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STATISTICAL METHODS FOR FORECASTING MORTALITY

By

Kyriaki Korakaki

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Κυριακή Κορακάκη

ΔΙΑΤΡΙΒΗ

Που υποβλήθηκε στο Τμήμα Στατιστικής

του Οικονομικού Πανεπιστημίου Αθηνών

ως μέρος των απαιτήσεων για την απόκτηση

Μεταπτυχιακού Διπλώματος Ειδίκευσης στη Στατιστική

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VITA

My name is Korakaki Kyriaki. I was born and grew up in Athens, where I live since today. In 2004, I graduated from 1st High School in Kessariani and I succeed at the exams to study in the Department of Mathematics of the University of Athens and I completed my studies in 2010. At that time, I was admitted to the Master Program in Statistics, full time, in the Athens University of Economics. The present work represents the completion of my studies at a Master level.

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ABSTRACT

Kyriaki Korakaki

Statistical Methods for Forecasting Mortality

June 2012

Time period 1980 – 2005 represents a period of growth for the demographic forecast. Previous years were years of inactivity in respect to demographic forecasts. Mortality, fertility and migration are the three components that affect the size, composition and change in population.

Fertility plays an important role in population forecast. However, there are also external factors that affect forecast. For instance, forecasts given by scholars in the twentieth century were inaccurate due to the Second World War. Both mortality and migration forecast play an important role.

Mortality forecast plays a vital role for a country, as a knowledge tool. Scholars have observed that populations are ageing.

At the present thesis, mortality forecast methods existed in literature since 1980 to today, are presented. According to literature, the most significant method for studying and forecasting mortality is the “Lee – Carter Method”. This method is applied in real data for three countries, Spain, France and Sweden, in order to forecast mortality for specific years. Finally, forecasted data are compared to the real ones, so as to verify the success of the selected method.

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ΠΕΡΙΛΗΨΗ

Κυριακή Κορακάκη

Στατιστικές Μέθοδοι για την Πρόβλεψη της Θνησιμότητας

Ιούνιος 2012

Τα έτη 1980 – 2005 αποτελούν περίοδο ανάπτυξης για τη δημογραφική πρόβλεψη. Τα προηγούμενα χρόνια ήταν χρόνια αδράνειας σε σχέση με τις δημογραφικές προβλέψεις. Η θνησιμότητα, η γεννητικότητα και η μετανάστευση είναι τα τρία στοιχεία που επηρεάζουν το μέγεθος, τη σύνθεση και τις αλλαγές του πληθυσμό.

Σημαντικό ρόλο στην πρόβλεψη του πληθυσμού έχει η γεννητικότητα. Ωστόσο, υπάρχουν και εξωτερικοί παράγοντες που την επηρεάζουν. Για παράδειγμα, η πρόβλεψη που γίνεται από επιστήμονες για τον εικοστό αιώνα ήταν ανακριβής λόγω του Δευτέρου Παγκοσμίου Πολέμου. Η πρόβλεψη της θνησιμότητας και της μετανάστευσης διαδραματίζουν σημαντικό ρόλο.

Η πρόβλεψη της θνησιμότητας αποτελεί σημαντικό στοιχείο γνώσης για μια χώρα. Οι μελετητές παρατήρησαν ότι οι πληθυσμοί γερνούν.

Στην παρούσα εργασία γίνεται αναφορά στις μεθόδους πρόβλεψης της θνησιμότητας που υπάρχουν στην βιβλιογραφία από το 1980 μέχρι σήμερα. Σύμφωνα με την βιβλιογραφία, η σημαντικότερη μέθοδος για την μελέτη και πρόβλεψη της θνησιμότητας είναι η «Lee – Carter». Ακόμα, γίνεται εφαρμογή της μεθόδου «Lee – Carter» σε πραγματικά δεδομένα για τρεις χώρες, Ισπανία, Γαλλία και Σουηδία, με σκοπό την πρόβλεψη της θνησιμότητας για συγκεκριμένα έτη. Τέλος, τα δεδομένα που προέκυψαν από την πρόβλεψη και τα πραγματικά δεδομένα συγκρίνονται για να επαληθευθεί η επιτυχία της επιλεγμένης μεθόδου.

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CHAPTER 1

Introduction

Mortality is a biological phenomenon with many social and economic implications. Mortality, fertility and migration are the three components that affect the size, composition and change in population. Historically, mortality has played a dominant role in determining population growth.

Mortality is studied based on Demography. This study differs by gender, age and according to many other biological, social, economic and environmental characteristics of the population. An important contribution of demography was the revelation of the fact that the sharp declines in mortality rates, rather than any increase in fertility rates, was responsible for ensuring the rapid population growth.

Studying mortality is necessary in demography research and in research in general, in private and public sector. More specifically, it helps to assess the level of population health and to shape health policy. Studying mortality, combined with the study of reproducibility, is the basis for estimating the population size in the future, data needed in order to formulate a policy for political, social and economic development.

To be more specific, public health administration strongly depends on the study of mortality, for statistics on death in the population cross – classified by age, sex and the cause of death are of great value for the formulation, implementation and evaluation of public health programs.

Statistics on deaths also form the basis of the policies of insurance companies.

This thesis is divided into two sections. The first section (Chapters 2 to 7) refers to methods of mortality prediction based on literature review from 1980 to today. Literature, the most significant method for studying and forecasting mortality is “Lee – Carter Method”. This is because the referred method is a relatively simple method that based on statistical methods and as its application to real data shown, it has a small error in forecasting mortality. In the second section (Chapter 8 – 10), “Lee – Carter Method” is applied to real data for three countries in order to forecast mortality for specific years. Finally, forecasted and real data were compared to verify the success of the selected method.

In more details, in Chapter 2, the methods existed before 1980 are presented. In Chapter 3, “Lee – Carter Method” is analyzed with all different ways. Many researchers have already studied this method as well. Two important modifications of the “Lee – Carter Method” are analyzed in Chapter 4 and in Chapter 5 the extensions of this method are presented. In Chapter 6, several methods for forecasting mortality are studied. Finally, in Chapter 7, another approach for “Lee – Carter Method” is analyzed.

In the second section of the present thesis, “Lee – Carter Method” is applied for three different countries. The selected countries are Spain, France and Sweden, a southern, a central and a northern country respectively. In Chapter 8, mortality for Spain is analyzed. Model’s parameters are calculated and assuming as known years 1970 – 2004, data for 2005 – 2009 were forecasted and then the resulting data were compared to real ones (as these are given from mortality database at www.mortality.org).

The same procedure is followed in Chapters 9 and 10 for Spain and Sweden, respectively. For Sweden, forecast was also done for 2010 as well, due to the availability of the respective data. In the last Chapter 11, conclusions from applying the “Lee – Carter Method” are presented.

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CHAPTER 2

Mortality Forecasting Before 1980

2.1 Introduction

In this chapter, known mortality forecasting methods before 1980 are studied. Many of these methods are very simple and they are not use technical method for mortality forecasting.

2.2 Graphical Period Forecast

In this method, the values of $\mu(x,t)$ and t for the various values of x are plotted. Then, these values are joined by a curve to the real values and, based on curves of other countries and ages of nearby, a curve forecasting is designed.

This is the simplest method for forecasting and it can be used for every age. However, this method does not follow any technical method for forecast and the prediction error can be large. Also, the prediction is not objective because it is deemed by the scholar. Finally, this method would not be politically correct because it is strongly dependent upon past data and a person dies independently of past.

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2.3 Graphical Generation Forecast

The values of ${}_{\theta}\mu_x$ and θ for the various values of x, where $\theta = t - x$ is the year of birth, are plotted. Same as before, the real points are joined with a curve and are extrapolated at the same direction.

As above, this method is simple and it can be used for every examined year. The examined method does not follow any technical method for forecast and the prediction error can be large. Also, the prediction is not objective because it is deemed by the scholar. Moreover, the curve is constructed according to the year of birth. Finally, the curve affected by temporary phenomena, such as an epidymia.

2.4 Rhodes's Method

For some countries, Kermack, McKendrick and McKinlay (1934) showed that ${}_{\theta}\mu_x$ is depended upon two factors: the age x and the year of birth θ. So ${}_{\theta}\mu_x$, can be found by using the following equation:

$$ {}_{\theta}\mu_x = Q(x) * R(\theta) $$

where, $Q(x)$ is a function of age and

$R(\theta)$ is a function of birth.

First, the truth of hypothesis is checked for any given data. Let 1860 be the base year of birth. Then, the ratio $\frac{{}_{\theta}\mu_x}{{}_{1860}\mu_x}$ is calculated for every x. If the hypothesis is true, the ratio will be constant for every x and the given value of θ. Then, the following $\frac{R(\theta)}{R(1860)}$ and $Q(x) * R(1860)$ are computed.

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The trend of the line for $\frac{R(\theta)}{R(1860)}$ follows the respective trend of $\frac{{}_{\theta}\mu_x}{{}_{1860}\mu_x}$. As a result, based on hypothesis of Kermack, McKendrick and McKinlay (1934), $Q(x) * R(1860)$ is the result of the division ${}_{\theta}\mu_x$ with the $\frac{R(\theta)}{R(1860)}$, i.e. ${}_{\theta}\mu_x \div \frac{R(\theta)}{R(1860)} = Q(x) * R(1860)$.

This method depends on hypothesis of Kermack, McKendrick and McKinlay (1934) and it yields a particularly simple formula. Also, it is applicable to financial problems for ages over 30 and can not be influenced by the view of the scholar.

2.5 Makeham Period Method

The Makeham curve is given by the formula "$A + B \times c^{x-30}$". For every year t_1, for which $\mu(x,t)$ is known, $\mu(x,t_1)$ is given by the Makeham curve. The constants are calculated as functions of time.

Literally, the method is shown to have a certain theoretical appeal, very extensive data are required and if the method is to be used frequently with any confidence, it cannot be used at all.

2.6 Makeham Generation Method

In this method the Makeham curve is again used. In this case, for each year of birth θ, ${}_{\theta}\mu_x$ is calculated from the Makeham curve. The constants are calculated as functions of θ.

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CHAPTER 3

The Lee – Carter Method

3.1 Introduction

The Lee – Carter Method (henceforth LC) is a method by which mortality can be forecasted. Although other methods have been developed as well, the LC method is often considered as the benchmark method.

The basic model is very simple and although its use for forecasting involves a number of steps, each is simple in itself. The method is "relational" in demographers' terminology. That is because it involves the transformation of actual existing mortality schedules for each study population. Therefore, on the one hand is largely non-parametric and on the other hand incorporates particular features of the mortality pattern of a given population (Lee and Miller, 2001).

Lee and Carter (1992) developed a method that uses standard methods for forecasting a stochastic time series, together with a simple model for the age – time surface of the log of mortality in order to model and forecast mortality. A forecast is produced for the probability distribution of each future age specific death. In this method, subjective judgment is reduces as standard diagnostic and modeling procedures for statistical time series analysis are applied. At the same time, the researcher must take decision as far as some issues are concerned, i.e. the beginning of history, the model to be

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used and the treatment of specific phenomena, such as wars, intense epidemics etc (Li, Lee and Tuljapurkar, 2004).

Lee and Carter (1992) used a principal component method to extract a single time – varying index of the level of mortality rates, from which the forecasts are obtained using a random walk with drift. Lee and Carter model combines a demographic model with statistical time series analysis. As it has been noticed, LC method has the potential for inflexibility with respect to age (Renshaw and Haberman, 2002). Lee and Carter model is fitted to historical data and the resulting estimate of the time – varying parameter is then forecasted as a stochastic time series using standard Box – Jenkins methods (Li, Lee and Tuljapurkar, 2004).

The main statistical tool of Lee and Carter (1992) is least-squares estimation. This implicitly means that the errors are assumed to be homoskedastic, which is quite unrealistic: the logarithm of the observed force of mortality is much more variable at older ages than at younger ages because of the much smaller absolute number of deaths at older ages (Delwarde, Denuit and Eilers, 2007).

The original LC uses Singular Value Decomposition (Lawson and Hanson, 1974). This method assumes that the model errors have the same variance over all ages, which is not always correct (Lee and Miller, 2001, Brouhns et al., 2002). Two alternative approaches to compute the LC parameters were proposed: a Weighted Least Square (Wilmoth, 1993) and a Maximum Likelihood Estimation (Wilmoth, 1993, Brouhns et al., 2002). Additionally, in Lee and Carter (1992) and major applications of this model, confidence intervals, for long-term forecast of death rates and life expectancy, were restricted solely to the uncertainty in the time – variable parameter. That approach raises criticisms because the other sources of variability in the model were ignored (Lee, 2000).

However, unlike theoretical methods, which take into account expert opinions and changes in the pattern of deaths by different causes, the Lee –

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Carter approach is based entirely on extrapolation, presuming that the forces of change, that were effective in the experience period, are going to be in effect in the future. Because of such an extrapolative nature, forecasters must be careful when specifying a time-series process for the time – varying component (parameter "k_t", often known as the mortality index) in the Lee – Carter model. Li and Chan (2005, 2007), for instance, point out that forecasters should be aware of potential outliers in the mortality index, because they may possibly lead to erroneous mortality forecasts, particularly if they are close to the forecast origin.

Modern time series methodologies, as this have been noticed by McNown and Rogers (1989), allow for more flexible modeling of trend and consequently offer greater possibilities for accommodating changes in long – run growth rates. Time series methods also offer the possibility of formally incorporating behavioral information in forecasts, such as responses to projected or hypothesized changes in socioeconomic variables.

The method is also probabilistic, as it involves statistical fitting of models and the quality of the fit of the historical data can be used to provide probability intervals for the forecasts. As a matter of empirical fact, in the applications of the method to date, involving at least ten national data sets, the historical trend in 'k' has always been found to be highly linear with time and the random walk with drift has been found to give a good fit. This approximate linearity is useful for forecasting. It contrasts to the typically nonlinear trajectories of life expectancy, which rise at a decelerating rate when age – specific mortality rates decline at constant exponential rates. Nonetheless, it is clear that if a data series extends sufficiently far back in time, the linearity of decline would cease to hold (Lee and Miller, 2001).

Finally, the method can also be used as the basis of a simple model life table system and indirect estimation methods can be developed to expand the mortality data available as the basis for forecasting (Lee and Miller, 2001).

There are many scholars who have studied this method for different countries as well: U.S. data from 1933 to 1987 (Lee and Carter, 1992), Chile data from 1952 to 1987 (Lee and Rofman, 1994), Canada data from 1922 to 1995 (Lee and Nault, 1993), Sweden 1860 – 2004 (Wang, 2007), Japan (Wilmoth, 1996), the seven most economically developed nations (G7) (Tuljapurkar et al., 2000), Belgium (Brouhns et al., 2002) (Brouhns and Denuit, 2001), Australia (Booth et al., 2002, De Jong and Tickle, 2006), Austria (Carter and Prskawetz, 2001), China (Lin, 1995), Finland (Alho, 1998), Norway (Keilman et al., 2002), Spain (Felipe et al., 2002, Debón et al., 2006), Sweden (Lundström and Qvist, 2004, Tuljapurkar, 2005), the U.K. (Renshaw and Haberman, 2003b), the Nordic countries (Koissi et al., 2006), China and South Korea with limited data (Li, Lee and Tuljapurkar, 2004), Taiwan (Wang and Liu, 2010), U.S. male mortality data: mortality rate forecasts are formed for the period 1990 – 1999 based on data from 1959 – 1989 (Pedroza, 2006), Italy 1950 – 2000 (Haberman and Russolillo, 2005), the Romanian female population, during 1970 – 2002 (Lazar), Portuguese mortality 1942 – 1999 (Coelho, 2001), Brazil (Fígoli,1998), Canada and the United States (Li and Chan, 2007). Unfortunately, the model did not succeed in Australian data (Booth et al., 2002) and the U.K. (Renshaw and Haberman, 2003a).

3.2 The model

The model is expressed as in the following equation:

$$m(x,t) = e^{a_x + k_t b_x + \varepsilon_{x,t}} \qquad (3.1)$$

where: $m(x,t)$ is the central death rate for age x in year t,

$\{a_x\}$ is the first age – specific constant interpreted as the general shape across age mortality schedule,

$\{b_x\}$ is the second age – specific constant, showing which rates decline more rapidly and which more slowly in response to changes in k_t,

k_t is time-varying index (is an index of the level of mortality) and
$\varepsilon_{x,t}$ is the error term

If "k_t" is the linear function of t, the age-specific rate will change as a constant exponential rate and the constants will vary according to the age. This model is applicable when one – parameter family of life tables is available.

Booth et al. (2002) noted that the time component "k_t" or the 'dominant temporal signal' (Tuljapurkare et al., 2000, p.789) follows the overall time trend in "$ln[m(x,t)]$" at all ages. In the base model no assumptions are made about the functional form of the trend in "k_t". The age component "b_x" modifies the main time trend according to whether the change at a particular age is faster or slower than the main trend (and towards to the same or opposite direction). The model assumes that "b_x" is invariant over time.

This method can be generated from two observed life tables. Firstly, "k_t" is considered a function of time. Let $k_t = 0$ when we have one life table and $k_t = 1$ otherwise. Also, $a_x = \ln[m(x,0)]$ and $b_x = \ln[m(x,0)] - \ln[m(x,1)]$ are assumed. If $0 < k_t < 1$, the model geometrically interpolates between the two life tables, else, it extrapolates from the two life tables. This approach is useful for countries with little data as some Third World countries.

When more than two life tables are available, it is preferable to estimate the "a_x" and "b_x" schedules along with "k_t" to minimize the squared deviations from a given matrix of age – specific rates.

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3.3 Fitting the model

From the following model (3.2), the least squares solution is to be found:

$$m(x,t) = e^{a_x + k_t b_x + \varepsilon_{x,t}} \Leftrightarrow \ln(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t} \quad (3.2)$$

where, $t = 1,2,\dots,T$ and $x = 1,2,\dots,n$.

If the vectors a, b and k are assumed to be one solution and c is one scalar, then "$a - bc, b, k + c$" is the other solution. Another solution is "$a, bc, k/c$".

The parameter a_x can be computed from the following equation:

$$a_x = \frac{1}{T}\sum_{t=1}^{T} \ln(m_{x,t}) \quad (3.3)$$

Observing the right side of equation (3.2), parameters "a_x", "b_x" must be estimated and index "k_t" is unknown. As a result the model is not fully determined and thus this model can not be fitted by ordinary regression methods. To overcome this problem, Lee and Carter (1992) suggested two stages to estimate the procedure.

In the first stage, Lee and Carter (1992) used the Singular Value Decomposition (SVD) method to find a least squares solution in the estimation of the parameters b_x and k_t. The SVD is applied to the matrix of $\{\ln(m_{x,t}) - a_x\}$ to obtain estimates of b_x and k_t.

In SVD Method, Lee and Carter (1992) used Gauss Method. It is assumed that $\sum_{x=1}^{n} b_x = 1$ and $\sum_{t=1}^{T} k_t = 0$. Then, a_x must be equal to the

average over time of $\ln(m_{x,t})$. So, k_t is almost equal to the sum over age of $(\ln(m_{x,t}) - a_x)$ since the sum of the b_x's is to unity. Thus, the b_x can be found by regressing without a constant term. Also, the SVD method can be calculated by mathematical/statistical packages (SVD method can be seen in more details in section 3.5.1).

In the second stage, the following equation is given:

$$D_t = \sum_x \{\exp(a_x + b_x k_t) N_{x,t}\} \qquad (3.4)$$

where D_t is the total number of deaths in time t and

$N_{x,t}$ is the exposure to risk of age x in time t.

In this stage, the time series of "k_t" is reestimated by solving for "k_t" the above equation. I.e., after "b_x" and "k_t" have been estimated, the parameter "k_t" be re-estimated using a different criterion. This reestimation step, often called "second stage estimation". Then, the ARIMA model is used to model the dynamics of k_t. Lee and Carter assumed that b_x is constant over time. Then, they forecasted the estimate of "k_t" from a standard univariate time series model. But, when they checked the ARIMA specification, they concluded that a random walk with drift was the most appropriate model. So, the Lee-Carter Model as a time series model can be written as follows:

$$\begin{cases} \ln(m_{x,t}) = a_x + k_t b_x + \varepsilon_{x,t} \\ k_t = k_{t-1} + c + \xi_t \end{cases} \qquad (3.5)$$

where c is constant annual change in k_t and

ξ_t are uncorrelated errors.

The combination of the standard errors in "c" and "ξ_t" represents the uncertainty associated with a one – year forecast. This is used to produce probabilistic prediction intervals for the forecast values of "k_t". Forecast age

– specific death rates are obtained using extrapolated "k_t" and fixed "a_x" and "b_x". In this case, the jump – off rates (i.e. the rates in the last year of the fitting period or jump – off year) are fitted rates (Booth et al., 2005).

The above two equations in (3.5) can be combined in one which is:

$$m_{x,t} = m_{x,t-1} + cb_x + (b_x\xi_t + \varepsilon_{x,t} - \varepsilon_{x,t-1}) \quad (3.6)$$

Parameter "c" is estimated as the average across all observed t and $t-1$ of $[k_t - k_{t-1}]$ (Li, Lee and Tuljapurkar, 2004).

$$\hat{c} = \frac{1}{T}\sum_{t=1}^{T}[k_t - k_{t-1}] = \frac{k_T - k_0}{T} \quad (3.7)$$

According to Lee and Carter (1992), this expression of D_t can be used for two purposes:

1. The original least – squares estimates of "k_t", "a_x", and "b_x" do not generate fitted life tables that imply the exact observed number of deaths for the actual historical population age distributions.

2. There may be periods for which population age distributions and total deaths are known but age – specific death rates are not available.

The model still can be fitted using this procedure. This is particularly useful for forecasting when, as in the United States, there is a lag of several years between publication of total deaths and publication of age – specific death rates. With this method, the base year for the forecast can be always taken to be the last year for which total deaths are available.

According to Lee and Carter (1992), this method differs from forecasting each age – specific rate independently in many ways:

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1. If each rate was forecasted independently, then $n(n-1)/2$ different covariances of errors would also need to be calculated, where n is the number of age groups; these rates are necessary to find the confidence bounds for life expectancy.

2. Each rate might be best modeled individually by a different order ARIMA process, requiring the estimation of many parameters. Only if each is well modeled by a random walk with drift, the number of parameters will be similar to the number in the examined method. This is true whether or not *"k"* happens to be modeled by a random walk with drift.

3. With individual forecasts, rates in the distant future might be combined to form highly implausible age profiles. Using the examined method, the individual age – specific rates are always constrained to belong to a life table system that fits the historical data.

4. This cohesion is obtained by forecasting the single parameter *"k"*, which is itself a kind of compromise among the trends in all the individual age – specific rates. This leads to different forecasts of the individual rates than the ones that would be obtained by modeling them individually.

3.4 Forecasting $\hat{k}_t$

Lee and Carter predicted the mortality index $\hat{k}_t$ in their original paper by a standard univariate time series model ARIMA(0,1,0). They demonstrated that other ARIMA models might be preferable for different data sets, but in practice the random walk with drift model (RWD) for "k_t" has been used almost exclusively. The model is as follows:

$$\hat{k}_t = \hat{k}_{t-1} + c + \xi_t \quad (3.8)$$

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where "c" is known as the drift parameter and

$$\hat{c} = \frac{\hat{k}_T - \hat{k}_1}{T - 1} \quad (3.9)$$

which means "$\hat{c}$" only depends on the first and last of the "k_t" estimates, while "ξ_t" is the error term. Then to forecast two periods ahead, the definition of $\hat{k}_{t-1}$ is just substituted for moved back in time one period:

$$\begin{aligned} \hat{k}_t &= \hat{k}_{t-1} + \hat{c} + \xi_t \\ &= \left(\hat{k}_{t-2} + \hat{c} + \xi_{t-1}\right) + \hat{c} + \xi_t \\ &= \hat{k}_{t-2} + 2\hat{c} + (\xi_{t-1} + \xi_t) \end{aligned} \quad (3.10)$$

To forecast "$\hat{k}_t$" at time $T + (\Delta t)$ with data available up to period T, the same procedure is followed and iterated for (Δt) times and the following equation is obtained:

$$\begin{aligned} \hat{k}_{T+(\Delta t)} &= \hat{k}_T + (\Delta t)\hat{c} + \sum_{n}^{(\Delta t)} \xi_{T+n-1} \\ &= \hat{k}_T + (\Delta t)\hat{c} + \sqrt{(\Delta t)}\xi_t \end{aligned} \quad (3.11)$$

If error term is ignored, forecast point estimates can be obtained, which follow a straight line as a function of (Δt), with slope $\hat{c}$:

$$\begin{aligned} \hat{k}_{T+(\Delta t)} &= \hat{k}_T + (\Delta t)\hat{c} \\ &= \hat{k}_T + (\Delta t)\frac{\hat{k}_T - \hat{k}_1}{T - 1} \end{aligned} \quad (3.12)$$

The forecasting of $\hat{k}_t$ is thus very simple: from a straight line drawn through the first $\hat{k}_1$ and the last $\hat{k}_T$ points are extrapolated and all other $\hat{k}_t$ points are ignored.

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3.5 The Singular Value Decomposition Method – The Weighted Least Square – The Maximum Likelihood Estimation

3.5.1 The Singular Value Decomposition Method (SVD)

The method can be described using the following steps:

1. The estimate of "a_x" is computed. The parameter vector "a_x" can be easily computed as the average over time of the logarithm of the central death rate. So "a_x" is given as follows,

$$\hat{a}_x = \frac{1}{T}\sum_{t=1}^{T} \ln(m_{x,t})$$

2. A matrix $Z_{x,t}$ for estimating "b_x" and "k_t" is created:

$$Z_{x,t} = \ln(m_{x,t}) - \hat{a}_x = b_x k_t$$

3. The Singular Value Decomposition to matrix $Z_{x,t}$ is applied:

$$ULV' = SVD(Z_{x,t}) = L_1 U_1 V_1 + \ldots + L_X U_n V_T$$

where U represents the age component,
L are the singular values and
V represents the time component.

4. Singular Value Decomposition Dialog from Biplot in Microsoft Excel is selected, by running the program: $\hat{k}_t$ is derived from the first vector of the time – component matrix and the first singular value $(\hat{k}_t = L_1 V_1)$

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and $\hat{b}_x$ are derived from the first vector of the age – component matrix $(\hat{b}_x = U_1)$.

5. Approximate, a new matrix $\hat{Z}_{x,t}$ is generated by multiplying the estimated parameters $\hat{b}_x$ and $\hat{k}_t$ and applying the result for $x = 1$ and $t = 1$, $\hat{Z}_{1,1} = \hat{b}_1\hat{k}_1$ is got (Lee – Carter).

6. The logarithm of the central death rate is estimated as follows:

$$\ln\widehat{(m_{x,t})} = \hat{a}_x + \hat{Z}_{x,t} = \hat{a}_x + \hat{b}_x\hat{k}_t$$

3.5.2 The Weighted Least Square (WLS)

The SVD approach is not suitable for very low central death rates because the logarithms are not defined. A Weighted Least Square (Wilmoth, 1993) solves this problem by minimizing the following squared error:

$$\sum_{x=1}^{n}\sum_{t=1}^{T} d_{x,t}(m_{x,t} - a_x - b_x k_t)^2$$

A null number of deaths can then be assigned to low central death rates. The choice of "$d_{x,t}$" is statistically justified by $var[\ln m_{x,t}] \approx {}^{1}/_{d_{x,t}}$(Wilmoth, 1993).With the Weighted Least Square, no second stage estimation is needed for k_t (see details in Chapter 7).

3.5.3 The Maximum Likelihood Estimation (MLE)

Under the LC method, the errors are expected to have the same variance over all ages (homoscedastic), which is not always correct (Lee and Miller, 2001). Alho (2000) suggested using a Maximum Likelihood Estimation (Wilmoth, 1993) based on a Poisson number of deaths $D_{x,t}$ ($D_{x,t} \sim Poisson\left(\widehat{m}_{x,t}E_{x,t}\right)$ where $\widehat{m}_{x,t} = \exp\left(\hat{a}_x + \hat{b}_x\hat{k}_t\right)$) (Brillinger, 1986), because it allows heteroscedasticity.

Maximizing the log – likelihood function below $\hat{a}_x$, $\hat{b}_x$ and $\hat{k}_t$ are provided (Wilmoth, 1993)

$$l = \sum_{x=x_1}^{x_A} \sum_{t=t_1}^{t_1+T-1} [\ D_{x,t} \ln\left(\widehat{m}_{x,t}E_{x,t}\right) - E_{x,t} \exp(\ a_x + b_x k_t) - \ln\left(D_{x,t}!\right)\ ]$$

where $E_{x,t}$ is the number of exposures – to – risk ages x and $x + 1$ in year t.

This approach is also called Poisson log – bilinear modeling and it is fully described in Brouhns et al. (2002).

3.6 Advantages of Lee – Carter Method

Literally, this method has been found to have a number of appealing features:

- The Lee – Carter model is widely adopted because of its advantages as no subjective judgment is involved; forecasting is based on long term trends and there is availability of confidence intervals (Lee and Carter, 1992).

- The use of the model indicates the dynamic and the static structure of the mortality process (i.e. the level of mortality over time and the age profile as estimated across the entire time series). It can be used to generate missing age profiles of mortality (Tabeau, Van Den Berg Jeths and Heathcote, 2001).

- The traditional method, if it is to be used by a skilled forecaster, has the advantage of drawing on the full range of relevant knowledge for the middle forecast and the high-low range (Li, Lee and Tuljapurkar, 2004).

- Its strengths are simplicity and robustness especially in the case of largely linear time trends in age-specific death rates (Booth et al., 2006). In particular, it involves a two-factor (age and time) model and uses a decomposition to extract a single time – varying mortality index with a time series model (Wang and Liu, 2010).

- Lee and Carter (1992) referred that the advantage of this new method is that it combines a rich, demographic model with a model of time series, allowing probabilistic intervals to the respective forecasts and at the same time it is based in a relatively long set of historical information. Given the underlying demographic model, the method allows the mortality rates to decrease exponentially, not being necessary to establish an arbitrary superior limit or rationalize in some way the deceleration of the life expectancy gains, since this deceleration happens without any additional restrictions. They also referred at the possibility of obtaining indirectly the mortality for periods to which the age specific mortality rates are not available (Coelho, 2001)

- Another advantage of the LC approach is that once the data are fitted to the model and the values of the vectors $\hat{a}_x, \hat{b}_x$ and $\hat{k}_t$ are found only the mortality index $\hat{k}_t$ needs to be predicted (Wang, 2007).

- Finally, Li, Chan and Cheung (2011) summarized various reasons why the Lee – Carter model is particularly suitable for actuarial applications. Firstly, the model has a relatively small number of parameters and the parameters are fairly easy to interpret. Secondly, it attaches probabilistic confidence intervals to central mortality forecasts so that actuaries can assess how light (and how heavy) future mortality improvements may turn out to be. Thirdly, sample paths of future mortality can be generated via the stochastic components of the model, allowing actuaries to quantify the risk of unanticipated mortality improvement by using prevalent risk measures such as value at risk (VaR) and conditional tail expectation (CTE).

3.7 Disadvantages of Lee – Carter Method

Literally, this method has been also found to have a number of disadvantages:

- Booth et al. (2002) and Di Cesare and Murphy (2009) noted that a major problem with the Lee – Carter method is the assumption that the age component is invariant over time and hence no age – time interaction is taken into account (Lee and Miller, 2001). Lee and Miller (2001,p .545) suggested that a 'simple and satisfactory solution' to this problem is to base the forecast on data since 1950, as Tuljapurkaert al. (2000) had done, so that the assumption that the age component is invariant applies to only half the century.

- Also, Li, Lee and Tuljapurkar (2004) noted that official mortality projections in low mortality countries have been found to under – predict mortality, which declines and gains in life expectancy when compared to the subsequent outcomes (Lee and Miller, 2001).

- Moreover, it is not clear how to interpret a variable's high – low range unless a corresponding probability for the range is stated. The traditional method, unfortunately, cannot provide such a probabilistic interpretation. Nor is it clear whether the range is supposed to refer to annual variations or to some sort of general trend or long run average (Li, Lee and Tuljapurkar, 2004).

- Finally, the combination of the high – low range with other uncertainties is not certain (Li, Lee and Tuljapurkar, 2004).

3.8 Advantages of ARIMA Models

A general category of models representative of a wide class of time series is the autoregressive integrated moving average (ARIMA) model, as it has been noted by McNown and Rogers (1989) gather a number of advantages, which make them attractive candidates for time series modeling, among them the most significant are represented as follows:

- They can approximate most time series observed in practice, incorporating either local stochastic trends or universal fixed trends as the data indicate.

- ARIMA models have been demonstrated to be successful forecasting models, even in comparisons with large simultaneous equation models (Ascher, 1978, McNown, 1986).

- Unlike simple extrapolative models, ARIMA models are actually reduced forms of some underlying structural model (McDonald, 1979, Zellner and Palm, 1974); therefore, they implicitly embody a theoretical structure that mayor may not be expressed explicitly.

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- There are generalizations of the univariate ARIMA models to multivariate forms, which can also be related to an underlying structural model (Tiao and Box, 1981), providing thereby instruments for examining directly the relations among variables using the time series methodology (Bohara, Bradley, and McNown, 1987, Sims, 1980) and for generating forecasts of potentially greater accuracy than those of univariate models (Kling and Bessler, 1985).

3.9 Lee – Carter Method with Limited Data

In this section, it will be represented the ways in which the LC method can be used for countries with limited mortality data. Li, Lee and Tuljapurkar (2004) showed that the four following parameters are required to produce a LC forecast, where the LC notation is given in parentheses:

1. A baseline age schedule of mortality (a_x)
2. The relative pace of change by age (b_x)
3. The overall rate of change (drift in the random walk model for (k_t)
4. Variability about the trend in mortality decline (the variance of the innovation term in the random walk model).

In order to apply the LC method to countries with limited mortality data, at least two questions need to be answered. The first is how to apply the LC method to mortality data collected at unequal intervals a number of years apart. The second question is what quality of results can be expected to derive from the LC method, when the historical data are only available for a small number of time points, as in the case of China.

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To answer the first question, it is noted that the standard statistical time series analysis cannot be used in this case. However, if we make the assumption that k_t follows a random walk with drift, then as few as three observed $m(x,t)$ schedules can be used to find all the parameters of the LC model. In this way, it is not necessary to figure out the appropriate model, which would require much more data. Since the condition for the k_t in the LC model is to be a random walk with drift that mortality declines stably, which has already been observed for many countries, both developed and Third World, the strong assumption is defensible.

For mortality data be collected at times $u(0), u(1), \dots, u(T)$, parameters a_x are calculated as follows:

$$\sum_{t=0}^{T} \frac{\ln[m(x,u(t)]}{T}$$

Applying SVD on $[\ln[m(x,u(t)] - a_x]$, b_x and $k(u(0)), k(u(1)), \dots, k(u(T))$ are obtained.

For the standard LC model, the random walk with drift is expressed as follows:

$$k(t) = k(t-1) + c + e(t)\sigma,$$

where $e(t) \sim N(0,1), E\big(e(s), e(t)\big) = 0$.

For $k(u(t))$, we have

$$k\big(u(t)\big) - k\big(u(t-1)\big) = c[u(t) - u(t-1)] + \sigma\big[e(u(t-1)+1) + \dots + e\big(u(t)\big)\big] \quad (3.14)$$

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Thus, for different t, $\left[k\left(u(t)\right) - k\left(u(t-1)\right)\right]$ are no longer identically distributed. Consequently, estimated c and σ from (3.14) cannot be as simple as that for i.i.d. variables.

Because the means of the second term in the right – hand side of (3.14) are still zero, the unbiased estimate of c is obtained as in the following equation:

$$\hat{c} = \frac{\sum_{t=1}^{T}\left[k\left(u(t)\right) - k\left(u(t-1)\right)\right]}{\sum_{t=1}^{T}[u(t) - u(t-1)]} = \frac{k\left(u(T)\right) - k\left(u(0)\right)}{u(T) - u(0)}$$

To answer the second question is sufficient to consider the following two cases:

1. The Mean Forecasts Based on Data at Few Time Points:

A special feature of the LC method is that it converts the task of forecasting an age – specific vector $\ln[m(x,t)]$ into that of forecasting a scalar "k_t".

2. The Probability Intervals for Forecasts Based on Data at Few Time Points – How Accurate Can They Be? :

For only two years of data, the LC method cannot provide uncertainty forecasts, since there is no deviation from the linear change of "k_t".

3.10 Literature Review In Reference to Lee – Carter Method

Many scholars have studied LC Method and the most significant references are presented in the following pages.

First of all, Lee and Carter (1992) fitted the total mortality in United States in 1990 to 2065 and Carter – Lee 1992 fitted the total mortality in

United States in 1900 – 1989 as they used Box – Jenkins techniques to estimate and forecast "k_t".

Renshaw and Haberman (2000) described a methodology based on generalized linear modeling (GLM) regression for the construction of a class of mortality rate reduction factors for representing the time trends in mortality rates. Their methodology was initially designed with a view to the retrospective study of such factors, but it is also capable of a prospective interpretation and application, so that forecasts may be generated. Renshaw and Haberman (2001) noted that this formulation also encompasses the model structure underpinning the Lee – Carter (LC) approach to forecasting mortality (Lee and Carter, 1992, Lee, 2000, Lee and Miller, 2001) and they investigated how the LC approach might be adapted to construct reduction factors for use in an actuarial context

Lee (2000) provides a summary of the model's development, extensions and applications such as stochastic forecasts of social security system finances.

Tuljapurkar, Li and Boe (2000:792) used this method to project for a number of industrial nations that "e_0" will be 1 to 4 years higher in 2050 than indicated by official projections, with larger discrepancies for Japan. On the basis of this method, Tuljapurkar, Li and Boe (2000) identified a universal pattern of constant rates of mortality that decline in the world's most developed countries, with rates of decline higher than those incorporated in official projections, leading to higher forecast levels of life expectancy.

Lee and Miller (2001) examined hypothetical forecast errors and compared them with Social Security forecast errors. Hypothetical historical projections suggested that LC tended to under project gains, but by less than did Social Security. The main purpose is to make a careful and detailed assessment of the performance of the Lee – Carter method for forecasting mortality. They also conducted less detailed experiments using the method to produce forecasts for Japan, Canada, France and Sweden, with jump – off

year in 1950. They also examined age patterns of decline during the 20th century and considered the possibility that the age pattern has changed over time contrary to the assumptions of the method. Finally, the results suggested that the LC method produces surprisingly good forecasts over rather long time periods. Used for long – term forecasts within the 20th century, it would generally have tended to under – predict future gains. The probability intervals, despite some problems, also do a surprisingly good job of containing the true outcomes.

