}{E_l} \right\rfloor$, for each lower district l.

     (b) computation of E_{il}:

     – if $E_l \neq 1$, then calculate the number of the seats occupied by the party i, in the lower district l: $E_{il} = \left\lfloor \frac{\Psi_{il}}{M_a} \right\rfloor$

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- If $E_l = 1$, then set $\Psi i^* l = \max_i \{\Psi_{il}\}$ and $E_{il} = 0$ if $i \neq i^*$ and $E_{il} = 1$ if $i = i^*$

- if $n_i = 0$ then $E_{i^*l} = 1$ only if $\sum_{l=1}^{v} \Psi_{i^*l} > \sum_{l=1}^{v} \Psi_{il}, \forall\, i \neq i^*$.

2. Calculate the total number of seats given to all parties, in each district l: $Z_{al} = \sum_{i=1}^{p} E_{il}$
3. If there is l such that $Z_{al} \leq E_l$ then stop.
4. If there is l such that $Z_{al} > E_l$ then
   - Find i^* such that $\{\Psi_{i^*l} - M_a * E_{i^*l}\} = \min\, \{\Psi_{il} - M_a * E_{il}\}, \forall i \neq i^*$ and then set $E_{i^*l} = E_{i^*l} - 1$,
   - If (4) holds the procedure continuous until $Z_{al} = E_l$, $\forall\, l$.
5. Calculate the total number of seats given to each party i, $Z_{ia} = \sum_{l=1}^{v} E_{il}$.
6. Calculate the total number of seats given to each district l, $Z_{al} = \sum_{i=1}^{p} E_{il}$.
7. Calculate the total number of seats given to all parties, $Z_a = \sum_{i=1}^{p} \sum_{l=1}^{v} E_{il}$.

Secondary Distribution of Seats (b)

1. First we define the set I
   - if $n_i = 0 \Rightarrow i \notin I$
   - if $n_i \neq 0 \Rightarrow i \in I$.
2. Calculate the quota $M_b = \left\lfloor \frac{\sum_{i \in I} \Psi_{im}}{R_a} \right\rfloor$ $\forall m$, where $R_a = E_m - \sum_{i=1}^{p} E_{im}$ are the available seats in each major district m.
3. Calculate $E_{im} = \left\lfloor \frac{\Psi_{im}}{M_b} \right\rfloor$, $\forall\, i, \in m$.
4. Calculate $Z_b = \sum_{m=1}^{u} \sum_{i \in I} E_{im}$.
5. Calculate $Z_{ib} = \sum_{m=1}^{u} E_{im}$, for each party i, $i \in I$.

https://doi.org/10.26219/heal.aueb.6403

6. Allocation of seats of major districts to lower districts

- Compute the remaining available seats of the primary distribution, in each lower district l, $R_{a_l} = E_l - Z_{al}$.
- Compute the quota $M_{b_l} = \left\lfloor \frac{\sum\limits_{i \in I} \Psi_{il}}{R_{a_l}} \right\rfloor$, for each lower district l.
- The number of seats, for each party, in each lower district is given by $E_{im}^{elas} = \left\lfloor \frac{\Psi_{il}}{M_{b_l}} \right\rfloor$.
- The remaining available seats of the above distribution are given by $R_l^{elas} = R_{a_l} - \sum\limits_{i \in I} E_{il}^{elas}$
  (a) if $R_l^{elas} \neq 0$ then, in each lower district l are distributed R_l^{elas} seats. The first seat is given to the party i^* such that, $\forall i \neq i^* : \left\{\Psi_{i^*l} - M_{b_l} * E_{i^*l}^{elas}\right\} = \max\left\{\Psi_{il} - M_{b_l} * E_{il}^{elas}\right\}$. The second seat is given to the party i' such that $\left\{\Psi_{i'l} - M_{b_l} * E_{i'l}^{elas}\right\} = \max\left\{\Psi_{il} - M_{b_l} * E_{il}^{elas}\right\}, \forall i \neq i^*, i'$ and so on. Let E_{il}^{upol} the seats that are distributed with this procedure.

  (b) if $R^{elas} = 0$ then stop.
- If $(\exists l : E_l = 2)$and $(\exists i^*$such that$Z_{ai^*} = 1)$ then the one and only seat is given to the party i^* only if $\frac{\sum\limits_{l} \Psi_{i^*l}}{2} = \max\left\{\sum\limits_{l} \Psi_{il}\right\}, \forall i \neq i^*$.
- Calculate the total number of the seats that are distributed to parties $E_{im} = E_{im}^{elas} + E_{im}^{upol}$

  (a) if $E_{im} = E_{im,}^{elas}$ $\forall$ i, m then stop.

  (b) if $E_{iM} > E_{iM,}^{elas}$ $\forall$ i, M then compute the $\beta_{il} = \frac{\Psi il}{E_{il} + E_{il}^{upol}}$, for each party i and for each lower district l. If $\exists$ i : $\sum\limits_{l} E_{il}^{upol} > \sum\limits_{M} E_{iM}$ then seats are subtracted from this party i,in the lower district l^*. The districts l^* are those which satisfy the: $\beta_{il^*} = \min\{\beta_{il^*}\}$. Seats are substracted until $\sum\limits_{l} E_{il}^{upol} > \sum\limits_{M} E_{iM}$. The subtracted seats are distributed in the same districts l^* to the parties i^*. the parties $i^*$are those that satisfy the: $\beta_{i^*l^*} = \max\{\beta_{i^*l^*}\}$ only if $\sum\limits_{l \in M} E_{i^*l}^{upol} < \sum\limits_{M} E_{i^*M}$.

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Tertiary Distribution of Seats (c)

1. Evaluate $R_{ab} = \sum_l E_l - Z_a - Z_b$.

2. In each l : if there is party i_0 such that $\Psi_{i_0 l} > \Psi_{il}, \forall\, i \neq i_0$ and $\sum_l \Psi_{i_0 l} > \sum_l \Psi_{il}, \forall\, i \neq i_0$, this party takes all the available seats ϵ_l in l.

3. Calculate all the seats given with the above procedure $\epsilon_{\,i_o} = \sum_l \epsilon_{l\,i_o}$.

4. The remaining seats are $R'_{ab} = R_{ab} - \epsilon_{\,i_o}$.

5. Calculate quota $M_c = \left\lfloor \dfrac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{R'_{ab}} \right\rfloor$ and $E_{ih} = \left\lfloor \dfrac{\sum_{l=1}^{v} \Psi_{il}}{M_c} \right\rfloor$

6. Calculate $Z_c = \sum_{i \in I} E_{ih} + \epsilon_{\,i_o}$ and $Z_{ic} = E_{ih} + \epsilon_{i0}$

7. Calculate $R_c = R'_{ab} - Z_c$ If $R_c \neq 0$ we find the party i_0 such that $P_{i_0} = \max_i \{Pi\}$. We define $R_{ic} = R_c$ for $i = i_0$ and $R_{ic} = 0$ for $i \neq i_0$. Each party takes $Z_{ic} + R_{ic}$ seats.

Distribution of the State Deputies' Seats (s)

1. Calculate quota $M_s = \left\lfloor \dfrac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{12} \right\rfloor$ and $E_{si} = \left\lfloor \dfrac{\sum_{l=1}^{v} \Psi_{il}}{M_s} \right\rfloor$

2. Calculate the remaining seats $R_s = 12 - \sum_{i \in I} E_{is}$.

3. If $R_s \neq 0$ then compute $\lambda_i = \frac{\sum_{l=1}^{v} \Psi il}{1 + \sum_{i \in I} E_{is}}$

   - the first seat is given to the party i^*such that $\lambda_{i^*} = \max \{\lambda_i\}, \forall\, i \neq i^*$. The second seat is given to the party i' :$\lambda_{i'} = \max \{\lambda_i\}, \forall\, i \neq i^*, i'$ and so on. From this procedure each party i obtains R_{is} seats.

4. Computation of the total number of seat given to each party from the distribution (s) : $Z_{is} = E_{is} + R_{is}$.

Finally, $T_i = Z_{ia} + Z_{ib} + Z_{ic} + R_{ic} + Z_{is}$.

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3.3.5 1989 algorithm

Primary Distribution of Seats (a)

1. The distribution of seats depends on the value of the n_i :

   - if $n_i > 0$ then

     (a) calculate the Hare quota: $M_a = \left\lfloor \frac{\Psi_l}{E_l} \right\rfloor$, for each lower district l.

     (b) computation of E_{il}:

     - if $E_l \neq 1$, then calculate the number of the seats occupied by the party i, in the lower district l: $E_{il} = \left\lfloor \frac{\Psi_{il}}{M_a} \right\rfloor$
     - If $E_l = 1$, then set $\Psi i^*l = \max_i \{\Psi il\}$ and $E_{il} = 0$ if $i \neq i^*$ and $E_{il} = 1$ if $i = i^*$

   - if $n_i = 0$ then $\mathrm{Ei}^* l = 1$ only if $\sum_{l=1}^{v} \Psi_{i^*l} > \sum_{l=1}^{v} \Psi_{il}$, $\forall\, i \neq i^*$.

2. Calculate the total number of seats given to all parties, in each district l: $Z_{al} = \sum_{i=1}^{p} E_{il}$.

3. If there is l such that $Z_{al} \leq E_l$ then stop.

4. If there is l such that $Z_{al} > E_l$ then

   - find i^* such that $\{\Psi_{i^*l} - M_{al} * E_{i^*l}\} = \min \{\Psi_{il} - M_{al} * E_{il}\}, \forall i \neq i^*$ and then $E_{i^*l} = E_{i^*l} - 1$,
   - if (4) holds the procedure continuous until $Z_{al} = E_l$, $\forall\, l$.

5. Calculate the total number of seats given to each party i, $Z_{ia} = \sum_{l=1}^{v} E_{il}$.

6. Calculate the total number of seats given to each district l, $Z_{al} = \sum_{i=1}^{p} E_{il}$.

7. Calculate the total number of seats given to all parties, $Z_a = \sum_{i=1}^{p} \sum_{l=1}^{v} E_{il}$.

Secondary Distribution of Seats (b)

1. First we define the set I

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- if $n_i = 0 \Rightarrow i \notin I$
- if $n_i \neq 0 \Rightarrow i \in I$.

2. Calculate the quota $M_b = \left\lfloor \frac{\sum_{i \in I} \Psi^*_{im}}{R_a} \right\rfloor$, for each major district, where $R_a = E_m - \sum_{i=1}^{p} E_{im}$ are the available seats of the primary distribution, in each major district, and Ψ^*_{im} are the remaining valid votes (the votes that have not been used for the seats allocation in the primary distribution).

3. Calculate $E_{im} = \left\lfloor \frac{\Psi^*_{im}}{M_b} \right\rfloor, \forall\, i \in I$.

4. Calculate $Z_b = \sum_{m=1}^{u} \sum_{i \in I} E_{im}$.

5. Calculate $Z_{ib} = \sum_{m=1}^{u} E_{im}$ for each party i, $i \in I$.

6. If there is m such that $E_{im} < R_a$ then
   - find i^* such that $\{\Psi^*_{i^*l} - M_b * E_M\} = \max\ \{\Psi^*_{il} - M_b * E_M\}, \forall i \neq i^*$ and then $E_{i^*m} = E_{i^*m} + 1$,
   - if (6) holds the procedure continuous until $E_{iM} = R_a$.

7. Let J be the set $\{i'\}$, where i' satisfies the $\left(\Psi_{i'} >= 0.02 \times \sum_{l \in} \Psi il / \sum_{l} \Psi_l\right)$ and the $(E_{i'm^*} = 0)$. If $i^* \in J$ then
   - if the party i^* : $\Psi_{i^*m} = \max\{\Psi_{im}\}, i \in J$, then $E_{i^*m} = E_{i^*m} + 1$.
   - This procedure continuous until $E_{i^*m} = 3$, for each $i^* \in J$.

8. Let H be the set $\{i'\}$, where i' satisfies the $\left(\Psi_{i'} >= 0.01 \times \sum_{l \in} \Psi il / \sum_{l} \Psi_l\right)$ and the $\left(\Psi_{i'} <= 0.02 \times \sum_{l \in} \Psi il / \sum_{l} \Psi_l\right)$. If $i^* \in H$ then
   - if there is party i^* which satisfies the $\Psi i^* M^* = \max\{\Psi iM\}, i \in J$, then $E_{i^*M^*} = 1$,in the major district m.

9. Let R_m be the available seats in each major district. If $R_m \neq 0$ then

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- Calculate $M'_b = \left\lfloor \frac{\sum_{i \in I} \Psi^*_{im}}{R_m} \right\rfloor$ and $E'_{im} = \left\lfloor \frac{\Psi^*_{im}}{M'_b} \right\rfloor$

10. If there are still available seats R'_m then find i^* such that $\{\Psi_{*i^*m} - M'_b * E'_{i^*m}\} = \max \{\Psi^*_{im} - M'_b * E_{im}\}, \forall i \neq i^*$ and then $E'_{i^*m} = E'_{i^*m} + 1$, until all seats are distributed.

11. The total seats occupied by the party i are $Z_{ib} = \sum_m E_{im} + E'_{im}$

Distribution of the State Deputies' Seats (s)

1. Calculate the quota $M_s = \left\lfloor \frac{\sum_{l=1}^{v} \sum_{i \in I} \Psi_{il}}{12} \right\rfloor$ and $E_{si} = \left\lfloor \frac{\sum_{l=1}^{v} \Psi_{il}}{M_s} \right\rfloor$

2. Calculate the remaining seats $R_s = 12 - \sum_{i \in I} E_{si}$.

3. If $R_s \neq 0$ then compute $\lambda_i = \frac{\sum_{l=1}^{v} \Psi_{il}}{1+\sum_{i \in I} E_{si}}$

   - the first seat is given to the party i^*such that $\lambda_{i^*} = \max\{\lambda_i\}$, $\forall\ i \neq i^*$. The second seat is given to the party i' :$\lambda_{i'} = \max\{\lambda_i\}$, $\forall\ i \neq i^*, i'$ and so on. From this procedure each party i takes R_{is} seats.

4. Computation of the total number of seat given to each party from the distribution (s) : $Z_{is} = E_{is} + R_{is}$.

Finally, $T_i = Z_{ia} + Z_{ib} + Z_{is}$.

3.3.6 1993 and 1996 algorithm

Primary Distribution of Seats (a)

1. The distribution of seats depends on the value of the n_i :

   - if $n_i > 0$ then

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(a) calculate the Hare quota: $M_a = \left\lfloor \frac{\Psi_l}{E_l} \right\rfloor$, for each lower district.

(b) computation of E_{il}:

- if $E_l \neq 1$, then calculate the total number of seats occupied by the party i, in the district l: $E_{il} = \left\lfloor \frac{\Psi_{il}}{M_a} \right\rfloor$
- if $E_l = 1$, then set $\Psi_{i^*l} = \max_i \{\Psi_{il}\}$ and $E_{il} = 0$ if $i \neq i^*$ and $E_{il} = 1$ if $i = i^*$

- if $n_i = 0$ then $E_{i^*l} = 1$ if and only if $\sum_{l=1}^{v} \Psi_{i^*l} > \sum_{l=1}^{v} \Psi_{il}, \forall\, i \neq i^*$.

2. Calculate the total number of seats given to all parties, in each district l: $Z_{al} = \sum_{i=1}^{p} E_{il}$.

3. If there is l such that $Z_{al} \leq E_l$ then stop.

4. If there is l such that $Z_{al} > E_l$ then

   - Find i^* such that $\{\Psi_{i^*l} - M_{al} * E_{i^*l}\} = \min \{\Psi_{il} - M_{al} * E_{il}\}, \forall i \neq i^*$ and then $E_{i^*l} = E_{i^*l} - 1$,
   - If (4) holds the procedure continuous until $Z_{al} = E_l, \forall\, l$.

