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Deterministic and Stochastic Claims Reserving methods in Non-life Insurance under Solvency II

By

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ΣΧΟΛΗ ΕΠΙΣΤΗΜΩΝ & ΤΕΧΝΟΛΟΓΙΑΣ ΤΗΣ ΠΛΗΡΟΦΟΡΙΑΣ

ΤΜΗΜΑ ΣΤΑΤΙΣΤΙΚΗΣ

ΜΕΤΑΠΤΥΧΙΑΚΟ ΠΡΟΓΡΑΜΜΑ

Κλασικές και στοχαστικές μέθοδοι αποθεματοποίησης στις Γενικές Ασφαλίσεις στο πλαίσιο του SOLVENCY II

Μαρία Ε. Μαλατάντη

ΔΙΑΤΡΙΒΗ

Που υποβλήθηκε στο Τμήμα Στατιστικής
του Οικονομικού Πανεπιστημίου Αθηνών
ως μέρος των απαιτήσεων για την απόκτηση
Διπλώματος Μεταπτυχιακών Σπουδών στη Στατιστική

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This present Master thesis is dedicated to my loving family
and my supportive friends

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ABSTRACT

Maria Malatanti

Deterministic and Stochastic Claims Reserving methods in Non-life Insurance under Solvency II

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It is a fact that life is full of uncertainties. Subsequently, it is crucial for insurance companies to continuously estimate the capital required to face losses from events that have transpired. In order to protect policyholders and the insurance companies themselves from bankruptcy, the EU has established a framework called Solvency II.

Thus, the prediction process of the required reserves has to follow some specific guidelines and it is supervised by the authorities. In this thesis, we present some of the basic claim reserving methods concerning the non-life insurance. The methods are covered both theoretically and practically with numeric examples.

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ΠΕΡΙΛΗΨΗ

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Σεπτέμβριος 2020

Είναι γεγονός πως ζούμε σε ένα κόσμο γεμάτο αβεβαιότητα. Συνεπώς, είναι κρίσιμο για τις ασφαλιστικές εταιρείες να εκτιμούν διαρκώς το απαραίτητο κεφάλαιο ώστε να είναι σε θέση να αντιμετωπίσουν τις ζημίες από γεγονότα που έχουν συμβεί. Προκειμένου να προστατέψουν τους ασφαλισμένους αλλά και τις ίδιες τις ασφαλιστικές εταιρείες από χρεωκοπία, η Ευρωπαϊκή Ένωση έχει θεσπίσει ένα πλαίσιο που ονομάζεται Φερεγγυότητα ΙΙ.

Έτσι, η διαδικασία εκτίμησης των απαραίτητων αποθεμάτων θα πρέπει να ακολουθεί συγκεκριμένες οδηγίες και να ελέγχεται από τις αρμόδιες αρχές. Στην παρούσα διατριβή, παρουσιάζονται ορισμένες από τις βασικές μεθόδους αποθεματοποίησης για τον κλάδο των γενικών ασφαλίσεων. Οι μέθοδοι θα καλυφθούν τόσο θεωρητικά όσο και πρακτικά με αριθμητικά παραδείγματα.

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1. Introduction

The purpose of this thesis is to explore various methodologies in order to ensure the financial security of an insurance company. More specifically, this thesis is about estimating the necessary reserves of the insurance company, under Solvency II guidelines, in non-life insurance.

Solvency II Directive is the current European law about insurance regulation. It came into effect from 1st January 2016 and its main concern is the amount of capital that insurance firms ought to have in order to minimize the risk of bankruptcy.

Non-life insurance or general insurance is basically an insurance policy to protect an individual against losses and damages other than those covered by Life insurance. For instance, it provides financial protection for building, machinery, equipment, furniture, and vehicle and merchandise items against the risk of fire, earthquake, accident and theft. It also includes fire insurance, marine insurance, and miscellaneous insurance.

In this context, this thesis will examine different techniques in order to find the most suitable estimation method. The different approaches will be covered both theoretically and practically, using real life data and making comparisons between the results.

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2. Capital Requirements in insurance industry

To begin with, the purpose of solvency margin's existence is to minimize the possibility that an insurance company will be unable to meet its obligations towards its customers if unpredictable financial conditions take place. That is the reason why a specific minimum capital should be defined by the authorities and the company should hold this for safety in order to prevent the above case from happening.

As already said, our focus is on Solvency II guidelines. However, the first official European guidelines concerning insurance supervision were introduced in 1973 for non-life insurance and in 1979 for life insurance.

The approach presented was pretty simplistic and easy to operate. The assessment mainly made use of simple factors that were applied to the results of the accounting books. To calculate the Minimum Solvency Margin (also called Required Solvency Margin) one should add the following two results:

- a result that reflected the Investment Risk (relative to mathematical provisions)
- a result that reflected the Technical Risk

Furthermore, a standard amount has been set as Guarantee fund. In 1979, the fund was set to 800.000 ECU.

Thus, the Minimum Solvency Margin was defined as:

$$Minimum\ Solvency\ Margin = 4\% \cdot Mathematical\ Reserves + 0{,}3\% \cdot Capital\ Sum\ at\ Risk$$

where 4% corresponded to 95% confidence interval.

Then, the Minimum Guarantee Fund was the maximum quantity between 800.000 ECU and the one third of the Minimum Solvency Margin.

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The above calculations paved the way to Solvency I. It was introduced in European Union in 2002 and it provided some additional parameters in the calculation of solvency margin. The regulation has certainly improved but it yet followed basic and easy calculations.

Solvency I suggested that the Minimum Guarantee Fund should be reexamined every five years. The Guaranteed fund was raised to 3.000.000 euro. It also suggested that the composition of the Minimum Solvency Margin and the Guarantee fund should be taken into consideration; not only the solvency margin.

Furthermore, it classified the risk to three different categories:

- Technical risk such as mortality and insufficient premiums
- Investment risk such as depreciation and liquidity
- Nontechnical risk such as management and regulations

Additionally, Solvency I's solvency requirements were set in order to always be met, in contrast with the previous guidelines that were bonded to a specific point in time.

All things considered, Solvency I has brought to the surface more realistic capital requirements compared to the past. Still, it became obvious over the years that there is the need for a procedure that will recognize the variety of the risk profiles that insurance companies face.

Thus, Solvency I was replaced by Solvency II, with the latter being a risk-sensitive system that provided better financial stability to the companies and better information to the insurance supervisors.

Some of the main differences between Solvency I and Solvency II are presented below:

- Instead of the Minimum Solvency Margin, Solvency II has a capital called "Minimum Capital Requirement" (MCR)

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- The Minimum Capital Requirement should be estimated and reported to the authorities at least four times in a year
- There is also an extra capital requirement which represents the target capital. It is called Solvency Capital Requirement (SCR) and it can be calculated either with a standard formula or with internal procedures.
- The SCR should be calculated and reported at least once a year.

Unlike Solvency I, Solvency II Directive do not force the companies to use an one-and-only model to calculate the solvency margin. It provided them with the liberty to pick the estimation method that best represents their company profile. That is the basic idea behind the new guidelines; that should the companies be able to calculate the solvency margin with their own internal models, the risk will be measured in a more holistic way.

However, it is firmly written in the Directive that whatever the chosen risk module may be, it must always be calibrated with a 99,5% confidence level. The insurance companies ought to be able to explain that their model is based on a risk management system and the model itself has to be approved by the supervisory authorities.

The Solvency II Directive has application to all European insurance companies that have gross premium income greater than 5.000.000€ or gross technical provisions greater than 25.000.000€.

As long as it concerns the Directive's structure, Solvency II is often compared to "Basel II" (three pillar Directive concerning banking regulations) since it is also based on three main pillars:

- Pillar 1 points out the Directive's quantitative capital requirements. Basically, there are two different quantities a company must hold to be safe of bankruptcy. As it has been mentioned before, the first quantity is the Minimum Capital Requirement (MCR). That is the minimum amount that the insurance companies have to hold at all times. Should the reserves fall down this number, the supervisory authorities have to intervene since policyholders are exposed to a considerable risk level.

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The second quantity is the previously mentioned Solvency Capital Requirement (SCR). That is the target capital level that the companies need to face the risks. This amount is calculated assuming that the insurance firm can encounter one event in 200 years and still be able to meet its liabilities towards policyholders. SCR can be computed either by the standard procedure written officially in the Directive or by an internal procedure that has been previously approved by the supervisory authorities.

Pillar I's approach is based on the balance sheet. It associates MCR and SCR with other quantities like assets and liabilities. In the figure below, one can notice the capital requirement differences between Solvency I and Solvency II.

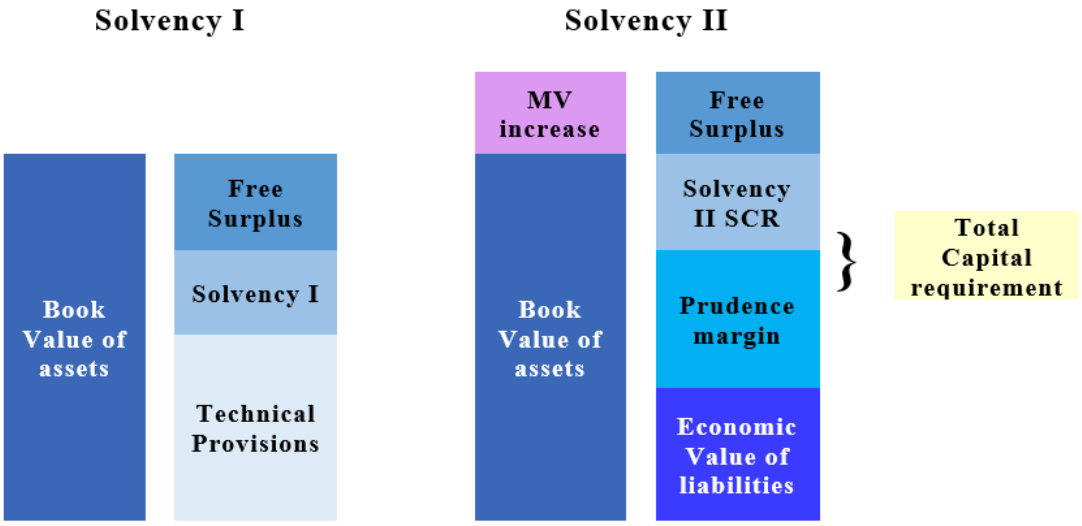

Figure 1: Solvency I vs Solvency II

As it can be observed, assets are now valued at their market value instead of their book value (as they used to under Solvency I's guidelines).

– Pillar 2 focuses on the qualitative review process. The insurance firms should develop specific risk management systems which form an internal evaluation procedure called Own Risk and Solvency Assessment (ORSA). The ORSA should be able to demonstrate that the systems are effective in the risk assessment and in the calculation of the necessary reserves, along with their maintenance. The risk assessment should be

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conducted in the scope of correctly predicting the future losses. In the case that the ORSA presented does not leave the authorities satisfied, Solvency II gives them the power to challenged it.

– Pillar 3 focuses on disclosure requirements. Insurance companies are obliged to publish a report (essentially analyzing Pillars 1 and 2) reflecting their solvency level. This report should include qualitative information about their financial condition as well as a description of the risk assessment for the purpose of absolute transparency. Then, the supervisory authorities will check for any deviation from the instructions of the regulation.

The content of the three pillars of Solvency II Directive can be summarized in the following diagram.

