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ATHENS UNIVERSITY OF ECONOMICS AND BUSINESS

MASTER OF SCIENCE IN FINANCE AND BANKING

DISSERTATION

VALUE AT RISK

SUPERVISING PROFESSOR: D. GEORGOUTSOS

STUDENT: ASSIMO AGELINA

ATHENS, MAY 2002

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ΟΙΚΟΝΟΜΙΚΟ ΠΑΝΕΠΙΣΤΗΜΙΟ ΑΘΗΝΩΝ

ΜΕΤΑΠΤΥΧΙΑΚΟ ΠΡΟΓΡΑΜΜΑ
ΣΤΗ ΧΡΗΜΑΤΟΟΙΚΟΝΟΜΙΚΗ ΚΑΙ ΤΡΑΠΕΖΙΚΗ
ΓΙΑ ΣΤΕΛΕΧΗ

VALUE AT RISK

ΑΓΓΕΛΙΝΑ ΑΣΗΜΩ

Διατριβή υποβληθείσα
προς μερική εκπλήρωση των απαραιτήτων προϋποθέσεων
για την απόκτηση του Μεταπτυχιακού Διπλώματος

Αθήνα, Μάιος 2002

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Εγκρίνουμε τη διατριβή της Αγγελίνα Ασήμως

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ΟΝΟΜΑ ΥΠΕΥΘΥΝΟΥ ΚΑΘΗΓΗΤΗ (Υπογραφή)

....

ΟΝΟΜΑ ΕΞΕΤΑΣΤΗ ΚΑΘΗΓΗΤΗ (Υπογραφή)

Ημερομηνία

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CHAPTER 1: DEFINING RISK AND VAR

1.1. DEFINING RISK

Corporations and banks are in the business of managing risks and returns the achievement of which is a major challenge.

Risks are usually defined by the adverse impact on profitability of several distinct sources of uncertainty.

Risk management is the process by which various risk exposures are identified, measured, and controlled.

The main objectives of risk management are

- To control the magnitude of potential losses
- To allocate capital among various business segments and charge appropriately
- To measure risk-adjusted performance
- To report to regulatory authorities and rating agencies

Risk measurement requires that both the uncertainty and its adverse effect on profitability are captured.

The main categories of risk are the following:

1.1.1 Market risk

The risk to an institution's financial condition arising from adverse movements in the level or volatility of market prices. There are several different classes of assets (interest rates, currencies, equities and commodities) and an almost infinite variety of financial products, all of which create exposure to market risk.

It is managed by

- Providing consistent information of market risk across the organization at all levels.
- Calculating consistent risk measures (volatilities, VaR)
- Establishing appropriate procedures and monitoring risk limits.
- Understanding where risk comes from across the organization

1.1.2 Credit risk

The risk of financial loss suffered when a company that the bank has dealt with (a counterparty) defaults, or when the market sentiment determines that a company is more likely to default (spread risk)

Credit risk is managed by

- Monitoring credit exposures relative to limits
- Resetting limits regularly
- Scenario analysis

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1.1.3 Settlement risk

Settlement risk is one form of credit risk, which occurs when two payments are exchanged the same day. This risk arises when the counterparty may default after the institution already made its payment. On settlement day, the exposure to counterparty default equals the value of the payment due. In contrast, the pre-settlement exposure is only the netted value of the two payments.

Settlement risk is managed by

- Monitoring counterparty activities and settlements limits
- Managing pre-settlement counterparty activities

1.1.4 Liquidity risk

Liquidity risk takes two forms, asset liquidity risk and funding liquidity risk. Asset liquidity risk, also know as market/product liquidity risk, arises when a transaction cannot be conducted because of inadequate market depth or because of disruptions in the market.

It is managed by

- Setting limits on certain markets or products and by means of diversification

Funding liquidity risk, also know as cash-flow risk, refers to the inability to meet payments obligations, which may force early liquidation. Cash-flow risk interacts with product liquidity risk if the portfolio contains illiquid assets that must be sold at less than fair market value.

It is managed by

- Planning of cash flow needs
- Setting limits on cash-flow gaps
- Diversification
- Consideration of how new funds can be raised to meet shortfalls.

1.1.5 Operational risk

Operational risks generally can be defined as arising from human and technical errors or accidents.

It is managed by

- Establishing proper supervision and segregation of duties
- Testing all systems in a comprehensive manner
- Establishing complete reconciliation between internal and external systems
- Setting up independent backup facilities and systems

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1.1.6 Legal risk

This is the risk that contracts are not legally enforceable or document correctly.

It is managed by

- Carefully contracting and conducting business with external parties and employees
- Establishing clear compliance and regulatory structures

1.2. CREATING A RISK MANAGEMENT PROCESS

In order to adequately control risk, it is necessary to create a multi-stage process that focuses on specific risks that the organization faces. There is no "one size fits all" risk management process. Indeed, it may be argued that, for central banks that frequently hold short-duration assets, market risk may pose less exposure than either liquidity risk or operational risk. On the other hand, for other financial institutions, especially those that depend on borrowed funds, invest in long duration assets, or hold securities in a wide variety of asset classes, market, credit and legal risk may take on far more significance.

The creation of a risk management process should contain each of the following components.

- Risk Measurement
- Risk Management Systems
- Rules of Behavior
- Mechanism to check compliance
- Performance attribution

1.2.1 Risk measurement

Traditional measurement of market risk

The quantification of market risk isn't a new thing. Ever since positions were first marked to market, traders wanted to understand the risk they are taking in terms of hedging parameters. The discussion below illustrates the traditional measures of interest rate risk.

The value of all interest rate products is the present value of the future cash flows that make up the instrument. The present value of any cash flow is dependent on the maturity of the cash flow and the relevant interest rate at that maturity.

The present value of each flow is given by:

$$PV = \text{cash flow} \times \text{discount factor}$$

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The discount factor, assuming annual yields, is given by:

$$\textit{Discount factor} = \frac{1}{(1+r)^n}$$

Where: r= annual interest rate
n=number of years

A typical portfolio of interest rate assets contains thousands of cash flows maturing on different dates. Each individual maturity date will have a unique yield associated with it. Therefore the value of the portfolio depends on several hundred yields and each yield can be considered as a risk factor that the portfolio is exposed to. What is needed is a way to approximate the behavior of all yields together, i.e. the behavior of the yield curve.

The movements of a yield curve can be described by a relatively small number of characteristics:

- Parallel shift
- Twist about a single maturity
- Wave or dip

There are further minor shifts, which are normally ignored. The first two movements, the parallel and the twist, taken together, account for between 80% and 90% of all yield curve movement. Most banks model their interest rate risk around the first two movements, ignoring the other types of shift. Of the two, the parallel shift is the dominant movement. A separate risk measure is used for each of the two main yield curve movements:

- The change in the present value of a portfolio due to a parallel basis point movement in the yield curve is know as PVBP, VBPM, PV01, etc. The use of one basis point parallel shift in the curve is a *de facto* industry standard.
- The portfolio value change due to a yield curve tilt or twist. For what is often confusingly called 'yield curve risk' there is *no de facto* industry standard yield curve movement.

Both these risk measures are "sensitivity" measures, as are the majority of traditional risk measures. Sensitivity risk measures quantify the profit and loss (P&L) impact on a portfolio of a specified change in a risk factor. Both the above measures quantify the sensitivity of the portfolio value to specified shifts in the yield curve.

Outright Interest Rate Risk

The term "Outright Interest Rate Risk" expresses the exposure of a portfolio to a change in the overall level of interest rates, i.e. a parallel shift in the yield curve. This is measured by the PVBP, or VBPM, or PV01 risk measure. For example, if the value of the PVBP for a loan or for a bond is $100 it means that in case of one basis point parallel shift in the yield curve the value of the asset will change by $100.

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An other risk measure tool, quite similar to the above mentioned, is the duration of the instrument. For example, if the duration of a bond is 4 years it means that in the case of 100 basis points change in the yield the value of the bond will change 4 percentage units to opposite side.

All the above mentioned risk measurement tools assume parallel shift in the yield curve, which is not always the case in the financial markets, and don't provide any information about the likelihood or the magnitude of an interest rate change. To get a useful measure of price risk, it is necessary to have both duration and expected yield change.

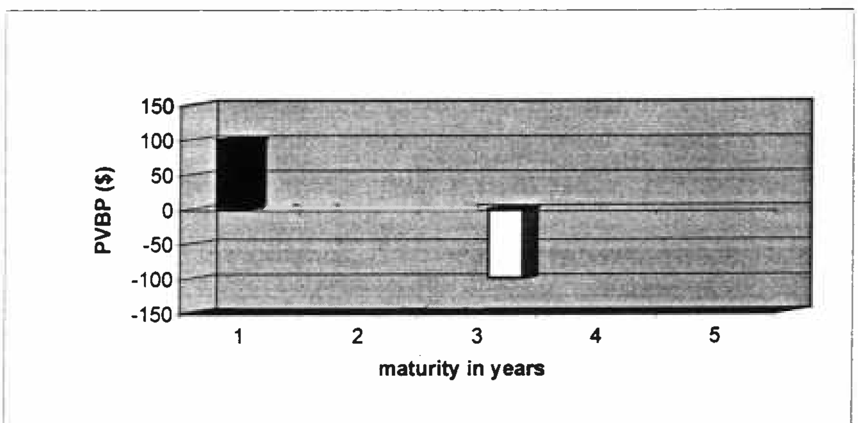
Yield curve risk

Some banks to measure yield curve risk use a twist around the one year maturity point, other banks simply tilt the yield curve upwards, starting at the beginning and increasing the by one basis for each successive year. A yield curve tilt is in fact a combination of a twist plus a parallel yield shift.

A yield curve twist will have the greatest impact on an interest rate that has significant maturity mismatches, i.e. long and short positions at different maturities.

Example: Consider a bond portfolio with two bonds as it illustrated in graph 1: the trader has purchased a Eurobond with a maturity of three years and has hedged it with a hedge maturity of one year (i.e. sold a government bond). The trader has 'duration' hedged the original position, i.e. the value of the portfolio is not affected by a parallel shift in interest rates. In others words, the right number of bonds are sold as a hedge, so that the PVBP of the original position are equal despite the different maturities.

Graph 1 shows that the PVBP of both positions are equal to $100, thus the PVBP of the portfolio is zero. Table 1 shows the sensitivity of this portfolio to a twist in the yield curve.



Graph 1. Duration hedged bond portfolio

Table 1 shows that, because there is maturity mismatch between the original position and the hedge, the portfolio will lose value if the slopes of the yield curve changes. In the example of the table 1 the portfolio will lose $200. This kind of yield curve measures is to control the amount of maturity

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mismatch that it is possible to create in the portfolio. As with PVBP, no feel is given by the yield curve measure of how likely it is that the bank will lose $200. It is extremely unlikely that a shift identical to the one described in the table 1 will occur.

Table 1. Impact of yield curve

Maturity (years)	Curve twist (bp)	Portfolio PVBP ($)	Value change ($)
1	0	100	0
2	1	0	0
3	2	-100	200
4	3	0	0
5	4	0	0
Portfolio value change ($)			**-200**

1.3 VALUE AT RISK – A DEFINITION

Value at risk is a statistical measure of the risk that estimates the maximum loss that may be experienced on a portfolio with a given level of confidence. VaR always is calculated under a probability that says how likely it is that the losses will be smaller than the amount given. VaR is a monetary amount, which may be lost over a certain period of time. The period of time is dependent on the period the portfolio is considered to be held constant.

Value at risk is the maximum amount of money that may be lost on a portfolio over a given period of time, with a given level of confidence.

VaR is usually calculated for one-day holding period and at 95% confidence level. This means that there is 95% chance the loss of the portfolio to be lower than the amount given by VaR. More formally, VaR describes the quantile of the projected distribution of gains or losses over the target horizon. The choice of the holding period and the confidence level depend on how the VaR results will be used. The longer the holding period or the confidence level the larger the VaR. Calculating the VaR with different confidence level doesn't change the distribution of possible outcomes nor does it changes the portfolio risk. It only changes the way the risk is stated by focusing on a different point of the distribution.

It is important to note that the VaR provides the expected maximum loss at a chosen confidence level but it says nothing about the magnitude of losses beyond that confidence level. In other words, for those occasions when the loss exceeds the VaR, the VaR doesn't tell us how much worse the actual outcome might be. To get more information about the points of the distribution beyond the VaR, it is necessary to perform a portfolio stress test that focuses only on the tails of the distribution.

Focusing on the tails of the distribution is critical because, in real – world financial markets, these distributions tend to have fat tails. The term

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"fat tails" refers to probability distribution where large changes (both positive and negative) have a greater probability of occurring than the normal distribution implies. In other words, extreme market moves, such as the 1987 U.S. stock market crash occur more frequently than they would if moves in financial markets could be accurately described by a normal distribution.

An appropriate portfolio *stress test* will quantify the magnitude of potential losses under simulated extreme market moves by combining historical market data drawn from periods of large market movements and assumptions based on scenarios analysis. If the results of the stress test show significant potential losses, then either the level of risk taken may be reduced or monitoring procedures and strategies to manage exposure may be developed.

An assessment of VaR

One of the major advantages of VaR is that it is a risk measure that can be applied to all traded products. Therefore, it is a standard risk benchmark, which allows the risk being taken by different trading areas to be compared directly. As VaR can be used to measure the risk on any product it can be combined across different trading areas to give a single figure for the risk being taken by all trading areas combined. Neither of these things can be achieved with traditional sensitivity-based measures of risk.

There are two other things that VaR can do which traditional risk measures cannot. The first is that VaR gives an estimate of the likelihood of a loss greater than a given figure occurring, i.e. VaR has a probability associated with it. This is far more useful than VBPM, which gives no feel for how likely the specified change in market prices is to occur. The second thing is that VaR takes account of how price changes of different assets are related to each other. This allows the reduction in risk through diversification (i.e. holding positions in a number of assets) to be measured.

However, VaR is no panacea, it only effectively measures market risk when the market is behaving "normally". This means that VaR is a measure of the day-to-day, or "business-as-usual" risk on the portfolio, with a given level of confidence. VaR does not deal adequately with the fairly frequent extreme price moves observed in the financial markets. Therefore, VaR must be combined with stress testing to provide a more comprehensive market risk measurement framework.

In addition to the internal requirement for better risk measurement techniques, there is also an external dimension to VaR. It is now a global de facto risk measurement standard and is expected, if not quite required, by most regulatory bodies in G10 countries. It should also be noted that there are potentially significant benefits for banks stemming from the use of VaR. Regulators now allow banks to use VaR as the basis for calculating the bank's regulatory capital requirement. The required regulatory capital is appreciably less if calculated using VaR in place of the standard method. This means that the bank's actual capital can be leveraged more highly.

Rating agencies also expect banks to have implemented comprehensive VaR systems and to be able to demonstrate how it is used as

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a risk management tool in the bank. Regulators, as well as rating agencies, are more concerned with the demonstrable process of risk management than they are in the precise definition or method of calculation of VaR that a bank has chosen (unless there is something deficient about it).

One of the ways regulators and rating agencies expect banks to use VaR is as the basis for risk adjusted performance measurement (RAPM). Risk capital is the day-to-day level of risk that the bank is running. VaR is a key component of the bank's risk capital. It is perhaps through the use of RAPM that banks can derive the greatest benefit from VaR.

CHAPTER 2: VAR METHODS

Although the term risk has come to mean "danger of loss", financial theory defines risk as the dispersion of unexpected outcomes due to movements in financial variables. Thus both positive and negative deviations should be viewed as sources of risk. An investor should realize that a superior performance of traders reflects great risks.

To measure risk, one has to define first the variable of interest, which could be portfolio value, earnings, capital, or a particular cash flow. Financial risks are created by the effects of financial factors on this variable.

Broadly there are four different types of financial market risks: interest rate risk, exchange rate risk, equity and commodity risk. Basic analytical tools apply to all these market risks. Risk is measured by the standard deviation of unexpected outcomes, or ***sigma* (σ)**, also called ***volatility***.

Losses can occur through a combination of two factors: the volatility in the underlying financial variable and the exposure to this source of risk. Whereas corporations have no control on the volatility of financial variables, they can adjust their exposures to these risks, for instance, through derivatives. Value at risk captures the combined effect of underlying volatility and exposure to financial risks.

Approaches to VaR can be classified into two groups. The first group uses local valuation. ***Local valuation methods*** measure risk by valuing the portfolio once, at the initial position, and using local derivatives to infer possible movements. The delta normal method uses linear or delta, derivatives and assumes normal distributions. Because the delta normal approach is easy to implement, a variant, called the "Greeks", is sometimes used. This method consists of analytical approximations to first and second order derivatives and is most appropriate for portfolios with limited sources of risk. The second group uses full valuation. ***Full valuation methods*** measure risk by fully repricing the portfolio over a range of scenarios.

2.1 VARIANCE-COVARIANCE METHOD

VaR assumes that the portfolio is "frozen" over the horizon and that the current portfolio will be marked-to-market on the target horizon. VaR can be derived by two ways, either from considering the actual empirical distribution or by approximating the distribution by a parametric approximation, such as the normal distribution, in which case VaR is derived from the standard deviation.

In this section is introduced the first widely used method of calculating value at risk: the variance-covariance or parametric method. This method assumes normality, i.e. that percentage price changes in financial markets are normally distributed. This assumption allows volatility to be described in terms of standard deviation (SD). Volatility is usually described in terms of a percentage change with the standard measure of volatility being change equal to one standard deviation. In other words, VaR figure can be derived

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directly from the portfolio standard deviation using a multiplicative factor that depends on the confidence level.

To compute VaR, the assumption that the daily revenues are identically and independently distributed is in force.

Steps in constructing VaR

The following steps are required to compute VaR:

- *Mark-to-market* of the current portfolio
- *Measure the variability of the risk factors*
- *Set the time horizon* or the holding period
- *Set the confidence level*
- *Report the worst loss* by processing all the preceding information

The formula for the calculation of the VaR is given by:

$$VaR = W_0 a \sigma \sqrt{\Delta t} \qquad (2.1.1)$$

Where:
W_0: is the initial investment
a: corresponds to the chosen confidence level, i.e. 99% confidence level gives a equal to 2.33, assuming normal distribution
σ: standard deviation of the prices returns, on an annual basis
Δt: is the time interval in years

2.1.1 PORTFOLIO VAR

A portfolio can be characterized by position on a certain number of assets, expressed in the base currency, say, dollars. If the positions are fixed over the selected horizon, the portfolio rate of return is the linear combination of the returns on underlying assets, where the weights are given by the relative amounts invested at the beginning of the period.

The rate of return for a portfolio, for a period of time t is given by:

$$R_p = \sum_{i=1}^{N} w_i R_i \qquad (2.1.2)$$

Where N is the number of assets, R_i is the rate of return on asset i and w_i is the weight.

The portfolio return can be written using matrix notation:

$$R_p = w_1 R_1 + w_2 R_2 + \ldots + w_N R_N = [w_1 w_2 \ldots\ldots w_N] \begin{bmatrix} R_1 \\ R_2 \\ \\ R_N \end{bmatrix} = w'R$$

Where w' is the horizontal vector of weights, and R is the vertical vector containing individual asset returns.

The expected return of the portfolio is

$$E(R_p) = \mu p = \sum_{i=1}^{N} w_i \mu_i \qquad (2.1.3)$$

And the variance is

$$V(R_p) = \sigma^2_p = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{N} \sum_{j<i}^{N} w_i w_j \sigma_{ij} \qquad (2.1.4)$$

As the number of assets increases, it is more convenient to use matrix notation. The variance can be written

$$\sigma_p = w'\Sigma w \qquad (2.1.5)$$

where Σ is the covariance matrix and w'/ w are the horizontal / vertical vectors of weights, respectively.

So far nothing has been said about the distribution of the portfolio return. In variance-covariance method, all individual security returns are assumed normally distributed. Therefore, the portfolio return as a linear combination of normal random variables is also normally distributed. If so, we can translate the confidence level c into a standard normal deviate *a* such that the probability of observing a loss worse than *–a* is *c*. Defining W as he initial value, the portfolio VaR is

$$VaR_p = a\,\sigma_p W = a\sqrt{w'\Sigma w}\,W \qquad (2.1.6)$$

Diversified VaR

The portfolio VaR, taking into account diversification benefits between assets.

The individual risk of the asset i

$$VaR_i = a\,\sigma_i |W_i| \qquad (2.1.7)$$

We take the absolute value of the W because it can be negative (short position), whereas the risk measure must be positive.