5. Calculate the total number of seats given to each party i, $Z_{ai} = \sum_{l=1}^{v} E_{il}$.

6. Calculate the total number of seats given to each district l, $Z_{al} = \sum_{i=1}^{p} E_{il}$.

7. Calculate the total number of seats given to all parties, $Z_a = \sum_{i=1}^{p} \sum_{l=1}^{v} E_{il}$.

Secondary Distribution of Seats (b)

1. First we define the set I

   - if $n_i = 0 \Rightarrow i \notin I$
   - if $n_i \neq 0 \Rightarrow i \in I$.

2. Calculate the quota $M_b = \left\lfloor \frac{\sum_{i \in I} \Psi_{im}}{R_a} \right\rfloor$, $\forall m$, where $R_a = E_m - \sum_{i=1}^{p} E_{im}$ are the available seats in each major district m.

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3. Calculate $E_{im} = \left\lfloor \frac{\Psi_{im}}{M_b} \right\rfloor$, $\forall\, i, m$.

4. Calculate $Z_b = \sum_{m=1}^{u} \sum_{i \in I} E_{im}$

5. calculate $Z_{ib} = \sum_{m=1}^{u} E_{im}$ for each party i, $i \in I$.

Allocation of seats of Major districts to Lower districts

- Compute the remaining available seats of the primary distribution, in each lower district l, $R_{a_l} = E_l - Z_{al}$.
- Compute the quota $M_{b_l} = \left\lceil \frac{\sum_{i \in I} \Psi_{il}}{R_{a_l}} \right\rceil$, for each lower district l.
- The number of seats, for each party, in each lower district is given by $E^{elas}_{im} = \left\lfloor \frac{\Psi_{il}}{M_{b_l}} \right\rfloor$.
- The remaining available seats of the above distribution are given by $R^{elas}_l = R_{a_l} - \sum_{i \in I} E^{elas}_{il}$
  (a) if $R^{elas}_l \neq 0$ then, in each lower district l are distributed R^{elas}_l seats. The first seat is given to the party i^* such that, $\forall i \neq i^* : \left\{\Psi_{i^*l} - M_{b_l} * E^{elas}_{i^*l}\right\} = \max\left\{\Psi_{il} - M_{b_l} * E^{elas}_{il}\right\}$. The second seat is given to the party i'such that $\left\{\Psi_{i'l} - M_{b_l} * E^{elas}_{i'l}\right\} = \max\left\{\Psi_{il} - M_{b_l} * E^{elas}_{il}\right\}, \forall i \neq i^*, i'$ and so on. Let E^{upol}_{il} the seats that are distributed with this procedure.

  (b) if $R^{elas} = 0$ then stop.
- If $(\exists l : E_l = 2)$and $(\exists i^*$such that$Z_{ai^*} = 1)$ then the one and only seat is given to the party i^* only if $\frac{\sum_l \Psi_{i^*l}}{2} = \max\left\{\sum_l \Psi_{il}\right\}, \forall i \neq i^*$.
- Calculate the total number of the seats that are distributed to parties $E_{im} = E^{elas}_{im} + E^{upol}_{im}$

  (a) if $E_{im} = E^{elas}_{im,}$ $\forall\, i, m$ then stop.

  (b) if $E_{iM} > E^{elas}_{iM,}$ $\forall\, i, M$ then compute the $\beta_{il} = \frac{\Psi il}{E_{il} + E^{upol}_{il}}$, for each party i and for each lower district l. If $\exists\, i : \sum_l E^{upol}_{il} > \sum_M E_{iM}$ then seats are subtracted

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from this party i,in the lower district l^*. The districts l^* are those which satisfy the: $\beta_{il^*} = \min\{\beta_{il^*}\}$. Seats are substracted until $\sum_l E_{il}^{upol} > \sum_M E_{iM}$. The subtracted seats are distributed in the same districts l^* to the parties i^*. the parties $i^*$are those that satisfy the: $\beta_{i^*l^*} = \max\{\beta_{i^*l^*}\}$ only if $\sum_{l\in M} E_{i^*l}^{upol} < \sum_M E_{i^*M}$.

Tertiary Primary Distribution of Seats (c)

1. Evaluate $R_{ab} = 288 - Z_a - Z_b$

2. Calculate the quota $M_c = \left\lfloor \frac{\sum_{l=1}^{v}\sum_{i\in I}\Psi_{il}}{R_{ab}} \right\rfloor$ and $E_{ih} = \left\lfloor \frac{\sum_{l=1}^{v}\Psi_{il}}{M_c} \right\rfloor$.

3. Calculate $Z_c = \sum_{i\in I} E_{ih}$ and $Z_{ic} = E_{ih}$.

4. Calculate $R_c = R_{ab} - \sum_{i\in I} E_{ih}$. If $R_c \neq 0$ we find the party i_0 such that $P_{i_0} = \max_i\{P_i\}$. We define $R_{ci} = R_c$ for $i = i_0$ and $R_{ic} = 0$ for $i \neq i_0$. Each party takes $Z_{ic} + R_{ic}$ seats.

Distribution of the State Deputies' Seats (s)

5. Calculate quota $M_s = \left\lfloor \frac{\sum_{l=1}^{v}\sum_{i\in I}\Psi_{il}}{12} \right\rfloor$ and $E_{is} = \left\lfloor \frac{\sum_{l=1}^{v}\Psi_{il}}{M_s} \right\rfloor$

6. Calculate the remaining seats $R_s = 12 - \sum_{i\in I} E_{is}$.

7. If $R_s \neq 0$ then compute $\lambda_i = \frac{\sum_{l=1}^{v}\Psi_{il}}{1+\sum_{i\in I} E_{si}}$

   - the first seat is given to the party i^*such that $\lambda_{i^*} = \max\{\lambda_i\}$, $\forall\ i \neq i^*$. The second seat is given to the party i' :$\lambda_{i'} = \max\{\lambda_i\}$, $\forall\ i \neq i^*, i'$ and so on. From this procedure each party i takes R_{is} seats.

8. Computation of the total number of seat given to each party from the distribution (s) : $Z_{is} = E_{is} + R_{is}$.

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9. Let K be the set $\{i\}$, where i satisfies the $\Psi_i <= 0.03 \sum_{l\in} \Psi_{il}$. If $i^* \in K$ then $T_{i^*} = 0$.

10. Let L be the set $\{i\}$, where i satisfies the $\Psi_i >= 0.03 \sum_{l\in} \Psi_{il}$. If $i^* \in L$ then $T_{i^*} >= 210\times\Psi_i$ $/ \sum_{l} \Psi_l = Ul_{i^*}$. For this purpose

    - If $i \in L$ and $T_i \leq Ul_i$ then set $T_i = Ul_i$.
    - $\sum_{i} Ul_i$ seats are sutracted from the parties. The first is subtracted from the party i' with $\Psi_{i'} = \max_{i} \{\Psi_i\}$, the second from the party i'' with $\Psi_{i''} = \max_{i \neq i'} \{\Psi_i\}$, and so on.

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https://doi.org/10.26219/heal.aueb.6403

Chapter 4

Modeling Voter preferences

In this study, we want to apply and compare the latest five Greek electoral systems. For this purpose we will use some measures of disproportionality. Each one of these measures uses the election result (seat shares) for the evaluation of a system. For each system under consideration, one or two election results are available, because each one of these systems has been applied once or twice in the Greek Parliamentary Elections. If we want to find a 'good' estimate for a particular measure, and for a particular system, we must use a large number of different election results. Thus, in order to have accurate results, it would be better to use a large number of datasets for each electoral system. These datasets will contain voters' preferences for the alternatives, and thus total votes for each party-candidate.

In this section we deal with the generation of voters' preferences on different alternatives, found in the literature. We examine how we can apply these methods in our study and if it is reasonable enough to use them. Finally, another idea for the generation of Greek electorate will be introduced and will be shown that this is an appropriate method for our study.

Let n be the size of the electorate. Each one of the n voters has to give his preference among m alternatives. We make some assumptions about individual preferences; see, for example, Bordley (1983).

- An individual has preferences among all possible alternatives.
- If an individual prefers alternative 1 to alternative 2 and alternative 2 to alternative 3,

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he also prefers 1 to 3. Krantz (1971) reached the conclusion that there exists such a utility function u that whenever the individual finds alternative 1 at least as preferable as alternative 2, we will have $u(alternative\ 1) \geq u(alternative\ 2)$. As the election offers the voter m alternatives, in the election, he will vote in such a way as to maximize his overall utility.

- Sincere-Honest voting: In any electoral system, many voters try to take into account the expected candidate strength, as well as their own preferences, in determining how to vote. Thus, their vote depends on their assessment of how other citizens intend to vote. Here we assume that the voters have no knowledge of the likelihood of success of the various candidates. Then the voter is said to be making a decision under uncertainty. Under uncertainty, the optimal strategy involves voting for one's favorite; see Merrill (1985). In simple words, we assume that, the voter cast his vote for the alternative to whom his preference is higher.

Given that each individual $i, i = 1, 2 \ldots n$ has utility $u_i(m)$ for each alternative m there are $n \times m$ values for $u_i(m)$. Those values consist the *utility matrix* U.

$$U = \begin{bmatrix} u_1(1) & \ldots & u_1(i) & \ldots & u_1(m) \\ u_2(1) & \ldots & u_2(i) & \ldots & u_2(m) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ u_j(1) & \ldots & u_j(i) & \ldots & u_j(m) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ u_n(1) & \ldots & u_n(i) & \ldots & u_n(m) \end{bmatrix}$$

The i^{th} row represents the i^{th} voter utilities for the m alternatives. The j^{th} column represents the utilities of all voters for the j^{th} candidate. If U is known, voter preferences among all possible alternatives will be known. Thus, election space will be known.[1] The utility matrix allows us

[1] Camberlin and Cohen (1978) use the term *election* to define the relative frequencies in an electorate of support, for each possible permutation (rank ordering) $= m!$ of the m alternatives. The *election space* is the set of all possible elections that could occur when there are m alternatives. It corresponds to all points in the space of $m!$ dimensions that have no negative coordinates and have coordinates summing to unity: $m!$ *dimensional simplex.*

Suppose a voter with preference ordering ABC. For each one of the alternatives A, B and C has a preference

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to know each individual vote, in this particular election with m alternatives. However, the alternatives that appear in a given election are not known before the election and thus utilities are random events. It is equivalent to make the elements of U random variables. But if the utilities are random variables we have to specify their distribution. In other words we need the probability density function on the election space.

4.1 Impartial Culture-Uniform assumption

The most prominent efforts for the definition of the probability density function on the election space come from the impartial culture assumption. The individual i's utility for the alternative m can be thought of ranging from strong preference to strong aversion:

Dislikes m		*Likes m*	
Strong Aversion	*Mild Aversion*	*Mild Preference*	*Strong Preference*

Suppose that the distribution is symmetric: The probability that an individual has a mild preference for m equals the probability that he has a mild aversion for m and the probability this individual has a strong preference for m equals the probability that he has a strong aversion for m.

Moreover, the uniform assumption demands that the chance an individual feels strongly about m equals the chance that he feels mildly about the same alternative. Under this assumption it is reasonable to model the utilities as *uniform random variables.* Thus, this method treats each voter as a sample point drawn from an infinite population in which all possible permutations are equally likely and it is equivalent to assume that all alternatives are equally attractive. For example, the probability of an individual i having various utilities for m is given

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(*utility*) such that $u(alternative\ A) > u(alternative\ B) > u(alternative\ C)$. For the voter with preference ordering BCA it is $u(alternative\ B) > u(alternative\ C) > u(alternative\ A)$,etc in Figure 1.

Figure 1: Uniform assumption

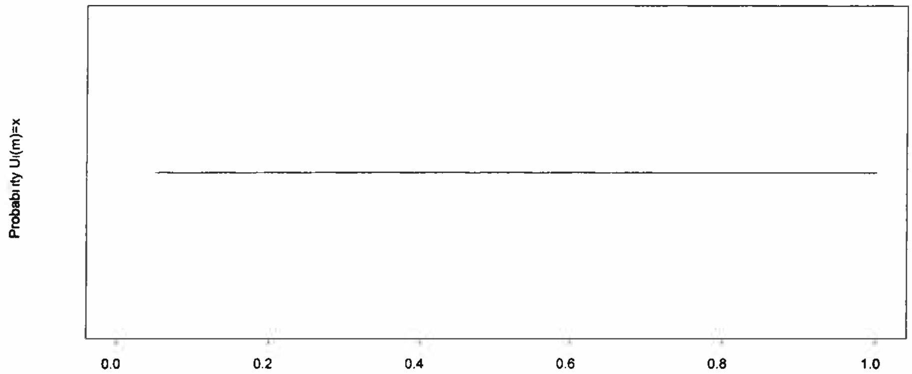
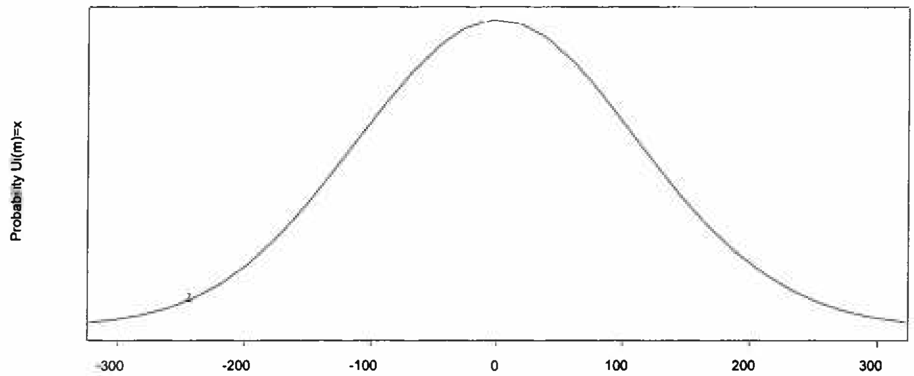


Figure 2: Normal assumption



Merrill (1984,1988) chose the utilities to be independent, uniformly distributed random variates on a common interval from 0 to 1, in order to generate a random society.

We point out that, if we generate an electorate with this method, and then compute the percentages of votes $v_1, v_2..., v_m$, for each alternative, the above method gives that $P(v_1) = P(v_2) = ... = P(v_m)$, which does not exist in reality. Although it is an important drawback, at first thought it leads us in the decision that this method must be taken into account, as it is the first and the most fundamental method for the generation of voter preferences. Furthermore, Merrill (1984) used the impartial culture and the spatial model in order to compare some well-known electoral systems, and he obtained almost the same results.

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As, our purpose is to study the results taken from different electoral formulas and not to study voter preferences and to model voter's behavior, a first thought was that it would be reasonable to use this method.

Ludwin (1976) compared six electoral systems (Plurality, Runoff, Alternate, Exhaustive, Alternate, Borda) based on Condorcet winner. In order to do this comparison, he modeled and he simulated a three candidate single-seat election many times. Ludwin (1976), instead of simulating voter utilities, simulated the frequencies of the possible permutations of the three candidates. As three candidates take part in his hypothetical election there are 3!=6 different permutations $(abc, acb.bac, bca, cab, cba)$. Six uniformly distributed pseudo random numbers generated the proportion of the electorate, which is matched with each one of these permutations. Each random number was matched with a permutation in the following way. The six random numbers were summed and each one of them was divided by the sum of all. Let $x_1, x_2, ..., x_n$ be the generated pseudo numbers. The $x_i \ / \ \sum_{i=1}^{6} x_i$, $i = 1, 2, ...6$ consist of the six percentages of permutation. As the percentage of each permutation comes from the uniform distribution, all possible permutations are equally likely. This is equivalent to the assumption that all candidates are equally attractive from voters. Consequently, Ludwin (1976) used another way of making the uniform assumption.