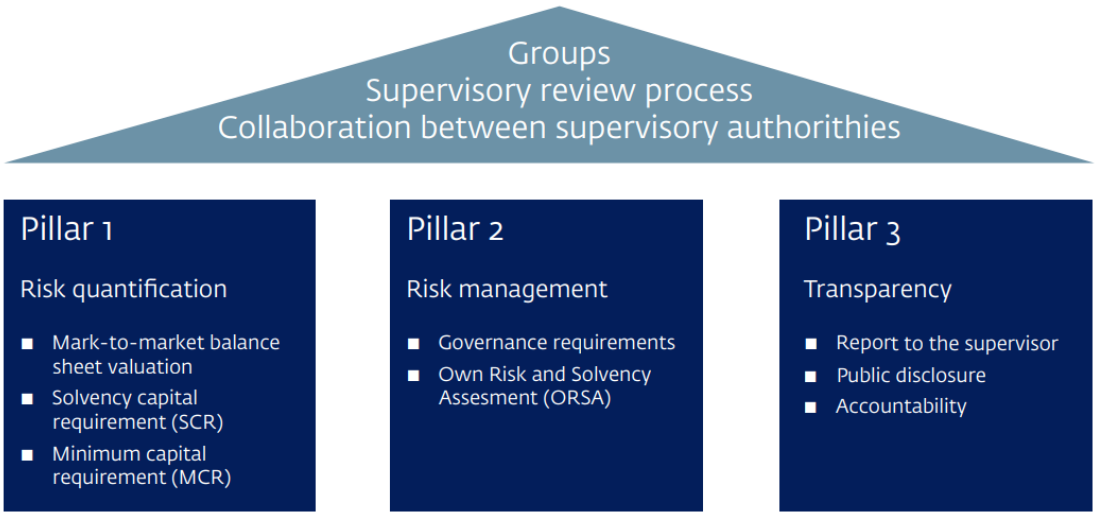

Figure 2: The three Pillars of Solvency II

More specifically, the SCR actually represents Value at Risk measured on a confidence interval of 99,5% over one-year time horizon. As it has been mentioned, it can be calculated either with a standard formula or with internal procedures.

Under the standard formula, it can be determined using standard prescribed parameters which will be later summed using specific correlation matrices. Instead of the predefined parameters, companies could alternatively utilize “undertaking specific parameters” (UPS).

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The SCR's structure (using the standard formula) can be illustrated in the diagram below:

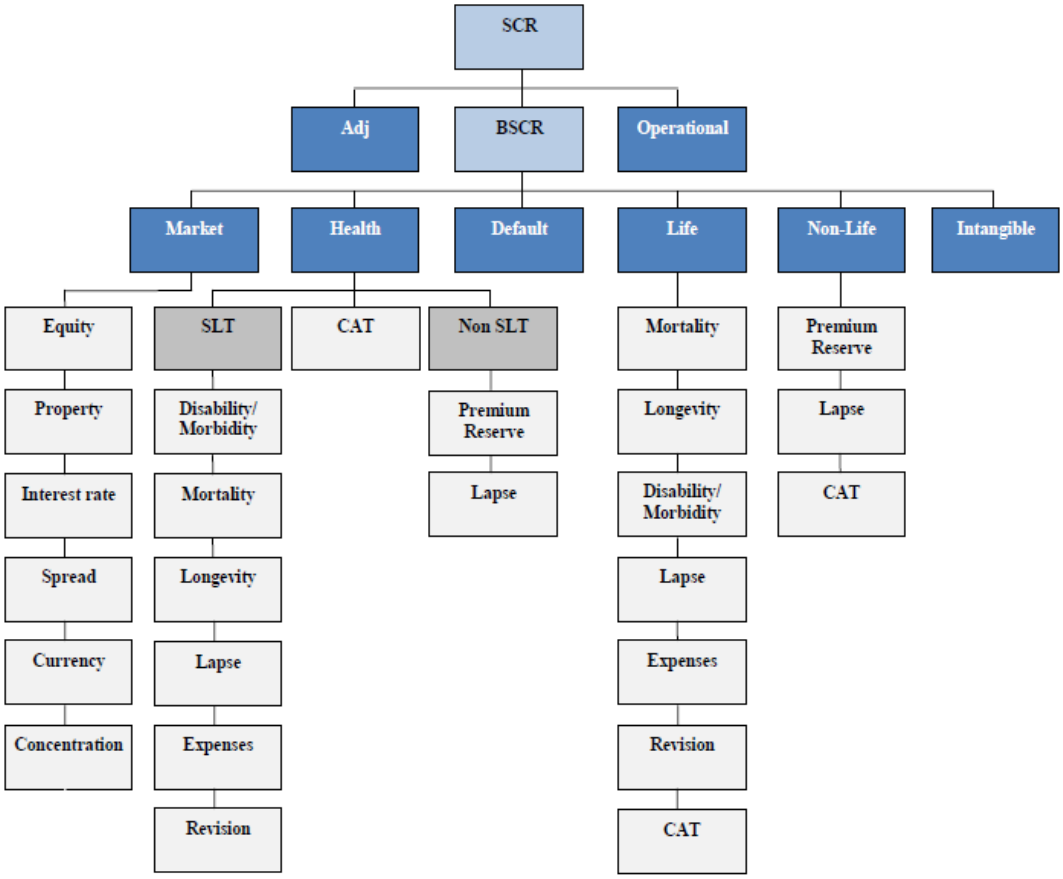

Figure 3: Calculating the SCR

The Basic SCR (BSCR) is determined by taking factors of different risk into consideration. These risks are:

- Default risk: It refers to the counterparty risk, which is the possibility that one of the members involved in the transaction may default on its contract.
- Market risk: It is related to the volatility of assets' market prices and includes the risk exposure due to changes in interest rates, equity, real estate prices and exchange rates. It consists of the following six risk subcategories: the equity risk, the property risk, the interest rate risk, the spread risk, the currency exchange risk and the concentration risk.
- Insurance risk: It is mainly connected to premium calculations, technical reserve requirements and assessment of the overall firm's ability to

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sustain their insurance policies. Separate calculations for life, non-life and health insurance risk should be made.

- Intangible assets

The prediction of the operational risk is then added to the basic SCR. Operational Risk is the risk of generating losses due to problems in the operational systems of the company, staff problems, fraud or problems in the internal management process.

Finally, to obtain the total SCR, a prediction should be made about the absorbing capacity of the losses in technical provisions or deferred taxes. This prediction along with the prediction of the operational risk and the basic SCR result to the overall SCR.

On the other hand, MCR has a linear formula which is based on a Value at Risk with 85% confidence over one year. The minimum MCR is 25% of SCR and the maximum MCR is 45% of SCR.

Finally, there are some categories of capital resources based on their quality. The first split of "own funds" is into basic own funds and ancillary own funds. Then, they are tiered into three categories based on their loss absorbency and their permanence:

- Tier 1: capital of the highest quality. This capital is considered to be the best in loss absorption and one of the most permanent forms of capital.
- Tier 2: capital of medium quality
- Tier 3 capital of the lowest quality

Solvency II Directive has placed specific restriction on the quality of the capital companies can hold to cover MCR and SCR. More specifically, the SCR should be covered by at least 50% of Tier 1 capital and should not use more than 15% of Tier 3 capital. Similarly, the MCR should be covered by at least 80% of Tier 1 and the use of Tier 3 capital is not accepted.

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3. Claims Reserving Methods

The majority of the loss reserving methods accumulate the data of individual claims' development into run–off triangles, as shown in the following figure.

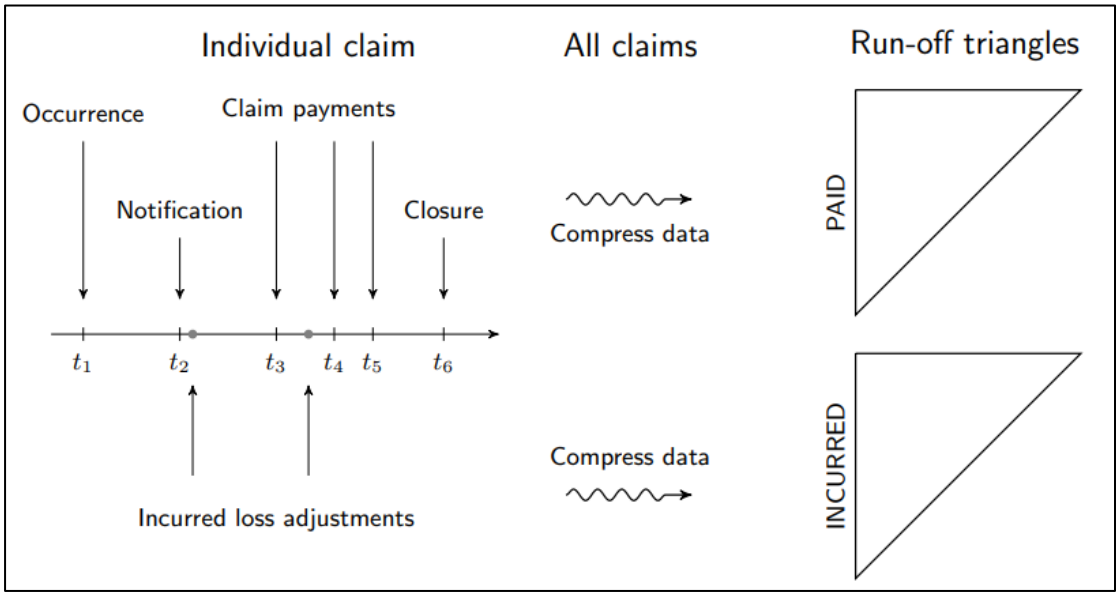

Figure 4: Timeline of a claim's development

In order to estimate the reserves, various categories of methods can be applied like:

- using paid claims data in a development triangle or
- using incurred claims data in a development triangle or
- using the average of paid claims data in a development triangle or
- using the average of incurred claims data in a development triangle.

Reserving actuaries typically analyze incurred losses and distinguish them into two cases:

- Incurred But Not Reported (IBNR) claims and
- Reported But Not Settled (RBNS) claims

The latter are also called Incurred But Not Enough Reported (IBNER) claims.

Over the years, various methodologies concerning the estimation of claims have arisen, both deterministic ones and stochastic ones.

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Deterministic reserving methods

One of the main advantages of the deterministic methods is that they result to one answer that is clear to understand and, then, explain to the management. Furthermore, they can often include the external knowledge or actuary's personal judgment.

Chain Ladder

The most popular loss reserving technique is the Chain Ladder method. Its aim is to calculate claims using run-off triangles with incurred and paid losses. The main reason of its popularity is its easiness to use and the fact that it is more like a calculation procedure than a model with a rigorous theoretical background.

The fact that it acts more like an algorithm than it acts as a model makes the method work almost without any assumptions. Nevertheless, there are some points that need to be highlighted. First of all, the Chain Ladder method assumes that history will repeat itself. In other words, it assumes that the patterns of claims observed in the previous years will be observed again in the future. Nonetheless, in order to make this hypothesis one must be certain of the accuracy of the data and that they are free of errors.

Furthermore, it is usually assumed that the estimation will be more accurate should all the given data is used. For example, it is preferred to make use of all the claims' occurrence years given, in order to calculate the claims, than to make use of only the last occurrence year.

However, this last assumption should not be made without consideration. If there has been an important legal change in the insurance sector or a change in ruling in the last few years, a selection should be made and a subset of the given years would give a more appropriate claim reserves' estimation than using all the given years.