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Individual VaR

The VaR of one component taken in isolation

Equation (2.1.4) shows that the portfolio VaR depends on variances, covariances, and the number of assets. Covariance is a measure of the extent to which two variables move linearly together. If the variables are independent, their covariance is equal to zero. A positive covariance means that the two variables tend to move in the same direction; a negative covariance means that they tend to move in opposite directions. The magnitude of covariance, however, depends on the variances of the individual components and isn't easily interpreted. The correlation coefficient is a more convenient, scale free measure of linear dependence:

$$\rho_{12} = \sigma_{12}/\sigma_1\sigma_2 \qquad (2.1.8)$$

where

σ_{12} the covariance between asset 1 & 2

σ_1 the standard deviation of asset 1

σ_2 the standard deviation of asset 2

The correlation coefficient ρ always lies between −1 and +1. When it is equal to unity we say that the two variables are perfectly correlated. When it is equal to zero then the two variables are uncorrelated.

Lower portfolio risk can be achieved through low correlations or large number of assets. When the correlations of the assets are lower than unity, the portfolio risk is lower than the sum of the individual VaRs. This reflects the fact that with assets that move independently, a portfolio will be less risky than either asset. Low correlations help to diversify portfolio risk.

When the correlations of the assets are equal to unity the portfolio risk is equal to the sum of the individual VaRs. In general, this will not be the case because correlations typically are imperfect. The benefit from the diversification can be measured by the difference between the *diversified* VaR and *undiversified* VaR.

Undiversified VaR

The sum of individual VaRs, or the portfolio VaR when there is no short position and all correlations are unity.

The undiversified VaR provides an upper bound on the portfolio VaR, an absolute worst-case scenario.

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2.1.2 VAR TOOLS

VaR tools could be used for active risk management and they are:

- **Marginal VaR**: is the change in portfolio VaR resulting from taking an additional dollar of exposure to a given component. It is also the partial (or linear) derivative with respect to the component weight.

$$\textbf{Marginal-VaR} = a \times \frac{\text{cov}(R_i, R_p)}{\sigma_p} \qquad (2.1.9)$$

The marginal VaR is closely related to the beta, defined as

$$\beta_i = \frac{\text{cov}(R_i, R_p)}{\sigma^2_p} \qquad (2.1.10)$$

which measures the contribution of one security to portfolio risk.
The relationship between the marginal VaR and beta is given by:

$$\textbf{Marginal-VaR} = \frac{VaR}{W}\beta_i \quad (2.1.11)$$

- **Incremental VaR**: is the change in VaR due to a new position. It differs from the marginal VaR in that the amount added or subtracted can be large, in which case VaR changes in a nonlinear fashion.
  Incremental VaR = $VaR_{p+a} - VaR_p \approx (\text{Marginal VaR})' \times a$
  where p is the portfolio and a is the new position
  If VaR is decreased, the new trade is risk reducing, or is a hedge; otherwise, the new trade is risk increasing.

- **Component VaR**: A partition of the portfolio VaR that indicates how much the portfolio VaR would change approximately if the given component was deleted.
  Component VaR = Marginal Var * w_i W
  Portfolio VaR = $\sum_{i=1}^{N} ComponentVaR_i$

Example: Portfolio with US$2 million invested in the Canadian dollar (CAD) and US$1 million invested in Euro (EUR). Assume that these two currencies are uncorelated with volatility against the dollar of 5% and 12% respectively.

1. VaR at 95% confidence level:

$$\sigma_p^2 = 0.05^2 * (\frac{2}{3})^2 + 0.12^2 * (\frac{1}{3})^2 = 0.002711$$

$\sigma_p = 0.052067$

VaR = 1.65 * 0.052067 * 3,000,000 = $257,738

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2. Individual VaRs:
$VaR_1 = 1.65 * 0.05 * 2,000,000 = \$165,000$
$VaR_2 = 1.65 * 0.12 * 2,000,000 = \$198,000$

3. Undiversified VaR =$165,000 + $198,000 = $363,000 > $257,738 = VaR

4. Marginal VaR Increase CAD position by US$ 1.

$\sigma^2_{p+1} = 0.05^2 * (0.666778)^2 + 0.12^2 * (0.333222)^2 = 0.00271042$

$\sigma_{p+1} = 0.05206165$

$VaR_{p+1} = 1.65 * 0.05206165 * 3,000,001 = \$257,738.2921$

Marginal VaR =$257,738.2921 - $257,738.2393 = $0.05281

5. Incremental VaR : Increase CAD position by US$ 10,000.
Incremental VaR = $VaR_{p+a} - VaR_p$

$\sigma^2_{p+a} = 0.05^2 * (0.6677)^2 + 0.12^2 * (0.3322)^2 = 0.002704$

$\sigma_{p+a} = 0.052002$

$VaR_{p+a} = 1.65 * 0.052002 * 3,010,000 = \$258,267$

Incremental VaR = $258,267 - $257,738 = $529

6. Component VaR:
For the Canadian dollar position :
Component VaR = Marginal Var * V_1 = 0.0528 *2,000,000 = $105,630
Percent contribution = Component VaR / VaR = 105,630/ $257,738 = 0.4097 = 40.97%
For the Euro position : percent contribution = 59.03%

2.2 HISTORICAL SIMULATION METHOD

Historical simulation provides an approach to calculate VaR that is not model based, although it is a statistical measure of potential loss. Historical simulation doesn't assume that price changes are normally distributed and it can cope with all vanilla exchange traded and over the counter (OTC) options. Historical simulation captures the characteristics of the price change distribution of the portfolio, as VaR is calculated from the actual distribution of portfolio changes. As a result of this, where portfolio distribution has fat tails, it will tend to produce slightly higher VaR number than the VaR calculated by variance-covariance method.

Calculating VaR using Historical Simulation

Historical simulation takes a portfolio of assets at a particular point in time and then revalues the portfolio a number of times, using a history of

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prices for the assets in the portfolio. The portfolio revaluations produce a distribution of profit and losses, which can be examined to determine the VaR of the portfolio with a chosen level of confidence. This approach becomes very computationally intensive when the portfolio contains a large number of products and when a reasonable length of historical prices is used.

There are a few different ways to calculate VaR using historical simulation. The simplest way is to revalue the portfolio using a specified history of prices. The portfolio value is then calculated for each day. The portfolio values can then be placed into percentiles. The VaR can then be read off from the percentile corresponding to the required confidence level.

The problem with this approach is that, as the value of the portfolio changes, the percentage value changes in the portfolio no longer refer to the original portfolio value. Also, over the given history, the prices of the assets making up the portfolio will change in relation to one another. This means that the composition of the portfolio is changing throughout the length of the price history used. Revaluing the portfolio using actual asset prices will not give the correct result. What is actually required is a history of portfolio value changes based on today' s portfolio with today' s portfolio value and composition.

The correct method of calculating VaR using historical simulation is to use a history of percentage price changes and apply these to today's portfolio, as follows:

- Obtain percentage price change series for every asset or risk factor needed to revalue the portfolio.
- Apply price changes to the portfolio, to generate a 'historical' series of portfolio values changes.
- Sort the series of portfolio value changes into percentiles.
- The VaR of the portfolio is the value change corresponding to the required level of confidence

There are two key questions to be answered with respect to setting up the required historical price series:

1. What length of price series should be used?
2. What should be done about products for which no price history exists?

The length of the time series is the biggest decision that must be made when using the historical simulation approach to calculate VaR. It is essential to ensure that a bank or trading area is happy with the VaR characteristics given by the length of time series chosen.

Apply price changes to a portfolio

Having created a history of percentage price changes for each component of the portfolio, the price changes can be applied to each portfolio component to generate a history of portfolio value changes.

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The value change of the portfolio is given:

$$V=\sum_{i} f(\delta_i, a_i) \qquad (2.2.1)$$

Where:
V= value change of the portfolio
f= function that determines the value of a portfolio component. For straightforward assets the value of the component is arrived by multiplying the price by the quantity of the asset. For other products, such as options, a valuation model must be used.
a_i = sensitivity of portfolio component to risk factor, i
δ_i = percentage change in risk factor, i

Once a series of portfolio value changes has been generated, VaR can be determined. The first step is to sort the portfolio value into percentiles. A percentile contains 1% of the value changes.

VaR is then derived from the entire distribution of hypothetical returns, where each historical scenario is assigned the same weight of 1/t.

2.3 MONTE CARLO SIMULATION METHOD

Monte Carlo (MC) simulations cover a wide range of possible values in financial variables and fully account for correlations. The MC method proceeds in two steps. ***First***, the risk manager specifies a stochastic process for financial variables as well as process parameters; parameters such as risk and correlations can be derived from historical or options data. ***Second,*** fictitious price paths are simulated for all variables of interest. At each horizon considered, the portfolio is marked-to-market using full valuation as in the historical simulation method. Each of these "pseudo" realizations is then used to compile a distribution of returns, from which a VaR figure can be measured.

The Monte Carlo method is thus similar to the historical simulation method, except that the hypothetical changes in the prices of the assets are created by random draws from a prespecified stochastic process instead of sampled from historical data.

A summary of the strengths and weaknesses of each methodology is given below in **Table 2.1.**

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Table 2.1

A summary of the strengths and weaknesses of each methodology

Methodology	Advantage	Disadvantage
Variance-covariance	• Fast & simple calculation • No need for extensive historical data (only volatility and correlation matrix are required	• Less accurate for nonlinear portfolios or for skewed distributions
Monte Carlo simulation	• Accurate* for all instruments • Provides a full distribution of potential portfolio values, not just a specific percentile • Permits use of various distributional assumptions (normal, T-distribution, normal mixture, etc.) and therefore has potential to address the issue of "fat tails" • No need for extensive historical data	• Computational intensive and time consuming (involves revaluing the portfolio under each scenario) • Quantifies fat-tailed risk *only if* ma market scenarios are generated from appropriate distributions
Historical simulation	• Accurate* for all instruments • Provides a full distribution of potential values, not just a specific percentile • No need to make distributional assumptions • Faster than Monte Carlo simulation because less scenarios are used	• Requires a significant amount of daily rate history • Difficult to scale far into the future (long horizons) • Coarse at high confidence levels (e.g. 99% and beyond) • Somewhat computationally intensive and time-consuming • Incorporates tail risk only if historical data set includes tail events

* Accurate if used with complete pricing algorithm

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CHAPTER 3: VAR FOR FIXED INCOME PORTFOLIOS

3.1 MAPPING APPROACHES

Bond positions describe the distribution of money flows over time by their amount, timing, and credit quality of issuer. This creates a continuum of risk factors, going from overnight to long maturities for various credit risks. In practice we have to restrict the number of risk factors to a small set. For some portfolios, one risk factor may be sufficient. For others, 15 maturities may be necessary. For portfolios with options, we need to model movements not only in yields but also in their implied volatilities.

Once the risk factors have been selected, the question is how to "map" or summarize, the portfolio positions into exposures on these risk factors. We can distinguish three mapping systems: principal, duration and cash flows. With the ***principal mapping,*** the bond risk is associated with the maturity of the principal only. With ***duration mapping***, the risk is associated with the zero coupon bond with maturity equal to the bond duration. With ***cash flow** mapping,* the risk of fixed income instruments is decomposed into the risk of each the bond cash flows. In each case, mapping should preserve the market value of the position as well its market risk.

3.2 COMPARISON OF MAPPING APPROACHES

As an example, Table 3.1 describes a two bond portfolio consisting of a \$100 million 5 year 6 percent issue and a \$100 million 1 year 4 percent issue. Both issues are selling at par, implying a market value of \$200 million. The portfolio has an average maturity of 3 years and duration of 2.733 years. The table shows the present value of all portfolio cash flows discounted at the appropriate zero coupon rates. Table 3.2 describes risk factors using Risk Metrics data. This table presents monthly VaRs of zero coupon bonds as well as the correlations for maturities going from 1 to 5 years. Here, VaR corresponds to 1.65 standard deviation movement.

Table 3.1

Mapping for a bond portfolio

	Cash flows			Mapping (present value)		
Term (year)	**5 year**	**1 year**	**Spot rate**	**Principal**	**Duration**	**Cash flow**
1	\$6	\$104 million	4,000	0	0	\$105.77
2	\$6	0	4.618	0	0	\$5.48
2.733	--			0	\$200	-
3	\$6	0	5.192	\$200	0	\$5.15
4	\$6	0	5.716	0	0	\$4.80
5	\$106	0	6.112	0	0	\$78.79
Total				**\$200 million**	**\$200 million**	**\$200 million**

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Table 3.2

Risk and correlations for U.S. Zeroes, monthly VaR at 95%

		Correlation Matrix				
Maturity	**VaR (%)**	**1y**	**2y**	**3y**	**4y**	**5y**
1y	0.470	1				
2y	0.987	0.897	1			
3y	1.484	0.886	0.991	1		
4y	1.971	0.866	0.976	0.994	1	
5y	2.426	0.855	0.966	0.988	0.998	1

Principal mapping considers the timing of redemption payments only. Since the average maturity of this portfolio is 3 years, the VaR can be found from the risk of a 3-year maturity, which is 1.484 %. The Var is then $200 million x 1.484% = $2.97 million. The only positive aspect of this method is its simplicity. This approach overstates the true risk because it ignores the coupon payments, It doesn't preserve the portfolio's risk.

At ***duration mapping*** approach, we replace the portfolio by a zero coupon bond with maturity equal to the duration of the portfolio, which is 2.733 years. Using a linear interpolation for duration for the VaRs, we find risk of 0.987+(1.484-0.987) x (2.733-2)=1.351% for this hypothetical zero bond. With a $200 million portfolio, the duration based VaR is $2.70 million, slightly less than before.

Finally, the ***cash flow mapping*** method consists of grouping all cash flows on term structure "vertices", which correspond to maturities for which volatilities are provided. Each cash flow is represented by the present value of the cash payment, discounted at the appropriate zero coupon rate

Coupon paying bond = Cash flow PV on vertex 1+ Cash flow PV on vertex 2+..........+ Principal PV on last vertex.

Table 3.3.

Computing VaR of a $200 million bond portfolio (monthly VaR at 95%)- Cash flow mapping approach

Term (Year)	Cash Flows X	VaR (%) V	Individual VaR (%) X x V	Correlation Matrix					Component VaR
				1y	2y	3y	4y	5Y	
1y	$105.77	0.470	49.66	1					$0.45
2y	$5.48	0.987	5.40	0.897	1				$0.05
3y	$5.15	1.484	7.65	0.886	0.991	1			$0.08
4y	$4.80	1.971	9.47	0.866	0.976	0.994	1		$0.09
5y	$78.79	2.426	191.15	0.855	0.966	0.988	0.998	1	$1.90
Total	**$200 million**		263.35						
Undiversified			**$2.63 million**						
Diversified									**$2.57million**

Table 3.3 shows how to compute the portfolio VaR using cash flow mapping. The second column reports the cash flows X from Table 3.1. The third column shows the monthly VaRs, V, of zero coupon bonds at 95% confidence level. The fourth column presents the individual VaRs of each cash flow and it is derived by the multiplication of each cash flow X by the risk factor V.

With perfect correlation across all cash flows the Var of the portfolio would be $2.63 million.

The right side of the table presents the correlation matrix of zeroes for maturities from 1 to 5 years. To obtain the portfolio VaR, we premultiply and postmultiply the matrix by the dollar amounts (xV) at each vertex. Taking the square root, we find a VaR measure of $2.57 million. This is the most the portfolio could lose over a 1-month horizon at 95% confidence level.

Note that the duration VaR was $2.70 million and the diversified VaR $2.57 million. These differences are due to two factors. First, risk measures are not perfectly linear with maturity, which should be the case if term structures shifts were strictly parallel. Second, correlations are below unity, which reduces risk even further. Thus, of the $130.000 difference in the extreme VaRs ($2.70 million – $2.57 million), $70.000 is due to differences in yield volatility ($2.70 million – $2.63 million), and $60.000 is due to imperfect correlations.

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CHAPTER 4: FINANCE KIT VALUE AT RISK PROCESS

4.1 INTRODUCING FINANCE KIT

Finance Kit is a real time and cashflow based, integrated front, middle and back office system that is open and flexible. It provides sophisticated tools for effective transaction management, decision support, risk management, performance measurement, and operations. Finance Kit supports all activities within trading, risk management, confirmation, settlement, cash management and reporting and can be used for single or multiple site locations. Finance Kit for the time being between the other orga is implemented by some major European Central Banks.

4.2 OVERALL FINANCE KIT VAR PROCESS

Variance-covariance or Parametric Value at Risk is the method, which is implemented in Finance Kit

The VaR calculation in Finance Kit involves the following steps:

- **Volatilities & Correlations Datasets:** The statistical data come from Risk Metrics of J.P.Morgan.
- **Scenario Mapping:** This function is used to map the volatility and correlation data for certain market variables provided by Risk Metrics to market variables recognized by Finance Kit (fx rates, yield curves, IR instruments and equities).
- **Confidence level & Time Horizon:** One week time horizon and 95% confidence level have been specified in the Portfolio Editor area.
- **Position Data:** It is self evident that one must have some transactions entered into the system in order to calculate Value at Risk.
- **Market Data:** Real time market data changes to modify market values, exposures and finally the value at risk of your positions.
- **Structural Mapping:** Parametric Value at Risk of financial instruments is based on the volatilities and correlations of sub-set of primitive instruments. Each instrument is therefore mapped to a risk-equivalent position of one or more basic assets.
- **Position Delta:** The Parametric Value at Risk is calculated using linear or delta approximations of actual positions upon market variable changes. Deltas, Volatilities and Correlations should be based on the same type of market data changes (relative).
- **Cashflow Mapping:** Portfolios are made up of numerous cashflows for which we need to calculate the risk exposure. In order to be consistent, we chose the Risk Metric's volatilities vertex periods as the common vertex periods structure for both risk exposure and variance-covariance matrix.

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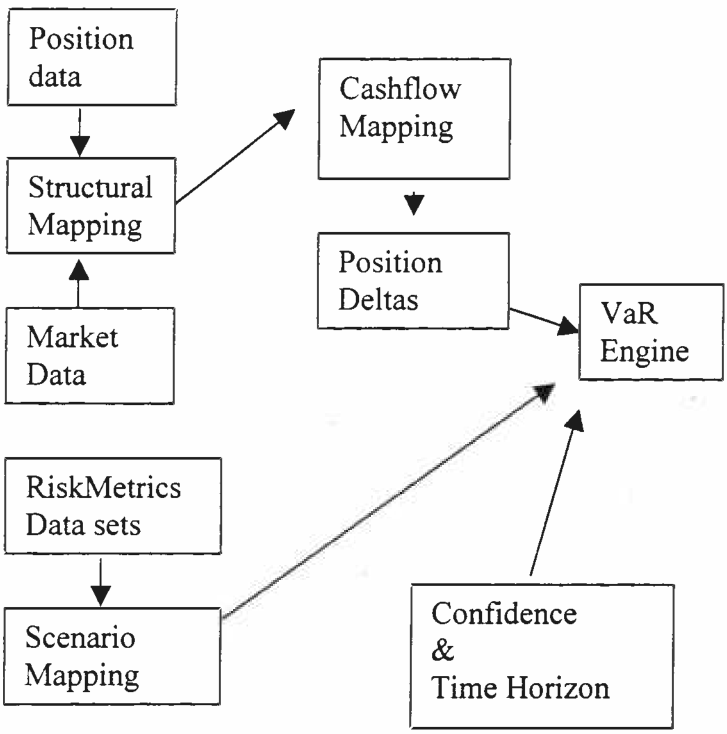
- **VaR Engine:** Once volatilities & correlations are entered into the system and the position's deltas are calculated, Finance Kit aggregates data to calculate the Value at Risk using formula based on the normal distribution theory.

The above-mentioned steps are demonstrated in **Figure 4.1**

Figure 4.1: Finance Kit VaR process



4.3 STRUCTURAL MAPPING

In general volatilities and correlations are provided only for a sub-set of basic instruments by the RiskMetrics dataset. More complex instruments need to be mapped to an equivalent position of one or more basic assets. In this section we describe the structural mapping principles and methods for various instruments.