Coelho (2001), for the first time, has applied the Lee – Carter method to Portuguese mortality data for 1942 to 1999. His work, focusing mainly on the aspects of stability and stationarity of the general mortality index, points to the existence of a break in the overall trend in mortality and consequently for the selection of a time series model different from the random walk proposed by Lee and Carter. An out – of – sample forecasting experiment confirms the superiority of a model that explicitly allows for a structural break in the trend occurring in 1976 which coincides with a period of abrupt politic, economic and social changes. The application of Lee – Carter method to the Portuguese mortality, with the enclosed adaptations, according to the tests on forecasting performance, presented, in general, a sufficiently reasonable behaviour.

Booth et al. (2002) wanted to improve the validity and reliability of the Lee – Carter method by improving the fit of the base model and by addressing departures from linearity in the dominant time component and the violation of the invariance assumption in the age component. Specifically, this involved developing methods for improving the estimation of the time component, for identifying the most appropriate period to which to apply the Lee – Carter method with a linear ARIMA and for incorporating age – time interactions into the base model.

Renshaw and Haberman (2003a) presented a reinterpretation of the model underpinning the Lee – Carter methodology for forecasting mortality (and other vital) rates. A parallel methodology based on generalized linear

modelling is introduced. They investigated further how this GLM – based approach to the construction of mortality reduction factors might be adapted along parallel lines to the LC approach for forecasting mortality. A key difference between the methods concerns the treatment of time. In the LC approach, time is modelled as a multiplicative factor and estimated together with the b_x, by SVD. In the GLM approach, time is modelled as a known covariate, having first established a well – defined origin and only the "b_x" is estimated accordingly. Finally, they investigated further (Renshaw and Haberman, 2002) whether the LC approach can also be adapted to capture such features, how the LC approach in general might be reformulated as a Poisson response generalized bilinear model.

Li, Lee and Tuljapurkar (2004) discussed ways in which the LC method can be used for countries with limited mortality data. This has been already discussed in details in section 3.9.

Booth et al. (2005) presented the results of an evaluation of the Lee – Carter, Lee – Miller and Booth – Maindonald – Smith variants based on data by sex for ten countries. The evaluation involved fitting the different variants to data up to 1985, forecasting for the period since that date and comparing the forecasts with actual mortality in that period. It has been shown that the LM and BMS variants are superior to LC in both forecast accuracy and width of prediction interval. The decomposition of error has demonstrated that jump – off bias is a significant source of error for LC. These results confirm the findings of Lee and Miller (2001). In addition, the LC adjustment by fitting to D_t has been shown to be marginally inferior to the other two adjustment methods. It has been also shown that in a majority of cases, BMS is marginally superior to LM in accuracy and uncertainty. Finally, these results are limited to the forecasting period and countries adopted. It is likely that they may be generalized to other developed countries. The extent to which they may be generalized to other forecasting periods is less clear.

Haberman and Russolillo (2005) investigated the feasibility of using the Lee – Carter methodology to construct mortality forecasts for the Italian population. They fitted the model to the matrix of Italian death rates for each gender from 1950 to 2000. A time – varying index of mortality is forecasted in an ARIMA framework and is used to generate projected life tables. In particular, they focused on life expectancies at birth and (for the purpose of comparison) they introduced an alternative approach for forecasting life expectancies on a period basis. Finally, the difference in results is evident for both genders.

Booth et al. (2006) compared the short – to medium – term accuracy of five variants or extensions of the Lee – Carter method for mortality forecasting. These included the original Lee – Carter, the Lee – Miller and Booth – Maindonald – Smith variants and the more flexible Hyndman – Ullah and De Jong – Tickle extensions. These methods are compared by applying them to sex – specific populations of 10 developed countries using data for 1986 – 2000 for evaluation. The referred countries were: Australia, Canada, Denmark, England and Wales, Finland, France, Italy, Norway, Sweden and Switzerland. All variants and extensions are more accurate than the original Lee – Carter method for forecasting log death rates, by up to 61%.

The results of the comparative evaluation of forecasts for the period 1986 – 2000 showed that while each of the four variants and extensions is more accurate in forecasting log death rates than the original Lee – Carter method, none is consistently more accurate than the others. It was found that on average Hyndman – Ullah and De Jong – Tickle provided the most accurate forecasts of log death rates; however, the differences among the four methods are small and are not significant. BMS provided marginally more accurate forecasts of life expectancy but there were no significant differences between the five methods for this measure.

De Jong and Tickle (2006) aimed to improve on the LC approach by strengthening both the modeling and estimation framework.

Kiossi et al. (2006) studied the performance of the LC model on data from the Nordic countries. Three approaches were implemented: Singular Value Decomposition (SVD), Weighted Least Square (WLS) and Maximum Likelihood Estimation (MLE). Also, they proposed a residual bootstrapped approach in order to compute the confidence interval of forecasted life expectancies and death rates. Uncertainties produced with this method incorporate the variability from all parameters, while the original Lee – Carter method focuses on the variability in the model time – varying parameter.

Pedroza (2006) presented a Bayesian approach to forecast mortality rates. This approach formalizes the Lee – Carter method as a statistical model accounting for all sources of variability. Markov chain Monte Carlo Methods are used to fit the model and to sample from the posterior predictive distribution. Pedroza showed how multiple imputations can be readily incorporated into the model to handle missing data and presents some possible extensions to the model. The Methodology is applied to U.S. male mortality data. Finally, Pedroza compared results from the Bayesian model and the Lee – Carter model for U.S. male data and showed that confidence interval widths are different. For some age groups, the Lee – Carter intervals do not cover the observed values. In contrast, the Bayesian intervals cover the observed values and reflect the true uncertainty of these forecasts.

Delwarde, Denuit and Eilers (2007) demonstrated that it is possible to smooth the estimated b_x's in the Lee – Carter and Poisson logbilinear models for mortality projection. To this end, a penalized least – squares/maximum likelihood analysis is performed. The optimal value of the smoothing parameter is selected with the help of cross validation.

An important aspect of the methodology presented in the reference article is that the time factor $\hat{k}_t$ is intrinsically viewed as a stochastic process as initially proposed by Lee and Carter (1992). Box – Jenkins techniques are used to estimate and forecast k_t within an ARIMA times series model. These forecasts in turn yield projected age – specific mortality rates, life expectancies and annuities single premiums. The random walk with drift (or

ARIMA(0,1,0) process) features prominently in the published applications of the Method proposed by Lee and Carter (1992). Here, the optimal ARIMA model is selected for projecting the k_t's to the future. The ARIMA(1,1,0) is selected for the k_t's estimated by penalized Least squares, whereas the ARIMA(0,1,1) is preferred when the k_t's are estimated by penalized Poisson likelihood.

A naive solution consists of smoothing the estimated b_x's, without modifying the a_x's or the k_t's. This may however produce large discrepancies between observations and model predictions. To avoid this problem, the smoothing of the b_x's could be performed in the estimation phase. A convenient way to impose some smoothness is to introduce a roughness penalty via penalized least squares in the Lee – Carter model and penalized maximum likelihood in the Poisson log – bilinear model.

3.10.1 Lee – Carter Model: Penalized least squares

It is assumed that $\widehat{m(x,t)}$ are the empirical death rates at age x in year t. The estimated Lee – Carter parameters are traditionally obtained by minimizing the following

$$\sum_{x,t}\left(\ln \widehat{m(x,t)} - a_x - k_t b_x\right)^2$$

This produces estimated b_x's and k_t's with irregular shape in the majority of empirical studies. To smooth the estimated b_x's, the following objective function is used

$$\sum_{x=x_{min}}^{x_{max}} \sum_{t=t_{min}}^{t_{max}} \left(\ln \widehat{m(x,t)} - a_x - k_t b_x\right)^2 + \boldsymbol{b}' P_b \boldsymbol{b}$$

where $P_b = \pi_b \boldsymbol{\Delta}'\boldsymbol{\Delta}$ with $\boldsymbol{\Delta} = \begin{bmatrix} 1 & -2 & 1 & 0 \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & 1 & -2 & 1 \end{bmatrix}$ and

π_b is the smoothing parameter.

It must be also referred that the term $\boldsymbol{b}' P_b \boldsymbol{b}$ penalizes irregular b_x's.

3.10.2 Poisson Log – Bilinear Model: Penalized Log Likelihood

In the Poisson log – bilinear model, the number of deaths D_{xt} recorded at age x in calendar year t is assumed to be Poisson distributed, with mean $er_{xt}m(x,t)$ (recall that er_{xt} is the exposure – to – risk for age x and year t). As far as the force of mortality $m(x,t)$ is concerned, the same form $\exp(a_x + k_t b_x)$ as in the Lee – Carter model is kept in the Poisson model.

In this case, penalized least squares become penalized log likelihood. Specifically, denoting as $\mathcal{L}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{k})$ the likelihood associated to the Poisson model, is expressed as follows:

$$\ln \mathcal{L}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{k}) = \sum_{x,t} \{\, D_{xt}(a_x + k_t b_x) - er_{xt} \exp(a_x + k_t b_x) \,\} + constant$$

and the estimations are obtained by maximizing the following equation

$$\ln \mathcal{L}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{k}) - \frac{1}{2}\boldsymbol{b}' P_b \boldsymbol{b} = \ln \mathcal{L}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{k}) - \frac{1}{2}\pi_b \boldsymbol{\Delta}'\boldsymbol{\Delta}\boldsymbol{b}.$$

Delwarde et al. (2007) compared the above models with Hyndman and Ullah model: these authors proposed a new method for forecasting age – specific mortality and fertility rates observed over time. Their approach extends the Lee – Carter model and allows for smooth functions of age and their approach seems robust for outlying years due to wars and epidemics. Basically, the time – dependent function linking age to forces of mortality is

decomposed in an appropriate functional basis and the coefficients are projected to the future using ARIMA models. However, this approach is restricted to the modelling of death rates, whereas the present contribution also deals with observed death counts.

Girosi and King (2007) demonstrated several previously unrecognized or insufficiently appreciated properties of the Lee – Carter model and used these properties to suggest where and when the model would be most applicable. Unlike Lee and Carter, however, in this study it is set $\sum_x b_x^2 = 1$ (they set $\sum_x b_x = 1$). This last choice is done only to simplify some calculations later on and has no bearing on empirical applications.

Li and Chan (2007) performed a systematic outlier analysis of the mortality index built in the Lee – Carter model. Through outlier detection they found that mortality levels in the United States and Canada are vulnerable to events like pandemics and wars. By incorporating the effect of outliers, they created an outlier – adjusted Lee – Carter model. Given the better fit and the enhancement in forecast efficiency, the outlier – adjusted version seems to be an attractive alternative to the original model in predicting the long – term trend of mortality rates.

Di Cesare and Murphy (2009) applied a range of alternative forecasting techniques to examples of causes of death with different underlying age and time patterns to assess which method copes better with the specificities of each case. Results showed major differences among the three forecasting techniques, Lee – Carter and its Booth – Maindonald – Smith variant, Age – Period – Cohort model and Bayesian approach.

Wang and Liu (2010) applied the four stochastic mortality models, Lee – Carter (LC) model together with its modifications and extensions including Lee and Miller (LM), Booth, Maindonald and Smith (BMS) and Hyndman and Ullah (HU) models and they investigated the goodness of fit in mortality forecasting of Taiwan mortality data spanning the period 1970 – 2007. To compare the goodness – of – fit for all the fitted models along the years, they

used five measures: ME, MAE, MSE, MPE and MAPE statistics, which are defined respectively, as follows:

$$ME(\hat{\mu}_{xt}) = \frac{1}{n}\sum_{i=1}^{n}(\mu_{xt} - \hat{\mu}_{xt})$$

$$MAE(\hat{\mu}_{xt}) = \frac{1}{n}\sum_{i=1}^{n}|\mu_{xt} - \hat{\mu}_{xt}|$$

$$MSE(\hat{\mu}_{xt}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mu_{xt} - \hat{\mu}_{xt})^2}$$

$$MPE(\hat{\mu}_{xt}) = \frac{1}{n}\sum_{i=1}^{n}\frac{\mu_{xt} - \hat{\mu}_{xt}}{\mu_{xt}}$$

$$MAPE(\hat{\mu}_{xt}) = \frac{1}{n}\sum_{i=1}^{n}\frac{|\mu_{xt} - \hat{\mu}_{xt}|}{\mu_{xt}}$$

where $\hat{\mu}_{xt}$ is the prediction,

μ_{xt} is the true value,

n is the number of observer and

t is the time.

The empirical results demonstrated that, when the fitted period is longer, the BMS model exhibits a better fits in Taiwan male mortality data and the LC model attains a better fit in Taiwan female mortality data. When the fitted period becomes shorter, however, the HU model has the best good of fit for Taiwan mortality data. In sum, different from the suggestion of Booth et al. (2006), the LC model does not perform least well and there are still some differences between the variants and extensions of LC model in forecasting accuracy. Besides, when the fitted period is longer, the best

forecasting model for Taiwan death rates is BMS model for male and LC model for female; when the fitted period is shorter, however, the best one is HU model for both genders.

Li, Chan and Cheung (2011) used Zivot and Andrews' procedure to detect any possible structural changes in the mortality indexes for the general populations of England and Wales and the United States. The results favor, for these two populations, a broken – trend stationary model rather than a simple random walk with drift that postulates pure linearity.

Lazar made some remarks regarding forecasting mortality rates using Lee – Carter model. Lazar has chosen to restrict his study for those people aged 63 and over, because mortality of this segment of population being important in pricing the immediate life annuities. The model is estimated using the approach proposed by authors (Lee and Carter, 1992), based on singular value decomposition method and also like a space state model. Moreover, it is useful to examine whether the time series $\ln(m_{xt})$ and k_t are cointegrated. If these two time series are cointegrated, the equation from Lee – Carter model will be the cointegration relation, thus it will be necessary to capture both short run and long run effects through an error correction model. Numerical results are obtained on the basis of mortality rates for the Romanian female population, during 1970 – 2002 periods.

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CHAPTER 4

Modification of Lee – Carter Method

4.1 Introduction

Many researchers have proposed various modifications for the LC method. These modifications include the LC method without adjustment and the methods proposed by Tuljapurkar et al. (2000), Lee and Miller (2001) and Booth et al. (2002). The forecast accuracy of the LC Method and its variants were first evaluated by Booth et al. (2005) and further studied by Booth et al. (2006).

4.2 The Lee – Miller (LM) Method

This method is a modification of the Lee – Carter method. Lee and Miller (2001) note that the Lee – Carter method has a major problem: the age component is invariant over time. This problem can be solved by observing the mortality experience of the world's leading industrial nations, which suggests the substantial age – time interaction. So, in this method, the age component, which applies to only half the century, is considered unchanged.

Lee and Miller (2001) observe that the US data did not entirely satisfactory fit with Lee-Carter model when using the fitting period 1900-1989 to forecast the period 1990-1997. This is due to the last year of the

fitting period and the actual rates in the respective year. Thus, Lee and Miller (2001) adopted 1950 as the first year of the fitting period. The same pattern was also adopted by Tulipurkar et al. (2000). Finally, the adjustment of "k_t" by fitting to "$e(0)$" was adopted to avoid the use of population data as required for fitting to "D_t" (Lee and Miller, 2001).

So, there are three differentiations from the Lee – Carter method:

1. The fitting period is reduced to begin in 1950.

2. The adjustment of "k_t" involves fitting to e(0) in year t

3. The jump-off rates are taken to be the actual rates in the jump-off year.

In this method, k_t is calculated by fitting a Poisson regression model, shown in equation 4.1:

$$\ln D(x,t) = \ln N(x,t) + \ln m'(x,t) + \varepsilon'(x,t) \quad (4.1)$$

where $D(x,t)$ denotes the annual number of deaths for age x, at time t and $N(x,t)$ denotes the population aged x, at time t.

The minimization criterion is given by the following equation 4.2:

$$deviance(t) = 2\sum_x \{ D(x,t) \ln \left[{D(x,t)}/{D'(x,t)} \right] - (D(x,t) - D'(x,t))\} \quad (4.2)$$

Where $D'(x,t)$ denotes fitted deaths and is expressed by equation 4.3.

$$D'(x,t) = N(x,t) \sum_x \{ \exp[a(x) + b(x)k'(t)]\} \quad (4.3)$$

Finally, this method gathers many advantages:

1. The Lee – Miller (2001) method is a (conditional) maximum likelihood procedure which assumes quasi-Poisson errors.

2. In the Lee – Miller (2001) method the minimization is easier to be calculated than Lee – Carter, because it does not require finding the root of a non – linear function.

3. The minimization criterion is a deviance, which is a standard statistical practice and is useful in comparisons, especially for determining the fitting period.

Among the Lee – Carter (1992) method, the Lee – Miller (2001) method, the Booth – Maindonald – Smith (2002) method, the Hyndman – Ullah (2007) method and the weighted Hyndman – Ullah (2011) method, Shang et al. (2010) showed that the Lee – Miller (2011) method gives the best point forecast accuracy for life expectancy.

4.3 The Booth – Maindonald – Smith (BMS) Model

The Booth – Maindonald – Smith (BMS) (2002) model is a variant of the Lee – Carter (1992) method for understanding the age – time interactions. This model aims to improve the validity and reliability of the Lee – Carter (1992) model.

The BMS (2002) model was developed after the analysis of mortality decline in Australia during the twentieth century, by the research of Booth et al. (2006). In the reference study, it is found that with BMS (2002) model the Australian mortality forecast was more reliable than the Lee – Cartel (1992) model. It is also found that there are not important differences to other

methods like Lee and Miller (2001), Hyndman and Ullah (2007) and De Jong and Tickle (2006).

Booth, Maindonald and Smith (2002) made three modifications, as following referred (Booth et al., 2002):

1. The fit of the base model to observed deaths is improved by adjusting "$k(t)$" by fitting it to the age distribution of deaths rather than to total deaths.

2. The departure from linearity in "$k'(t)$" is addressed by developing a method for identifying the most appropriate fitting period.

3. The base model is expanded to include higher terms.

The adjustment of "k_t" involves fitting to the age distribution of deaths, "$D_{x,t}$", rather than the total number of deaths, "D_t". The age distribution of deaths, "$D_{x,t}$", fitted by Poisson distribution to model deaths and by using deviance statistics to measure the 'goodness of fit' (Booth et al., 2002).

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CHAPTER 5

Extension of Lee – Carter Method

5.1 Introduction

There have been several extensions of the LC (1992) method. Of these, the extension proposed by Hyndman and Ullah (2007) has been receiving an increasing amount of attention in the fields of demography and statistics. Their method combined the ideas of nonparametric smoothing, functional principal component regression and functional data analysis, in order to forecast mortality and fertility rates. This method has been applied by Erbas et al. (2007) for forecasting breast cancer mortality rates in Australia. Furthermore, this method has been extended by Hyndman and Booth (2008) to improve the estimation of the variance and to include the forecasting of migration rates in Australia. Recently, Hyndman and Shang (2009) extended the referred method to allow the weighting of more recent data.

As it is literally referred, Lee and Carter (1992) have also proposed three different ways of extending the Lee – Carter analysis to male and female subpopulations or to any subpopulations:

1. Examine every subpopulation separately and then, if it is desirable, the researcher will search for dependence between the separate k – series.

2. Estimate a single k which drives changes in all the age – specific rates of both subpopulations.

3. Estimate the subpopulations jointly as a co – integrated process.

5.2 The De Jong – Tickle LC (smooth) Method

The De Jong – Tickle LC (smooth) (2006) method is a modification of the Lee – Carter method.

The Lee – Carter model may be written in the form as follows

$$y_t = \alpha + bk_t + \varepsilon_t, \qquad (5.1)$$

where y_t is the vector of the log – central death rates at each age in year t,

a and b are vectors of the corresponding Lee – Carter parameters for each age,

k_t is an index of the level of mortality in year t as in the Lee – Carter model and

ε_t is a vector of error terms at each age in year t.

De Jong and Tickle (2006) developed the more general specification as it is expressed as follows:

$$y_t = X\alpha + Xbk_t + \varepsilon_t, \qquad (5.2)$$

where X is a known "design" matrix with more rows than columns, unless $X = I$ in which case the model reduces to Lee – Carter model.

De Jong and Tickle (2006) referred to the equation (5.2) as the "model LC (smooth)" because in the model (5.2), X has fewer columns than rows.

This means that "a" and "b" parameters are fewer than the created age groups. Thus, the effects of the "k_t" time series are not independent across age but are constrained by the structure of X, imposing across – age smoothness.

The matrix X has various forms and the time series "k_t" is possible. The matrix X is based on B – splines. A single random walk with drift time series has been used. Maximum likelihood estimates of the model are derived using Kalman filtering and smoothing.

5.3 The Hyndman – Ullah (HU) Method

The Hyndman – Ullah (HU) (2007) method is a generalization of the Lee – Carter (1992) method. This method is widely known for forecasting age – specific mortality and fertility rates observed over time. Also it must be referred that HU (2007) method is a nonparametric method.

Hyndman and Ullah (2007) noted that this approach differs from the existing proposals in that it treats the underlying process as functional and provides estimation and forecasting which is robust to outliers. This methodology also applies to the fertility forecasting problem and to other areas as well, where forecasts of functional data are required. Hyndman and Ullah (2007) used this method to french mortality data and Australian fertility data and the forecasts obtained are shown to be superior to those from the Lee – Carter (1992) method and several of its variants.

The HU (2007) method includes the following steps:

1. Smooth the log mortality rates. The index $m_t(x)$ is used rather than $\ln m_{x,t}$ in order to represent the log mortality rates for age $x \in [x_1, x_p]$ in year t. The resulting equation is given below:

$$m_t(x_i) = f_t(x_i) + \sigma_t(x_i)\varepsilon_{t,i},\ i = 1, \dots, p, t = 1, \dots, n \qquad (5.3)$$

where $m_t(x_i)$ denotes the log of the observed mortality rate for age x_i in year *t*,

$\sigma_t(x_i)$ allows the amount of noise to vary with x_i in year *t*, thus rectifying the assumption of homoskedastic error in the LC model and

$\varepsilon_{t,i}$ is an independent and identically distributed standard normal random variable.

Typically $\{x_1, \dots, x_p\}$ are single years of age $(x_1 = 0, x_2 = 1, \dots)$ or denote 5 – years age groups.

2. Decompose the fitted curves via a basis function expansion using the following model:

$$m_t(x) = a(x) + \sum_{j=1}^{J} b_j(x)\, k_{t,j} + e_t(x) + \sigma_t(x)\varepsilon_t, \qquad (5.4)$$

where $m_t(x)$ is the functional form, which can be reconstructed using a linear interpolation technique,

$a(x)$ is the mean function which can be estimated by $\hat{a}(x) = \frac{1}{n}\sum_{t=1}^{n} f_t(x)$, $\{b_1(x), \dots, b_J(x)\}$, which is a set of the first *J* functional principal components,

$\{k_{t,1}, \dots, k_{t,J}\}$ is a set of uncorrelated principal component scores satisfying $\sum_{j=1}^{J} k_{t,j}^2 < \infty$,

$e_t(x)$ is the residual function with mean zero and

$J < n$ is the number of principal components to be used.

The decomposition is achieved using functional principal component analysis (FPCA).

3. Fit univariate time series models to each of the coefficients $\{k_{t,j}\}$, $j = 1, \dots, J$. This can be done using a robust method for principal components to avoid problems with outlying observations.

4. Forecast the coefficients $\{k_{t,j}\}$, $j = 1, \dots, J$ for $t = 1, \dots, n$ using the fitted time series models.

5. Use the forecasted coefficients derived from (5.4) to obtain forecasts of $m_t(x)$, $t = 1, \dots, n$. From (5.3), forecasts of $m_t(x)$ are also forecasts of $m_t(x)$.

6. The estimated variances of the error terms in (5.4) and (5.3) are used to compute prediction intervals for the forecasts.

5.3.1 Forecasting the functions

If the equations (5.3) and (5.4) are combined, the resulting equation will be as follows:

$$m_t(x_i) = a(x_i) + \sum_{j=1}^{J} b_j(x_i)\, k_{t,j} + e_t(x_i) + \sigma_t(x_i)\varepsilon_{t,i}$$

By conditioning on the observed data $\boldsymbol{I} = \{m_t(x), t = 1, \dots, n\}$ and the set of functional principal components $\boldsymbol{B} = \{b_1(x), \dots, b_J(x)\}$, the h – step – ahead forecast of $m_{n+h}(x)$ can be obtained by:

$$\hat{m}_{n+h|n}(x) = E[m_{n+h}(x)|I, B] = \hat{a}(x) + \sum_{j=1}^{J} b_j(x)\, \hat{k}_{n+h|n,j},$$

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where $\widehat{k}_{n+h|n,j}$ denotes the h – step – ahead forecast of $k_{n+h,j}$ using a univariate time series model, such as an ARIMA model (Box et al., 2008), or an exponential smoothing state space model (Hyndman et al., 2008).

In conclusion, the HU (2007) method can be used to forecast functional time series data. This method is also used in epidemiology and finance. According to Hyndman and Ullah (2007), the HU (2007) method has better results in fertility and mortality forecasting for several reason:

1. Compared to other methods, more complex dynamics is allowed by setting $J > 1$, thus allowing higher order terms to be included.

2. Nonparametric smoothing reduces the observational noise.

3. The use of robust methods avoids problems of outlying years, especially around the world wars. It has the added advantage of providing interesting historical interpretations of dynamic changes by separating out the effects of several orthogonal components.

4. Finally, the method can be easily realized by R packages which methodology is available from Hyndman.

Among the Lee – Carter (1992) method, the Lee – Miller (2001) method, the Booth – Maindonald – Smith (2002) method, the Hyndman – Ullah (2007) method and the weighted Hyndman – Ullah (2011) method (more details are given in section 5.4), Shang et al. (2010) showed that the robust Hyndman – Ullah (2007) method provides the best interval forecast accuracy of life expectancy.

5.4 Weighted Hyndman – Ullah (HUw) Method

The weighted Hyndman – Ullah (HUw) (2011) method follows the same smoothing technique as the Hyndman – Ullah (2007) method. In this method, to estimate $a(x)$ and $b_j(x)$, geometrically decaying weights are used.

HUw (2011) differs from HU (2007) method at three points:

1. The estimation of $a^*(x)$ is computed as a weighted average, i.e.

$$\hat{a}^*(x) = \sum_{t=1}^{n} w_t f_t(x)$$

where $\{w_t = \lambda(1-\lambda)^{n-t}, t = 1, \dots, n\}$ denotes a set of weights and
$0 < \lambda < 1$ denotes a geometrically decaying weight parameter.

The parameter λ can be estimated from data like Hyndman and Shang (2009).

2. Using functional principal component analysis (FPCA), the following equation is given:

$$m_t(x) = a^*(x) + \sum_{j=1}^{J} b_j^*(x)\, k_{t,j} + e_t(x) + \sigma_t(x)\varepsilon_t$$

where $a^*(x)$ is the weighted functional mean and
$\{b_1^*(x), \dots, b_j^*(x)\}$ is a set of weighted functional principal components.

3. By conditioning on the observed data $\boldsymbol{I} = \{m_t(x), t = 1, \dots, n\}$ and the set of functional principal components $\boldsymbol{B}^* = \{b_1^*(x), \dots, b_j^*(x)\}$, the h – step – ahead forecast of $m_{n+h}(x)$ can be obtained by:

$$\widehat{m}_{n+h|n}(x) = E[m_{n+h}(x)|\boldsymbol{I}, \boldsymbol{B}^*] = a^*(x) + \sum_{j=1}^{J} b_j^*(x)\, \hat{k}_{n+h|n,j}.$$

Among the Lee – Carter (1992) method, the Lee – Miller (2001) method, the Booth – Maindonald – Smith (2002) method, the Hyndman – Ullah (2007) method and the weighted Hyndman – Ullah (2011) method, Shang et al. (2010) showed that the weighted Hyndman – Ullah (2011) method provides the most accurate point forecasts and the most accurate interval forecasts of mortality rates.

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CHAPTER 6

Other Methods to Examine Mortality

6.1 Introduction

There are many methods to study the mortality. The most important methods are chosen to be discussed in this chapter.

6.2 The Age – Period – Cohort (APC) Model

The Age – Period – Cohort (APC) (1980) model has been developed to estimate age, period and cohort patterns in time series. The model has the following form:

$$\ln(m_{atc}) = f(a) + g(t) + h(c) \qquad (6.1)$$

where m_{atc} is a log – death rates at age a in time period t for persons in birth cohort $c = t - a$ (Di Cesare M. and Murphy M., 2009). This model has a limit: if two of a, t or c are known, the third component will be known exactly as well from the equation $c = t - a$. So, as it can be seen, there is linear association between the components a, t and c.

According to Tabeau et al. (2001, page 16): “an APC model can be classified as symptomatic: period effects approximate contemporary factors, such as health status of the population and cohort effects approximate historical factors”. The APC (1980) model can be estimated using Poisson maximum likelihood (or weighted least – squares) methods. Finally, this model is popular in epidemiology.

6.3 The Bayesian Model

Many scholars have studied the Bayesian model (2006). For instance, Girosi and King (2008) studied a Bayesian hierarchical model which is able to pool information from similar cross – sections such as age groups or time in an efficient way. With the Bayesian model (2006) it is possible to incorporate a priori information about the data. This prior information is then used along with the observed data to define the posterior distribution on which inference is based (Pedroza, 2006). The prior knowledge used in this case is that the expected value of the dependent variable μ_{at} – where it is assumed that the observed value $m_{at} \sim N(\mu_{at}, \sigma^2)$ – varies smoothly over age, 0 to A, time, 0 to T, and age/ time according to the following prior (Di Cesare M. and Murphy M., (2009)):

$$H\left[\mu, \theta_{age}, \theta_{time}, \theta_{age/time}\right] \equiv \frac{\theta_{age}}{AT} \int_0^T dt \int_0^A da \left(\frac{d^2}{da^2} \left(\mu(a,t) - \bar{\mu}(a)\right) \right) + \frac{\theta_{time}}{AT} \int_0^T dt \int_0^A da \left(\frac{d^2}{da^2} \mu(a,t) \right) + \frac{\theta_{age/time}}{AT} \int_0^T dt \int_0^A da \left(\frac{\vartheta^3 \mu(a,t)}{\vartheta a \, \vartheta t^2} \right) \quad (6.2)$$

In the equation (6.2), the first term smoothes over age groups, the second term smoothes the time series over time and the third term is a mixed

smoothness functional ensuring that the curvature of the time series varies smoothly over age groups (Girosi and King, 2008).

As it can be concluded, the Bayesian model (2006) allows incorporation of levels of uncertainty in the model. With the Bayesian hierarchical model better results are exported when applicable a priori knowledge is incorporated in the model. Last but not least, if the prior information is unknown, nothing will be lost in forecasting.

Pedroza (2006) realized that implementation of the Bayesian model (2006) is more complex. However, Pedroza believed that it is important for forecasts and their estimated errors to reflect the underlying uncertainty of the model and data. Furthermore, a Bayesian analysis allows the investigator to test the validity of the model through the use of posterior predictive checks. A second disadvantage of the Bayesian approach is in the specification of prior information. If true prior information is known, it should be incorporated into the Bayesian model. However, it is not always easy to quantify prior information in terms of a prior distribution.

6.4 Parameterized Model mortality schedules

6.4.1 The Heligman – Pollard Model

The age profile of mortality can be described by the following equation:

$$f(x) = f_1(x) + f_2(x) + f_3(x) \quad (6.3)$$

where $f(x)$ denotes the mortality rate, probability or intensity at age x,

$f_1(x)$ denotes the mortality probability in childhood,

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$f_2(x)$ denotes the mortality probability in middle life and
$f_3(x)$ denotes the mortality probability in older ages

According to Thiele (1872), the following equations are given:

$$f_1(x) = a_1 \exp(-b_1 x) \qquad (6.4)$$

and

$$f_3(x) = a_3 \exp(b_3 x) \qquad (6.5)$$

According to Rogers and Planck (1983) and Rogers and Watkins (1986), $f_2(x)$ can be expressed from the double exponential distribution of Coale and McNeil (1972) as follows:

$$f_2(x) = a_2 \exp\{-a_2(x - \mu_2) - \exp[\gamma_2(x - \mu_2)]\} \qquad (6.6)$$

As a result, replacing the respective equations (6.4), (6.5) and (6.6) in the initial equation (6.3), the equation of the mortality rate, probability or intensity at age x is given.

The alternative equation given by Heligman and Pollard (1980) is as follows:

$$f(x) = A^{(x+B)^C} + D \exp[-E(\ln x - \ln F)^2] + \frac{GH^x}{1 + GH^x} \qquad (6.7)$$

This equation called “the eight-parameter Heligman – Pollard model”.

The advantage of this method is that it offers a representation of the age distribution of mortality compared with ‘relational’ methods or the Brass logit model. However, Carter and Lee (1992) believed that the method also may suffer from a number of disadvantages:

1. While the number of parameters necessary to express a single age distribution is relatively small, the number that must be forecasted is rather large ranging to eight in McNown and Rogers (1989).

2. This range of parameters means that probability intervals for the forecasts must take into account the covariances of the parameter forecast errors. This complication may be the reason for neither article presents probability intervals.

3. There is some evidence that the time series of parameter values behave erratically, at least for the United States (McNown and Rogers (1989), making them difficult to be forecasted.

4. This could be one reason that the forecasts in McNown and Rogers (1989) were only for a 15 – year horizon, so it is not clear whether longer term forecasts would be well behaved.

5. Perhaps also related to the third point, many of the coefficients in the time series models in the paper are insignificant, with some standard errors exceeding the estimate.

McNown R. and Rogers A., (1992), fitted the above model with ARIMA models. Their study has demonstrated the use of a parameterized mortality schedule to represent age patterns of mortality by cause and the viability of forecasting cause – specific and total mortality by the application of time series methods to the parameter histories of such schedules. The multiexponential function adequately approximates the variety of age patterns displayed by the leading causes of death in the United States and projection of these parameters into a hold – out sample yields reasonably accurate forecasted age patterns. The structure of the mortality age pattern embodied in the multiexponential model becomes a source of increased accuracy as the forecast horizon is lengthened.

The power of the technique lies in the ability of the multiexponential model to reproduce the age pattern of mortality for each cause of death. The relative constancy of these profiles over time allows one to use a smaller set of time varying parameters to project changes in the level of the curve over time. With a small number of varying parameters, simple multivariate time series methods, such as vector autoregressions, become feasible and forecasts from such models may show even greater accuracy. Such a multivariate extension also would allow the introduction of socioeconomic variables to reflect the various causal influences on mortality.

6.4.2 Generalized Linear Modelling (GLM)

Renshaw (1991) observed that many of the one – factor parameterization functions are special cases within the GLM framework. Placing such models in the GLM context provides a unified framework and facilitates generalizations such as the extension of the models to include period (time) for forecasting.

Renshaw, Haberman and Hatzoupoulos (1996) proposed a two – factor model based on an over – dispersed Poisson distribution of deaths and comprising a Gompertz – Makeham graduation term in combination with a multiplicative age – specific trend adjustment term as follows:

$$\log \mu_{x,t} = \beta_0 + \sum_{j=1}^{s} \beta_j L_j(x') + \sum_{i=1}^{r} a_i t'^i + \sum_{i=1}^{r}\sum_{j=1}^{s} \gamma_{ij} L_j(x') t'^i \quad (6.8)$$

where $\mu_{x,t}$ is the force of mortality at age x in year t, a_i, b_j and γ_{ij} are unknown parameters,

$L_j(x')$ are denote Legendre polynomials of degree j and

x' and t' are x and t rescaled over the interval $[-1, 1]$.

They found that the model provided an adequate fit to mortality for UK male assured lives aged 22 to 89 and of duration five and over.

Other researchers have also found that two – or higher – degree polynomials in time are required to attain a satisfactory fit, but lead to implausible forecasts.

Renshaw and Haberman (2000) used GLM to model mortality reduction factors and identified the conditions under which the underlying structure of their model is the same as that of the Lee – Carter model. They developed a GLM – based approach that parallels the Lee – Carter method including matching observed and expected total deaths (Renshaw and Haberman, 2003c) and adapted the Lee – Carter method for use it in forecasting mortality reduction factors (Renshaw and Haberman, 2003b). Renshaw and Haberman (2003a) extended the Lee – Carter model to include the second SVD term to allow for ‘age – specific enhancement’ and compared its forecast with similarly – enhanced GLM and Poisson log – bilinear forecasts.

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CHAPTER 7

Wilmoth's Weighted Least Squares (WLS)

7.1 Introduction

Lee and Carter proposed this method for use in forecasting total mortality, but it can also be used to forecast trends in mortality by cause of death. Since some cause-specific death rates are zero at certain ages, however, the fitting procedure must be modified in order to avoid taking logarithms of zero. This problem is avoided if the model is fit by weighted least squares (WLS), with weights equal to the observed number of deaths in each cell of the data matrix (since the weights are zero when $m(x,t) = 0$, an arbitrary value can be assigned to $\ln m(x,t)$ in this case without affecting the results) (Wilmoth, 1993). This is also the appropriate choice from a statistical point of view, since the variance of $\ln m(x,t)$ is approximately $1/d_{xt}$, where d_{xt} equals the number of deaths observed at age x and time t (Wilmoth, 1989).

Wilmoth (1993) notes that the WLS technique has other advantages in addition to avoiding the "zero-cell" problem. Since the model is fit by WLS, the predicted values are closest to observed deaths rates for those ages and years when the raw number of deaths was highest, thus at younger ages. Conversely, the fit is noticeably imperfect only for ages and years where few deaths occur anyway. For this reason, the Lee – Carter model reproduces summary indicators of mortality, such as life expectancy at birth, almost perfectly when fit by WLS.

Rather than using the second stage estimation of k_t as described in Lee and Carter (1992), Wilmoth (1993) proposes two alternative estimation strategies: a weighted least square (WLS) and a maximum likelihood (MLE) technique. Both techniques have the significant advantage, over the original Lee-Carter formulation, that they deal naturally with the case in which the observed number of deaths is zero, which occurs when analyzing cause-specific data and/or when dealing with small countries.

The WLS technique is based on the recognition that one could weight the first stage of Lee-Carter in such a way that observed and predicted deaths are closer to each other. To be specific, Wilmoth suggests finding the parameters a_x, b_x and k_t of the Lee – Carter model of (3.1) as the solution of the weighted least squares (WLS) problem

$$\min_{a_x,b_x,k_t} \sum_{x,t} d_{x,t} \, (m_{x,t} - a_x - b_x k_t)^2, \qquad (7.1)$$

where d_{xt} is the observed number of deaths in age group x at time t, and where k_t and b_x satisfy the usual constraints of the Lee - Carter model. By weighting the error by the observed number of deaths, the expression above gives more weight to those age groups and years with large numbers of deaths, and the resulting estimates are more likely to fit the total number of deaths in each year.