4.2 Application of the Impartial Culture

First of all, we have to point out that Merrill (1984, 1988) as well as Bordley (1983) used this method for the comparison of electoral systems such as Approval voting, Borda, etc., where voters have to decide between single candidates and not between both political parties and independent candidates. Furthermore, these systems do not demand the division of the electorate in districts and finally the result of the procedure is the number of votes obtained by each candidate and not the number of seats in the parliament obtained by each party (or independent candidate, if he receives a seat).

As we mentioned before, the main drawback of this method is the assumption that all alternatives are equally attractive. Thus, the generation of voter's preferences for an electorate based on the uniform distribution, leads to a generation of a number of votes obtained by each

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alternative that is almost the same. In the case of a small electorate, it is reasonable that the variation in these numbers would be sufficiently large. However, in a case of a large electorate, those numbers would be almost the same.

Greek electoral systems demand the generation of a large electorate, as we have to generate voter's preferences for each lower district separately, and the number of them is large (56). After all, the 300 seats correspond to an electorate that ranges from 6 up to more than 8 million people, and an electorate of such range must be used if we want to use the real number of seats. The application of this method gave 56 districts where, in each one of them, all political parties obtained the same percentage of votes. Table 4 presents the results of the above application, in the case of a small electorate ($n = 1000$). Table 5 presents the results of the above application in the case of a large electorate ($n = 6000000$) indicating that the uniform assumption is unrealistic and not suitable for our study.

Table 4: The results for the first 8 lower districts in case of 1000 voters and 5 political parties for the uniform distribution $U(0,1)$. m: political parties, l: lower districts.

$m \backslash l$	1	2	3	4	5	6	7	8
1	17	14	8	4	2	3	3	3
2	13	39	6	2	9	1	2	2
3	11	24	4	5	2	1	4	6
4	14	33	8	5	4	5	9	3
5	8	15	6	8	10	2	3	4

Table 5: The results for the first 8 lower districts in case of 6000000 voters and 5 political parties for the uniform distribution $U(0,1)$.

$m \backslash l$	1	2	3	4	5	6	7	8
1	76211	150516	39141	29152	32406	14818	25638	21821
2	75800	150093	38899	29476	32619	14881	25593	22020
3	75554	150122	38815	29378	32670	14959	25600	21788
4	76025	150512	38909	29570	32341	14798	25432	21594
5	75610	149957	39236	29421	32564	14944	25537	21799

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4.3 Normal assumption

The assumption that the distribution is symmetric remains. That is, the probability that an individual has a mild preference for m, equals the probability that the individual has a mild aversion for m and the probability that the individual has a strong preference or a strong aversion are identical. In other words, P(mild preference) = P(mild aversion) and P(strong preference) = P(strong aversion). Furthermore, it is assumed that the chance an individual feels strongly about m equals the chance that he feels mildly about him. Under this assumption it is reasonable to model the utilities as normal random variables. The probability an individual i having various utilities for m is given in Figure 2.

It seems that with this method we surpass the main problem of the uniform assumption, all alternatives are not assumed to be equally attractive. Bordley (1983) used this method for the comparison of six well-known systems. Those are: Borda, Copeland, Approval, One person/One vote, Dictatorial and Random choice system, systems where single candidates compete.

4.3.1 Application of the Normal assumption

We applied this method by using a Normal distribution with zero mean and standard deviation equal to 1, $N\ (0,1)$. Voters' preferences were generated by this distribution. By using this assumption the percentages of all political parties were almost the same in each district, when the size of the electorate, in each district, was very large. Table 6 presents the results of this simulation. On the other hand, we notice a small difference in the percentage of total votes of the political parties, in each district, when the size of the electorate was small. Table 7 shows these results. As we have already explained in order to allocate 300 seats in 56 lower districts, to a number of political parties, we have to use an electorate of a large size.

From the Table 6 and the Table 7 is odvious that the Normal distribution is not a suitable distribution for the simulation of the Greek electorate, because the chance that all political parties are almost equally attractive is unrealistic.

Bordley (1983) used both methods in his study (Uniform and Normal assumption) and he found out that the result does not change a lot as we move from communities with uniform distribution to communities with normal distribution. Bordley (1983) after this reached the

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conclusion that the utility distribution may not matter too much.

Table 6: The results for the first 8 lower districts in case of 6000000 voters and 5 political parties for the normal distribution $N(0,100)$.

$m \backslash l$	1	2	3	4	5	6	7	8
1	75694	150212	38927	29166	32710	14859	25643	21785
2	76008	150299	38959	29401	32412	15030	25365	21798
3	75984	149811	39055	29595	32604	14982	25446	21790
4	75807	150306	39010	29381	32465	14742	25515	21956
5	75707	150572	39049	29457	32409	14787	25831	21871

Table 7: The results for the first 8 lower districts in case of 1000 voters and 5 political parties for the normal distribution $N(0,100)$. m: political parties, l: lower districts.

$m \backslash l$	1	2	3	4	5	6	7	8
1	1	10	2	4	0	1	2	1
2	4	6	3	1	2	1	1	3
3	4	7	1	0	4	0	0	1
4	9	5	2	0	2	1	1	0
5	0	9	1	2	0	2	2	0

4.4 Spatial model

Originally, it was developed in the field of economics by Hotteling (1929) who was interested in explaining market locations. The essential model was developed, independently by Coombs (1950) a phycologist interested in individual choice behavior. It was first used to represent electoral competitions by Downs (1957) and have been studied extensively in the past decade.

The main ideas of the Spatial model are:

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- Each voter can be represented by a point in the hypothetical space, so that the point reflects the person's ideal set of policies.
- The policy position can be represented by a point in the same space.
- A voter chooses the candidate whose policy position is closer to his position in the hypothetical space.

Models of this type are also called *Proximity models,* because preference follows directly from 'closeness'. From the above is obvious that the spatial model assumes that both the voters and the alternatives to be distributed in the same space, either unidimensional or multidimensional. Each dimension represents a specific issue. Such issues might be sectors such as political ideology, etc. The voter's and the candidate's positions in the space, depends on their own perception on each one of the issues.

In order to present the main assumptions of the model we give the following multidimensional notation: $x = (x_1, x_2, ..., x_n)$ is the representation of a voter preferences for all issues. Thus, x_i is the voter's position on issue i (dimension i), $i = 1, 2, ..., n$, where n is the number of issues taking into account, $v_j = (v_{j1}, v_{j2}, ..., v_{jn})$ is the position which the candidate j advocate, $j = 1, 2, ..., m$, where m is the number of the candidates. This vector is called candidate j's strategy.

Assumption 1: Although a complete description of the electorate requires several dimensions, a voter may be interested in only some of these dimensions. Hence, he does not assign values for all elements of x. We assume, nevertheless, that every element of x has a numeric value. Every voter acts as if he gives his preferred position on each issue.

Assumption 2: We assume that the vectors x, v_j are continuous variables, although there are issues where preference on them implies continuous or discrete values. The continuous assumption is convenient because it facilitates the use of continuous calculus.

Assumption 3: We assume that all voters act as if they make identical estimates of v_j, which implies that the voters' perceptions are the same for each candidate position.[2] This assumption implies that citizens estimate a position for each candidate on every dimension.

[2] Analysis of real data indicated that these is a substantial disagreement between different individual perception's of candidates. Aldrich and McKelvey (1977) gave an estimation of the true candidate positions, by the positions each voter reports, for each candidate.

Assumption 4: Voter preferences are characterized by a density function $f(x)$. Candidate strategies are represented by points in the coordinate space of this density. The number of densities that might characterize the preferences is infinite. It is assumed that $f(x)$ is symmetric and unimodal.

This model makes some other, more complicated, assumptions about citizen's evaluation of candidate's strategies. For details see Hinich and Ordeshook (1970).

In order to clarify the above model we give a simple example. Voter i has to rank issues in a pre-specified interval scale. If we work in the two dimensional space and the voter i gives for the first issue the value x_i and for the second issue the value y_i , his position in the two dimensional space is given by the point with coordinates (x_i, y_i). Suppose that, alternative A is described by the point with coordinates (x_A, y_A) and B is described by the point (x_B, y_B). According to this model the voters chose the candidate whose issue position on the space is the closest to his issue position. The most common utility function for measuring the 'closeness' is the Euclidean distance d. Thus, voter's i distance from alternative A is given by $d_A = [(x_i - x_A)^2 + (y_i - y_A)^2]^{1/2}$ and his distance from B is given by $d_B = [(x_i - x_B)^2 + (y_i - y_B)^2]^{1/2}$. Furthermore the model assumes that the voter's utility for the candidates decreases linearly with the Euclidean distance d. Thus, if $d_A \geq d_B$ then i prefers B to A. Thus, the use of this special utility function (Loss function) assigns to the voter a single comparable measurement of the phycological distance between his point of view and the location of each alternative. In this model as well as in the uniform and in the normal assumption utilities are used only to determine preference order.

Chamberlin and Cohen (1978) used this classical approach to issue voting and generated voter and candidate positions in the space via simulation. Voters were represented in the four dimensional space. Thus, for each voter four numbers had to be simulated representing voter's position in the four dimensions-issues. The four numbers for voter i were generated in the following way:

- Voter i's position on the first dimension Y_{1i} was chosen from the standard Gaussian distribution with zero mean and standard deviation equal to 1.0

- Voter i's position on the second dimension Y_{2i} was generated from the position in the

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first dimension by perturbing it with Gaussian noise of zero mean and standard deviation equal to 1.4

- The third was produced from the second with fresh noise of similar character
- The fourth coordinate was generated from the third in the same fashion.
- The values on all dimensions were normalized in order to have unit variance

Also Merrill (1984), (1985) used the spatial model in his study so that both voters and candidates were generated via simulation from the multivariate normal distribution. Although the utility function that he used is the Euclidean distance, he suggested some other Loss functions like

a) *City block metric* given by $\sum |v_i - c_i|$, where $v = (v_1, v_2, ...v_\delta)$ and $v = (c_1, c_2, ...c_\delta)$ are the voter's and the candidate's positions, respectively and δ is the number of dimensions.

b) *Shelpsle utility function* given by $u(d) = exp(-d^2/2)$, where d is the Euclidean distance from voter to candidate. For more details see Merrill (1988) .

c) *Negative of distance* given by $u(d) = -d$, where d is the Euclidean distance from voter to candidate.

Spatial model has been studied a lot and many extensions of it has been found in the literature. For example, Rabinowitz and MacDonald (1989) gave an extension of the spatial model. Sustaining that the traditional spatial theory of elections is seriously flawed, introduced two new components of issues. Those are the direction and the intensity.

Instead of generating voter preferences for alternatives based on a distribution, we can generate voter preferences for specific issues, using the Spatial model. A hypothetical distribution that estimates the distribution of voter preferences on issues has to be used. This is, usually, the normal distribution, see, Chamberlin and Cohen (1978). The use of this distribution leads us to the conclusion that the Spatial model will not produce the 'appropriate' data. These are the data that can be thought as citizen's preferences for political parties and independent candidates, for the Greek electorate.

Consequently, we realize that none of the above methods would be useful in our study.

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4.5 The proposed data generation method

Due to the fact that the previous methods are not suitable for our study, we suggest to use real data taken from the latest Greek Parliamentary elections, and to simulate/generate a large number of other datasets by introducing noise (random error) in the initial dataset.

It would be far more interesting to study Greek Electorate in our days, and not its behavior in the past. So a first good step is to use the data from the latest Parliamentary Elections. These are the elections that took place in the year 1996. These data could be easily taken from the Ministry of Internal Affairs. This data set includes vote totals for each political party or independent candidate for each one of the 56 Lower districts. The magnitude for each one of the Lower districts is included. The total number of political parties and independent candidates is 32. In fact the greatest majority of them (26 of 32) obtained a total percentage of votes less than 1%. So it is useless to study each one of them separately. We presume that there are 6 important political parties. The first is P.A.S.O.K with a percentage of total votes in the entire state equal to 41.49%, the second is New Democracy with 38.12%, the third is Political Spring with 2.94%, the fourth is K.K.E. with 5,60%, the fifth is Synaspismos with 5.12, and the sixth is D.I.K.K.I with 4.44%. Despite its small percentage of total votes (2.94%), we include Political Spring in our study, because it is a percentage which allows Political Spring to enter the Parliament only when some of the electoral systems are used. The rest of the parties and the candidates took such a small number of votes that none of the systems under consideration can occupy a seat.

It is very well known that the political party, which governs, for a period of time, losses some voters due to political activities. These voters are divided to other political parties or independent candidates. The second most powerful political party usually increases its total number of votes. The above phenomenon characterizes the Greek electorate. Therefore, it is reasonable to examine the possibility of generating datasets according to the characteristic described above. We generate a large number of datasets with the use of the real dataset of total votes, of each political party, in each lower district, and by using the following procedure.

The total number of votes for the first political party, in each district, is eliminated according to a value which is taken from a Normal distribution with zero mean and variance which is related to the real number of votes of this party, in each district. The total number of votes,

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for the second political party, in each district, is increased according to a value which is taken from a Normal distribution with zero mean and variance which is related to the real number of votes of this party, in each district. For the rest of the political parties we change the total number of votes, in each district, by introducing noise in the initial number of votes. That is, these votes are taken from a Normal distribution with mean value equal to the real number of votes and variance based on the votes, of each political party, in each district. Therefore, the algorithm that we use is the following.

Let n be the real number of districts and m the real number of political parties. For each district i, $i = 1, ..., n$ do

- For the first political party, $m = 1$
  - take a value ε_i from the normal distribution $N(0, c_1 \Psi_{1,i})$
  - calculate the absolute value of ε_i
  - the new number of votes for the first political party, $m = 1$, for the i^{th} district is given by $\Psi_{new,1,i} = \Psi_{1,i} - abs(\varepsilon_i)$, where $\Psi_{1,i}$ is the real number of votes of the first political party for the i^{th} district, c_1 is a number which changes the number of votes for the first political party in each district by the percentage we want, and $\Psi_{new,1,i}$ is the generated number of votes for the first political party for the i^{th} district.
- For the second political party, $m = 2$
  - take a value ε_i from the normal distribution $N(0, c_2 \Psi_{1,i})$
  - calculate the absolute value of ε_i
  - the new number of votes for the second political party, $m = 2$, for the i^{th} district is given by $\Psi_{new,2,i} = \Psi_{2,i} + abs(\varepsilon_i)$, where $\Psi_{2,i}$ is the real number of votes of the second political party for the i^{th} district, c_2 is a number which changes the number of votes for the second political party in each district by the percentage we want, and $\Psi_{new,2,i}$ is the generated number of votes for the second political party for the i^{th} district.
- For the rest of the political parties, $m = 3, ..., 7$

- take a value ε_i from the normal distribution $N\left(\Psi_{m,i}, c_3\Psi_{m,i}\right)$
- the new number of votes for the rest political parties, $m \neq 1, 2$, for the i^{th} district is given by $\Psi_{new,m,i} = \varepsilon_i$, where $\Psi_{m,i}$ is the real number of votes of the rest political parties for the i^{th} district, c_3 is a number which changes the number of votes for the rest political parties in each district by the percentage we want, and $\Psi_{new,m,i}$ is the generated number of votes for these political parties for the i^{th} district.