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4.3.1 Analyzing Risks of Financial Instruments

Analyzing Risks of Financial Instruments may be performed using three methods of different complexity:

- Modified Duration Equivalent
- Cash flow analysis
- Risk- equivalent Portfolio

Modified Duration Equivalent method simply calculates the modified duration (or sensitivity) of the instrument. As far as interest rate instruments are concerned this method considers only parallel shift exposure of the instrument. As an example, this method states that being long of EUR 10000 (per basis point) of either a 10 years IR swap or a 3-month MM Depo is equivalent on term of risk.
Cash flow analysis takes into account each basic risk factor (cashflows) involved in the instrument structure. For instance, this method will consider only one 3 month cash flow for the MM Depo and 11 for the 10 years IR Swap.
Risk- equivalent Portfolio method is a trade off between the simple modified duration and the more complex cash-flow methods. For a given financial instrument, this method tries to determine the best risk-equivalent portfolio using generally simpler financial instruments. For instance, this method will consider the long position in 10 years IR Swap as equivalent to a long position in a 10-year Par Bond plus a 6-month MM loan.

4.3.1.1 Risk- equivalent Portfolio Method

The variance-covariance method calculates Value at Risk of financial instruments based on the volatilities and correlation of their risk-equivalent portfolios. Each instrument is therefore mapped to an equivalent position of one or more basic assets. Structural mapping for various instruments is shown in the table below:

Table 4.1

Instrument Class	Risk Equivalent Portfolio
MM Depo/Loan **Zero Coupon Bond**	Zero Coupon MM or Bond A single cashflow at a specified future date
FX Spot	FX Spot
Government Bond	Government Bond A series of future cashflows at specified futures dates
Equity	Equity or equity Index using a β approximation $N_{INDEX} = \beta_{equity} * N_{equity}$
Interest Rate Swap	Floating Leg: MM ZC Depo/Loan Fixed Leg: Par Bond (Swap Zero Rates)

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Instrument Class	Risk Equivalent Portfolio
FX Forward	FX Spot CCY1/CCY2 MM Depo in CCY1 value forward date MM Loan in CCY2 value forward date
FX Option	$\Delta_{fxoption}$ * FX Forward

4.4 CALCULATING MARKET EXPOSURE (Deltas)

The variance-covariance method assumes that the position market value is linear to changes in market variables. In Finance Kit, deltas are calculated using the same type of market data changes (relative) as RiskMetrics's volatilities & correlations. Three different types of exposures are calculated in Finance Kit: FX Exposures, IR Exposures and EQ Exposures.

4.4.1 The Linearity Assumption

The method used in Finance Kit to calculate value at risk is the linear (delta) method. Let V be the value of the position we are considering. This value depends on several market variables:

$$V = V(x_1, x_2, \ldots\ldots, x_n)$$

In order to calculate the vector exposure we use a linear approximation of the market value changes ΔV upon market variables changes Δx_i:

$$\Delta V = \frac{\theta V}{\theta x_1} \times \Delta x_1 + \frac{\theta V}{\theta x_2} \times \Delta x_2 + \ldots\ldots\ldots + \frac{\theta V}{\theta x_n} \times \Delta x_n$$

We can rewrite this formula as the scalar product of the vector of exposure with vector of market variable changes using matrix notations:

$$\Delta V = \delta_{\Delta x} * [\Delta x]$$

with,

$$\delta_{\Delta x} = \left(\frac{\theta V}{\theta x_1}, \frac{\theta V}{\theta x_2}, \ldots\ldots, \frac{\theta V}{\theta x_n}\right)$$

and $[\Delta x]$ is the matrix of $\Delta x_1 \ldots \; \Delta x_n$

RiskMetrics volatilities & correlations are not calculated on market variables changes but on market variable returns. So, we have to calculate deltas consistent with market variables returns:

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$$\Delta V = \delta_{\frac{\Delta x}{x}} * \left[\frac{\Delta x}{x}\right]$$

with:

$$\delta_{\frac{\Delta x}{x},t} = x_t * \delta_{\Delta x,t}$$

In Finance Kit, exposures are calculated for absolute market variables not for relatives one's. As a consequence, we have to multiply the absolute delta given by Finance Kit by the market variable.

Once the general formula has been derived, we can go in more detail into Foreign Exchanges, Interest Rates & Equity risk exposures (deltas). In particular, we have to specify which FK key figures are used to calculate these different instruments type risk exposures: FX Delta, IR Delta & Equity Delta.

We will see below the calculation of the IR Delta:

4.4.2 IR Delta

The quote market value of a simple cashflow of amount A (in quote currency) is given by:

$$V_Q = \frac{A}{DF(r_t, t)}$$

where DF (r_t, t) is the discount factor for the period t calculated with market interest rate, for the period t, r_t.

The IR Delta is the first derivative of the quote market value V_Q with respect to the interest rate r_t:

$$\delta_{\Delta r_t} = \frac{\theta V_Q}{\theta r_t} = D\bmod(r_t, t) \times V_Q$$

where $D\bmod(r_t, t)$ is the modified duration or the sensitivity of the quote market value with respect to interest rate changes. For instance if r_t is a periodic rate then the discount factor is:

$$DF(r_t, t) = (1 + t \times r_t)$$

and the modified duration:

$$D\bmod(r_t, t) = -\frac{t}{1 + t x r_t}$$

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In Finance Kit, IR Exposure (1bp) is the sensitivity of the market value of the position to 1 basis point changes in the interest rate:

$$\text{IR Exposure (1bp)} = D\,\text{mod}\,(r_t, t) \times V_Q$$

As a consequence the IR Delta is calculated by multiplying the IR Exposure (1 bp) by Interest Rate for the quote currency:

$$\delta_{\frac{\Delta r_t}{r_t}} = \text{IR Exposure (1bp)} \times \text{Interest Rate}$$

4.4.3 Risk Exposure Mapping

Finance Kit portfolios are made up of numerous cashflows for which we need to calculate the risk exposure. In order to be consistent, we have to use common vertex period structure for both risk exposure and the variance-covariance matrix. In each case the crucial point is the dimension of both the vector exposure & the variance covariance matrix, which will be used in the value at risk calculation. We have to choose between the two alternatives:

- Finance Kit's financial position vertex periods
- RiskMetrics volatilities vertex periods

In the first case, the number of market variables is of the same order as the number of cash flow dates in the portfolio that is under scrutiny. This dimension changes for each portfolio and, more importantly, it can lead to a very large variance-covariance matrix.

In the second case, this dimension depends only on the RiskMetrics vertex periods, which are fixed. As a consequence this dimension cannot exceed a given number dependent on the RiskMetrics dataset structure.

IR Delta with respect to the interest rate for period t is not necessarily a VaR vertex. If so, we need to map δ_{IR} on the closest VaR vertices t_1 and t_2 in order to generate deltas δ_1 and δ_2 compatibles with RiskMetrics vertex periods:

$$\delta_1 = a\,\delta_r$$
$$\delta_2 = b\,\delta_r$$

where

$$a = \frac{t_2 - t}{t_2 - t_1}$$

$$b = \frac{t - t_1}{t_2 - t_1}$$

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Example:

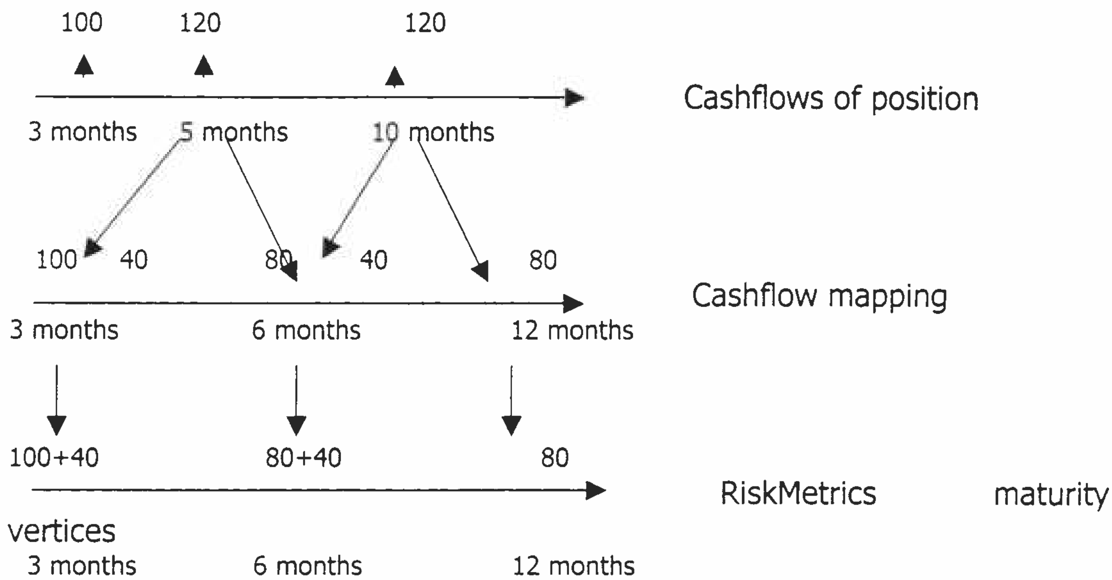

Figure 4-2: Risk Exposure Mapping

For example, in the Figure above, the 10 months Finance Kit IR Exposure will be split into the 6 months & 12 months RiskMetrics vertex according to the following formulas:

- 6 months Exposure: 120 * (12 months – 10 months)/(12 months – 6 months)= 40
- 12 months Exposure: 120 * (10 months - 6 months)/ (12 months – 6 months) = 80

4.5 VALUE AT RISK

Once volatilities and correlations are entered into the system bad the position's deltas are calculated, Finance Kit is ready to aggregate these data and calculate the Value at Risk. The general Value at Risk formula is theoretically derived from both normality and linearity assumptions.

According to the ***Normality Hypothesis (1)***, we state that the joint distribution of the relative changes in all market values is assumed to follow multivariate normal distribution with a zero mean vector and variance covariance matrix Σ:

$$\left(\frac{\Delta x_1}{x_1}, \frac{\Delta x_2}{x_2}, \ldots\ldots, \frac{\Delta x_n}{x_n}\right) \rightarrow N\left(0, \Sigma_{\frac{\Delta x}{x}}\right)$$

According to the ***Linear Hypothesis (2)***, we were able to describe the changes in the market value of given portfolio as a scalar product of its vector of exposure and the vector of market variables returns:

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$$\Delta V = \delta_{\frac{\Delta x}{x}} * \left[\frac{\Delta x}{x}\right]$$

VaR Matrix Formula

According to hypotheses 1 & 2 and the mathematical theory of normal distribution, we know that the change in market value ΔV follows a one-dimensional normal distribution with a zero mean and standard deviation $\sigma_{\Delta V}$:

$$\Delta V \rightarrow N(0, \sigma^2_{\Delta V})$$

with:

$$\sigma_{\Delta V} = \sqrt{\delta_{\frac{\Delta X}{X}}} * \sqrt{\Sigma_{\frac{\Delta X}{X}}} * \sqrt{\delta_{\frac{\Delta X}{X}}}$$

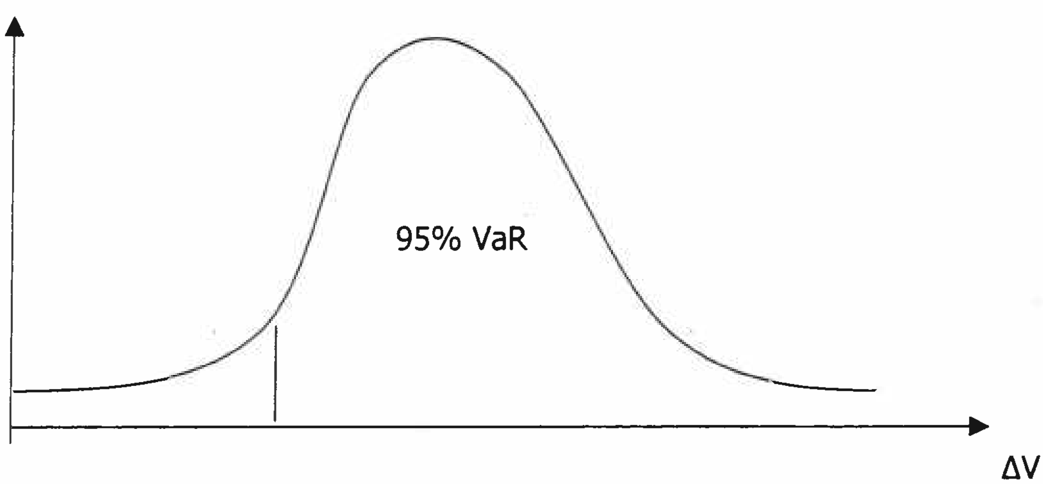
We can rewrite this formula using extended matrix formulas:

$$\sigma_{\Delta V} = \sqrt{[\delta_1 \delta_2 \ldots \delta_n] * \begin{bmatrix} \sigma^2{}_1 \sigma^2{}_{1,2} \ldots\ldots \sigma^2{}_{1,n} \\ \sigma^2{}_{1,2} \sigma^2{}_2 \ldots\ldots \sigma^2{}_{2,n} \\ \ldots \\ \sigma^2{}_{1,n} \sigma^2{}_{2,n} \ldots\ldots \sigma^2{}_n \end{bmatrix} * \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_n \end{bmatrix}}$$

Note that if the variance covariance matrix Σ_R is calculating using RiskMetrics volatilities 1.65 * σ_R instead of standard deviations itself, the result of the previous equation will be directly the 95% Value at Risk of the portfolio.

In the figure below, the area under the curve on the left of the vertical line represents the probability that the value of the portfolio will move below the Value-at-Risk figure 1.65*σ_R. This probability is 5%.

Figure 4.3 : Value at Risk



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CHAPTER 5: PORTFOLIOS VALUE AT RISK DATA ANALYSIS

We entered in Finance Kit system three US Treasury fixed income portfolios in order to compare the VaR results of them. All the portfolios have almost the same modified duration and market value but different distribution along the yield curve. Portfolios A and B include five and thirteen instruments respectively, with maturity from one month to nine years, and portfolio C includes only six instruments with maturity from one month to 4.5 years.

In Tables 5.1, 5.2, 5.3 the above-mentioned portfolios are presented.

Table 5.1

PORTFOLIO A (in USD)				
Instrument		Nominal Amount	Market Value*	Mod. Duration
BILLS	USB20020502	180,000,000	179.919.024	0,0278
	USB20020801	165,000,000	164.210.555	0,2792
BONDS	UST2003123.25	126,000,000	127.516.758	1,6170
	UST2006113.5	70,000,000	68.129.357	4,0905
	UST2011085	515,000,000	511.261.128	7,2860
Total		**1,056,000,000**	**1.051.036.821**	**4,0541**

Table 5.2

PORTFOLIO B (in USD)				
Instrument		Nominal Amount	Market Value*	Mod. Duration
BILLS	USB20020502	30,000,000	29.986.279	0,028
	USB20020801	50,000,000	49.760.769	0,279
BONDEURO	EIB2004044.75	100,000,000	104.454.520	1,838
BONDFRN	FIT20020512	110,000,000	110.374.379	0,058
BONDS	UST2003044	10,000,000	10.352.385	0,982
	UST2003123.25	80,000,000	80.938.022	1,617
	UST2005086.5	45,000,000	48.863.749	2,957
	UST2005115.75	50,000,000	53.783.529	3,139
	UST2006054.625	45,000,000	46.313.995	3,601
	UST2006113.5	70,000,000	68.052.798	4,090
	UST2010085.75	80,000,000	83.922.133	6,527
	UST2011025	120,000,000	119.419.417	6,979
	UST2011085	248,000,000	245.502.067	7,282
Total		**1,038,000,000**	**1.051.724.042**	**4,072**

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Table 5.3

PORTFOLIO C (in USD)				
	Instrument	Nominal Amount	Market Value*	Mod. Duration
BILLS	USB20020516	5,000,000	4.994.470	0,0666
	USB20020718	5,000,000	4.979.392	0,2407
	USB20021017	5,000,000	4.953.664	0,4899
BONDS	UST2004033.625	10,000,000	10.070.520	1,8538
	UST2005115.75	10,000,000	10.755.143	3,1393
	UST2006113.5	1,045,000,000	1.015.931.052	4,0897
Total		**1,080,000,000 USD**	**1.051.684.242**	**4,0043**

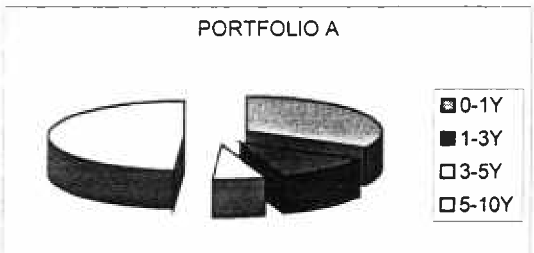
* the market values as on 23/4/02

In Table 5.4 and in Graphs 5.1, 5.2 and 5.3 the distribution of instruments per specific maturity segment (0-1years, 1-3years, 3-5years and 5-10years) for each one of the portfolios is presented. The graph 5.4 shows a comparison of the instrument distribution of the portfolios.

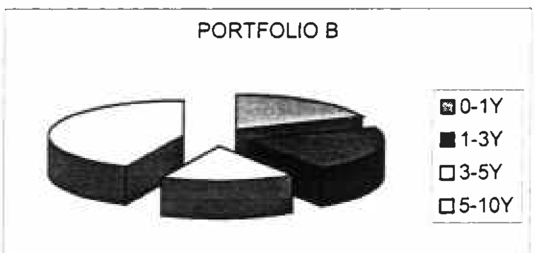
Table 5.4

	DISTRIBUTION OF INSTRUMENTS PER MATURITY SEGMENT		
	SEGMENT %		
MATURITY	**PORTFOLIO A**	**PORTFOLIO B**	**PORTFOLIO C**
0-1Y	32,67	19,27	1,39
1-3Y	11,93	21,68	0,93
3-5Y	6,63	15,90	0,93
5-10Y	48,77	43,16	96,76
TOTAL	**100**	**100**	**100**

Graph 5.1



Graph 5.2



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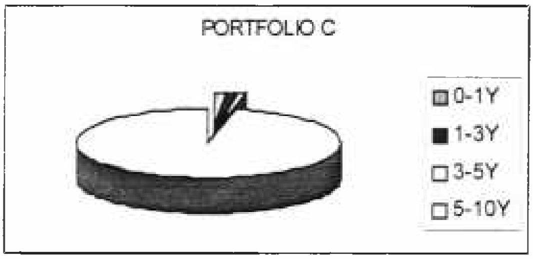
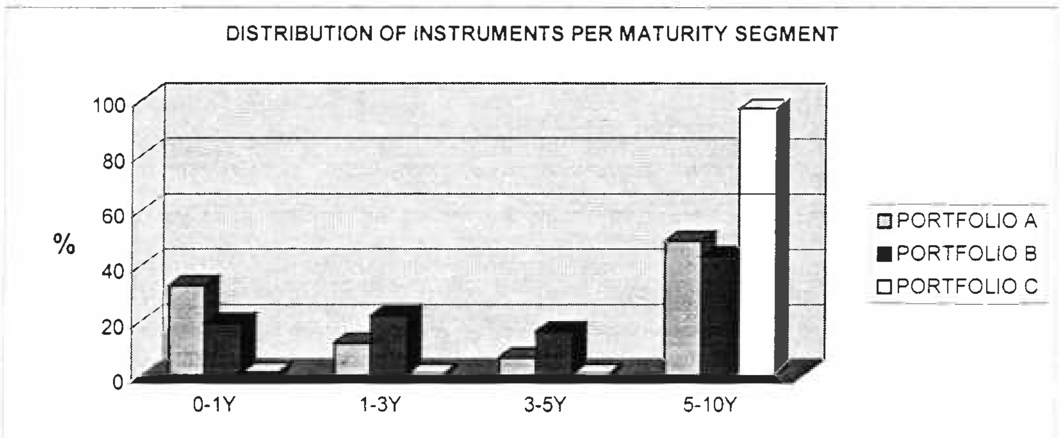
Graph 5.3



Graph 5.4



From the above tables and graphs we can see that the portfolio C has the lowest diversification as the 97% of the total capital amount is invested only in one bond, with maturity 4.5 years. For portfolio A, the majority of the capital is invested in segments 0-1years and 5-10years while for portfolio B the 81% of the total capital is invested from 1 to 10 years maturity.

Initially, the Diversified and Undiversified Value at Risk were calculated for each one portfolio, for 10 working days (23/4- 7/5/02) and after that five different scenarios were applied on the portfolios. Each scenario concerns a specific movement of the yield curve. The scenarios are demonstrated in Table 5.5.