To solve the equation (7.1) it is necessary to compute its first derivatives with respect to a_x, b_x and k_t, and to set these equal to zero. Then, solving for the required parameters yields three sets of “normal equations”, which must be solved numerically:

$$\hat{a}_x = \frac{\sum_t d_{xt}(m_{x,t} - \hat{b}_x \hat{k}_t)}{\sum_t d_{xt}} \quad (7.2)$$

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$$\hat{b}_x = \frac{\sum_t d_{xt}\hat{k}_t(m_{x,t} - \hat{a}_x)}{\sum_t d_{xt}\,\hat{k}_t^{\,2}} \quad (7.3)$$

$$\hat{k}_x = \frac{\sum_t d_{xt}\hat{b}_x(m_{x,t} - \hat{a}_t)}{\sum_t d_{xt}\,\hat{b}_t^{\,2}} \quad (7.4)$$

Simultaneous solutions of these equations are found most easily by an iterative procedure: after choosing a set of starting values (typically the parameters of the OLS fit), equations (7.2), (7.3), (7.4) are computed sequentially using the most recent set of parameter estimates available on the right-hand side of each equation. This process continues until successive computations yield little or no change in parameter values.

The WLS procedure has two additional advantages. The first is that it eliminates the problem that log-mortality is not defined when the number of deaths is zero. The second is that the equation (7.1) is easy to write down the corresponding first order conditions (Wilmoth, 1993: 3).

Unfortunately, the procedure is not statistically sound, and these advantages may be outweighed by the bias induced by the estimator. In particular, Wilmoth interprets equation (7.1) as the log-likelihood of a model with normal, heteroskedastic disturbances, and variance proportional to $1/d_{xt}$. This claim is incorrect, since the variance of the disturbances cannot be (inversely) proportional to the observed number of deaths. A valid weighted likelihood must use exogenous weights, but obviously the number of deaths is a random variable. As such, estimates resulting from this minimization problem have no known statistical properties.

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CHAPTER 8

Mortality Forecasting in Spanish Data

8.1 Introduction

In this chapter, we fit the Lee – Carter method to Spanish data (the referred data were obtained from www.mortality.org), from year 1970 to 2004. The purpose is to forecast the mortality rate $m_{x,t}$ for years 2005, 2006, 2007, 2008 and 2009 for female, male and total population. So, we have $T = 35$.

The ages are grouped in classes, as $[0, 1-4, 5-9, \dots, 100-104, 105-109, 110+]$. As a result, we have $x = 1$ for age class 0, $x = 2$ for age class $1-4, \dots, x = 23$ for age class $105-109$ and $x = 24$ for age class $110+$.

In this chapter, the results will be presented in diagrams. The raw results, as these have been given using Microsoft Excel and R Packages, are given in details in Appendix A.

8.2 Fitting The Model

According to Lee – Carter (1992), the parameter $\hat{a}_x$ is computed first from equation (3.3) and we have the following image as a result.

https://doi.org/10.26219/heal.aueb.5981

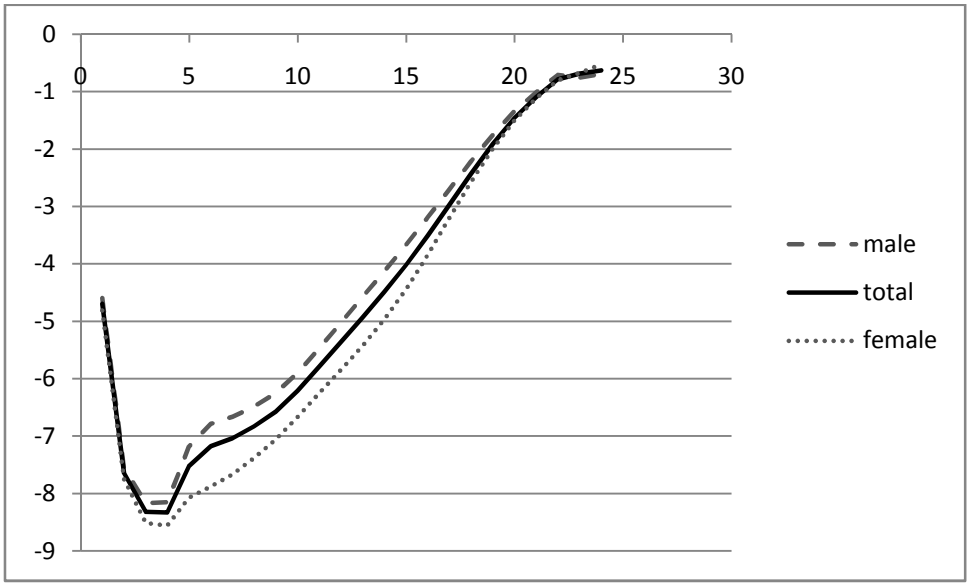

Figure 8.1: Estimation of $\hat{a}_x$ for female, male and total population

Then, we estimate the parameter k_t, for all $t = 1970, \dots 2004$, from the following equation:

$$k_t = \sum_{x=1}^{24} \big(\ln(m_{x,t}) - a_x \big)$$

The results are plotted as follows:

https://doi.org/10.26219/heal.aueb.5981

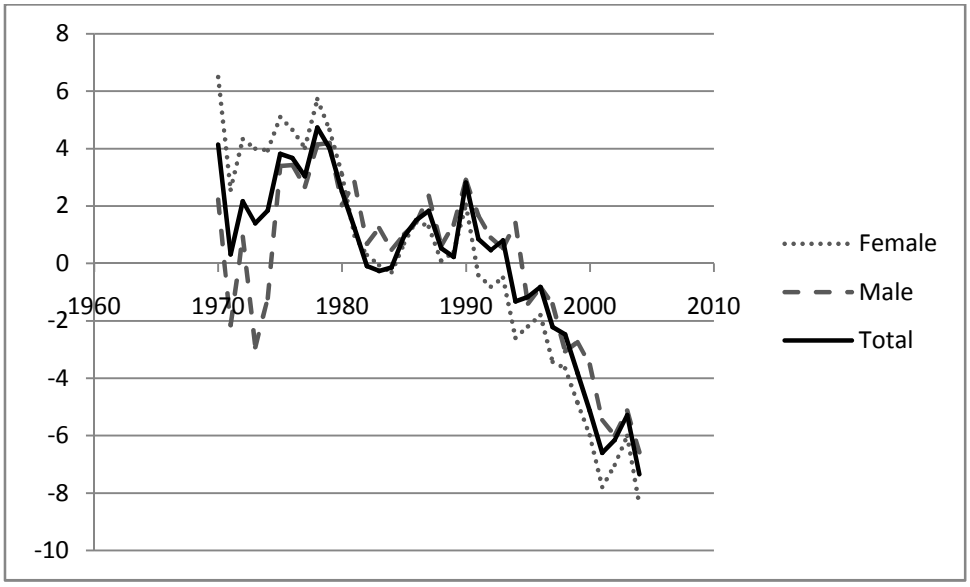

Figure 8.2: Estimation of $\hat{k}_t$ for female, male and total population

To complete the first step, we estimate the parameter b_x by regressing without a constant term using R packages and we have the following figure.

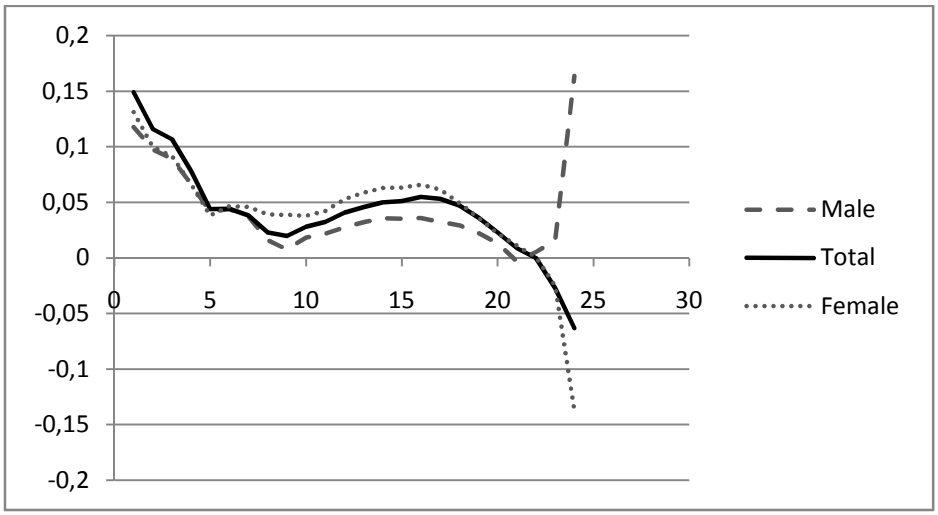

Figure 8.3: Estimation of $\hat{b}_x$ for female, male and total population

In the second step, we re-estimate the parameter k_t by solving equation (3.4) for k_t and we have the figure as follows.

https://doi.org/10.26219/heal.aueb.5981

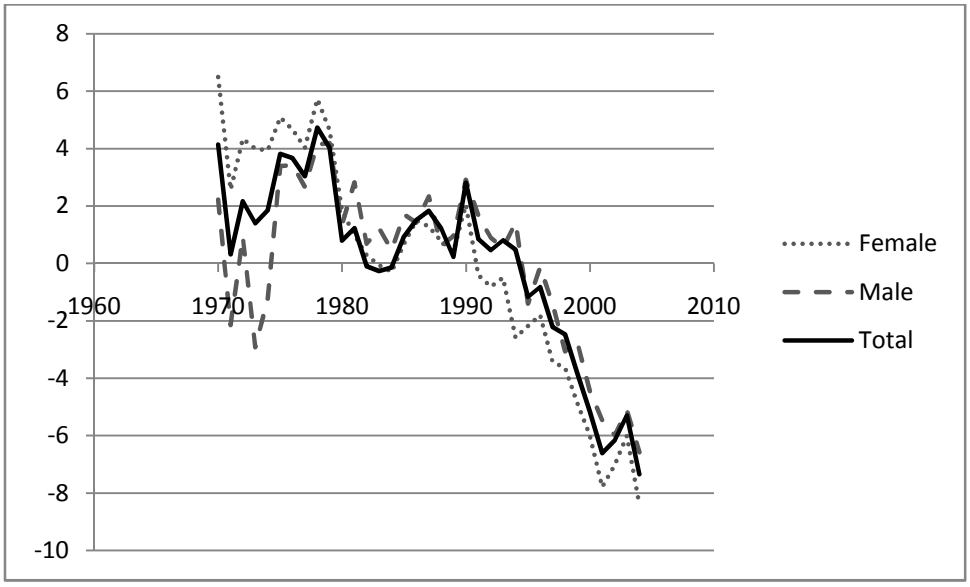

Figure 8.4: Re-estimation of $\hat{k}_t$ for female, male and total population

If we take a look to Appendix A, we will notice that the first and the second computation do not differentiate a lot. Especially, we have the following figures.

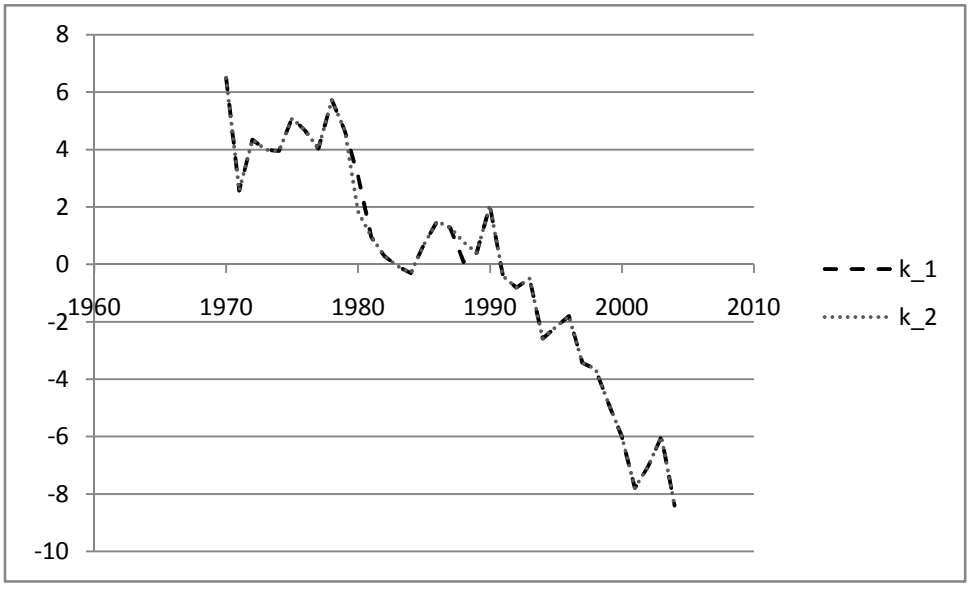

Figure 8.5: First and second estimation of k_t for female

https://doi.org/10.26219/heal.aueb.5981

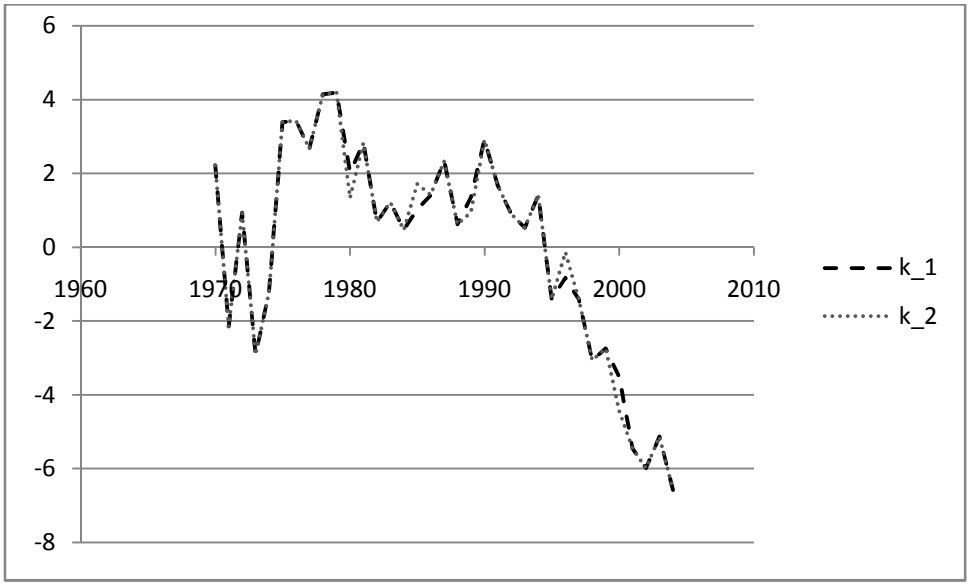

Figure 8.6: First and second estimation of k_t for male

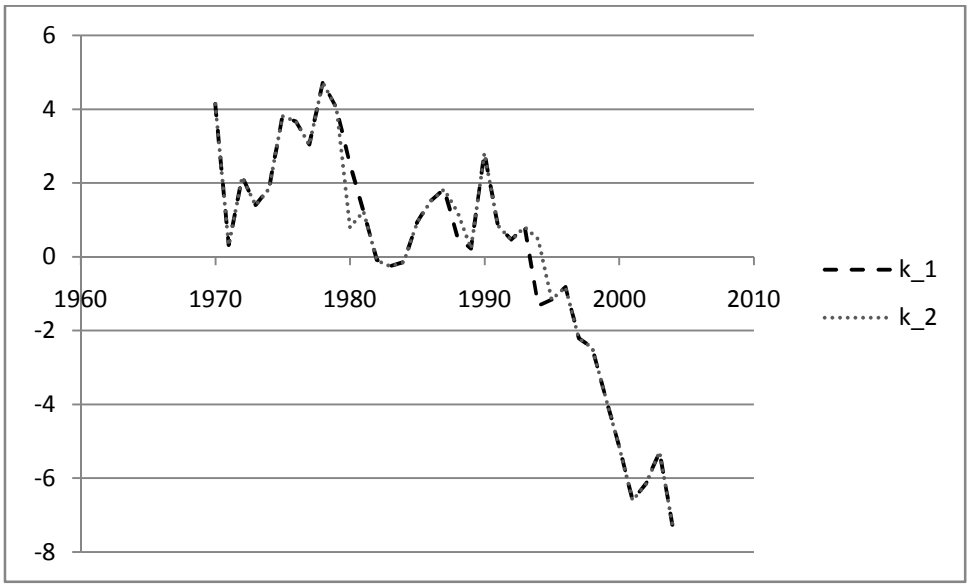

Figure 8.7: First and second estimation of k_t for total population

8.3 Forecast Mortality Rate

The advantage of Lee – Carter method is that the only index which needs forecast is k_t. From the equation (3.9), the following results are given.

Table 8.1: Estimation of $\hat{c}$

	Female	***Male***	***Total***
$\hat{c}$	-0,437975203	-0,258676997	-0,337458993

And respectively, from (3.12) we have

Table 8.2: Forecasting for $\hat{k}_t$

Year	***Female***	***Male***	***Total***
2005	-8,840135188	-6,833941957	-7,67538177
2006	-9,278110391	-7,092618953	-8,012840763
2007	-9,716085594	-7,35129595	-8,350299757
2008	-10,1540608	-7,609972946	-8,68775875
2009	-10,592036	-7,868649943	-9,025217743

The mortality rate, $(\ln m_{x,t})$, is now forecasted for years 2005, 2006, 2007, 2008 and 2009 from the following equation

$$\ln(m_{x,T+\Delta t}) \approx \hat{a}_x + \hat{b}_x \hat{k}_{T+\Delta t}$$

and we have the following figures.

https://doi.org/10.26219/heal.aueb.5981

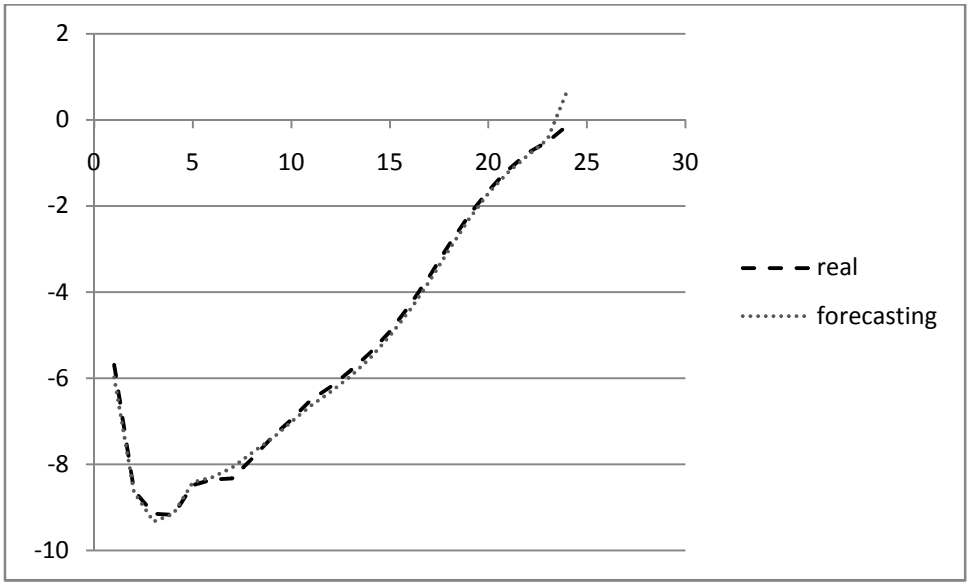

Figure 8.8: Comparison of real and forecasted data for female (2005)

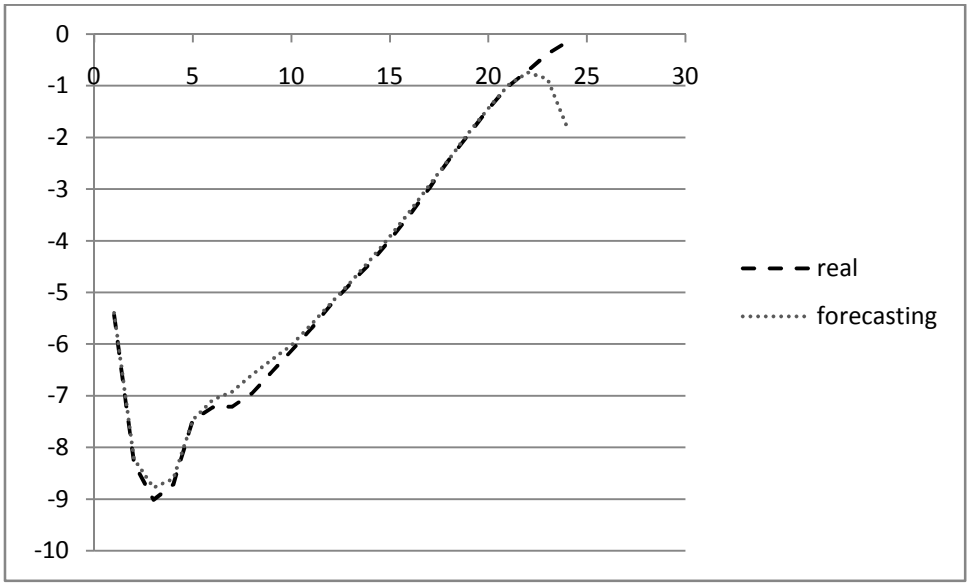

Figure 8.9: Comparison of real and forecasted data for male (2005)

https://doi.org/10.26219/heal.aueb.5981

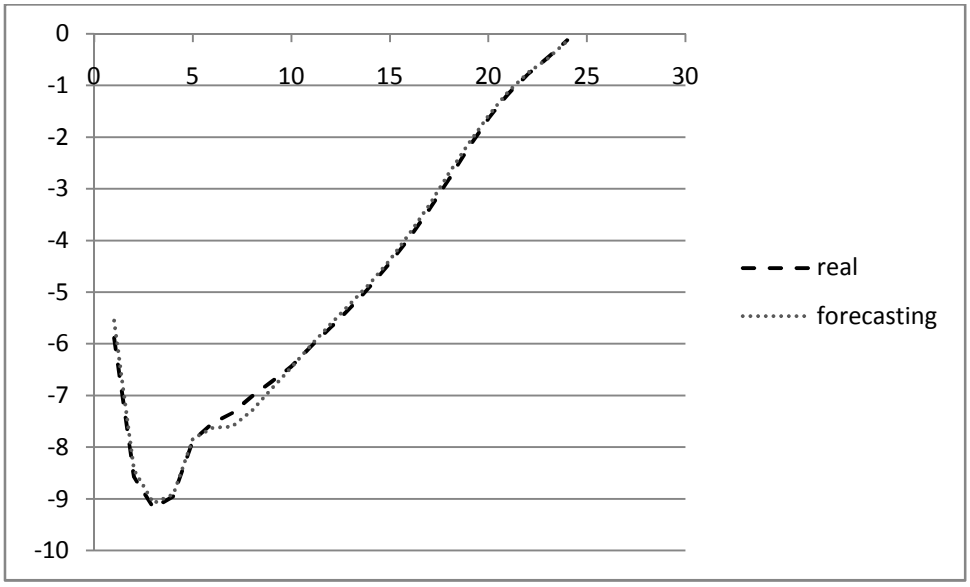

Figure 8.10: Comparison of real and forecasted data for total population (2005)

As it can be seen from the above figures, the forecasted data are very close to the real ones.

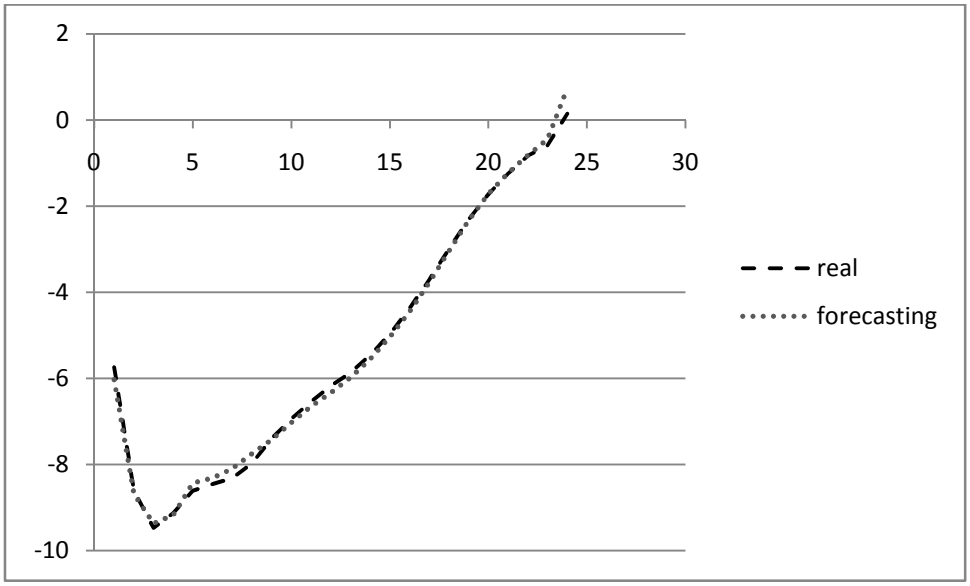

Figure 8.11: Comparison of real and forecasted data for female (2006)

https://doi.org/10.26219/heal.aueb.5981

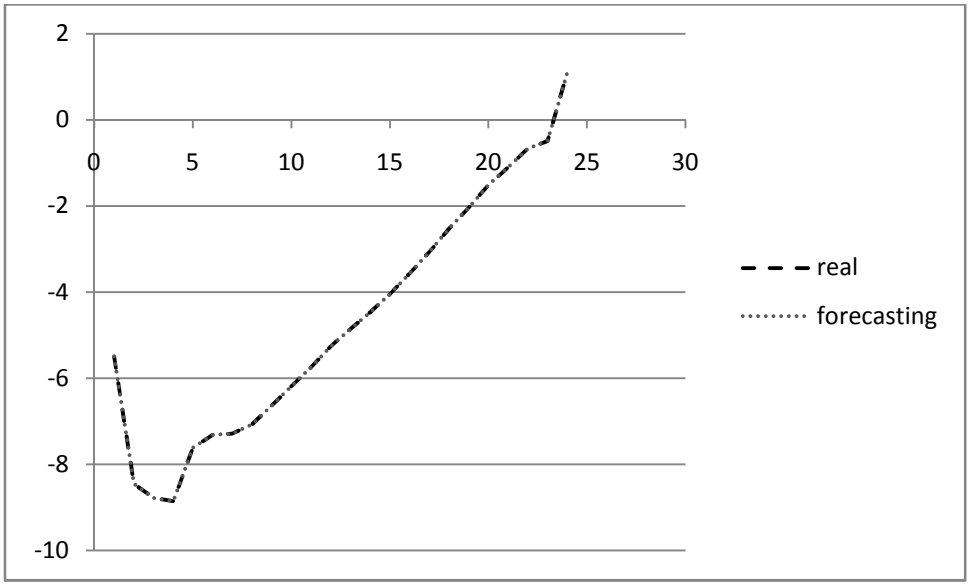

Figure 8.12: Comparison of real and forecasted data for male (2006)

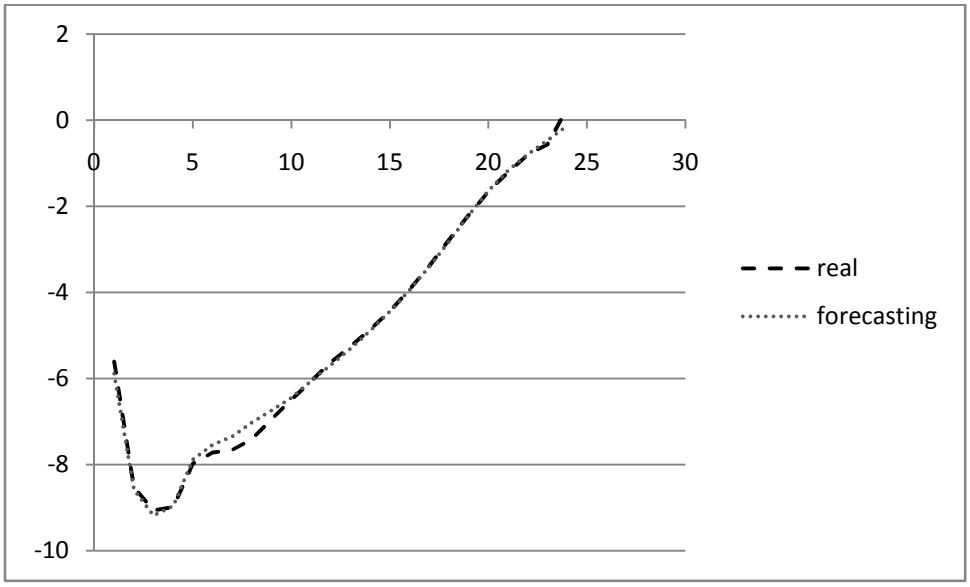

Figure 8.13: Comparison of real and forecasted data for total population (2006)

Similarly to 2005, in 2006 the forecasted data are very close to the real ones.

https://doi.org/10.26219/heal.aueb.5981

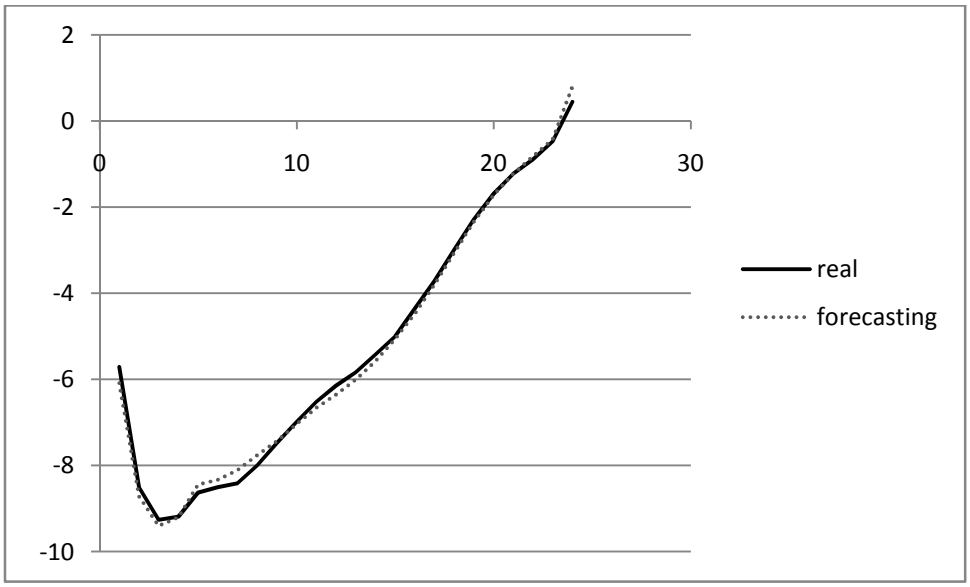

Figure 8.14: Comparison of real and forecasted data for female (2007)

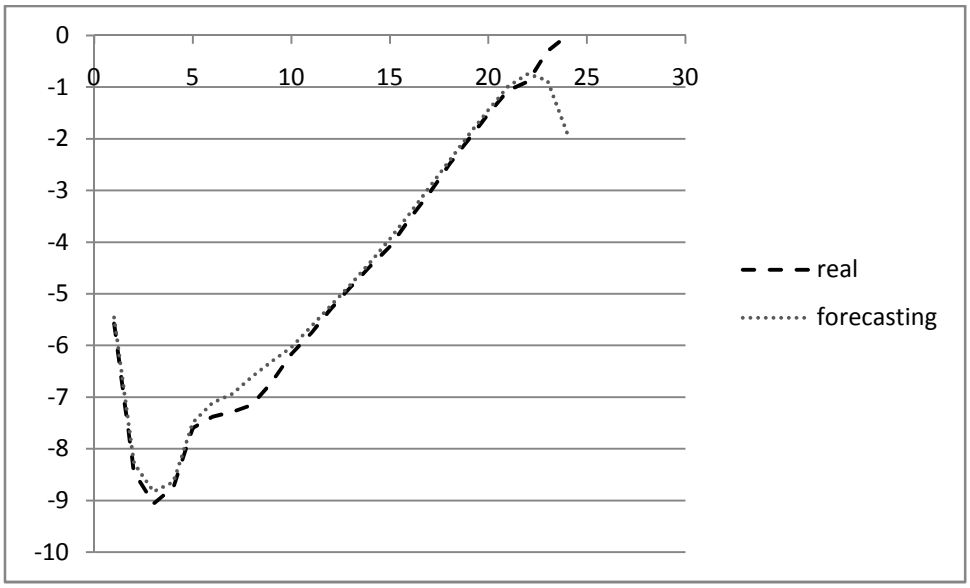

Figure 8.15: Comparison of real and forecasted data for male (2007)

https://doi.org/10.26219/heal.aueb.5981

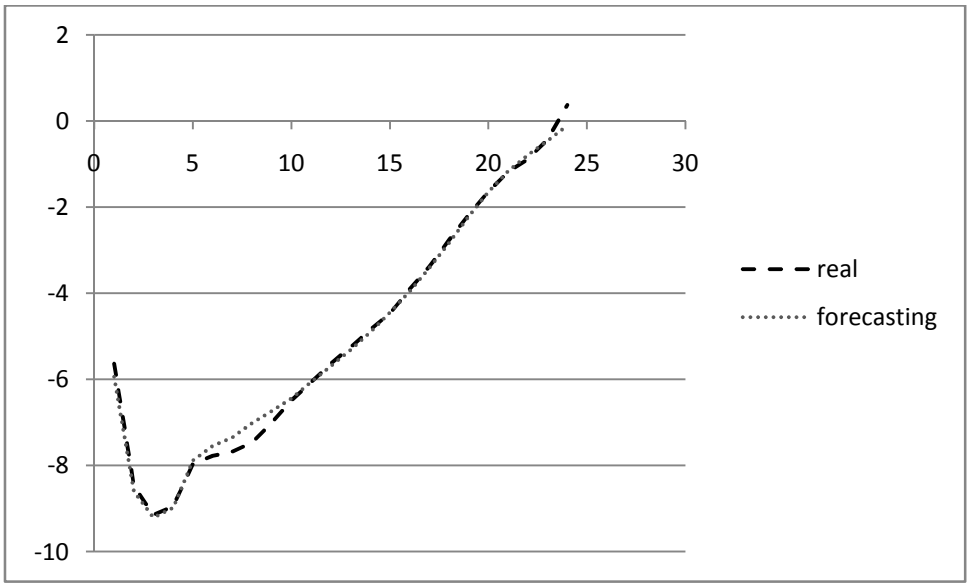

Figure 8.16: Comparison of real and forecasted data for total population (2007)

The same trend as in 2005 and 2006 is shown in 2007 as well, where the forecasted data and the real ones are very close to each other.