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Chapter 5

Measures of disproportionality

In order to evaluate the analyzed Greek Parliamentary Electoral systems, we examine the concept of electoral disproportionality, performing some different operational measures that have been proposed in the literature. The measures of disproportionality, we use, are described in detail in Gallagher (1991) and Lijphart (1994). We, also, discuss the advantages and the disadvantages of these measures.

An important political consequence of the electoral systems is the effect on the proportionality or disproportionality of the electoral outcomes. Disproportionality means the deviation of the parties' seats shares from their votes shares. Perfect proportionality is the situation in which every party receives exactly the same share of seats with the share of votes it receives. We will use measures of disproportionality in order to evaluate the Greek electoral systems. These systems can be included in the category of PR systems, which means that they try to minimize the disproportionality and to produce an outcome that is close to perfect proportionality, as possible. It is obvious that, although these systems 'seek' for proportional results, the situation of perfect proportionality is impossible. Some systems achieve more proportional results than other systems. For this purpose, we study the concepts of proportionality and disproportionality. Although, they seem to be simple concepts, the question of finding the best way to measure the proportionality or the disproportionality is much more difficult. All these measures have the same point of departure: they begin by noting the differences between the percentages of seats and the percentages of votes receiving by each alternative (political party or independent candidate). They differ on the way that seat and vote deviations are aggregated. When we use measures of proportionality we seek for large values of the measure, in order to obtain the fairest (the most proportional) system. When we use measures of disproportionality we seek for small values of the measure, in order to obtain the fairest system. In fact, measures of proportionality and disproportionality are two sides of exactly the same coin. We will generally use the term 'measures of disproportionality' because the values of all these indices increase when the disproportionality increases. Therefore, the indices of disproportionality that we will present in this chapter are alternative ways of measuring the same phenomenon. However, we have to take into account the fact that although PR systems 'seek' for proportional results, the notion proportionality or disproportionality is not always the same for the different electoral systems. That is why Gallagher (1991) say that every method of seat allocation generates its own measure of disproportionality, and that many measures of disproportionality implicitly endorse a method of seat allocation.

The above phenomenon consists the main objection that has been raised to the entire family of the disproportionality measures. However, this is a serious problem only if one focuses on the different outcomes exclusively at the district level. As Cox and Shugart (1991) concede 'whether national seat totals will be proportional to national vote totals depends on many factors - such as additional seats, thresholds, malapportionment, and the geographical distribution of party support - in addition to the formula used to allocate seats within districts'.

In case of PR systems there are two broad categories of measures of disproportionality, corresponding to the two main types of allocation methods, which we have already discussed. The first category of measures concentrates on the absolute difference between parties' seats and votes as the Largest Remainders methods do. Methods in the second category focus on the ratio between parties' seats and votes, just as the Highest Averages methods do. We perform eight measures of disproportionality, which have been proposed in the literature. The first five indices (Rae's index, L-H, LSq, L-H adj., S-L) belong to the first category, the Lijphart's index and d' Hondt index belong to the second category, while the Regression index, is a satisfactory measure of big parties bias.

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5.1 Rae's index

It is the oldest measure of disproportionality and has been proposed by Rae (1967). It uses the average of the deviations. In fact, it sums the absolute differences between vote percentages (v_i) and seat percentages (s_i) and the outcome is divided by the number of the political parties (n):

$$I = \frac{1}{n}\sum |v_i - s_i|$$

The problem with this index is, that it is sensitive to the presence of small parties. Because of the presence of small parties this index underestimates the real disproportionality measure of the systems. In order to make this characteristic more clear we present an example. Suppose that there are only two political parties. Party A with total percentage of votes equal to 69,96 and Party B with total percentage of votes equal to 30,04. Suppose also that, the total percentage of seat shares is equal to 53 for the first party and the total percentage of seat shares is equal to 25 for the second party. In this case (case (a)) the index proposed by Rae is $I = 7,5$. The existence of one additional small party (case (b)) with total percentage of votes equal to 1 and total percentage of seats in the parliament equal to zero, reduces the Rae's index to $I = 5,3$. Therefore, there is a notable decrease in this index, which means more proportional results, because of just a very small party. Furthermore the existence of additional small parties with small percentage of votes and no seats in the parliament causes an additional decrease of the Rae index. This is the case in Greek parliamentary elections, as there are many small parties which do not gain seats in the parliament. Rae's index has the tendency to underestimate the disproportionality of PR systems with many small parties. The difference in the value of the Rae index in cases (a), (b) is quite large, although the real difference between the two cases is due to the existence of only one small political party. Therefore, this index has the tendency to give more proportional results. Rae in order to avoid this problem excludes from the study the small parties. For this purpose, he uses a cutoff point usually 5% of votes. Also, he considers all small parties as 'other' in the election statistics.

This index is trying to measure the total disproportionality per election. As an overall measure of disproportionality it is flawed since a plethora of small parties, each of whose total votes exceeds Rae's cutoff point, will bring down the value to an artificial level.

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5.2 Loosemore-Hanby's index

An index that avoids Rae's index disadvantage was proposed by Loosemore and Hanby (1971). Loosemore Hanby's index has become the most widely used measure of disproportionality. This index (L-H) is given by the sum of the absolute differences between vote percentages (v_i) and seat percentages (s_i), as it happens in the case of Rae's Index I, but now the sum is divided by 2 instead of the number (n) of the political parties. Thus, it is given by

$$D = \frac{1}{2}\sum |v_i - s_i| .$$

Mackie and Rose (1982, 1991) subtracted D from 100 and called the result the index of proportionality.

It is obvious that, except from the case of the two parties system ($n_i = 2$), where we take the same result with both measures (Rae and L-H), L-H index gives higher values than the Rae's index.

It is

$$D = \frac{1}{2}|v_i - s_i| = \frac{n}{2}\frac{1}{n}|v_i - s_i| = \frac{n}{2}(\frac{1}{n}|v_i - s_i|) = \frac{n}{2}I \Longrightarrow D = \frac{n}{2}I$$

For $n > 2$, we have that $D > I$. Thus, the Loosemore-Hanby index will always yield higher values than the Rae's Index. In the previous example the difference between the two cases $(a), (b)$ is represented more satisfactory as the index goes up. The advantage of this index is that it does not have to disaggregate the 'other' small parties as in the case of Rae's Index. In contrast with Rae's Index, this index is trying to measure the total disproportionality per party.

L-H index may lead to other paradoxes. Suppose that there are 90 voters, 2 seats and two parties, A and B, each one received 68 and 22 votes respectively. When the L.R.-Hare system is applied the two seats are awarded to party A, because the Hare quota is equal to 45. When the Sainte Lague system is applied again the two seats are awarded to party A. In both cases the L-H index is equal to 0,25. Suppose that, a third party C joins the fray and wins 10 uncast votes, which means that the distribution of the seats becomes 68-22-10. When the L.R.-Hare system is applied the first seat is awarded to party A and the second is awarded to party B. In

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case of Sainte Lague the distribution of the seats remain 2-0. L-H index is equal to 0,3 in the case of the L.R.-Hare system and 0,32 in the case of Sainte Lague system. In this example the L-H index indicates that 2-0 is the least disproportional allocation in case of 90 votes, but 1-1 is least disproportional in case of 100 votes. This index always, by definition, slavishly follows the Largest Remainders method.

Although this method is easy to understand, it is weakened by its vulnerability to paradoxes. These and other doubts have lead to the development of other difference-based indices.

5.3 Least Square index

Although, there is a good idea behind the Rae's proposal, as its rational is that the vote-seat differences are not on its own enough to convey reliable information of the proportionality of an election outcome, we want to know more about how this sum was reached. Does it derive from many parties each having a small vote-seat difference or from a few parties each having a large difference? Gallagher (1991) gave the solution with the introduction of the least squares measure. The key feature of this index is that registers a few large deviations much more strongly than a lot of small one's.

In order to make the above problem more clear we present the following example. Consider two elections (a) and (b). In election (a) there are only two parties: the first wins 60% of votes and 64% of seats and the second 40% of votes and 36% of seats. In election (b), there are eight parties: four win 15% of votes and 16% of seats, while the other four win 10% of votes and 9% of seats. According to the Loosemore and Hanby's index these two elections are equally disproportional as in both cases the index is equal to 4. Thus, in this case this index is insensitive to the number of parties. Rae's measure gives the first outcome less proportional ($I = 4$) with respect to the second outcome ($I = 1$). In order to take into account the Rae's idea without encountering the above problem Gallagher (1991) offered the following solution: the method of least squares. It is widely used in the social sciences, for example, in fitting a least square regression line to a set of data. A least square index would entail squaring the vote-seat difference for each party, adding these values, dividing the sum by two and taking its square root:

$$LSq = \sqrt{\frac{1}{2}\sum (v_i - s_i)^2}$$

This gives an index that measures disproportionality per election rather than per party and runs from 0 to 100. Another way of thinking about what this index does is that it weights the deviations by their own values, making the larger deviations account for a great deal, than small deviations. In case of only 2 parties this index yields exactly the same values as the Rae and Loosemore-Hanby indices. In other cases it gives a medium value between these two measures. Lijphart (1994) characterizes this index as 'the most faithful reflection of disproportionality of election results'. Gallagher (1991) describes this fact is as 'a happy medium'. This is clear from the results of the three hypothetical situations (see, Lijphart (1994)) presented in Table 8.

situation	n_i	v_i	s_i	**Index**	
A	1	55	60	**I**	5
	1	45	40	**D**	5
				LSq	5
B	1	50	55	I	1,67
	1	40	45	D	10
	10	1	0	LSq	5,48
C	5	15	16	I	1
	5	5	4	D	5
				LSq	2,24

Table 8: The Rae (I), the L-H (D) and the Least Square (LSq) index for different values of n_i (number of political parties), v_i (number of total votes), and s_i (number of total seats).

Situation A: existence of only two parties, in this case all of the three indices take the same value. Situation B: existence of many small parties with no seats. In this case Rae's

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disproportional index is low indicating that the system is proportional and Loosemore-Hanby's index is high indicating that the system is disproportional. Least Square Index takes a value almost in the middle of those two. Analogous results can be seen also in situation C.

5.4 Adjusted Loosemore-Hanby index (Grofman's index)

It was suggested by Groffman (1985) and is another effort to find a middle course between Rae's and Loosemore-Hanby's indices. The difference from the Rae index is that it divides the total amount of disproportionality by the effective number of parties (N) rather than the real number of parties (n). The effective number of parties weights the parties by their relative sizes and almost always takes a value between 2 and the raw number of parties. It can be calculated either on bases of vote share: $N_v = 1/\sum v_i^2$ or seats shares: $N_s = 1/\sum s_i^2$. In order to clarify the concept of the effective number (N) of parties, we give N in the form: $N = \frac{1}{1-F}$, where F can be based on both parties' vote shares (F_v) and parties' seat shares. F_v is equal to $F_v = 1 - \sum v_i^2$, and F_s is equal to $F_s = 1 - \sum s_i^2$. F_v represents the frequency with which pairs of voters would disagree on their choice of parties if an entire electorate interacted randomly. For more details on the effective number of parties see, Laakso and Taagepera (1979).

Consequently, L-H adj. is given by

$$
\begin{aligned}
\frac{1}{N}\sum |v_i - s_i| &= \frac{1}{\frac{1}{1-F}}\sum |v_i - s_i| \\
&= (1-F)\sum |v_i - s_i| \\
&= \left\langle \begin{array}{l} [1-(1-\sum v_i^2)]\sum |v_i - s_i| \\ [1-(1-\sum s_i^2)]\sum |v_i - s_i| \end{array} \right. \\
&= \left\langle \begin{array}{l} \sum v_i^2 \sum |v_i - s_i| \\ \sum s_i^2 \sum |v_i - s_i| \end{array} \right.
\end{aligned}
$$

Like Rae's index, the L-H adj. index measures the amount of disproportionality per party rather than per election. It is an improvement on Rae's index, but is more complicated to calculate rather than the least square index. Also, it does not have the property of the least square index of 'penalizing' a few large disproportionalities more than a host of small ones.

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5.5 Lijphart's index

Lijphart (1994), in addition to the above well-known indices, introduces in his study another index. He simply uses the largest deviation in an election result as an overall index of disproportionality. This largest deviation comes from the percentage of overrepresentation of the most overrepresented party, and this is usually one of the largest parties. Thus, the index is given by

$$\max |v_i - s_i|$$

Where v_i is the total percentage of votes for the most over-represented party and s_i the total percentage of seats obtained by the most over-represented party. The minimum value of the measure is equal to zero and it happens in case of perfect proportionality of the most overrepresented party. Lijphart (1994) argues that the beauty of this index is that it not only makes good sense but it is also the simplest possible way of measuring disproportionality. The idea come from the following fact: The discrepancy between Rae's and Loosemore-Hanby indices can be alleviated by averaging the vote-seat share differences of the larger parties only. For example, we can exclude from the study the parties which win less than 5 or 10 percent of the vote. In order to be able to apply this measure in different elections and in both two-party and multi-party systems, he uses the two largest parties. Then he took this line of reasoning one step furthermore, this step was simply to use the largest deviation, in an election result, as the overall index of disproportionality.

5.6 Sainte-Lague index

It belongs to the set of indices that focus not on the absolute differences between votes and seats for each party, but on the parties' seats and votes ratio, as the highest averages methods do. The relationship between ratio measures and highest averages electoral formulas can be illustrated by performing the concept of disproportionality that Sainte-Lague formula tries to minimize, and it is defined in the following way. For each party calculate the difference $s_i/v_i - TS/TV$, where v_i is the total number of votes of the i^{th} party, s_i is the number of seats received by

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the i^{th} party, TV, is the total number of votes and TS the total number of seats. Then the difference is squared and the resulting square is weighted by the size of the party. If TV and TS are expressed in percentages, the error term for each party equals the $v_i(\frac{s_i}{v_i} - 1)^2$.

$$\text{It is } v_i(\frac{s_i}{v_i} - 1)^2 = \frac{v_i^2}{v_i}(\frac{s_i}{v_i} - 1)^2 = \frac{1}{v_i}[v_i(\frac{s_i}{v_i} - 1)]^2 = \frac{1}{v_i}(s_i - v_i)^2$$

The index involves simply adding the error terms for each party. Thus, the index is given by

$$\sum \frac{1}{v_i}(s_i - v_i)^2,$$

where v_i is the total percentage of votes for the i^{th} party and s_i the total percentage of seats for the i^{th} party. It differs from the previous indices (Rae, L-H and Lijphart) in that it uses the *relative* difference between parties' shares of seats and votes and not the *absolute* difference. Sainte Lague index is a measure whose minimum value is zero, in case of full proportionality, and whose maximum value is infinity, when a party with no votes somehow wins a seat. The open ended nature of the range of this index makes it less easily interpreted.

5.7 D'Hondt index

As we have already mentioned in chapter 2, the aim of the d'Hondt formula is to keep to a minimum the overrepresentation of the most over-represented party. Consequently, if there was to be a d'Hondt index, it would have to be simply this: The seats percentage to votes percentage ratio of the most over-represented party. Thus, it is given by

$$\max(s_i \,/\, v_i)\,,$$

where v_i is the total percentage of votes for the over-represented party and s_i the total percentage of seats obtained by the most over-represented party. The minimum value is 1 in case of the ideal proportionality, and the maximum is the infinity, attained if a party with no votes somehow wins a seat. The disadvantage of this index is that it gives unreliable results in the case where a small party gains some degree of overrepresentation. For example, at Italy's 1983

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general election, a small party (Val d'Aost Union) with 0,076 per cent of votes won 0,159 per cent of the parliamentary seats. The d'Hondt index is equal to 2,085, a value that yields to this election more disproportionality than the real one. However, in most cases the party in question is the largest one and thus the index gives reliable results. If the most overrepresented party is a small one we can refine the index using a cutoff point, e.g. 5% or 10%.