Table 5.5

	YIELD CURVE MOVEMENT (in bps)				
	SCENARIOS				
MATURITY	**PARALLEL**	**FLATTER**	**FLATTER 2**	**STEEPER**	**STEEPER 2**
1 year	200	200	300	5	-
2 years	200	160	250	20	-
3 years	200	120	200	80	-
4 years	200	100	150	100	50
5 years	200	-	-	150	80
6 years	200	-	-	180	100
7 years	200	-	-	210	120
8 years	200	-	-	250	150
9 years	200	-	-	280	200
10 years	200	-	-	300	250

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In first scenario an upward parallel shift of the yield curve (200 bps) is assumed. The next two scenarios concern a bearish flattening of the yield curve. In other words an increase in the yields from 1 to 4 years is occurred. The pivot point is the modified duration of the portfolios (4 years). The last two scenarios assume a bearish steepening of the yield curve. The "steeper" scenario shows an upward movement from 1 year to 10 years and the "steeper 2" scenario concerns a bearish steepening with pivot the 4 years maturity.

Notice:

- All scenarios concern only movements of the yield curve and not change of the volatilities or correlations.
- The VaR has been calculated with confidence level of 95% and time horizon one week.

Each Graph, from 5.5 to 5.10, concerns a specific scenario and compares the movement of diversified VaR between the three portfolios, from 23/4 to7/5/02.

As we can see from these graphs, despite the same modified duration and the same market value the portfolios A, B, C have different Value at Risk figures during the time period under consideration. The portfolio C has the higher diversified VaR while the A portfolio has the lowest. These differences in VaR figures become stronger in the cases of the applied scenarios.

In **Appendix A,** in Tables from A.1 to A.10 an analytical presentation per day of VaR figures is given. It is clear, if we see the numbers under the columns "diversification benefit as %", that the portfolio C has the lowest diversification benefit while the portfolio B has the biggest. Despite the better diversification effect in portfolio B than in portfolio A, finally the lowest diversified VaR (as percentage of the total portfolio market value) is noticed in portfolio A. This could be explained by the fact that the exposure of portfolio A in the maturity area with the highest volatility (1 to 5 years) is lower than the corresponding exposure in portfolio B. The exposure of portfolio A, in this maturity area, is 18.56% of its total market value while the exposure of portfolio B is 37.58% (see above the Table 5.4).

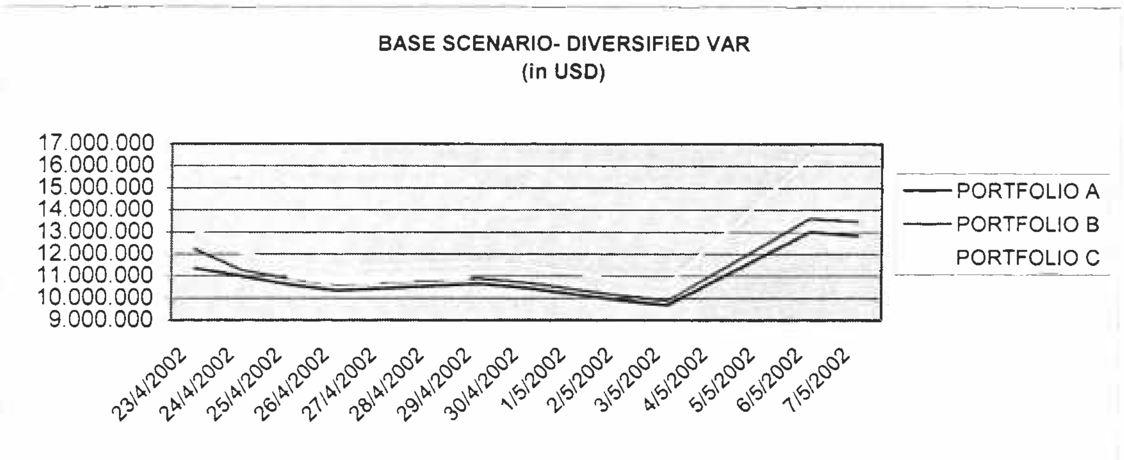
In **Appendix B,** in Table B.11, the volatilities per maturity for each day are given and it is more than clear that the sectors from 1 o 5 years have the higher volatility. Also, from the same Table we notice that the day with the highest volatility is the 6th of May and the result of that is the upward movement of VaR in all portfolios, in all scenarios (see graphs 5.5 to 5.10).

Also, in **Appendix B,** the volatilities, correlations and covariances from RiskMetrics are given (Tables B.1 to B.10).

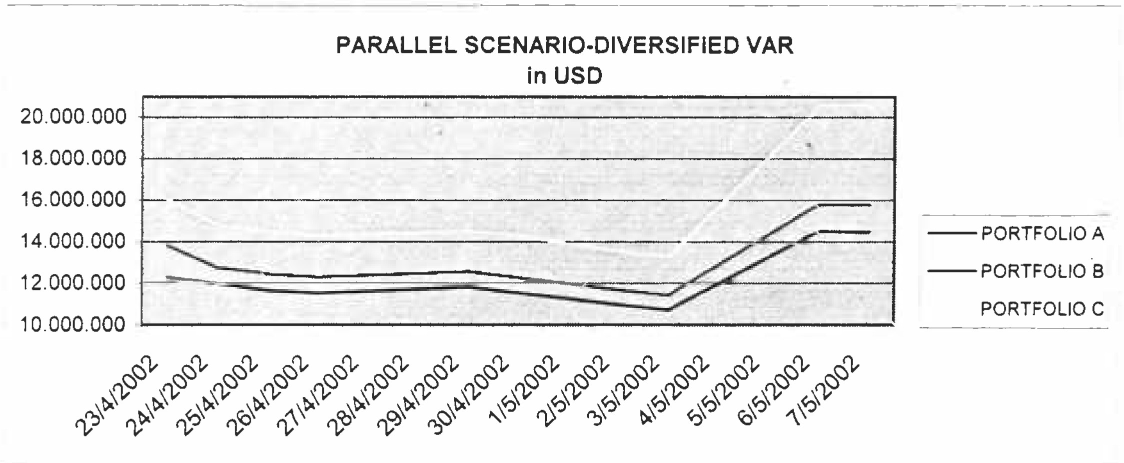
The higher volatility of the short end (3months to 5 years) of the US yield curve appeared when the market begun to discount that the cycle of rate cuts by Federal Reserve was at the end and the phase of rate increase is ahead. In **Appendix C,** the graph C.1 and the table C.1 show the movement of the US yield curve from 15/1/01 to 15/4/02. The flatter slope of the yield curve is obvious.

https://doi.org/10.26219/heal.aueb.1203

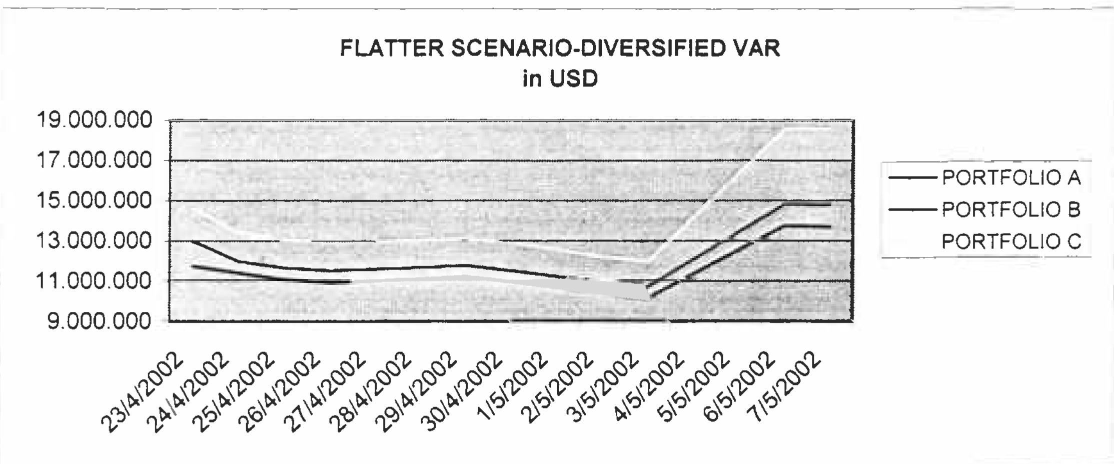
GRAPH 5.5



GRAPH 5.6



GRAPH 5.7



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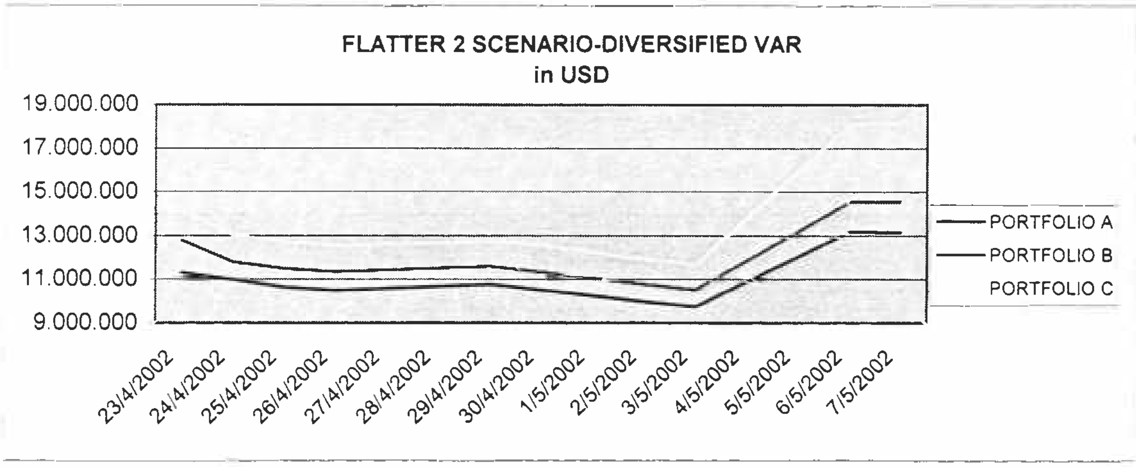
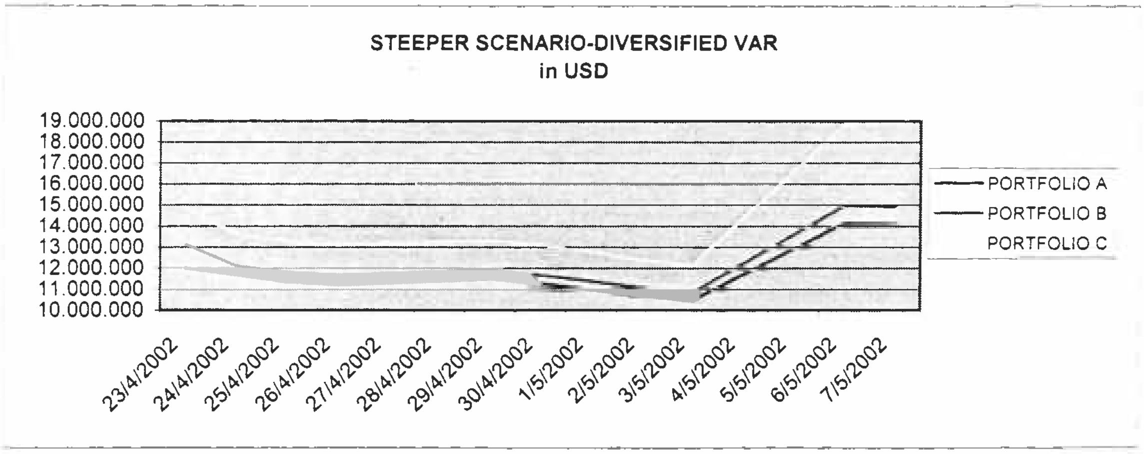
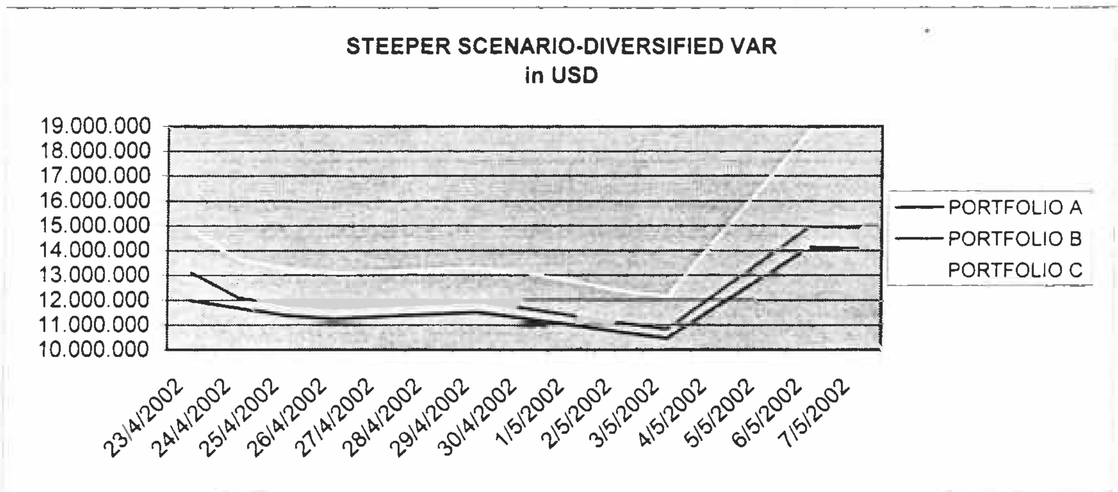
GRAPH 5.8



GRAPH 5.9



GRAPH 5.10



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The conclusion from the above analysis is that duration hedging provides only a first approximation to interest rate risk management. If the goal is to minimize tracking error relative to an index, it is essential to use a fine decomposition of the index by maturity. The lowest tracking error is attained for a portfolio with positions that are closest to those of the index.

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Appendices

Appendix A:

1. Analytical presentation per day of VaR figures: Table A.1 –Table A.10

Appendix B:

1. Volatilities, correlations and covariances by RiskMetrics from 23/4/02 to 7/5/02: Table B.1 – Table B.10
2. Volatilities figures per maturity from 23/4/02 to 7/5/02: Table B.11

Appendix C:

1. US yield curve changes: Graph C.1
2. US yield curve changes: Table C.1

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Appendix A

- Analytical presentation per day of VaR figures: Table A.1 –Table A.10

https://doi.org/10.26219/heal.aueb.1203

TABLE A.1

23-04-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,051,036,821	1,051,724,042	1,051,684,242	11,350,399	12,215,538	13,074,426	1.08	1.16	1.24
PARALLEL	950,732,938	951,892,700	951,170,163	12,281,985	13,782,358	16,028,862	1.29	1.45	1.69
FLATTER	982,916,080	984,335,698	987,500,503	11,686,015	12,977,017	14,530,475	1.19	1.32	1.47
FLATTER 2	1,010,261,273	1,003,959,301	997,155,770	11,290,549	12,793,075	14,155,394	1.12	1.27	1.42
STEEPER	943,021,208	954,515,253	980,244,048	11,980,168	13,107,461	14,785,481	1.27	1.37	1.51
STEEPER 2	959,840,990	971,449,839	1,002,136,892	11,762,187	12,802,339	13,866,748	1.23	1.32	1.38

23-04-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	11,590,338	12,510,663	13,143,142	239,939	295,125	68,716	2.07	2.36	0.52
PARALLEL	12,622,012	14,179,966	16,127,244	340,027	397,608	98,382	2.69	2.80	0.61
FLATTER	12,012,661	13,356,058	14,623,182	326,646	379,041	92,707	2.72	2.84	0.63
FLATTER 2	11,653,432	13,204,225	14,256,945	362,883	411,150	101,551	3.11	3.11	0.71
STEEPER	12,233,726	13,422,476	14,860,919	253,558	315,015	75,438	2.07	2.35	0.51
STEEPER 2	12,030,157	13,128,183	13,943,720	267,970	325,844	76,972	2.23	2.48	0.55

	VOLATILITY
1M	4.141046
3M	3.629223
6M	5.257351
1Y	7.361232
2Y	7.985191
3Y	6.8426
4Y	6.041368
5Y	5.471549
7Y	4.92233
10Y	4.288931

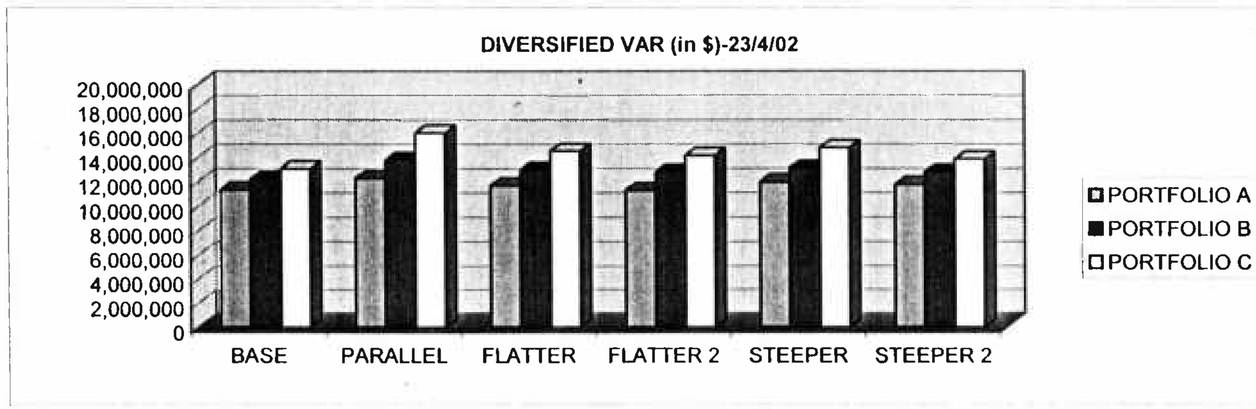

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TABLE A.2

24-04-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,052,404,402	1,053,784,178	1,053,598,388	10,995,572	11,243,899	12,022,475	1.04	1.07	1.14
PARALLEL	952,899,624	953,872,414	952,536,374	11,961,662	12,722,980	14,759,381	1.26	1.33	1.55
FLATTER	985,261,484	986,453,164	988,910,545	11,369,914	11,969,326	13,369,923	1.15	1.21	1.35
FLATTER 2	1,012,803,054	1,006,215,234	998,432,331	10,972,198	11,790,838	13,027,912	1.08	1.17	1.30
STEEPER	945,069,470	956,426,222	981,690,421	11,673,159	12,100,969	13,604,498	1.24	1.27	1.39
STEEPER 2	962,007,560	973,457,350	1,003,592,433	11,455,156	11,814,293	12,753,265	1.19	1.21	1.27

24-04-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	11,228,766	11,517,766	12,085,854	233,194	273,867	63,379	2.08	2.38	0.52
PARALLEL	12,293,165	13,090,651	14,850,220	331,503	367,671	90,839	2.70	2.81	0.61
FLATTER	11,688,253	12,319,665	13,455,504	318,339	350,339	85,581	2.72	2.84	0.64
FLATTER 2	11,325,861	12,170,847	13,121,717	353,663	380,009	93,805	3.12	3.12	0.71
STEEPER	11,920,093	12,391,924	13,674,026	246,934	290,955	69,528	2.07	2.35	0.51
STEEPER 2	11,716,173	12,115,292	12,824,228	261,017	300,999	70,963	2.23	2.48	0.55

	VOLATILITY
1M	4.033895
3M	3.596091
6M	5.164156
1Y	7.222432
2Y	7.787514
3Y	6.665274
4Y	5.882719
5Y	5.32635
7Y	4.792832
10Y	4.198276