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As long as it concerns the representation of the data, Buchwalder, Bóhlmann, Merz and Wóthrich (2006) consider a $K \times J$ rectangle of claims observations Y_{kj} as seen in the image below:

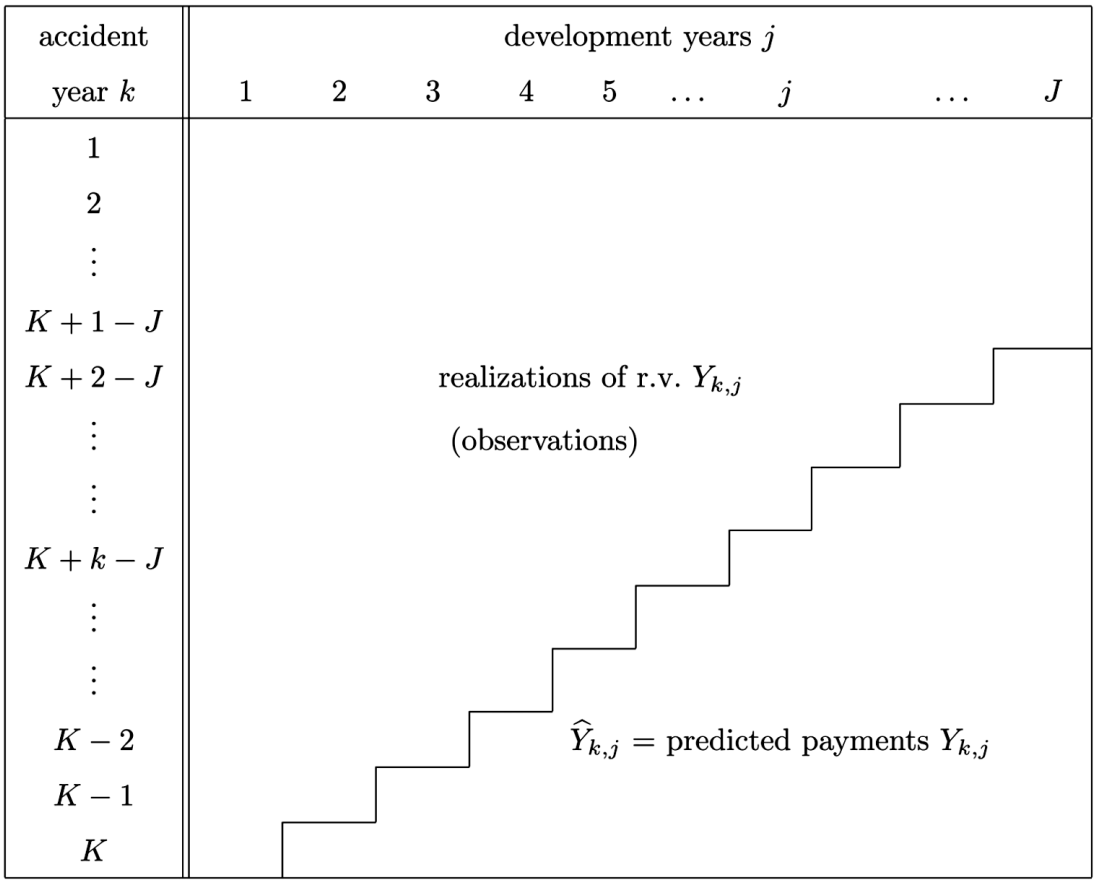

Figure 5: Run-off triangle structure

In this trapezoid:

- the accident periods represented by rows and labelled $k = 1, 2, \dots, K$
- the development periods represented by columns and labelled by $j = 1, 2, \dots, J \leq K$

Usually, within the rectangle we are given a development trapezoid of past observations:

$$D_K = \{Y_{kj} \mid 1 \leq k \leq K \text{ and } 1 \leq j \leq min(J, K - k + 1)\}$$

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and there is a need to estimate the future observations is (which are the completement of the above set):

$$D_K^c = \{Y_{kj} \mid 1 \le k \le K \; and \; min(J, K-k+1) < j \le J\}$$
$$= \{Y_{kj} \mid K-J+1 < k \le K \; and \; K-k+1 < j \le J\}$$

Also, each diagonal contains the claims experience for a specific d-th period, where $d = k + j - 1$. Thus, the diagonals are being called experience periods. Also, the case usually is that $J = K$ and, thus, the trapezoid is a triangle.

As long as it concerns the nature of the observations, the data Y_{kj} could be interpreted either on a paid claims' basis or on an occurred claims' basis. Either way, the structure of the chain ladder method remains the same and the nature of Y_{kj} does not require further specification.

While Y_{kj} represent the incremental claims, we use X_{kj} to represent the cumulative claims. Thus:

$$X_{kj} = \sum_{i=1}^{j} Y_{kj}$$

Furthermore, the total amount of losses in a specific accident period k is defined as:

$$R_k = \sum_{j=K-k+2}^{J} Y_{kj} = X_{kJ} - X_{k,K-k+1}$$

So, the problem every insurance company faces each year is the correct estimation of the claim reserves R_k.

A numerical example will follow in order to visualize the data and make the procedure more comprehensible.

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The table below presents some incremental claims in fire insurance.

Table 1: Example of incremental claims' triangle

		Incremental Losses in Development Year									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	5.960.020	9.036.970	1.848.743	525.132	1.003.130	986.916	2.392.273	79.624	180.800	19.037
2004	**2**	7.510.865	12.627.652	2.236.282	512.114	485.030	42.881	87.967	2.124.467	0	
2005	**3**	25.092.681	14.273.063	1.388.363	1.276.939	78.277	203.041	677.679	1.671.684		
2006	**4**	26.950.816	34.483.829	2.322.247	363.892	700.567	72.161	80.914			
2007	**5**	38.615.756	24.826.200	5.531.906	960.694	479.243	78.435				
2008	**6**	35.901.247	26.814.296	11.240.204	8.789.311	2.882.068					
2009	**7**	47.734.726	21.648.139	7.753.540	900.709						
2010	**8**	18.329.702	23.454.387	3.283.457							
2011	**9**	21.124.374	25.298.735								
2012	**10**	24.241.030									

The above triangle is called "incremental paid losses triangle" and it contains the claims observations Y_{kj}.

A description of the Chain ladder algorithm will follow. Firstly, we are about to define the development factors using the cumulative claims X_{kj} :

$$\hat{f}_{kj} = \frac{X_{k,j+1}}{X_{kj}} \quad , k = 1, 2, \dots, K-1; j = 1, 2, \dots, min(J-1, K-k)$$

and the weighted average development factors:

$$\hat{f}_j = \sum_{k=1}^{K-j} w_{kj} \hat{f}_{kj} \quad , j = 1, 2, \dots, J-1$$

where w_{kj} represents a set of weights with $\sum_{k=1}^{K-j} w_{kj} = 1$ and $w_{kj} \geq 0$ for every (k, j).

We will assume that the weighting procedure will be done with the formula below:

$$w_{kj} = \frac{X_{kj}}{\sum_{k=1}^{K-j} X_{kj}}$$

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Then, the weighted average development factors will become:

$$\hat{f}_j = \frac{\sum_{k=1}^{K-j} X_{k,j+1}}{\sum_{k=1}^{K-j} X_{kj}}$$

Having calculated the above values, we will move on to the estimation of the future cumulative claims X_{kj} corresponding to the $Y_{kj} \in D_K^c$:

$$\hat{X}_{kj} = X_{k,K-k+1}\hat{f}_{K-k+1} \cdots \hat{f}_{j-1}$$

Thus, the estimations of Y_{kj} will be:

$$\hat{Y}_{kj} = X_{k,K-k+1}\hat{f}_{K-k+1} \cdots \hat{f}_{j-2}(\hat{f}_{j-1} - 1)$$

As a result, the total amount of losses in a specific accident period k will be:

$$\hat{R}_k = \hat{X}_{kJ} - X_{k,K-k+1} = X_{k,K-k+1}(\hat{f}_{K-k+1} \cdots \hat{f}_{J-1} - 1)$$

and the estimated total claims in all accident periods will be:

$$\hat{R} = \sum_{k=1}^{K-1} \hat{R}_k$$

Furthermore, a detail that needs to be pointed out is what comes next to the estimations of the last development year. The estimated cumulative losses of the development year J using the chain ladder method will be:

$$\hat{X}_{kJ} = X_{k,K-k+1}\hat{f}_{K-k+1} \cdots \hat{f}_{J-1}$$

However, the loss development should not necessarily end at age J. Hence, the actuaries sometimes make use of a tail factor $\hat{f}_{ult} > 1$ in their attempt to estimate the total losses $X_{k,ult}$ of the accident year k.

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Thus, the ultimate losses are given by multiplying the cumulative losses of development year J with the tail factor $\hat{f}_{ult}$, as seen below:

$$\hat{X}_{k,ult} = \hat{X}_{kJ} \cdot \hat{f}_{ult}$$

According to Mack (1999), a potential approach to estimate the tail factor would be an extrapolation of $ln(\hat{f}_t - 1)$ by a line $a \cdot t + b, a < 0$, along with the relation:

$$\hat{f}_{ult} = \prod_{t=J}^{\infty} \hat{f}_t$$

Nevertheless, the final tail factor should always be revised and chosen by the actuary's personal judgement.

To sum up, the main advantage of this method is that is really easy to be applied and simple to be understood. These facts are the reasons for its popularity. However, there are some drawbacks. Firstly, it provides no information on the prediction error. In addition, the results are not credible if there are different factors that affect each year (e.g. inflation).

Next, we will apply the chain ladder algorithm to our numerical example. First of all, the table below represents the "cumulative paid losses triangle" and it contains the claims observations X_{kj} :

Table 2: Example of cumulative claims' triangle

		Cumulative Losses in Development Year									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	5.960.020	14.996.990	16.845.733	17.370.865	18.373.995	19.360.911	21.753.184	21.832.807	22.013.607	22.032.644
2004	**2**	7.510.865	20.138.517	22.374.799	22.886.914	23.371.944	23.414.825	23.502.792	25.627.259	25.627.259	
2005	**3**	25.092.681	39.365.744	40.754.107	42.031.046	42.109.323	42.312.364	42.990.043	44.661.727		
2006	**4**	26.950.816	61.434.645	63.756.892	64.120.784	64.821.352	64.893.512	64.974.427			
2007	**5**	38.615.756	63.441.956	68.973.861	69.934.555	70.413.798	70.492.233				
2008	**6**	35.903.255	62.717.551	73.957.755	82.747.067	85.629.135					
2009	**7**	47.734.726	69.382.865	77.136.406	78.037.115						
2010	**8**	18.329.702	41.784.089	45.067.546							
2011	**9**	21.124.374	46.423.109								
2012	**10**	24.241.030									

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Then, we will calculate the weighted average development factors:

Table 3: Calculating the development factors between periods

	Development Year								
	1 - 2	2 – 3	3 - 4	4 - 5	5 - 6	6 - 7	7 - 8	8 - 9	9 - 10
Average development factors	1,8470	1,0954	1,0366	1,0188	1,0063	1,0216	1,0439	1,0038	1,0009

Having calculated the development factors, we can now estimate the future cumulative claims. The estimated values $\hat{X}_{kj}$ are being presented in the table below in the grey area.