This index has been used by Laakso and Taagepera (1980) in constructing 'proportionality profiles' of the results produced by the electoral systems. It has also been used by Katz (1984) as a measure of disproportionality.

5.8 Regression index

Cox and Shugart (1991) argued that it should be focus attention on the 'political character' of disproportionality. That is, the extend to which different methods of PR systems favor the large parties over the small ones. However, the development of a satisfactory measure of big parties' bias turned out to be very hard. Cox and Shugart (1991) offer an intriguing proposal. They regress the parties' seat percentages on the vote percentages. The slope (b) of the regression line (the regression coefficient) provides a simple index of the big parties' bias. Suppose that, the x_i represents the vote percentages and the y_i represents the seat percentages. Then, the regression index is given by

$$b = \frac{\sum x_i \, y_i}{\sum x_i^2}$$

We use a regression line with zero intercept because parties with zero percentages of votes obtain no seats. For more details, on Simple Linear Regression, see, Panaretos (1994). Suppose, for example the regression lines of the following figure.

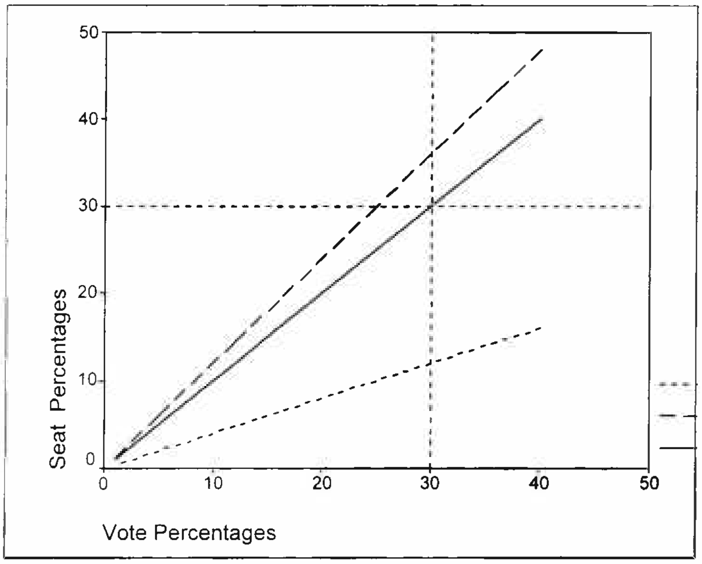

Figure 3: Regression lines of vote percentages on seat percentages

The regression line with slope equal to one indicates an absence of bias, because the ratio of vote percentages to seat percentages is equal to one, for all parties. This case is represented by the second line, where the ratio of vote percentages to seat percentages is equal to one ($\frac{\%v_i}{\%s_i} = 1$). The first line, where the slope is greater than one ($b > 1$) indicates a systematic advantage for larger parties, as the ratio of seat percentages to vote percentages is greater than one ($\frac{\%s_i}{\%v_i} > 1$), which means seat percentages are greater than vote percentages. Furthermore, as we move from small values of the vote percentages, which represent the small parties, to greater values, which represent larger parties, more seats are given with respect to votes. That is why this line indicates large parties bias. In the same sense the line with slope smaller than one ($b < 1$) indicates a small parties bias.

Chapter 6

Application

6.1 Application with real dataset

As we have already mentioned, we will use the data of the latest Greek Parliamentary Elections which took place on the 22th of September in 1996. 32 parties competed, included single parties, cartels of two or more parties and also independent candidates. The total number of valid votes was 6.738.445. Our purpose is to find out what would have happened, with respect to vote shares, if other systems have been applied to these data.

Five computer programs have been set up for each one of the five electoral systems studied in chapter 3. These computer programs take as an input the valid votes for each party and for each lower district, and give as an output the number of the seats obtained from each party. In fact the input file is an $m \times n$ matrix of valid votes, where m is the number of the parties and n is the number of the lower districts. In our study the number of the lower districts is $n = 56$, while the number of the parties that competed in the 1996 elections is $m = 32$. In fact 26 of the 32 parties obtained total percentage of valid votes less than 1% each. We found impractical to study each one of them separately, as such a small percentage does not allow to the small political parties to obtain seats in the Parliament. We included Political Spring in our analysis despite its small percentage (2,94%), because it can gain seats when some of the analysed systems are used, while with other it can not (e.g. the 1996 system). The problem was how to treat the 26 small parties: to ignore them completely or to treat them as 'other' party. The input $m \times n$ matrix consists of the total valid votes, for each party and for each

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lower district, which means that we have to use the valid votes for *all* the parties which take part in the elections. This is very important because the quota that is used in the primary distribution of seats, in all systems studied, uses the total number of valid votes for all the parties. Thus, the complete exclusion of some parties will lead to wrong results. In order to solve this problem, we treated the 26 small parties as 'other', which means that we treat these parties as one party, the 7th party. In that case the input matrix consists of 7 rows. The first 6 correspond to the first 6 parties and the last one to 'other'. The votes of the 26 parties were summed in each lower district. The sum of the votes, of all these parties, gave a percentage of 2,29%. This is a percentage which might allow to 'other' to obtain seats, while in fact no one of them would obtain seats. For this reason the computer programs was updated such that the seventh party never obtain seats.

In order to analyse the electoral systems we have to specify if each one of the competitors is an independent candidate, a single party or a cartel of two or more parties. In the elections studied, the 1996 Parliamentary Elections, all of the six parties under consideration are characterized as single parties.

Each formula also needs the district magnitude $E[l]$ of each lower district l, in order to compute the parties seats shares. The district magnitude, for the first 30 lower districts is given below:

$E[1] = 19$	$E[2] = 38$	$E[3] = 7$	$E[4] = 8$	$E[5] = 9$	$E[6] = 4$
$E[7] = 6$	$E[8] = 6$	$E[9] = 2$	$E[10] = 3$	$E[11] = 4$	$E[12] = 4$
$E[13] = 3$	$E[14] = 6$	$E[15] = 8$	$E[16] = 9$	$E[17] = 1$	$E[18] = 1$
$E[19] = 6$	$E[20] = 1$	$E[21] = 3$	$E[22] = 2$	$E[23] = 5$	$E[24] = 3$
$E[25] = 1$	$E[26] = 2$	$E[27] = 1$	$E[28] = 5$	$E[29] = 5$	$E[30] = 8$

The secondary distribution of seats is done in k major districts. The lower districts are aggregated and k major districts are produced. From 1974 up to 1985 nine major districts was used, while from 1989 up to now thirteen major districts are used. In order to investigate significant differences between these two distinctions we 'run' each system twice. One for $k = 9$ and one for $k = 13$. The seats that would have been distributed to parties, for each system,

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using 13 major districts, for the 1996 election results is given in Table 9. The seats that would have been distributed to parties using 9a major districts are presented in Table 9b.

Table 9a: The seats that would have been distributed to parties ,for each system, and for 13 major districts, for the 1996 election results.

System	1st party	2nd party	3rd party	4rth party	5th party	6th party
1974	152	137	1	4	4	2
1977 (1981)	156	133	1	4	4	2
1985	164	125	1	4	4	2
1989	136	118	11	12	12	11
1993 (1996)	164	106	0	11	10	9

Table 9**b:** The seats that would have been distributed to parties , for each system, and for 9 major districts, for the 1996 election results.

System	1st party	2nd party	3rd party	4rth party	5th party	6th party
1974	154	135	1	4	4	2
1977 (1981)	153	136	1	4	4	2
1985	161	127	1	4	5	2
1989	136	117	11	13	12	11
1993 (1996)	160	110	0	11	10	9

We notice that, the allocation of the seats to the parties differs a lot when a different system is applied. As we can notice from the Table 9a the first party takes 152 parliamentary seats

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when the 1974 system is used and 164 when the 1993 system is used. Therefore, there is a difference of 12 parliamentary seats, which is a significant number. As we see from Table 9b the first party takes 154 seats with the 1974 system and 160 seats with the 1993 system, when 9 major districts are used. The number of the seats that are distributed to the second party, according to the different systems, varies more than the first one. The second party takes much more less seats with the 1993 system than with any one of the other systems, when we use k=9 or k=13 major districts. The small parties, which are the third, the fourth, the fifth, and the sixth take the same number of seats with the first three systems, the 1974, the 1981 and the 1985. The system of the 1889 gives them much more seats in both cases, see Tables 9a and 9b. We also present these results graphically. Figures 4a and 4b summarize the results, with respect to seat shares, for the 'big' parties which are the first two parties. Figures 5a and 5b summarize the results, with respect to seat shares, for the 'small' parties which are the rest parties.

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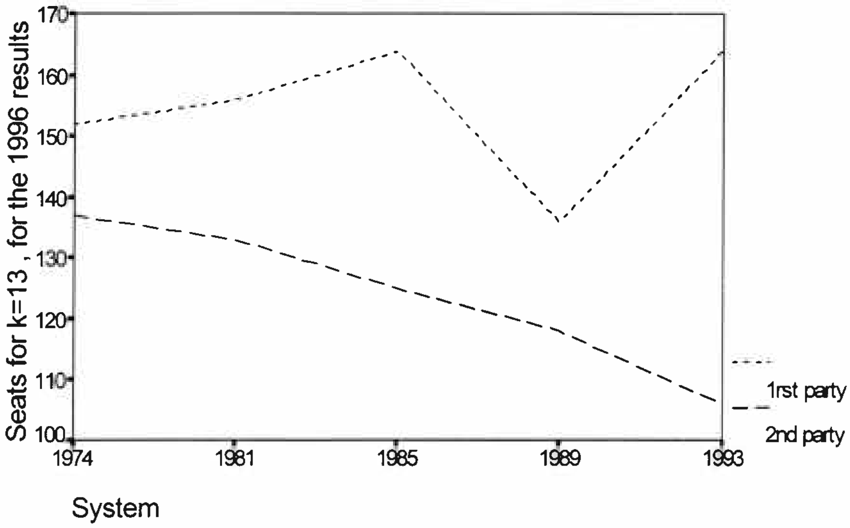

Figure 4a: The seats that would have been obtained by the first two parties, when each one of the studied systems has been applied, and if 13 major districts have been used.

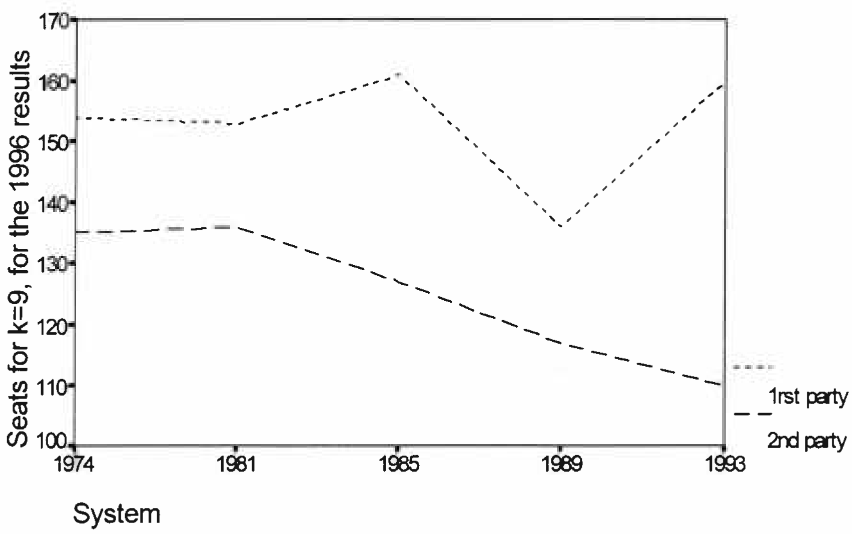

Figure 4b: The seats that would have been obtained by the first two parties, when each one of the studied systems has been applied, and if 9 major districts have been used.

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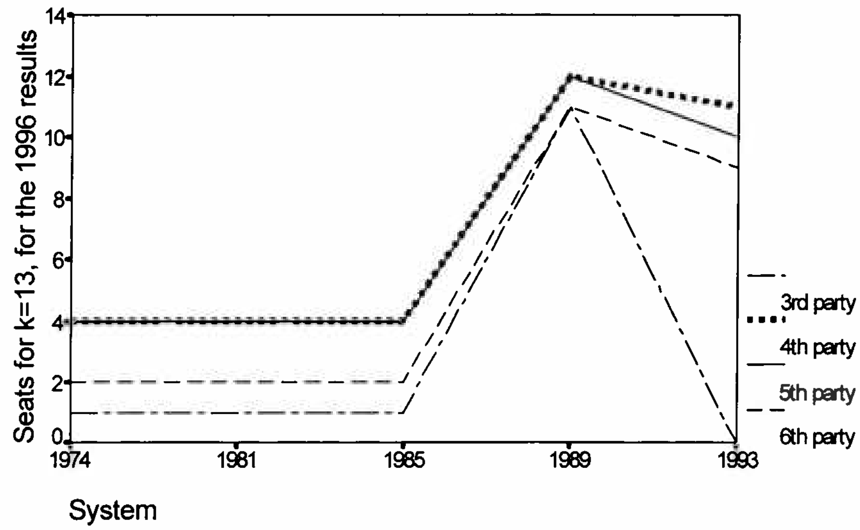

Figure 5a: The seats that would have been obtained by the 'small' parties, when each one of the studied systems has been applied, and if 13 major districts have been used.

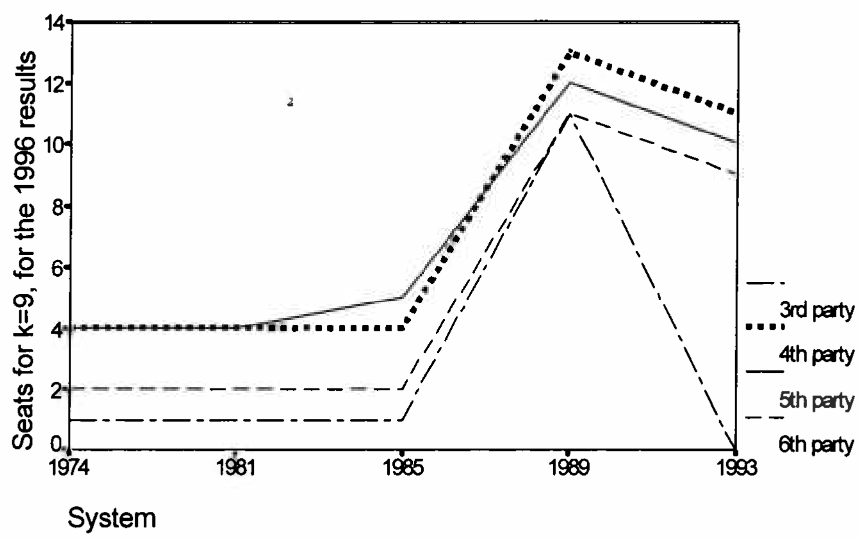

Figure 5b: The seats that would have been obtained by the 'small' parties, when each one of the studied systems has been applied, and if 9 major districts have been used.

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Figures 4a and 4b show that in both cases (k=9 and k=13) there is an important variation in the number of seats that each one of the first two parties obtains, when the different systems are used. In both cases the first party is enforced with the 1985 and the 1993 system. The second party takes fewer seats as we move from the oldest to the newest systems, in both cases. A first look at the two graphs shows no significant differences between the two cases studied, the one with the 13 major districts and the one with the 9 major districts. This will be studied in detail in the next section.