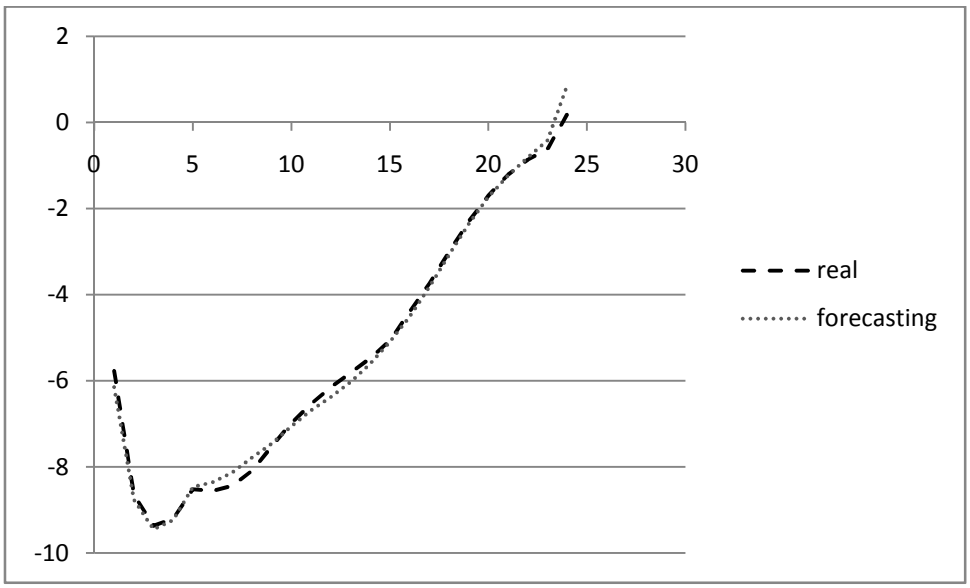

Figure 8.17: Comparison of real and forecasted data for female (2008)

https://doi.org/10.26219/heal.aueb.5981

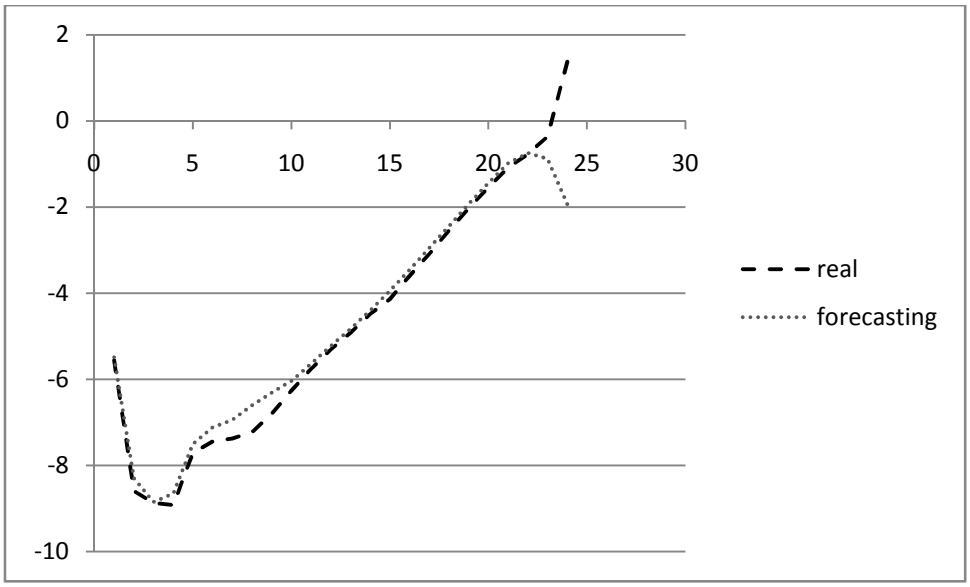

Figure 8.18: Comparison or real and forecast data for male (2008)

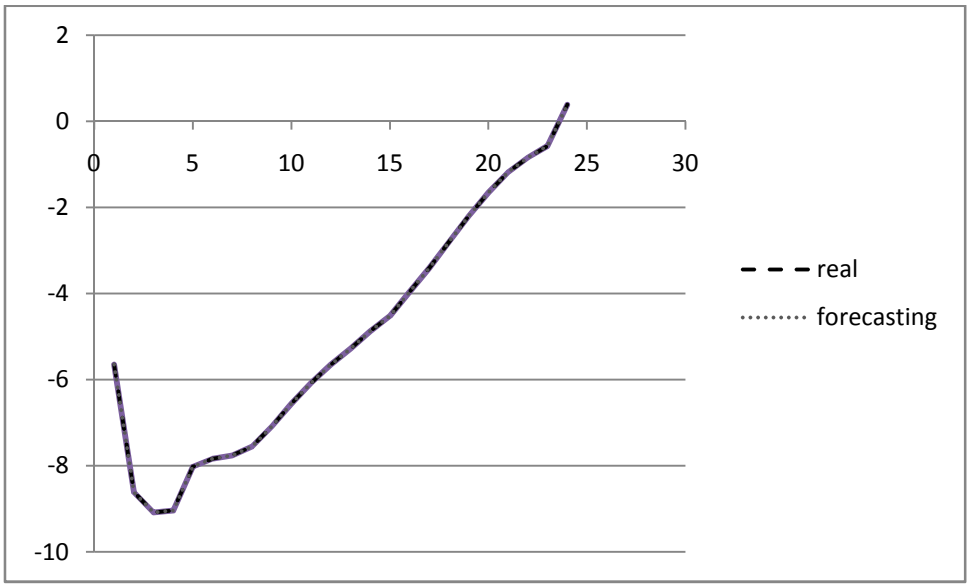

Figure 8.19: Comparison or real and forecast data for male (2008)

For 2008, the actual and the forecasted data are very close to each other for female and total population. For male, we can see a differentiation for $x = 5 - 10$ and $x = 21 - 24$.

https://doi.org/10.26219/heal.aueb.5981

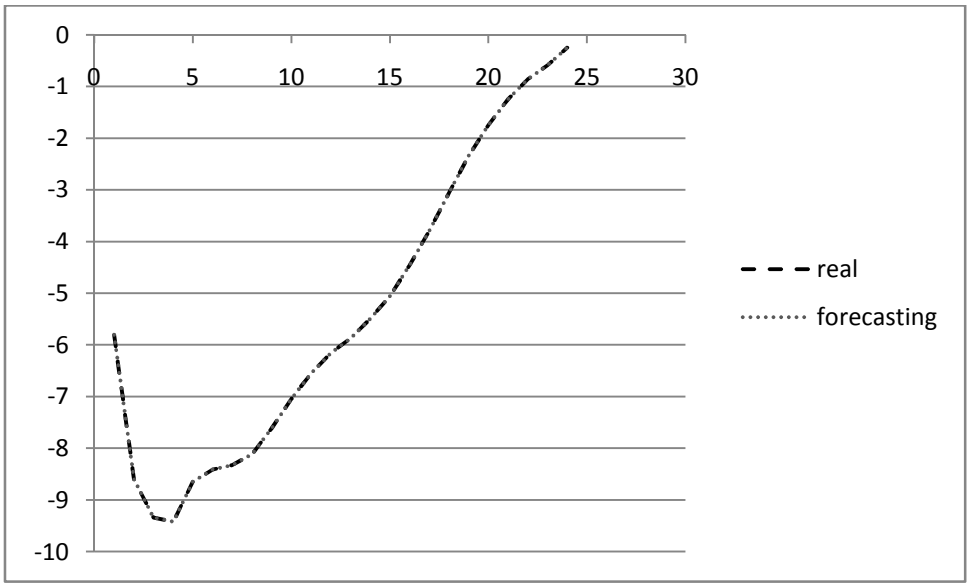

Figure 8.20: Comparison of real and forecasted data for female (2009)

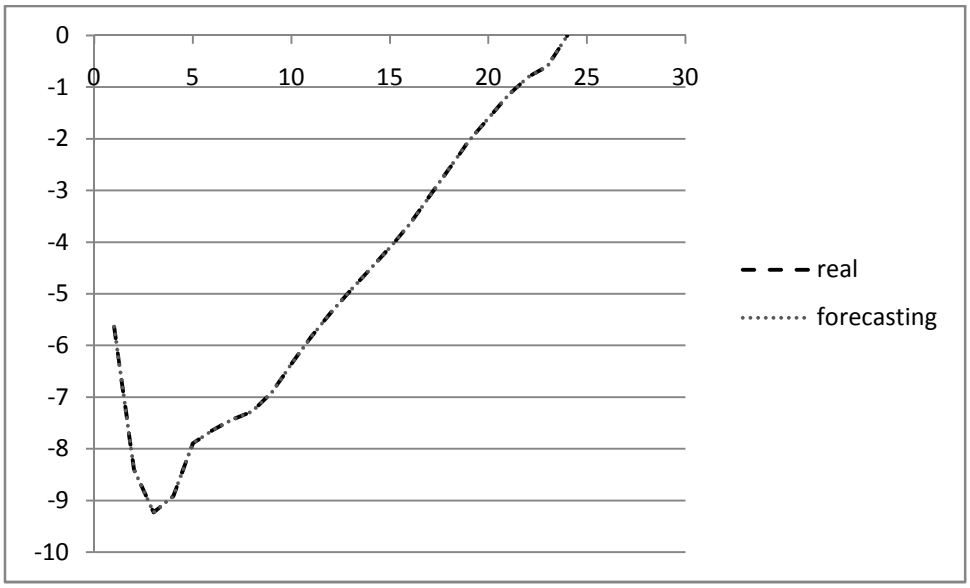

Figure 8.21: Comparison of real and forecasted data for male (2009)

https://doi.org/10.26219/heal.aueb.5981

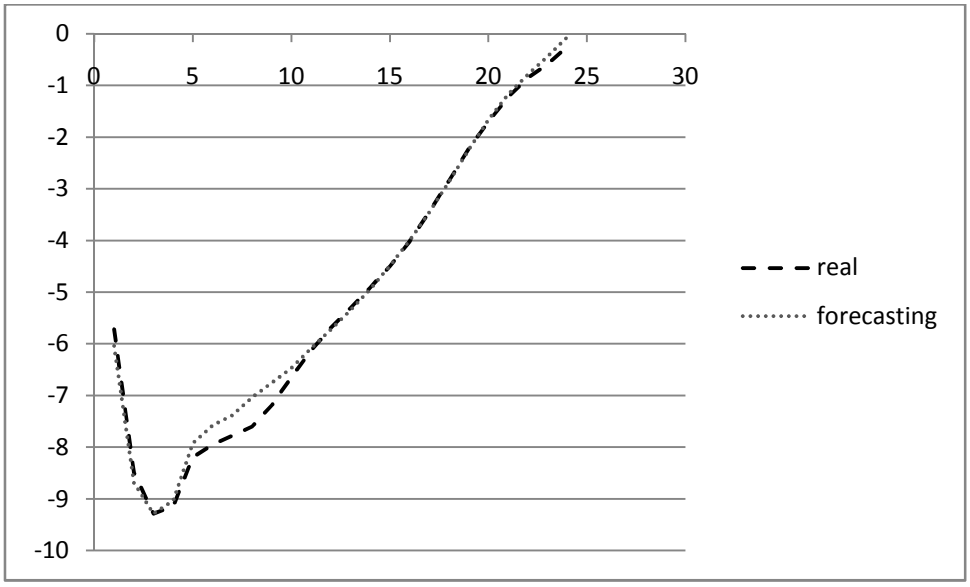

Figure 8.22: Comparison of real and forecasted data for total population (2009)

For 2009, the forecasted data are very close to the real ones.

https://doi.org/10.26219/heal.aueb.5981

CHAPTER 9

Mortality Forecasting in French Data

9.1 Introduction

In this chapter, we fit the Lee – Carter method to French data (the referred data were obtained from www.mortality.org), from year 1970 to 2004. The purpose is to forecast the mortality rate $m_{x,t}$ for years 2005, 2006, 2007, 2008 and 2009 for female, male and total population. So, we have $T = 35$.

The ages are grouped in classes, as $[0, 1-4, 5-9, \dots, 100-104, 105-109, 110+]$. As a result, we have $x = 1$ for age class 0, $x = 2$ for age class $1-4, \dots, x = 23$ for age class $105-109$ and $x = 24$ for age class $110+$.

In this chapter, similarly to the Chapter 8, the results will be presented in diagrams. The raw results, as these have been given using Microsoft Excel and R Packages, are given in details in Appendix B.

9.2 Fitting The Model

According to Lee – Carter (1992), the parameter $\hat{a}_x$ is computed first from equation (3.3) and we have the following image as a result.

https://doi.org/10.26219/heal.aueb.5981

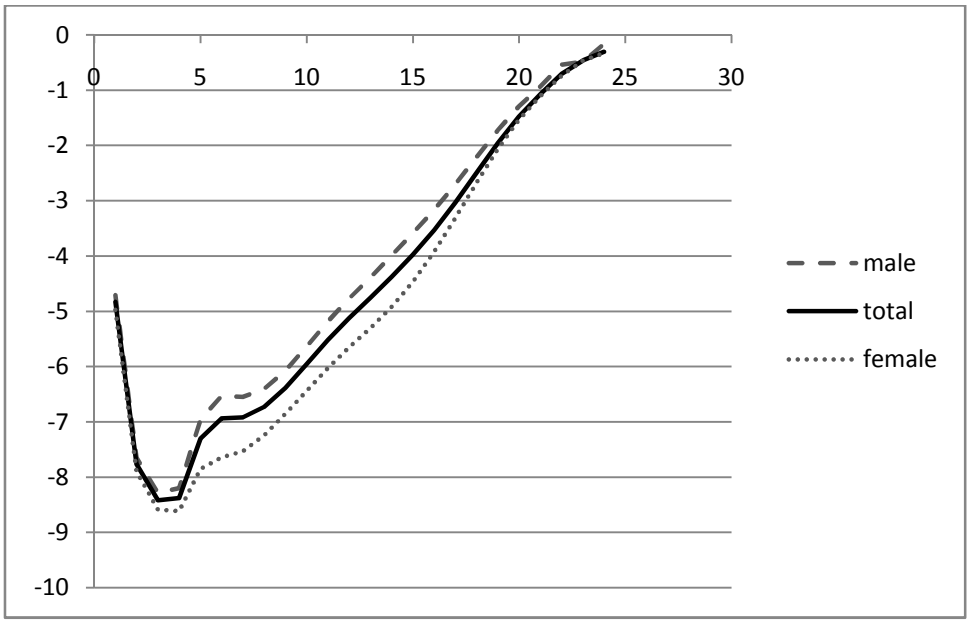

Figure 9.1: Estimation of $\hat{a}_x$ for female, male and total population

Then, we estimate the parameter k_t, for all $t = 1970, \dots 2004$, from the following equation:

$$k_t = \sum_{x=1}^{24} \left(\ln(m_{x,t}) - a_x \right)$$

The results are plotted as follows:

https://doi.org/10.26219/heal.aueb.5981

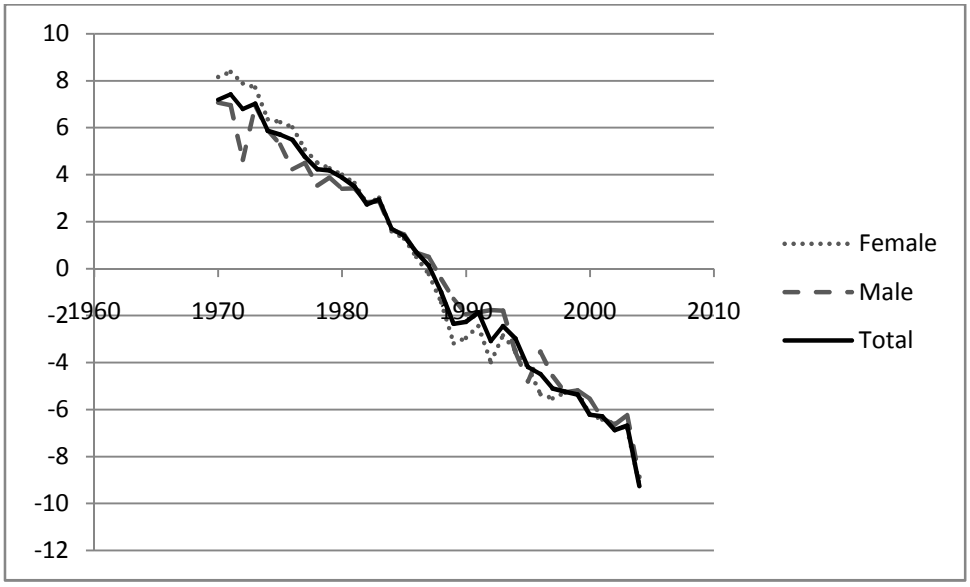

Figure 9.2: Estimation of $\hat{k}_t$ for female, male and total population

To complete the first step, we estimate the parameter b_x by regressing without a constant term using R packages and we have the following figure.

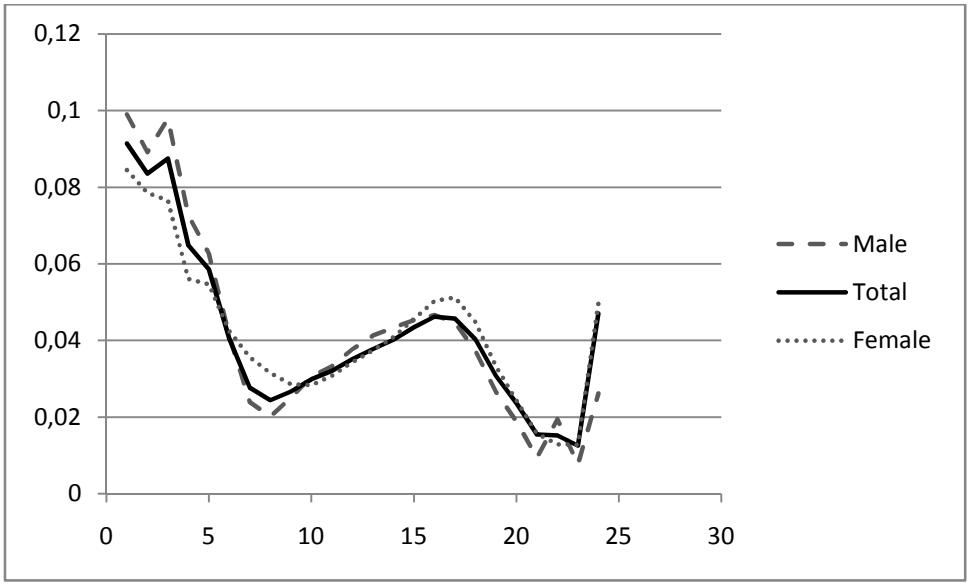

Figure 9.3: Estimation of $\hat{b}_x$ for female, male and total population

In the second step, we re-estimate the parameter k_t by solving equation (3.4) for k_t and we have the figure as follows.

https://doi.org/10.26219/heal.aueb.5981

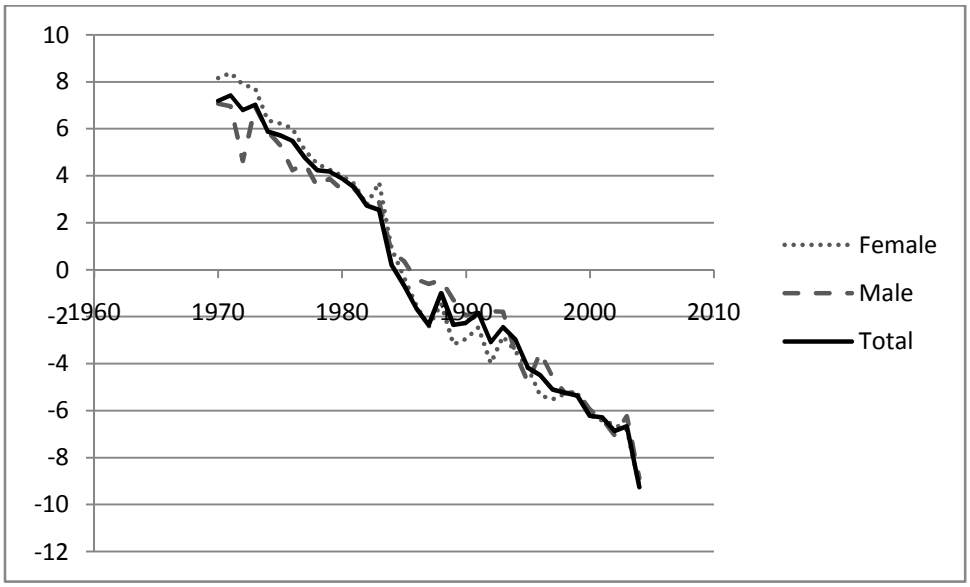

Figure 9.4: Re-estimation of $\hat{k}_t$ for female, male and total population

If we take a look to Appendix B, we will notice that the first and the second computation do not differentiate a lot. Especially, we have the following figures.

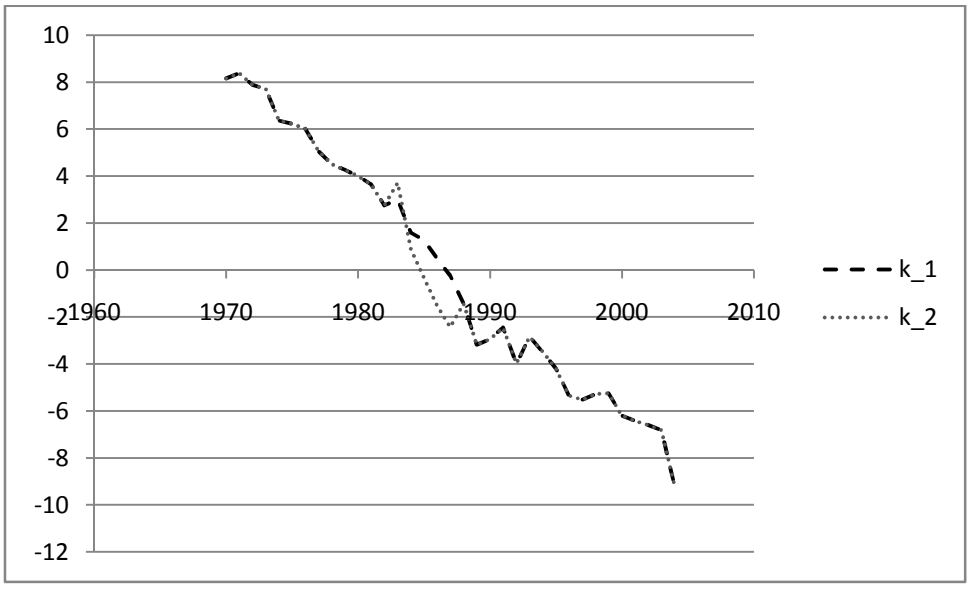

Figure 9.5: First and second estimation of k_t for female

https://doi.org/10.26219/heal.aueb.5981

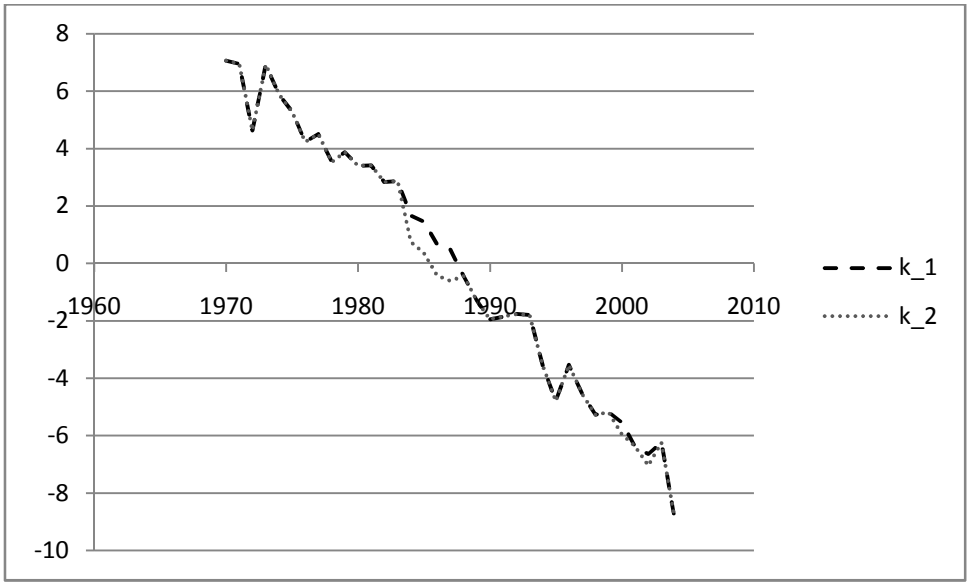

Figure 9.6: First and second estimation of k_t for male

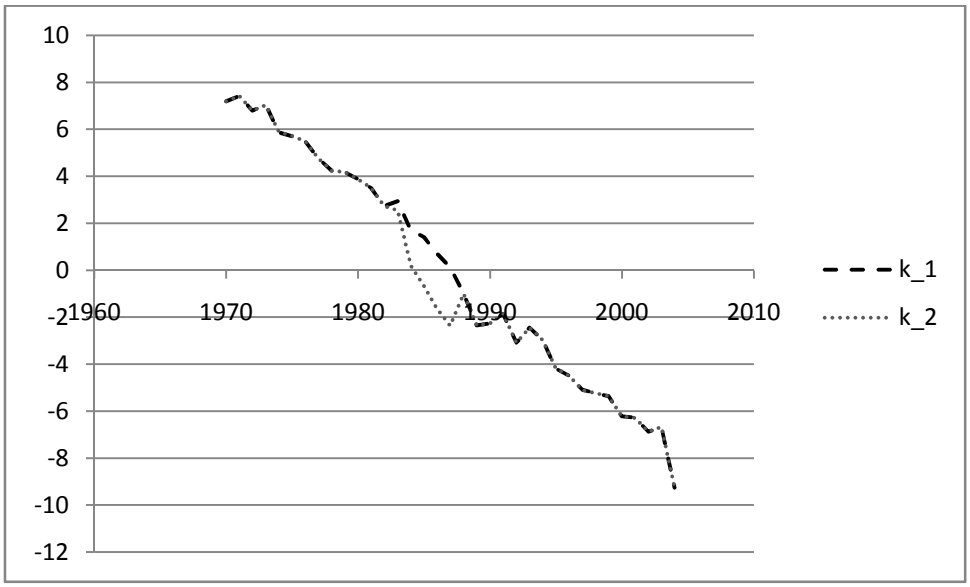

Figure 9.7: First and second estimation of k_t for total population

https://doi.org/10.26219/heal.aueb.5981

9.3 Forecast Mortality Rate

The advantage of Lee – Carter method is that the only index which needs forecast is k_t. From the equation (3.9), the following results are given.

Table 9.1: Estimation of $\hat{c}$

	Female	***Male***	***Total***
$\hat{c}$	-0,509754494	-0,468654942	-0,483822842

And respectively, from (3.12) we have

Table 9.2: Forecasting for $\hat{k}_t$

Year	***Female***	***Male***	***Total***
2005	-9,682880261	-9,341744577	-9,743400802
2006	-10,19263476	-9,810399519	-10,22722364
2007	-10,70238925	-10,27905446	-10,71104649
2008	-11,21214374	-10,7477094	-11,19486933
2009	-11,72189824	-11,21636435	-11,67869217

The mortality rate, $(\ln m_{x,t})$, is now forecasted for years 2005, 2006, 2007, 2008 and 2009 from the following equation

$$\ln(m_{x,T+\Delta t}) \approx \hat{a}_x + \hat{b}_x \hat{k}_{T+\Delta t}$$

and we have the following figures.

https://doi.org/10.26219/heal.aueb.5981

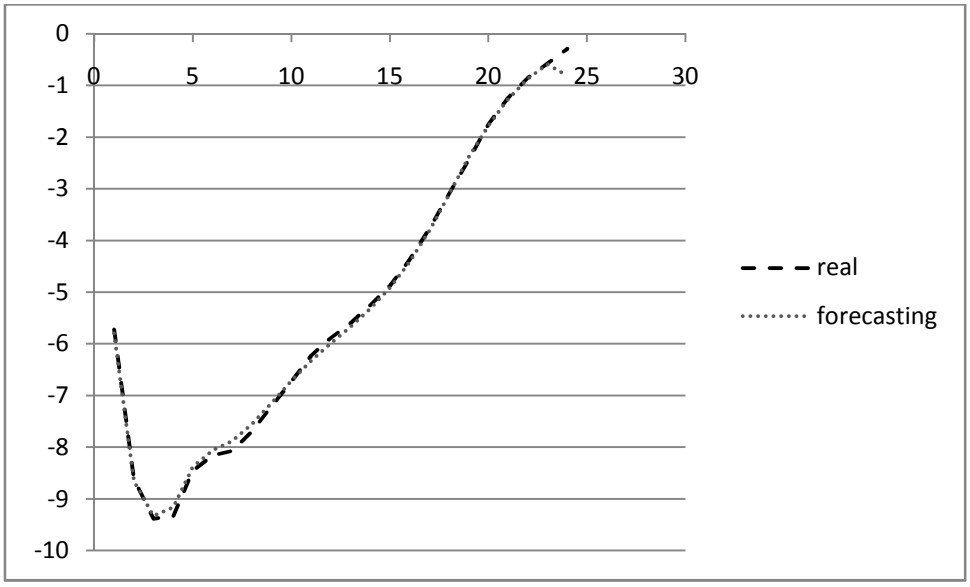

Figure 9.8: Comparison of real and forecasted data for female (2005)

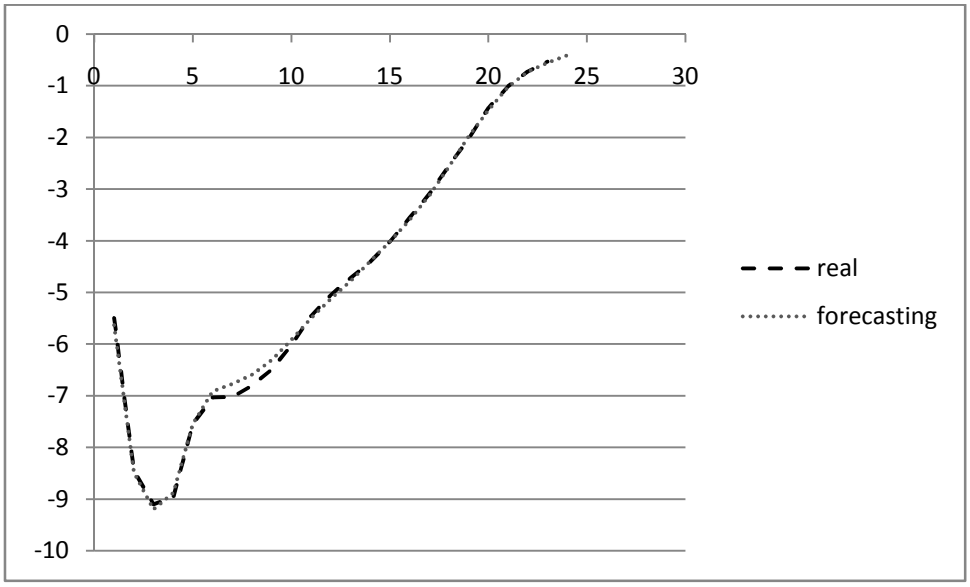

Figure 9.9: Comparison of real and forecasted data for male (2005)

https://doi.org/10.26219/heal.aueb.5981

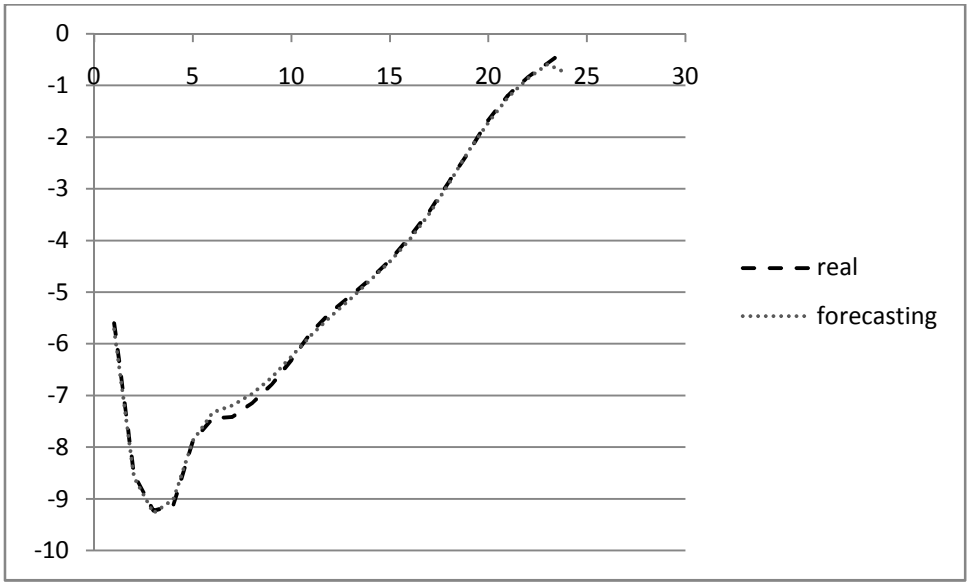

Figure 9.10: Comparison of real and forecasted data for total population (2005)

As it can be seen from the above figures, the forecasted data are very close to the real ones.

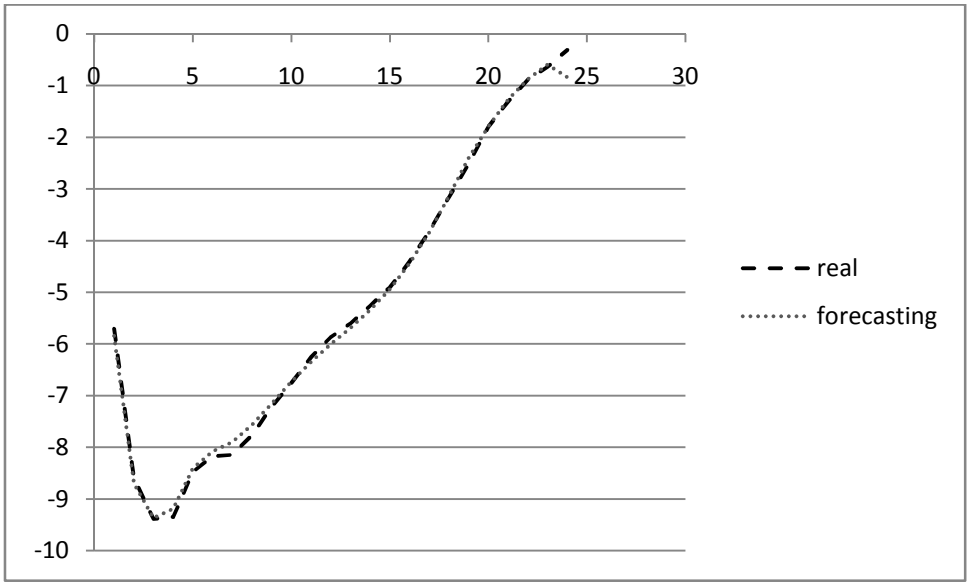

Figure 9.11: Comparison of real and forecasted data for female (2006)

https://doi.org/10.26219/heal.aueb.5981

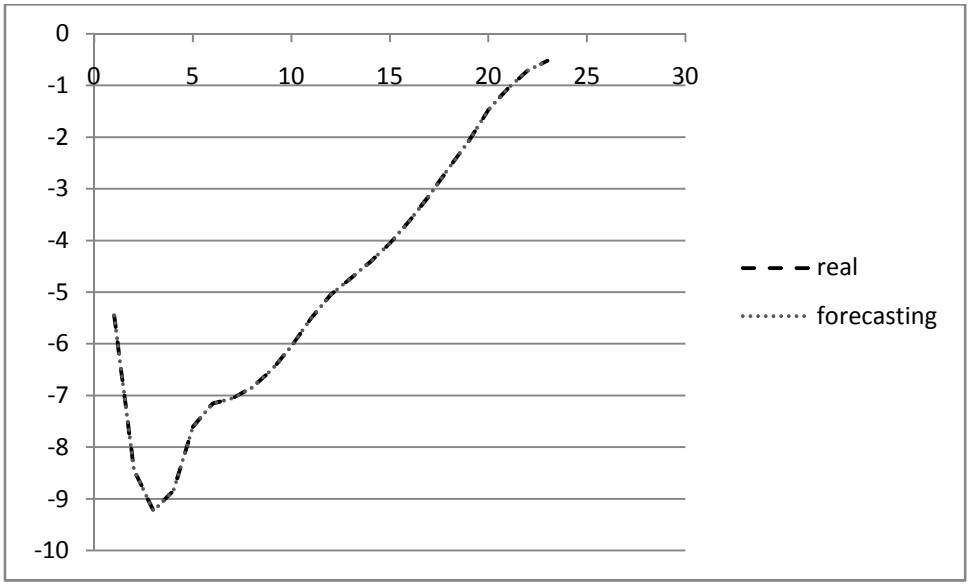

Figure 9.12: Comparison of real and forecasted data for male (2006)

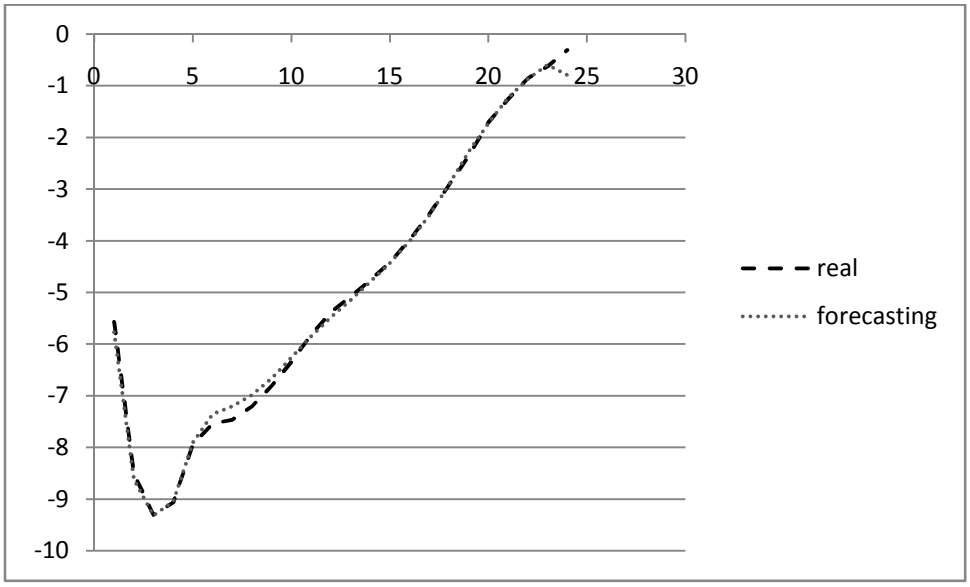

Figure 9.13: Comparison of real and forecasted data for total population (2006)

Similarly to 2005, in 2006 the forecasted data are very close to the real ones.

https://doi.org/10.26219/heal.aueb.5981

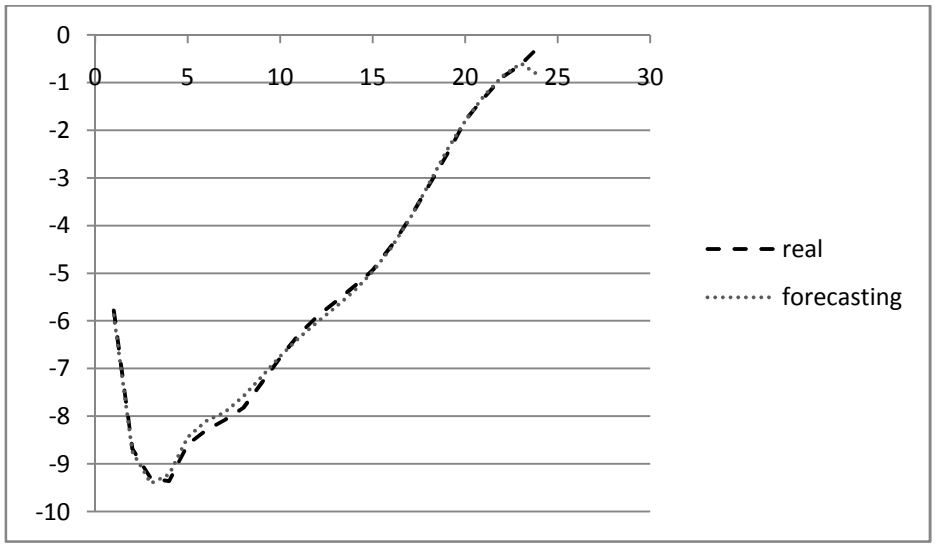

Figure 9.14: Comparison of real and forecasted data for female (2007)

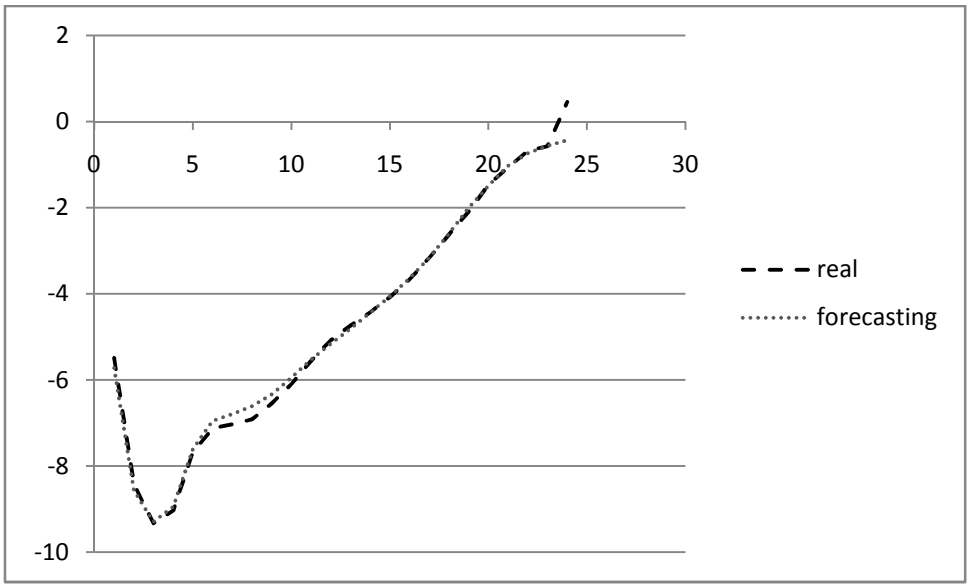

Figure 9.15: Comparison of real and forecasted data for male (2007)

https://doi.org/10.26219/heal.aueb.5981

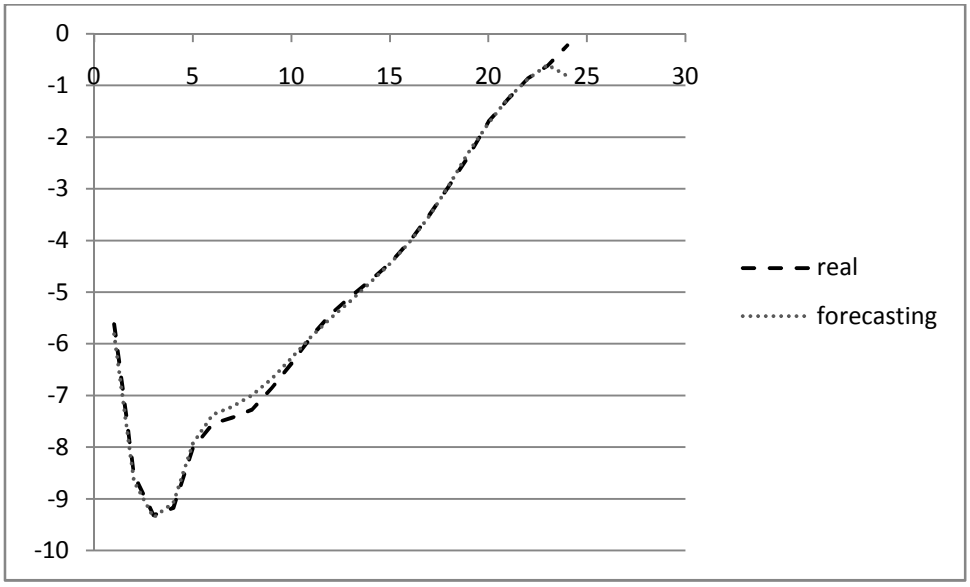

Figure 9.16: Comparison of real and forecasted data for total population (2007)

The same trend as in 2005 and 2006 is shown in 2007 as well, where the forecasted data and the real ones are very close to each other.