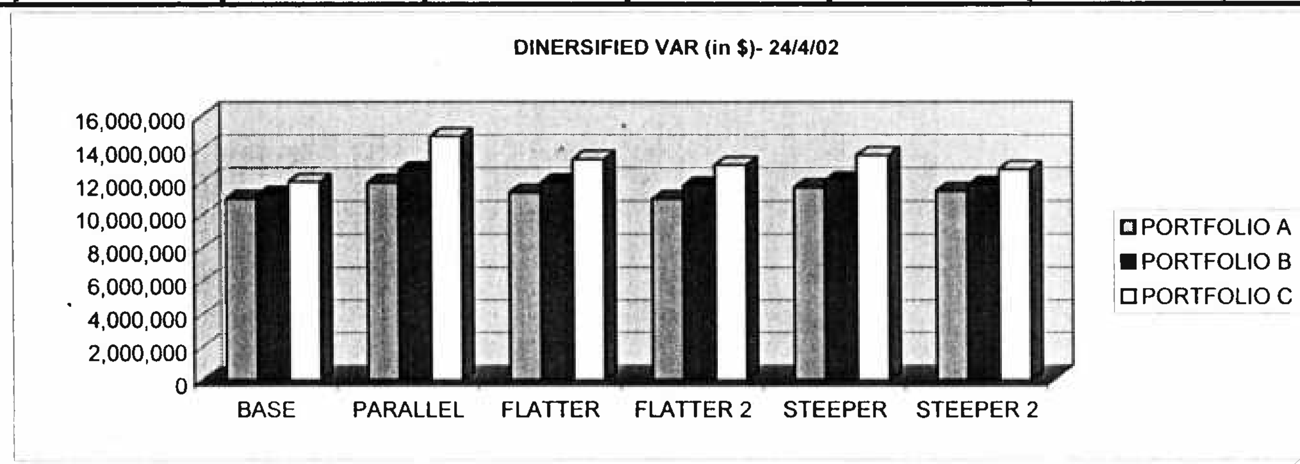

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TABLE A.3

25-04-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,057,476,083	1,059,168,340	1,059,761,785	10,593,015	10,861,444	11,623,021	1.00	1.03	1.10
PARALLEL	958,324,542	959,580,211	959,159,196	11,644,107	12,436,641	14,432,286	1.22	1.30	1.50
FLATTER	990,963,733	992,411,314	995,773,392	11,039,731	11,668,166	13,024,893	1.11	1.18	1.31
FLATTER 2	1,018,844,258	1,012,406,479	1,004,644,530	10,632,560	11,489,670	12,707,998	1.04	1.13	1.26
STEEPER	950,313,413	962,089,157	988,798,659	11,360,423	11,802,690	13,250,338	1.20	1.23	1.34
STEEPER 2	967,494,080	979,317,676	1,010,749,512	11,134,079	11,510,071	12,392,178	1.15	1.18	1.23

25-04-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	10,837,692	11,149,787	11,684,028	244,677	288,343	61,007	2.26	2.59	0.52
PARALLEL	11,990,556	12,823,731	14,520,740	346,449	387,090	88,454	2.89	3.02	0.61
FLATTER	11,371,620	12,036,212	13,108,100	331,889	368,046	83,207	2.92	3.06	0.63
FLATTER 2	10,999,309	11,887,169	12,799,460	366,749	397,499	91,462	3.33	3.34	0.71
STEEPER	11,620,876	12,110,704	13,317,500	260,453	308,014	67,162	2.24	2.54	0.50
STEEPER 2	11,408,215	11,827,857	12,460,785	274,136	317,786	68,607	2.40	2.69	0.55

	25-04-02
1M	3.829693
3M	3.499092
6M	5.000104
1Y	7.096372
2Y	7.958185
3Y	6.732194
4Y	5.858719
5Y	5.274181
7Y	4.718725
10Y	4.096311

DIVERSIFIED VAR (in $)- 25/4/02

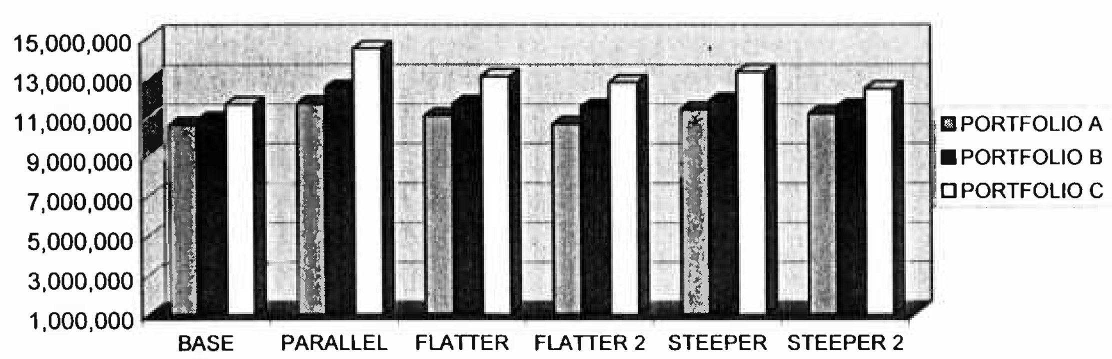

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TABLE A.4

26-04-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,055,821,345	1,057,346,593	1,057,449,882	10,344,296	10,614,657	11,359,522	0.98	1.00	1.07
PARALLEL	960,604,173	962,096,242	962,383,209	11,504,180	12,292,437	14,267,868	1.20	1.28	1.48
FLATTER	993,521,418	995,162,530	999,121,695	10,892,362	11,516,446	12,851,828	1.10	1.16	1.29
FLATTER 2	1,021,600,554	1,015,308,603	1,008,073,111	10,478,669	11,333,935	12,530,731	1.03	1.12	1.24
STEEPER	952,587,165	964,632,708	992,141,993	11,223,138	11,657,311	13,078,416	1.18	1.21	1.32
STEEPER 2	969,875,533	981,956,082	1,014,165,428	10,992,666	11,361,076	12,215,046	1.13	1.16	1.20

26-04-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	10,583,689	10,896,922	11,419,378	239,393	282,265	59,856	2.26	2.59	0.52
PARALLEL	11,845,620	12,674,492	14,355,352	341,440	382,055	87,484	2.88	3.01	0.61
FLATTER	11,219,191	11,879,350	12,934,049	326,829	362,904	82,221	2.91	3.05	0.64
FLATTER 2	10,839,816	11,725,937	12,621,144	361,147	392,002	90,413	3.33	3.34	0.72
STEEPER	11,479,506	11,960,734	13,144,691	256,368	303,423	66,275	2.23	2.54	0.50
STEEPER 2	11,262,474	11,674,132	12,282,732	269,808	313,056	67,686	2.40	2.68	0.55

	26-04-02
1M	3.737287
3M	3.452828
6M	5.067318
1Y	7.039269
2Y	7.943015
3Y	6.720597
4Y	5.844798
5Y	5.251709
7Y	4.687845
10Y	4.049983

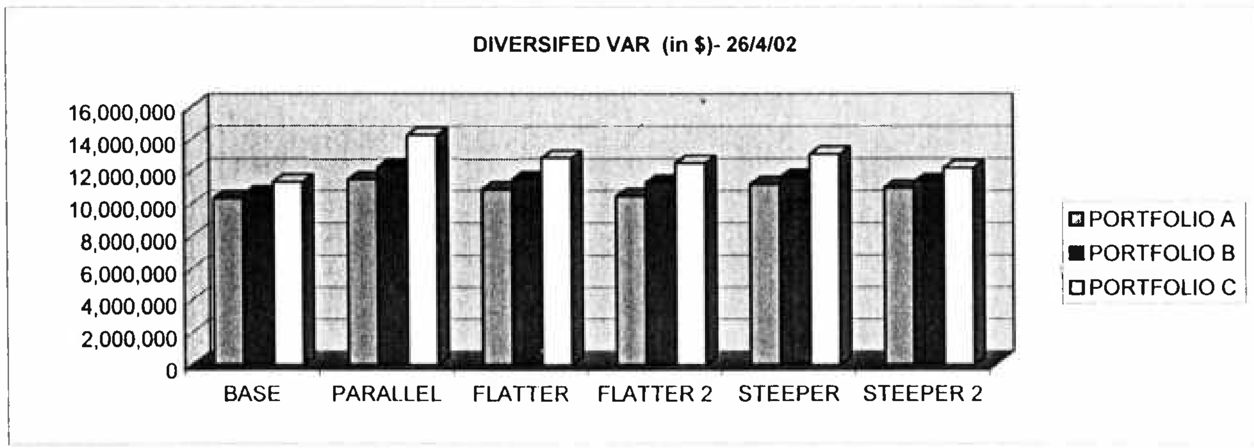

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TABLE A.5

29-04-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,056,381,806	1,058,031,915	1,058,193,026	10,638,929	10,888,889	11,584,544	1.01	1.03	1.09
PARALLEL	960,057,862	961,463,715	961,426,003	11,797,917	12,559,747	14,489,235	1.23	1.31	1.51
FLATTER	992,894,439	994,446,827	998,101,048	11,176,962	11,775,742	13,060,803	1.13	1.18	1.31
FLATTER 2	1,020,936,761	1,014,544,526	1,006,824,494	10,763,730	11,591,855	12,746,280	1.05	1.14	1.27
STEEPER	951,972,671	963,919,656	991,089,394	11,498,204	11,911,972	13,287,861	1.21	1.24	1.34
STEEPER 2	969,252,315	981,231,224	1,013,068,525	11,268,145	11,614,867	12,417,925	1.16	1.18	1.23

29-04-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	10,807,434	11,098,141	11,625,513	168,505	209,252	40,969	1.56	1.89	0.35
PARALLEL	12,033,757	12,838,243	14,547,792	235,840	278,496	58,557	1.96	2.17	0.40
FLATTER	11,402,293	12,039,479	13,115,493	225,331	263,737	54,690	1.98	2.19	0.42
FLATTER 2	11,010,702	11,874,813	12,805,738	246,972	282,958	59,458	2.24	2.38	0.46
STEEPER	11,678,929	12,137,093	13,333,386	180,725	225,121	45,525	1.55	1.85	0.34
STEEPER 2	11,456,916	11,845,505	12,463,835	188,771	230,638	45,910	1.65	1.95	0.37

	VOLATILITY
1M	3.641716
3M	3.363985
6M	5.013827
1Y	6.90614
2Y	7.875483
3Y	6.672928
4Y	5.853187
5Y	5.363252
7Y	4.78805
10Y	4.129549

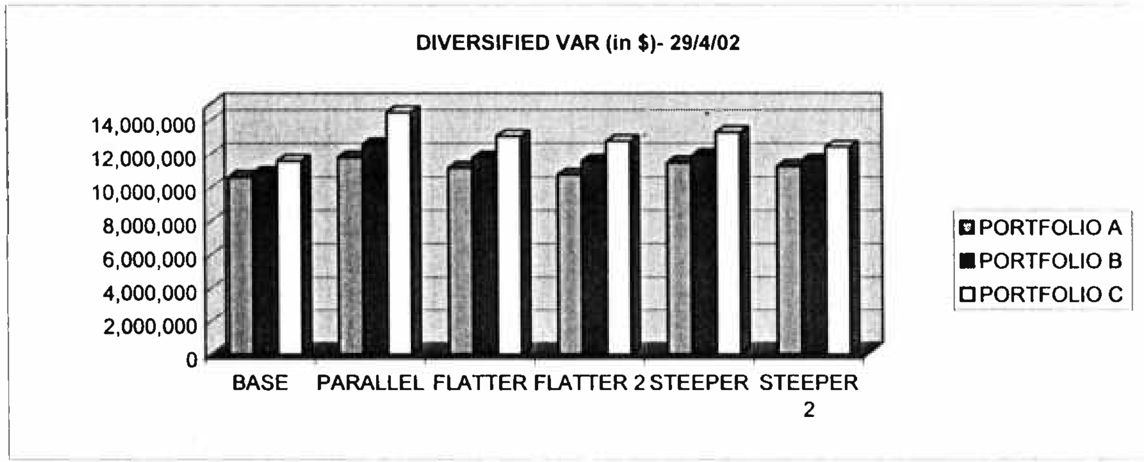

TABLE A.6

30-04-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,056,587,388	1,058,340,287	1,058,303,732	10,451,816	10,691,359	11,459,843	0.99	1.01	1.08
PARALLEL	959,187,787	960,541,244	960,096,204	11,516,489	12,260,742	14,266,178	1.20	1.28	1.49
FLATTER	991,924,381	993,425,302	996,672,870	10,915,436	11,500,784	12,870,396	1.10	1.16	1.29
FLATTER 2	1,019,871,634	1,013,440,147	1,005,221,274	10,500,952	11,312,759	12,564,985	1.03	1.12	1.25
STEEPER	951,142,535	963,019,207	989,732,739	11,250,092	11,650,701	13,097,199	1.18	1.21	1.32
STEEPER 2	968,387,554	980,291,712	1,011,632,942	11,021,287	11,357,662	12,245,876	1.14	1.16	1.21

30-04-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	10,691,800	10,984,749	11,540,934	239,984	293,390	81,091	2.24	2.67	0.70
PARALLEL	11,854,684	12,653,259	14,379,284	338,195	392,517	113,106	2.85	3.10	0.79
FLATTER	11,238,495	11,872,738	12,975,581	323,059	371,954	105,185	2.87	3.13	0.81
FLATTER 2	10,856,785	11,713,416	12,677,732	355,833	400,657	112,747	3.28	3.42	0.89
STEEPER	11,506,800	11,965,736	13,187,545	256,708	315,035	90,346	2.23	2.63	0.69
STEEPER 2	11,290,121	11,680,915	12,335,907	268,834	323,253	90,031	2.38	2.77	0.73

VOLATILITY

1M	3.545186
3M	3.279615
6M	4.893233
1Y	6.725966
2Y	7.730217
3Y	6.561086
4Y	5.763535
5Y	5.285015
7Y	4.716263
10Y	4.062685

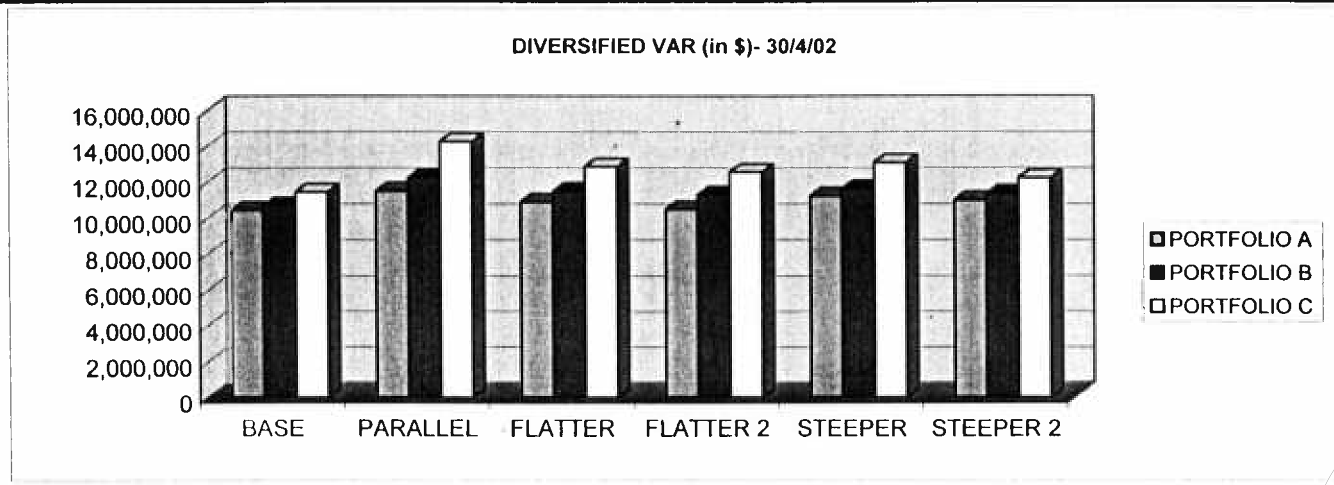

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TABLE A.7

02-05-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,058,701,786	1,057,936,959	1,060,579,444	9,888,636	10,108,919	10,784,224	0.93	0.96	1.02
PARALLEL	962,139,355	961,227,975	963,442,715	10,966,393	11,666,971	13,497,922	1.14	1.21	1.40
FLATTER	995,016,265	994,195,087	1,000,027,387	10,381,243	10,931,432	12,158,470	1.04	1.10	1.22
FLATTER 2	1,023,060,958	1,014,260,934	1,007,690,752	9,980,433	10,752,024	11,900,452	0.98	1.06	1.18
STEEPER	954,128,579	963,700,023	993,368,515	10,707,489	11,074,718	12,362,995	1.12	1.15	1.24
STEEPER 2	971,496,209	981,071,004	1,015,164,136	10,484,439	10,791,673	11,550,690	1.08	1.10	1.14

02-05-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	10,092,229	10,354,037	10,857,868	203,593	245,118	73,644	2.02	2.37	0.68
PARALLEL	11,258,930	11,999,166	13,601,476	292,537	332,195	103,554	2.60	2.77	0.76
FLATTER	10,660,831	11,246,012	12,254,771	279,588	314,580	96,301	2.62	2.80	0.79
FLATTER 2	10,290,502	11,092,219	12,003,976	310,069	340,195	103,524	3.01	3.07	0.86
STEEPER	10,925,677	11,338,798	12,445,192	218,188	264,080	82,197	2.00	2.33	0.66
STEEPER 2	10,714,116	11,063,163	11,632,722	229,677	271,490	82,032	2.14	2.45	0.71

	VOLATILITY
1M	3.222797
3M	3.073713
6M	5.181052
1Y	6.773521
2Y	7.383061
3Y	6.244253
4Y	5.491606
5Y	5.033159
7Y	4.495514
10Y	3.870258

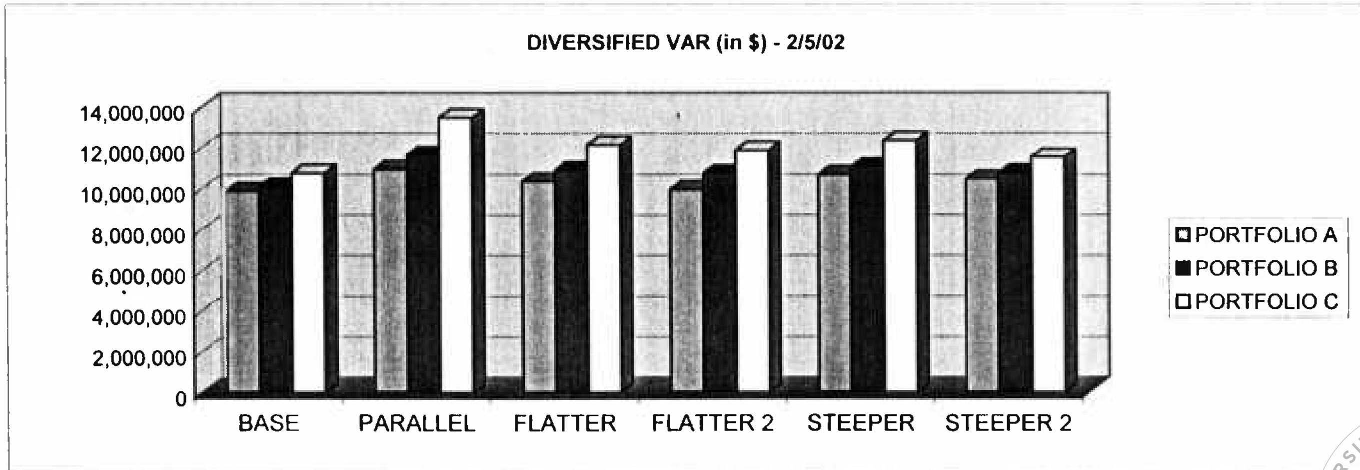

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TABLE A.8

03-05-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,059,298,153	1,057,204,239	1,062,019,432	9,673,867	9,882,620	10,608,301	0.91	0.93	1.00
PARALLEL	961,937,009	961,163,615	963,434,514	10,705,843	11,403,620	13,252,469	1.11	1.19	1.38
FLATTER	994,762,383	994,087,642	999,989,751	10,136,052	10,685,542	11,938,948	1.02	1.07	1.19
FLATTER 2	1,022,755,520	1,014,110,628	1,007,497,905	9,748,300	10,515,417	11,692,324	0.95	1.04	1.16
STEEPER	953,953,864	963,655,750	993,385,938	10,450,060	10,819,009	12,135,985	1.10	1.12	1.22
STEEPER 2	971,314,857	981,020,638	1,015,145,156	10,232,891	10,543,390	11,340,486	1.05	1.07	1.12

03-05-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	9,890,964	10,143,375	10,684,703	217,097	260,755	76,402	2.19	2.57	0.72
PARALLEL	11,018,125	11,758,110	13,360,089	312,282	354,490	107,620	2.83	3.01	0.81
FLATTER	10,434,656	11,021,497	11,340,486	298,604	335,955	-598,462	2.86	3.05	-5.28
FLATTER 2	10,079,868	10,879,056	11,800,181	331,568	363,639	107,857	3.29	3.34	0.91
STEEPER	10,682,205	11,099,899	12,221,111	232,145	280,890	85,126	2.17	2.53	0.70
STEEPER 2	10,477,669	10,832,705	11,425,568	244,778	289,315	85,082	2.34	2.67	0.74

VOLATILITY	
1M	3.139242
3M	3.041238
6M	5.393185
1Y	7.072039
2Y	7.236235
3Y	6.13302
4Y	5.387237
5Y	4.942871
7Y	4.397371
10Y	3.769449