Analogous results for the 'small' parties are shown in Figures 5a and 5b. It is obvious that in both cases the 1989 system is the one that enforces more the 'small' parties. We also notice that the third party takes no seats with the 1993 system while with the other systems gains a notable number of seats and especially in the case of the 1989 system. Again a simple look at the two graphs shows no significant differences between the two cases studied, the one with the 13 major districts and the one with the 9 major districts.

Also we notice the first party obtains the majority of the parliamentary seats (more than 150 seats) when the 1974, the 1981, the 1985 and the 1993 system are used. All of them are different forms of the reinforced PR systems. Only the 1989 system gives different results. It does not provide the majority of the parliamentary seats to the first party for both k=9 (Table 9b) and k=13 (Table 9a).

In order to evaluate and compare the electoral systems eight well-known measures of disproportionality will be used. We will now present these indices evaluated for the seats distributed to each one of the six parties, according the analysed systems. We will again present the results for both k=9 and k=13, as we did before with the seat shares.

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Table 10a: The measures of disproportionality evaluated for the seats obtained by each party, according to the analysed systems (13 major districts), for the 1996 election results.

	1974	1981	1895	1989	1993
Rae index	0,0519	0,0519	0,0519	0,0154	0,0401
LH	0,1557	0,1557	0,1557	0,0463	0,1203
LSq	0,0986	0,1006	0,1094	0,0325	0,0997
LH-adjusted	0,3115	0,3115	0,3115	0,0926	0,1873
Lijphart	0,0917	0,1050	0,1317	0,0383	0,1317
Sainte-Lague	0,1509	0,1524	0,1608	0,0140	0,0908
d'Hondt	0,2210	1,2531	1,3174	1,2455	0,3174
Regression	1,1841	1,1855	1,1882	1,0580	1,2442

Table 10b: The measures of disproportionality evaluated for the seats obtained by each party, according to the analysed systems (9 major districts), for the 1996 election results.

	1974	1981	1985	1989	1993
Rae index	0,0519	0,0519	0,0508	0,0143	0,356
LH	0,1557	0,1557	0,1524	0,0429	0,1069
LSq	0,0994	0,0989	0,1041	0,0312	0,0894
LH-adjusted	0,3115	0,3115	0,3048	0,0859	0,2139
Lijphart	0,0983	0,0950	0,1217	0,0383	0,1138
Sainte-Lague	0,1514	0,1511	0,1513	0,0121	0,0813
d'Hondt	1,2371	1,2290	1,2933	1,2455	1,2853
Regression	1,1848	1,1844	1,1838	1,0547	1,1228

The measures of disproportionality[1] , presented in Table 10a, give us some interesting results: a) All indices except d'Hondt give almost the same result. The 1989 system seems to be the

[1]The interpretation of the results is given in the second part of this Chapter (paragraph 6.2) where the results of 10 additional generated datasets are are performed.

most proportional. It gives significant smallest values of these indices when it is compared with the other systems. b) The d' Hondt index shows that the 1993 is the system which provides the smallest overrepresentation of the most overrepresented party. In other words, the values of the d'Hondt index correspond to the most overrepresented party, and the smallest overrepresentation appears in case of 1993. c) After the 1989 system the most proportional is the 1993 system. d) The other three systems seem to give similar results. We notice also that the indices Rae, LH, LH-adj. and Lijphart give the same value for two or more systems. This happens because the computations are based only on one dataset. We will see, in the next section, that this does not happen when we use 10 datasets. e) A regression coefficient greater than 1 shows that all systems favor the big parties. Less biased in favor of large parties is given by the 1989 system and the next less biased in favor of large parties is the 1993 system. The other three systems do not differ with respect to the regression index. However, as we move from older to newer systems (1974, 1981 and 1985) more bias is given in favor of large parties. f) We notice no significant differences in the results with respect to seat shares and to indices, when 9 major districts are applied. All these results are illustrated in the following figures. Figure 6a presents the indices for k=9 and Figure 6b presents the indices for k=13. All the above mentioned results are presented in Kalogirou and Panaretos (1999).

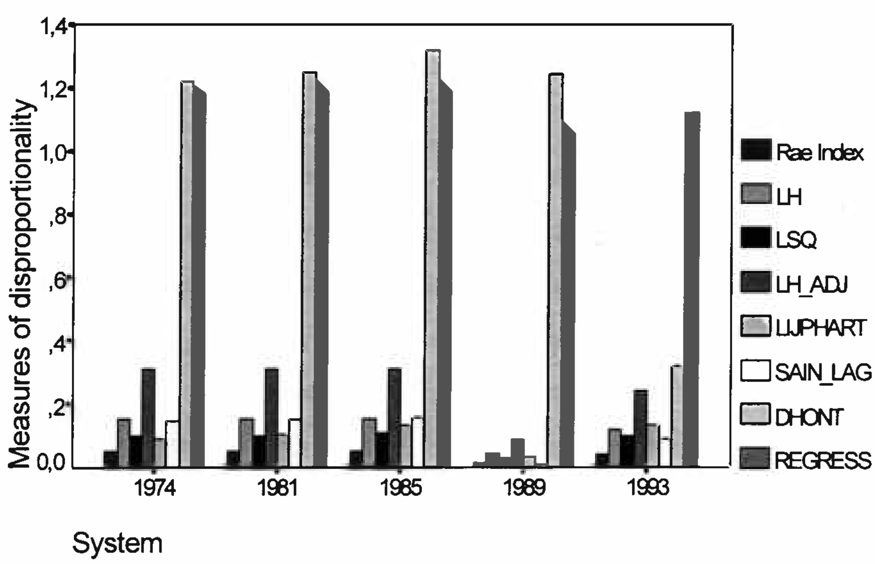

Figure 6a: The eight measures of disproportionality for the five electoral systems, for 13 major

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districts, and for the 1996 election results

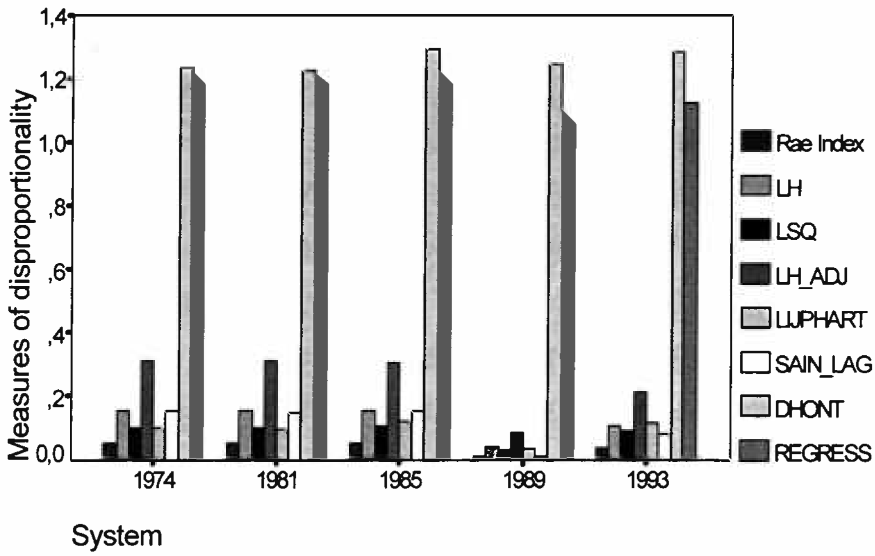

Figure 6b: The eight measures of disproportionality for the five electoral systems, for 9 major districts, and for the 1996 election results

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6.2 Application with generated data sets

In order to confirm, the results taken from only *one* dataset, the one of the 1996 Greek Parliamentary Elections, we use a large number of generated datasets. In this way we find the allocation of the seats, according to each system when other possible datasets have been considered for the elections. We can see the sensitivity of the systems analysed by noticing the allocation of the seats to each party and the sensitivity of the measures of disproportionality we use.

The way that the new datasets are generated has been described in details in section 4.4. We applied the proposed data generation method for $c_1 = c_2 = 350$. We generated values from the Normal distribution $N(0, 350\Psi_{m,i})$, for the first party ($m = 1$) and the second party ($m = 2$), for each lower district i. Lets say these values $\varepsilon_{m,i}$. The $\varepsilon_{m,i}$ differs for each party in each lower district, and it depends on the party's votes in this district ($\Psi_{m,i}$). We have tried different values for c_1 and finally we choose the value of 350, because it is a value that permits different election results, without producing significant differences in the total number of the valid votes, in each lower district, in the generated datasets. This is what we wanted to achieve: datasets based on the initial real dataset,which produce different election results, with respect to seat shares, but not very significant differences with respect to vote shares. This permits us to use, to the generated datasets, the same values for the lower district magnitudes with the real dataset. In fact the computation of the district magnitude is based on the vote totals.[2]Furthermore, we noticed that the value of 350 produces datasets that sometimes permit absolute majority of seats, for the first party, and sometimes not. Thus, it will help us to investigate which systems favor the absolute majority of the first party and which systems not. The parameter c_i determines the variation in the vote totals in the generated datasets, for each district and for each party. For this reason a smaller variation in vote totals was selected for the smaller parties. For the small parties we choose $c_3 = 100$.

It is obvious that the number of the parties is again seven with the seventh party being the

[2]The district magnitude is computed according to the following procedure: the legal number of the state population is divided by the total number of the parliamentary seats, which is equal to 300. This quotitient is called National Divisor. The legal number of the state population consists of all the people that are registered to the municipal rolls and not only the people that vote. Then the legal number of the state population is divided by the National Divisor. The new quotitient is the district mangitude for each lower district.

'other'. Furthermore, the number of the lower districts is again 56, and again 9 and 13 major districts are considered.

We have generated 10 datasets which contain the valid votes for each one of the six parties and for each lower district. For each dataset we have computed the number of the seats that each party obtains when each one of the five electoral systems, under consideration, is used. The mean value and the standard deviation of the number of the parties' seats, for each system, and for all these datasets, are given bellow.

Table 11: Mean values and standard deviations (in parenthesis) of the seats, for each party, and for each system, for the generated datasets.

System	1st party	2nd party	3rd party	4rth party	5th party	6th party
1974	139.8	150.6	0.6	3.90	4.10	2.0
	(4.64)	(2.46)	(0.5)	(0.31)	(0.31)	(0.0)
1977 (1981)	139.3	149.10	0.7	4.30	4.50	2.10
	(2.11)	(2.63)	(0.47)	(0.47)	(0.51)	(0.31)
1985	130.9	156.05	0.7	4.90	5.20	2.10
	(2.45)	(3.02)	(0.47)	(0.64)	(0.89)	(0.31)
1989	121.7	131.4	10.75	12.25	12	10.30
	(2.25)	(5.33)	(1.29)	(1.62)	(1.62)	(1.03)
1993 (1996)	112.20	156.20	1.20	11.20	10.20	9
	(3.78)	(3.14)	(2.46)	(0.41)	(0.41)	(0.0)

In order to find the fairest system we computed the eight measures of disproportionality, for each dataset and for each one of the electoral systems under consideration. Thus, for each one of the electoral systems there are $8 \times 10 = 80$ values of indices. It is obvious that such a large number of values are difficult to interpret. For this reason we use the mean values, and the standard deviations of the measures, for the 10 datasets. The results are illustrated in the Table 12.

Table 12: Mean values and standard deviations (in parenthesis) of the indices for each system, when 13 major districts are used.

	1974	**1981**	**1985**	**1989**	**1993**
Rae index	0,0524 (0,0010)	0,0515 (0,012)	0,0502 (0,0008)	0,0149 (0,0033)	0,0359 (0,0027)
LH	0,1521 (0,0180)	0,1548 (0,0035)	0,1507 (0,0026)	0,0450 (0,0101)	0,1173 (0,0254)
LSq	0,1008 (0,0017)	0,0983 (0,0024)	0,1023 (0,0039)	0,0301 (0,0057)	0,0903 (0,0064)
LH-adjusted	0,3163 (0,0047)	0,3096 (0,0071)	0,3016 (0,0052)	0,0895 (0,0205)	0,2159 (0,0168)
Lijphart	0,0992 (0,0556)	0,0929 (0,0065)	0,1182 (0,0085)	0,0308 (0,0068)	0,1192 (0,0084)
Sainte-Lague	0,1564 (0,0048)	0,1495 (0,0075)	0,1482 (0,0075)	0,0154 (0,0058)	0,0780 (0,0112)
d'Hondt	1,1447 (0,3168)	1,1296 (0,3239)	1,0916 (0,04151	1,3016 (0,1454)	1,1940 (0,3083)
Regression	1,1866 (0,0028)	1,1832 (0,0044)	1,1799 (0,0033)	1,0537 (0,0122)	1,1214 (0,0253)

We can visually see each one of the above indices in the following graphs:

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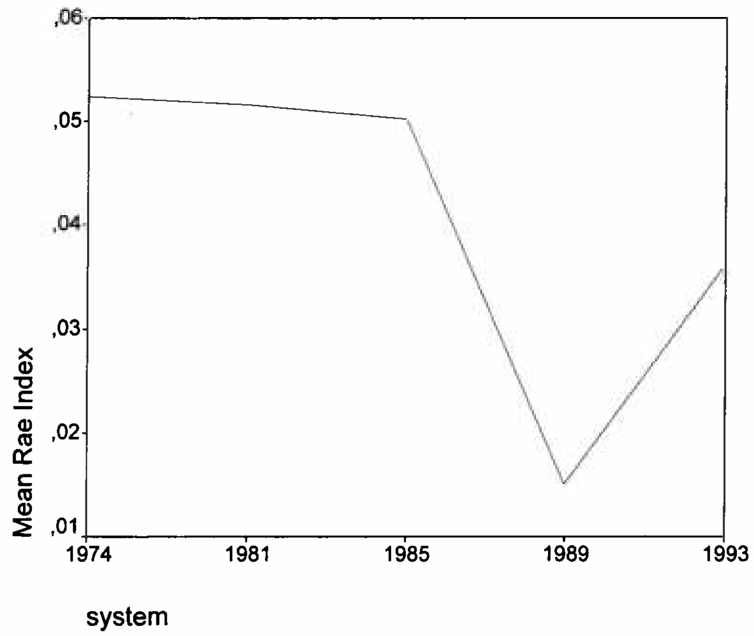

Figure 7.1: The mean value of the Rae index for the 10 datasets, for k=13.

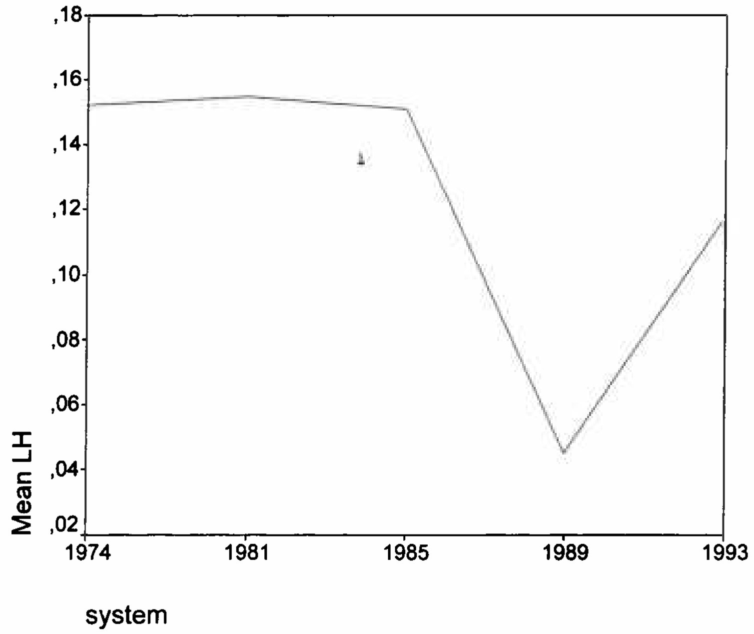

Figure 7.2: The mean value of the LH index for the 10 datasets, for k=13.