Table 4: Prediction of the future claims using CL method

		Forecast Cumulative Losses in Development Year									
Accident Year		1	2	3	4	5	6	7	8	9	10
2003	**1**	5.960.020	14.996.990	16.845.733	17.370.865	18.373.995	19.360.911	21.753.184	21.832.807	22.013.607	22.032.644
2004	**2**	7.510.865	20.138.517	22.374.799	22.886.914	23.371.944	23.414.825	23.502.792	25.627.259	25.627.259	25.649.427
2005	**3**	25.092.681	39.365.744	40.754.107	42.031.046	42.109.323	42.312.364	42.990.043	44.661.727	44.831.888	44.870.668
2006	**4**	26.950.816	61.434.645	63.756.892	64.120.784	64.821.352	64.893.512	64.974.427	67.828.104	68.086.529	68.145.423
2007	**5**	38.615.756	63.441.956	68.973.861	69.934.555	70.413.798	70.492.233	72.014.513	75.177.390	75.463.816	75.529.092
2008	**6**	35.903.255	62.717.551	73.957.755	82.747.067	85.629.135	86.169.797	88.030.634	91.896.939	92.247.067	92.326.860
2009	**7**	47.734.726	69.382.865	77.136.406	78.037.115	79.505.617	80.007.615	81.735.380	85.325.198	85.650.287	85.724.374
2010	**8**	18.329.702	41.784.089	45.067.546	46.718.730	47.597.883	47.898.416	48.932.783	51.081.910	51.276.533	51.320.887
2011	**9**	21.124.374	46.423.109	50.851.316	52.714.407	53.706.387	54.045.489	55.212.601	57.637.539	57.857.138	57.907.184
2012	**10**	24.241.030	44.773.837	49.044.724	50.841.625	51.798.362	52.125.417	53.251.066	55.589.853	55.801.650	55.849.918

Using the estimated future cumulative claims, we can easily extract the estimated outstanding losses removing from the last column the observations of the 10th experience period.

The results are being presented in the following table:

Table 5: Estimated Losses with CL method (annually and total reserves)

	Accident Year										
	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	Total
Estimated Outstanding Claims	0	22.168	208.941	3.170.997	5.036.859	6.697.726	7.687.260	6.253.341	11.484.075	31.608.888	**72.170.254**

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As it can be observed, the estimated claims increase over the accident years. That fact is reasonable since the expected claims for each subsequent year are higher than the paid ones (or occurred ones) until today.

Thus, the application of the basic Chain ladder algorithm in our numeric example estimates the reserves to be equal to 72.170.254 . This amount represents the necessary reserves that the insurance company should have raised in order to meet its customers' obligations.

However, as it should be pointed out, this amount does not have to be kept as a reserve at once, but gradually over the years. Thus, it is important to observe the reserves needed for each year, as shown in the following table.

Table 6: Estimated Cash Flows

	Forecast Cumulative Losses in Development Year									
Accident Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003										
2004										22.168
2005									170.161	38.780
2006								2.853.677	258.425	58.895
2007							1.522.280	3.162.877	286.426	65.276
2008						540.662	1.860.837	3.866.305	350.127	79.794
2009					1.468.502	501.998	1.727.764	3.589.818	325.089	74.087
2010				1.651.185	879.153	300.533	1.034.366	2.149.128	194.622	44.354
2011			4.428.208	1.863.091	991.980	339.102	1.167.112	2.424.937	219.599	50.046
2012		20.532.807	4.270.887	1.796.901	956.738	327.055	1.125.648	2.338.787	211.797	48.268
	2013	**2014**	**2015**	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**Total**
	33.189.649	12.836.048	9.028.804	6.335.428	4.048.178	3.819.295	2.602.740	261.844	48.268	**72.170.254**

So, we calculated the reserves needed for each year between 2013 and 2021. Nevertheless, it is naive to believe that the insurance company just keeps the reserves without investing them. That is almost never the case. Insurance companies can invest in various investment products like bonds, options, derivatives, repos and others.

Thus, the actuary should have knowledge of the company's investment portfolio and strategic in order to use the proper yield curve. Using the proper

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rates, the actuary can calculate the discounting factors and, as a consequence, the reserves needed for each year. For simplicity's sake, in our example we will use a fixed mean rate of 1,5%.

Table 7: Discounting factors and reserves.

Year	Reserves before discounting	Discounting factors	Reserves after discounting
2013	33.189.649	0,985	32.699.162
2014	12.836.048	0,971	12.459.460
2015	9.028.804	0,956	8.634.399
2016	6.335.428	0,942	5.969.140
2017	4.048.178	0,928	3.757.763
2018	3.819.295	0,915	3.492.907
2019	2.602.740	0,901	2.345.138
2020	261.844	0,888	232.442
2021	48.268	0,875	42.215
Total	**72.170.254**	-	**69.632.626**

Hence, the application of the classical Chain ladder method in our numeric example estimates the discounted reserves to be equal to 69.632.626 instead of 72.170.254 , which is the amount of money the company would need without investing its funds.

Bornhuetter-Ferguson

The Bornhuetter-Ferguson (BF) method was published in 1972 by Bornhuetter and Ferguson. It is used worldwide and is usually preferred in new lines of business due to the fact that there is almost no experience and knowledge of the historical data.

The basic concept of the method is the calculation of reserves as a proportion of the ultimate losses estimated from before. The proportions that are more widely used are the ones that arise from the development factors calculated in the Chain Ladder method, although that is not always necessary in order for the method to work.

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This method has plenty advantages. First of all, the estimation of the reserves is quite easy to calculate. Moreover, like the Chain ladder method, the Bornhuetter-Ferguson method can be used with both incurred and paid data.

As it seems, fluctuations that happen randomly in volatile periods do not distort the estimated ultimate losses in a significant level while in the same case the Chain ladder method fails. That is why it is really useful in cases that asymmetry exists (when long-tailed lines are observed).

On the other hand, it is not really effective when it meets short-tailed lines of insurance. Furthermore, if a development factor is less than 1, then the Bornhuetter-Ferguson method will not do any good.

While the Chain ladder method supports that the claims are proportional to the amount of claims which is known until the present day, the Bornhuetter-Ferguson method assumes that claims will develop according to the expected losses; fact that gives kind of an independency to the BF method.

In order to estimate the ultimate losses, one must make use of external information. A knowledge of a company's earned premiums will be needed. This fact the Bornhuetter-Ferguson method to be considered by many as a Bayesian methodology.

In practice, to make use of the Bornhuetter-Ferguson method one should apply the following steps. First of all, expected Loss Ratio will be calculated (unless it is given as a fixed percentage from before). This can be done using the formula below:

$$(Expected\ Loss\ Ratio)_i = \frac{(Losses)_i}{(Earned\ Premium)_i}$$

We will first estimate the loss ratio for the first time period $(Expected\ Loss\ Ratio)_1$. Using this first estimation as a reference point, we will now proceed to the estimation of the Ultimate Losses:

$$(Expected\ Ultimate\ Losses)_i = (Earned\ Premium)_i \cdot (Expected\ Loss\ Ratio)_1$$

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Next, we are going to make use of the development factors that have already been calculated in the Chain Ladder method. We will calculate the product of the above development factors for each period in order to obtain the Ultimate development factors which project the claims' development in the remaining period of time.

Thus, we have:

$$f_{ult_i} = \left(\prod_j f_j \right)_i$$

where j represents development period.

For instance, in the numeric example we previously used in Chain Ladder method concerning fire insurance, we will obtain the following table:

Table 8: Moving from development factors to Ultimate development factors

	Development Year								
	1 - 2	**2 - 3**	**3 - 4**	**4 - 5**	**5 - 6**	**6 - 7**	**7 - 8**	**8 - 9**	**9 - 10**
Development factors	1,8470	1,0954	1,0366	1,0188	1,0063	1,0216	1,0439	1,0038	1,0009
Ultimate development factors	2,3039	1,2474	1,1388	1,0985	1,0782	1,0715	1,0488	1,0047	1,0009

Having calculated the above values, we are going to find the estimated Loss Reserves using the formula below:

$$(Estimated\ Loss\ Reserves)_i = (Estimated\ Ultimate\ Losses)_i \cdot \left(1 - \frac{1}{f_{ult_i}}\right)$$

So, the total reserves an insurance company will need is estimated taking the sum of the estimated loss reserves:

$$Total\ Estimated\ Loss\ Reserves = \sum_i (Estimated\ Loss\ Reserves)_i$$

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For instance, we will assume that the earned premiums in the fire insurance example are the ones presented below (next to the cumulative losses):

Table 9: Example of having both cumulative losses and earned premiums as data

Accident Year	Cumulative Losses	Earned Premiums
2003	22.032.644	25.000.000
2004	25.627.259	30.000.000
2005	44.661.727	45.000.000
2006	64.974.427	70.000.000
2007	70.492.233	72.000.000
2008	85.629.135	88.000.000
2009	78.037.115	90.000.000
2010	45.067.546	93.000.000
2011	46.423.109	95.000.000
2012	24.241.030	97.000.000

Then, the Expected Loss Ratio of the first period will be:

$$(Expected\ Loss\ Ratio)_1 = \frac{(Losses)_1}{(Earned\ Premium)_1} = \frac{22.032.644}{25.000.000} = 0{,}8813$$

Having 0,8813 as the original estimation of the loss ratio, we will then estimate the expected ultimate losses. Then, we will estimate the loss reserves using the ultimate development factors as seen below:

Table 10: Estimating Losses with BF method

Accident Year	Expected Utimate Losses	$\left(1 - \frac{1}{f_{ult_i}}\right)$	Estimated Loss Reserves
2003	22.032.644	0,0000	0
2004	26.439.172	0,0009	22.850
2005	39.658.758	0,0047	184.671
2006	61.691.402	0,0465	2.870.673
2007	63.454.013	0,0667	4.231.600
2008	77.554.905	0,0725	5.626.114
2009	79.317.517	0,0897	7.112.730
2010	81.961.434	0,1218	9.986.827
2011	83.724.045	0,1983	16.604.040
2012	85.486.657	0,5660	48.382.133
			95.021.640

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Thus, the application of Bornhuetter – Ferguson method in our numeric example estimates the reserves to be equal to 95.021.640 . This amount represents the necessary reserves that the insurance company should have raised in order to meet its customers' obligations.

Separation Method

The chain ladder method projects the inflation of the past into the future. Thus, it gives credible results when the inflation rate is constant. When it is not constant, an alternative method should be used that will incorporate the changes in the inflation during the years.

The separation method was developed in the 70s as a response to the above need. Unlike the chain ladder method, this method studies the inflation separately.

According to Taylor (1977) and Björkwall (2010) we can obtain the following equation for the conditional expectation of the average losses per claim given the total number of claims:

$$E\left(\frac{C_{ij}}{N_i}\middle|N_i\right) = r_j \cdot \lambda_\kappa$$

where C_{ij} represents the cumulative claims for accident year i and development year j, N_i represents the ultimate number of claims for accident year i, r_j represents the proportion of the claims that will occur in development year j $\left(\sum_{j=0}^{t} r_j = 1\right)$ and λ_κ represents the calendar year effect (incorporating the inflation).

The above equation forms the basic assumption of this method and, thus, we can create the following table:

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Table 11: Run-off triangle of expected average claim cost

	Development Year					
Accident Year	**0**	**1**	**2**	⋯	$t-1$	t
0	$r_0 \cdot \lambda_0$	$r_1 \cdot \lambda_0$	$r_2 \cdot \lambda_0$	⋯	$r_{t-1} \cdot \lambda_0$	$r_t \cdot \lambda_0$
1	$r_0 \cdot \lambda_1$	$r_1 \cdot \lambda_1$	$r_2 \cdot \lambda_1$	⋯	$r_{t-1} \cdot \lambda_1$	
2	$r_0 \cdot \lambda_2$	$r_1 \cdot \lambda_2$	$r_2 \cdot \lambda_2$	⋯		
⋮	⋮	⋮	⋮			
$t-1$	$r_0 \cdot \lambda_{t-1}$	$r_1 \cdot \lambda_{t-1}$				
t	$r_0 \cdot \lambda_t$					

Assuming that the proper information is provided (e.g. a claim counts' triangle), the N_i can be calculated and, thus, treated as known. As a consequence, we can obtain estimates of $\hat{r}_j$ and $\hat{\lambda}_\kappa$ using:

- the observed values: $s_{ij} = \frac{c_{ij}}{\widehat{N}_i}$
- the marginal sum equations for the diagonals of the upper triangle:

$$s_{k0} + s_{k-1,1} + \cdots + s_{0k} = (\hat{r}_0 + \cdots + \hat{r}_k) \cdot \hat{\lambda}_\kappa \ , \qquad k = 0, \dots, t$$

- the marginal sum equations for the columns of the upper triangle:

$$s_{0j} + s_{1j} + \cdots + s_{t-j,j} = \left(\hat{\lambda}_j + \cdots + \hat{\lambda}_t\right) \cdot \hat{r}_j \ , \qquad j = 0, \dots, t$$

Taylor proves that there is a unique solution to the above equations:

$$\hat{\lambda}_\kappa = \frac{\sum_{i=0}^{k} s_{i,k-i}}{1 - \sum_{j=k+1}^{t} \widehat{r_j}} \ , \qquad k = 0, \dots, t$$

and

$$\hat{r}_j = \frac{\sum_{i=0}^{t-j} s_{ij}}{\sum_{k=j}^{t} \hat{\lambda}_\kappa} \ , \qquad j = 0, \dots, t$$

where $\sum_{j=k+1}^{t} \widehat{r_j}$ is considered as zero when $k = t$.