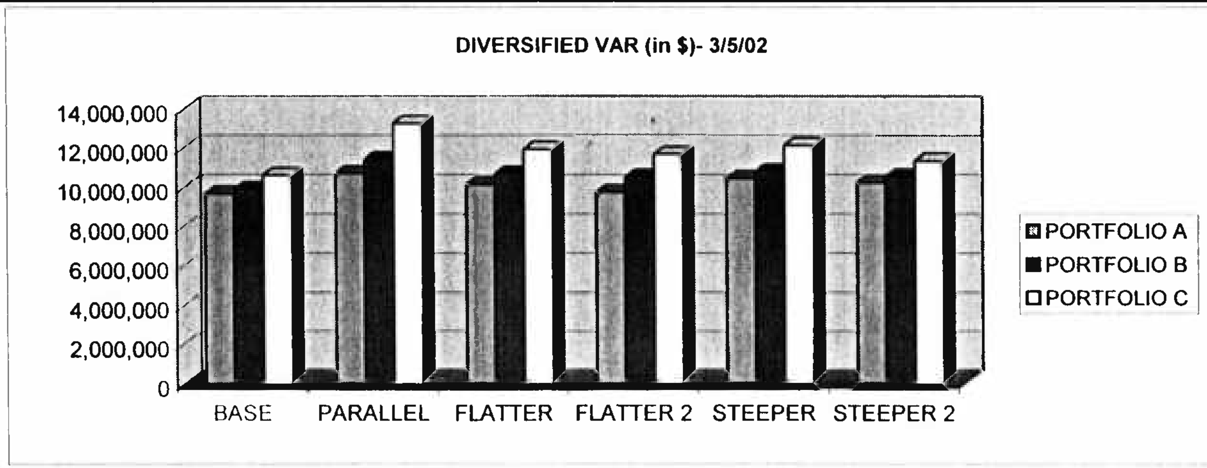

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TABLE A.9

06-05-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,056,436,670	1,055,496,726	1,059,722,658	13,003,126	13,610,308	16,434,056	1.23	1.29	1.55
PARALLEL	961,461,525	960,763,028	963,464,410	14,493,661	15,782,278	20,551,120	1.51	1.64	2.13
FLATTER	994,216,158	993,618,759	999,993,096	13,705,163	14,768,921	18,510,060	1.38	1.49	1.85
FLATTER 2	1,022,138,721	1,013,574,871	1,007,348,128	13,181,571	14,537,464	18,107,734	1.29	1.43	1.80
STEEPER	953,519,666	963,290,573	993,442,857	14,127,041	14,960,410	18,849,646	1.48	1.55	1.90
STEEPER 2	970,849,586	980,618,235	1,015,167,514	13,812,380	14,554,789	17,603,154	1.42	1.48	1.73

06-05-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	13,263,108	13,927,255	16,539,203	259,982	316,947	105,147	1.96	2.28	0.64
PARALLEL	14,861,613	16,209,562	20,697,415	367,952	427,284	146,295	2.48	2.64	0.71
FLATTER	14,056,183	15,173,240	18,645,722	351,020	404,319	135,662	2.50	2.66	0.73
FLATTER 2	13,568,158	14,973,385	18,252,553	386,587	435,921	144,819	2.85	2.91	0.79
STEEPER	14,406,266	15,302,605	18,967,376	279,225	342,195	117,730	1.94	2.24	0.62
STEEPER 2	14,104,557	14,905,669	17,719,890	292,177	350,880	116,736	2.07	2.35	0.66

VOLATILITY	
1M	3.057467
3M	3.060269
6M	6.329043
1Y	9.080704
2Y	9.272185
3Y	8.566567
4Y	8.162849
5Y	7.98172
7Y	6.482339
10Y	4.64304

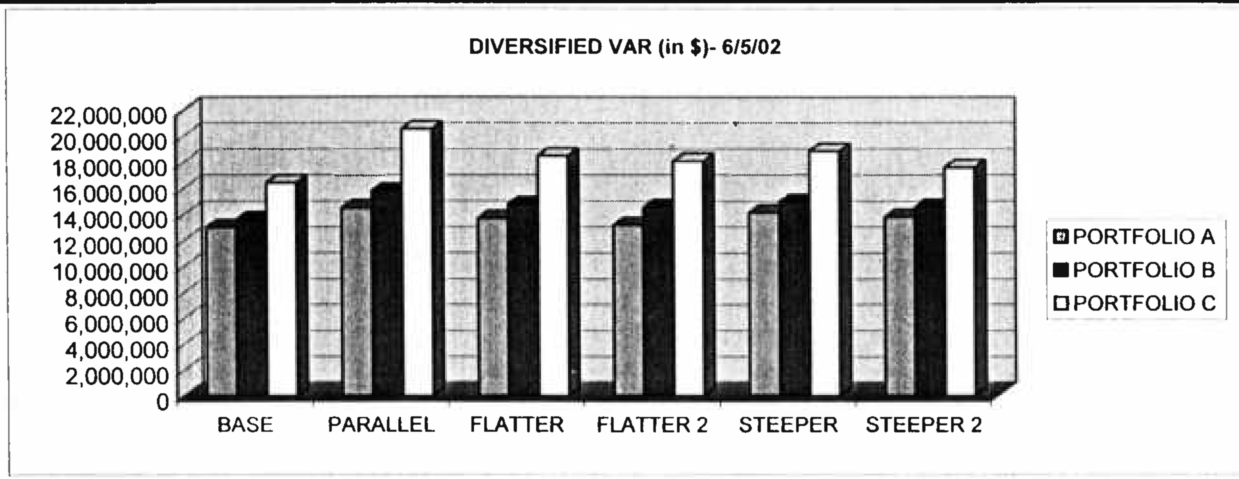

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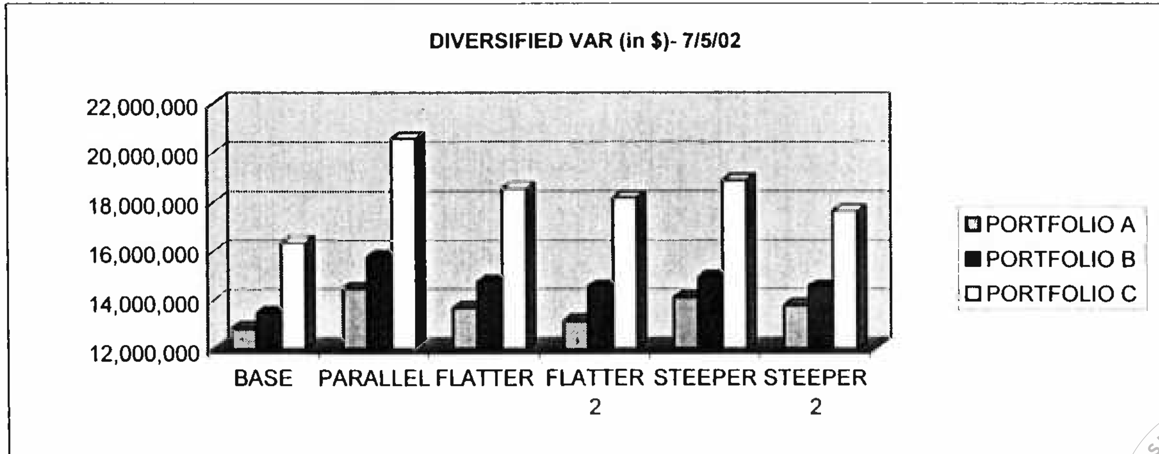
TABLE A.10

07-05-02	MARKET VALUE			DIVERSIFIED VAR			VAR as % of mkt value		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	1,051,857,493	1,050,673,907	1,054,859,820	12,834,667	13,467,902	16,346,871	1.22	1.28	1.55
PARALLEL	961,790,698	961,123,363	963,941,567	14,449,224	15,764,239	20,581,056	1.50	1.64	2.14
FLATTER	994,418,059	993,859,518	1,000,416,927	13,663,733	14,752,539	18,535,955	1.37	1.48	1.85
FLATTER 2	1,022,266,532	1,013,723,053	1,007,192,012	13,147,319	14,530,933	18,166,821	1.29	1.43	1.80
STEEPER	953,863,523	963,697,357	994,134,246	14,078,136	14,932,801	18,860,721	1.48	1.55	1.90
STEEPER 2	971,201,429	981,010,594	1,015,745,109	13,764,174	14,528,561	17,616,422	1.42	1.48	1.73

07-05-02	UNDIVERSIFIED VAR			DIVERSIFICATION BENEFIT			diversi. benefit as %		
	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C	PORTFOLIO A	PORTFOLIO B	PORTFOLIO C
BASE	13,094,438	13,784,680	16,452,144	259,771	316,778	105,273	1.98	2.30	0.64
PARALLEL	14,817,570	16,192,751	20,727,973	368,346	428,512	146,917	2.49	2.65	0.71
FLATTER	14,015,280	15,158,191	18,672,245	351,547	405,652	136,290	2.51	2.68	0.73
FLATTER 2	13,534,174	14,968,066	18,312,368	386,855	437,133	145,547	2.86	2.92	0.79
STEEPER	14,358,282	15,276,380	18,979,047	280,146	343,579	118,326	1.95	2.25	0.62
STEEPER 2	14,057,201	14,880,836	17,733,761	293,027	352,275	117,339	2.08	2.37	0.66

	VOLATILITY
1M	2.97778
3M	3.04838
6M	6.358654
1Y	9.057927
2Y	9.365701
3Y	8.63208
4Y	8.198054
5Y	7.985547
7Y	6.472772
10Y	4.606256



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Appendix B

- Volatilities, correlations and covariances by RiskMetrics from 23/4/02 to 7/5/02: Table B.1 – Table B.10

- Volatilities figures per maturity from 23/4/02 to 7/5/02: Table B.11

https://doi.org/10.26219/heal.aueb.1203

TABLE B.1 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

23-04-02		VOLATILITY	USD ZERO BOND 1M	USD ZERO BOND 3M	USD ZERO BOND 6M	USD ZERO BOND 1Y	USD ZERO BOND 2Y	USD ZERO BOND 3Y	USD ZERO BOND 4Y	USD ZERO BOND 5Y	USD ZERO BOND 7Y	USD ZERO BOND 10Y
VOLATILITY			4.141046	3.629223	5.257351	7.361232	7.985191	6.8426	6.041368	5.471549	4.92233	4.288931
USD ZERO BOND	1M	4.141046	1	0.589231	0.271392	0.37969	0.098678	0.092826	0.083385	0.071432	0.042198	-0.011961
USD ZERO BOND	3M	3.629223	0.589231	1	0.752254	0.762625	0.402124	0.379211	0.35565	0.324291	0.274034	0.175533
USD ZERO BOND	6M	5.257351	0.271392	0.752254	1	0.910318	0.768748	0.761716	0.744803	0.713402	0.66576	0.561843
USD ZERO BOND	1Y	7.361232	0.37969	0.762625	0.910318	1	0.809114	0.799229	0.779452	0.744537	0.70179	0.606287
USD ZERO BOND	2Y	7.985191	0.098678	0.402124	0.768748	0.809114	1	0.993575	0.972386	0.932686	0.916219	0.861244
USD ZERO BOND	3Y	6.8426	0.092826	0.379211	0.761716	0.799229	0.993575	1	0.992176	0.966501	0.953317	0.904203
USD ZERO BOND	4Y	6.041368	0.083385	0.35565	0.744803	0.779452	0.972386	0.992176	1	0.990969	0.98188	0.940465
USD ZERO BOND	5Y	5.471549	0.071432	0.324291	0.713402	0.744537	0.932686	0.966501	0.990969	1	0.99549	0.963086
USD ZERO BOND	7Y	4.92233	0.042198	0.274034	0.66576	0.70179	0.916219	0.953317	0.98188	0.99549	1	0.984071
USD ZERO BOND	10Y	4.288931	-0.011961	0.175533	0.561843	0.606287	0.861244	0.904203	0.940465	0.963086	0.984071	1

COVARIANCES 23-04-02		VOLATILITY	USD ZERO BOND 1M	USD ZERO BOND 3M	USD ZERO BOND 6M	USD ZERO BOND 1Y	USD ZERO BOND 2Y	USD ZERO BOND 3Y	USD ZERO BOND 4Y	USD ZERO BOND 5Y	USD ZERO BOND 7Y	USD ZERO BOND 10Y
VOLATILITY			4.141046	3.629223	5.257351	7.361232	7.985191	6.8426	6.041368	5.471549	4.92233	4.288931
USD ZERO BOND	1M	4.141046	17.148261	8.855427	5.908446	11.57417	3.262985	2.630261	2.086087	1.618492	0.860155	-0.212434
USD ZERO BOND	3M	3.629223	8.855427	13.171262	14.353077	20.37394	11.65358	9.417074	7.797805	6.439598	4.895411	2.732263
USD ZERO BOND	6M	5.257351	5.908446	14.353077	27.639741	35.22982	32.27275	27.40195	23.65613	20.52163	17.2288	12.66867
USD ZERO BOND	1Y	7.361232	11.574172	20.37394	35.229819	54.18774	47.56041	40.25712	34.66371	29.98799	25.42895	19.14159
USD ZERO BOND	2Y	7.985191	3.262985	11.653579	32.272753	47.56041	63.76328	54.28841	46.90932	40.75033	36.01269	29.49585
USD ZERO BOND	3Y	6.8426	2.630261	9.417074	27.40195	40.25712	54.28841	46.82118	41.01522	36.18542	32.10919	26.53604
USD ZERO BOND	4Y	6.041368	2.086087	7.797805	23.656132	34.66371	46.90932	41.01522	36.49813	32.75714	29.19876	24.36839
USD ZERO BOND	5Y	5.471549	1.618492	6.439598	20.521627	29.98799	40.75033	36.18542	32.75714	29.93785	26.81132	22.60084
USD ZERO BOND	7Y	4.92233	0.860155	4.895411	17.228802	25.42895	36.01269	32.10919	29.19876	26.81132	24.22933	20.77526
USD ZERO BOND	10Y	4.288931	-0.212434	2.732263	12.66867	19.14159	29.49585	26.53604	24.36839	22.60084	20.77526	18.39493

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TABLE B.2 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

24-04-02		VOLATILITY	USD ZERO BOND 1M	USD ZERO BOND 3M	USD ZERO BOND 6M	USD ZERO BOND 1Y	USD ZERO BOND 2Y	USD ZERO BOND 3Y	USD ZERO BOND 4Y	USD ZERO BOND 5Y	USD ZERO BOND 7Y	USD ZERO BOND 10Y
VOLATILITY			4.033895	3.596091	5.164156	7.222432	7.787514	6.665274	5.882719	5.32635	4.792832	4.198276
USD ZERO BOND	1M	4.033895	1	0.589231	0.271392	0.37969	0.098678	0.092826	0.083385	0.071432	0.042198	-0.011961
USD ZERO BOND	3M	3.596091	0.589231	1	0.752254	0.762625	0.402124	0.379211	0.35565	0.324291	0.274034	0.175533
USD ZERO BOND	6M	5.164156	0.271392	0.752254	1	0.910318	0.768748	0.761716	0.744803	0.713402	0.66576	0.561843
USD ZERO BOND	1Y	7.222432	0.37969	0.762625	0.910318	1	0.809114	0.799229	0.779452	0.744537	0.70179	0.606287
USD ZERO BOND	2Y	7.787514	0.098678	0.402124	0.768748	0.809114	1	0.993575	0.972386	0.932686	0.916219	0.861244
USD ZERO BOND	3Y	6.665274	0.092826	0.379211	0.761716	0.799229	0.993575	1	0.992176	0.966501	0.953317	0.904203
USD ZERO BOND	4Y	5.882719	0.083385	0.35565	0.744803	0.779452	0.972386	0.992176	1	0.990969	0.98188	0.940465
USD ZERO BOND	5Y	5.32635	0.071432	0.324291	0.713402	0.744537	0.932686	0.966501	0.990969	1	0.99549	0.963086
USD ZERO BOND	7Y	4.792832	0.042198	0.274034	0.66576	0.70179	0.916219	0.953317	0.98188	0.99549	1	0.984071
USD ZERO BOND	10Y	4.198276	-0.011961	0.175533	0.561843	0.606287	0.861244	0.904203	0.940465	0.963086	0.984071	1

COVARIANCES 24-04-02		VOLATILITY	USD ZERO BOND 1M	USD ZERO BOND 3M	USD ZERO BOND 6M	USD ZERO BOND 1Y	USD ZERO BOND 2Y	USD ZERO BOND 3Y	USD ZERO BOND 4Y	USD ZERO BOND 5Y	USD ZERO BOND 7Y	USD ZERO BOND 10Y
VOLATILITY			4.033895	3.596091	5.164156	7.222432	7.787514	6.665274	5.882719	5.32635	4.792832	4.198276
USD ZERO BOND	1M	4.033895	16.272313	8.547537	5.653537	11.0621	3.099868	2.495802	1.978744	1.534775	0.815854	-0.202563
USD ZERO BOND	3M	3.596091	8.547537	12.931868	13.969932	19.80728	11.26133	9.089286	7.52371	6.211478	4.723104	2.650094
USD ZERO BOND	6M	5.164156	5.653537	13.969932	26.668506	33.95281	30.9159	26.21867	22.62658	19.62291	16.47817	12.18107
USD ZERO BOND	1Y	7.222432	11.062097	19.807282	33.952809	52.16352	45.50845	38.47446	33.11698	28.64175	24.29309	18.38369
USD ZERO BOND	2Y	7.787514	3.099868	11.261332	30.915899	45.50845	60.64537	51.57242	44.54669	38.6869	34.1972	28.15764
USD ZERO BOND	3Y	6.665274	2.495802	9.089286	26.218671	38.47446	51.57242	44.42588	38.90314	34.3123	30.45424	25.302
USD ZERO BOND	4Y	5.882719	1.978744	7.52371	22.626577	33.11698	44.54669	38.90314	34.60638	31.05046	27.68399	23.22692
USD ZERO BOND	5Y	5.32635	1.534775	6.211478	19.622914	28.64175	38.6869	34.3123	31.05046	28.37001	25.41318	21.53604
USD ZERO BOND	7Y	4.792832	0.815854	4.723104	16.478168	24.29309	34.1972	30.45424	27.68399	25.41318	22.97124	19.80112
USD ZERO BOND	10Y	4.198276	-0.202563	2.650094	12.181067	18.38369	28.15764	25.302	23.22692	21.53604	19.80112	17.62552

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TABLE B.3 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

25-04-02			USD ZERO BON	USD ZERO BOND	USD ZERO BON	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BON
		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.829693	3.499092	5.000104	7.096372	7.958185	6.732194	5.858719	5.274181	4.718725	4.096311
USD ZERO BOND	1M	3.829693	1	0.56281	0.266879	0.371365	0.086717	0.081479	0.072039	0.060227	0.034164	-0.01483
USD ZERO BOND	3M	3.499092	0.56281	1	0.766409	0.770773	0.417746	0.390544	0.366263	0.334033	0.275729	0.166715
USD ZERO BOND	6M	5.000104	0.266879	0.766409	1	0.911649	0.768357	0.75811	0.740373	0.708355	0.654938	0.542035
USD ZERO BOND	1Y	7.096372	0.371365	0.770773	0.911649	1	0.806852	0.795075	0.774821	0.739666	0.690355	0.583393
USD ZERO BOND	2Y	7.958185	0.086717	0.417746	0.768357	0.806852	1	0.993397	0.972443	0.933251	0.91351	0.850185
USD ZERO BOND	3Y	6.732194	0.081479	0.390544	0.75811	0.795075	0.993397	1	0.992295	0.967005	0.951564	0.89503
USD ZERO BOND	4Y	5.858719	0.072039	0.366263	0.740373	0.774821	0.972443	0.992295	1	0.991104	0.980078	0.931299
USD ZERO BOND	5Y	5.274181	0.060227	0.334033	0.708355	0.739666	0.933251	0.967005	0.991104	1	0.993904	0.954293
USD ZERO BOND	7Y	4.718725	0.034164	0.275729	0.654938	0.690355	0.91351	0.951564	0.980078	0.993904	1	0.981056
USD ZERO BOND	10Y	4.096311	-0.01483	0.166715	0.542035	0.583393	0.850185	0.89503	0.931299	0.954293	0.981056	1