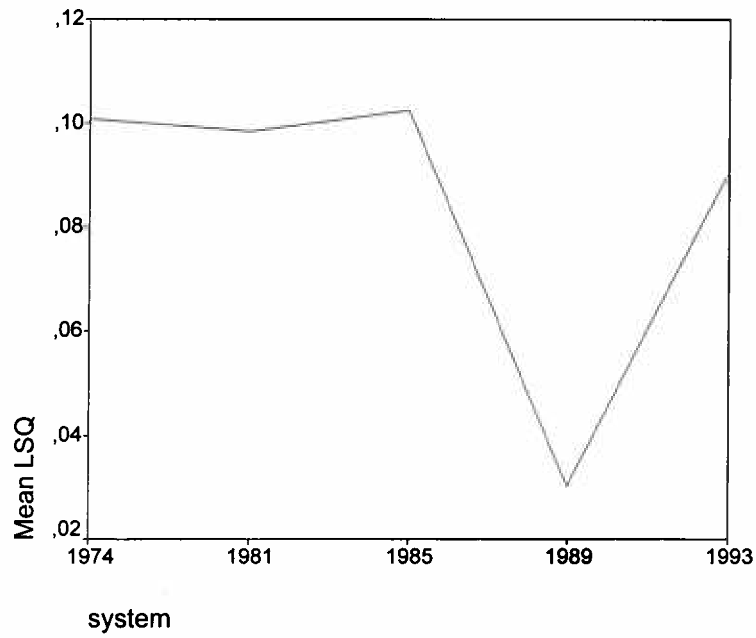

Figure 7.3: The mean value of the LSQ index for the 10 datasets, for k=13.

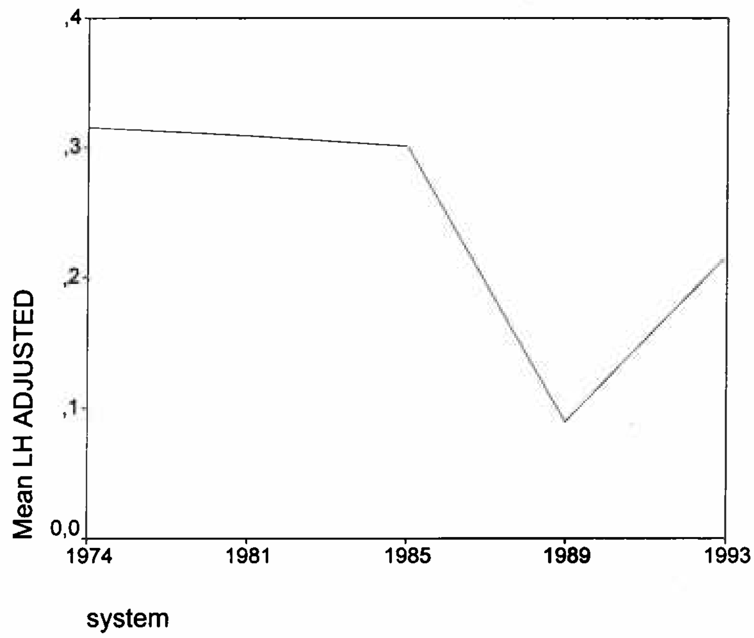

Figure 7.4: The mean value of the LH-adj. index for the 10 datasets, for k=13.

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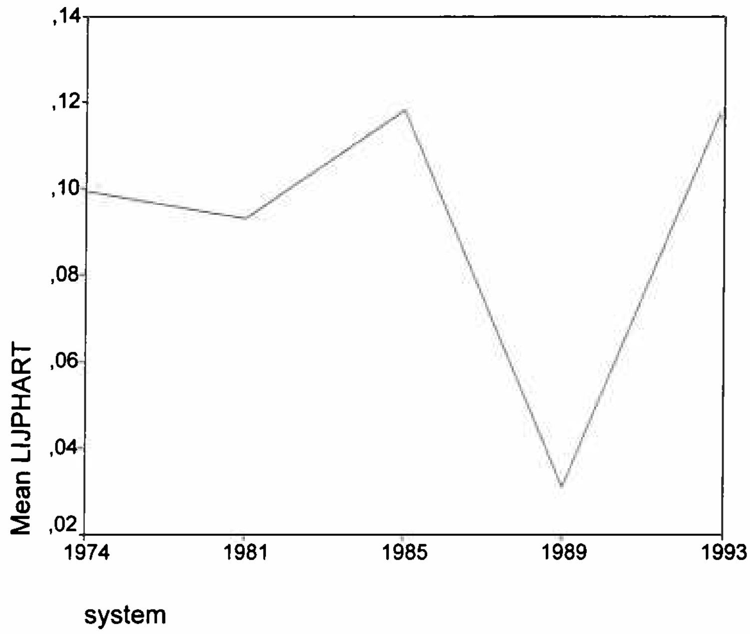

Figure 7.5: The mean value of the Lijphart index for the 10 datasets, for k=13.

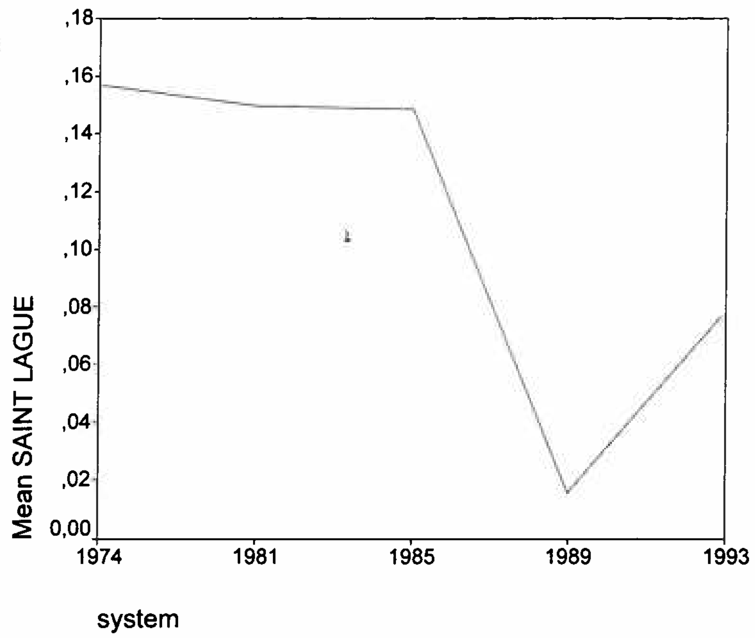

Figure 7.6: The mean value of the S-L index for the 10 datasets, for k=13.

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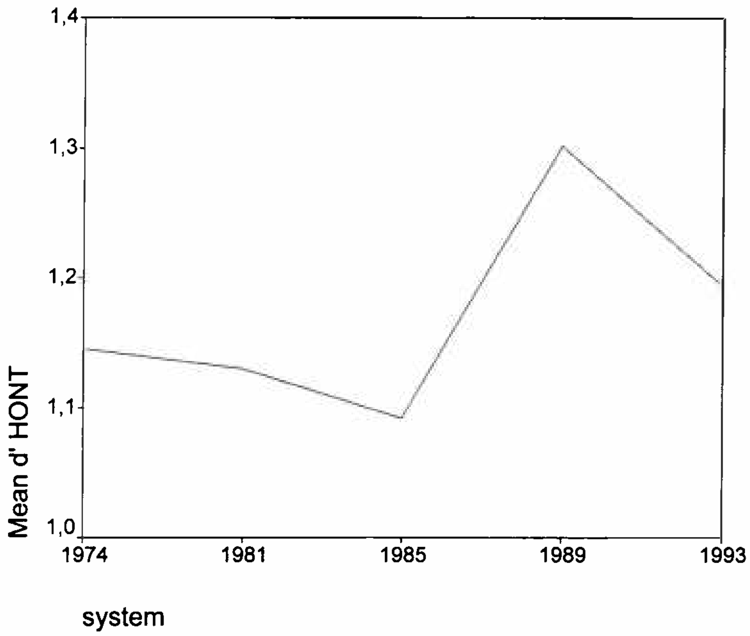

Figure 7.7: The mean value of the d Hont index for the 10 datasets, for k=13.

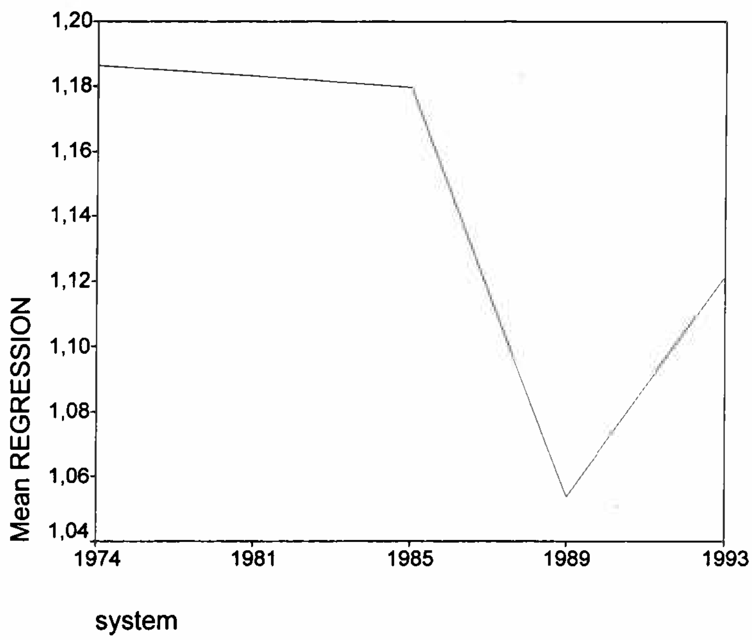

Figure 7.8: The mean value of the Regression index for the 10 datasets, for k=13.

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In order to find the fairest system we seek for the smallest values of the measures of disproportionality. From Table 12 and also the Figures 7.1 to 7.8 we see that all indices, except d'Hondt, take the smallest value in the case of the 1989 system. Furthermore, the above figures indicate that the 1989 system give significant small values for all the indices, except d'Hondt, with respect to the other systems. Thus, it is obvious that, according to the above indices, the 1989 system is much more fair than the other electoral systems under consideration. This result agrees with our comment in the section 3.3. It was mentioned that the 1989 system includes a completely different procedure in the secondary distribution of seats, which differentiates it with respect to the other systems. This procedure is much more proportional comparing to the procedure that the other systems use. It uses the remainders of votes and not the total votes. The use of the remaining votes has the result that large parties do not use all their votes in the secondary distribution and this means that they do not use again the votes used in the primary allocation of seats. When all votes are used the quota is much larger than the case where the remaining votes are used. However, larger quota allows over-representation for large parties. That is why this system gives the most proportional results. Furthermore, small parties are enforced more in the 1989 system because parties with at least 1% of votes take at least one seat and parties with at least 2% of votes take at least three seats. But why d'Hondt gives different results? This index expresses the over-representation of the most overrepresented party. Some parties, those are mainly the large parties, take more seats with respect to their vote shares. Those are the overrepresented parties. The d'Hondt index (as well as Lijphart index) is based only on the most over-represented party and not to all parties, as the other indices do. Consequently, according to the d'Hondt index, the 1985 system is the one with the smallest over-representation of the most overrepresented party. We have to note that although the real dataset gave the smallest value of the d'Hondt index when the 1993 system was used, in the generated datasets this happens for the 1985 system.

As we have seen all the above indices, except the d'Hondt give similar results. The high correlation between the seven indices is also illustrated in the Table 13.

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Table 13: The correlation between the eight measures of disproportionality.

	Rae	LH	LSq	LH-adj.	Lijph.	S-L	d'Hondt
Rae index	1						
LH	0,935	1					
LSq	0,954	0,919	1				
LH-adj.	0,999	0,936	0,955	1			
Lijphart	0,768	0,770	0,917	0,766	1		
S-L	0,986	0,922	0,913	0,986	0,702	1	
d'Hondt	-0,231	-0,221	-0,235	-0,229	-0,205	-0,220	1
Regr.	0,9741	0,917	0,915	0,974	0,720	0,977	-0,222

From the above table we notice that there is a high correlation between all the indices, except the d'Hondt. In fact the correlations between Rae, L-H, LSq, LH-adj., S-L and regression are very high, greater than 0,9 and sometimes they reach the 1; see for example, the correlation between the Rae and the LH-adj. index. Lijphart index presents small correlation with Rae, LH and LH-adj. However, it remains a significant correlation. The d'Hondt presents not only a low correlation with all the indices, but also this correlation is negative. This means that the fair systems, present large over-representation of the most overrepresented party.

After the 1989 system the one that gives the smallest values of almost all indices, except d'Hondt and Lijphart, is the 1993 system. Comparing the 1993 system with the 1985 the only difference is the thresholds that the first one uses. In fact they are very important as they give much more different results. As we have already mentioned, the important threshold of the 1993 system does not permit to 'small' parties to gain a seat, in any district, in any distribution. 'Small' parties are the parties with total percentage of votes smaller than 3%. This means that these parties do not take a seat not only in the primary distribution but also in any of the following distributions. On the other hand, the parties with percentage of valid votes greater or equal to 3% of the total valid votes of all the parties, in the entire state, they obtain a minimum number of seats. These seats are eliminated from other parties according to the total number of seats. Thus, this regulation take seats from large parties and allocates them to middle parties. This is illustrated in Table 11. The middle parties, which are the fourth,

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the fifth and the sixth, take much more seats when the 1993 system is used with respect to not only the 1985, but also the 1974 and 1981 systems. The fourth party occupies seven more seats, the fifth receives six more seats and the sixth party obtains seven more seats, when the 1993 system is used comparing with the 1974, the 1981 and the 1985 systems. As we have noticed only d'Hondt and Lijphart give different results. The index that has been proposed by Lijphart is quite similar with d'Hondt index. It also deals with the most overrepresented party. They differ on the way that they define the over-representation. Lijphart uses the difference between the vote and the seat shares, while the d'Hondt uses the ratio. That is why these two indices give similar results.

A simple look at the figures 7.1 to 7.8 reveals that the other systems (1974, 1981, 1985) give values, for the indices, which are very close. Comparing the first two systems (1974, 1981) the only difference is on the quota that is used in the primary distribution of the seats. The first one uses the Hare quota while the second uses the Droop quota. As it was mentioned in Chapter 3, Droop give more fair results with respect to Hare quota. This result agrees with our finding when the measures are used. All measures, except Lijphart and Sainte Lague give the same result: these systems seem to be more fair as we move from the oldest to the newest. Thus, the 1985 system is more fair than the 1974 and the 1981, while the 1974 system is less fair than the other systems. We have already explained why Droop is more fair than Hare: Hare divides total votes with the available seats, while Droop divides total votes with the available seats plus one. Thus, the value of Hare is greater than the value of Droop, when they are applied to the same number of votes and seats. Given that parties take as many seats as many times quota is contained in the number of votes, quota is covered easier, when Droop quota is used. More seats are allocated in the primary distribution, in this case, and fewer seats are available for the other distributions.

It is important to study the Regression index separately, as it is a measure of the large parties' bias. A Regression index greater to one reveals that the system favors the large parties. Table 11 indicates that all systems, under consideration, favor the large parties, and that less biased in favor of large parties is given by the 1989 system. Our results agree with Dimitras (1991) comment, who noted that the 1989 system bias much less in favor of large parties. The next less biased system in favor of large parties is the 1993 system. The other three systems

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give quite similar values for the regression index. Less biased in favor of large parties is given as we move to from older to newer systems (1974,1981,1985). All the above comments can be summarized to the following graph.