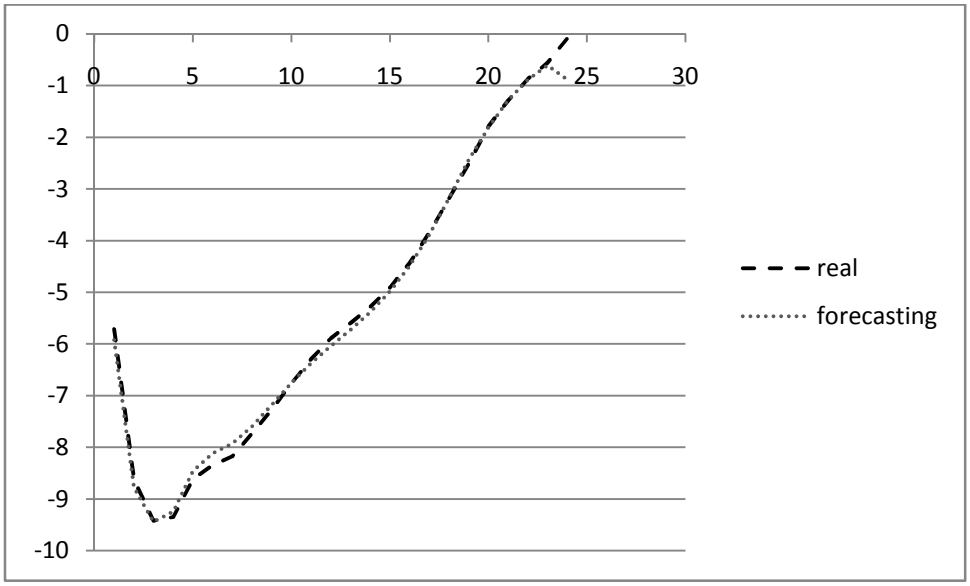

Figure 9.17: Comparison of real and forecasted data for female (2008)

https://doi.org/10.26219/heal.aueb.5981

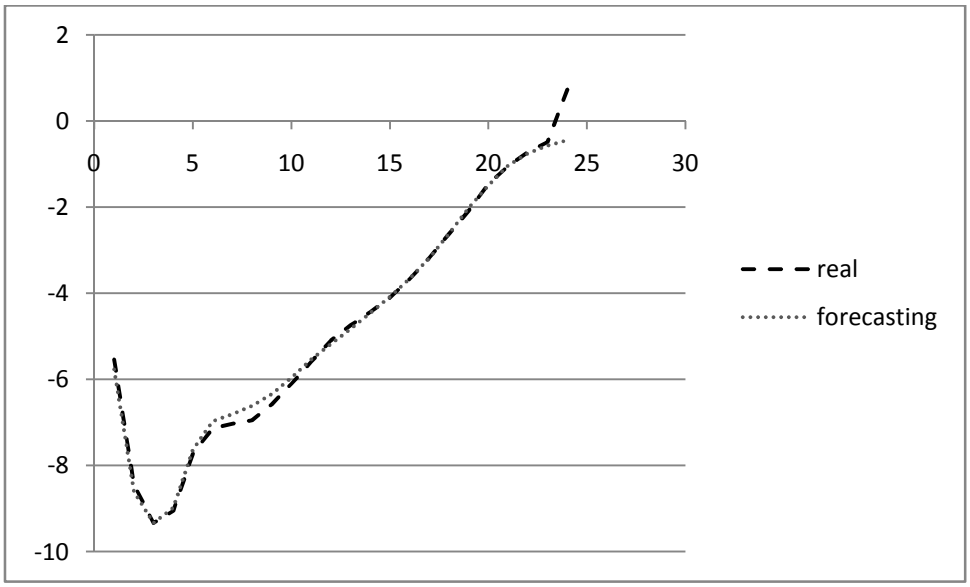

Figure 9.18: Comparison or real and forecast data for male (2008)

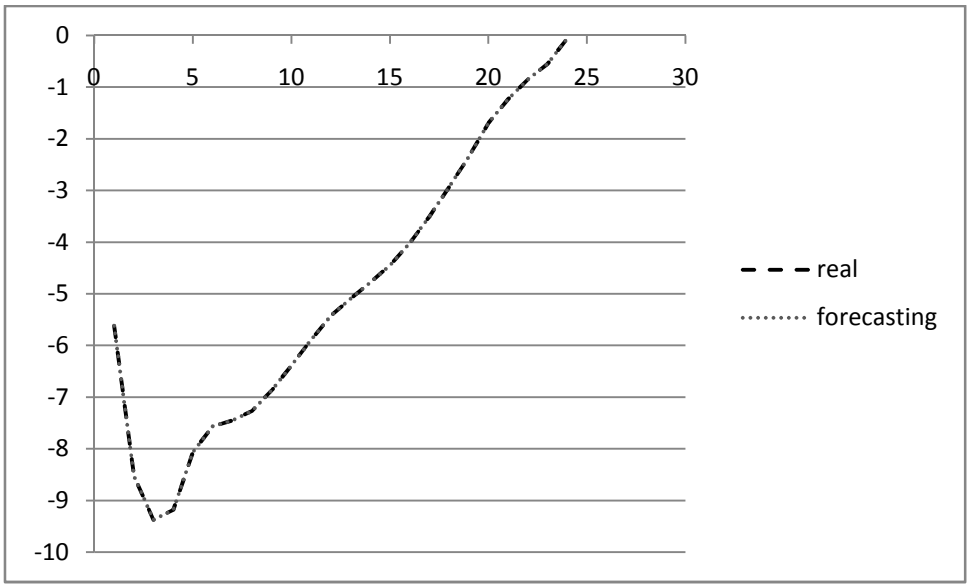

Figure 9.19: Comparison or real and forecast data for total population (2008)

For 2008, the actual and the forecasted data, as it can be seen in Figures 9.17 and 9.19 are very close to each other for female and total

https://doi.org/10.26219/heal.aueb.5981

population respectively. For male, as shown in Figure 9.18, we can see a differentiation for $x = 5 - 10$ and $x = 21 - 24$.

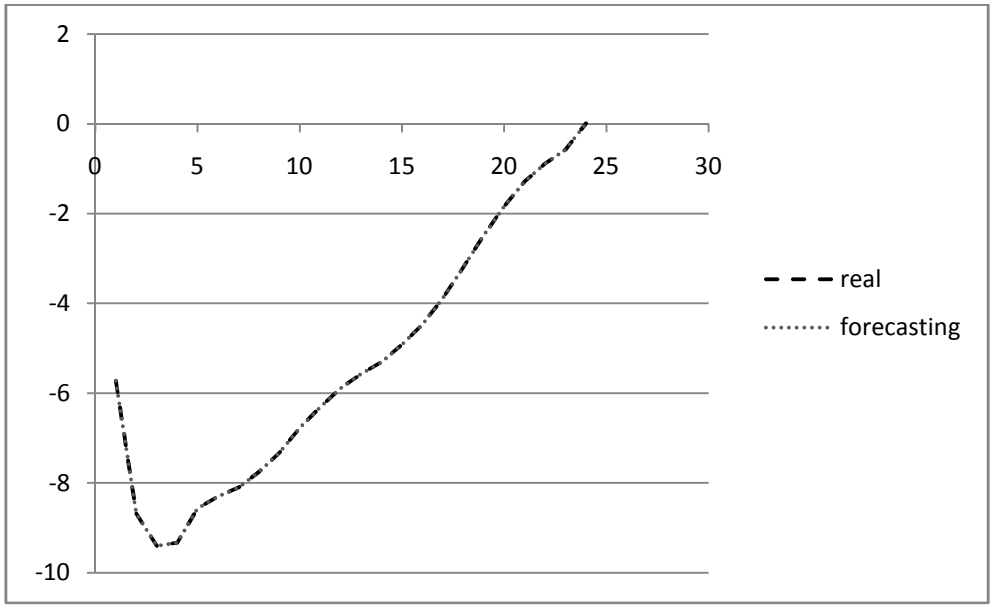

Figure 9.20: Comparison of real and forecasted data for female (2009)

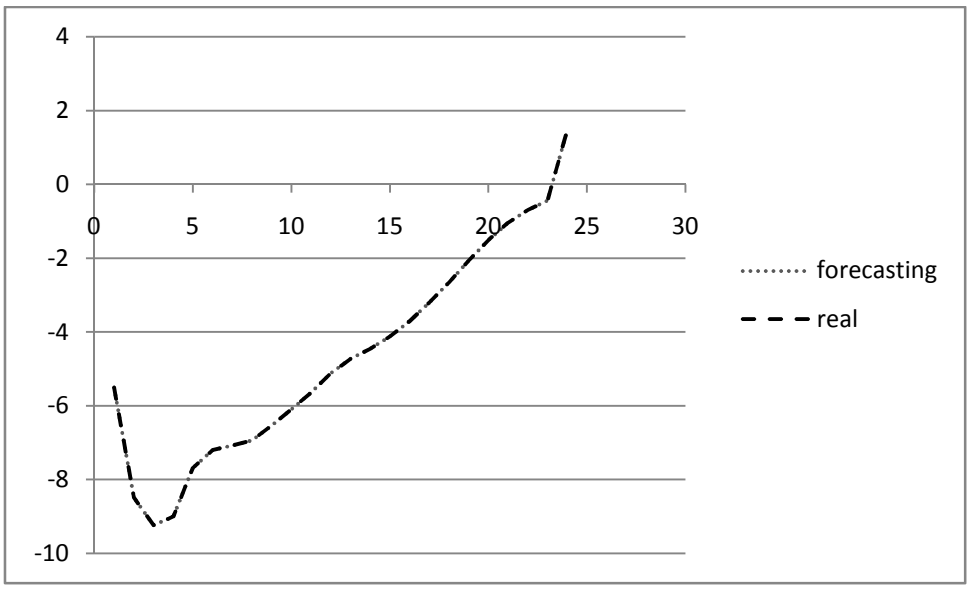

Figure 9.21: Comparison of real and forecasted data for male (2009)

https://doi.org/10.26219/heal.aueb.5981

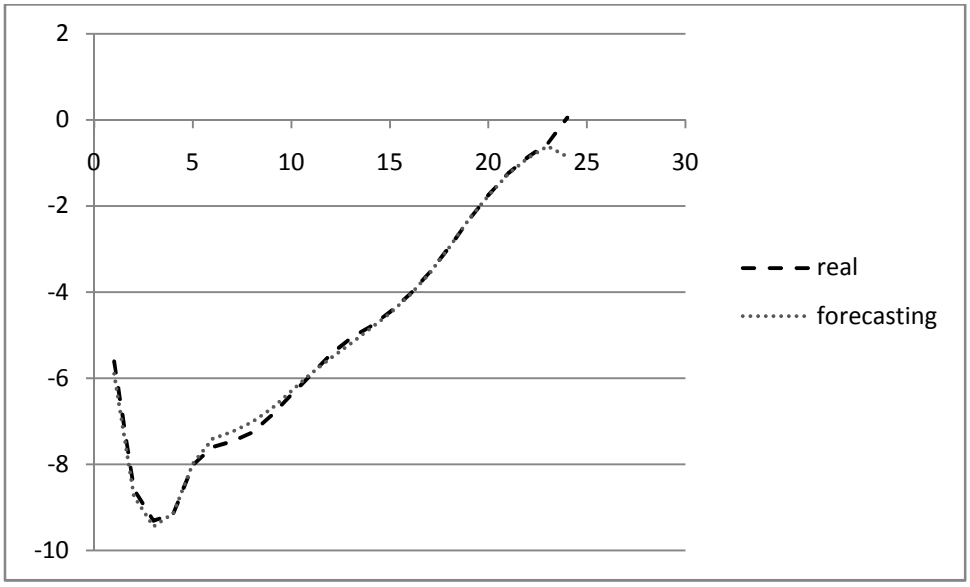

Figure 9.22: Comparison of real and forecasted data for total population (2009)

For 2009, the forecasted data are very close to the real ones.

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CHAPTER 10

Mortality Forecasting in Swedish Data

10.1 Introduction

In this chapter, we fit the Lee – Carter method to Spanish data (the referred data were obtained from www.mortality.org), from year 1970 to 2004. The purpose is to forecast the mortality rate $m_{x,t}$ for years 2005, 2006, 2007, 2008, 2009 and 2010 for female, male and total population. So, we have $T = 35$.

The ages are grouped in classes, as $[0, 1-4, 5-9, \dots, 100-104, 105-109, 110+]$. As a result, we have $x = 1$ for age class 0, $x = 2$ for age class $1-4, \dots, x = 23$ for age class $105-109$ and $x = 24$ for age class $110+$.

In this chapter, similarly to Chapters 8 and 9, the results will be presented in diagrams. The raw results, as these have been given using Microsoft Excel and R Packages, are given in details in Appendix C.

10.2 Fitting The Model

According to Lee – Carter (1992), the parameter $\hat{a}_x$ is computed first from equation (3.3) and we have the following image as a result.

https://doi.org/10.26219/heal.aueb.5981

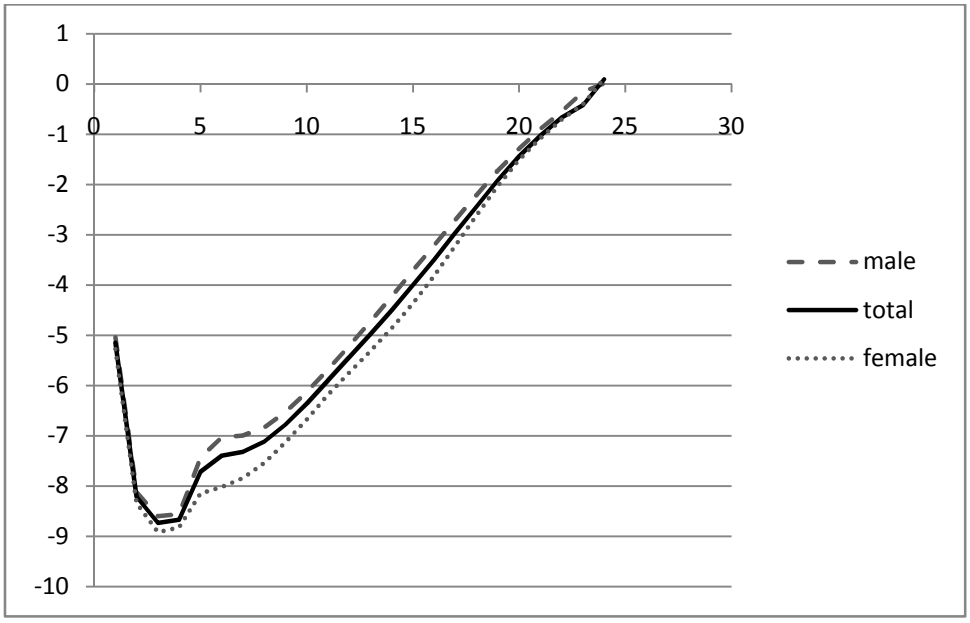

Figure 10.1: Estimation of $\hat{a}_x$ for female, male and total population

Then, we estimate the parameter k_t, for all $t = 1970, \dots 2004$, from the following equation:

$$k_t = \sum_{x=1}^{24} \left(\ln(m_{x,t}) - a_x \right)$$

The results are plotted as follows:

https://doi.org/10.26219/heal.aueb.5981

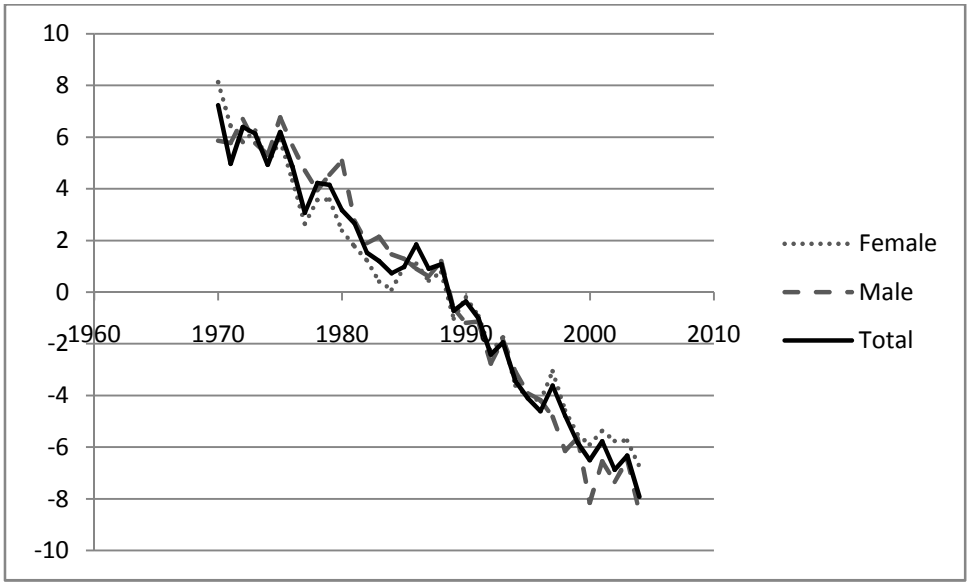

Figure 10.2: Estimation of $\hat{k}_t$ for female, male and total population

To complete the first step, we estimate the parameter b_x by regressing without a constant term using R packages and we have the following figure.

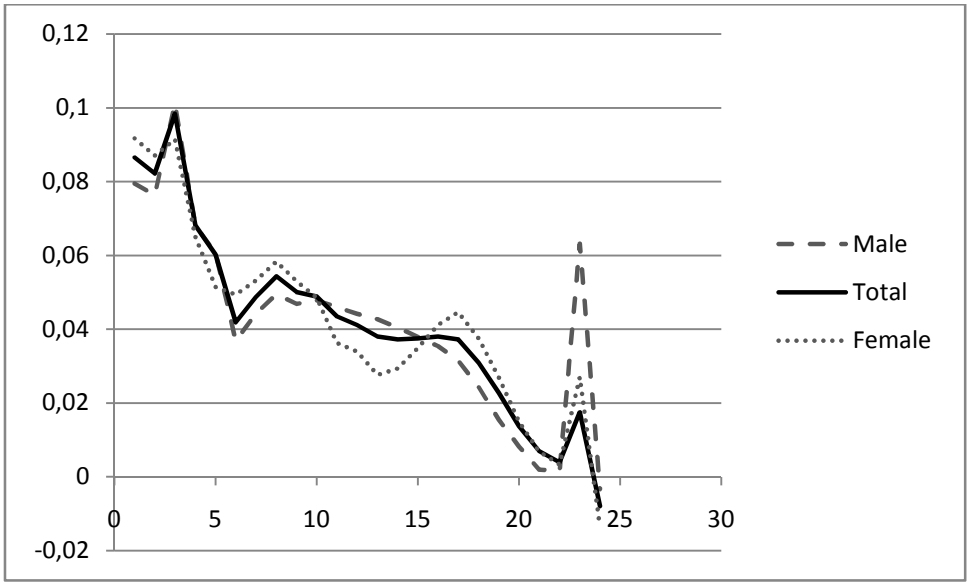

Figure 10.3: Estimation of $\hat{b}_x$ for female, male and total population

https://doi.org/10.26219/heal.aueb.5981

In the second step, we re-estimate the parameter k_t by solving equation (3.4) for k_t and we have the figure as follows.

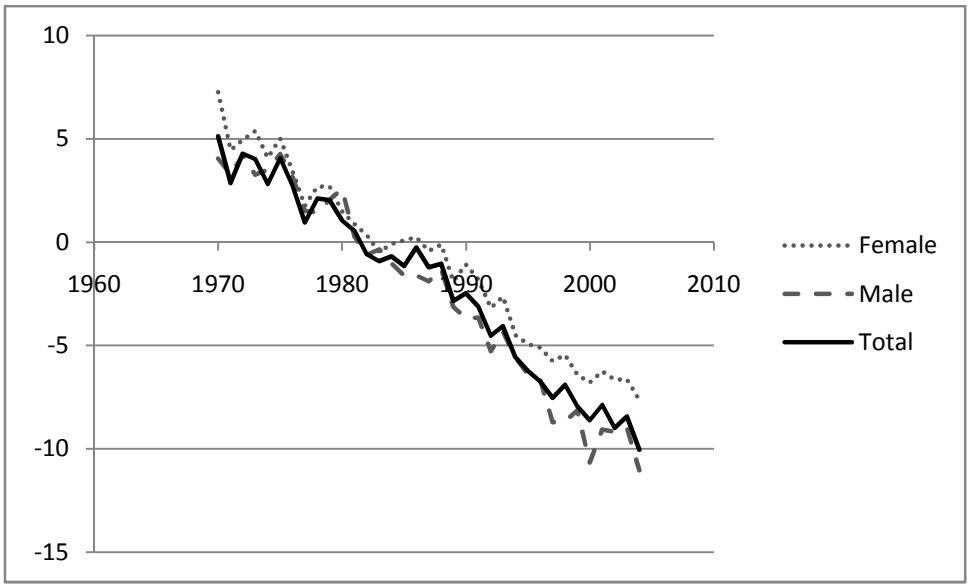

Figure 10.4: Re-estimation of $\hat{k}_t$ for female, male and total population

If we take a look to Appendix C, we will notice that the first and the second computation do not differentiate a lot. Especially, we have the following figures.

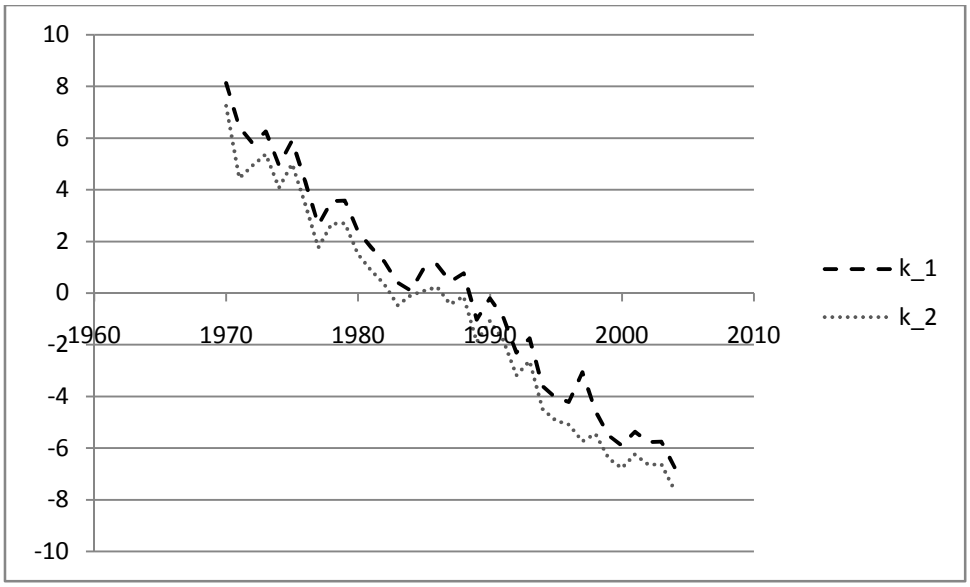

Figure 10.5: First and second estimation of k_t for female

https://doi.org/10.26219/heal.aueb.5981

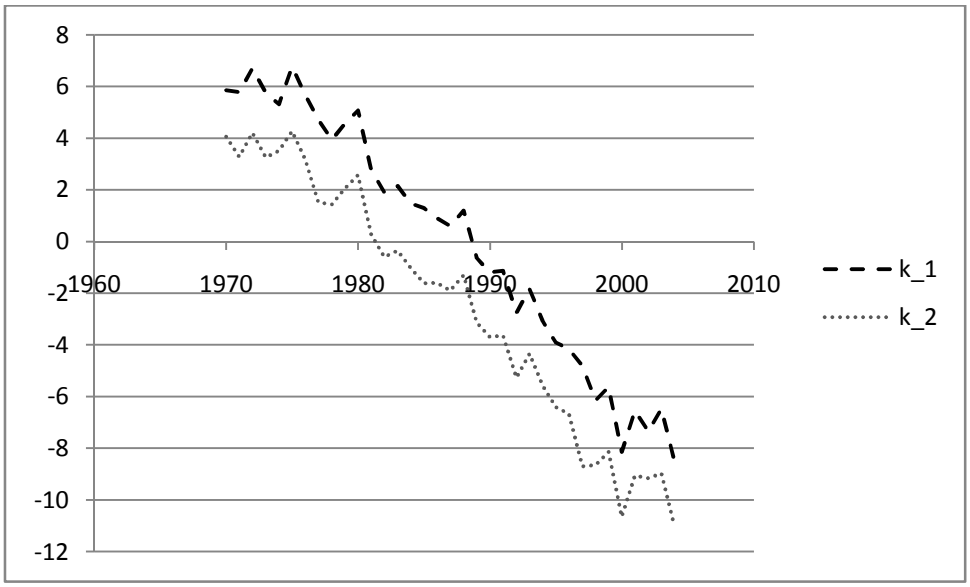

Figure 10.6: First and second estimation of k_t for male

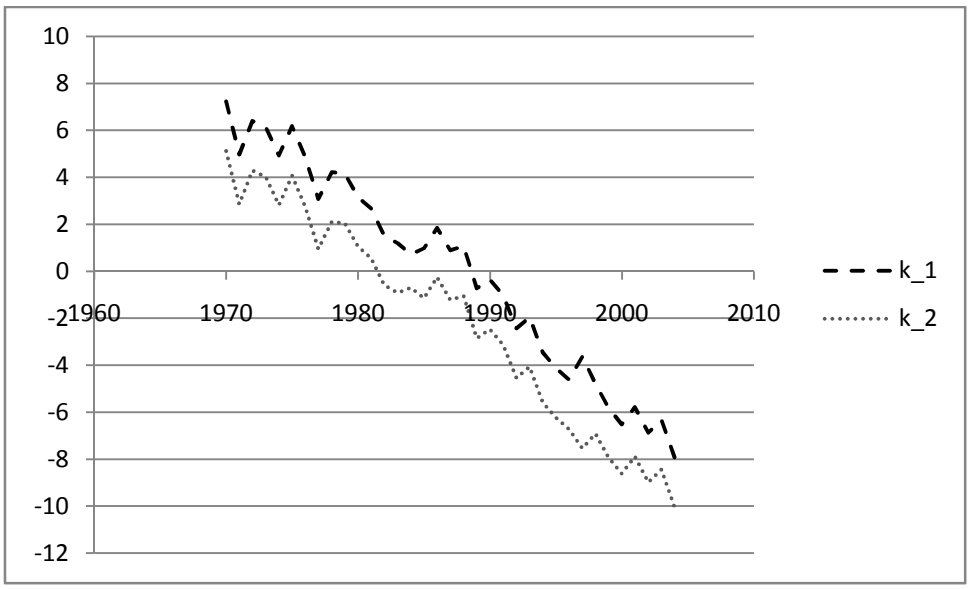

Figure 10.7: First and second estimation of k_t for total population

As it can be seen from Figures 10.5 – 10.7, re – estimating of k_t differs significantly from the first estimation of k_t, compared to the respective estimation for Spain and France.

https://doi.org/10.26219/heal.aueb.5981

10.3 Forecast Mortality Rate

The advantage of Lee – Carter method is that the only index which needs forecast is k_t. From the equation (3.9), the following results are given.

Table 10.1: Estimation of $\hat{c}$

	Female	***Male***	***Total***
$\hat{c}$	-0,437606424	-0,443354816	-0,445631454

And respectively, from (3.12) we have

Table 10.2: Forecasting for $\hat{k}_t$

Year	***Female***	***Male***	***Total***
2005	-8,070141812	-11,46857266	-10,48460678
2006	-8,507748236	-11,91192747	-10,93023824
2007	-8,945354659	-12,35528229	-11,37586969
2008	-9,382961083	-12,79863711	-11,82150115
2009	-9,820567507	-13,24199192	-12,2671326
2010	-10,25817393	-13,68534674	-12,71276406

The mortality rate, $(\ln m_{x,t})$, is now forecasted for years 2005, 2006, 2007, 2008, 2009 and 2010 from the following equation

$$\ln(m_{x,T+\Delta t}) \approx \hat{a}_x + \hat{b}_x \hat{k}_{T+\Delta t}$$

and we have the following figures.

https://doi.org/10.26219/heal.aueb.5981

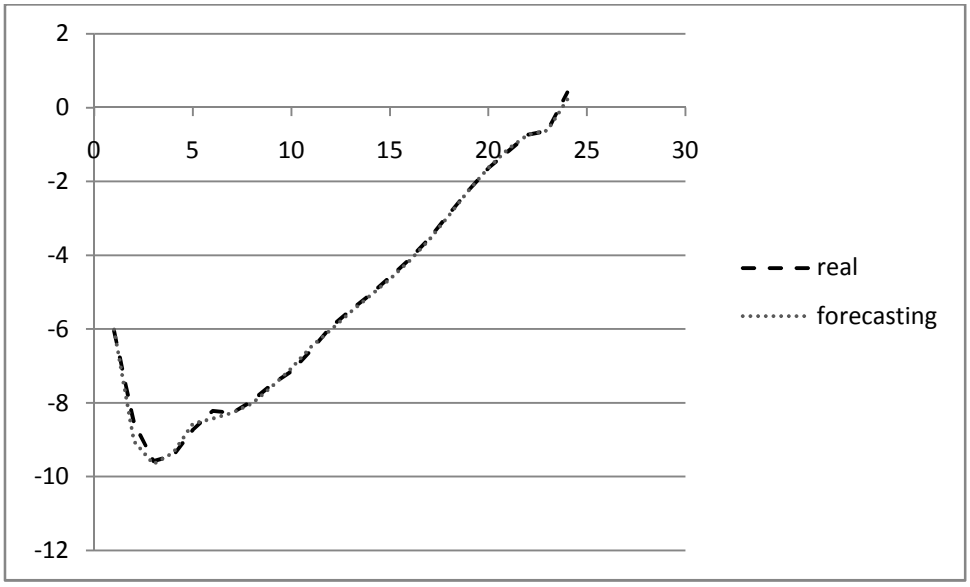

Figure 10.8: Comparison of real and forecasted data for female (2005)

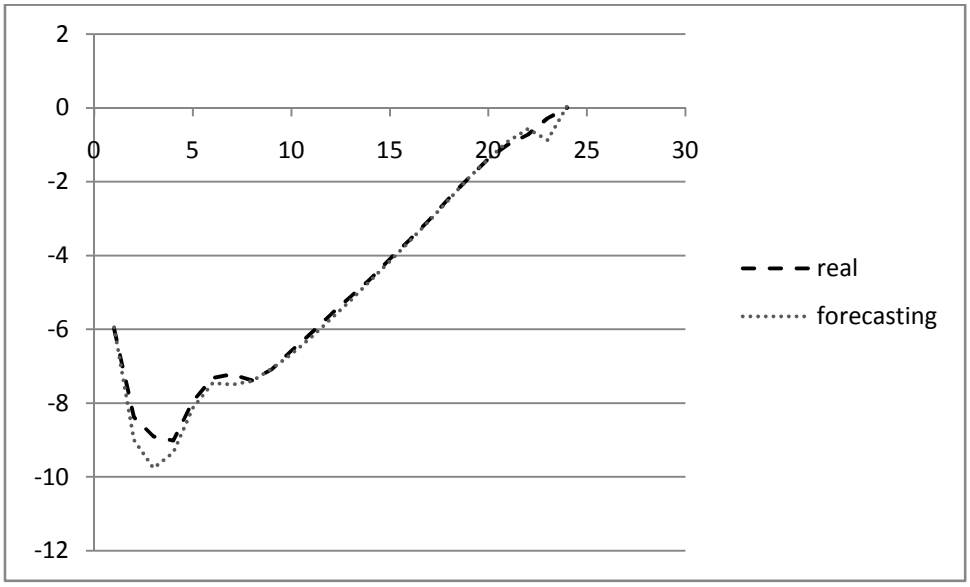

Figure 10.9: Comparison of real and forecasted data for male (2005)

https://doi.org/10.26219/heal.aueb.5981

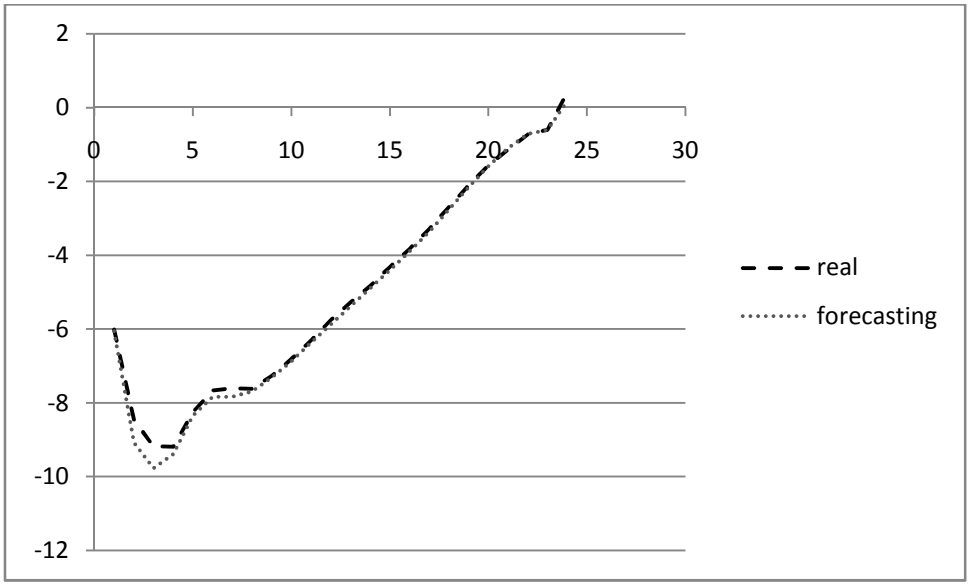

Figure 10.10: Comparison of real and forecasted data for total population (2005)

As it can be seen from the above figures, the forecasted data are very close to the real ones.