COVARIANCES 25-04-02			USD ZERO BON	USD ZERO BOND	USD ZERO BON	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BON
		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.829693	3.499092	5.000104	7.096372	7.958185	6.732194	5.858719	5.274181	4.718725	4.096311
USD ZERO BOND	1M	3.829693	14.666549	7.541912	5.110435	10.09256	2.642895	2.100707	1.616335	1.216493	0.617379	-0.232645
USD ZERO BOND	3M	3.499092	7.541912	12.243645	13.408957	19.13894	11.63272	9.199875	7.508462	6.164521	4.552633	2.389589
USD ZERO BOND	6M	5.000104	5.110435	13.408957	25.00104	32.34769	30.57427	25.51927	21.68862	18.68035	15.45268	11.10194
USD ZERO BOND	1Y	7.096372	10.092555	19.138944	32.347691	50.3585	45.56637	37.98404	32.21369	27.68388	23.11712	16.95861
USD ZERO BOND	2Y	7.958185	2.642895	11.632722	30.574269	45.56637	63.33271	53.22228	45.33995	39.17127	34.30457	27.71536
USD ZERO BOND	3Y	6.732194	2.100707	9.199875	25.519266	37.98404	53.22228	45.32244	39.13812	34.33526	30.2287	24.6824
USD ZERO BOND	4Y	5.858719	1.616335	7.508462	21.688623	32.21369	45.33995	39.13812	34.32459	30.62506	27.09493	22.35037
USD ZERO BOND	5Y	5.274181	1.216493	6.164521	18.68035	27.68388	39.17127	34.33526	30.62506	27.81699	24.73569	20.6172
USD ZERO BOND	7Y	4.718725	0.617379	4.552633	15.452684	23.11712	34.30457	30.2287	27.09493	24.73569	22.26636	18.96318
USD ZERO BOND	10Y	4.096311	-0.232645	2.389589	11.10194	16.95861	27.71536	24.6824	22.35037	20.6172	18.96318	16.77976

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TABLE B.4 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

26-04-02			USD ZERO BON	USD ZERO BOND	USD ZERO BON	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BON
		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.737287	3.452828	5.067318	7.039269	7.943015	6.720597	5.844798	5.251709	4.687845	4.049983
USD ZERO BOND	1M	3.737287	1	0.56281	0.266879	0.371365	0.086717	0.081479	0.072039	0.060227	0.034164	-0.01483
USD ZERO BOND	3M	3.452828	0.56281	1	0.766409	0.770773	0.417746	0.390544	0.366263	0.334033	0.275729	0.166715
USD ZERO BOND	6M	5.067318	0.266879	0.766409	1	0.911649	0.768357	0.75811	0.740373	0.708355	0.654938	0.542035
USD ZERO BOND	1Y	7.039269	0.371365	0.770773	0.911649	1	0.806852	0.795075	0.774821	0.739666	0.690355	0.583393
USD ZERO BOND	2Y	7.943015	0.086717	0.417746	0.768357	0.806852	1	0.993397	0.972443	0.933251	0.91351	0.850185
USD ZERO BOND	3Y	6.720597	0.081479	0.390544	0.75811	0.795075	0.993397	1	0.992295	0.967005	0.951564	0.89503
USD ZERO BOND	4Y	5.844798	0.072039	0.366263	0.740373	0.774821	0.972443	0.992295	1	0.991104	0.980078	0.931299
USD ZERO BOND	5Y	5.251709	0.060227	0.334033	0.708355	0.739666	0.933251	0.967005	0.991104	1	0.993904	0.954293
USD ZERO BOND	7Y	4.687845	0.034164	0.275729	0.654938	0.690355	0.91351	0.951564	0.980078	0.993904	1	0.981056
USD ZERO BOND	10Y	4.049983	-0.01483	0.166715	0.542035	0.583393	0.850185	0.89503	0.931299	0.954293	0.981056	1

COVARIANCES			USD ZERO BON	USD ZERO BOND	USD ZERO BON	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BON
26-04-02		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.737287	3.452828	5.067318	7.039269	7.943015	6.720597	5.844798	5.251709	4.687845	4.049983
USD ZERO BOND	1M	3.737287	13.967315	7.262625	5.054167	9.76978	2.574209	2.046487	1.573587	1.182083	0.598539	-0.224463
USD ZERO BOND	3M	3.452828	7.262625	11.922022	13.409536	18.73392	11.45704	9.062597	7.391583	6.057097	4.463041	2.331327
USD ZERO BOND	6M	5.067318	5.054167	13.409536	25.677713	32.51873	30.9262	25.81775	21.92795	18.8508	15.55793	11.12393
USD ZERO BOND	1Y	7.039269	9.76978	18.733923	32.518728	49.55131	45.11354	37.61347	31.87855	27.34411	22.78103	16.6319
USD ZERO BOND	2Y	7.943015	2.574209	11.457037	30.926197	45.11354	63.09148	53.02931	45.14599	38.93001	34.01511	27.34967
USD ZERO BOND	3Y	6.720597	2.046487	9.062597	25.817754	37.61347	53.02931	45.16642	38.97785	34.13005	29.97915	24.3612
USD ZERO BOND	4Y	5.844798	1.573587	7.391583	21.927947	31.87855	45.14599	38.97785	34.16167	30.42212	26.85367	22.04509
USD ZERO BOND	5Y	5.251709	1.182083	6.057097	18.850796	27.34411	38.93001	34.13005	30.42212	27.58045	24.46911	20.29717
USD ZERO BOND	7Y	4.687845	0.598539	4.463041	15.557925	22.78103	34.01511	29.97915	26.85367	24.46911	21.97589	18.62603
USD ZERO BOND	10Y	4.049983	-0.224463	2.331327	11.123932	16.6319	27.34967	24.3612	22.04509	20.29717	18.62603	16.40236

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TABLE B.5 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

29-04-02		VOLATILITY	USD ZERO BOND 1M	USD ZERO BOND 3M	USD ZERO BOND 6M	USD ZERO BOND 1Y	USD ZERO BOND 2Y	USD ZERO BOND 3Y	USD ZERO BOND 4Y	USD ZERO BOND 5Y	USD ZERO BOND 7Y	USD ZERO BOND 10Y
VOLATILITY			3.641716	3.363985	5.013827	6.90614	7.875483	6.672928	5.853187	5.363252	4.78805	4.129549
USD ZERO BOND	1M	3.641716	1	0.715394	0.583283	0.438173	0.366106	0.340905	0.310631	0.274217	0.244093	0.180368
USD ZERO BOND	3M	3.363985	0.715394	1	0.830251	0.732952	0.564261	0.552241	0.532177	0.504425	0.471399	0.396884
USD ZERO BOND	6M	5.013827	0.583283	0.830251	1	0.961724	0.856864	0.848262	0.82891	0.797568	0.765076	0.684203
USD ZERO BOND	1Y	6.90614	0.438173	0.732952	0.961724	1	0.913907	0.910851	0.896923	0.870438	0.843372	0.7722
USD ZERO BOND	2Y	7.875483	0.366106	0.564261	0.856864	0.913907	1	0.994699	0.977885	0.947485	0.930498	0.874249
USD ZERO BOND	3Y	6.672928	0.340905	0.552241	0.848262	0.910851	0.994699	1	0.994027	0.974876	0.961917	0.913596
USD ZERO BOND	4Y	5.853187	0.310631	0.532177	0.82891	0.896923	0.977885	0.994027	1	0.993341	0.984845	0.945571
USD ZERO BOND	5Y	5.363252	0.274217	0.504425	0.797568	0.870438	0.947485	0.974876	0.993341	1	0.99633	0.967098
USD ZERO BOND	7Y	4.78805	0.244093	0.471399	0.765076	0.843372	0.930498	0.961917	0.984845	0.99633	1	0.985023
USD ZERO BOND	10Y	4.129549	0.180368	0.396884	0.684203	0.7722	0.874249	0.913596	0.945571	0.967098	0.985023	1

COVARIANCES 29-04-02		VOLATILITY	USD ZERO BOND 1M	USD ZERO BOND 3M	USD ZERO BOND 6M	USD ZERO BOND 1Y	USD ZERO BOND 2Y	USD ZERO BOND 3Y	USD ZERO BOND 4Y	USD ZERO BOND 5Y	USD ZERO BOND 7Y	USD ZERO BOND 10Y
VOLATILITY			3.641716	3.363985	5.013827	6.90614	7.875483	6.672928	5.853187	5.363252	4.78805	4.129549
USD ZERO BOND	1M	3.641716	13.262093	8.764058	10.650119	11.02013	10.500028	8.284305	6.621303	5.355861	4.256178	2.712483
USD ZERO BOND	3M	3.363985	8.764058	11.316395	14.003372	17.028052	14.948968	12.396496	10.478582	9.100783	7.592787	5.513417
USD ZERO BOND	6M	5.013827	10.650119	14.003372	25.138461	33.300831	33.834411	28.380239	24.325907	21.446936	18.366752	14.166313
USD ZERO BOND	1Y	6.90614	11.02013	17.028052	33.300831	47.694772	49.706652	41.975816	36.256271	32.240481	27.887742	22.022566
USD ZERO BOND	2Y	7.875483	10.500028	14.948968	33.834411	49.706652	62.023237	52.273952	45.077257	40.020042	35.087428	28.432505
USD ZERO BOND	3Y	6.672928	8.284305	12.396496	28.380239	41.975816	52.273952	44.527972	38.824616	34.88943	30.733534	25.175209
USD ZERO BOND	4Y	5.853187	6.621303	10.478582	24.325907	36.256271	45.077257	38.824616	34.259798	31.183083	27.600629	22.855431
USD ZERO BOND	5Y	5.363252	5.355861	9.100783	21.446936	32.240481	40.020042	34.88943	31.183083	28.76447	25.585268	21.419109
USD ZERO BOND	7Y	4.78805	4.256178	7.592787	18.366752	27.887742	35.087428	30.733534	27.600629	25.585268	22.925424	19.476361
USD ZERO BOND	10Y	4.129549	2.712483	5.513417	14.166313	22.022566	28.432505	25.175209	22.855431	21.419109	19.476361	17.053177

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TABLE B.6 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

30-04-02			USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND
		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.545186	3.279615	4.893233	6.725966	7.730217	6.561086	5.763535	5.285015	4.716263	4.062685
USD ZERO BOND	1M	3.545186	1	0.540831	0.247032	0.33848	0.066574	0.06178	0.051815	0.037728	0.014157	-0.030999
USD ZERO BOND	3M	3.279615	0.540831	1	0.762587	0.754389	0.4608	0.43286	0.401468	0.352681	0.296942	0.191043
USD ZERO BOND	6M	4.893233	0.247032	0.762587	1	0.913741	0.761456	0.757951	0.749771	0.723061	0.672775	0.56441
USD ZERO BOND	1Y	6.725966	0.33848	0.754389	0.913741	1	0.790079	0.787329	0.781551	0.759248	0.712712	0.610091
USD ZERO BOND	2Y	7.730217	0.066574	0.4608	0.761456	0.790079	1	0.993493	0.96874	0.912142	0.8935	0.83357
USD ZERO BOND	3Y	6.561086	0.06178	0.43286	0.757951	0.787329	0.993493	1	0.989984	0.950155	0.935558	0.881854
USD ZERO BOND	4Y	5.763535	0.051815	0.401468	0.749771	0.781551	0.96874	0.989984	1	0.98443	0.973987	0.927323
USD ZERO BOND	5Y	5.285015	0.037728	0.352681	0.723061	0.759248	0.912142	0.950155	0.98443	1	0.994213	0.956196
USD ZERO BOND	7Y	4.716263	0.014157	0.296942	0.672775	0.712712	0.8935	0.935558	0.973987	0.994213	1	0.98175
USD ZERO BOND	10Y	4.062685	-0.030999	0.191043	0.56441	0.610091	0.83357	0.881854	0.927323	0.956196	0.98175	1

COVARIANCES			USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND
30-04-02		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.545186	3.279615	4.893233	6.725966	7.730217	6.561086	5.763535	5.285015	4.716263	4.062685
USD ZERO BOND	1M	3.545186	12.568342	6.288157	4.285369	8.07098	1.824452	1.437015	1.058717	0.706891	0.2367	-0.446477
USD ZERO BOND	3M	3.279615	6.288157	10.755877	12.237945	16.640752	11.682269	9.314223	7.588618	6.112949	4.592963	2.545463
USD ZERO BOND	6M	4.893233	4.285369	12.237945	23.94373	30.072795	28.802656	24.333959	21.145289	18.69894	15.526146	11.220275
USD ZERO BOND	1Y	6.725966	8.07098	16.640752	30.072795	45.23862	41.078734	34.744545	30.297097	26.988846	22.608232	16.671042
USD ZERO BOND	2Y	7.730217	1.824452	11.682269	28.802656	41.078734	59.756258	50.388607	43.160653	37.264948	32.575	26.178649
USD ZERO BOND	3Y	6.561086	1.437015	9.314223	24.333959	34.744545	50.388607	43.047854	37.436287	32.947049	28.949744	23.506362
USD ZERO BOND	4Y	5.763535	1.058717	7.588618	21.145289	30.297097	43.160653	37.436287	33.218339	29.986106	26.475256	21.71367
USD ZERO BOND	5Y	5.285015	0.706891	6.112949	18.69894	26.988846	37.264948	32.947049	29.986106	27.931386	24.781281	20.530829
USD ZERO BOND	7Y	4.716263	0.2367	4.592963	15.526146	22.608232	32.575	28.949744	26.475256	24.781281	22.24314	18.811011
USD ZERO BOND	10Y	4.062685	-0.446477	2.545463	11.220275	16.671042	26.178649	23.506362	21.71367	20.530829	18.811011	16.505412

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TABLE B.7 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

02-05-02			USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND
		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.222797	3.073713	5.181052	6.773521	7.383061	6.244253	5.491606	5.033159	4.495514	3.870258
USD ZERO BOND	1M	3.222797	1	0.498539	0.198246	0.300141	0.054569	0.054186	0.047892	0.036403	0.016919	-0.020733
USD ZERO BOND	3M	3.073713	0.498539	1	0.766666	0.772426	0.503679	0.475186	0.443085	0.39577	0.353088	0.268666
USD ZERO BOND	6M	5.181052	0.198246	0.766666	1	0.927755	0.782039	0.770603	0.757776	0.727817	0.695522	0.619957
USD ZERO BOND	1Y	6.773521	0.300141	0.772426	0.927755	1	0.80781	0.799698	0.788878	0.760986	0.729431	0.654938
USD ZERO BOND	2Y	7.383061	0.054569	0.503679	0.782039	0.80781	1	0.993674	0.970744	0.918003	0.903057	0.853688
USD ZERO BOND	3Y	6.244253	0.054186	0.475186	0.770603	0.799698	0.993674	1	0.990709	0.95306	0.941054	0.896423
USD ZERO BOND	4Y	5.491606	0.047892	0.443085	0.757776	0.788878	0.970744	0.990709	1	0.984951	0.976208	0.937372
USD ZERO BOND	5Y	5.033159	0.036403	0.39577	0.727817	0.760986	0.918003	0.95306	0.984951	1	0.995311	0.964207
USD ZERO BOND	7Y	4.495514	0.016919	0.353088	0.695522	0.729431	0.903057	0.941054	0.976208	0.995311	1	0.985045
USD ZERO BOND	10Y	3.870258	-0.020733	0.268666	0.619957	0.654938	0.853688	0.896423	0.937372	0.964207	0.985045	1

COVARIANCES			USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND	USD ZERO BOND
02-05-02		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.222797	3.073713	5.181052	6.773521	7.383061	6.244253	5.491606	5.033159	4.495514	3.870258
USD ZERO BOND	1M	3.222797	10.386421	4.938508	3.310202	6.551988	1.298417	1.09044	0.847606	0.590495	0.24512	-0.258609
USD ZERO BOND	3M	3.073713	4.938508	9.447711	12.209212	16.08179	11.430195	9.12026	7.479101	6.122753	4.878942	3.196066
USD ZERO BOND	6M	5.181052	3.310202	12.209212	26.843302	32.558605	29.914582	24.930396	21.560466	18.979324	16.199741	12.431376
USD ZERO BOND	1Y	6.773521	6.551988	16.08179	32.558605	45.880582	40.398049	33.82368	29.344279	25.943686	22.211503	17.169383
USD ZERO BOND	2Y	7.383061	1.298417	11.430195	29.914582	40.398049	54.509595	45.810062	39.358671	34.113125	29.973068	24.393571
USD ZERO BOND	3Y	6.244253	1.09044	9.12026	24.930396	33.82368	45.810062	38.990691	33.972383	29.953079	26.416447	21.663743
USD ZERO BOND	4Y	5.491606	0.847606	7.479101	21.560466	29.344279	39.358671	33.972383	30.157739	27.22417	24.100229	19.922842
USD ZERO BOND	5Y	5.033159	0.590495	6.122753	18.979324	25.943686	34.113125	29.953079	27.22417	25.332694	22.520546	18.782396
USD ZERO BOND	7Y	4.495514	0.24512	4.878942	16.199741	22.211503	29.973068	26.416447	24.100229	22.520546	20.209647	17.138602
USD ZERO BOND	10Y	3.870258	-0.258609	3.196066	12.431376	17.169383	24.393571	21.663743	19.922842	18.782396	17.138602	14.978895

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TABLE B.8 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

03-05-02			USD ZERO BONI	USD ZERO BOND	USD ZERO BONI	SD ZERO BOI	SD ZERO BOI	SD ZERO BOI	SD ZERO BOI	SD ZERO BOI	SD ZERO BOI	SD ZERO BON
		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.139242	3.041238	5.393185	7.072039	7.236235	6.13302	5.387237	4.942871	4.397371	3.769449
USD ZERO BOND	1M	3.139242	1	0.517449	0.206946	0.309429	0.036687	0.03125	0.01981	0.003768	-0.017306	-0.057384
USD ZERO BOND	3M	3.041238	0.517449	1	0.764047	0.760428	0.466946	0.439252	0.406883	0.356676	0.305421	0.207193
USD ZERO BOND	6M	5.393185	0.206946	0.764047	1	0.914789	0.76401	0.759081	0.750041	0.722818	0.678251	0.581417
USD ZERO BOND	1Y	7.072039	0.309429	0.760428	0.914789	1	0.790492	0.785981	0.777263	0.750038	0.708131	0.615474
USD ZERO BOND	2Y	7.236235	0.036687	0.466946	0.76401	0.790492	1	0.994007	0.97073	0.915992	0.899222	0.845423
USD ZERO BOND	3Y	6.13302	0.03125	0.439252	0.759081	0.785981	0.994007	1	0.990313	0.950691	0.937555	0.889534
USD ZERO BOND	4Y	5.387237	0.01981	0.406883	0.750041	0.777263	0.97073	0.990313	1	0.984181	0.974884	0.93354
USD ZERO BOND	5Y	4.942871	0.003768	0.356676	0.722818	0.750038	0.915992	0.950691	0.984181	1	0.995027	0.961859
USD ZERO BOND	7Y	4.397371	-0.017306	0.305421	0.678251	0.708131	0.899222	0.937555	0.974884	0.995027	1	0.984009
USD ZERO BOND	10Y	3.769449	-0.057384	0.207193	0.581417	0.615474	0.845423	0.889534	0.93354	0.961859	0.984009	1

COVARIANCES			USD ZERO BONI	USD ZERO BOND	USD ZERO BONI	SD ZERO BOI	SD ZERO BOI	SD ZERO BOI	SD ZERO BOI	SD ZERO BOI	SD ZERO BOI	SD ZERO BON
03-05-02		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.139242	3.041238	5.393185	7.072039	7.236235	6.13302	5.387237	4.942871	4.397371	3.769449
USD ZERO BOND	1M	3.139242	9.854842	4.940181	3.503696	6.869594	0.833383	0.601653	0.335025	0.058463	-0.238894	-0.67904
USD ZERO BOND	3M	3.041238	4.940181	9.249131	12.531864	16.355109	10.276125	8.192925	6.666327	5.361716	4.084538	2.375218
USD ZERO BOND	6M	5.393185	3.503696	12.531864	29.086448	34.890816	29.816535	25.107742	21.791972	19.268752	16.085296	11.819831
USD ZERO BOND	1Y	7.072039	6.869594	16.355109	34.890816	50.013732	40.453361	34.090318	29.61276	26.21845	22.021722	16.407108
USD ZERO BOND	2Y	7.236235	0.833383	10.276125	29.816535	40.453361	52.36309	44.113986	37.842287	32.762997	28.613625	23.060266
USD ZERO BOND	3Y	6.13302	0.601653	8.192925	25.107742	34.090318	44.113986	37.613931	32.719964	28.819924	25.28509	20.564337
USD ZERO BOND	4Y	5.387237	0.335025	6.666327	21.791972	29.61276	37.842287	32.719964	29.022322	26.207178	23.09469	18.957313
USD ZERO BOND	5Y	4.942871	0.058463	5.361716	19.268752	26.21845	32.762997	28.819924	26.207178	24.431976	21.627545	17.921259
USD ZERO BOND	7Y	4.397371	-0.238894	4.084538	16.085296	22.021722	28.613625	25.28509	23.09469	21.627545	19.336873	16.310601
USD ZERO BOND	10Y	3.769449	-0.67904	2.375218	11.819831	16.407108	23.060266	20.564337	18.957313	17.921259	16.310601	14.208745