Therefore, cumulative claims of upper triangle can now be estimated using the following equation:

$$\hat{C}_{ij} = \widehat{N}_i \cdot \hat{r}_j \cdot \hat{\lambda}_\kappa$$

Nevertheless, in order to estimate the claims of the lower triangle, the prediction of λ_k for $t+1 \leq k \leq 2t$ should be done in advance.

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To sum up, the separation method is an upstanding claims' reserving method that can isolate the effects that various factors (like inflation) have on a calendar year. Still, it remains quite unpopular compared to the previous two methods.

Stochastic reserving methods

The main weakness of the methods mentioned before is that they do not provide adequate information about the uncertainty associated with the final estimation. For this reason, a stochastic procedure is required.

Mach Model

One of the first people that managed to successfully reproduce the reserve estimates of the chain-ladder method was Mack in 1993. He proposed a stochastic model that was distribution-free with no assumptions. That is the reason why it is one of the most famous stochastic claims' reserving methods.

In this model, Mack suggests a specific formula for the calculation of the mean squared error of the estimated reserves. His model provides us with unbiased estimators for the development factors used in the chain-ladder method. Having calculated the development factors, we can move on to the estimation of the ultimate claims, with the result being an unbiased estimate. Basically, he uses the weighted average of the development factors to estimate the "fitted" claims, while using the actual claims as the observed values. Hence, he scientifically supports the calculations that the basic chain-ladder has already made by logic but without theoretical background.

To understand the model, we can denote as D_{ij} the cumulative claims for the accident year i and the development year j. Then, Mack made the following assumptions:

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- For each $j = 1, \dots, n-1$, development factors f_j exist for which: $E\left[D_{i,j+1} | D_{i1}, \dots, D_{ij}\right] = D_{ij} \cdot f_j$
- The accident years i are independent i.e. for each $s \neq t$, $\{D_{s1}, \dots, D_{sn}\}, \{D_{t1}, \dots, D_{tn}\}$ are independent
- For each $j = 1, \dots, n-1$, parameters ${\sigma_j}^2$ exist for which: $Var\left[D_{i,j+1} | D_{i1}, \dots, D_{ij}\right] = D_{ij} \cdot {\sigma_j}^2$

As it can be observed, Mack makes use of conditional expectations. That is used as a solution to a quite fundamental problem; the fact that both his explanatory and response variables are stochastic. Since standard linear regression's assumption is that only the response variables are stochastic (the explanatory ones are always considered known), conditions expectations should be used in the formula to overcome that obstacle.

Furthermore, Mack calculated the following estimators for the unknown parameters f_j and ${\sigma_j}^2$:

$$\hat{f_j} = \frac{\sum_{i=1}^{n-j} D_{i,j+1}}{\sum_{i=1}^{n-j} D_{ij}}$$

$${\hat{\sigma}_j}^2 = \frac{1}{n-j-1} \sum_{i=1}^{n-j} D_{ij} \cdot \left(f_{ij} - \hat{f_j}\right)^2$$

where $f_{ij} = \frac{D_{i,j+1}}{D_{ij}}$ and ${\hat{\sigma}_{n-1}}^2 = min\left(\frac{{\hat{\sigma}_{n-2}}^4}{{\hat{\sigma}_{n-3}}^2}, min\left({\hat{\sigma}_{n-3}}^2, {\hat{\sigma}_{n-2}}^2\right) \right)$

In Mack's model, the estimator $\hat{f_j}$ is the same with the development factor already seen in the chain-ladder method. According to Mack, the estimators $\hat{f_j}$ are unbiased estimators of the development factors f_j and there is no correlation between them. This means that if claims happen to be exceptionally low in an accident year, the chain-ladder's development factor will not securely predict the future claims since it assumes that history will repeat itself, without that always being the case.

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So, the estimated future claims $\widehat{D}_{ik}$ will be calculated in the exact way as they were calculated in the chain-ladder method; multiplying the observations of the n-th experience period with the future development factors. Thus, we have:

$$\widehat{D}_{ik} = D_{i,n-i+1} \prod_{j=n-i+1}^{n} \hat{f}_j$$

while the estimation of the claims reserves in each accident year emerges from the following relation:

$$\hat{R}_i = \widehat{D}_{i,k} - D_{i,n-i+1}$$

Finally, the total claims reserves are obtained from the equation:

$$\hat{R} = \hat{R}_1 + \hat{R}_2 + \cdots + \hat{R}_n$$

Assuming independence between the accident years, we can calculate the standard error of the claims' prediction in a specific year. Thus, the Mean Square Error of Prediction is given by the following equation:

$$MSEP\left[\widehat{D}_{ij}\right] = E\left[\left(D_{ij} - \widehat{D}_{ij}\right)^2\right] = E\left[D_{ij} - E\left[D_{ij}\right]^2\right] + E\left[\widehat{D}_{ij} - E\left[\widehat{D}_{ij}\right]^2\right]$$

which can be assumed approximately similar to:

$$MSEP\left[\widehat{D}_{ij}\right] = Var\left[D_{ij}\right] + Var\left[\widehat{D}_{ij}\right]$$

where $Var\left[D_{ij}\right]$ represents the standard error and $Var\left[\widehat{D}_{ij}\right]$ the estimation error.

According to the above, Mack calculated the standard error of reserves R_i for the accident year i as:

$$Var[R_i] = \widehat{D}^2{}_{in} \cdot \sum_{k=n-i+1}^{n-1} \frac{\hat{\sigma}^2{}_{k+1}}{\hat{f}^2{}_{k+1} \cdot \widehat{D}_{ik}}$$

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and the estimation error of reserves R_i for the accident year i as:

$$Var\left[\hat{R}_i\right] = \hat{D}^2{}_{in} \cdot \sum_{k=n-i+1}^{n-1} \frac{\hat{\sigma}^2{}_{k+1}}{\hat{f}^2{}_{k+1} \cdot \sum_{q=1}^{n-k} D_{qk}}$$

Thus, the standard error for the reserve estimate for the accident year i is given by the following equation:

$$MSEP\left[\hat{R}_i\right] = E\left[\left(D_{ij} - \hat{D}_{ij}\right)^2\right] = \hat{D}^2{}_{in} \sum_{k=n-i+1}^{n-1} \frac{\hat{\sigma}^2{}_k}{\hat{f}^2{}_k}\left(\frac{1}{\hat{D}_{ik}} + \frac{1}{\sum_{q=1}^{n-k} D_{qk}}\right)$$

Finally, the standard error for the total reserves estimate is given by the following equation:

$$MSEP\left[\hat{R}\right] = \sum_{i=2}^{n}\left\{MSEP\left[\hat{R}_i\right] + \hat{D}_{in}\left(\sum_{q=i+1}^{n} D_{qn}\right)\sum_{k=n-i+1}^{n-1} \frac{2 \cdot \hat{\sigma}^2{}_k}{\hat{f}^2{}_k \cdot \sum_{q=1}^{n-k} D_{qk}}\right\}$$

To sum up, Mack demonstrated the steps in order to calculate the prediction error in Chain-ladder method. Thus, he completes the basic chain-ladder algorithm by making a stochastic approach.

Bootstrapping

According to England and Verall, incremental claims can be modeled by an over-dispersed Poisson distribution. Considering the incremental paid claims as C_{ij}, with i being the accident year and j being the development year, they proposed the following relations:

$$E\left[C_{ij}\right] = m_{ij} \text{ and } Var\left[C_{ij}\right] = \varphi \cdot E\left[C_{ij}\right] = \varphi \cdot m_{ij}$$

$$log\left(m_{ij}\right) = n_{ij}$$

$$n_{ij} = c + a_i + \beta_j\,, a_1 = \beta_1 = 0$$

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The above equations define a generalized linear model in which the claims C_{ij} are being modeled with a logarithmic link function and the variance is greater than the mean value. The logarithmic link function is being used in order to change the model's parameters so that the logarithm of the mean value has a linear form instead of a multiplicative one. Thus, the model has a form similar to the Chain ladder method, since there is a parameter corresponding to each raw i and a parameter corresponding to each column j. The φ is an unknown scale parameter which is about to be estimated.

The model is durable for a small number of negative incremental claims, provided that the sum of incremental claims of each raw and column of the run-off triangle will always give a positive sign. Due to the logarithmic link function, the adjusted values that will occur will always be positive numbers.

Furthermore, the residuals should have a specific form in order to be suitable for the generalized linear model. The most common residuals that are being tested are the Deviance residuals and Pearson residuals, whereas there also exist the Anscombe residuals which are not particularly useful to Bootstrap procedure.

More specifically, the equations of interest are:

- Unscaled Deviance Residual:

$$r_D = sign(C - m)\sqrt{2(C\log(C/m) - C + m}$$

- Unscaled Pearson Residual:

$$r_P = \frac{C - m}{\sqrt{m}}$$

- Unscaled Anscombe Residual:

$$r_A = \frac{\frac{3}{2}\left(C^{\frac{2}{3}} - m^{\frac{2}{3}}\right)}{m^{1/6}}$$

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In the Bootstrap process, resampling is being used with replacement from the residuals. A new data sample is created using the inverted formula and the resampled residuals. Since Anscombe residuals are not used so widely, our focus will be given into solving Deviance residuals and Pearson residuals for C. Seeing that Pearson residuals can be solved more easily for C, the following relation arises:

$$C^* = r_P^* \sqrt{m} + m$$

where r_P^* represents the Pearson residuals that result from the resampling, m the fitted values and C^* the bootstrap incremental claim.

Therefore, having obtained our new bootstrap sample with the incremental claims, we can adjust our data in the run-off triangle and get a new bootstrap sample with the cumulative claims and the new development factors. Then, we apply the Chain ladder method in the Bootstrap data. Thus, the future claims that will emerge and the calculation of the reserves will be calculated in the way that has been mentioned before.

The whole process is completed using resampling from the residuals many times (N times). Each time a new bootstrap sample is being created and, as a result, a new bootstrap residual. Bootstrap standard error is the standard deviation of the N bootstrap reserves' estimation.

Another significant part that must be pointed out is the scale parameter φ. While it is not used in the bootstrap process, it is necessary in the calculation of the process error. Therefore, the Pearson scale parameter is given by:

$$\varphi_P = \frac{\sum r_P^2}{n - p}$$

where n is the number of data points in the sample and p is the number of estimated parameters.

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As it can be easily observed, increasing the number of parameters in the model leads to a smaller scale parameter and acts like a penalty (ceteris paribus).