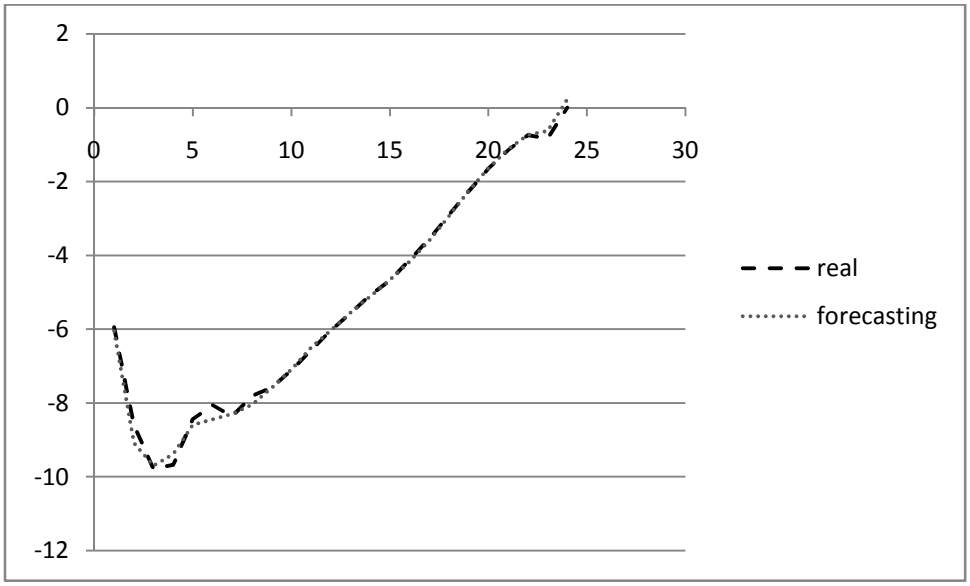

Figure 10.11: Comparison of real and forecasted data for female (2006)

https://doi.org/10.26219/heal.aueb.5981

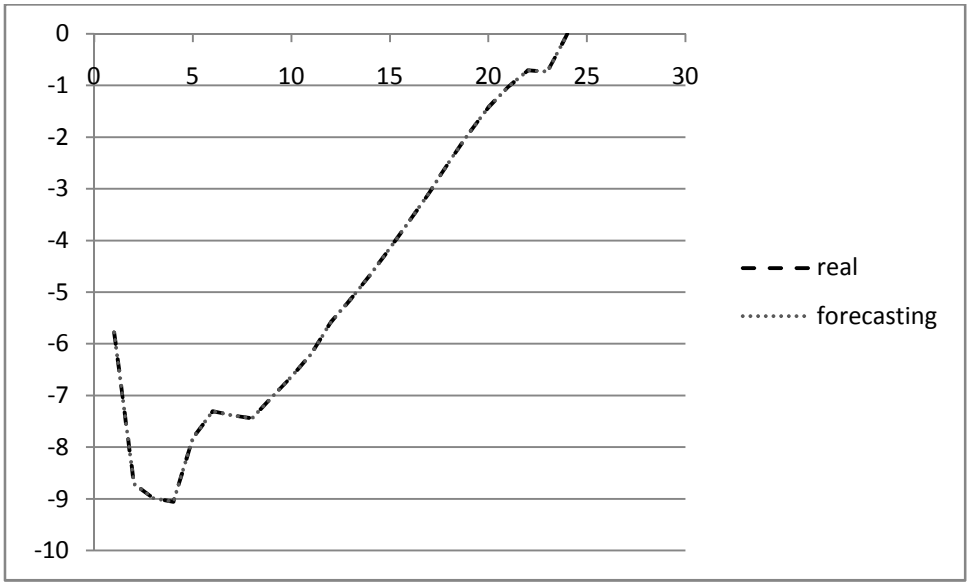

Figure 10.12: Comparison of real and forecasted data for male (2006)

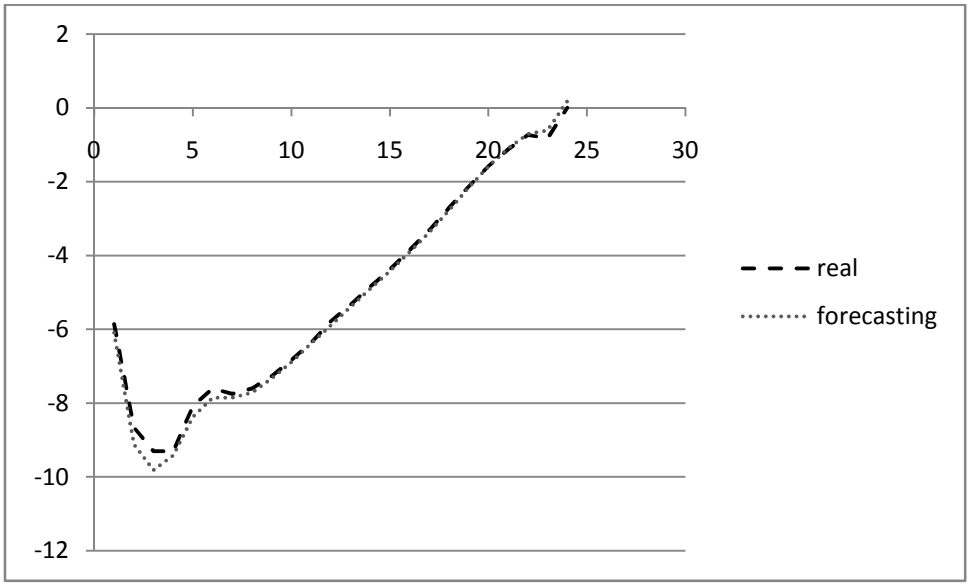

Figure 10.13: Comparison of real and forecasted data for total population (2006)

Similarly to 2005, in 2006 the forecasted data are very close to the real ones.

https://doi.org/10.26219/heal.aueb.5981

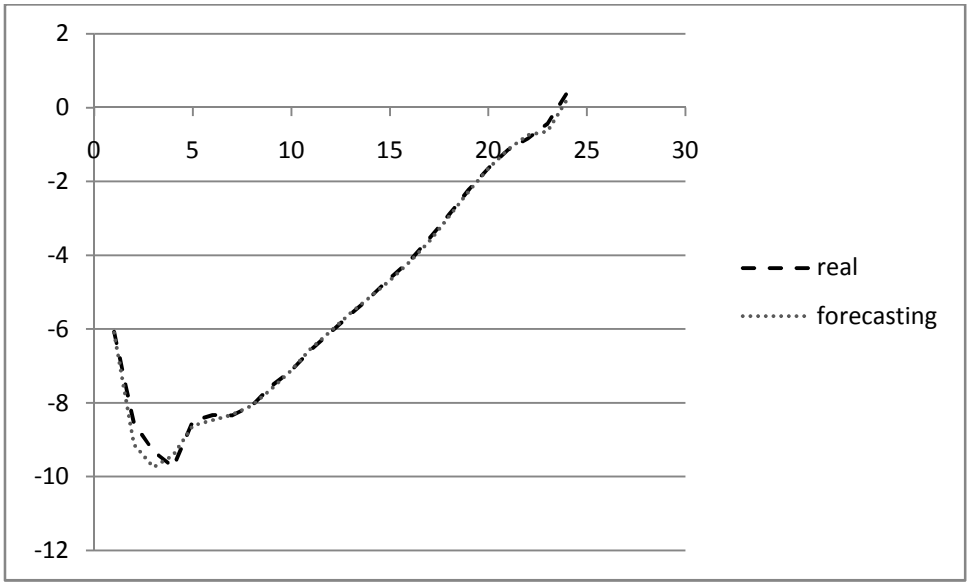

Figure 10.14: Comparison of real and forecasted data for female (2007)

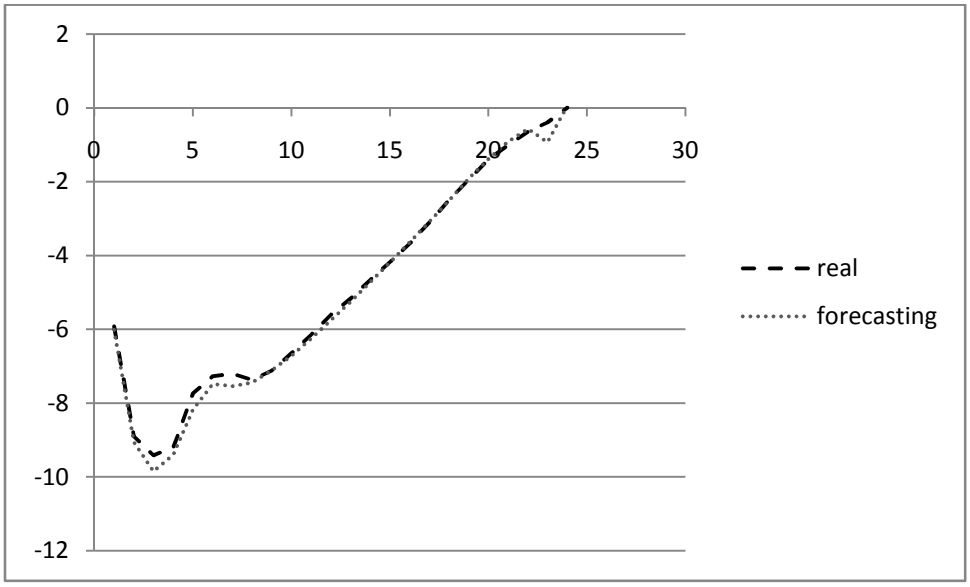

Figure 10.15: Comparison of real and forecasted data for male (2007)

https://doi.org/10.26219/heal.aueb.5981

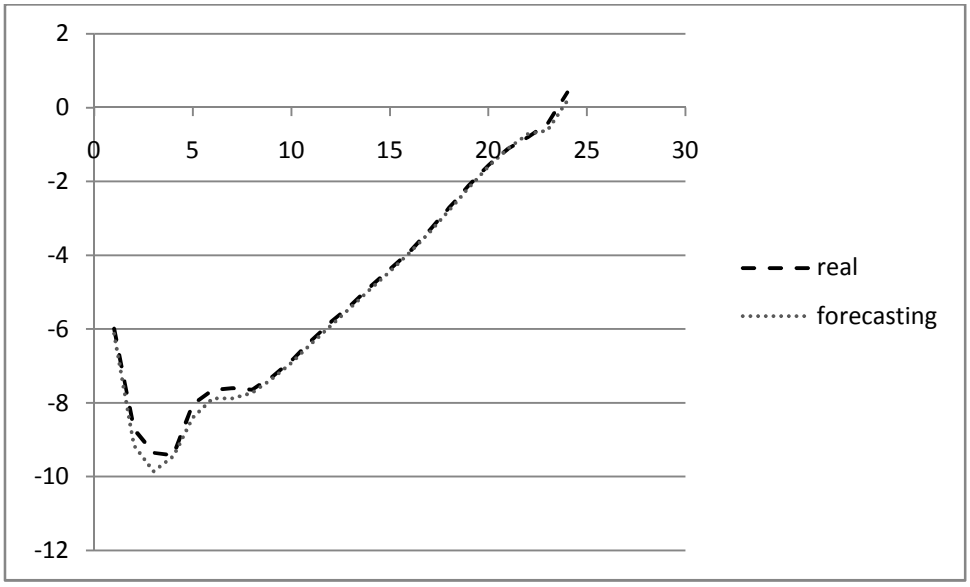

Figure 10.16: Comparison of real and forecasted data for total population (2007)

The same trend as in 2005 and 2006 is shown in 2007 as well, where the forecasted data and the real ones are very close to each other.

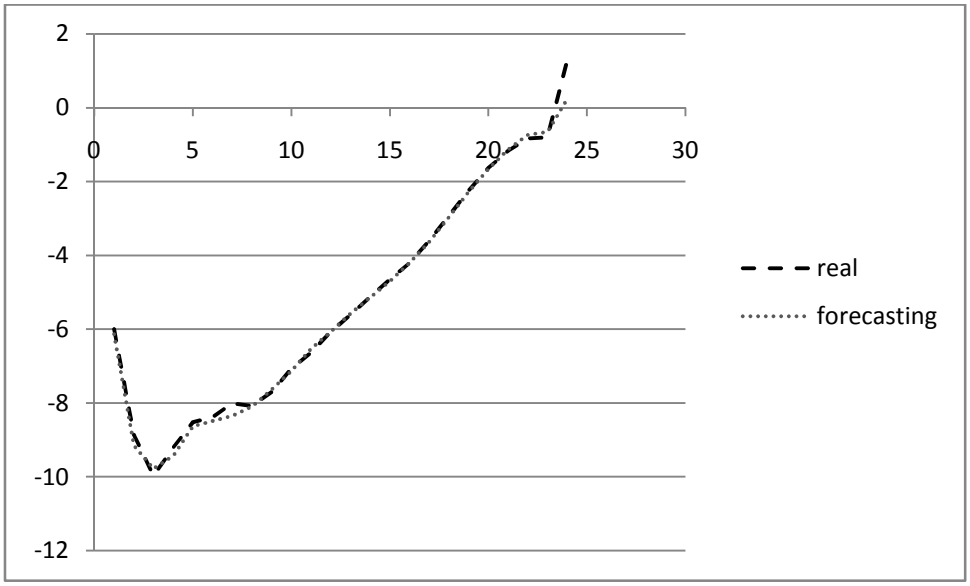

Figure 10.17: Comparison of real and forecasted data for female (2008)

https://doi.org/10.26219/heal.aueb.5981

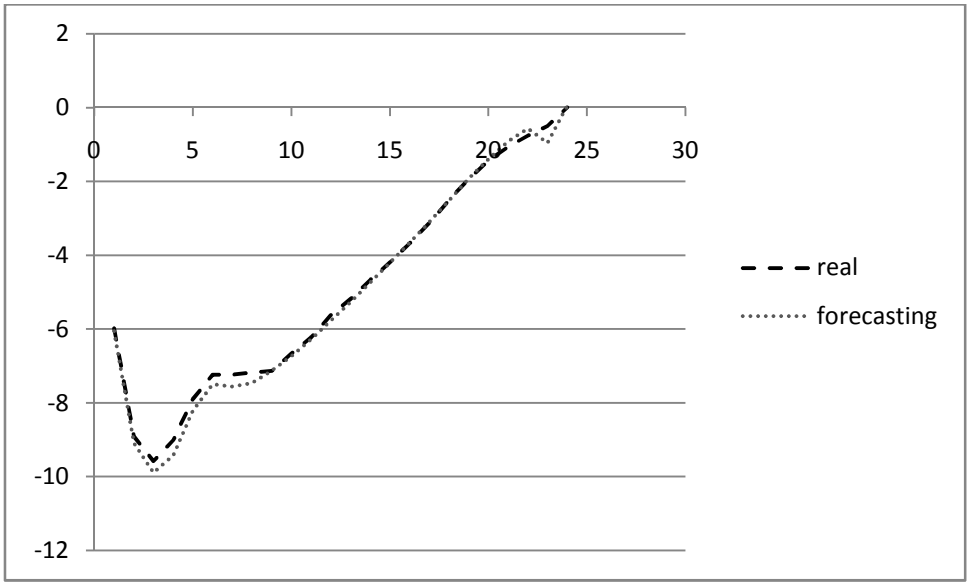

Figure 10.18: Comparison or real and forecast data for male (2008)

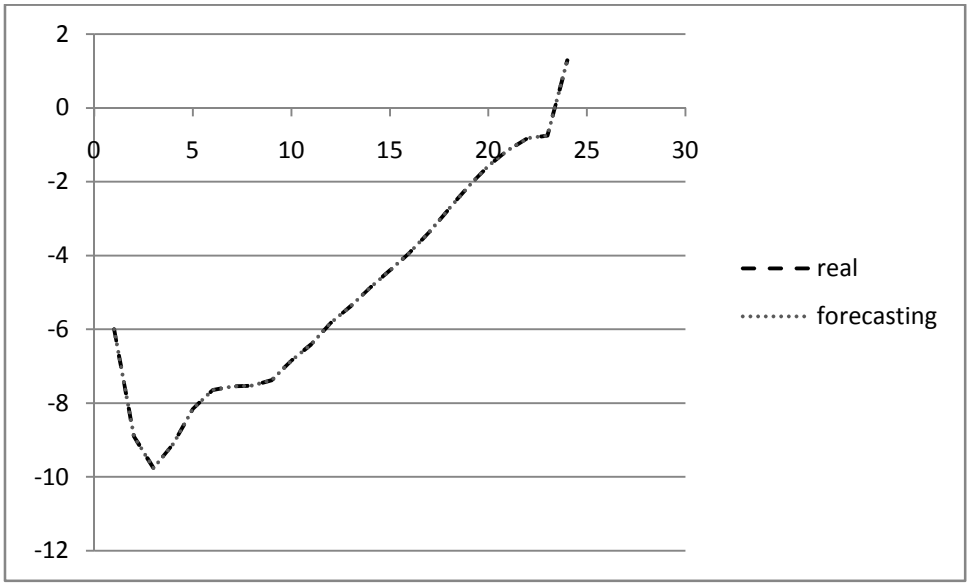

Figure 10.19: Comparison or real and forecast data for total population (2008)

For 2008, the actual and the forecasted data are very close to each other for female, male and total population.

https://doi.org/10.26219/heal.aueb.5981

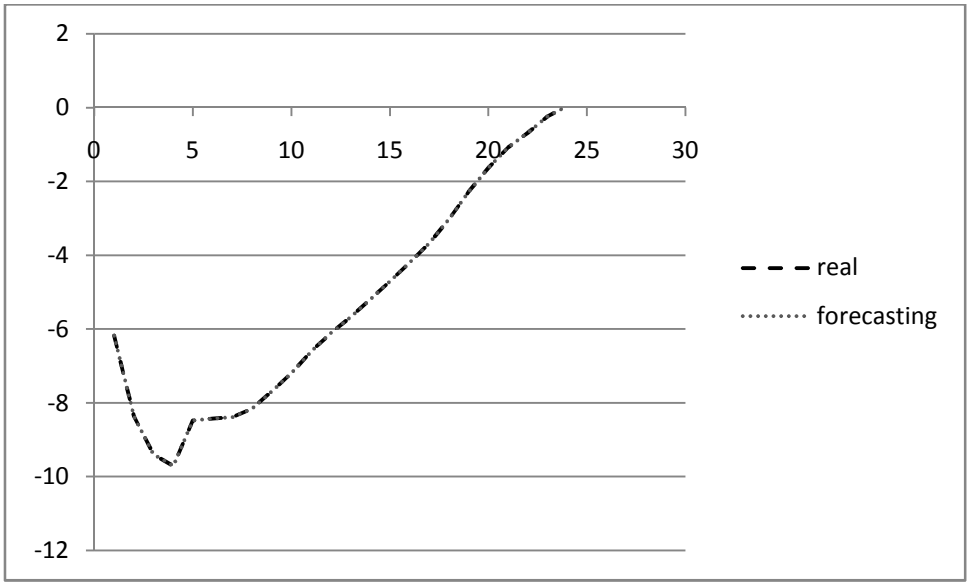

Figure 10.20: Comparison of real and forecasted data for female (2009)

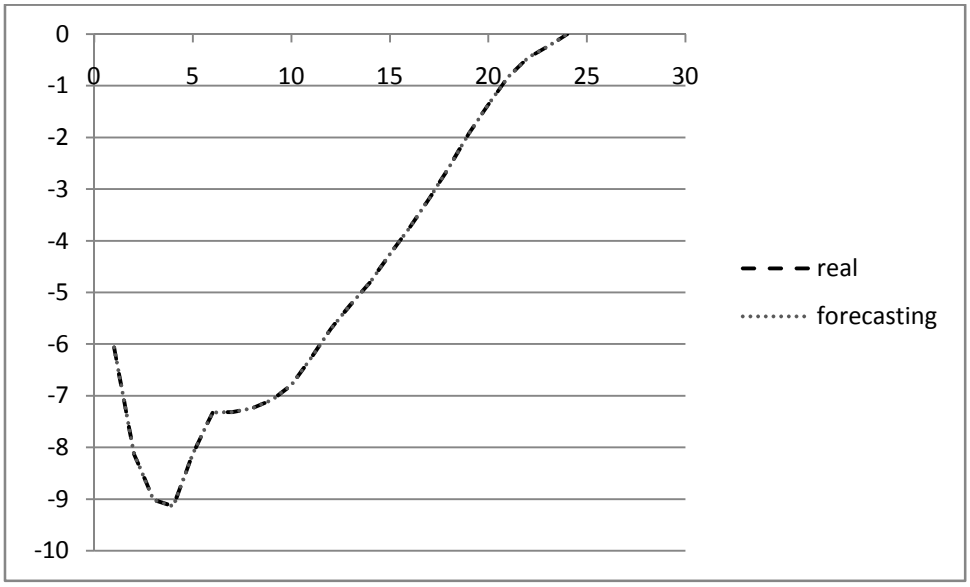

Figure 10.21: Comparison of real and forecasted data for male (2009)

https://doi.org/10.26219/heal.aueb.5981

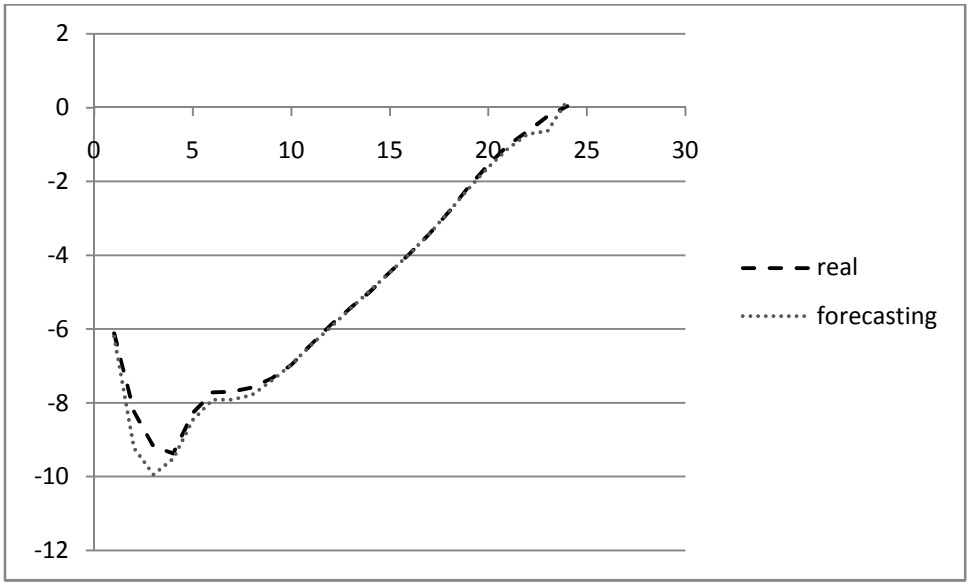

Figure 10.22: Comparison of real and forecasted data for total population (2009)

For 2009, the forecasted data are very close to the real ones.

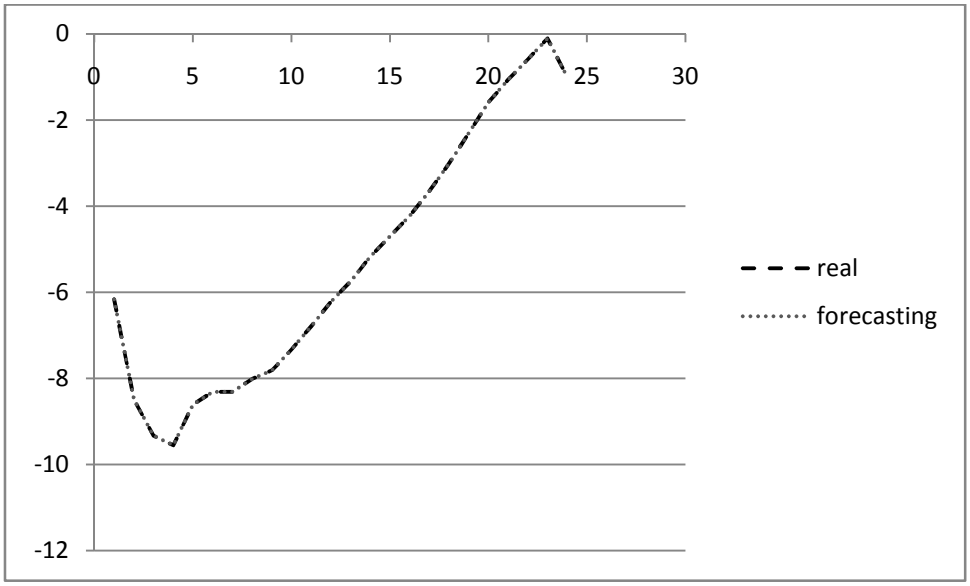

Figure 10.23: Comparison of real and forecasted data for female (2010)

https://doi.org/10.26219/heal.aueb.5981

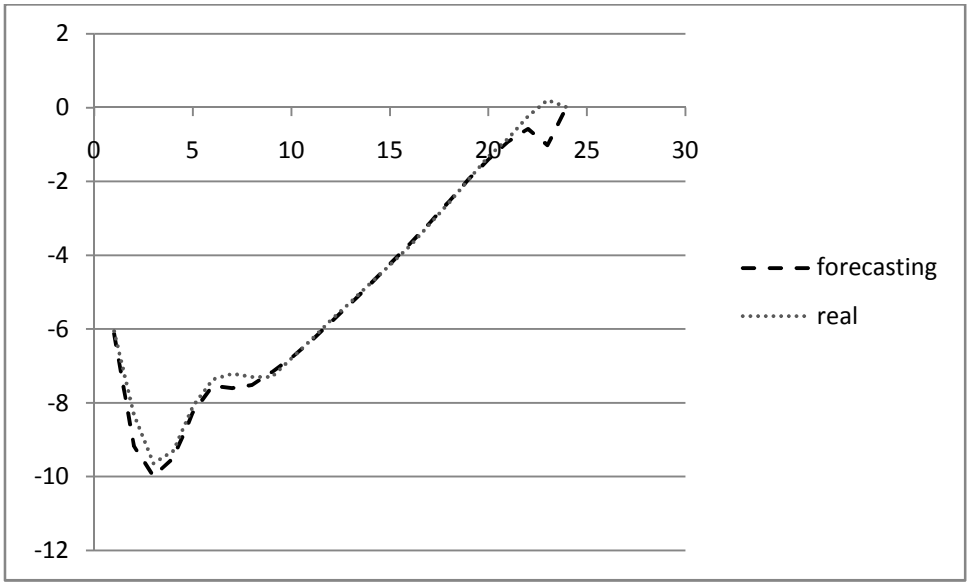

Figure 10.24: Comparison of real and forecasted data for male (2010)

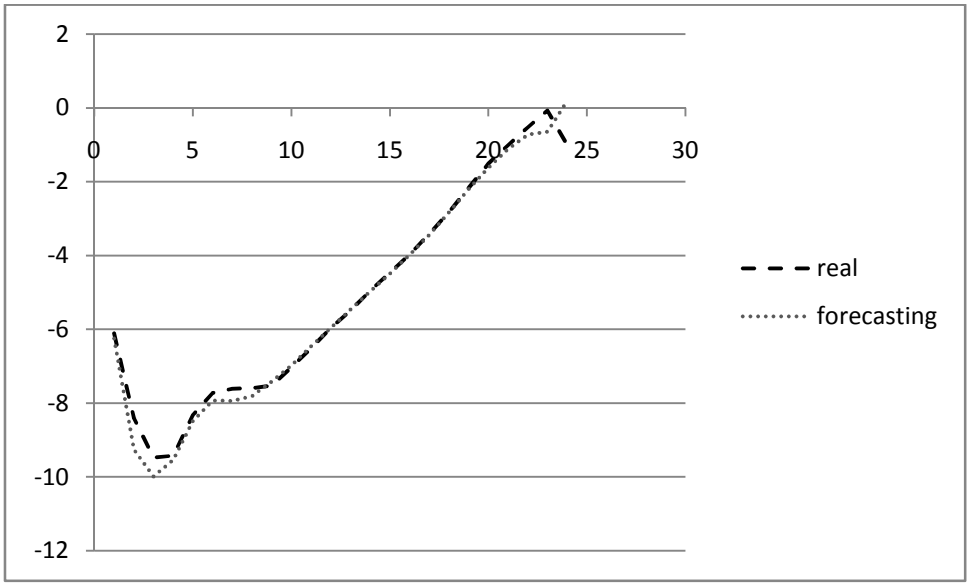

Figure 10.25: Comparison of real and forecasted data for total population (2010)

For 2010, the forecasted data and the real ones seem to be identical. For male and total population between forecasted and real data there seem to be more differences.

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CHAPTER 11

Conclusions

The purpose of this thesis is the study of mortality and its forecast. As a result, in chapters 2 to 7, the existed mortality methods from 1980 to today were studied. From this study, the “Lee – Carter” method was found to be the most significant.

The “Lee – Carter” method was implemented for three countries: Spain, France and Sweden. Mortality rates for the time period 1970 to 2004 are considered as known and mortality was predicted using “Lee – Carter” method for 2005 to 2009 for Spain and France and for 2005 to 2010 for Sweden. The aim was to examine any discrepancy between actual and forecast data using “Lee – Carter” method. The results are presented in this chapter.

In chapters 8, 9 and 10, Lee – Carter Method was applied for Spain, France and Sweden respectively. All the estimations were based to the reference article of Lee – Carter (1992).

According to Lee and Carter (1992), the calculations are divided in two stages: in the first stage parameter “a_x” is estimated from equation (3.3) After that, indexes k_t are calculated and as it is found are almost equal to the sum over age of $\left(\ln(m_{x,t}) - a_x\right)$, since the sum of the "$b_x$"’s is to unity. Finally, the "b_x" can be found by regressing without a constant term.

In the second stage, the time series of "k_t" is re – estimated by solving for "k_t" the equation (3.4).

For the first and second estimation of "k_t" for Spain and France, estimations seemed to not differentiate a lot and decline more in specific years (numerical data are given in details and can be seen in Tables A.5, A.6, A.7, B.5, B.6 and B.7).

At the following table, the years for which a larger differentiation is observed for each examined country is represented. It must be mentioned that for Sweden all the estimations differ a lot for all the examined years.

Table 11.1: Years with noticeable decline in k_t estimation

	Female	**Male**	**Total Population**
Spain	1980 1988	1980 1985 1989 1996 2000	1980 1988 1994
France	1984 1987	1985 1986 1987 2002	1984 1987

After that, index $\ln m_{x,t}$ was forecasted. Lee – Carter method has the advantage that only parameter "k_t" needs to be forecasted. Thus, from equation (3.9) parameter $\hat{c}$ was forecasted and from (3.12) "k_t" was calculated. As a result, the mortality rate, $(\ln m_{x,t})$, is now forecasted for years 2005, 2006, 2007, 2008 and 2009 for Spain, France and Sweden and for 2010 for Sweden only, using the respective equation ($\ln(m_{x,T+\Delta t}) \approx \hat{a}_x + \hat{b}_x \hat{k}_{T+\Delta t}$).

After, indexes $\ln \hat{m}_{x,t}$ were calculated, forecasted and real data were compared and the data were very similar to each other. To conclude, it can be said that Lee – Carter method is a satisfying method for forecasting mortality.

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APPENDIX A

SPAIN

Table A.1: Estimation of a_x

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-4,813888773	-4,594055918	-4,69428485
2	*1-4*	-7,752329372	-7,55466432	-7,645020496
3	*5-9*	-8,516387445	-8,167907675	-8,321335524
4	*10-14*	-8,559762042	-8,152492378	-8,329791802
5	*15-19*	-8,075129564	-7,182787972	-7,523504983
6	*20-24*	-7,879903079	-6,783872519	-7,178670018
7	*25-29*	-7,666107064	-6,6657631	-7,037403078
8	*30-34*	-7,38228702	-6,484895219	-6,829033153
9	*35-39*	-7,060568614	-6,250516619	-6,572550968
10	*40-44*	-6,672154725	-5,90038536	-6,214188681
11	*45-49*	-6,254532716	-5,46844913	-5,788566095
12	*50-54*	-5,844168008	-5,023288169	-5,358390747
13	*55-59*	-5,434985427	-4,581230706	-4,933859191
14	*60-64*	-4,969065407	-4,13082593	-4,488057389
15	*65-69*	-4,448138103	-3,672779078	-4,021489746
16	*70-74*	-3,84349707	-3,190684897	-3,508894746
17	*75-79*	-3,199731477	-2,701689322	-2,969504782
18	*80-84*	-2,572521236	-2,21689243	-2,429785758
19	*85-89*	-1,997386144	-1,758645586	-1,915059519
20	*90-94*	-1,503591613	-1,342192709	-1,455987106
21	*95-99*	-1,119868169	-1,01335166	-1,093101239
22	*100-104*	-0,808308335	-0,709617095	-0,785536651
23	*105-109*	-0,675270967	-0,761356833	-0,685193673
24	*110+*	-0,527540768	-0,693816313	-0,631634141

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Table A.2: First estimation of k_t

Year	*Female*	*Male*	*Total*
1970	6,488352543	2,22000233	4,135994862
1971	2,56864808	-2,14874815	0,316189946
1972	4,333249649	0,932155064	2,160709248
1973	3,995705265	-2,91901039	1,397285573
1974	3,942920216	-1,21715174	1,847198621
1975	5,092067407	3,391537284	3,811775998
1976	4,653093072	3,429931639	3,664524502
1977	4,022729452	2,668345003	3,03451633
1978	5,723022633	4,143420434	4,727929871
1979	4,647866192	4,196314495	4,019881884
1980	3,088045371	2,03131663	2,503577424
1981	0,964300248	2,829811391	1,232367204
1982	0,294537756	0,682948894	-0,09583853
1983	-0,06569546	1,227229764	-0,26060711
1984	-0,30517381	0,471518753	-0,14242917
1985	0,690805531	1,020453894	0,936032049
1986	1,495356294	1,416102039	1,510388225
1987	1,275770818	2,350816317	1,832442494
1988	0,078838127	0,620803458	0,522519084
1989	0,404093957	1,355689155	0,224049318
1990	2,039235734	2,91245505	2,810017219
1991	-0,40771674	1,666468668	0,84740427
1992	-0,80636837	0,897591827	0,465572686
1993	-0,49366011	0,526616157	0,80792342
1994	-2,58928265	1,410302504	-1,32254291
1995	-2,18882421	-1,40508191	-1,16411553
1996	-1,80646288	-0,8254776	-0,81618019
1997	-3,4315411	-1,41833786	-2,21154488
1998	-3,64855632	-3,06118135	-2,46728715
1999	-4,86006365	-2,74337962	-3,81904145
2000	-5,98071503	-3,51640853	-5,13882802
2001	-7,78389734	-5,46072697	-6,59959174
2002	-7,02254799	-5,98407607	-6,14747266
2003	-6,00641615	-5,12471972	-5,2831212
2004	-8,40171658	-6,57753086	-7,33969969

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Table A.3: Estimation of b_x

X	*Age*	*Female*	*Male*	*Total*
1	*0*	0,1311842	0,117684	0,1490
2	*1-4*	0,0997978	0,097562	0,1158
3	*5-9*	0,0915147	0,089199	0,1067
4	*10-14*	0,0661936	0,0661	0,07784
5	*15-19*	0,0381088	0,041591	0,04403
6	*20-24*	0,0465278	0,043794	0,04416
7	*25-29*	0,0460289	0,036891	0,03825
8	*30-34*	0,0391139	0,015788	0,02296
9	*35-39*	0,0388976	0,007641	0,01995
10	*40-44*	0,03789	0,018094	0,02827
11	*45-49*	0,0422611	0,021785	0,03239
12	*50-54*	0,0529149	0,027453	0,04072
13	*55-59*	0,0585373	0,032099	0,04556
14	*60-64*	0,0629704	0,035442	0,05011
15	*65-69*	0,0631604	0,035182	0,05131
16	*70-74*	0,0659178	0,035855	0,05489
17	*75-79*	0,0614084	0,032267	0,05321
18	*80-84*	0,0495568	0,029167	0,04710
19	*85-89*	0,0357965	0,022168	0,03610
20	*90-94*	0,0219585	0,013048	0,02273
21	*95-99*	0,0113016	-0,003146	0,008974
22	*100-104*	0,0009383	0,005111	0,00003258
23	*105-109*	-0,0249687	0,015483	-0,02701
24	*110+*	-0,1370108	0,163743	-0,06306

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Table A.4: Re-estimation k_t

Year	*Female*	*Male*	*Total*
1970	6,488996922	2,219752922	4,135682993
1971	2,569846793	-2,14798031	0,314450079
1972	4,332524658	0,931214408	2,161425953
1973	3,994605999	-2,919233741	1,398551443
1974	3,94112594	-1,218707542	1,84779355
1975	5,089259656	3,391173359	3,812398317
1976	4,655986165	3,431807163	3,665410337
1977	4,022277719	2,667320076	3,035248161
1978	5,723545228	4,14370509	4,725501922
1979	4,647460401	4,196742036	4,021569627
1980	1,837813904	1,338067695	0,797713977
1981	0,969302128	2,82400656	1,23305684
1982	0,287675436	0,682635224	-0,096513268
1983	-0,065837449	1,224218669	-0,259760998
1984	-0,302816064	0,470080195	-0,143500446
1985	0,690664801	1,712564252	0,938099072
1986	1,491987405	1,424708816	1,511243533
1987	1,288786178	2,330136258	1,831982104
1988	0,770716982	0,623658026	1,215953845
1989	0,405368258	0,950772916	0,22076763
1990	2,041579684	2,909013153	2,809128375
1991	-0,412487534	1,665334969	0,849389582
1992	-0,807977696	0,899337495	0,460210997
1993	-0,502541382	0,527626398	0,80300716
1994	-2,584465711	1,418875802	0,479345568
1995	-2,188060448	-1,404260163	-1,162005457
1996	-1,806551915	-0,133762129	-0,819515032
1997	-3,430159007	-1,417192494	-2,210120427
1998	-3,644416549	-3,060779772	-2,466344214
1999	-4,862167156	-2,742421673	-3,81977074
2000	-5,982728758	-4,428986212	-5,143488108
2001	-7,784389681	-5,457444564	-6,599827109
2002	-7,017833314	-5,985100121	-6,153301219
2003	-6,004766951	-5,127792955	-5,283886724
2004	-8,402159985	-6,57526496	-7,337922777

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Table A.5: First and second estimation of k_t for female

	1st estimation	***2nd estimation***
Year	***Female***	***Female***
1970	6,488352543	6,488996922
1971	2,56864808	2,569846793
1972	4,333249649	4,332524658
1973	3,995705265	3,994605999
1974	3,942920216	3,94112594
1975	5,092067407	5,089259656
1976	4,653093072	4,655986165
1977	4,022729452	4,022277719
1978	5,723022633	5,723545228
1979	4,647866192	4,647460401
1980	3,088045371	1,837813904
1981	0,964300248	0,969302128
1982	0,294537756	0,287675436
1983	-0,06569546	-0,065837449
1984	-0,30517381	-0,302816064
1985	0,690805531	0,690664801
1986	1,495356294	1,491987405
1987	1,275770818	1,288786178
1988	0,078838127	0,770716982
1989	0,404093957	0,405368258
1990	2,039235734	2,041579684
1991	-0,40771674	-0,412487534
1992	-0,80636837	-0,807977696
1993	-0,49366011	-0,502541382
1994	-2,58928265	-2,584465711
1995	-2,18882421	-2,188060448
1996	-1,80646288	-1,806551915
1997	-3,4315411	-3,430159007
1998	-3,64855632	-3,644416549
1999	-4,86006365	-4,862167156
2000	-5,98071503	-5,982728758
2001	-7,78389734	-7,784389681
2002	-7,02254799	-7,017833314
2003	-6,00641615	-6,004766951
2004	-8,40171658	-8,402159985

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Table A.6: First and second estimation of k_t for male

	1st estimation	***2nd estimation***
Year	***Male***	***Male***
1970	2,22000233	2,219752922
1971	-2,14874815	-2,14798031
1972	0,932155064	0,931214408
1973	-2,91901039	-2,919233741
1974	-1,21715174	-1,218707542
1975	3,391537284	3,391173359
1976	3,429931639	3,431807163
1977	2,668345003	2,667320076
1978	4,143420434	4,14370509
1979	4,196314495	4,196742036
1980	2,03131663	1,338067695
1981	2,829811391	2,82400656
1982	0,682948894	0,682635224
1983	1,227229764	1,224218669
1984	0,471518753	0,470080195
1985	1,020453894	1,712564252
1986	1,416102039	1,424708816
1987	2,350816317	2,330136258
1988	0,620803458	0,623658026
1989	1,355689155	0,950772916
1990	2,91245505	2,909013153
1991	1,666468668	1,665334969
1992	0,897591827	0,899337495
1993	0,526616157	0,527626398
1994	1,410302504	1,418875802
1995	-1,40508191	-1,404260163
1996	-0,8254776	-0,133762129
1997	-1,41833786	-1,417192494
1998	-3,06118135	-3,060779772
1999	-2,74337962	-2,742421673
2000	-3,51640853	-4,428986212
2001	-5,46072697	-5,457444564
2002	-5,98407607	-5,985100121
2003	-5,12471972	-5,127792955
2004	-6,57753086	-6,57526496

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Table A.7: First and second estimation of k_t for total population

	1st estimation	***2nd estimation***
Year	***Total***	***Total***
1970	4,135994862	4,135682993
1971	0,316189946	0,314450079
1972	2,160709248	2,161425953
1973	1,397285573	1,398551443
1974	1,847198621	1,84779355
1975	3,811775998	3,812398317
1976	3,664524502	3,665410337
1977	3,03451633	3,035248161
1978	4,727929871	4,725501922
1979	4,019881884	4,021569627
1980	2,503577424	0,797713977
1981	1,232367204	1,23305684
1982	-0,09583853	-0,096513268
1983	-0,26060711	-0,259760998
1984	-0,14242917	-0,143500446
1985	0,936032049	0,938099072
1986	1,510388225	1,511243533
1987	1,832442494	1,831982104
1988	0,522519084	1,215953845
1989	0,224049318	0,22076763
1990	2,810017219	2,809128375
1991	0,84740427	0,849389582
1992	0,465572686	0,460210997
1993	0,80792342	0,80300716
1994	-1,32254291	0,479345568
1995	-1,16411553	-1,162005457
1996	-0,81618019	-0,819515032
1997	-2,21154488	-2,210120427
1998	-2,46728715	-2,466344214
1999	-3,81904145	-3,81977074
2000	-5,13882802	-5,143488108
2001	-6,59959174	-6,599827109
2002	-6,14747266	-6,153301219
2003	-5,2831212	-5,283886724
2004	-7,33969969	-7,337922777

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Table A.8: Forecasting for $\ln(m_{x,t})$ for 2005