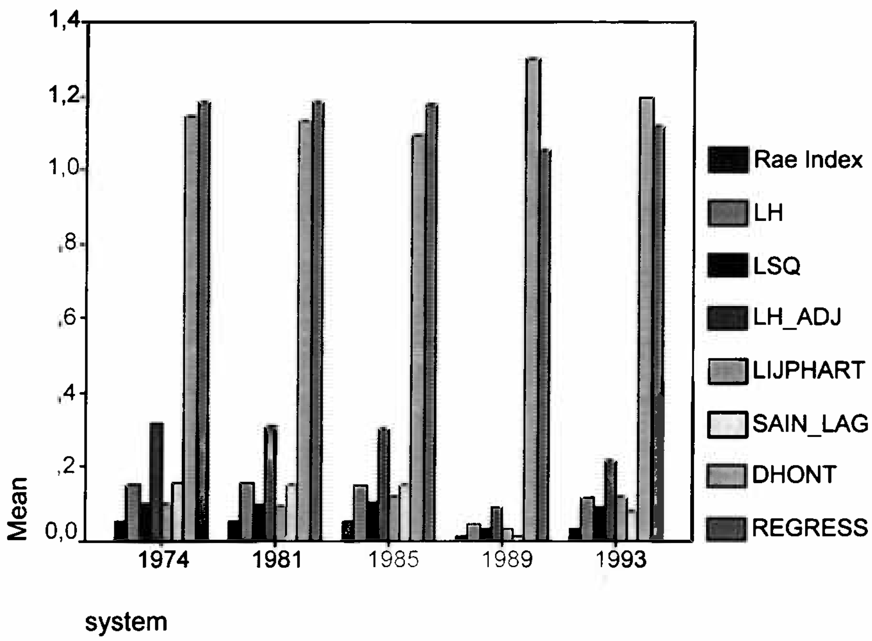

Figure 8: The mean value for each index, and for each system for k=13

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	1974	1981	1985	1989	1993
Rae index	0.052	0.0515	0.0496	0.0175	0.0336
	(0.0009)	(0.0012)	(0.0008)	(0.0025)	(0.0020)
LH	0.1581	0.1548	0.1491	0.0526	0.1010
	(0.0023)	(0.0035)	(0.0026)	(0.0076)	0.0061)
LSq	0.1002	0.0980	0.0992	0.0376	0.0843
	(0.0016)	(0.0024)	(0.0032)	(0.0046)	(0.0056)
LH-adjusted	0.3163	0.0309	0.2957	0.1053	0.2021
	(0.0047)	(0.0071)	(0.0088)	(0.0152)	(0.0123)
Lijphart	0.0938	0.0908	0.1112	0.0435	0.1112
	(0.0068)	(0.0064)	(0.0078)	(0.0053)	(0.0087)
Sainte-Lague	0.1459	0.1493	0.1608	0.0183	0.0727
	(0.0300)	(0.0075)	(0.0544)	(0.0049)	(0.0112)
d'Hondt	1.2321	1.2247	1.2623	1.1624	1.2743
	(0.0152)	(0.0141)	(0.0507)	(0.0693)	(0.0190)
Regression	1.1864	1.1828	1.1779	1.0679	1.1141
	(0.0027)	(0.0041)	(0.0032)	(0.0095)	(0.0098)

Table 14: Mean values and standard deviations (in parenthesis) of the indices for each system, when 9 major districts are used.

In Table 14, we present the results for 9 major districts when the 10 generated datasets are used. From the Tables 3 and 5 we see that the use of the nine major districts give quite similar results, on the indices, with the use of the thirteen major districts. This means that the result of an electoral systems is not affected from the choice of the major districts (k=9 or k=13). This result can be confirmed by using Hypothesis Test that the mean of the values of the different measures of disproportionality we analyse are equal, when we take into account 9 and

13 major districts (Two sample T-Test); for more details on hypothesis testing, see, Panaretos (1992). When we applied this test we found that there are no statistical significant differences (at a=0.05) between the mean values of the measures of disproportionality for 9 and 13 major districts, when each one of the eight indices is used.

Table 15: Two-sample t-test, for the number of the major districts, for each index.

	T	**p-value**
Rae index	0.000670	0.979
LH	0.009636	0.922
LSq	0.008075	0.929
LH-adjusted	0.001928	0.965
Lijphart	0,108441	0.743
Sainte-Lague	0.000063	0.994
d'Hondt	1.745972	0.189
Regression	0.006914	0.934

Finally, we will present eight graphs presenting the mean values and 95% confidence interval for each one of the eight measures of disproportionality. From the following graphs we notice that the confidence interval for the d'Hondt index is quite large, and therefore we can not rely on this index in order to compare the electoral systems. On the other hand the results and the 95% confidence intervals for the other indices are more informative for the comparison of the systems.

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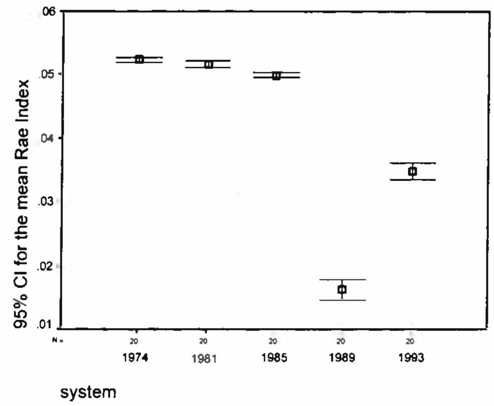


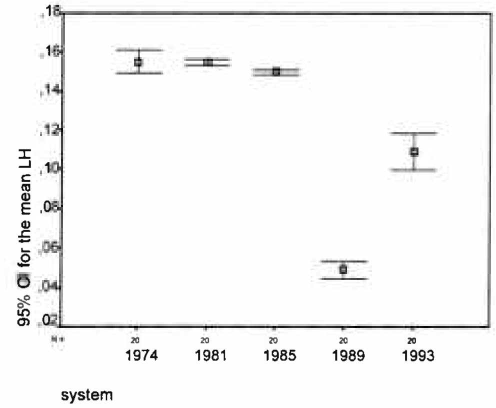
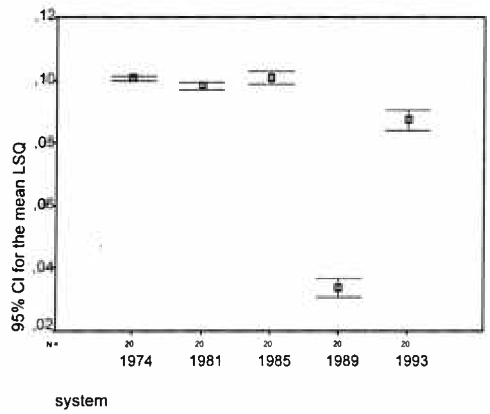


Figure 9.1: 95% Confidence Interval for the mean Rae index

Figure 9.2: 95% Confidence Interval for the mean L-H index



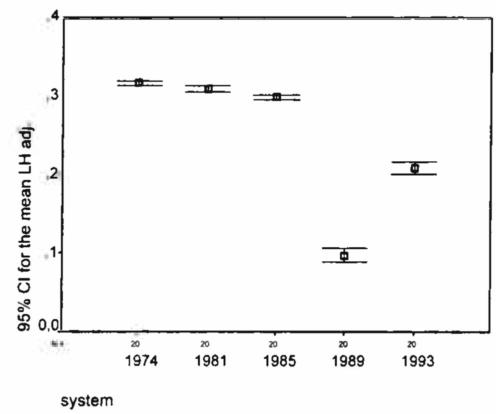
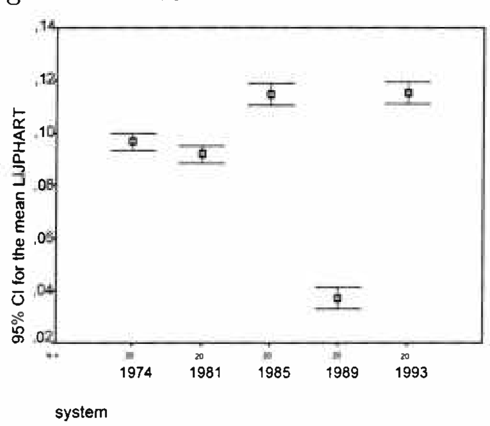


Figure 9.3: 95% Confidence Interval for the mean LSq index

Figure 9.4: 95% Confidence Interval for the mean L-H adj. index



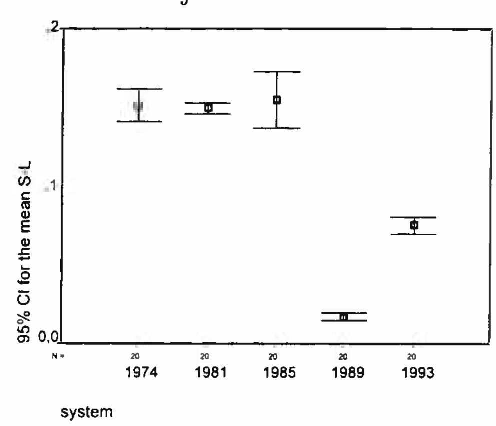
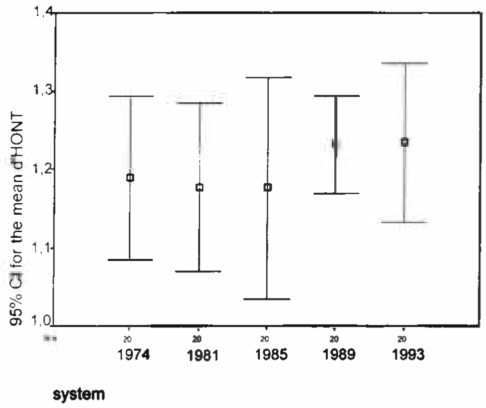


Figure 9.5: 95% Confidence Interval for the mean Lijphart index

Figure 9.6: 95% Confidence Interval for the mean S-L index



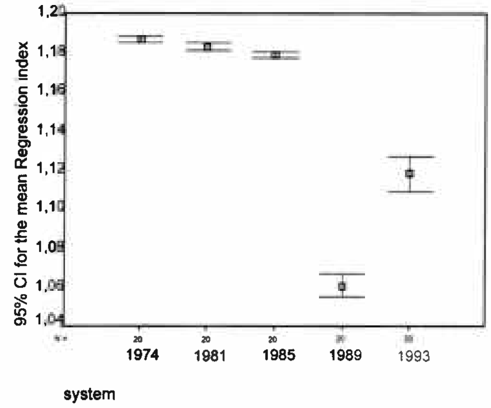


Figure 9.7: 95% Confidence Interval for the mean d' Hondt index

Figure 9.8: 95% Confidence Interval for the mean Regression index

6.3 Application with the data of the 1999 Elections for the European Parliament

In this section, we present the results of the 1993 (1996) electoral system when the dataset of the 1999 elections for the European Parliament is used.

The latest elections for the European Parliament took place on the 13th of June of 1999 in all the counties of the European Community. For this reason Greek voters voted for their 25 representatives in the European Parliament. Each one of these representatives belongs to a Greek political party. We suppose that, the Greek citizens vote for the same political parties (representatives) for the election of their representatives in the Greek Parliament (not for the European Parliament). We want to find out the results of this **hypothetical** election, when the current electoral system (for the Greek Parliamentary Elections) is used.

The data have been taken from the Ministry of Internal Affairs. In our analysis we have not included the Greek voters who live abroad. It is impossible for us to include them, in the analysis, because we do not know the lower district that each one of them belongs to.

The number of the votes and the respective percentages for the six large parties is given in

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the following table.

Party	votes	%votes (% used in the analysis)	%votes (real percentages)
PASOK	2090762	32,69	32,92
NEW DEMOCRACY	2301866	35,99	36
POLOTICAL SPRING	146039	2,28	2,27
K.K.E.	554915	8,67	8,66
SYNAPSISMOS	330589	5,16	5,16
DH.K.KI	439712	6,87	6,84

Table 16: The total number of votes and the percentages of votes used in the analysis, taken from the 1999 elections for the European Parliament in Greece. The last column gives the real percentages of votes.

Party	seats	%seats	%seats / %votes
PASOK	93	31	0.94
NEW DEMOCRACY	165	55	1.52
POLOTICAL SPRING	0	0	-
K.K.E.	18	6	0,69
SYNAPSISMOS	10	3,33	0,63
DH.K.KI	14	4,66	0,66

Table 17: The seats that would have been distributed to parties, using the 1993 (1996) electoral system, for the 1999 election results. It also includes the ratio of the percentage of seats divided by the percentage of votes.

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Table 17 confirms all the comments and the results that we have presented for the 1993 (1996) system. We note that the first party is overrepresented a lot while this does not happen for the second party: PASOK which obtained the 32,69% of the total votes, would have obtained the 31% of the total seats, if National Parliamentary Elections would have been applied. On the other hand the first party, NEW DEMOCRACY, which obtained the 35,99% of the total votes, would have obtained the 55% of the total seats, if National Parliamentary Elections would have been applied. This fact consists the most important disadvantage of this system. Middle parties like K.K.E., SYNASPISMOS and DH.K.KI. are enforced by taking seats from the second party. Finally the party POLITICAL SPRING takes no seats. These results are summarized in the following graph.

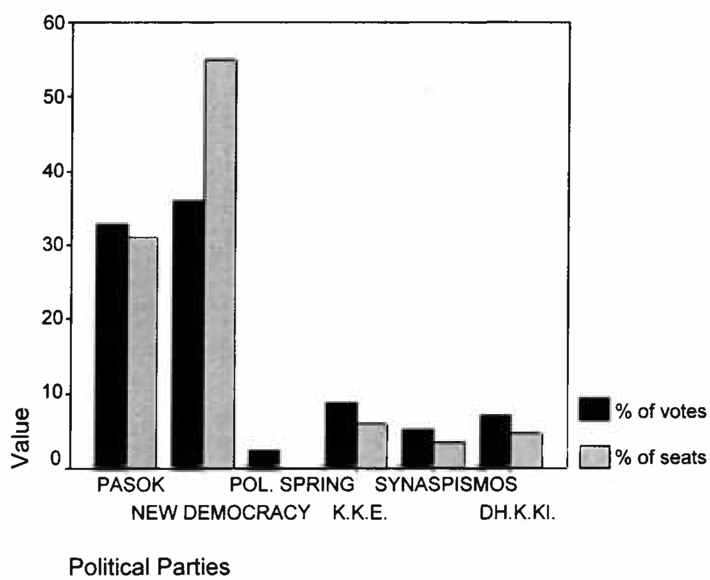

Figure 10: The percentages of votes and the respective percentages of seats that would have been obtained by the six political parties if the 1996 electoral system has been applied, for the 1999 election results for the European Parliament, in Greece.

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Chapter 7

Conclusions

In this dissertation, we described and analysed the Greek Electoral systems of the period 1974-1999. We also, provided a systematic quantitative analysis in order to compare and evaluate these electoral systems. The comparison of electoral systems by using quantitative measures is not broadly used in the literature, and has not been applied to Greek Parliamentary Elections before.

The analysis of the Electoral systems led to some important conclusions. The system of 1989 seems to be the fairest according to the measures of disproportionality that we analysed. The second fairest system is the 1993 one, while the other three systems seem to give similar results. The system of 1989, in contrast to the other systems, does not provide the first party with the majority of the parliamentary seats, when used in the election data set of 1996 and the other generated data sets.

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