To calculate the bootstrap prediction error, it is essential to use an estimate of the variance. This can be easily calculated since it is given by the reserve estimates multiplied by the scale parameter. Then, the prediction error is given by:

$$PE_{bs} = \sqrt{\varphi_p R + \frac{n}{n-p}(SE_{bs}(R))^2}$$

where R is total reserve of one accident year and $SE_{bs}(R)$ is the standard error of the bootstrap estimated reserves.

To sum up, in order to make use of the bootstrap process certain steps should be followed. Firstly, we put the cumulative claims into a run-off triangle and we calculate the development factors. Then, we estimate future cumulative claims and calculate the incremental claims (m) taking the differences between two development years. We calculate Pearson residuals and Pearson scale parameter.

Next, we start a loop which we are going to repeat N times. In the unscaled Pearson residuals, we apply a random resampling with replacement, thus creating a new residuals' triangle. Solving for C, we obtain a new set of incremental claims C^*. Then, we use the Chain ladder method in the bootstrap incremental claims and estimate the future claims. Then, for each cell (i, j) we find the incremental claims taking the differences, which are going to be used as mean value during the simulation. For each cell of the triangle we simulate a claim using Over-Dispersed Poisson. Finally, we add the future payments and take each year's reserve leading to the total reserves. We keep the results and repeat the process.

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In the end, the results that we collected will give us the estimated distribution of the future claims. Then, a comparison should be made between the mean value of the bootstrap reserves and the reserves calculated by plain Chain ladder method. The standard deviation of the results should give us an estimation of the prediction error.

Stochastic Bornhuetter-Ferguson

We have previously become familiar with the Bornhuetter – Ferguson method. Since this method uses some external prior information in order to make the prediction, it shows similarities to a Bayesian method. In 2004, this method was studied as a Bayesian method by Verrall in the scope of generalized linear models.

As it has already been mentioned, incremental claims can be modeled by an over-dispersed Poisson distribution. So, this time Verall analyzes the connection between chain-ladder technique (claims following ODP) and the external information about company's earned premiums used in Bornhuetter – Ferguson method (used as a prior distribution).

Thus, Verrall considers the following model:

$$C_{ij}|x, y, \varphi \sim independent\ over-dispersed\ Poisson$$
$$with\ mean\ x_i \cdot y_i\ and\ \sum_{k=1}^{n} y_k = 1$$

and

$$x_i|\alpha_i, \beta_i \cdot \sim independent\ \Gamma(\alpha_i, \beta_i)$$
$$and\ improper\ prior\ distributions\ are\ used\ for\ y_j$$

Then, he derives the distribution $C_{ij}|C_{i1}, C_{i2}, \cdots, C_{i,j-1}, y, \varphi$. It is shown that this is a negative binomial distribution. The distribution's mean for the Bayesian method could be written as:

$$\left[Z_{ij} \cdot \frac{D_{i,j-1}}{S_{j-1}} + \left(1 - Z_{ij}\right)\frac{\alpha_i}{\beta_i}\right] y_i$$

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where $Z_{ij} = \frac{S_{j-1}}{\beta_i \cdot \varphi + S_{j-1}}$, $\frac{\alpha_i}{\beta_i}$ is the prior mean of the cumulative claims (BF method) and $\frac{D_{i,j-1}}{S_{j-1}}$ is the estimate of cumulative claims from the data (CL method).

Thus, we have a combination of two extreme situations:

- Chain-ladder method making use of the data and assuming no prior information
- Bornhuetter-Ferguson method assuming exclusively prior information and making no use of the data

Verrall uses two approaches. In the first approach, prior distributions are applied to the rows' parameters in advance and, as a result, the column parameters' estimation of chain-ladder will be affected. In the second approach, development factors are calculated first and, then, prior distributions are applied to the rows' parameters. Between the two approaches, the one that recreates the Bornhuetter-Ferguson method is the second one.

For the implementation of the two models, Markov Chain Monte Carlo approach is being used. Thus, the prediction errors come from the iterations and not from a specific given formula.

The Verrall's study of stochastic Bornhuetter-Ferguson model is an interesting approach combining two of the most famous claim reserving methods in a stochastic way. A drawback of this approach is that it may collapse if negative incremental claims appear.

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4. Application of the methods in a real-life example

The claims reserving methods mentioned before have been presented mostly in a theoretical way. We are about to apply them in a real-life example. Be doing so, different results will occur and there comes the basic problem that actuaries face each time; choosing the method that best suits their data.

Thus, we obtained 11 IBNR run-off triangles each containing real-life data of a different insurance sector. The different sectors are presented in the table below:

Table 12: Different insurance sectors of interest

Code	Insurance Sector
10	Personal Accidents
12	Land Transport
15	Marine
16	Transported goods
17	Fire
18	Other property damage
19	Land Transport Liability
21	Marine Liability
22	General liability
25	Financial losses
26	Legal protection

Next, we are going to use the previous methodologies and comment on the different outputs. More specifically, we are about to apply the basic Chain Ladder method and the Mack's model. These two methods give us same results with the difference that Mack's model provides us with information on the standard error. Then, we are about to use Bootstrap Chain Ladder running 1.000 iterations and assuming that claims follow the over-dispersed Poisson distribution.

The results that follow have been computed using the statistic programming language R.

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Insurance Sector 10

The observations of the cumulative claims in this sector are presented in the table that follows:

Table 13: Run-off triangle containing cumulative claims of Sector 10

Sector 10		Cumulative Losses in Development Year									
Accident Year		1	2	3	4	5	6	7	8	9	10
2003	1	2.205.148	3.610.926	4.040.310	4.266.151	4.337.853	4.238.327	4.270.133	4.275.592	4.315.592	4.352.449
2004	2	3.279.940	4.453.472	5.580.249	6.097.660	6.306.551	6.555.568	6.841.163	6.924.622	6.970.531	
2005	3	5.034.676	6.741.909	7.828.080	9.090.894	9.643.974	10.480.053	10.537.618	10.557.846		
2006	4	5.211.477	7.152.673	8.014.106	8.673.165	9.531.665	9.850.442	10.260.685			
2007	5	5.654.183	7.955.319	8.875.829	9.585.017	9.933.080	10.320.729				
2008	6	4.149.360	5.776.320	6.214.188	6.867.370	7.268.383					
2009	7	3.276.961	5.000.580	5.544.967	6.569.379						
2010	8	2.728.236	4.677.924	6.101.055							
2011	9	2.420.559	4.138.315								
2012	10	2.053.287									

The above table containing cumulative claims can be visualized with the following plot:

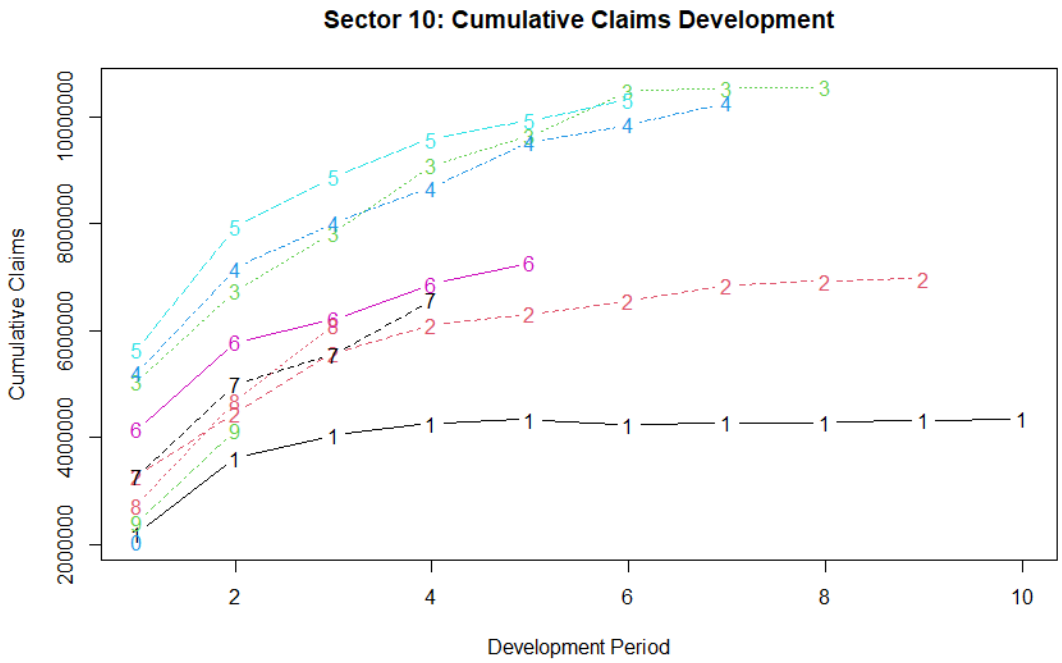

Figure 6: The development of cumulative claims across the periods

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Firstly, we obtain that the total paid claims' amount which is 68.592.659 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 77.179.255,65. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *8.586.596,65*.

Then, using the Mack's model, we can also obtain the standard error of the estimations of each accident year. The results are displayed in *Table 11*.

Table 14: IBNR estimation using Mack's model

Accident Year	Paid Losses	Total Estimated Losses	IBNR	Mack S.E.
2003	4.352.449	4.352.449	0	0
2004	6.970.531	7.030.062	59.531	5.232
2005	10.557.846	10.729.688	171.842	21.565
2006	10.260.685	10.480.263	219.578	66.789
2007	10.320.729	10.807.536	486.807	231.831
2008	7.268.383	7.935.171	666.788	356.653
2009	6.569.379	7.564.788	995.409	416.006
2010	6.101.055	7.795.436	1.694.381	548.144
2011	4.138.315	6.083.577	1.945.262	676.461
2012	2.053.287	4.400.287	2.347.000	819.435
	68.592.659	**77.179.255,65**	**8.586.596,65**	

Thus, the IBNR through Mack's model is also equal to *8.586.596,65*. This time we have the additional information that the standard error of this value is *1.528.171,03* and, subsequently, the coefficient of variation is 18%.

A symmetric 95% confidence interval for the reserves is given by:

$$\hat{R} - 1{,}96 \cdot s.e.\left(\hat{R}\right) < R < \hat{R} + 1{,}96 \cdot s.e.\left(\hat{R}\right)$$

$$\Rightarrow 5.591.381 < R < 11.581.811$$

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Mack results can also be visualized in the *Figure 4* below:

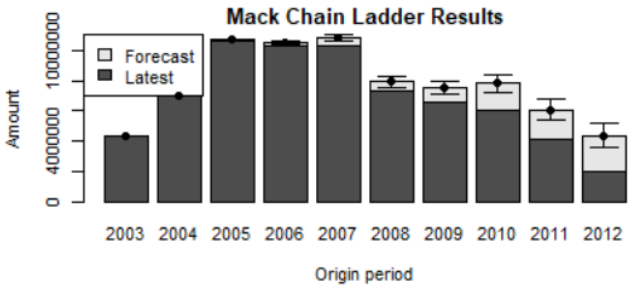

Figure 7: Boxplot of paid vs future claims

Furthermore, in the next plot we can observe how logarithms of Mack's estimators $\hat{\sigma}_j^2$ alter over the accident years:

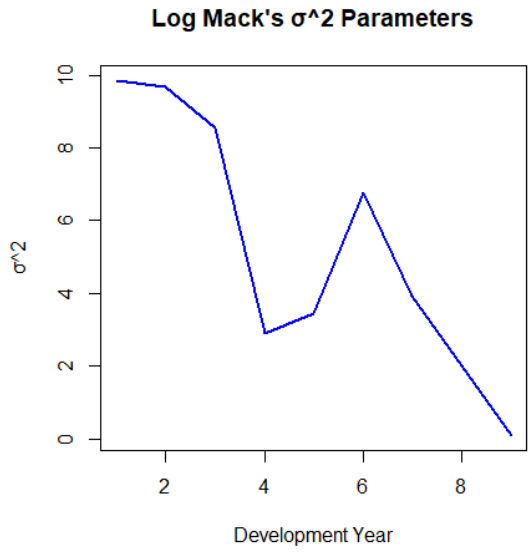

Figure 8: The logarithm of Mack's variance parameter

Next, we move on to the implementation of Bootstrap. As mentioned before, we chose to run 1000 iterations and assumed that the incremental claims follow the over-dispersed Poisson distribution.