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,973574836	-5,398301543	-5,837916734
2	*1-4*	-8,634555415	-8,221397365	-8,533829705
3	*5-9*	-9,325389765	-8,777488464	-9,140298759
4	*10-14*	-9,144922415	-8,604215941	-8,927243519
5	*15-19*	-8,412016508	-7,467018452	-7,861452042
6	*20-24*	-8,291215121	-7,083158173	-7,517614877
7	*25-29*	-8,073008763	-6,917874053	-7,330986431
8	*30-34*	-7,728059184	-6,592789495	-7,005259918
9	*35-39*	-7,404428656	-6,302734769	-6,725674834
10	*40-44*	-7,007107447	-6,024038706	-6,431171724
11	*45-49*	-6,628126553	-5,617326556	-6,037171711
12	*50-54*	-6,311942877	-5,210900378	-5,670932293
13	*55-59*	-5,952463073	-4,800593409	-5,283549584
14	*60-64*	-5,525732256	-4,373034501	-4,872670769
15	*65-69*	-5,006484578	-3,913210824	-4,415313585
16	*70-74*	-4,426219333	-3,435715886	-3,930196451
17	*75-79*	-3,742590035	-2,922200127	-3,377911846
18	*80-84*	-3,010610047	-2,416218015	-2,791296239
19	*85-89*	-2,313832043	-1,910140411	-2,192140801
20	*90-94*	-1,697707722	-1,431361984	-1,630448534
21	*95-99*	-1,219775841	-0,991852079	-1,161980115
22	*100-104*	-0,816603034	-0,744545372	-0,785786715
23	*105-109*	-0,454544284	-0,867166756	-0,477881611
24	*110+*	0,683653226	-1,812826471	-0,147624567

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Table A.9: Real value of $\ln(m_{x,t})$ for 2005

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,686335561	-5,441033669	-5,552435722
2	*1-4*	-8,595154733	-8,302081812	-8,435613204
3	*5-9*	-9,142681724	-9,011489513	-9,07931211
4	*10-14*	-9,171119659	-8,71564413	-8,91023578
5	*15-19*	-8,477972478	-7,464624841	-7,834096347
6	*20-24*	-8,343239884	-7,22520951	-7,623148068
7	*25-29*	-8,322449115	-7,207509933	-7,60090246
8	*30-34*	-7,859673188	-6,948577274	-7,2904809
9	*35-39*	-7,372970391	-6,536191723	-6,861826347
10	*40-44*	-6,955895654	-6,125681889	-6,454135129
11	*45-49*	-6,494322001	-5,702083918	-6,022748837
12	*50-54*	-6,19040341	-5,244397525	-5,614124603
13	*55-59*	-5,81751124	-4,829314238	-5,216553065
14	*60-64*	-5,406125319	-4,439316822	-4,826315735
15	*65-69*	-4,911423644	-3,98427117	-4,372313956
16	*70-74*	-4,304029094	-3,506391245	-3,86614183
17	*75-79*	-3,6301201	-2,982835792	-3,305305553
18	*80-84*	-2,887254621	-2,428318398	-2,686029447
19	*85-89*	-2,218694414	-1,944672915	-2,121597759
20	*90-94*	-1,632455803	-1,450598243	-1,579107563
21	*95-99*	-1,158078894	-1,01655039	-1,123253403
22	*100-104*	-0,767579821	-0,709754319	-0,75611041
23	*105-109*	-0,504833612	-0,391881781	-0,484136631
24	*110+*	-0,13248965	-0,154150513	-0,136859209

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Table A.10: Forecasting for $\ln(m_{x,t})$ for 2006

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,031030262	-5,428743687	-5,888198124
2	*1-4*	-8,678264377	-8,24663441	-8,572907456
3	*5-9*	-9,365470934	-8,800562193	-9,176305633
4	*10-14*	-9,17391357	-8,621314491	-8,953511327
5	*15-19*	-8,428707217	-7,477777087	-7,876310362
6	*20-24*	-8,311593144	-7,094486673	-7,532517066
7	*25-29*	-8,093168279	-6,927416906	-7,343894237
8	*30-34*	-7,745190102	-6,596873487	-7,013007977
9	*35-39*	-7,421464841	-6,30471132	-6,732407141
10	*40-44*	-7,023702328	-6,028719207	-6,440711689
11	*45-49*	-6,646635867	-5,622961834	-6,048102007
12	*50-54*	-6,335118292	-5,218001837	-5,684673623
13	*55-59*	-5,978100958	-4,808896682	-5,298924216
14	*60-64*	-5,55331173	-4,382202531	-4,88958084
15	*65-69*	-5,034147267	-3,922311598	-4,432628606
16	*70-74*	-4,455089695	-3,44499075	-3,948719575
17	*75-79*	-3,769485391	-2,930546858	-3,395868039
18	*80-84*	-3,032314697	-2,423762847	-2,807190558
19	*85-89*	-2,329510023	-1,915874763	-2,204323071
20	*90-94*	-1,707325	-1,434737201	-1,638118977
21	*95-99*	-1,224725661	-0,991038281	-1,165008472
22	*100-104*	-0,817013986	-0,74586747	-0,785797709
23	*105-109*	-0,443608612	-0,871171852	-0,468766844
24	*110+*	0,743660559	-1,855183018	-0,126344402

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Table A.11: Real value of $\ln(m_{x,t})$ for 2006

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,738684527	-5,492873461	-5,604570825
2	*1-4*	-8,606024405	-8,454218392	-8,527243527
3	*5-9*	-9,471705136	-8,778557956	-9,061920367
4	*10-14*	-9,133379331	-8,852665928	-8,979228651
5	*15-19*	-8,606024405	-7,612975041	-7,977780111
6	*20-24*	-8,454218392	-7,315723517	-7,719685996
7	*25-29*	-8,322449115	-7,28609172	-7,660652464
8	*30-34*	-7,971966141	-7,067924031	-7,405335676
9	*35-39*	-7,392263594	-6,633918613	-6,934099254
10	*40-44*	-6,933073087	-6,186506668	-6,488387266
11	*45-49*	-6,536191723	-5,736822329	-6,059315214
12	*50-54*	-6,174426309	-5,259096653	-5,620176891
13	*55-59*	-5,857633199	-4,855171191	-5,246294338
14	*60-64*	-5,459660517	-4,470027592	-4,86381139
15	*65-69*	-4,96728849	-4,04612599	-4,431216879
16	*70-74*	-4,370414319	-3,571629894	-3,931358742
17	*75-79*	-3,702897246	-3,06326193	-3,38145358
18	*80-84*	-2,982145	-2,517735673	-2,777046644
19	*85-89*	-2,301066247	-2,03429993	-2,205086401
20	*90-94*	-1,713998748	-1,513137314	-1,654440509
21	*95-99*	-1,241501616	-1,104389947	-1,207766659
22	*100-104*	-0,827229582	-0,666312474	-0,794655127
23	*105-109*	-0,586239063	-0,495393645	-0,566939289
24	*110+*	0,148887137	1,098612289	0,279713659

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Table A.12: Forecasting for $\ln(m_{x,t})$ for 2007

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,088485689	-5,459185831	-5,938479514
2	*1-4*	-8,721973339	-8,271871455	-8,611985208
3	*5-9*	-9,405552103	-8,823635922	-9,212312508
4	*10-14*	-9,202904725	-8,63841304	-8,979779135
5	*15-19*	-8,445397927	-7,488535722	-7,891168681
6	*20-24*	-8,331971166	-7,105815174	-7,547419255
7	*25-29*	-8,113327796	-6,936959759	-7,356802044
8	*30-34*	-7,76232102	-6,600957479	-7,020756035
9	*35-39*	-7,438501025	-6,306687871	-6,739139448
10	*40-44*	-7,040297208	-6,033399709	-6,450251655
11	*45-49*	-6,665145181	-5,628597112	-6,059032304
12	*50-54*	-6,358293706	-5,225103297	-5,698414953
13	*55-59*	-6,003738844	-4,817199955	-5,314298848
14	*60-64*	-5,580891203	-4,391370561	-4,90649091
15	*65-69*	-5,061809956	-3,931412372	-4,449943627
16	*70-74*	-4,483960057	-3,454265613	-3,9672427
17	*75-79*	-3,796380748	-2,938893588	-3,413824232
18	*80-84*	-3,054019347	-2,431307679	-2,823084877
19	*85-89*	-2,345188002	-1,921609115	-2,21650534
20	*90-94*	-1,716942279	-1,438112419	-1,645789419
21	*95-99*	-1,229675482	-0,990224483	-1,168036829
22	*100-104*	-0,817424938	-0,747189569	-0,785808704
23	*105-109*	-0,432672941	-0,875176948	-0,459652077
24	*110+*	0,803667892	-1,897539566	-0,105064238

https://doi.org/10.26219/heal.aueb.5981

Table A.13: Real value of $\ln(m_{x,t})$ for 2007

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,70508332	-5,574861695	-5,635750064
2	*1-4*	-8,507242861	-8,449534543	-8,477972478
3	*5-9*	-9,261633666	-9,061920367	-9,152071464
4	*10-14*	-9,190537745	-8,746606356	-8,940313235
5	*15-19*	-8,628124752	-7,596910438	-7,971966141
6	*20-24*	-8,502304579	-7,374564017	-7,775255847
7	*25-29*	-8,412833176	-7,28609172	-7,682112515
8	*30-34*	-7,998399398	-7,14982684	-7,471630124
9	*35-39*	-7,480456306	-6,70074111	-7,005368108
10	*40-44*	-6,990050522	-6,164390382	-6,489703055
11	*45-49*	-6,517742275	-5,75908379	-6,068345591
12	*50-54*	-6,14694945	-5,28995245	-5,633790103
13	*55-59*	-5,838228571	-4,867834495	-5,249908227
14	*60-64*	-5,428653868	-4,463497028	-4,850559346
15	*65-69*	-5,006095614	-4,083820139	-4,467933168
16	*70-74*	-4,365838374	-3,546963312	-3,913474058
17	*75-79*	-3,717237777	-3,056586922	-3,382513026
18	*80-84*	-2,977715554	-2,497870316	-2,764176207
19	*85-89*	-2,282507994	-2,005462721	-2,180756917
20	*90-94*	-1,687859019	-1,513991378	-1,636480284
21	*95-99*	-1,225465459	-1,060940852	-1,184539526
22	*100-104*	-0,889405403	-0,8937541	-0,890216148
23	*105-109*	-0,47401647	-0,303063766	-0,438382489
24	*110+*	0,445766647	0	0,367687779

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Table A.14: Forecasting for $\ln(m_{x,t})$ for 2008

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,145941116	-5,489627974	-5,988760904
2	*1-4*	-8,765682301	-8,297108501	-8,651062959
3	*5-9*	-9,445633273	-8,846709652	-9,248319383
4	*10-14*	-9,231895881	-8,65551159	-9,006046943
5	*15-19*	-8,462088636	-7,499294357	-7,906027001
6	*20-24*	-8,352349189	-7,117143674	-7,562321444
7	*25-29*	-8,133487313	-6,946502612	-7,36970985
8	*30-34*	-7,779451939	-6,605041472	-7,028504094
9	*35-39*	-7,455537209	-6,308664422	-6,745871755
10	*40-44*	-7,056892089	-6,03808021	-6,459791621
11	*45-49*	-6,683654495	-5,634232391	-6,069962601
12	*50-54*	-6,38146912	-5,232204756	-5,712156283
13	*55-59*	-6,02937673	-4,825503228	-5,32967348
14	*60-64*	-5,608470677	-4,400538591	-4,92340098
15	*65-69*	-5,089472645	-3,940513146	-4,467258647
16	*70-74*	-4,512830419	-3,463540477	-3,985765824
17	*75-79*	-3,823276104	-2,947240319	-3,431780425
18	*80-84*	-3,075723996	-2,438852511	-2,838979195
19	*85-89*	-2,360865981	-1,927343466	-2,22868761
20	*90-94*	-1,726559557	-1,441487636	-1,653459862
21	*95-99*	-1,234625303	-0,989410685	-1,171065186
22	*100-104*	-0,81783589	-0,748511667	-0,785819698
23	*105-109*	-0,421737269	-0,879182044	-0,450537309
24	*110+*	0,863675225	-1,939896113	-0,083784074

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Table A.15: Real value of $\ln(m_{x,t})$ for 2008

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,76448835	-5,550374928	-5,64844238
2	*1-4*	-8,639360825	-8,589763884	-8,611503871
3	*5-9*	-9,361163262	-8,866750668	-9,07931211
4	*10-14*	-9,190537745	-8,917670758	-9,036387065
5	*15-19*	-8,51220565	-7,704043218	-8,016417904
6	*20-24*	-8,547652399	-7,437084374	-7,834096347
7	*25-29*	-8,435613204	-7,372970391	-7,756387362
8	*30-34*	-8,085410775	-7,221097098	-7,548310009
9	*35-39*	-7,507412116	-6,79531985	-7,0785436
10	*40-44*	-6,978177743	-6,260652037	-6,551480415
11	*45-49*	-6,526582863	-5,754339198	-6,067482034
12	*50-54*	-6,140893061	-5,308771824	-5,644759134
13	*55-59*	-5,809476379	-4,913463212	-5,273624754
14	*60-64*	-5,474100347	-4,48312956	-4,877897967
15	*65-69*	-5,057884147	-4,132919084	-4,516701538
16	*70-74*	-4,396774857	-3,600282668	-3,958747804
17	*75-79*	-3,737313682	-3,084257523	-3,406139574
18	*80-84*	-3,00388542	-2,53701712	-2,796455276
19	*85-89*	-2,295887572	-2,031932197	-2,197864782
20	*90-94*	-1,696129341	-1,537382402	-1,649099123
21	*95-99*	-1,211044416	-1,085446341	-1,180324639
22	*100-104*	-0,859178152	-0,7705035	-0,841491749
23	*105-109*	-0,61706801	-0,368078287	-0,57246524
24	*110+*	0,194394221	1,373049028	0,384437571

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Table A.16: Forecasting for $\ln(m_{x,t})$ for 2009

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,203396542	-5,520070118	-6,039042294
2	*1-4*	-8,809391262	-8,322345546	-8,690140711
3	*5-9*	-9,485714442	-8,869783381	-9,284326257
4	*10-14*	-9,260887036	-8,672610139	-9,032314751
5	*15-19*	-8,478779346	-7,510052992	-7,92088532
6	*20-24*	-8,372727212	-7,128472175	-7,577223634
7	*25-29*	-8,15364683	-6,956045465	-7,382617657
8	*30-34*	-7,796582857	-6,609125464	-7,036252152
9	*35-39*	-7,472573394	-6,310640973	-6,752604062
10	*40-44*	-7,073486969	-6,042760712	-6,469331587
11	*45-49*	-6,702163809	-5,639867669	-6,080892898
12	*50-54*	-6,404644534	-5,239306216	-5,725897613
13	*55-59*	-6,055014616	-4,833806501	-5,345048111
14	*60-64*	-5,636050151	-4,409706621	-4,94031105
15	*65-69*	-5,117135334	-3,94961392	-4,484573668
16	*70-74*	-4,541700781	-3,472815341	-4,004288948
17	*75-79*	-3,850171461	-2,95558705	-3,449736618
18	*80-84*	-3,097428646	-2,446397343	-2,854873514
19	*85-89*	-2,376543961	-1,933077818	-2,24086988
20	*90-94*	-1,736176836	-1,444862853	-1,661130305
21	*95-99*	-1,239575123	-0,988596887	-1,174093543
22	*100-104*	-0,818246842	-0,749833765	-0,785830693
23	*105-109*	-0,410801598	-0,88318714	-0,441422542
24	*110+*	0,923682558	-1,982252661	-0,06250391

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Table A.17: Real value of $\ln(m_{x,t})$ for 2009

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,806479873	-5,638837786	-5,716563809
2	*1-4*	-8,595154733	-8,399410156	-8,487634389
3	*5-9*	-9,338173743	-9,230543079	-9,282911065
4	*10-14*	-9,421061403	-8,917670758	-9,133379331
5	*15-19*	-8,645026563	-7,893932138	-8,191493052
6	*20-24*	-8,412833176	-7,639643288	-7,946213645
7	*25-29*	-8,322449115	-7,437084374	-7,775255847
8	*30-34*	-8,115066985	-7,280269287	-7,598904457
9	*35-39*	-7,608934631	-6,90475977	-7,184827172
10	*40-44*	-7,040144467	-6,359056003	-6,637728142
11	*45-49*	-6,545197672	-5,826967575	-6,12248481
12	*50-54*	-6,158700466	-5,36402991	-5,688696837
13	*55-59*	-5,869600959	-4,929792906	-5,302927979
14	*60-64*	-5,495062444	-4,520185862	-4,91060898
15	*65-69*	-5,053490604	-4,103546773	-4,494434144
16	*70-74*	-4,459685519	-3,657729685	-4,017661337
17	*75-79*	-3,79060215	-3,114178029	-3,444898517
18	*80-84*	-3,048584677	-2,574538184	-2,837003518
19	*85-89*	-2,334003523	-2,043433677	-2,224125176
20	*90-94*	-1,73946118	-1,597746524	-1,697029458
21	*95-99*	-1,246358521	-1,159053615	-1,225094301
22	*100-104*	-0,85831196	-0,803870942	-0,84743887
23	*105-109*	-0,587942477	-0,591762759	-0,58855476
24	*110+*	-0,248294707	,	-0,248294707

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APPENDIX B

FRANCE

Table B.1: Estimation of a_x

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-4,977722481	-4,706351851	-4,829428833
2	*1-4*	-7,899253176	-7,651148796	-7,763798896
3	*5-9*	-8,584851933	-8,284780338	-8,41908368
4	*10-14*	-8,616027145	-8,195397546	-8,378265821
5	*15-19*	-7,849957279	-6,960557315	-7,301225073
6	*20-24*	-7,64317499	-6,531246358	-6,935184333
7	*25-29*	-7,531828304	-6,549226464	-6,91985849
8	*30-34*	-7,244497589	-6,402124222	-6,732093523
9	*35-39*	-6,855154694	-6,073795661	-6,386082021
10	*40-44*	-6,43871092	-5,631774573	-5,952922567
11	*45-49*	-6,026911479	-5,184280821	-5,518445729
12	*50-54*	-5,654264759	-4,774867222	-5,124453196
13	*55-59*	-5,30700284	-4,388410199	-4,758244319
14	*60-64*	-4,926157553	-3,992922327	-4,380562489
15	*65-69*	-4,471533386	-3,594694963	-3,98245073
16	*70-74*	-3,928506455	-3,164327023	-3,534085565
17	*75-79*	-3,314384134	-2,706524498	-3,036405356
18	*80-84*	-2,671402856	-2,211388115	-2,493480666
19	*85-89*	-2,062283415	-1,730901341	-1,958381267
20	*90-94*	-1,538618902	-1,294620423	-1,478399643
21	*95-99*	-1,110290226	-0,935699089	-1,077473143
22	*100-104*	-0,74258026	-0,543065105	-0,712187189
23	*105-109*	-0,466139542	-0,489045699	-0,466999571
24	*110+*	-0,333406558	-0,16649373	-0,311916039

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Table B.2: First estimation of k_t

Year	*Female*	*Male*	*Total*
1970	8,155647608	7,060363954	7,190744197
1971	8,378179408	6,953990927	7,419436856
1972	7,886797676	4,624376079	6,793560638
1973	7,711244771	6,923766137	7,024756531
1974	6,356537057	5,888925854	5,872095791
1975	6,227979485	5,285962433	5,708098461
1976	6,020250214	4,224905019	5,487906548
1977	5,071782551	4,51032146	4,756680866
1978	4,506829255	3,532970777	4,229760606
1979	4,267777839	3,872670113	4,179182798
1980	3,997250056	3,399504878	3,880147783
1981	3,646906593	3,416301928	3,500889959
1982	2,747456216	2,830306836	2,738451592
1983	3,014785438	2,867976684	2,938918769
1984	1,59499049	1,662599596	1,703521591
1985	1,276885077	1,454685217	1,403905942
1986	0,494212842	0,674611349	0,70709565
1987	-0,239755885	0,492428412	0,124919384
1988	-1,440087304	-0,448252533	-0,998446592
1989	-3,183615191	-1,292880331	-2,348481431
1990	-2,950810606	-1,945844323	-2,269740184
1991	-2,444834805	-1,856307196	-1,853743767
1992	-3,985784963	-1,764228568	-3,086426803
1993	-2,839778794	-1,794356156	-2,44992178
1994	-3,491119344	-3,554604961	-2,977506564
1995	-4,18464833	-4,797956223	-4,186069401
1996	-5,352194655	-3,53072543	-4,493384459
1997	-5,523290294	-4,575067653	-5,095156327
1998	-5,272349194	-5,266920672	-5,242371958
1999	-5,251765715	-5,184865062	-5,360865966
2000	-6,186047683	-5,542604249	-6,215639101
2001	-6,420701157	-6,369685249	-6,279716141
2002	-6,596612595	-6,641990475	-6,86862424
2003	-6,8171909	-6,236610945	-6,676063653
2004	-9,174925165	-8,873767628	-9,257915597

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Table B.3: Estimation of b_x

X	*Age*	*Female*	*Male*	*Total*
1	*0*	0,08443	0,098995	0,09139
2	*1-4*	0,07849	0,089158	0,0835
3	*5-9*	0,07646	0,097839	0,0875
4	*10-14*	0,05600	0,072682	0,06485
5	*15-19*	0,05461	0,062671	0,05857
6	*20-24*	0,04243	0,041123	0,04046
7	*25-29*	0,03559	0,023969	0,02764
8	*30-34*	0,03148	0,020155	0,02437
9	*35-39*	0,02855	0,025139	0,02666
10	*40-44*	0,02832	0,030515	0,0298
11	*45-49*	0,03076	0,03327	0,03198
12	*50-54*	0,03435	0,037674	0,0351
13	*55-59*	0,03742	0,041327	0,03779
14	*60-64*	0,04080	0,043352	0,04018
15	*65-69*	0,04561	0,045295	0,04345
16	*70-74*	0,05027	0,046679	0,04623
17	*75-79*	0,05121	0,044639	0,04572
18	*80-84*	0,04478	0,037272	0,04023
19	*85-89*	0,03320	0,026577	0,03079
20	*90-94*	0,02433	0,018909	0,02361
21	*95-99*	0,01567	0,009368	0,01546
22	*100-104*	0,01290	0,019522	0,01519
23	*105-109*	0,01278	0,007654	0,01254
24	*110+*	0,04959	0,026213	0,04701

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Table B.4: Re-estimation k_t

Year	*Female*	*Male*	*Total*
1970	8,158527036	7,061178398	7,190398671
1971	8,377940852	6,952786578	7,419000053
1972	7,886553727	4,62526579	6,792010543
1973	7,711324985	6,922315176	7,022952355
1974	6,357004483	5,88858621	5,872837175
1975	6,224332078	5,28411968	5,709490118
1976	6,018042601	4,226383035	5,490619245
1977	5,073714616	4,511555712	4,757090422
1978	4,505441332	3,531857945	4,229714374
1979	4,268296109	3,873103656	4,178473855
1980	3,997230153	3,400360006	3,880678337
1981	3,643089443	3,418165039	3,500706584
1982	2,748708319	2,830402812	2,735228801
1983	3,711999926	2,870489405	2,534928735
1984	0,902907916	0,748115975	0,202837298
1985	-0,335965927	0,356135515	-0,675860863
1986	-1,52008951	-0,421506305	-1,642308124
1987	-2,434079846	-0,606177699	-2,359510981
1988	-1,437322301	-0,448578773	-1,000219193
1989	-3,181618922	-1,289104817	-2,349867237
1990	-2,9502698	-1,945107938	-2,267644943
1991	-2,44977551	-1,851196396	-1,853568174
1992	-3,984244328	-1,764991941	-3,085874628
1993	-2,838601478	-1,792057205	-2,445672722
1994	-3,482819736	-3,557529831	-2,983085327
1995	-4,18912364	-4,800090483	-4,18014734
1996	-5,350258002	-3,527437364	-4,491684671
1997	-5,51946804	-4,573967161	-5,097383679
1998	-5,272737288	-5,265975417	-5,243493488
1999	-5,254877439	-5,179687704	-5,359456994
2000	-6,188825716	-5,947377663	-6,214895561
2001	-6,419032668	-6,368095745	-6,282643751
2002	-6,592382848	-7,048503348	-6,873965135
2003	-6,814665879	-6,236371158	-6,673744676
2004	-9,173125767	-8,873089635	-9,25957796

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Table B.5: First and second estimation of k_t for female

	1st estimation	***2nd estimation***
Year	***Female***	***Female***
1970	8,155647608	8,158527036
1971	8,378179408	8,377940852
1972	7,886797676	7,886553727
1973	7,711244771	7,711324985
1974	6,356537057	6,357004483
1975	6,227979485	6,224332078
1976	6,020250214	6,018042601
1977	5,071782551	5,073714616
1978	4,506829255	4,505441332
1979	4,267777839	4,268296109
1980	3,997250056	3,997230153
1981	3,646906593	3,643089443
1982	2,747456216	2,748708319
1983	3,014785438	3,711999926
1984	1,59499049	0,902907916
1985	1,276885077	-0,335965927
1986	0,494212842	-1,52008951
1987	-0,239755885	-2,434079846
1988	-1,440087304	-1,437322301
1989	-3,183615191	-3,181618922
1990	-2,950810606	-2,9502698
1991	-2,444834805	-2,44977551
1992	-3,985784963	-3,984244328
1993	-2,839778794	-2,838601478
1994	-3,491119344	-3,482819736
1995	-4,18464833	-4,18912364
1996	-5,352194655	-5,350258002
1997	-5,523290294	-5,51946804
1998	-5,272349194	-5,272737288
1999	-5,251765715	-5,254877439
2000	-6,186047683	-6,188825716
2001	-6,420701157	-6,419032668
2002	-6,596612595	-6,592382848
2003	-6,8171909	-6,814665879
2004	-9,174925165	-9,173125767

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Table B.6: First and second estimation of k_t for male

	1st estimation	***2nd estimation***
Year	***Male***	***Male***
1970	7,060363954	7,061178398
1971	6,953990927	6,952786578
1972	4,624376079	4,62526579
1973	6,923766137	6,922315176
1974	5,888925854	5,88858621
1975	5,285962433	5,28411968
1976	4,224905019	4,226383035
1977	4,51032146	4,511555712
1978	3,532970777	3,531857945
1979	3,872670113	3,873103656
1980	3,399504878	3,400360006
1981	3,416301928	3,418165039
1982	2,830306836	2,830402812
1983	2,867976684	2,870489405
1984	1,662599596	0,748115975
1985	1,454685217	0,356135515
1986	0,674611349	-0,421506305
1987	0,492428412	-0,606177699
1988	-0,448252533	-0,448578773
1989	-1,292880331	-1,289104817
1990	-1,945844323	-1,945107938
1991	-1,856307196	-1,851196396
1992	-1,764228568	-1,764991941
1993	-1,794356156	-1,792057205
1994	-3,554604961	-3,557529831
1995	-4,797956223	-4,800090483
1996	-3,53072543	-3,527437364
1997	-4,575067653	-4,573967161
1998	-5,266920672	-5,265975417
1999	-5,184865062	-5,179687704
2000	-5,542604249	-5,947377663
2001	-6,369685249	-6,368095745
2002	-6,641990475	-7,048503348
2003	-6,236610945	-6,236371158
2004	-8,873767628	-8,873089635

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Table B.7: First and second estimation of k_t for total population

	1st estimation	***2nd estimation***
Year	***Total***	***Total***
1970	7,190744197	7,190398671
1971	7,419436856	7,419000053
1972	6,793560638	6,792010543
1973	7,024756531	7,022952355
1974	5,872095791	5,872837175
1975	5,708098461	5,709490118
1976	5,487906548	5,490619245
1977	4,756680866	4,757090422
1978	4,229760606	4,229714374
1979	4,179182798	4,178473855
1980	3,880147783	3,880678337
1981	3,500889959	3,500706584
1982	2,738451592	2,735228801
1983	2,938918769	2,534928735
1984	1,703521591	0,202837298
1985	1,403905942	-0,675860863
1986	0,70709565	-1,642308124
1987	0,124919384	-2,359510981
1988	-0,998446592	-1,000219193
1989	-2,348481431	-2,349867237
1990	-2,269740184	-2,267644943
1991	-1,853743767	-1,853568174
1992	-3,086426803	-3,085874628
1993	-2,44992178	-2,445672722
1994	-2,977506564	-2,983085327
1995	-4,186069401	-4,18014734
1996	-4,493384459	-4,491684671
1997	-5,095156327	-5,097383679
1998	-5,242371958	-5,243493488
1999	-5,360865966	-5,359456994
2000	-6,215639101	-6,214895561
2001	-6,279716141	-6,282643751
2002	-6,86862424	-6,873965135
2003	-6,676063653	-6,673744676
2004	-9,257915597	-9,25957796

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Table B.8: Forecasting for $\ln(m_{x,t})$ for 2005

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,795248061	-5,631137855	-5,719878232
2	*1-4*	-8,659262448	-8,484040059	-8,577372863
3	*5-9*	-9,325204958	-9,198767286	-9,27163125
4	*10-14*	-9,15826844	-8,874374225	-9,010125363
5	*15-19*	-8,37873937	-7,546013789	-7,871896058
6	*20-24*	-8,054019599	-6,91540692	-7,329402329
7	*25-29*	-7,876442012	-6,77313874	-7,189166088
8	*30-34*	-7,54931466	-6,590407084	-6,969540201
9	*35-39*	-7,131600925	-6,308637778	-6,645841086
10	*40-44*	-6,712930089	-5,916837909	-6,243275911
11	*45-49*	-6,324756876	-5,495080663	-5,830039687
12	*50-54*	-5,986871696	-5,126808107	-5,466446564
13	*55-59*	-5,669336219	-4,774476477	-5,126447435
14	*60-64*	-5,321219068	-4,397905638	-4,772052333
15	*65-69*	-4,913169555	-4,017829284	-4,405801495
16	*70-74*	-4,415264846	-3,600390318	-3,984522984
17	*75-79*	-3,810244432	-3,123530634	-3,481873641
18	*80-84*	-3,105002234	-2,559573619	-2,88545768
19	*85-89*	-2,38375504	-1,979176887	-2,258380578
20	*90-94*	-1,774203379	-1,471263471	-1,708441336
21	*95-99*	-1,26202096	-1,023212552	-1,228106119
22	*100-104*	-0,867489415	-0,725434643	-0,860189447
23	*105-109*	-0,589886752	-0,560547412	-0,589181817
24	*110+*	-0,81358059	-0,411368881	-0,769953311

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Table B.9: Real value of $\ln(m_{x,t})$ for 2005

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,72787027	-5,489720032	-5,599152226
2	*1-4*	-8,611503871	-8,44487253	-8,522205733
3	*5-9*	-9,384693759	-9,097011687	-9,230543079
4	*10-14*	-9,326874188	-8,963480294	-9,124162676
5	*15-19*	-8,454218392	-7,540748537	-7,888584532
6	*20-24*	-8,159518747	-7,026538815	-7,43878361
7	*25-29*	-8,07250737	-7,02316613	-7,418580903
8	*30-34*	-7,695213139	-6,797108759	-7,151101538
9	*35-39*	-7,19810758	-6,4903616	-6,785537646
10	*40-44*	-6,725433722	-6,017397929	-6,314981073
11	*45-49*	-6,228707023	-5,455673177	-5,7766758
12	*50-54*	-5,886024035	-5,049896008	-5,390871348
13	*55-59*	-5,599152226	-4,715766607	-5,069748559
14	*60-64*	-5,25736738	-4,406483275	-4,755411849
15	*65-69*	-4,874488574	-4,011953844	-4,381306894
16	*70-74*	-4,375408582	-3,556098343	-3,932940254
17	*75-79*	-3,776618368	-3,087781775	-3,438961556
18	*80-84*	-3,090680519	-2,542493391	-2,856005114
19	*85-89*	-2,434780236	-2,019762308	-2,286190226
20	*90-94*	-1,74930361	-1,432895523	-1,663060233
21	*95-99*	-1,239427285	-1,011247224	-1,194676152
22	*100-104*	-0,852762095	-0,725120936	-0,836260332
23	*105-109*	-0,577852572	-0,532398317	-0,574037026
24	*110+*	-0,294200605		-0,294200605

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Table B.10: Forecasting for $\ln(m_{x,t})$ for 2006

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,838286634	-5,677532351	-5,764094801
2	*1-4*	-8,699273078	-8,525824396	-8,61777207
3	*5-9*	-9,364180787	-9,244620017	-9,313965749
4	*10-14*	-9,186814692	-8,908437004	-9,041501274
5	*15-19*	-8,406577063	-7,575384863	-7,900233562
6	*20-24*	-8,075648483	-6,934679417	-7,348977801
7	*25-29*	-7,894584175	-6,78437193	-7,202538951
8	*30-34*	-7,565361731	-6,599852824	-6,981330963
9	*35-39*	-7,146154416	-6,320419295	-6,658739803
10	*40-44*	-6,727366336	-5,931138914	-6,257693831
11	*45-49*	-6,340436924	-5,510672813	-5,845512341
12	*50-54*	-6,004381763	-5,144464213	-5,483428746
13	*55-59*	-5,688411233	-4,79384458	-5,1447311
14	*60-64*	-5,342017051	-4,418222767	-4,791492335
15	*65-69*	-4,936419457	-4,039057009	-4,426823597
16	*70-74*	-4,440890204	-3,622266662	-4,006890114
17	*75-79*	-3,83634896	-3,144450922	-3,503994021
18	*80-84*	-3,127829041	-2,577041326	-2,904921873
19	*85-89*	-2,400678889	-1,991632329	-2,273277483
20	*90-94*	-1,786605706	-1,480125268	-1,719864393
21	*95-99*	-1,270008813	-1,027602912	-1,23558602
22	*100-104*	-0,874065248	-0,734583724	-0,867538716
23	*105-109*	-0,596401414	-0,564134497	-0,595248955
24	*110+*	-0,838859316	-0,423653733	-0,792697822

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Table B.11: Real value of $\ln(m_{x,t})$ for 2006

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,707790496	-5,451934855	-5,568814614
2	*1-4*	-8,6005748	-8,435613204	-8,51220565
3	*5-9*	-9,384693759	-9,230543079	-9,304651051
4	*10-14*	-9,349602439	-8,845697258	-9,061920367
5	*15-19*	-8,473176306	-7,610952795	-7,943392768
6	*20-24*	-8,173603487	-7,153655817	-7,536989134
7	*25-29*	-8,14908387	-7,058578169	-7,464624841
8	*30-34*	-7,761071212	-6,849486371	-7,206161315
9	*35-39*	-7,212922666	-6,50696776	-6,800696207
10	*40-44*	-6,755034192	-6,046554513	-6,34414747
11	*45-49*	-6,253868812	-5,509285342	-5,820541035
12	*50-54*	-5,867125273	-5,050676344	-5,386711517
13	*55-59*	-5,599963146	-4,734026808	-5,083689975
14	*60-64*	-5,260443714	-4,416452462	-4,762994271
15	*65-69*	-4,90344146	-4,056239856	-4,418856608
16	*70-74*	-4,407303418	-3,621033907	-3,984163683
17	*75-79*	-3,81762233	-3,134167097	-3,481605156
18	*80-84*	-3,155384727	-2,59961741	-2,916274352
19	*85-89*	-2,513664213	-2,081066862	-2,355300451
20	*90-94*	-1,791823471	-1,476923912	-1,706290281
21	*95-99*	-1,313627713	-1,058090499	-1,262552227
22	*100-104*	-0,883646887	-0,71451383	-0,861246357
23	*105-109*	-0,643342293	-0,524725109	-0,632345633
24	*110+*	-0,313864528		-0,313864528

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Table B.12: Forecasting for $\ln(m_{x,t})$ for 2007

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,881325205	-5,723926847	-5,808311372
2	*1-4*	-8,739283708	-8,567608734	-8,658171278
3	*5-9*	-9,403156615	-9,290472747	-9,356300248
4	*10-14*	-9,215360943	-8,942499782	-9,072877186
5	*15-19*	-8,434414756	-7,604755937	-7,928571066
6	*20-24*	-8,097277366	-6,953951915	-7,368553274
7	*25-29*	-7,912726337	-6,79560512	-7,215911815
8	*30-34*	-7,581408803	-6,609298565	-6,993121726
9	*35-39*	-7,160707907	-6,332200811	-6,67163852
10	*40-44*	-6,741802584	-5,94543992	-6,272111752
11	*45-49*	-6,356116972	-5,526264963	-5,860984996
12	*50-54*	-6,02189183	-5,16212032	-5,500410928
13	*55-59*	-5,707486246	-4,813212683	-5,163014766
14	*60-64*	-5,362815034	-4,438539896	-4,810932337
15	*65-69*	-4,95966936	-4,060284735	-4,4478457
16	*70-74*	-4,466515563	-3,644143006	-4,029257244
17	*75-79*	-3,862453487	-3,16537121	-3,526114402
18	*80-84*	-3,150655847	-2,594509033	-2,924386066
19	*85-89*	-2,417602738	-2,004087771	-2,288174388
20	*90-94*	-1,799008032	-1,488987064	-1,731287451
21	*95-99*	-1,277996666	-1,031993271	-1,243065922
22	*100-104*	-0,880641081	-0,743732806	-0,874887985
23	*105-109*	-0,602916077	-0,567721582	-0,601316094
24	*110+*	-0,864138041	-0,435938585	-0,815442334

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Table B.13: Real value of $\ln(m_{x,t})$ for 2007

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,784125227	-5,481279128	-5,617696526
2	*1-4*	-8,685611843	-8,417347856	-8,537395899
3	*5-9*	-9,304651051	-9,326874188	-9,315700888
4	*10-14*	-9,361163262	-9,028018815	-9,171119659
5	*15-19*	-8,589763884	-7,675626006	-8,02249695
6	*20-24*	-8,29404964	-7,120948499	-7,542633551
7	*25-29*	-8,07893826	-7,027665576	-7,425269891
8	*30-34*	-7,824046011	-6,914779894	-7,271598712
9	*35-39*	-7,29341776	-6,543112165	-6,851374946
10	*40-44*	-6,774098894	-6,093718781	-6,381253176
11	*45-49*	-6,28771657	-5,551147228	-5,859734776
12	*50-54*	-5,904553175	-5,078218445	-5,417776807
13	*55-59*	-5,593225484	-4,735620615	-5,08352862
14	*60-64*	-5,275382395	-4,429453728	-4,776670504
15	*65-69*	-4,928963145	-4,069495864	-4,434078111
16	*70-74*	-4,435933869	-3,660758578	-4,017494638
17	*75-79*	-3,850363625	-3,157169435	-3,506991325
18	*80-84*	-3,179342661	-2,613576735	-2,934035286
19	*85-89*	-2,515209169	-2,085942628	-2,357664493
20	*90-94*	-1,786401847	-1,462014093	-1,697826601
21	*95-99*	-1,318168532	-1,064500759	-1,267171779
22	*100-104*	-0,889096373	-0,676349062	-0,860350004
23	*105-109*	-0,626558263	-0,569265446	-0,62130105
24	*110+*	-0,238287646	0,453474015	-0,223628669

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Table B.14: Forecasting for $\ln(m_{x,t})$ for 2008