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TABLE B.9 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

06-05-02			USD ZERO BONE	USD ZERO BOND	USD ZERO BONI	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BON
		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.057467	3.060269	6.329043	9.080704	9.272185	8.566567	8.162849	7.98172	6.482339	4.64304
USD ZERO BOND	1M	3.057467	1	0.498539	0.198246	0.300141	0.054569	0.054186	0.047892	0.036403	0.016919	-0.020733
USD ZERO BOND	3M	3.060269	0.498539	1	0.766666	0.772426	0.503679	0.475186	0.443085	0.39577	0.353088	0.268666
USD ZERO BOND	6M	6.329043	0.198246	0.766666	1	0.927755	0.782039	0.770603	0.757776	0.727817	0.695522	0.619957
USD ZERO BOND	1Y	9.080704	0.300141	0.772426	0.927755	1	0.80781	0.799698	0.788878	0.760986	0.729431	0.654938
USD ZERO BOND	2Y	9.272185	0.054569	0.503679	0.782039	0.80781	1	0.993674	0.970744	0.918003	0.903057	0.853688
USD ZERO BOND	3Y	8.566567	0.054186	0.475186	0.770603	0.799698	0.993674	1	0.990709	0.95306	0.941054	0.896423
USD ZERO BOND	4Y	8.162849	0.047892	0.443085	0.757776	0.788878	0.970744	0.990709	1	0.984951	0.976208	0.937372
USD ZERO BOND	5Y	7.98172	0.036403	0.39577	0.727817	0.760986	0.918003	0.95306	0.984951	1	0.995311	0.964207
USD ZERO BOND	7Y	6.482339	0.016919	0.353088	0.695522	0.729431	0.903057	0.941054	0.976208	0.995311	1	0.985045
USD ZERO BOND	10Y	4.64304	-0.020733	0.268666	0.619957	0.654938	0.853688	0.896423	0.937372	0.964207	0.985045	1

COVARIANCES			USD ZERO BONE	USD ZERO BOND	USD ZERO BONI	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BO	SD ZERO BON
06-05-02		VOLATILITY	1M	3M	6M	1Y	2Y	3Y	4Y	5Y	7Y	10Y
VOLATILITY			3.057467	3.060269	6.329043	9.080704	9.272185	8.566567	8.162849	7.98172	6.482339	4.64304
USD ZERO BOND	1M	3.057467	9.348102	4.664669	3.836218	8.333106	1.546994	1.419244	1.195267	0.888384	0.33532	-0.29433
USD ZERO BOND	3M	3.060269	4.664669	9.365247	14.849232	21.46524	14.292085	12.457471	11.068483	9.667157	7.004456	3.817461
USD ZERO BOND	6M	6.329043	3.836218	14.849232	40.056782	53.320093	45.893229	41.780689	39.148992	36.766861	28.535177	18.218047
USD ZERO BOND	1Y	9.080704	8.333106	21.46524	53.320093	82.459188	68.015999	62.208866	58.47509	55.155965	42.93737	27.613553
USD ZERO BOND	2Y	9.272185	1.546994	14.292085	45.893229	68.015999	85.973417	78.928323	73.473113	67.939585	54.27868	36.752219
USD ZERO BOND	3Y	8.566567	1.419244	12.457471	41.780689	62.208866	78.928323	73.386077	69.277908	65.166379	52.258051	35.655162
USD ZERO BOND	4Y	8.162849	1.195267	11.068483	39.148992	58.47509	73.473113	69.277908	66.632104	64.173065	51.655432	35.526811
USD ZERO BOND	5Y	7.98172	0.888384	9.667157	36.766861	55.155965	67.939585	65.166379	64.173065	63.707848	51.497614	35.732985
USD ZERO BOND	7Y	6.482339	0.33532	7.004456	28.535177	42.93737	54.27868	52.258051	51.655432	51.497614	42.020725	29.647657
USD ZERO BOND	10Y	4.64304	-0.29433	3.817461	18.218047	27.613553	36.752219	35.655162	35.526811	35.732985	29.647657	21.557822

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TABLE B.10 VOLATILITIES, CORRELATIONS & COVARIANCES FROM RISKMETRICS

07-05-02		VOLATILITY	USD ZERO BOND 1M	USD ZERO BOND 3M	USD ZERO BOND 6M	USD ZERO BOND 1Y	USD ZERO BOND 2Y	USD ZERO BOND 3Y	USD ZERO BOND 4Y	USD ZERO BOND 5Y	USD ZERO BOND 7Y	USD ZERO BOND 10Y
VOLATILITY			2.97778	3.04838	6.358654	9.057927	9.365701	8.63208	8.198054	7.985547	6.472772	4.606256
USD ZERO BOND	1M	2.97778	1	0.498539	0.198246	0.300141	0.054569	0.054186	0.047892	0.036403	0.016919	-0.020733
USD ZERO BOND	3M	3.04838	0.498539	1	0.766666	0.772426	0.503679	0.475186	0.443085	0.39577	0.353088	0.268666
USD ZERO BOND	6M	6.358654	0.198246	0.766666	1	0.927755	0.782039	0.770603	0.757776	0.727817	0.695522	0.619957
USD ZERO BOND	1Y	9.057927	0.300141	0.772426	0.927755	1	0.80781	0.799698	0.788878	0.760986	0.729431	0.654938
USD ZERO BOND	2Y	9.365701	0.054569	0.503679	0.782039	0.80781	1	0.993674	0.970744	0.918003	0.903057	0.853688
USD ZERO BOND	3Y	8.63208	0.054186	0.475186	0.770603	0.799698	0.993674	1	0.990709	0.95306	0.941054	0.896423
USD ZERO BOND	4Y	8.198054	0.047892	0.443085	0.757776	0.788878	0.970744	0.990709	1	0.984951	0.976208	0.937372
USD ZERO BOND	5Y	7.985547	0.036403	0.39577	0.727817	0.760986	0.918003	0.95306	0.984951	1	0.995311	0.964207
USD ZERO BOND	7Y	6.472772	0.016919	0.353088	0.695522	0.729431	0.903057	0.941054	0.976208	0.995311	1	0.985045
USD ZERO BOND	10Y	4.606256	-0.020733	0.268666	0.619957	0.654938	0.853688	0.896423	0.937372	0.964207	0.985045	1

COVARIANCES 07-05-02		VOLATILITY	USD ZERO BOND 1M	USD ZERO BOND 3M	USD ZERO BOND 6M	USD ZERO BOND 1Y	USD ZERO BOND 2Y	USD ZERO BOND 3Y	USD ZERO BOND 4Y	USD ZERO BOND 5Y	USD ZERO BOND 7Y	USD ZERO BOND 10Y
VOLATILITY			2.97778	3.04838	6.358654	9.057927	9.365701	8.63208	8.198054	7.985547	6.472772	4.606256
USD ZERO BOND	1M	2.97778	8.867174	4.525445	3.753715	8.095564	1.521871	1.392825	1.169136	0.865645	0.326098	-0.284388
USD ZERO BOND	3M	3.04838	4.525445	9.292621	14.860747	21.328216	14.380146	12.503972	11.073034	9.634219	6.966945	3.772504
USD ZERO BOND	6M	6.358654	3.753715	14.860747	40.432478	53.435184	46.572971	42.297171	39.501787	36.956592	28.626364	18.158273
USD ZERO BOND	1Y	9.057927	8.095564	21.328216	53.435184	82.046033	68.529655	62.52737	58.579978	55.044001	42.766451	27.326069
USD ZERO BOND	2Y	9.365701	1.521871	14.380146	46.572971	68.529655	87.71636	80.334049	74.534211	68.657709	54.745188	36.82878
USD ZERO BOND	3Y	8.63208	1.392825	12.503972	42.297171	62.52737	80.334049	74.512804	70.108777	65.696225	52.579966	35.643192
USD ZERO BOND	4Y	8.198054	1.169136	11.073034	39.501787	58.579978	74.534211	70.108777	67.208095	64.480744	51.801641	35.397357
USD ZERO BOND	5Y	7.985547	0.865645	9.634219	36.956592	55.044001	68.657709	65.696225	64.480744	63.768963	51.446262	35.46689
USD ZERO BOND	7Y	6.472772	0.326098	6.966945	28.626364	42.766451	54.745188	52.579966	51.801641	51.446262	41.896776	29.369358
USD ZERO BOND	10Y	4.606256	-0.284388	3.772504	18.158273	27.326069	36.82878	35.643192	35.397357	35.46689	29.369358	21.21759

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TABLE B.11 VOLATILITIES FROM RISKMETRICS

	23-04-02	24-04-02	25-04-02	26-04-02	29-04-02	30-04-02	02-05-02	03-05-02	06-05-02	07-08-02
1M	4.141046	4.033895	3.829693	3.737287	3.641716	3.545186	3.222797	3.139242	3.057467	2.97778
3M	3.629223	3.596091	3.499092	3.452828	3.363985	3.279615	3.073713	3.041238	3.060269	3.04838
6M	5.257351	5.164156	5.000104	5.067318	5.013827	4.893233	5.181052	5.393185	6.329043	6.358654
1Y	7.361232	7.222432	7.096372	7.039269	6.90614	6.725966	6.773521	7.072039	9.080704	9.057927
2Y	7.985191	7.787514	7.958185	7.943015	7.875483	7.730217	7.383061	7.236235	9.272185	9.365701
3Y	6.8426	6.665274	6.732194	6.720597	6.672928	6.561086	6.244253	6.13302	8.566567	8.63208
4Y	6.041368	5.882719	5.858719	5.844798	5.853187	5.763535	5.491606	5.387237	8.162849	8.198054
5Y	5.471549	5.32635	5.274181	5.251709	5.363252	5.285015	5.033159	4.942871	7.98172	7.985547
7Y	4.92233	4.792832	4.718725	4.687845	4.78805	4.716263	4.495514	4.397371	6.482339	6.472772
10Y	4.288931	4.198276	4.096311	4.049983	4.129549	4.062685	3.870258	3.769449	4.64304	4.606256

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Appendix C

- US yield curve changes: Graph C.1
- US yield curve changes: Table C.1

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TABLE C 1

<HELP> for explanation. P218 Govt **IYC**

YIELD CHANGES FOR

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US TREASURY ACTIVES 4/15/2002 vs 1/15/2002

	TKR / CPN / MTY	YIELD	TKR / CPN / MTY	YIELD	CHANGES
3MO	1) B 0 07/11/02	1.710 BGN	16) B 0 04/18/02	1.598 BGN	0.112
6MO	2) B 0 10/10/02	1.945 BGN	17) B 0 07/18/02	1.635 BGN	0.310
1YR	3)		18)		
2YR	4) T 3 5/8 03/31/04	3.342 BGN	19) T 3 1/4 12/31/03	2.770 BGN	0.572
3YR	5)		20)		
4YR	6)		21)		
5YR	7) T 3 1/2 11/15/06	4.479 BGN	22) T 3 1/2 11/15/06	4.093 BGN	0.386
6YR	8)		23)		
7YR	9)		24)		
8YR	10)		25)		
9YR	11)		26)		
10YR	12) T 4 7/8 02/15/12	5.140 BGN	27) T 5 08/15/11	4.837 BGN	0.303
15YR	13)		28)		
20YR	14)		29)		
30YR	15) T 5 3/8 02/15/31	5.619 BGN	30) T 5 3/8 02/15/31	5.338 BGN	0.281

To change price source for securities, use <FMPS>.

Yields are based on STANDARD settlement and are Conventional.

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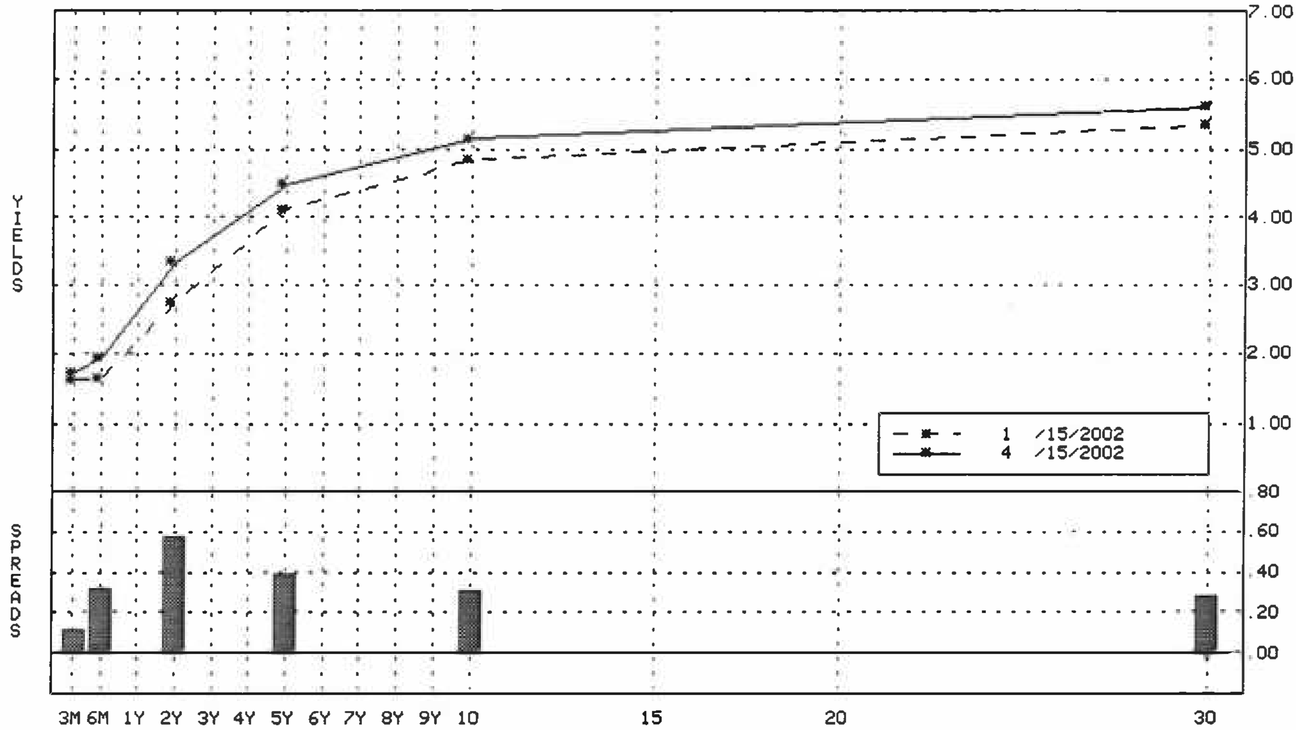
GRAPH C1

<HELP> for explanation. P218 Govt **IYC**

Hit <PAGE> for more info or <MENU> for list of curves.

YIELD CHANGES FOR Page 1/2

US TREASURY ACTIVES 4/15/2002 vs 1/15/2002 3M - 30



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BASIC DEFINITIONS

Beta: is a factor for each individual stock, which describes how its price moves in relation to the market. Beta measures the contribution of one security to total portfolio risk. This is also called the *systematic risk* of a security i vis-à-vis portfolio p. The actual definition of beta is:

$$\beta_i = \frac{Cov(R_i, R_p)}{\sigma_p^2} \quad \text{where}$$

$Cov(R_i, R_p)$ is the covariance between the return on asset in and the return in the market portfolio p and σ_p^2 is the variance of the market portfolio.

A stock with a beta of one will move exactly in line with the market. A stock with bets of two will respond aggressively to changes in the value of the market, i.e. for a 1% change in the value of the index, the price of a stock with a beta of two will change by 2%.

Convexity: is a measure of the rate of change of duration with respect to yield. Because of convexity, the market value changes are not linear. The effect of a decrease in rates from 9% to 8% is less than the change generated by a move from 7% to 6%. For small variations of rates, the duration captures the change of the asset value. But when moderate or large changes in interest rates are considered, the convexity alters the sensitivity. Visually, convexity is the change in the slope that measures duration at various levels of interest rates.

Correlation: in general, is a linear statistical measure of the co-movement.

1) Between two random variables X, Y and is described by the following type:

$$\text{Correlation: } \rho = \frac{Cov(X,Y)}{\sqrt{Var(X) * VaR(Y)}}$$

where Cov (X, Y) is the covariance of the two variables X, Y

VaR (X) is the variance of the variable X and VaR (Y) is the variance of the variable Y.

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2) For a set of n paired observations (x_1, y_1), (x_2, y_2),. . . . , (x_n, y_n), the sample correlation coefficient r is given by: $r = \dfrac{s_{xy}}{\sqrt{s_{xx} * s_{yy}}}$

where s_{xy} denotes the sum of products of deviations of the x_i, y_i from their means and s_{xx}, s_{yy} are sums of squares of deviations from the respective means.

The correlation coefficient ρ always lies between –1 and +1.

- **Cases: a)** ρ = 0: the assets are uncorrelated meaning that there is no relationship between the price changes of the two assets, i.e. they are entirely independent of one another.

  b) ρ = 1: the variables are said to be perfect correlated. If two assets have correlation 1 they behave as they are the same assets.

  c) ρ = -1: the variables are said to be perfect negatively correlated. The price changes of one asset are always opposite to that of the other asset.

Covariance: If X, Y are two random variables and $\overline{X}$, $\overline{Y}$ are their means, then the covariance is defined by the following type:

$$Cov(X,Y) = E[(X-\overline{X})*(Y-\overline{Y})]=E(XY) - \overline{X}\,\overline{Y}$$

- For a sample of n paired observations (x_i, y_i), the sample covariance is:

$$c_{xy} = \sum_i (x_i - \bar{x})(y_i - \bar{y})/n$$ where $\bar{x}$ & $\bar{y}$ are the respective means.

The sign of the covariance will indicate the direction of covariation of X & Y. Its magnitude depends on the scales of measurement so a preferable measure is the *correlation coefficient.*

Modified Duration: is the approximate percentage change in a bond's price for a small change in yield.

Normal Distribution: plays a central role in a large body of statistics because it has the familiar bell shape whose symmetry makes it an appealing choice for many validation models. The Normal distribution has two parameters, usually denoted by μ & σ^2, which are its mean & variance. The probability distribution function (pdf) of the normal distribution with mean μ & variance σ^2 (usually denoted as n (μ, σ^2)) is given by:

$$f(x) = \frac{1}{\sqrt{2\pi}} \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sigma} \qquad -\infty < x < \infty$$

If the mean is equal to 0 & variance equals to 1 the normal distribution is called standard normal distribution.

The normal distribution is somewhat special in the sense that the its two parameters μ (mean) & σ^2 (variance), provide us with complete information about the exact shape & location of the distribution. Straightforward calculus shows that the normal pdf has its maximum at x=μ & inflection points (where the curve changes from concave to convex) at μ±σ. Furthermore the probability content within 1,2,3 standard deviations of the mean is:

$P(|x-\mu|)<\sigma) = 0{,}6826$

$P(|x-\mu|)<2\sigma) = 0{,}9544$

$P(|x-\mu|<3\sigma) = 0.9974$

Among the many uses of the normal distribution, an important one its use as an approximation to other distributions. For example, if X~binomial (n, p), then the mean is equal to EX= np & the variance σ^2 = np (1-p). Under suitable conditions, this distribution can be approximated with that of a normal distribution with mean μ = np & the variance σ^2 = np (1-p).

Standard Deviation : is the square root of variance and is always positive:

$$\sigma_x = \sqrt{VaR(X)}$$

Variance: Let $X_1, X_2, \ldots X_n$ be a random sample of size n with the random variable. Then the variance of the random variable X is measured by the type:

$$VaR(X) = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 \text{ where } \overline{X} \text{ is the mean.}$$

Variance characterizes the dispersion around the mean & can be defined as the weighted sum of the squared deviations around the mean.

Volatility: See Standard Deviation

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Yield-to-maturity: is the interest rate that will make the present value of the cash flows from the bond equals to the cash flows that the investor would realize by holding the bond to maturity.

The yield to maturity considers the coupon income & any capital gain or loss that the investor will realize by holding the bond to maturity. It also considers the timing of the cash flows, & interest-to-interest assuming that the coupon payments can be reinvested at an interest rate equal to the yield to maturity.

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