The results are displayed in the following table:

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Table 15: Predicting IBNR through Bootstrap procedure (number of iterations = 1000)

Accident Year	Paid Losses	Total Estimated Losses	IBNR	Bootstrap S.E.
2003	4.352.449	4.352.449	0	0
2004	6.970.531	7.039.450	68.919	396.641
2005	10.557.846	10.742.858	185.012	700.289
2006	10.260.685	10.474.263	213.578	709.680
2007	10.320.729	10.815.479	494.750	1.142.365
2008	7.268.383	7.935.583	667.200	1.318.666
2009	6.569.379	7.585.805	1.016.426	1.832.988
2010	6.101.055	7.826.029	1.724.974	2.800.598
2011	4.138.315	6.056.616	1.918.301	3.067.193
2012	2.053.287	4.394.365	2.341.078	3.749.528
	68.592.659	**77.222.897**	**8.630.238**	

Thus, the IBNR through Bootstrap procedure is equal to *8.630.238*. The standard error of the above calculation is equal to *2.037.896*.

A symmetric 95% confidence interval for the reserves is given by:

$$\hat{R} - 1{,}96 \cdot s.e.\left(\hat{R}\right) < R < \hat{R} + 1{,}96 \cdot s.e.\left(\hat{R}\right)$$

$$\Rightarrow 4.635.962 < R < 12.624.514$$

The coefficient of variation is 24% which is higher than Mack's model.

As it can be observed, both stochastic procedures have quite similar outputs with Bootstrap giving a little higher IBNR and 6% higher CV.

Below, we present a Histogram of the different values of IBNR through the iterations.

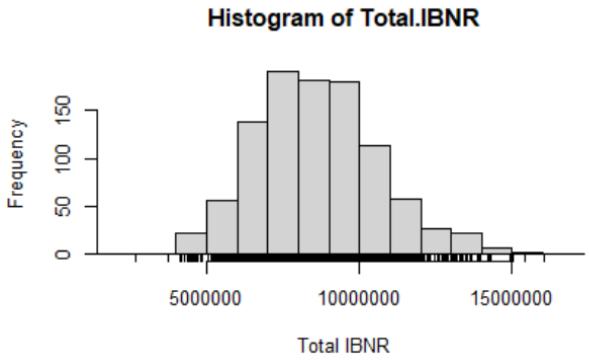

Figure 9: Histogram presenting the different IBNR values emerged from the 1000 iterations

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Furthermore, in the next plot we can notice that the IBNR results from Mack and Bootstrap method pretty much give similar results.

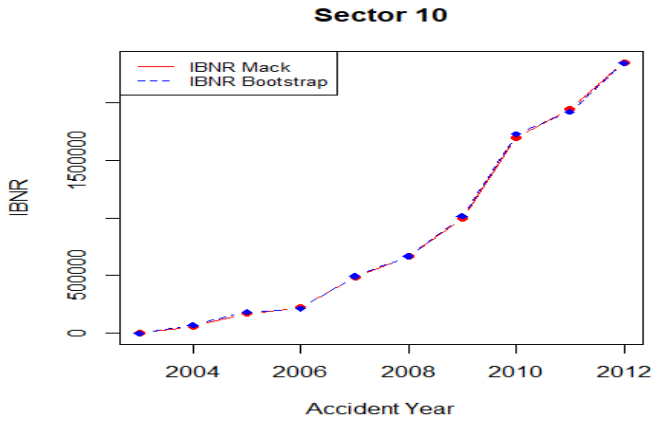

Figure 10: Comparing the IBNR results from Mack's model with Bootstrap procedure

Finally, the above methods give us estimations about the Incurred But Not Reported claims. The results correspond to the expected value of IBNR, meaning the one that is more probably to occur.

However, under Solvency II Directive, insurance firms should not just keep the reserve amount that is more probable to be needed. Instead, they should hold an amount that will have them covered in 99.5% of the cases. Thus, it is necessary to calculate the 99.5% quantile of the estimated IBNR claims.

Table 16: Mean value of Bootstrap IBNR vs 99,5% quantile

Accident Year	Bootstrap IBNR	99,5% Quantile
2003	0	0
2004	68.919	718.297
2005	185.012	1.273.269
2006	213.578	1.183.593
2007	494.750	1.660.053
2008	667.200	2.097.376
2009	1.016.426	2.395.409
2010	1.724.974	3.248.748
2011	1.918.301	3.782.903
2012	2.341.078	4.832.851
	8.630.238	**14.093.382**

As a result, the reserves that an insurance firm must hold in order to be safe in the 99,5% of the cases should be equal to *14.093.382*.

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Insurance Sector 12

In the Sector 12, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 17: Run-off triangle containing cumulative claims of Sector 12

Sector 12		**Cumulative Losses in Development Year**									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	31.001.859	41.918.112	42.757.623	42.965.422	43.012.487	43.185.041	43.206.302	43.220.458	43.220.378	43.201.772
2004	**2**	32.038.869	45.090.185	46.079.915	46.262.400	46.402.465	46.556.894	46.575.043	46.571.269	46.578.231	
2005	**3**	44.403.019	57.447.060	58.056.335	58.268.079	58.322.316	58.336.289	58.460.439	58.503.506		
2006	**4**	48.413.684	65.951.694	66.016.307	66.302.334	66.386.392	66.390.322	66.401.920			
2007	**5**	70.482.722	90.644.444	91.894.616	92.187.836	92.086.643	92.167.058				
2008	**6**	56.413.962	82.132.585	83.252.938	83.924.407	84.300.232					
2009	**7**	87.436.036	121.802.612	123.156.350	123.706.403						
2010	**8**	97.766.067	126.953.618	128.360.442							
2011	**9**	77.063.935	103.317.670								
2012	**10**	58.342.729									

Firstly, we obtain that the total paid claims' amount which is 804.879.963 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 830.778.734,25. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *25.898.771,25*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

Table 18: Comparison between Mack's and Bootstrap's results in Insurance Sector 12

Mack's IBNR	**Mack's S.E.**	**Bootstrap IBNR**	**Bootstrap S.E.**
25.898.771,25	3.845.968,14	26.210.978	3.064.773

As we can observe, there is no great difference between the different results.

Finally, the 99,5% quantile of bootstrap IBNR is *34.536.712*. Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

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Insurance Sector 15

In the Sector 15, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 19: Run-off triangle containing cumulative claims of Sector 15

Sector 15		**Cumulative Losses in Development Year**									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	1.550.898	2.340.028	2.462.480	2.544.035	2.544.035	2.561.074	2.700.543	2.710.823	2.762.671	2.762.671
2004	**2**	476.434	1.373.611	1.418.332	1.575.854	1.575.854	1.680.068	1.680.068	1.681.168	1.681.168	
2005	**3**	1.298.345	4.053.511	4.974.374	5.573.488	5.596.039	5.672.857	5.697.681	5.827.242		
2006	**4**	3.498.351	7.334.842	7.953.964	7.961.579	7.974.558	7.993.501	8.338.243			
2007	**5**	5.897.667	8.925.432	9.304.181	9.479.948	9.502.760	9.550.834				
2008	**6**	8.725.220	13.896.421	14.808.332	14.875.595	15.088.201					
2009	**7**	9.344.299	16.190.973	16.828.140	17.332.895						
2010	**8**	8.914.067	11.823.189	12.394.175							
2011	**9**	3.389.865	6.544.897								
2012	**10**	2.311.723									

Firstly, we obtain that the total paid claims' amount which is 81.832.049 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 89.518.247,17. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *7.686.198,17*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

Table 20: Comparison between Mack's and Bootstrap's results in Insurance Sector 15

Mack's IBNR	**Mack's S.E.**	**Bootstrap IBNR**	**Bootstrap S.E.**
7.686.198,17	2.459.511,47	8.334.492	3.601.807

As we can observe, there is no great difference between the different results.

Finally, the 99,5% quantile of bootstrap IBNR is *20.707.218*. Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

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Insurance Sector 16

In the Sector 16, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 21: Run-off triangle containing cumulative claims of Sector 16

Sector 16		**Cumulative Losses in Development Year**									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	3.023.408	3.620.272	3.722.044	3.767.153	3.861.507	3.848.614	3.829.539	3.835.284	3.841.768	3.844.910
2004	**2**	944.796	2.003.065	2.059.700	2.083.301	2.099.752	2.101.203	2.105.591	2.106.746	2.107.277	
2005	**3**	1.311.722	2.324.364	2.421.485	2.433.414	2.452.634	2.440.307	2.443.954	2.466.199		
2006	**4**	1.552.307	2.549.987	2.770.160	2.802.921	2.842.165	2.820.314	2.905.681			
2007	**5**	1.226.437	2.606.122	2.892.793	2.981.640	2.973.698	3.030.903				
2008	**6**	2.862.701	4.753.996	5.024.253	5.153.529	5.262.995					
2009	**7**	2.071.951	3.790.874	4.103.223	4.002.234						
2010	**8**	2.813.133	4.478.384	4.733.200							
2011	**9**	1.305.468	2.336.876								
2012	**10**	1.269.701									

Firstly, we obtain that the total paid claims' amount which is 31.959.976 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 33.666.424,69. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *1.706.448,69*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

Table 22: Comparison between Mack's and Bootstrap's results in Insurance Sector 16

Mack's IBNR	**Mack's S.E.**	**Bootstrap IBNR**	**Bootstrap S.E.**
1.706.448,69	601.757,24	1.691.676	722.650

As we can observe, there is no great difference between the different results.

Finally, the 99,5% quantile of bootstrap IBNR is *3.782.205*. Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

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Insurance Sector 17

In the Sector 17, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 23: Run-off triangle containing cumulative claims of Sector 17

Sector 17		**Cumulative Losses in Development Year**									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	5.960.020	14.996.990	16.845.733	17.370.865	18.373.995	19.360.911	21.753.184	21.832.807	22.013.607	22.032.644
2004	**2**	7.510.865	20.138.517	22.374.799	22.886.914	23.371.944	23.414.825	23.502.792	25.627.259	25.627.259	
2005	**3**	25.092.681	39.365.744	40.754.107	42.031.046	42.109.323	42.312.364	42.990.043	44.661.727		
2006	**4**	26.950.816	61.434.645	63.756.892	64.120.784	64.821.352	64.893.512	64.974.427			
2007	**5**	38.615.756	63.441.956	68.973.861	69.934.555	70.413.798	70.492.233				
2008	**6**	35.903.255	62.717.551	73.957.755	82.747.067	85.629.135					
2009	**7**	47.734.726	69.382.865	77.136.406	78.037.115						
2010	**8**	18.329.702	41.784.089	45.067.546							
2011	**9**	21.124.374	46.423.109								
2012	**10**	24.241.030									

Firstly, we obtain that the total paid claims' amount which is 507.186.225 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 579.356.209,91. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *72.169.984,91*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

Table 24: Comparison between Mack's and Bootstrap's results in Insurance Sector 17

Mack's IBNR	**Mack's S.E.**	**Bootstrap IBNR**	**Bootstrap S.E.**
72.169.984,91	21.787.373,83	73.863.099	24.153.342

As we can observe, there is no great difference between the different results.