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,924363777	-5,770321343	-5,852527941
2	*1-4*	-8,779294338	-8,609393071	-8,698570485
3	*5-9*	-9,442132443	-9,336325478	-9,398634746
4	*10-14*	-9,243907194	-8,976562561	-9,104253097
5	*15-19*	-8,462252449	-7,634127011	-7,95690857
6	*20-24*	-8,118906249	-6,973224412	-7,388128746
7	*25-29*	-7,9308685	-6,806838311	-7,229284678
8	*30-34*	-7,597455874	-6,618744305	-7,004912489
9	*35-39*	-7,175261398	-6,343982328	-6,684537237
10	*40-44*	-6,756238831	-5,959740925	-6,286529673
11	*45-49*	-6,37179702	-5,541857113	-5,87645765
12	*50-54*	-6,039401896	-5,179776426	-5,517393109
13	*55-59*	-5,726561259	-4,832580785	-5,181298431
14	*60-64*	-5,383613018	-4,458857025	-4,830372339
15	*65-69*	-4,982919262	-4,08151246	-4,468867802
16	*70-74*	-4,492140921	-3,66601935	-4,051624374
17	*75-79*	-3,888558015	-3,186291498	-3,548234782
18	*80-84*	-3,173482653	-2,61197674	-2,943850259
19	*85-89*	-2,434526587	-2,016543214	-2,303071294
20	*90-94*	-1,811410359	-1,49784886	-1,742710508
21	*95-99*	-1,285984518	-1,036383631	-1,250545823
22	*100-104*	-0,887216914	-0,752881888	-0,882237254
23	*105-109*	-0,609430739	-0,571308667	-0,607383232
24	*110+*	-0,889416766	-0,448223437	-0,838186846

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Table B.15: Real value of $\ln(m_{x,t})$ for 2008

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,71050502	-5,539879503	-5,619625167
2	*1-4*	-8,622553707	-8,44487253	-8,527243527
3	*5-9*	-9,421061403	-9,338173743	-9,372859301
4	*10-14*	-9,349602439	-9,053336623	-9,18078157
5	*15-19*	-8,617013527	-7,728735831	-8,069307367
6	*20-24*	-8,343239884	-7,137168443	-7,565535316
7	*25-29*	-8,173603487	-7,031053495	-7,450759801
8	*30-34*	-7,728735831	-6,95066278	-7,267291455
9	*35-39*	-7,263002671	-6,577013717	-6,863738394
10	*40-44*	-6,767124149	-6,107099891	-6,387177364
11	*45-49*	-6,289870583	-5,591883382	-5,887104532
12	*50-54*	-5,889630241	-5,103902562	-5,430706555
13	*55-59*	-5,594031611	-4,746157879	-5,092279283
14	*60-64*	-5,284414461	-4,435427411	-4,783698945
15	*65-69*	-4,906680766	-4,107672992	-4,450219826
16	*70-74*	-4,438808649	-3,675726337	-4,025975886
17	*75-79*	-3,843010134	-3,175098203	-3,512508903
18	*80-84*	-3,179078355	-2,617336935	-2,93469362
19	*85-89*	-2,509191146	-2,080514117	-2,351817355
20	*90-94*	-1,785292426	-1,460423257	-1,69596577
21	*95-99*	-1,288993437	-1,048080784	-1,240215024
22	*100-104*	-0,871573135	-0,720732172	-0,85123828
23	*105-109*	-0,561135192	-0,490962541	-0,554504125
24	*110+*	-0,088251054	0,728063451	-0,06101301

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Table B.16: Forecasting for $\ln(m_{x,t})$ for 2009

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,967402349	-5,81671584	-5,89674451
2	*1-4*	-8,819304969	-8,651177409	-8,738969692
3	*5-9*	-9,481108272	-9,38217821	-9,440969245
4	*10-14*	-9,272453446	-9,01062534	-9,135629008
5	*15-19*	-8,490090142	-7,663498085	-7,985246073
6	*20-24*	-8,140535132	-6,992496909	-7,407704218
7	*25-29*	-7,949010662	-6,818071501	-7,242657542
8	*30-34*	-7,613502946	-6,628190045	-7,016703251
9	*35-39*	-7,189814889	-6,355763844	-6,697435954
10	*40-44*	-6,770675078	-5,974041931	-6,300947594
11	*45-49*	-6,387477069	-5,557449263	-5,891930305
12	*50-54*	-6,056911964	-5,197432533	-5,534375291
13	*55-59*	-5,745636272	-4,851948888	-5,199582096
14	*60-64*	-5,404411001	-4,479174154	-4,84981234
15	*65-69*	-5,006169165	-4,102740186	-4,489889905
16	*70-74*	-4,51776628	-3,687895694	-4,073991504
17	*75-79*	-3,914662543	-3,207211786	-3,570355162
18	*80-84*	-3,196309459	-2,629444447	-2,963314452
19	*85-89*	-2,451450437	-2,028998656	-2,317968199
20	*90-94*	-1,823812686	-1,506710656	-1,754133565
21	*95-99*	-1,293972371	-1,04077399	-1,258025724
22	*100-104*	-0,893792747	-0,76203097	-0,889586523
23	*105-109*	-0,615945402	-0,574895752	-0,613450371
24	*110+*	-0,914695492	-0,460508289	-0,860931358

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Table B.17: Real value of $\ln(m_{x,t})$ for 2009

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,72235386	-5,505095564	-5,605930074
2	*1-4*	-8,679712121	-8,487634389	-8,579068595
3	*5-9*	-9,39666995	-9,230543079	-9,304651051
4	*10-14*	-9,326874188	-8,995228992	-9,142681724
5	*15-19*	-8,56323713	-7,675626006	-8,013392183
6	*20-24*	-8,298057661	-7,192774234	-7,594920388
7	*25-29*	-8,098482857	-7,069098429	-7,459402897
8	*30-34*	-7,749402468	-6,931023906	-7,260153666
9	*35-39*	-7,323270723	-6,526582863	-6,849486371
10	*40-44*	-6,767993337	-6,09018052	-6,375951249
11	*45-49*	-6,302347013	-5,635469834	-5,919702181
12	*50-54*	-5,885304351	-5,11849894	-5,439420208
13	*55-59*	-5,57012609	-4,722940722	-5,069748559
14	*60-64*	-5,307963744	-4,454855425	-4,804475148
15	*65-69*	-4,912510896	-4,121633542	-4,459771983
16	*70-74*	-4,465582172	-3,711370492	-4,05647091
17	*75-79*	-3,881687806	-3,202608852	-3,543049006
18	*80-84*	-3,195281376	-2,6520291	-2,958687002
19	*85-89*	-2,480504354	-2,065680658	-2,32911389
20	*90-94*	-1,834264129	-1,512265831	-1,744438956
21	*95-99*	-1,290635151	-1,044317304	-1,240412056
22	*100-104*	-0,888580725	-0,695293482	-0,862036949
23	*105-109*	-0,575396145	-0,443860362	-0,562920994
24	*110+*	0,006321974	1,470175845	0,049906683

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APPENDIX C

SWEDEN

Table C.1: Estimation of a_x

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,273893554	-5,036952842	-5,144304333
2	*1-4*	-8,340808201	-8,117643679	-8,214279037
3	*5-9*	-8,919633971	-8,596841693	-8,733681732
4	*10-14*	-8,825455116	-8,556104684	-8,669940943
5	*15-19*	-8,150929104	-7,430380925	-7,715520804
6	*20-24*	-8,023903628	-7,029199212	-7,396579832
7	*25-29*	-7,844468769	-6,992089829	-7,319283543
8	*30-34*	-7,544823597	-6,83117721	-7,11563563
9	*35-39*	-7,130039174	-6,525818583	-6,775258634
10	*40-44*	-6,674263329	-6,123450059	-6,356232163
11	*45-49*	-6,18697385	-5,682598709	-5,899828652
12	*50-54*	-5,747409434	-5,209429452	-5,441010158
13	*55-59*	-5,315739792	-4,724495834	-4,978059413
14	*60-64*	-4,861428685	-4,226057147	-4,501004003
15	*65-69*	-4,368109554	-3,714523789	-4,005623508
16	*70-74*	-3,816506355	-3,210687787	-3,49675334
17	*75-79*	-3,216544517	-2,699246426	-2,964270422
18	*80-84*	-2,60719965	-2,20168624	-2,431158833
19	*85-89*	-2,024213147	-1,722356435	-1,911073658
20	*90-94*	-1,51002833	-1,285012599	-1,438288367
21	*95-99*	-1,0735425	-0,891698707	-1,024424075
22	*100-104*	-0,706819822	-0,551831451	-0,670792313
23	*105-109*	-0,40304443	-0,154214422	-0,420547624
24	*110+*	0,108592477	0,011584717	0,095163802

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Table C.2: First estimation of k_t

Year	*Female*	*Male*	*Total*
1970	8,12742588	5,857405723	7,232499948
1971	6,424026459	5,773658577	4,966579414
1972	5,812435466	6,700487558	6,396289652
1973	6,257534686	5,757780779	6,134806301
1974	4,959806455	5,312883469	4,928733132
1975	5,891520172	6,756368554	6,187354834
1976	4,319304237	5,678106587	4,855975299
1977	2,632948048	4,708994873	3,071601505
1978	3,565552718	3,933998194	4,223706913
1979	3,579201425	4,548230242	4,151462279
1980	2,372179091	5,076429859	3,165995916
1981	1,764929631	2,780230931	2,664886327
1982	1,226400167	1,894740228	1,522581203
1983	0,406713677	2,146310832	1,197797773
1984	0,099407204	1,462994021	0,72513463
1985	0,971452683	1,290669739	0,974131062
1986	1,10709181	0,901168855	1,849650319
1987	0,44798562	0,601218839	0,900048618
1988	0,765930607	1,197238356	1,072631702
1989	-1,02363462	-0,63701472	-0,72040215
1990	-0,19998159	-1,19366264	-0,35337247
1991	-0,91881026	-1,13485177	-1,00815621
1992	-2,31182703	-2,75845925	-2,42093096
1993	-1,74754231	-1,84131826	-1,94633536
1994	-3,61086559	-3,0857427	-3,44241905
1995	-4,04884425	-3,91064773	-4,11113339
1996	-4,20871943	-4,16754277	-4,60178238
1997	-3,05976532	-4,81636583	-3,62226907
1998	-4,56759035	-6,14722071	-4,78522164
1999	-5,51423978	-5,63392867	-5,8217117
2000	-5,8984854	-8,1519938	-6,50610823
2001	-5,36784009	-6,55343433	-5,77105493
2002	-5,76230867	-7,34853773	-6,86939167
2003	-5,74939417	-6,47896218	-6,325755
2004	-6,74199723	-8,51923315	-7,91582257

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Table C.3: Estimation of b_x

X	*Age*	*Female*	*Male*	*Total*
1	*0*	0,091699	0,079479	0,086568
2	*1-4*	0,086981	0,076327	0,082154
3	*5-9*	0,091475	0,101113	0,098415
4	*10-14*	0,064974	0,067892	0,068111
5	*15-19*	0,051490	0,060616	0,060258
6	*20-24*	0,049411	0,036788	0,041879
7	*25-29*	0,053192	0,044211	0,048699
8	*30-34*	0,058342	0,049444	0,054309
9	*35-39*	0,052987	0,046852	0,050095
10	*40-44*	0,048555	0,047769	0,048891
11	*45-49*	0,036289	0,046004	0,043476
12	*50-54*	0,033972	0,04415	0,041139
13	*55-59*	0,027635	0,042714	0,038025
14	*60-64*	0,029321	0,040414	0,037263
15	*65-69*	0,034873	0,037939	0,037516
16	*70-74*	0,041183	0,035403	0,038017
17	*75-79*	0,044583	0,031503	0,037216
18	*80-84*	0,037477	0,024323	0,030935
19	*85-89*	0,026960	0,01543	0,022790
20	*90-94*	0,014970	0,008223	0,013750
21	*95-99*	0,006790	0,001928	0,006991
22	*100-104*	0,003379	0,001673	0,003929
23	*105-109*	0,027020	0,063149	0,017512
24	*110+*	-0,013559	-0,003345	-0,007937

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Table C.4: Re-estimation k_t

Year	*Female*	*Male*	*Total*
1970	7,24608302	4,048845919	5,112494122
1971	4,445869091	3,277520044	2,852496926
1972	4,93555603	4,198304214	4,278064561
1973	5,37802818	3,254461806	4,012774734
1974	4,080458597	3,504135961	2,810076279
1975	5,007915801	4,265505007	4,065725367
1976	3,443485999	3,172795833	2,741419355
1977	1,75392135	1,517346785	0,950333281
1978	2,684531641	1,433062153	2,103642479
1979	2,703682747	2,046933235	2,034805108
1980	1,491902834	2,571828548	1,053601833
1981	0,880204584	0,277061912	0,548082055
1982	0,3511858	-0,605216646	-0,589639139
1983	-0,471067923	-0,353558802	-0,920628669
1984	-0,087944489	-1,031518409	-0,697437333
1985	0,087258665	-1,612236798	-1,147973112
1986	0,235415468	-1,597063364	-0,254845314
1987	-0,429842884	-1,901492749	-1,216351539
1988	-0,11996483	-1,307508681	-1,052160866
1989	-1,901416686	-3,137367282	-2,844816263
1990	-1,075783114	-3,69514108	-2,47123704
1991	-1,797914722	-3,638463407	-3,127413424
1992	-3,184918614	-5,262607112	-4,534832918
1993	-2,630168775	-4,341896949	-4,063846114
1994	-4,497948319	-5,581946848	-5,569646562
1995	-4,931121776	-6,405566059	-6,232133662
1996	-5,092899648	-6,668682793	-6,720938989
1997	-5,740649656	-8,696913457	-7,527357588
1998	-5,448881888	-8,650725519	-6,902184965
1999	-6,392982184	-8,137463675	-7,931684045
2000	-6,778446727	-10,64883263	-8,615580569
2001	-6,243221929	-9,05599607	-7,878016944
2002	-6,638062128	-9,160521889	-8,979500484
2003	-6,632351163	-8,975052659	-8,436561967
2004	-7,632535388	-11,02521784	-10,03897533

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Table C.5: First and second estimation of k_t for female

	1st estimation	***2nd estimation***
Year	***Female***	***Female***
1970	8,12742588	7,24608302
1971	6,424026459	4,445869091
1972	5,812435466	4,93555603
1973	6,257534686	5,37802818
1974	4,959806455	4,080458597
1975	5,891520172	5,007915801
1976	4,319304237	3,443485999
1977	2,632948048	1,75392135
1978	3,565552718	2,684531641
1979	3,579201425	2,703682747
1980	2,372179091	1,491902834
1981	1,764929631	0,880204584
1982	1,226400167	0,3511858
1983	0,406713677	-0,471067923
1984	0,099407204	-0,087944489
1985	0,971452683	0,087258665
1986	1,10709181	0,235415468
1987	0,44798562	-0,429842884
1988	0,765930607	-0,11996483
1989	-1,02363462	-1,901416686
1990	-0,19998159	-1,075783114
1991	-0,91881026	-1,797914722
1992	-2,31182703	-3,184918614
1993	-1,74754231	-2,630168775
1994	-3,61086559	-4,497948319
1995	-4,04884425	-4,931121776
1996	-4,20871943	-5,092899648
1997	-3,05976532	-5,740649656
1998	-4,56759035	-5,448881888
1999	-5,51423978	-6,392982184
2000	-5,8984854	-6,778446727
2001	-5,36784009	-6,243221929
2002	-5,76230867	-6,638062128
2003	-5,74939417	-6,632351163
2004	-6,74199723	-7,632535388

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Table C.6: First and second estimation of k_t for male

	1st estimation	***2nd estimation***
Year	***Male***	***Male***
1970	5,857405723	4,048845919
1971	5,773658577	3,277520044
1972	6,700487558	4,198304214
1973	5,757780779	3,254461806
1974	5,312883469	3,504135961
1975	6,756368554	4,265505007
1976	5,678106587	3,172795833
1977	4,708994873	1,517346785
1978	3,933998194	1,433062153
1979	4,548230242	2,046933235
1980	5,076429859	2,571828548
1981	2,780230931	0,277061912
1982	1,894740228	-0,605216646
1983	2,146310832	-0,353558802
1984	1,462994021	-1,031518409
1985	1,290669739	-1,612236798
1986	0,901168855	-1,597063364
1987	0,601218839	-1,901492749
1988	1,197238356	-1,307508681
1989	-0,63701472	-3,137367282
1990	-1,19366264	-3,69514108
1991	-1,13485177	-3,638463407
1992	-2,75845925	-5,262607112
1993	-1,84131826	-4,341896949
1994	-3,0857427	-5,581946848
1995	-3,91064773	-6,405566059
1996	-4,16754277	-6,668682793
1997	-4,81636583	-8,696913457
1998	-6,14722071	-8,650725519
1999	-5,63392867	-8,137463675
2000	-8,1519938	-10,64883263
2001	-6,55343433	-9,05599607
2002	-7,34853773	-9,160521889
2003	-6,47896218	-8,975052659
2004	-8,51923315	-11,02521784

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Table C.7: First and second estimation of k_t for total population

	1st estimation	***2nd estimation***
Year	***Total***	***Total***
1970	7,232499948	5,112494122
1971	4,966579414	2,852496926
1972	6,396289652	4,278064561
1973	6,134806301	4,012774734
1974	4,928733132	2,810076279
1975	6,187354834	4,065725367
1976	4,855975299	2,741419355
1977	3,071601505	0,950333281
1978	4,223706913	2,103642479
1979	4,151462279	2,034805108
1980	3,165995916	1,053601833
1981	2,664886327	0,548082055
1982	1,522581203	-0,589639139
1983	1,197797773	-0,920628669
1984	0,72513463	-0,697437333
1985	0,974131062	-1,147973112
1986	1,849650319	-0,254845314
1987	0,900048618	-1,216351539
1988	1,072631702	-1,052160866
1989	-0,72040215	-2,844816263
1990	-0,35337247	-2,47123704
1991	-1,00815621	-3,127413424
1992	-2,42093096	-4,534832918
1993	-1,94633536	-4,063846114
1994	-3,44241905	-5,569646562
1995	-4,11113339	-6,232133662
1996	-4,60178238	-6,720938989
1997	-3,62226907	-7,527357588
1998	-4,78522164	-6,902184965
1999	-5,8217117	-7,931684045
2000	-6,50610823	-8,615580569
2001	-5,77105493	-7,878016944
2002	-6,86939167	-8,979500484
2003	-6,325755	-8,436561967
2004	-7,91582257	-10,03897533

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Table C.8: Forecasting for $\ln(m_{x,t})$ for 2005

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,013917488	-5,948463528	-6,051935773
2	*1-4*	-9,042757206	-8,993005424	-9,075631423
3	*5-9*	-9,657850193	-9,75646348	-9,765524309
4	*10-14*	-9,34980451	-9,334729019	-9,384057996
5	*15-19*	-8,566460706	-8,125559925	-8,34730224
6	*20-24*	-8,422657405	-7,451105063	-7,83566468
7	*25-29*	-8,273735752	-7,499126895	-7,829873409
8	*30-34*	-8,015651811	-7,398229316	-7,68504414
9	*35-39*	-7,557651778	-7,063144149	-7,300485011
10	*40-44*	-7,066109065	-6,671292306	-6,868835073
11	*45-49*	-6,479831226	-6,210198925	-6,355657417
12	*50-54*	-6,021568292	-5,715766935	-5,872336397
13	*55-59*	-5,538758161	-5,214364446	-5,376736586
14	*60-64*	-5,098053313	-4,689548042	-4,891691906
15	*65-69*	-4,649539609	-4,149629967	-4,398964016
16	*70-74*	-4,148859005	-3,616709665	-3,895346636
17	*75-79*	-3,576335649	-3,06054087	-3,354465548
18	*80-84*	-2,909644355	-2,480636333	-2,755500144
19	*85-89*	-2,24178417	-1,899316511	-2,150017847
20	*90-94*	-1,630838353	-1,379318672	-1,58245171
21	*95-99*	-1,128338763	-0,913810115	-1,097721961
22	*100-104*	-0,734088831	-0,571018373	-0,711986333
23	*105-109*	-0,621099662	-0,878443317	-0,604154058
24	*110+*	0,21801553	0,049947093	0,178380126

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Table C.9: Real value of $\ln(m_{x,t})$ for 2005

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,059743388	-5,985880169	-6,021099352
2	*1-4*	-8,542510999	-8,377431249	-8,454218392
3	*5-9*	-9,581404053	-8,902855672	-9,18078157
4	*10-14*	-9,421061403	-9,011489513	-9,190537745
5	*15-19*	-8,734106193	-7,943392768	-8,250990151
6	*20-24*	-8,220799178	-7,317228408	-7,660652464
7	*25-29*	-8,278176291	-7,219730044	-7,606920532
8	*30-34*	-7,949042501	-7,37936019	-7,61906643
9	*35-39*	-7,503775749	-7,076173931	-7,263002671
10	*40-44*	-7,143477613	-6,580612138	-6,817914575
11	*45-49*	-6,573428199	-6,097714347	-6,30398591
12	*50-54*	-5,953013361	-5,600233599	-5,759718107
13	*55-59*	-5,491174225	-5,108523795	-5,280870453
14	*60-64*	-5,070066878	-4,646617376	-4,834834954
15	*65-69*	-4,595516929	-4,093604836	-4,319314781
16	*70-74*	-4,117449436	-3,577159205	-3,830443018
17	*75-79*	-3,542046915	-3,029351107	-3,286753247
18	*80-84*	-2,883207483	-2,444770971	-2,685193419
19	*85-89*	-2,242949195	-1,896127145	-2,108582031
20	*90-94*	-1,639639421	-1,374077294	-1,558193619
21	*95-99*	-1,187479569	-0,994071209	-1,143621161
22	*100-104*	-0,736042156	-0,725139521	-0,734423445
23	*105-109*	-0,632821898	-0,287682072	-0,610113872
24	*110+*	0,405465108	,	0,405465108

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Table C.10: Forecasting for $\ln(m_{x,t})$ for 2006

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,054045559	-5,983700926	-6,090513197
2	*1-4*	-9,08082065	-9,026845367	-9,112241829
3	*5-9*	-9,697880241	-9,801292416	-9,809381128
4	*10-14*	-9,37823755	-9,364829264	-9,4144104
5	*15-19*	-8,588993061	-8,152434321	-8,3741551
6	*20-24*	-8,444279976	-7,4674152	-7,854327279
7	*25-29*	-8,297012913	-7,518728055	-7,851575215
8	*30-34*	-8,041182645	-7,420150552	-7,709245939
9	*35-39*	-7,58083923	-7,083916209	-7,322808919
10	*40-44*	-7,087357045	-6,692470922	-6,890622441
11	*45-49*	-6,495711526	-6,23059502	-6,37503169
12	*50-54*	-6,036434657	-5,73534105	-5,890669229
13	*55-59*	-5,550851414	-5,233301904	-5,393681722
14	*60-64*	-5,110884371	-4,707465784	-4,90829747
15	*65-69*	-4,664800258	-4,166450405	-4,415682326
16	*70-74*	-4,166880951	-3,632405755	-3,912288207
17	*75-79*	-3,595845457	-3,074507877	-3,371050168
18	*80-84*	-2,926044531	-2,491420052	-2,769285753
19	*85-89*	-2,253582039	-1,906157476	-2,160173787
20	*90-94*	-1,637389321	-1,382964379	-1,588579143
21	*95-99*	-1,131310111	-0,914664903	-1,100837371
22	*100-104*	-0,735567503	-0,571760106	-0,713737219
23	*105-109*	-0,632923787	-0,90644073	-0,611957956
24	*110+*	0,223949035	0,051430114	0,181917103

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Table C.11: Real value of $\ln(m_{x,t})$ for 2006

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,947256292	-5,780231647	-5,857983156
2	*1-4*	-8,6005748	-8,70352277	-8,650724584
3	*5-9*	-9,77245929	-8,987196821	-9,293721981
4	*10-14*	-9,672375832	-9,053336623	-9,304651051
5	*15-19*	-8,426438828	-7,824046011	-8,07250737
6	*20-24*	-8,053459175	-7,299317482	-7,598904457
7	*25-29*	-8,355925044	-7,382570465	-7,74708497
8	*30-34*	-7,809157398	-7,435388021	-7,60090246
9	*35-39*	-7,573287293	-7,029922913	-7,260153666
10	*40-44*	-7,114779448	-6,629366253	-6,838229216
11	*45-49*	-6,567006486	-6,189915486	-6,357901268
12	*50-54*	-6,029789661	-5,576973628	-5,776030639
13	*55-59*	-5,563585844	-5,136198517	-5,326511101
14	*60-64*	-5,079181837	-4,660682896	-4,846987003
15	*65-69*	-4,676344005	-4,141750685	-4,378911492
16	*70-74*	-4,120031669	-3,615368803	-3,852528434
17	*75-79*	-3,553160456	-3,068840745	-3,312911817
18	*80-84*	-2,897898009	-2,472140485	-2,705393733
19	*85-89*	-2,258769185	-1,923114945	-2,128152909
20	*90-94*	-1,644712821	-1,416271211	-1,574852928
21	*95-99*	-1,15573835	-1,026186516	-1,126647632
22	*100-104*	-0,742602166	-0,702973299	-0,736332382
23	*105-109*	-0,841164707	-0,731164778	-0,833418953
24	*110+*	0	,	0

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Table C.12: Forecasting for $\ln(m_{x,t})$ for 2007

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,094173631	-6,018938323	-6,129090621
2	*1-4*	-9,118884095	-9,06068531	-9,148852236
3	*5-9*	-9,737910288	-9,846121351	-9,853237948
4	*10-14*	-9,40667059	-9,394929509	-9,444762804
5	*15-19*	-8,611525415	-8,179308716	-8,40100796
6	*20-24*	-8,465902547	-7,483725337	-7,872989879
7	*25-29*	-8,320290074	-7,538329214	-7,873277021
8	*30-34*	-8,066713479	-7,442071788	-7,733447737
9	*35-39*	-7,604026681	-7,104688269	-7,345132826
10	*40-44*	-7,108605024	-6,713649539	-6,912409808
11	*45-49*	-6,511591825	-6,250991115	-6,394405963
12	*50-54*	-6,051301022	-5,754915165	-5,909002061
13	*55-59*	-5,562944668	-5,252239362	-5,410626858
14	*60-64*	-5,123715429	-4,725383525	-4,924903035
15	*65-69*	-4,680060907	-4,183270844	-4,432400635
16	*70-74*	-4,184902896	-3,648101846	-3,929229778
17	*75-79*	-3,615355264	-3,088474884	-3,387634789
18	*80-84*	-2,942444707	-2,502203771	-2,783071362
19	*85-89*	-2,265379909	-1,912998441	-2,170329728
20	*90-94*	-1,643940289	-1,386610085	-1,594706575
21	*95-99*	-1,134281458	-0,915519691	-1,10395278
22	*100-104*	-0,737046175	-0,572501838	-0,715488105
23	*105-109*	-0,644747913	-0,934438143	-0,619761854
24	*110+*	0,229882541	0,052913136	0,18545408

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Table C.13: Real value of $\ln(m_{x,t})$ for 2007

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,070940749	-5,919329949	-5,990265267
2	*1-4*	-8,547652399	-8,895529632	-8,709565084
3	*5-9*	-9,304651051	-9,408791311	-9,349602439
4	*10-14*	-9,737973114	-9,190537745	-9,421061403
5	*15-19*	-8,487634389	-7,735577363	-8,031685376
6	*20-24*	-8,326572832	-7,270160898	-7,652195754
7	*25-29*	-8,330713624	-7,200784958	-7,602904462
8	*30-34*	-8,066117572	-7,363461604	-7,647994067
9	*35-39*	-7,529512463	-7,114779448	-7,297839285
10	*40-44*	-7,125911289	-6,646160541	-6,852320572
11	*45-49*	-6,547287537	-6,142287437	-6,321636671
12	*50-54*	-6,081826987	-5,609472795	-5,815496464
13	*55-59*	-5,599692766	-5,161167528	-5,355946479
14	*60-64*	-5,135858439	-4,657200594	-4,867184512
15	*65-69*	-4,617850239	-4,17273863	-4,373979103
16	*70-74*	-4,160356168	-3,684369639	-3,908379651
17	*75-79*	-3,544328995	-3,101270583	-3,325737467
18	*80-84*	-2,898678161	-2,49115815	-2,71442548
19	*85-89*	-2,22475406	-1,948960798	-2,118789623
20	*90-94*	-1,634550613	-1,396983285	-1,560785853
21	*95-99*	-1,147321556	-1,012322672	-1,116343563
22	*100-104*	-0,844470195	-0,651746154	-0,811358808
23	*105-109*	-0,446796607	-0,40011793	-0,442131686
24	*110+*	0,405465108	,	0,405465108

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Table C.14: Forecasting for $\ln(m_{x,t})$ for 2008

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,134301702	-6,054175721	-6,167668044
2	*1-4*	-9,156947539	-9,094525253	-9,185462642
3	*5-9*	-9,777940336	-9,890950287	-9,897094767
4	*10-14*	-9,435103629	-9,425029754	-9,475115208
5	*15-19*	-8,63405777	-8,206183112	-8,42786082
6	*20-24*	-8,487525118	-7,500035474	-7,891652479
7	*25-29*	-8,343567235	-7,557930374	-7,894978827
8	*30-34*	-8,092244313	-7,463993023	-7,757649536
9	*35-39*	-7,627214133	-7,125460329	-7,367456734
10	*40-44*	-7,129853004	-6,734828155	-6,934197176
11	*45-49*	-6,527472125	-6,27138721	-6,413780236
12	*50-54*	-6,066167388	-5,77448928	-5,927334894
13	*55-59*	-5,575037922	-5,271176819	-5,427571994
14	*60-64*	-5,136546487	-4,743301267	-4,9415086
15	*65-69*	-4,695321556	-4,200091282	-4,449118945
16	*70-74*	-4,202924841	-3,663797936	-3,946171349
17	*75-79*	-3,634865071	-3,102441891	-3,404219409
18	*80-84*	-2,958844883	-2,51298749	-2,796856971
19	*85-89*	-2,277177778	-1,919839406	-2,180485669
20	*90-94*	-1,650491257	-1,390255792	-1,600834008
21	*95-99*	-1,137252806	-0,916374479	-1,10706819
22	*100-104*	-0,738524847	-0,573243571	-0,717238991
23	*105-109*	-0,656572038	-0,962435557	-0,627565752
24	*110+*	0,235816046	0,054396158	0,188991057

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Table C.15: Real value of $\ln(m_{x,t})$ for 2008

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-5,998287772	-5,984290343	-5,991064627
2	*1-4*	-8,866750668	-8,917670758	-8,895529632
3	*5-9*	-9,965362956	-9,581404053	-9,755067547
4	*10-14*	-9,200390041	-9,011489513	-9,097011687
5	*15-19*	-8,517193191	-7,899308495	-8,152550078
6	*20-24*	-8,377431249	-7,239040989	-7,641724454
7	*25-29*	-8,004369565	-7,240434717	-7,542633551
8	*30-34*	-8,07250737	-7,179564002	-7,522091279
9	*35-39*	-7,69741336	-7,133401961	-7,371379301
10	*40-44*	-7,062072639	-6,667951287	-6,841967538
11	*45-49*	-6,627097821	-6,221632713	-6,400937677
12	*50-54*	-6,069209896	-5,628489491	-5,822566011
13	*55-59*	-5,60294216	-5,189283606	-5,37411908
14	*60-64*	-5,122852597	-4,671309988	-4,871351727
15	*65-69*	-4,645159194	-4,194850215	-4,397262107
16	*70-74*	-4,208006103	-3,683533768	-3,925767023
17	*75-79*	-3,591099599	-3,131300024	-3,361851676
18	*80-84*	-2,919586379	-2,503306901	-2,730387839
19	*85-89*	-2,240418132	-1,933839296	-2,120396878
20	*90-94*	-1,623896942	-1,436489312	-1,565837382
21	*95-99*	-1,169259027	-1,065251983	-1,145122304
22	*100-104*	-0,830948247	-0,764686148	-0,819951536
23	*105-109*	-0,79999992	-0,506531523	-0,755605733
24	*110+*	1,284941914	,	1,284941914

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Table C.16: Forecasting for $\ln(m_{x,t})$ for 2009

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,174429774	-6,089413118	-6,206245468
2	*1-4*	-9,195010983	-9,128365196	-9,222073049
3	*5-9*	-9,817970384	-9,935779222	-9,940951587
4	*10-14*	-9,463536669	-9,45513	-9,505467612
5	*15-19*	-8,656590125	-8,233057507	-8,45471368
6	*20-24*	-8,509147689	-7,516345611	-7,910315078
7	*25-29*	-8,366844396	-7,577531534	-7,916680634
8	*30-34*	-8,117775146	-7,485914259	-7,781851335
9	*35-39*	-7,650401584	-7,146232389	-7,389780642
10	*40-44*	-7,151100984	-6,756006771	-6,955984543
11	*45-49*	-6,543352424	-6,291783305	-6,433154509
12	*50-54*	-6,081033753	-5,794063395	-5,945667726
13	*55-59*	-5,587131175	-5,290114277	-5,44451713
14	*60-64*	-5,149377545	-4,761219009	-4,958114165
15	*65-69*	-4,710582205	-4,216911721	-4,465837255
16	*70-74*	-4,220946787	-3,679494027	-3,96311292
17	*75-79*	-3,654374878	-3,116408898	-3,420804029
18	*80-84*	-2,975245058	-2,52377121	-2,81064258
19	*85-89*	-2,288975647	-1,92668037	-2,19064161
20	*90-94*	-1,657042226	-1,393901499	-1,60696144
21	*95-99*	-1,140224153	-0,917229267	-1,110183599
22	*100-104*	-0,74000352	-0,573985303	-0,718989877
23	*105-109*	-0,668396164	-0,99043297	-0,63536965
24	*110+*	0,241749552	0,05587918	0,192528033

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Table C.17: Real value of $\ln(m_{x,t})$ for 2009

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,173946117	-6,062745749	-6,115215355
2	*1-4*	-8,364472104	-8,118417071	-8,232014249
3	*5-9*	-9,39666995	-9,011489513	-9,18078157
4	*10-14*	-9,704636694	-9,133379331	-9,372859301
5	*15-19*	-8,473176306	-8,118417071	-8,274247013
6	*20-24*	-8,417347856	-7,318735568	-7,712951963
7	*25-29*	-8,386164929	-7,312720512	-7,699618432
8	*30-34*	-8,156028342	-7,236259346	-7,58502911
9	*35-39*	-7,688641374	-7,074991198	-7,330875322
10	*40-44*	-7,194104906	-6,789083749	-6,967505283
11	*45-49*	-6,603216089	-6,260652037	-6,414279294
12	*50-54*	-6,103961578	-5,703582495	-5,88207217
13	*55-59*	-5,671702581	-5,234967051	-5,428881737
14	*60-64*	-5,197929193	-4,800090778	-4,979427367
15	*65-69*	-4,704438188	-4,254302069	-4,455888483
16	*70-74*	-4,195181989	-3,748247366	-3,957700519
17	*75-79*	-3,671786377	-3,177014371	-3,421604191
18	*80-84*	-3,015118987	-2,568805784	-2,809817181
19	*85-89*	-2,272094686	-1,923026	-2,133011109
20	*90-94*	-1,622710606	-1,355586705	-1,536438412
21	*95-99*	-1,074445668	-0,8272616	-1,013484685
22	*100-104*	-0,676282197	-0,455673202	-0,636464435
23	*105-109*	-0,240808665	-0,241311341	-0,240868464
24	*110+*	0,040822315	,	0,040822315

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Table C.18: Forecasting for $\ln(m_{x,t})$ for 2010

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,214557845	-6,124650515	-6,244822892
2	*1-4*	-9,233074428	-9,16220514	-9,258683455
3	*5-9*	-9,858000431	-9,980608158	-9,984808407
4	*10-14*	-9,491969709	-9,485230245	-9,535820016
5	*15-19*	-8,67912248	-8,259931903	-8,481566541
6	*20-24*	-8,53077026	-7,532655748	-7,928977678
7	*25-29*	-8,390121557	-7,597132694	-7,93838244
8	*30-34*	-8,14330598	-7,507835494	-7,806053133
9	*35-39*	-7,673589036	-7,167004448	-7,412104549
10	*40-44*	-7,172348964	-6,777185387	-6,977771911
11	*45-49*	-6,559232724	-6,3121794	-6,452528782
12	*50-54*	-6,095900119	-5,813637511	-5,964000559
13	*55-59*	-5,599224429	-5,309051735	-5,461462266
14	*60-64*	-5,162208603	-4,77913675	-4,97471973
15	*65-69*	-4,725842853	-4,233732159	-4,482555564
16	*70-74*	-4,238968732	-3,695190118	-3,980054491
17	*75-79*	-3,673884685	-3,130375904	-3,437388649
18	*80-84*	-2,991645234	-2,534554929	-2,824428189
19	*85-89*	-2,300773516	-1,933521335	-2,200797551
20	*90-94*	-1,663593194	-1,397547205	-1,613088873
21	*95-99*	-1,143195501	-0,918084056	-1,113299009
22	*100-104*	-0,741482192	-0,574727036	-0,720740763
23	*105-109*	-0,68022029	-1,018430383	-0,643173548
24	*110+*	0,247683057	0,057362202	0,19606501

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Table C.19: Real value of $\ln(m_{x,t})$ for 2010

X	*Age*	*Female*	*Male*	*Total*
1	*0*	-6,16059351	-6,058887223	-6,107099891
2	*1-4*	-8,492500579	-8,322449115	-8,403864506
3	*5-9*	-9,326874188	-9,641123288	-9,471705136
4	*10-14*	-9,552830681	-9,304651051	-9,421061403
5	*15-19*	-8,606024405	-8,101777752	-8,314252347
6	*20-24*	-8,314252347	-7,366621164	-7,721940788
7	*25-29*	-8,310179022	-7,214280439	-7,604910481
8	*30-34*	-8,013392183	-7,302280447	-7,588973889
9	*35-39*	-7,81409568	-7,302280447	-7,522091279
10	*40-44*	-7,333933429	-6,799798137	-7,026538815
11	*45-49*	-6,788196044	-6,290409812	-6,504292174
12	*50-54*	-6,22264027	-5,761941389	-5,963460351
13	*55-59*	-5,750874082	-5,267012517	-5,479120077
14	*60-64*	-5,176276741	-4,771697137	-4,954160447
15	*65-69*	-4,700250392	-4,265417621	-4,460550496
16	*70-74*	-4,230233016	-3,761966409	-3,978910951
17	*75-79*	-3,654555325	-3,175912125	-3,411793321
18	*80-84*	-3,008837777	-2,577982927	-2,810381975
19	*85-89*	-2,303645655	-1,949480515	-2,161788903
20	*90-94*	-1,586214672	-1,327768882	-1,502280013
21	*95-99*	-1,065466723	-0,837657479	-1,010769008
22	*100-104*	-0,592825941	-0,24047284	-0,528081996
23	*105-109*	-0,107358065	0,200740057	-0,073107396
24	*110+*	-1,00795656	,	-1,00795656

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