Finally, the 99,5% quantile of bootstrap IBNR is *152.845.442*. Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

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Insurance Sector 18

In the Sector 18, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 25: Run-off triangle containing cumulative claims of Sector 18

Sector 18		**Cumulative Losses in Development Year**									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	11.547.727	13.177.924	13.636.643	13.815.667	13.991.431	14.015.885	14.373.825	17.050.101	17.095.793	17.134.087
2004	**2**	3.584.488	6.484.280	7.434.305	7.557.539	8.307.854	8.964.619	9.085.891	9.129.697	9.151.938	
2005	**3**	4.797.466	6.682.963	7.406.318	7.749.873	7.886.905	7.945.687	7.946.194	7.956.989		
2006	**4**	7.029.237	14.743.102	15.602.901	16.126.638	16.322.641	16.351.642	16.450.259			
2007	**5**	4.757.188	11.733.974	13.166.813	13.771.184	13.936.381	14.359.446				
2008	**6**	9.499.805	36.381.707	39.415.588	44.206.319	44.610.097					
2009	**7**	15.089.428	26.674.425	31.877.893	36.536.440						
2010	**8**	9.094.191	19.888.791	22.821.828							
2011	**9**	8.820.205	14.937.744								
2012	**10**	12.888.602									

Firstly, we obtain that the total paid claims' amount which is 196.847.430 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 246.002.328,25. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *49.154.898,25*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

Table 26: Comparison between Mack's and Bootstrap's results in Insurance Sector 18

Mack's IBNR	**Mack's S.E.**	**Bootstrap IBNR**	**Bootstrap S.E.**
49.154.898,25	19.311.183,23	50.416.751	17.214.913

As we can observe, there is no great difference between the different results.

Finally, the 99,5% quantile of bootstrap IBNR is *101.338.307*. Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

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Insurance Sector 19

In the Sector 19, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 27: Run-off triangle containing cumulative claims of Sector 19

Sector 19		Cumulative Losses in Development Year									
Accident Year		1	2	3	4	5	6	7	8	9	10
2003	1	96.471.534	120.602.746	148.905.834	168.077.362	195.932.051	216.501.875	229.323.438	235.338.012	240.612.734	245.318.101
2004	2	61.070.002	103.082.439	125.521.816	168.618.941	196.483.761	217.846.090	225.437.429	234.036.707	239.101.320	
2005	3	83.015.051	120.008.533	160.006.749	188.110.566	211.976.671	227.263.929	237.443.666	245.761.582		
2006	4	69.164.896	133.545.204	161.628.753	186.997.880	207.567.618	220.676.371	231.765.073			
2007	5	152.073.835	215.970.417	245.477.463	269.979.536	291.560.841	317.575.267				
2008	6	131.475.998	199.923.291	231.232.854	255.117.103	280.080.883					
2009	7	136.727.994	208.041.462	237.712.857	268.567.513						
2010	8	155.165.619	226.508.393	262.881.456							
2011	9	138.745.791	227.714.875								
2012	10	136.793.549									

Firstly, we obtain that the total paid claims' amount which is 2.455.559.619 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 3.291.385.836,28. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *835.826.217,28*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

Table 28: Comparison between Mack's and Bootstrap's results in Insurance Sector 19

Mack's IBNR	Mack's S.E.	Bootstrap IBNR	Bootstrap S.E.
835.826.217,28	80.512.093,37	841.137.288	106.207.521

As we can observe, there is no significant difference between the IBNR but there is a difference between the two standard errors which seems important. Thus, among the two methods we would prefer using Mack's model.

Finally, the 99,5% quantile of bootstrap IBNR is *1.152.658.136*. Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

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Insurance Sector 21

In the Sector 21, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 29: Run-off triangle containing cumulative claims of Sector 21

Sector 21		**Cumulative Losses in Development Year**									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	8.788	15.890	17.960	18.007	46.071	70.688	70.688	70.688	70.688	70.688
2004	**2**	8.594	50.192	50.981	58.451	109.299	117.399	124.800	130.685	130.685	
2005	**3**	36.811	205.277	292.099	309.345	309.345	329.828	332.108	336.903		
2006	**4**	158.854	720.737	819.683	822.897	829.023	829.023	829.023			
2007	**5**	95.887	559.723	622.829	633.095	633.406	637.157				
2008	**6**	409.516	887.318	1.005.237	1.035.869	1.050.185					
2009	**7**	925.583	1.530.329	1.575.436	1.599.321						
2010	**8**	478.475	958.089	1.014.855							
2011	**9**	175.114	563.687								
2012	**10**	211.267									

Firstly, we obtain that the total paid claims' amount which is 6.443.771 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 7.343.504,89. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *899.733,89*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

Table 30: Comparison between Mack's and Bootstrap's results in Insurance Sector 21

Mack's IBNR	**Mack's S.E.**	**Bootstrap IBNR**	**Bootstrap S.E.**
899.733,89	620.692,88	1.089.814	616.944

As we can observe, there is no great difference between the standard errors but between IBNR there is a difference of approximately *200.000* which seems important. Thus, since Mack's model has the higher coefficient of variation, we would prefer using Bootstrap among the two methods.

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Finally, the 99,5% quantile of bootstrap IBNR is *2.822.692*. Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

Insurance Sector 22

In the Sector 22, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 31: Run-off triangle containing cumulative claims of Sector 22

Sector 22		**Cumulative Losses in Development Year**									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	730.785	1.558.899	1.996.583	2.464.448	2.669.530	2.941.073	3.062.880	3.891.837	4.353.940	4.744.848
2004	**2**	674.169	1.593.431	1.901.487	2.639.580	3.371.904	3.836.161	4.048.174	4.326.946	4.484.687	
2005	**3**	982.539	2.258.594	2.580.688	3.060.007	3.440.920	3.725.365	4.418.742	4.579.982		
2006	**4**	987.456	2.293.914	3.017.001	4.221.021	4.984.828	5.330.054	5.625.275			
2007	**5**	1.673.280	4.370.385	5.647.079	7.514.627	8.475.779	9.083.054				
2008	**6**	1.630.263	3.477.703	4.794.190	5.898.446	6.544.227					
2009	**7**	1.737.666	3.897.400	5.215.399	6.443.757						
2010	**8**	1.415.558	2.655.922	2.953.412							
2011	**9**	1.109.206	1.806.491								
2012	**10**	924.804									

Firstly, we obtain that the total paid claims' amount which is 47.190.537 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 73.954.243,09. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *26.763.706,09*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

Table 32: Comparison between Mack's and Bootstrap's results in Insurance Sector 22

Mack's IBNR	Mack's S.E.	Bootstrap IBNR	Bootstrap S.E.
26.763.706,09	5.844.815,58	27.209.802	5.073.650

As we can observe, there is no great difference between the different results.

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Finally, the 99,5% quantile of bootstrap IBNR is *40.444.597*. Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

Insurance Sector 25

In the Sector 25, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 33: Run-off triangle containing cumulative claims of Sector 25

Sector 25		**Cumulative Losses in Development Year**									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	511.876	720.748	736.131	735.971	729.094	721.833	734.633	733.150	733.150	713.384
2004	**2**	460.225	707.120	701.150	702.076	701.254	705.593	720.593	720.603	720.603	
2005	**3**	483.478	793.462	989.183	997.324	1.025.397	1.023.454	1.038.089	1.733.958		
2006	**4**	629.012	2.056.344	2.042.864	2.043.473	2.044.951	2.031.262	2.030.703			
2007	**5**	627.886	864.020	859.827	855.561	845.464	845.953				
2008	**6**	479.438	1.636.530	1.765.316	1.759.039	1.765.000					
2009	**7**	1.263.947	1.733.342	1.989.362	2.030.150						
2010	**8**	554.078	1.136.575	2.048.260							
2011	**9**	1.369.295	1.983.149								
2012	**10**	1.018.231									

Firstly, we obtain that the total paid claims' amount which is 14.889.391 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 19.609.051,97. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *4.719.660,97*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

Table 34: Comparison between Mack's and Bootstrap's results in Insurance Sector 25

Mack's IBNR	**Mack's S.E.**	**Bootstrap IBNR**	**Bootstrap S.E.**
4.719.660,97	3.551.466,89	4.796.852	9.144.872

As we can observe, there is no significant difference between the IBNR but there is a great difference between the two standard errors. Thus, among the

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two methods we would prefer using Mack's model. Still, Mack's coefficient of variation remains pretty high which lead us to feel insecurity about the predictions.

Finally, the 99,5% quantile of bootstrap IBNR is *21.586.024*. Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

Insurance Sector 26

In the Sector 26, we are about to work similarly as before. The observations of the cumulative claims in this sector are presented in the table that follows:

Table 35: Run-off triangle containing cumulative claims of Sector 26

Sector 26		**Cumulative Losses in Development Year**									
Accident Year		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
2003	**1**	723.785	822.895	962.176	990.789	1.046.186	1.149.665	1.160.935	1.164.891	1.169.014	1.171.104
2004	**2**	756.989	934.884	998.402	1.074.270	1.088.743	1.116.443	1.123.105	1.139.913	1.136.163	
2005	**3**	933.025	1.041.609	1.144.050	1.186.844	1.220.350	1.237.402	1.262.030	1.275.552		
2006	**4**	992.796	1.213.623	1.275.337	1.320.039	1.332.749	1.365.542	1.382.742			
2007	**5**	1.075.479	1.175.540	1.256.805	1.285.797	1.328.271	1.358.694				
2008	**6**	1.129.739	1.256.370	1.344.656	1.410.494	1.448.229					
2009	**7**	1.428.736	1.562.249	1.708.488	1.767.565						
2010	**8**	2.133.739	2.331.660	2.467.922							
2011	**9**	3.310.101	3.483.014								
2012	**10**	2.348.224									

Firstly, we obtain that the total paid claims' amount which is 17.839.209 and represents the sum of values in the diagonal of the above table. Using the Chain Ladder method in the above data, we calculate the final estimated losses to 20.061.977,68. Thus, the total reserves' amount that the insurance firm should hold to meet its customers obligations is equal to *2.222.768,68*.

Then, we are about to Mack's model and Bootstrap. The estimated IBNR and their standard errors are displayed in the following table:

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Table 36: Comparison between Mack's and Bootstrap's results in Insurance Sector 26

Mack's IBNR	Mack's S.E.	Bootstrap IBNR	Bootstrap S.E.
2.222.768,68	353.649,83	2.283.719	400.462

As we can observe, there is no great difference between the different results.

Finally, the 99,5% quantile of bootstrap IBNR is *3.407.509.* Thus, this is the necessary amount that an insurance firm must hold in order to be safe in the 99,5% of the cases.

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5. Conclusions

The purpose of the above applications was to calculate and properly estimate the necessary reserves that the company would need to hold, according to Solvency II framework.

In order to do so, we explored various methodologies like traditional Chain Ladder, Mack's model and Bootstrap process. Then, we compared the results of the above models to find out the differences.

As it turned out, the differences between the methods were negligible in most cases. We examined eleven different real data sets and, on less than the half, there was a bias for a specific method. On these few cases, sometimes the results were in favor of the Mack's model and the others in favor of Bootstrap. Traditional chain ladder gives the same results as Mack's model.

To conclude, although we successfully applied some of the methods in our data, we still cannot officially draw any conclusions about which estimation method is the catholic best. Certainly, the traditional Chain Ladder is the most popular for its convenience and it is widely preferred by many.

Nevertheless, a safe approach would be to firstly examine the data in detail and, then, attempt to find the model that suits best to the given data instead of using the same method all the time. This approach should be used to both deterministic and stochastic reserving models.

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